

# Sistemi Concorrenti e di Rete LS

Il Facoltà di Ingegneria - Cesena

a.a 2008/2009

**[module 2.4]**

**VISUAL FORMALISMS FOR  
CONCURRENT SYSTEMS**

# VISUAL FORMALISMS

- Formalisms for rigorously describing models of concurrent systems by means of some kind of visual diagrams
  - structural and behavioural aspects
- Useful for both requirement specification / analysis and design
  - formal analysis when formally specified
- Formalisms considered in this module
  - Petri Nets
  - Statecharts
  - Activity Diagrams

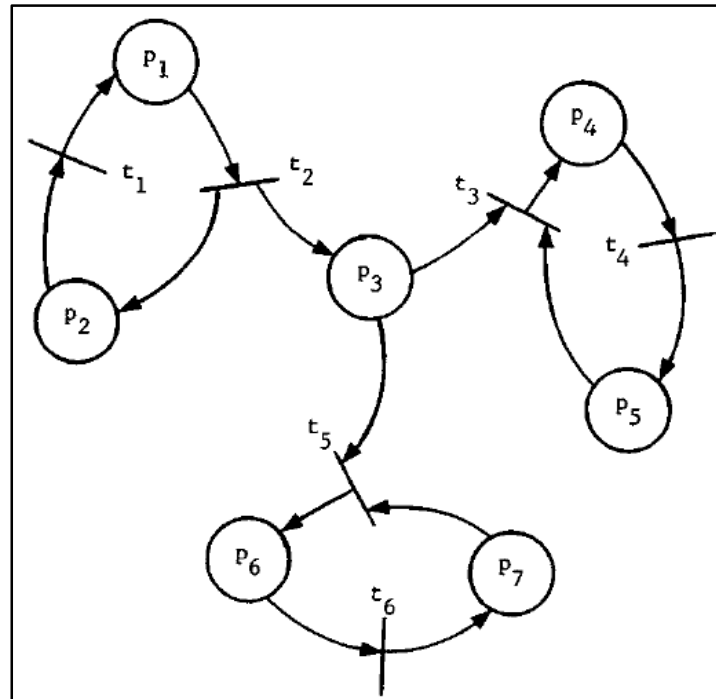
# PETRI NETS

# PETRI NETS

- Abstract, formal model of information flow
  - describing and analyzing the *flow of information* and *control* in systems
  - particularly systems that may exhibit asynchronous and concurrent activities
- Introduced by Carl Adam Petri ~ 1965
  - further developed, extended and adopted in many computer science contexts
- Major use
  - modelling of systems of events in which it is possible for some events to occur concurrently but there are constraints on the concurrence, precedence, or frequency of these occurrences

# PETRI NET GRAPH

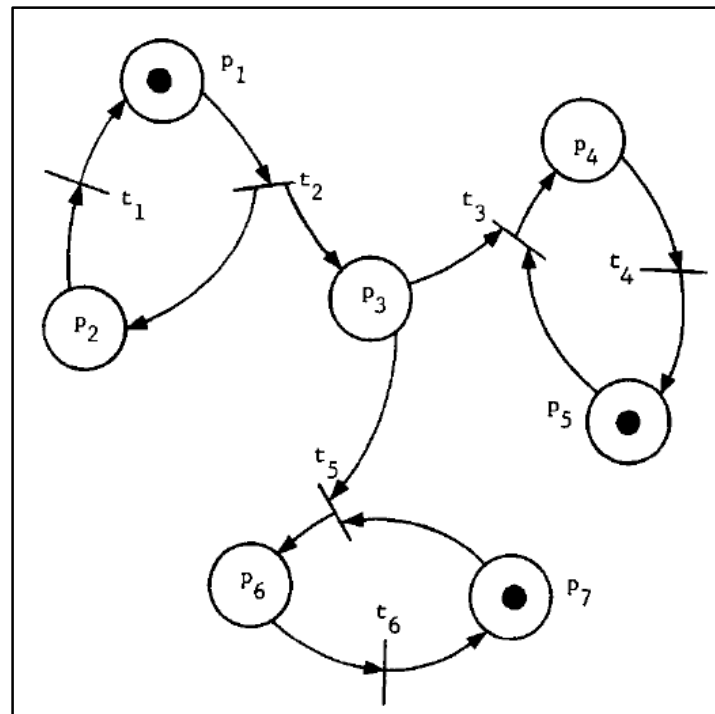
- Bi-partite graphs representing a Petri Net
  - two types of *nodes*
    - **places** (the circles) and **transitions** (the bars)
  - connected by directed arcs
    - from node  $i$  to node  $j$ :  $i$  is an input to  $j$  and  $i$  is an output of  $j$



- Models the static properties of the system

# TOKENS

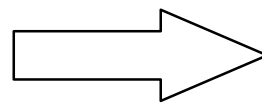
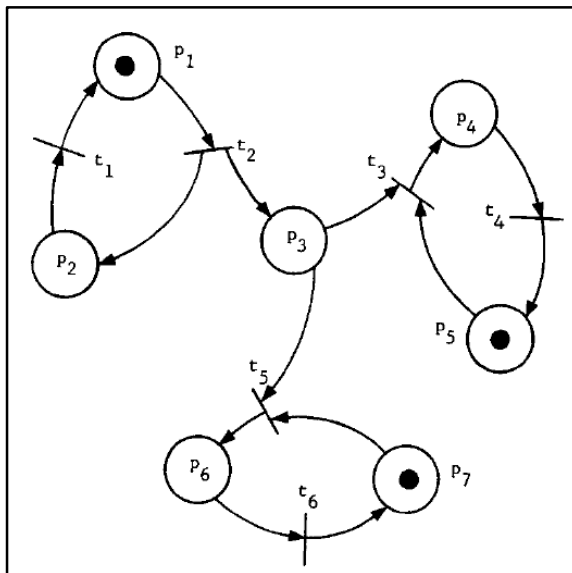
- In addition to the static properties represented by the graph, a Petri Net has *dynamic properties* that result from its *execution*
  - the execution of a Petri net is controlled by the position and movement of markers called **tokens** in the net.



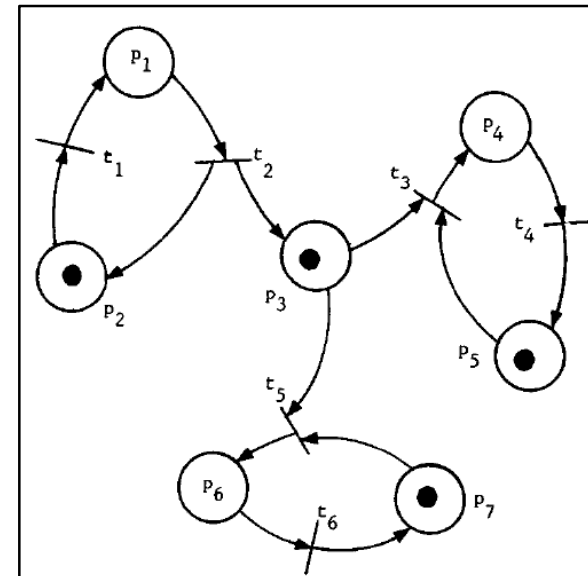
- tokens are indicated by black dots, residing in places
- A Petri Net with tokens is called **marked Petri Net**

# EXECUTION RULES

- Tokens are moved by the *firing* of transitions of the net
  - a transition must be *enabled* in order to fire
    - a transition is enabled when all of its input places have a token in them
  - the transition fires by removing the enabling tokens from their input places and generating new tokens which are deposited in the output place of the transition

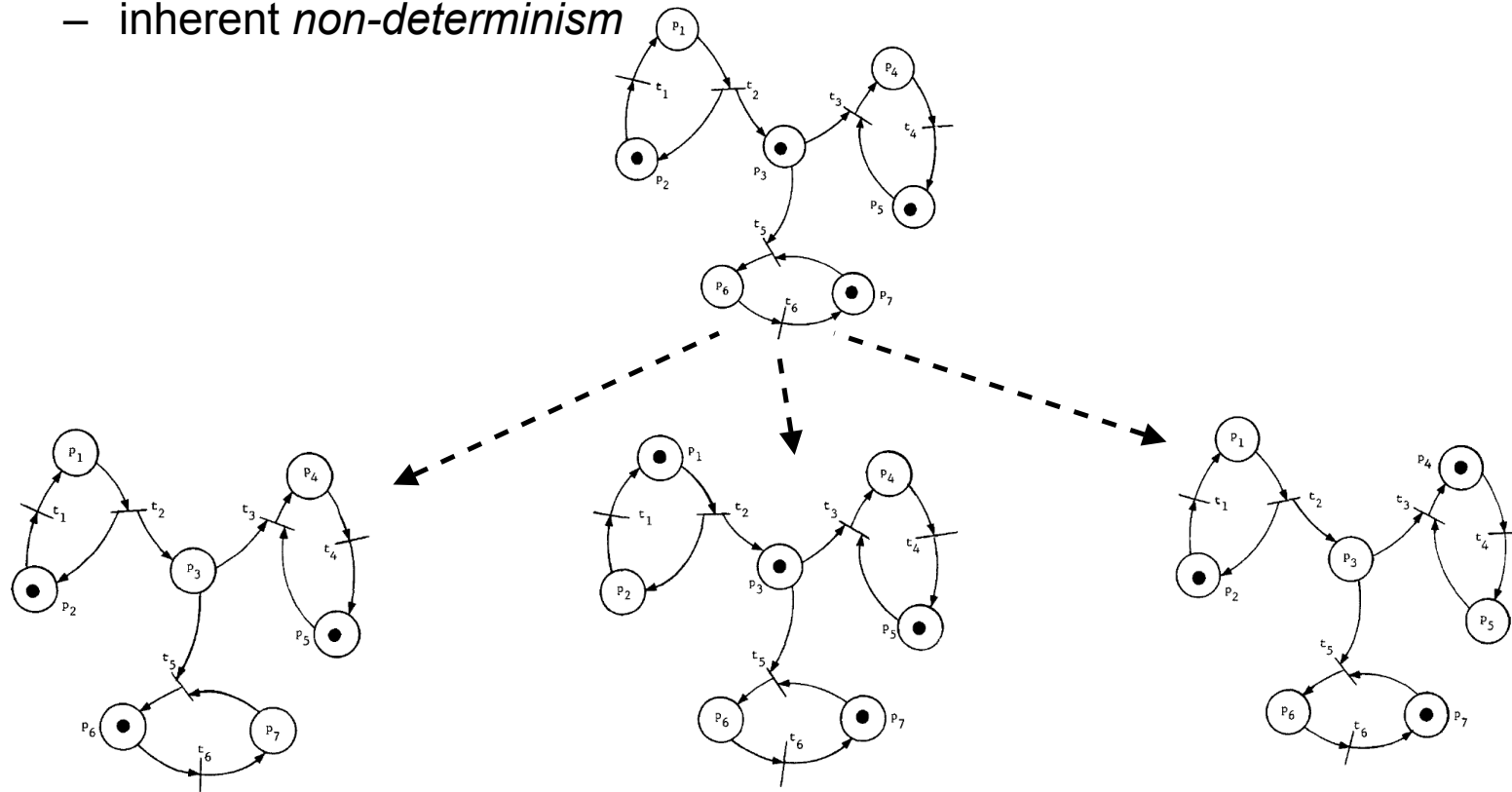


firing  
t2



# MARKINGS

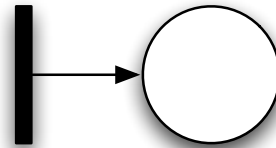
- The distribution of tokens in a marked Petri Net defines the state of the net and is called **marking**
  - the marking may change as a result of firing transitions
- Different transitions may fire, with different result markings
  - inherent *non-determinism*



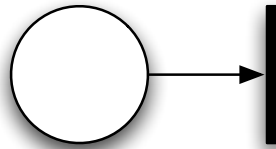


# TRANSITIONS ON THE BOUNDARY

- *source* transition
  - without any input place
  - just produce tokens

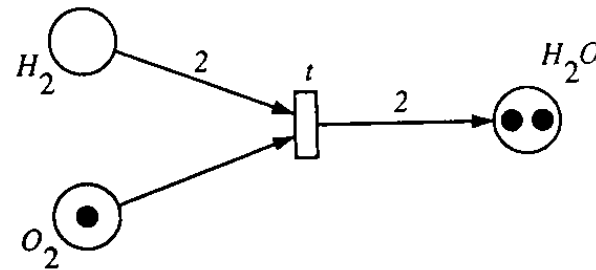
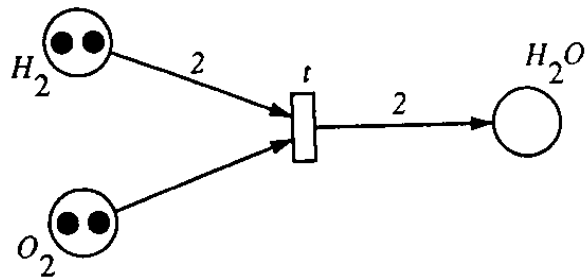


- *sink* transition
  - without any output place
  - just consume tokens



# WEIGHTED ARCS

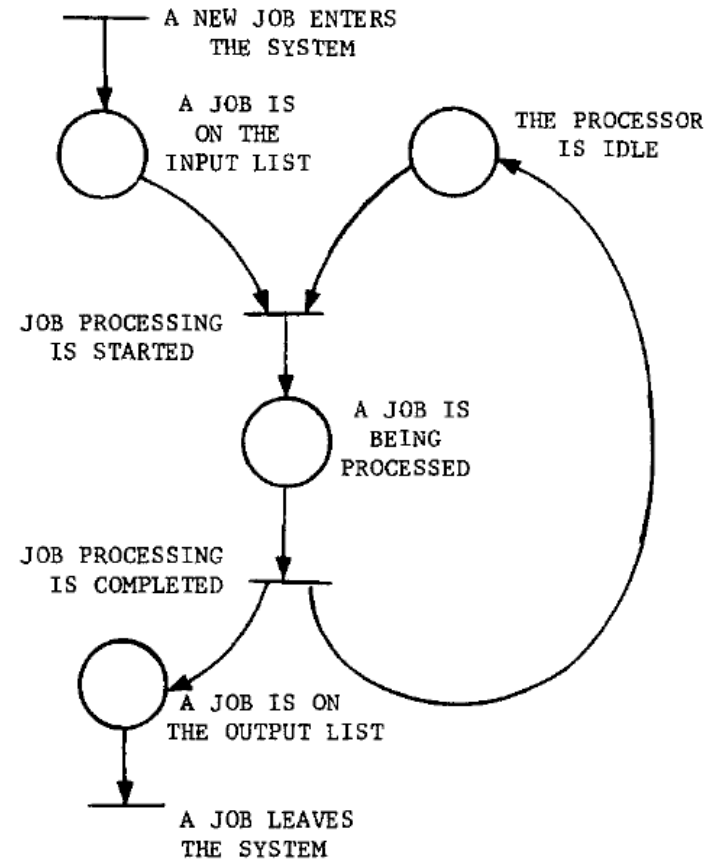
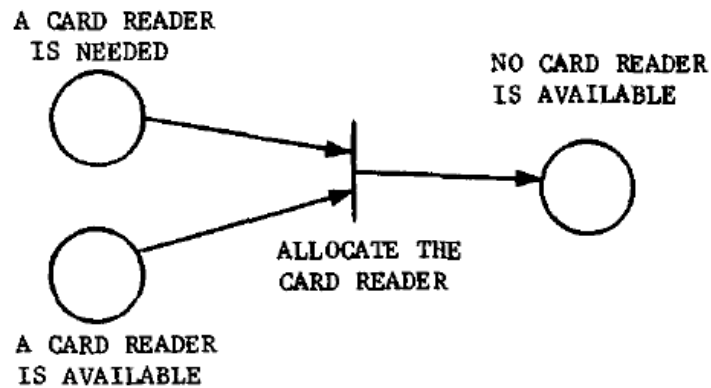
- A variant consider also weights for arcs:
  - a transition is enabled if each input place  $p$  of  $t$  is marked with at least  $w(p,t)$  tokens, where  $w(p,t)$  is the weight of the arc from  $p$  to  $t$
  - a firing of an enabled transition  $t$  removes  $w(p,t)$  tokens from each input place  $p$  of  $t$ , and adds  $w(t,p)$  tokens to each output place  $p$  of  $t$ , where  $w(t,p)$  is the weight of the arc from  $t$  to  $p$
- Reaction example
  - $2H_2 + O_2 \rightarrow 2H_2O$



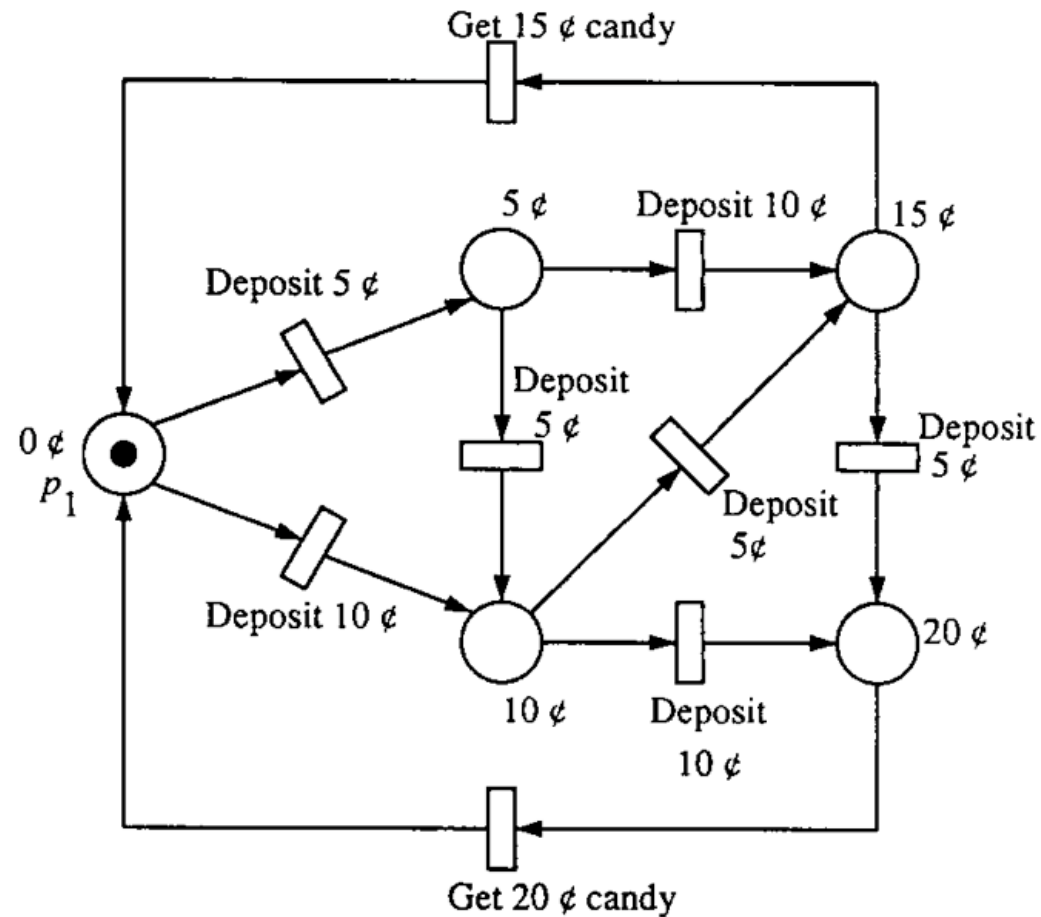
# MODELING WITH PETRI NETS

- Petri nets can be used to model quite naturally concurrent systems in terms of:
  - **events** and **conditions**
  - the relationships among them
- Interpretation
  - in a system at any given time certain conditions will hold
  - the fact that these conditions hold may cause the occurrence of certain events
  - the occurrence of events may change the state of the system
    - causing some of the previous conditions to cease holding and causing other conditions to begin to hold
  - firing of a transition = occurrence of an event
    - considered instantaneous or better: atomic change of the system

# EXAMPLE: RESOURCE ALLOCATION



# EXAMPLE: A VENDING MACHINE



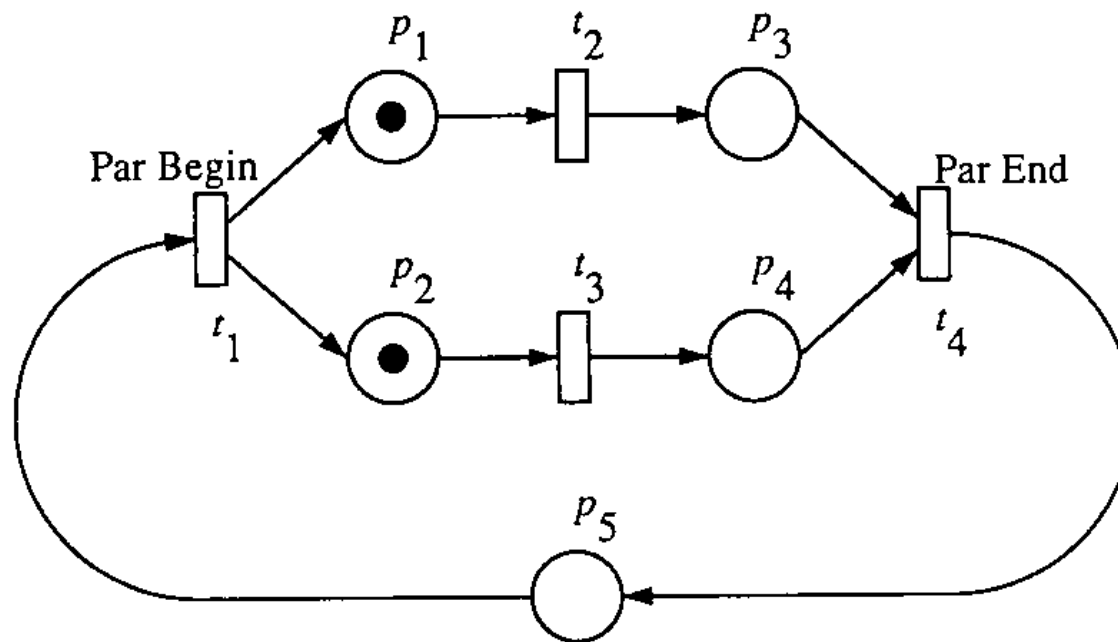
# INTERPRETATIONS

- Typical interpretations of places and transitions

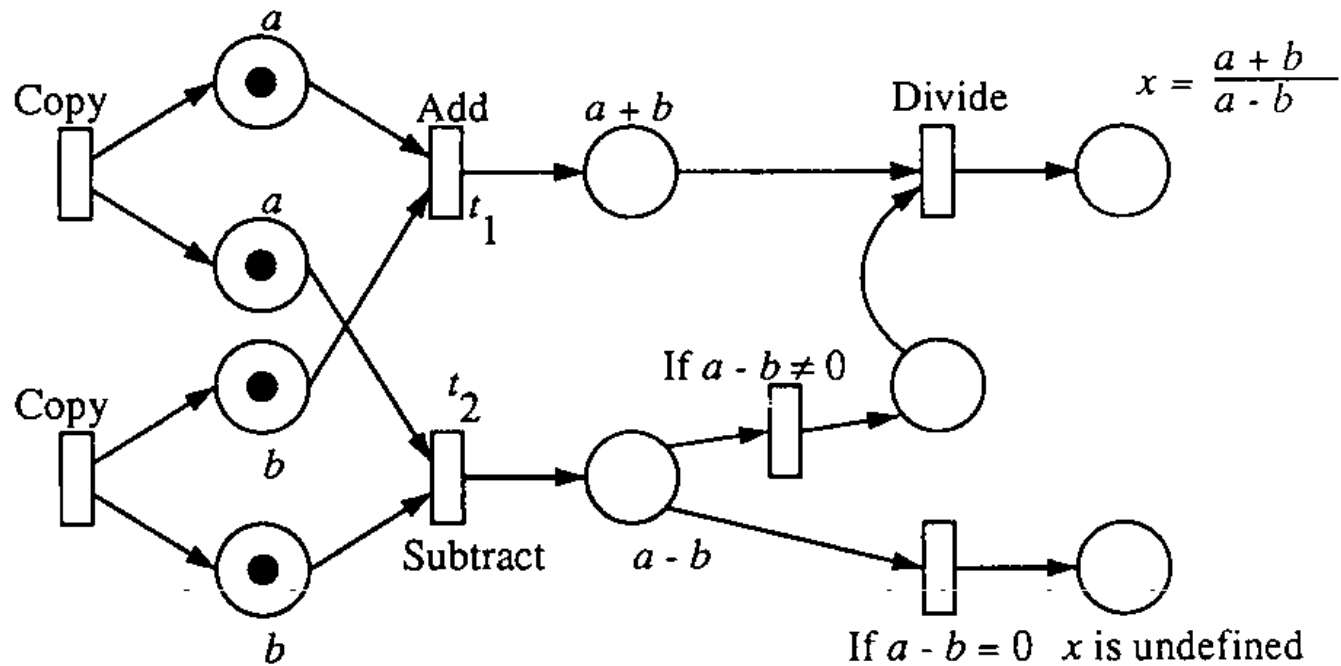
<b>Input places</b>	<b>Transition</b>	<b>Output places</b>
Pre-conditions	Event	Post-conditions
Input data	Computational step	output data
Input signals	Signal processor	Output signals
Resource needed	Task or Job	Resource Released
Conditions	Clause in Logic	Conclusion(s)
Buffers	Processor	Buffers

# MODELLING CONCURRENCY AND PARALLELISM

- PN are ideal for modelling systems of distributed control with multiple processes occurring concurrently



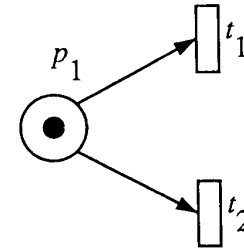
# DATA-FLOW COMPUTATION



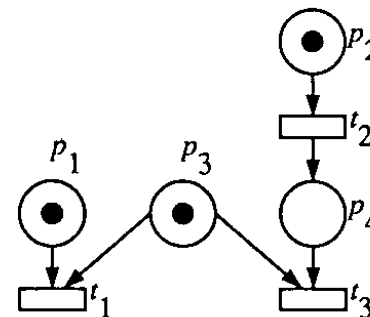
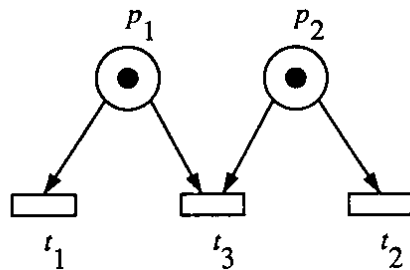


# MODELLING CONFLICT AND CONCURRENT EVENTS

- Representing conflicting or choice events:



- *Conflict vs. concurrent events*
  - two events  $e_1$  and  $e_2$  are *in conflict* if either  $e_1$  or  $e_2$  can occur but not both
  - two events  $e_1$  and  $e_2$  are *concurrent* if both events can occur in any order without conflicts
- A situation where conflict and concurrency are mixed is called a *confusion*



# ASYNCHRONY AND LOCALITY

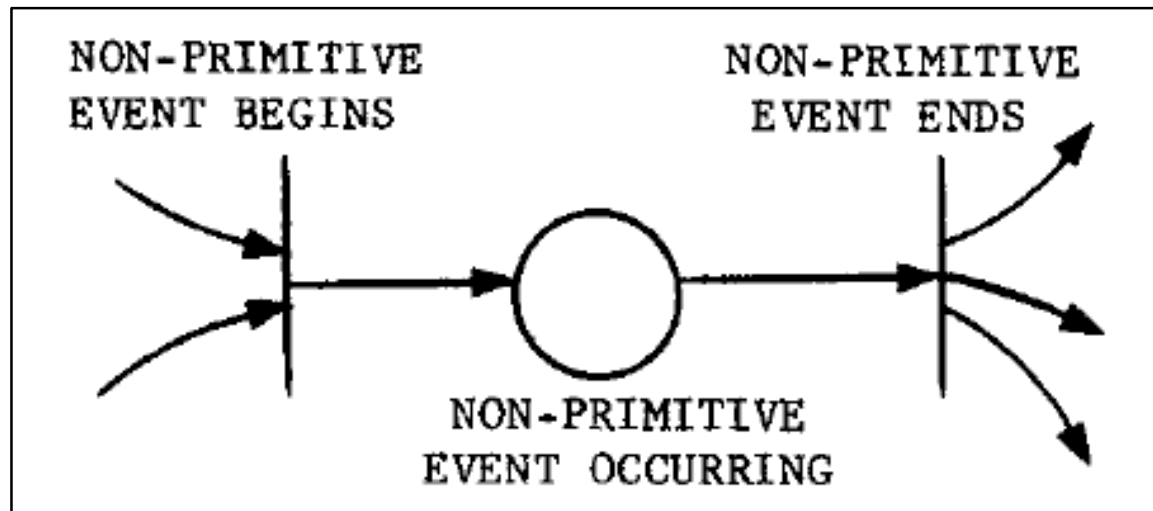
- In a PN there is no inherent measure of time or the flow of time
  - the only important property of time, from a logical point of view, is in defining a **partial ordering** of the occurrence of events
  - events which need not be constrained in terms of their relative order of occurrence are not constrained
- Locality
  - in a complex systems composed by independent asynchronously operating subparts each part can be modelled by a Petri Net
  - the enabling and firing of transitions are then affected by, and in turn affect only, *local changes* in the marking of the Petri Net

# NON-DETERMINISM

- Naturally modelling non deterministic behaviours
  - a PN is viewed as a sequence of discrete events whose order of occurrence is one of the possibly many allowed by the basic structure
  - if at any time more than one transition is enabled, then any of the several enabled transitions may fire
  - the choice as to which transition fires is made in a nondeterministic manner
    - randomly or by forces that are not modelled

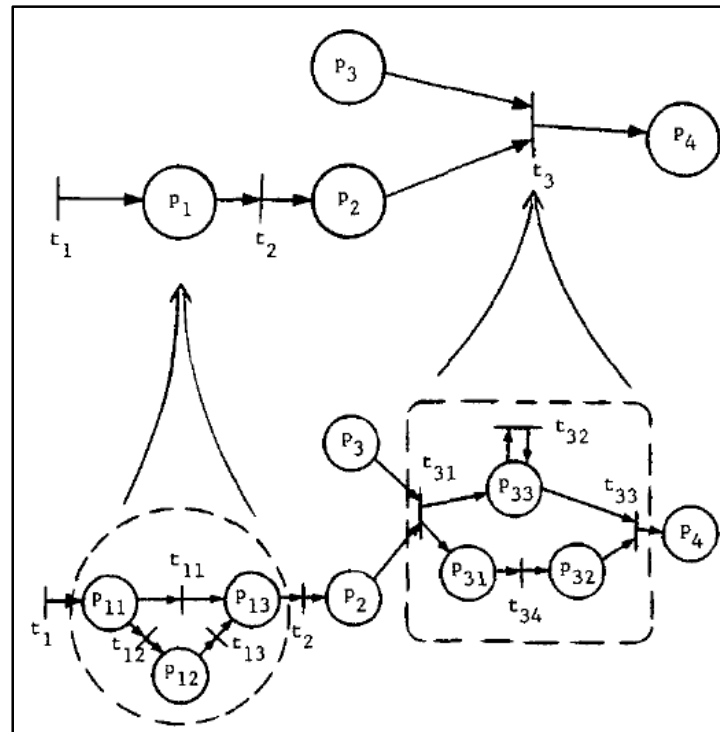
# ATOMIC VS. NON-ATOMIC EVENTS

- The occurrence of (primitive) events is instantaneous
  - non primitive events (with a duration) must be modelled by multiple events
  - e.g.: activity with a beginning and an ending event



# HIERARCHIES

- Natural support to model hierarchies
  - an entire net may be replaced by a single place or transition for modelling at a more abstract level (**abstraction**)
  - places and transitions may be replaced by subnets to provide more detailed modeling (**refinement**)

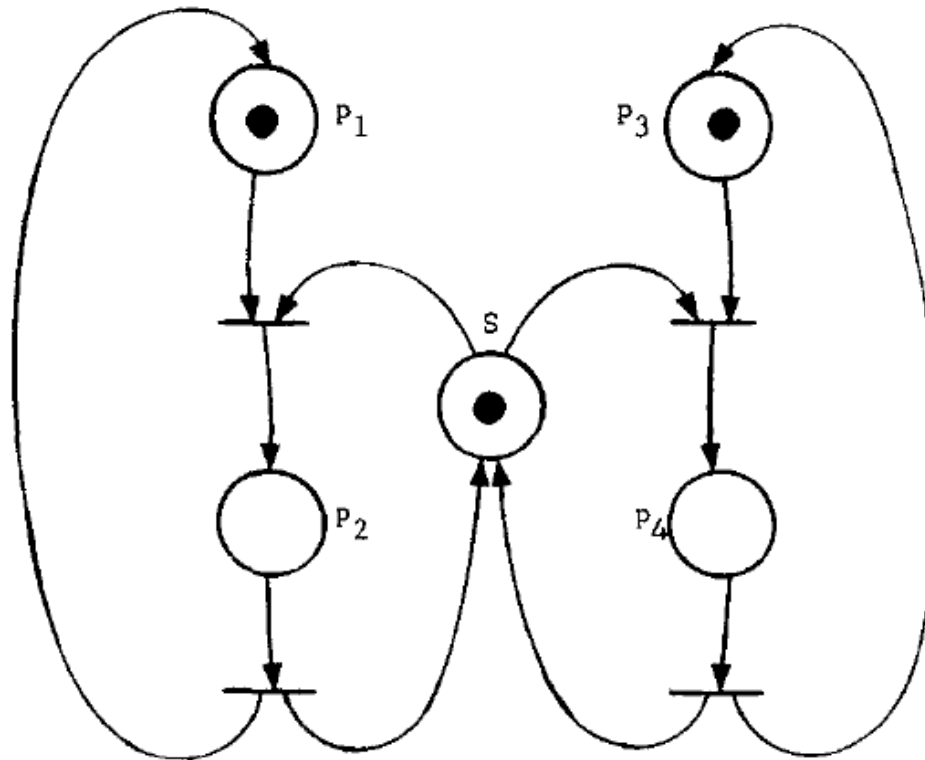


# APPLICATION TO CONCURRENT DESIGN AND PROGRAMMING

- PN can be naturally used to model software systems, concurrent software systems in particular
- Representing problems
  - critical sections and mutual exclusion problems
  - synchronization
- Representing mechanisms behaviour
  - semaphores
  - synchronizers
- Representing entire problems
  - Producers / Consumers
  - Readers-Writers
  - Dining Philosophers

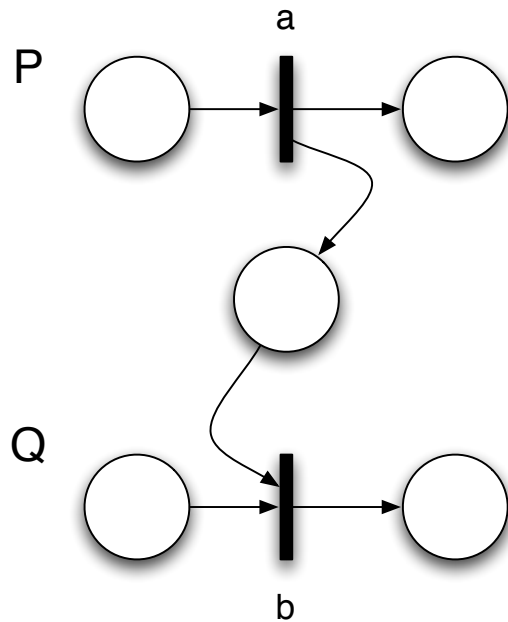
# CRITICAL SECTION AND MUTUAL EXCLUSION PROBLEMS

- p2 and p4 represent critical sections
  - s is the token needed for entering in CS

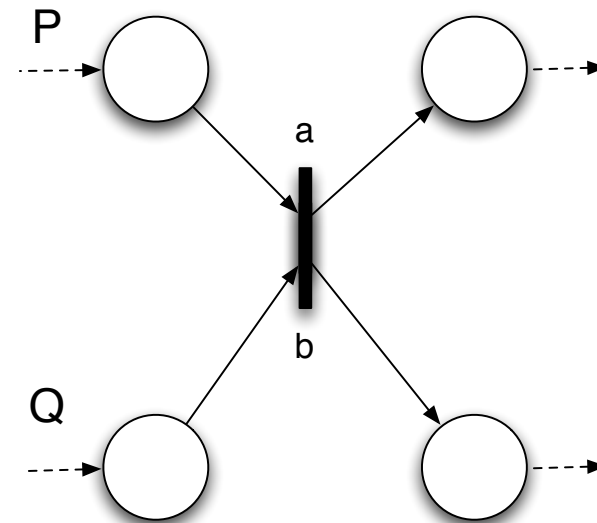


# SYNCHRONIZATION

- Imposing an order between process actions
  - process actions represented by transitions
  - relating actions (transitions) through conditions (places)



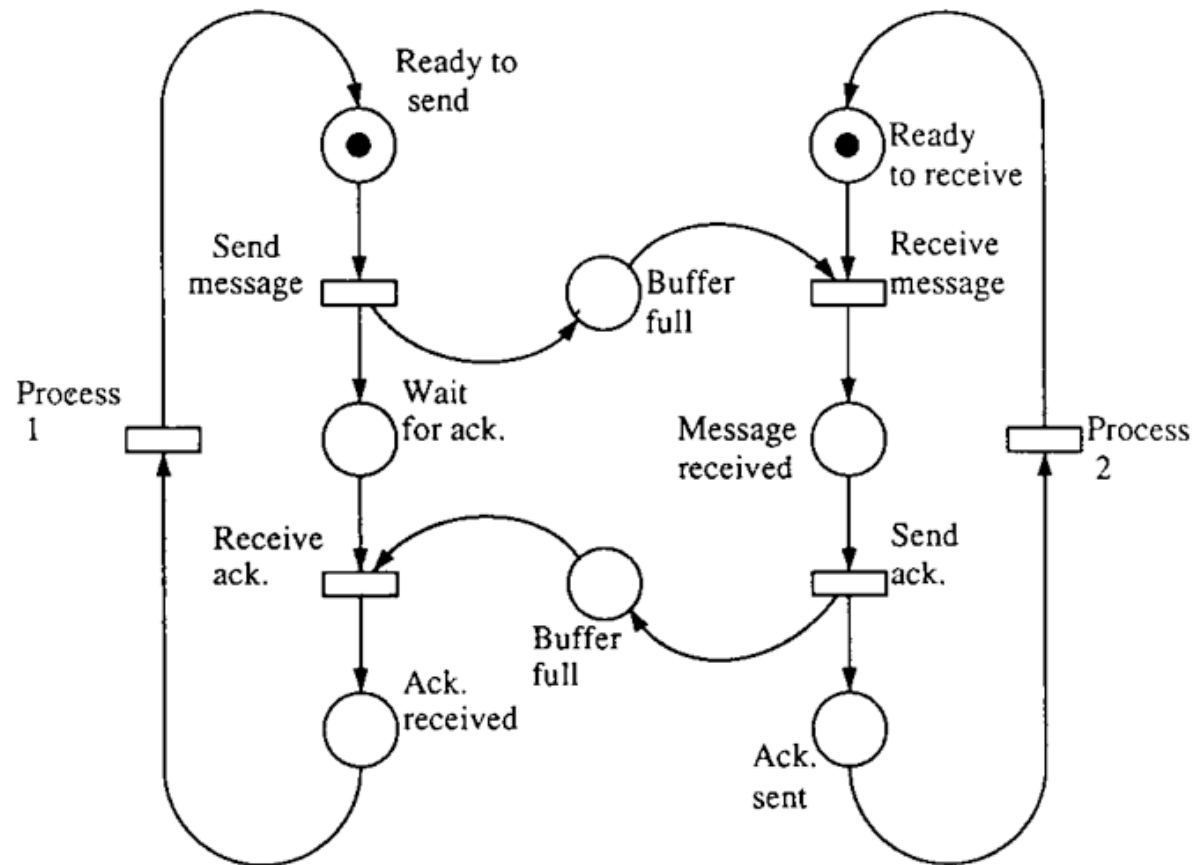
- b action of process Q can be executed after the execution of action a of process P



- b action of process Q and a action of process P must be executed synchronously

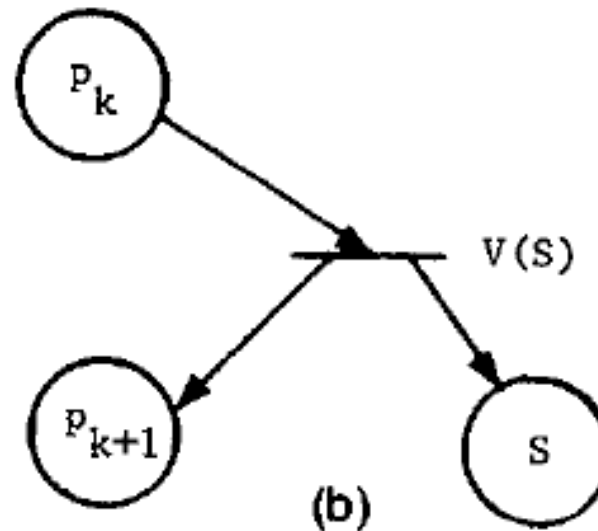
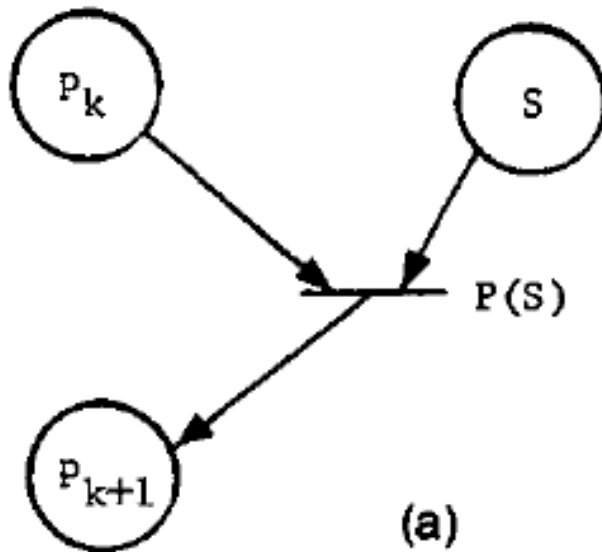


# COMMUNICATION

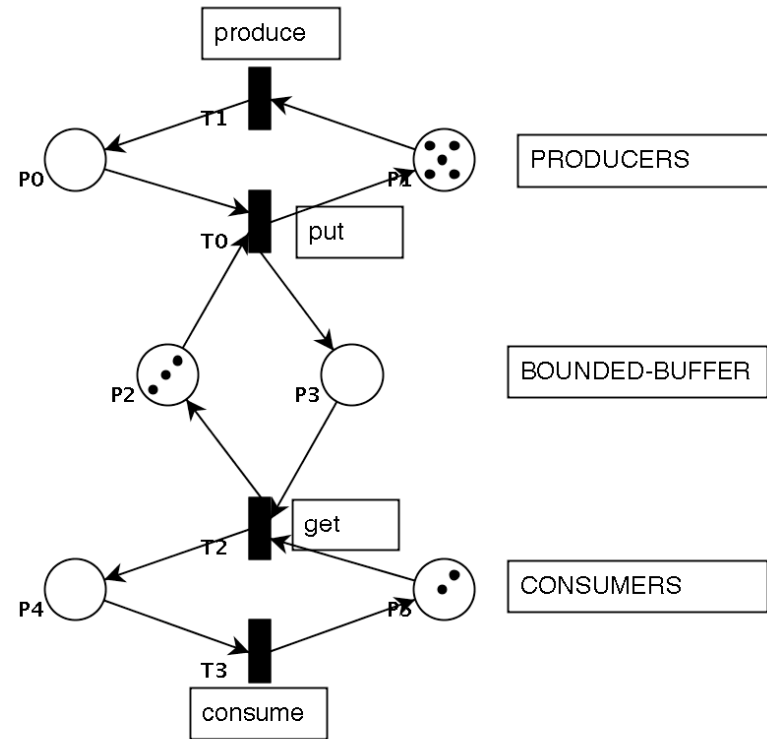


# SEMAPHORES

- Semaphore modeled as a shared resource (place)
  - modelling wait (P) as a transition with the semaphore res. as input place
  - modelling signal (Q) as a transition with the semaphore res. as output place

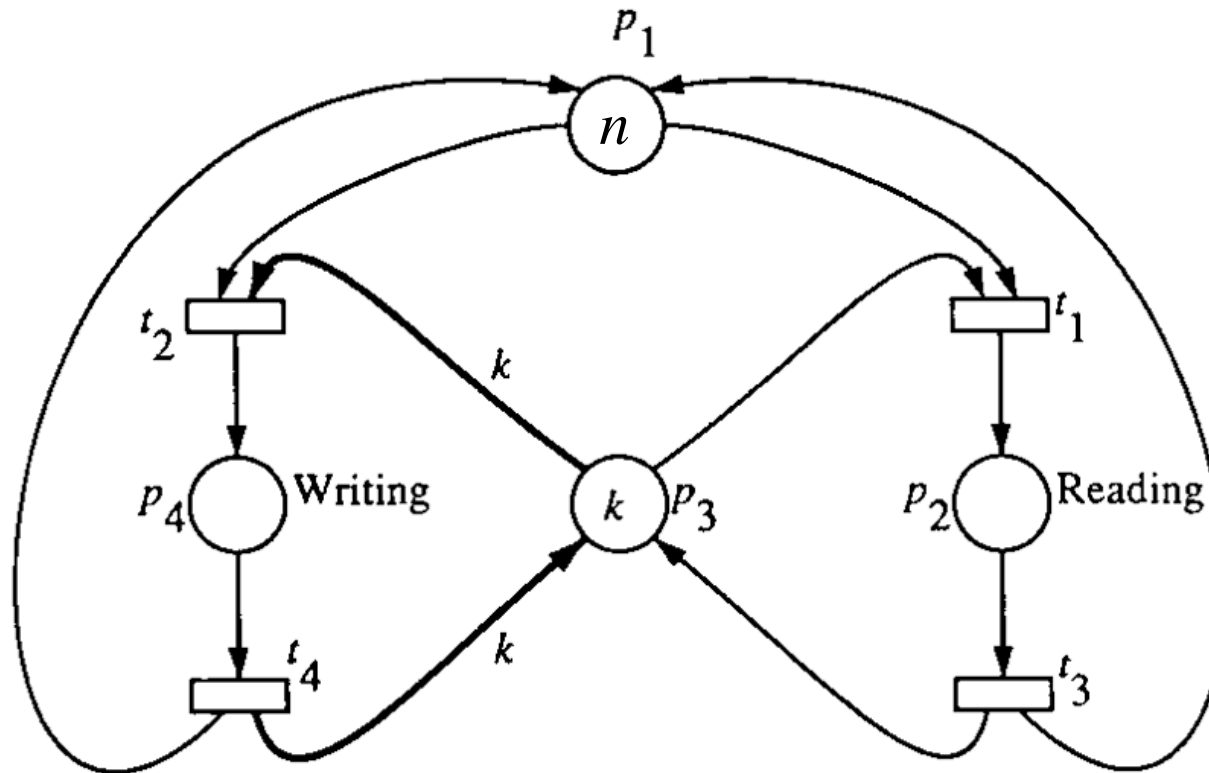


# PRODUCERS AND CONSUMERS



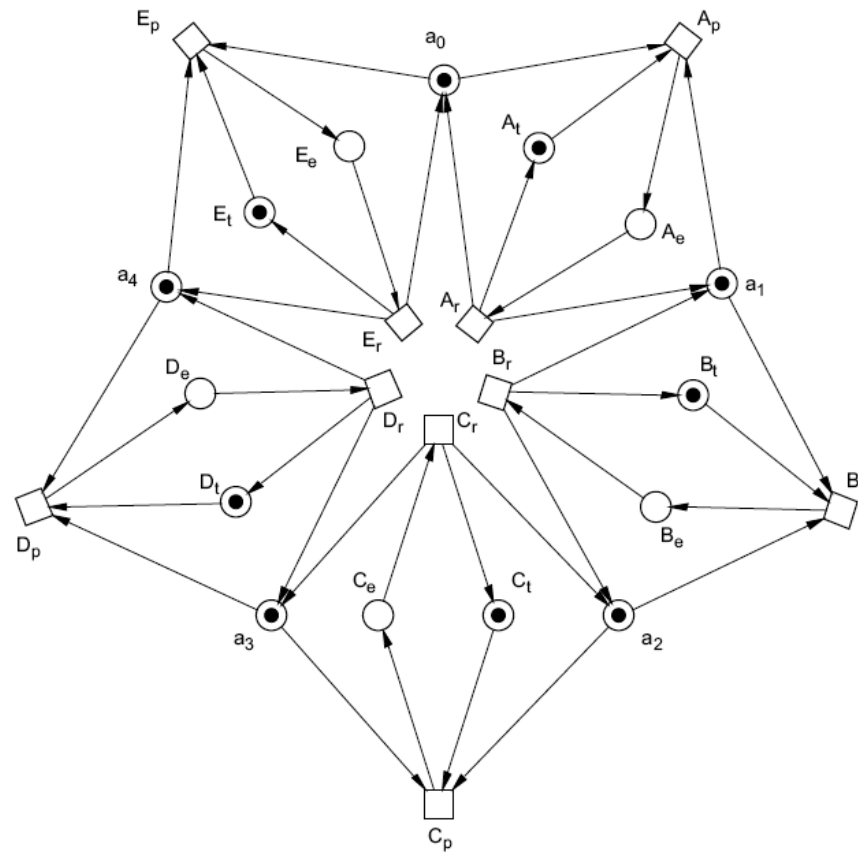
# READERS-WRITERS

- The  $n$  tokens in  $p_1$  represent  $n$  processes that may want to read or write a shared memory represented by  $p_3$



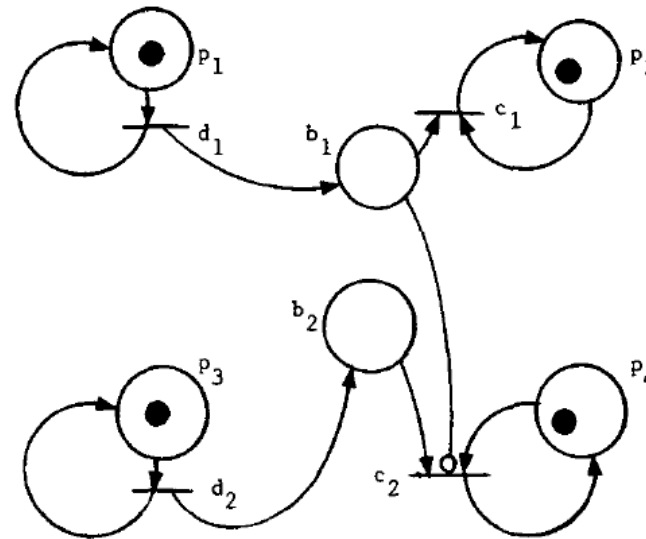
# DINING PHILOSOPHERS

- Forks are represented by places  $a_i$
- Philosophers thinking by  $A_t, B_t, C_t, D_t, E_t$  places and eating by  $A_e, B_e, C_e, D_e, E_e$  places



# EXTENDED PETRI NETS WITH INHIBITOR ARCS

- Zero-testing extension
  - extending the basic PN with the possibility of firing a transition only if a certain place has zero tokens
    - *inhibitor arc* represented by an arc with a small circle at the end



- PN+inhibitor arc = Turing-equivalent
  - expressiveness and undecidability problems

# FORMAL DESCRIPTION OF PETRI NETS

- PN can be formally described so as to enable a rigorous analysis of properties and problems of the system modeled
  - **structural** properties
    - independent from the initial marking
  - **behavioural** properties
    - dependent on the marking
- Mapping correctness of the systems on to structural / behavioural properties of the nets
  - safety and liveness properties

# PETRI NET STRUCTURE

- The structure of a Petri Net can be formally described as a tuple

$$C = (P, T, I, O)$$

- P is a set of places
- T is a set of transitions
- input function I defines the set of *input* places for each transition  $t_j$
- output function O defines the set of *output* places for each transition  $t_j$



# EXAMPLE

$P = \{ p_1, p_2, p_3, p_4, p_5 \}$

$T = \{ t_1, t_2, t_3, t_4 \}$

$I(t_1) = \{p_1\}$

$I(t_2) = \{p_2, p_3, p_5\}$

$I(t_3) = \{p_3\}$

$I(t_4) = \{p_4\}$

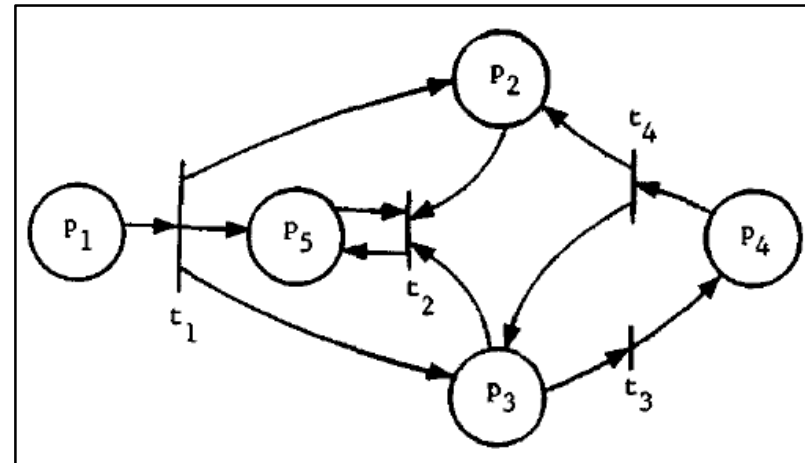
$O(t_1) = \{p_2, p_3, p_5\}$

$O(t_2) = \{p_5\}$

$O(t_3) = \{p_4\}$

$O(t_4) = \{p_2, p_3\}$

Corresponding  
Petri-Net graph:



# MARKING

- A **marking** is an assignement of tokens to the places of the net
- Can be formally represented either as
  - a vector of  $N$  elements, one for each place, representing the number of tokens for each place
  - a function  $\mu : P \rightarrow N$ 
    - $\mu(p_i)$  is the number of tokens in the place  $p_i$
- A marked Petri Net is represented by 5-tuple:  $M = (P, T, I, O, \mu)$

# EXECUTION RULE SEMANTICS

- The *state* of a Petri Net is defined by its marking
  - firing of a transition represents a change in the state of the net.
- Next-State partial function  $\delta(\mu, t_j)$ 
  - the function is undefined if the transition is not enabled in the marking
  - if  $t_j$  is enabled  $\mu' = \delta(\mu, t_j)$  is the marking that results from removing tokens from the input of  $t_j$  and adding tokens to the output of  $t_j$ .
- Given a PN and an initial marking, we can execute the PN by successive transition firings
  - firing a transition  $t_j$  in the initial marking produces a new marking  $\mu^1 = \delta(\mu^0, t_j)$
  - in this new marking we can fire any new enabled transition, say  $t_k$ , resulting in a new marking  $\mu^2 = \delta(\mu^1, t_k)$
  - this can continue as long as there is at least one enabled transition in each marking
  - if we reach a marking in which no transition is enabled, then no transition can fire and the PN must stop
- Non determinism
  - multiple sequence of markings  $(\mu^0, \mu^1, \mu^2, \dots)$  and related transitions  $(t_j^0, t_j^1, t_j^2, \dots)$  can result by executing a PN

# REACHABILITY SET

- Immediately reachable marking
  - a marking  $\mu'$  is *immediately reachable* from  $\mu$  if we can fire some enabled transition in  $\mu$  resulting in  $\mu'$
- *Reachable marking*
  - a marking  $\mu'$  is *reachable* from  $\mu$  if it is immediately reachable from  $\mu$  or is reachable from any marking  $\mu''$  which is immediately reachable from  $\mu$
- *Reachability set* of a PN
  - set of all states into which the PN can enter by any possible execution

# ANALYSIS OF PN MODELS

- *Safe nets*
  - Petri nets in which no more than one token can ever be in any place of the net at the same time.
  - Justification based on the original definition of events and conditions
    - a condition is represented by a place.
    - the fact that the condition holds is indicated by a token in the place
    - so a token should be either present or not: more than 2 token is pointless
- *Bounded net or k-bounded net (boundness)*
  - Nets in which the number of tokens in any place is bounded by  $k$
  - safe nets are 1-bounded net
  - Boundedness is a very important practical property:
- *Conservative net*
  - PN is conservative if the number of tokens in the net is conserved.
  - think for instance of using tokens to represent resources..

# LIVENESS

- Based on transition analysis
  - dead transition in a marking
    - if there is no sequence of transition firings that can enable it
    - related to deadlock situations
  - potentially firable
    - if there exists some sequence that enables it
    - related to starvation situation
  - live transition
    - if it is potentially firable in all reachable markings
- For liveness, it is important not only that a transition be firable in a given marking, but staying potentially firable in all markings reachable from that marking
  - if it is not true, then it is possible to reach a state in which the transition is dead
    - deadlocks

# PETRI NET EXTENSIONS

- *Timed Nets*
  - introducing time delays associated with transitions and/or places
    - useful for performance evaluation and scheduling problems
  - deterministic timed nets
    - delay are deterministically given
  - stochastic nets
    - delays are probabilistically specified
- *High-level Nets*
  - associate some kind of symbolic / numerical information to tokens and some computational rules to transition consuming and producing tokens
    - Coloured Petri Nets, Predicate Transitions Nets
  - **Coloured Petri Nets**
    - assigning typed values (“a color”) to each token

# PETRI NET TOOLS

- Many tools available for creating and analysing Petri Nets
  - check <http://www.informatik.uni-hamburg.de/TGI/PetriNets/>
- An example: PIPE 2
  - Platform Independent Petri net Editor 2
  - Java-based, open-source: <http://pipe2.sourceforge.net/>



# STATECHARTS

# STATECHARTS

- Introduced by David Harel in 1987 for modelling *complex reactive systems*
  - now part of UML with the name of state diagrams
- Reactive systems
  - systems being, to a large extent, *event-driven*, continuously having to react to external and internal stimuli
    - examples include automobiles, communication networks, operating systems, avionic systems, man-machine interface of many ordinary software
  - contrast to *transformational* systems
    - input / output, data-processing systems
- Main objective
  - introducing a way of describing reactive behaviour that is clear and realistic, and at the same time formal and rigorous
    - to be simulated and analyzed

# BEYOND BASIC STATE DIAGRAMS

- General agreement that **states** and **events** are a rather natural medium for describing the dynamic behaviour of a complex systems
  - state transition: "when event alfa occurs in state A, if condition C is true at that time, the system tranfers to state B"
- But finite state machine and state transition diagrams don't scale with complexity
  - unmanageable, exponentially growing moltitude of states, all of which have to be arranged in a "flat" unstratified manner
  - lead to unstructured, unrealistic and chaotic state diagram
- To be useful a state/event approach must be *modular*, *hierarchical* and *well-structured*
  - it must solve the exponential blow-up problem, by somehow relaxing the requirement that all cobinations of states have to be represented explicitly
- Statecharts proposal
  - extension of conventional state diagrams by mechanisms to enhance the descriptive power

# STATECHARTS FORMALISM

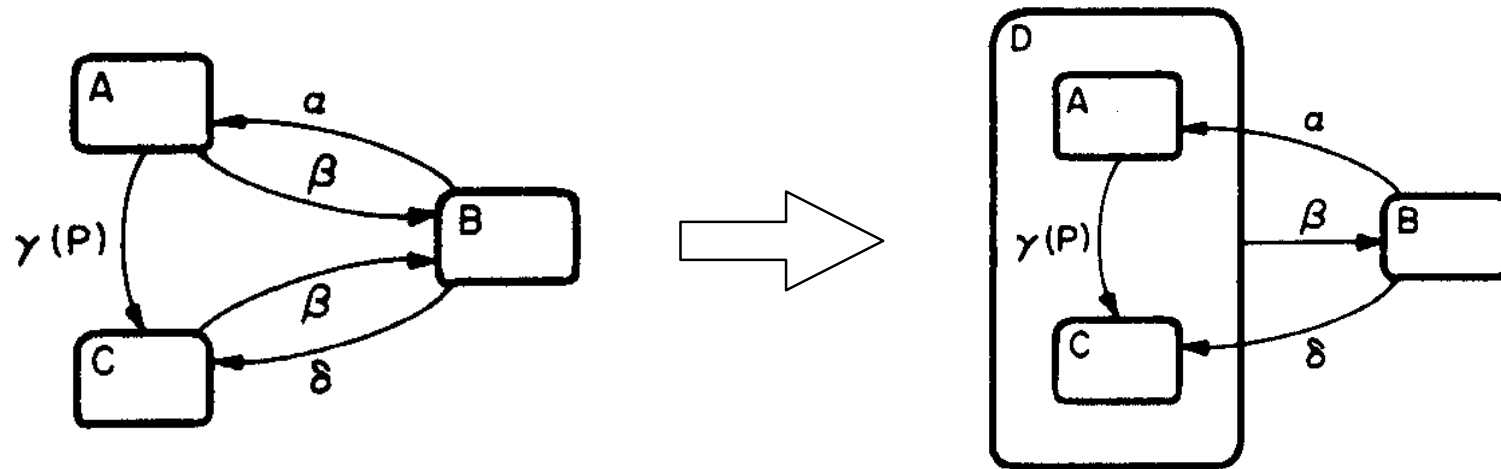
- Visual formalism to describe states and transitions in a modular fashion
  - **hierarchy**
    - clustering
    - refinement
    - promoting 'zoom' capabilities for moving easily back and forth between levels of abstractions
  - **orthogonality**
    - independence / concurrency of substates
    - synchronization among substates

# STATE AND EVENTS IN STATECHARTS

- States and events
  - boxes (rounded rectangle) denotes states
  - arrows labelled with event
    - optionally with a parenthesized conditions and an action (described later on)
- Different state levels ( = hierarchy support )
  - encapsulation express hierarchies
  - arrows can originate and terminate at any level

# HIERARCHY: CLUSTERING

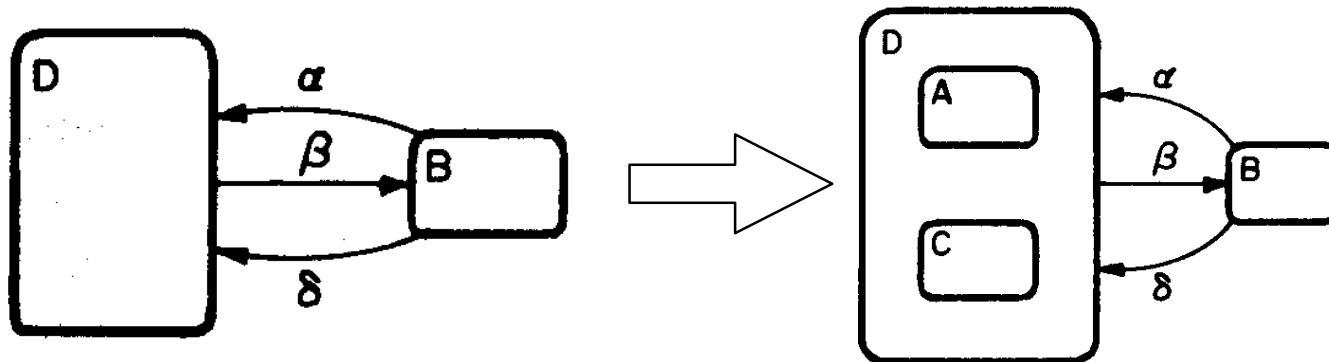
- XOR Decomposition
  - economizing arrows



- since beta takes the system to B from either A or C, we can cluster the latter into a new super-state D and replace the two arrows by one
  - the semantics of D is a XOR of A and C: to be in state D one must be either in A or in C, and not in both.
  - D is an *abstraction* of A and C
    - capturing common properties
- bottom-up approach

# REFINEMENT

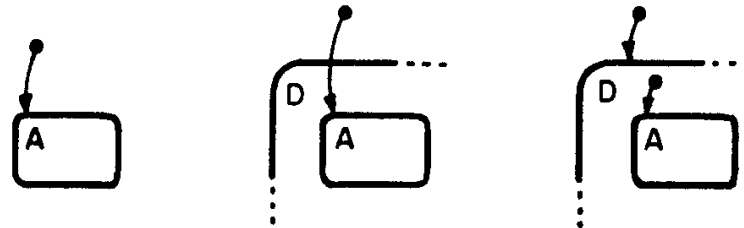
- We can proceed on the opposite direction, refining states:



- in this case the incoming alfa and beta arrows are underspecified
- top-down approach
- Zooming in and out support
  - zooming-in
    - by looking inside a state
  - zooming-out
    - abstracting from the inside of a state

# DEFAULT STATES

- Special arrow to explicitly represent the default entering state
  - at any level

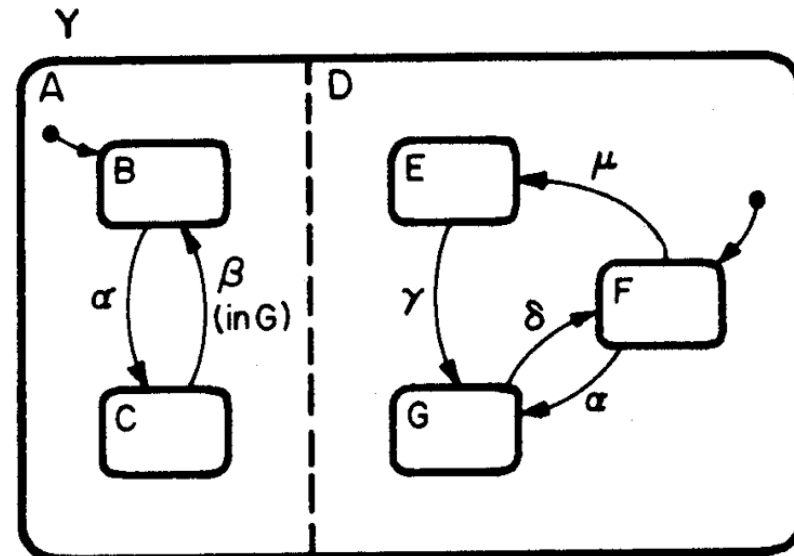


- Take into the account the history
  - entering the state most recently visited
  - H default-state arrows



# HIERARCHY 2: ORTHOGONALITY

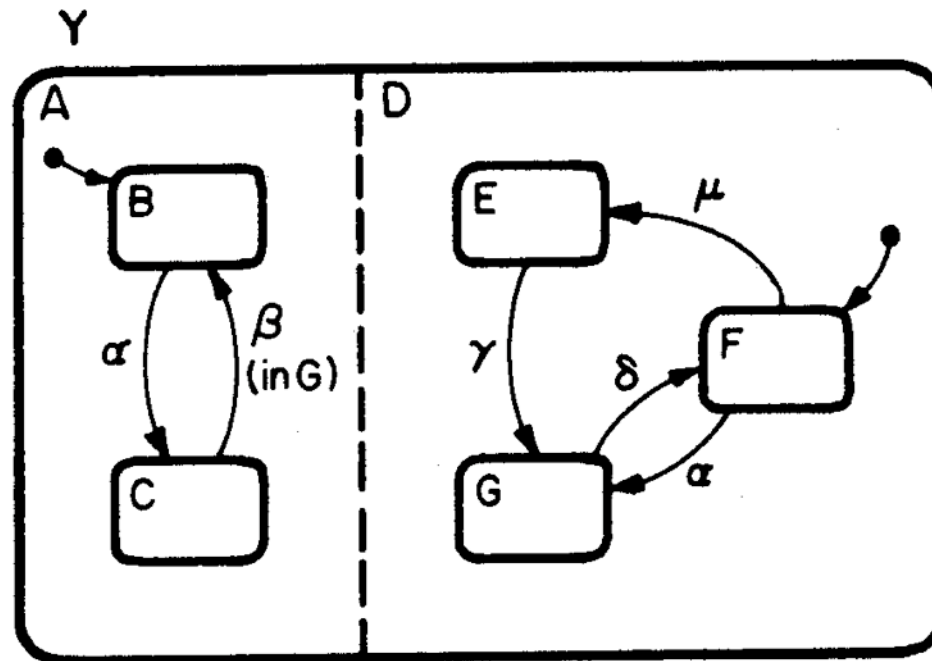
- AND decomposition
  - capturing the property that, being in a state, the system must be in all of its AND components
  - the notation used in statecharts is the physical splitting of a box into component using dashed lines



- state Y consists of AND components A and D
  - with the property that being in Y entails being in some combination of B or C with E, F or G.
  - Y is the orthogonal product of A and D
- Independency and / or Concurrency

# SYNCHRONIZATION

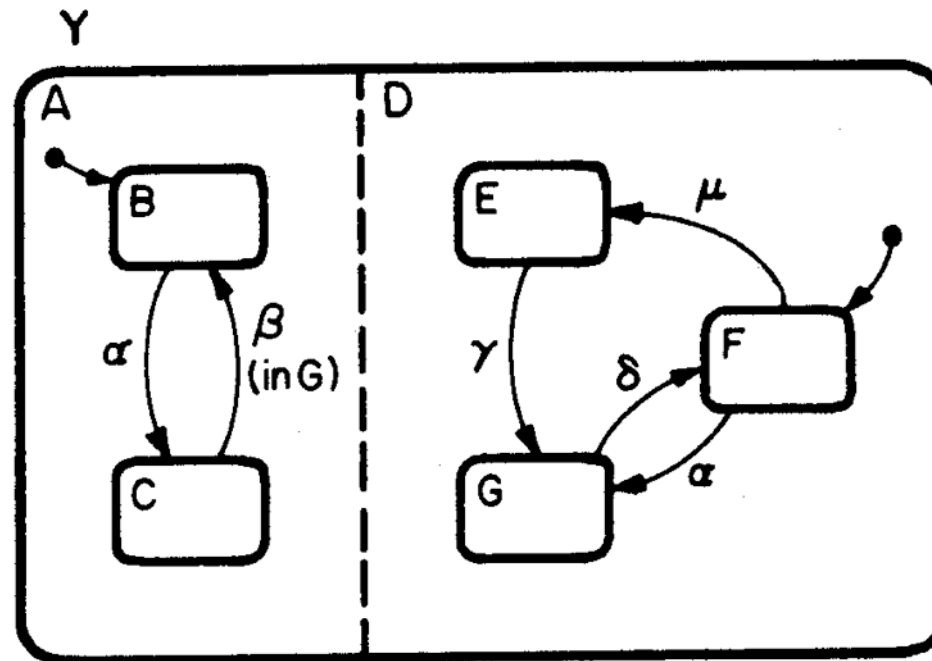
- In the example, if an event alfa occurs, it transfers B to C and F to G simultaneously, resulting in a new combined state (C,G)



- This illustrates a certain kind of synchronization
  - a single event causing simultaneous happenings

# INDEPENDENCE

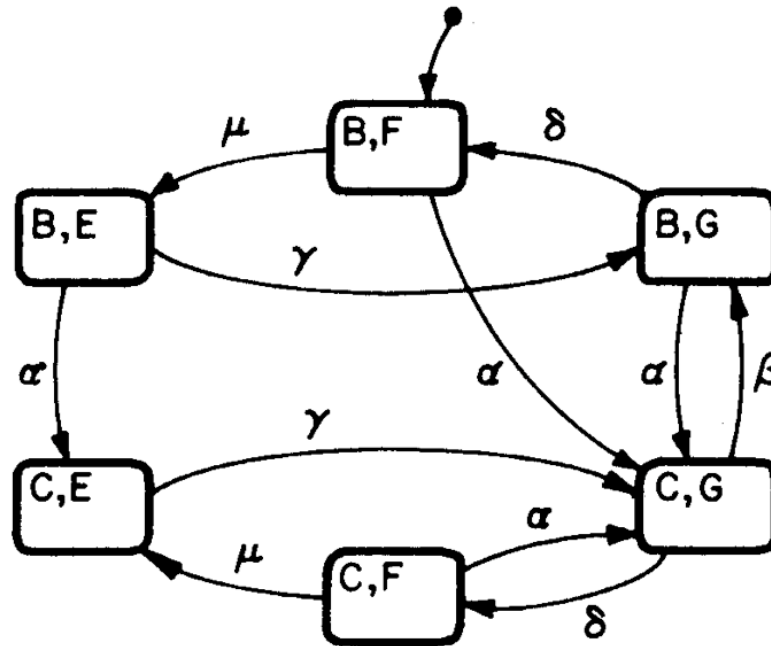
- On the other hand,  $\mu$  occurs at (B,F) it affects only the D component, resulting in (B,E)



- This illustrates a certain kind of independence
  - the transition is the same whether the system is in  $B$  or  $C$  in its  $A$  components

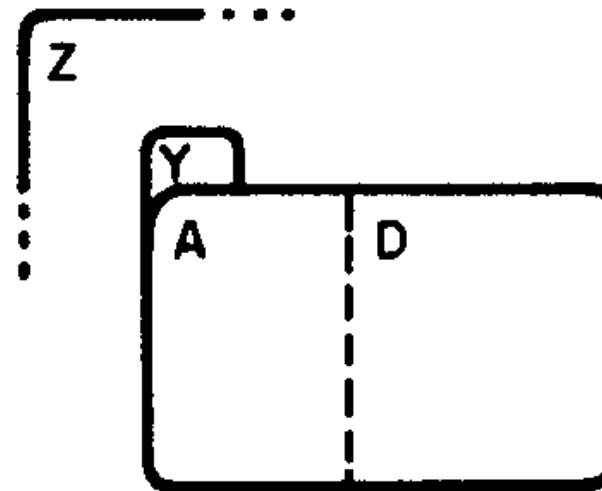
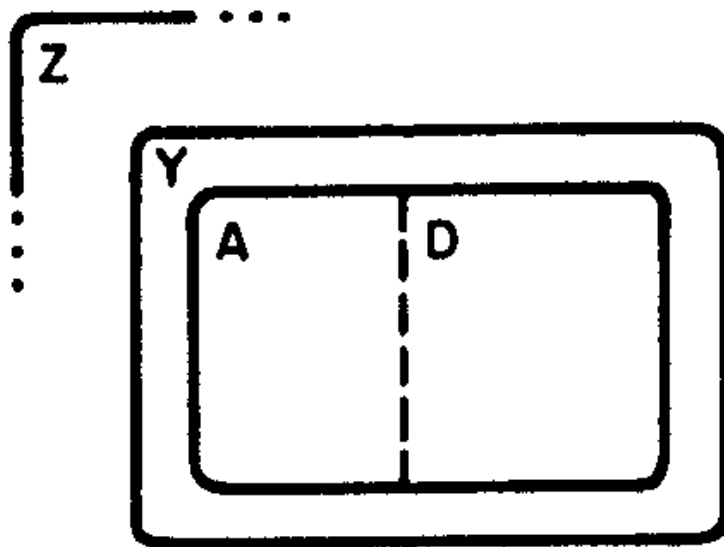
# AND-FREE EQUIVALENT

- The AND-Free equivalent diagram has the product of the states



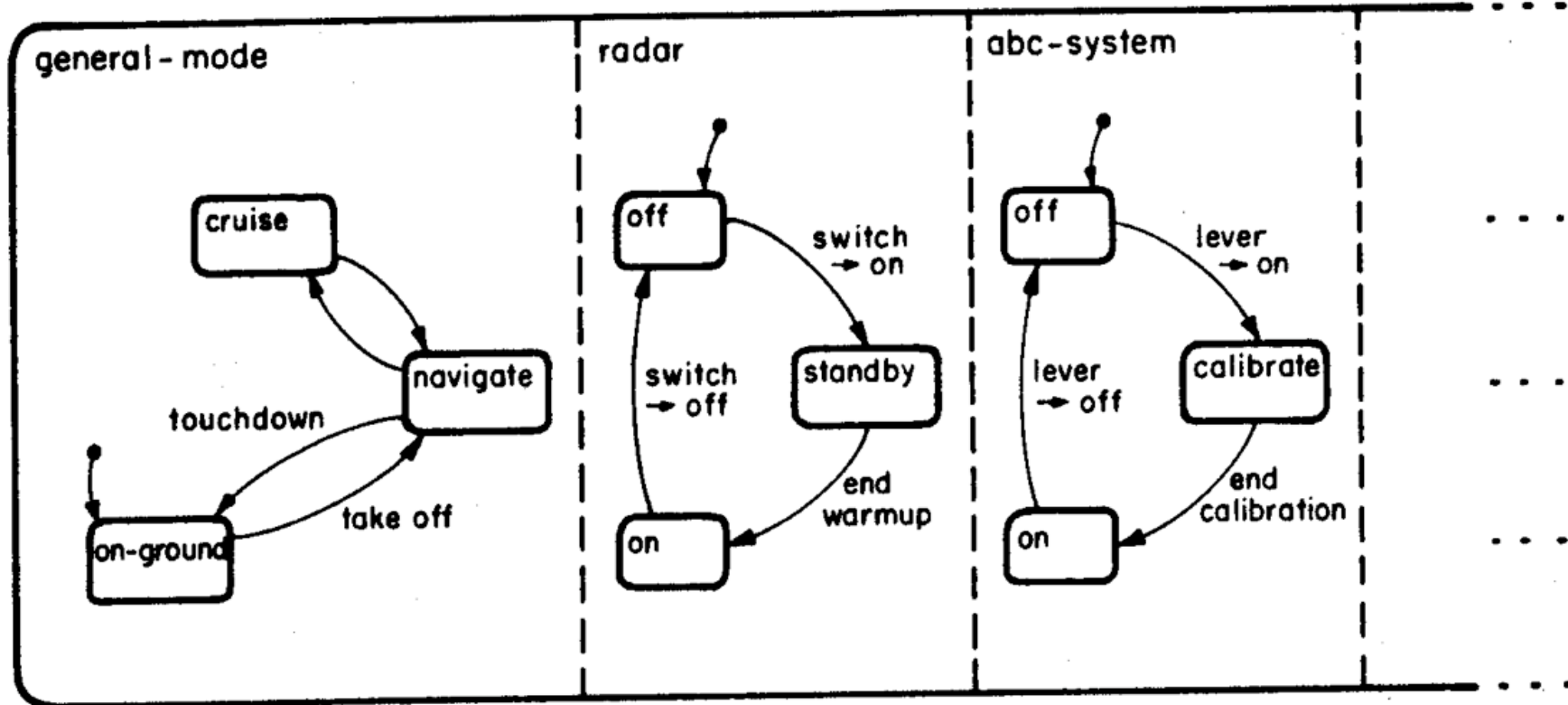
- That is: if we have two components with 1000 states, we have one million of states in the product
  - if we have 3 components:  $10^9$  states..

# NOTATIONS FOR AND-DECOMPOSITION



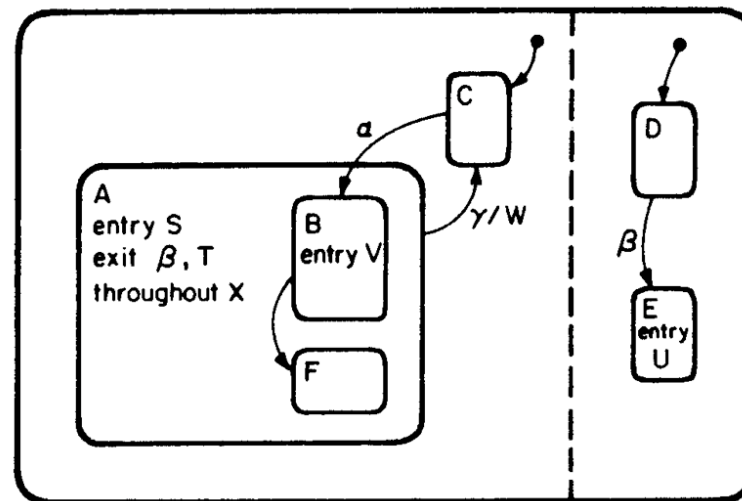
# AN EXAMPLE

## AVIONICS SYSTEM



# ACTIONS

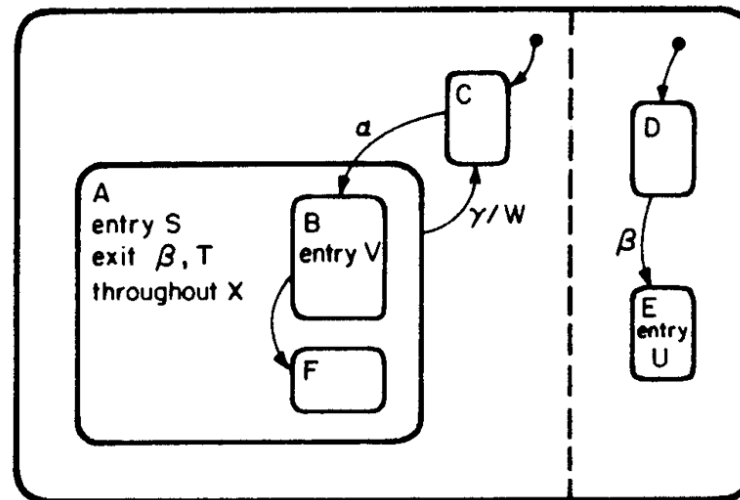
- Actions represents the ability of the statecharts to generate events and to change the value of conditions
  - influencing other components of the system
  - influencing the environment of the system



- Expressed by the notation ".../S" that can be attached to the label of the transition
  - S is an action carried out by the system
  - actions have instantaneous occurrences that take ideally 0 time.

# ACTIVITIES

- Activities
  - are to actions what conditions are to events
  - an activity always takes a nonzero amount of time (like beeping, displaying, executing lengthy computations..)
  - activities are durable

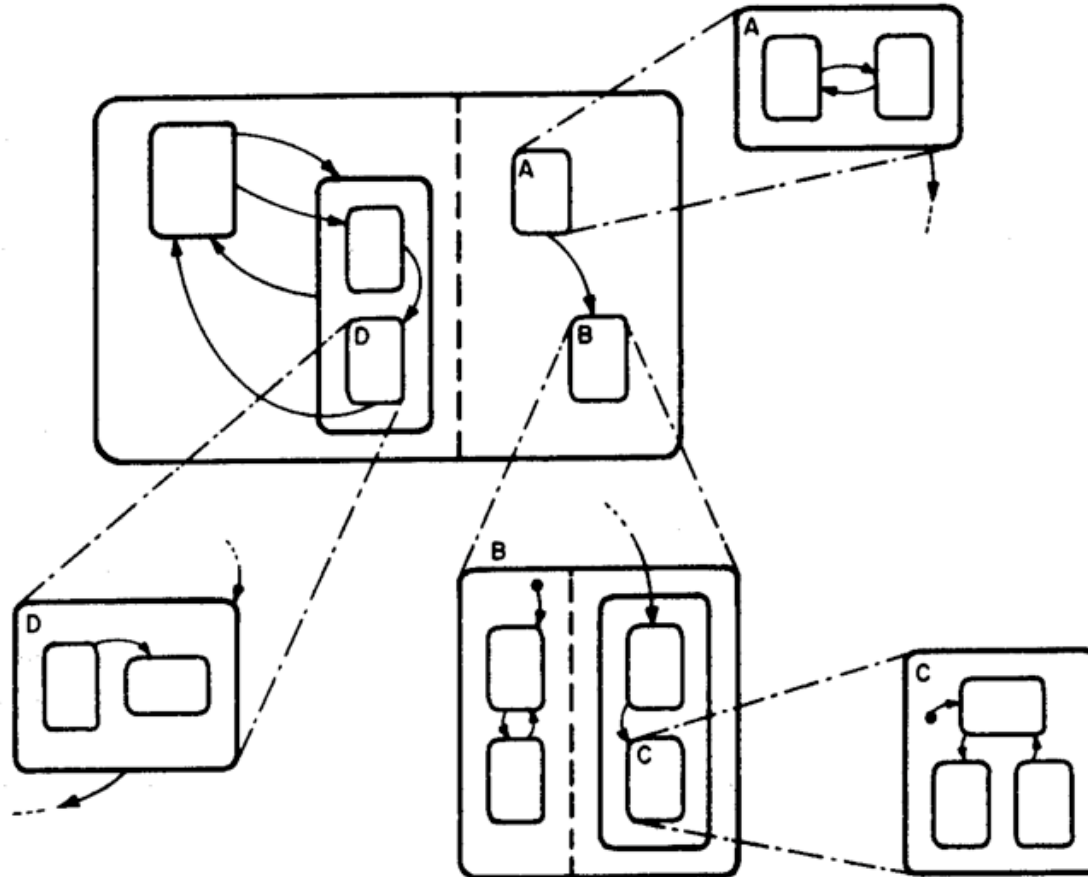


- Activities are associated with states
  - entry and exit actions



# FURTHER FEATURES: UNCLUSTERING

- Laying out parts outside the natural neighborhood



# THE STATEMATE TOOL

- Statemate is a comprehensive graphical modeling and simulation tool for the rapid development of complex embedded systems based on statecharts
  - using a combination of traditional graphical design notations combined with some of the Unified Modeling Language (UML) diagrams
- Statemate provides a direct and formal link between user requirements and software implementation by allowing the user to create a complete, executable specification
  - this specification may be executed, or graphically simulated, so the system engineer can explore what-if scenarios to determine if the behavior and the interactions between system elements are correct
  - these scenarios can be captured and included in Test Plans which are later run on the embedded system to ensure that what gets built meets what was specified.
  - this executable specification is also used to communicate with the customer or end user to confirm that the specification meets their requirements

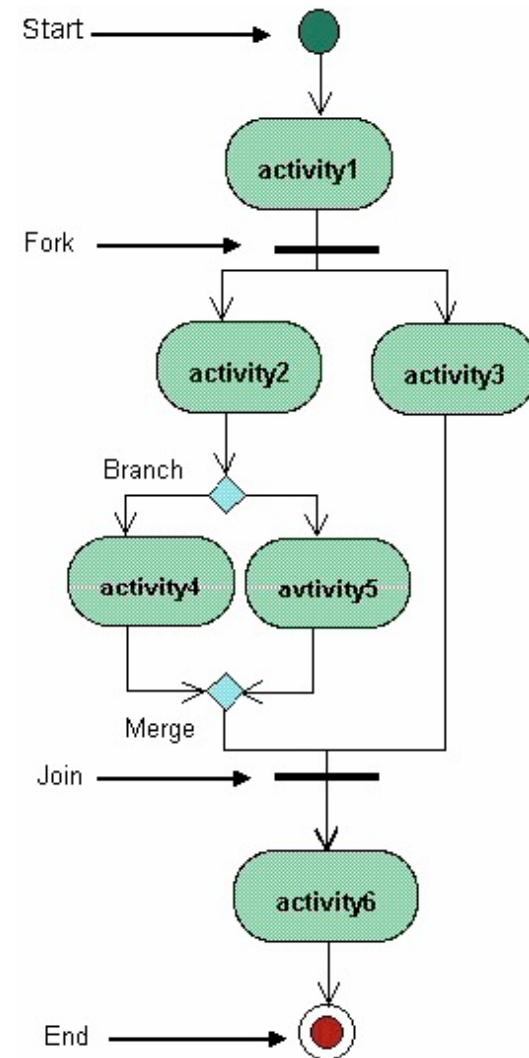
# ACTIVITY DIAGRAMS

# ACTIVITY DIAGRAMS

- Activity diagrams are one of the diagrams adopted in UML to represent the business and operational workflows of software system
  - an activity diagram is a dynamic diagram that shows the activity and the event that causes the object to be in the particular state
- Activity diagrams vs. state diagrams
  - a state diagram shows the different states an object is in during the lifecycle of its existence in the system, and the transitions in the states of the objects
    - these transitions depict the activities causing these transitions, shown by arrows
  - an activity diagram talks more about these transitions and activities causing the changes in the object states

# ACTIVITY DIAGRAM OVERVIEW (\*)

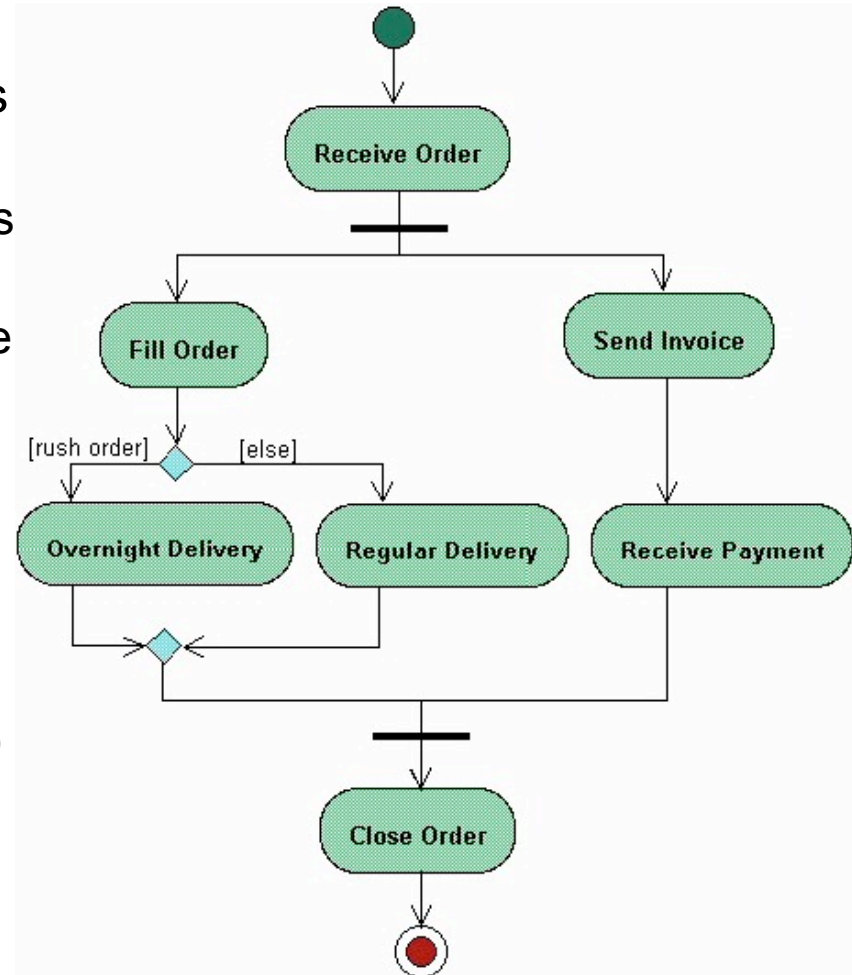
- Showing the flow of activities through the system
  - diagrams are read from top to bottom and have branches and forks to describe conditions and parallel activities. A fork is used when multiple activities are occurring at the same time.
- The diagram below shows a fork after activity1
  - this indicates that both activity2 and activity3 are occurring at the same time. After activity2 there is a branch. The branch describes what activities will take place based on a set of conditions. All branches at some point are followed by a merge to indicate the end of the conditional behavior started by that branch. After the merge all of the parallel activities must be combined by a join before transitioning into the final activity state.



(\*) taken from [http://atlas.kennesaw.edu/~dbraun/csis4650/A&D/UML\\_tutorial/activity.htm](http://atlas.kennesaw.edu/~dbraun/csis4650/A&D/UML_tutorial/activity.htm)

# PROCESSING ORDER EXAMPLE (\*)

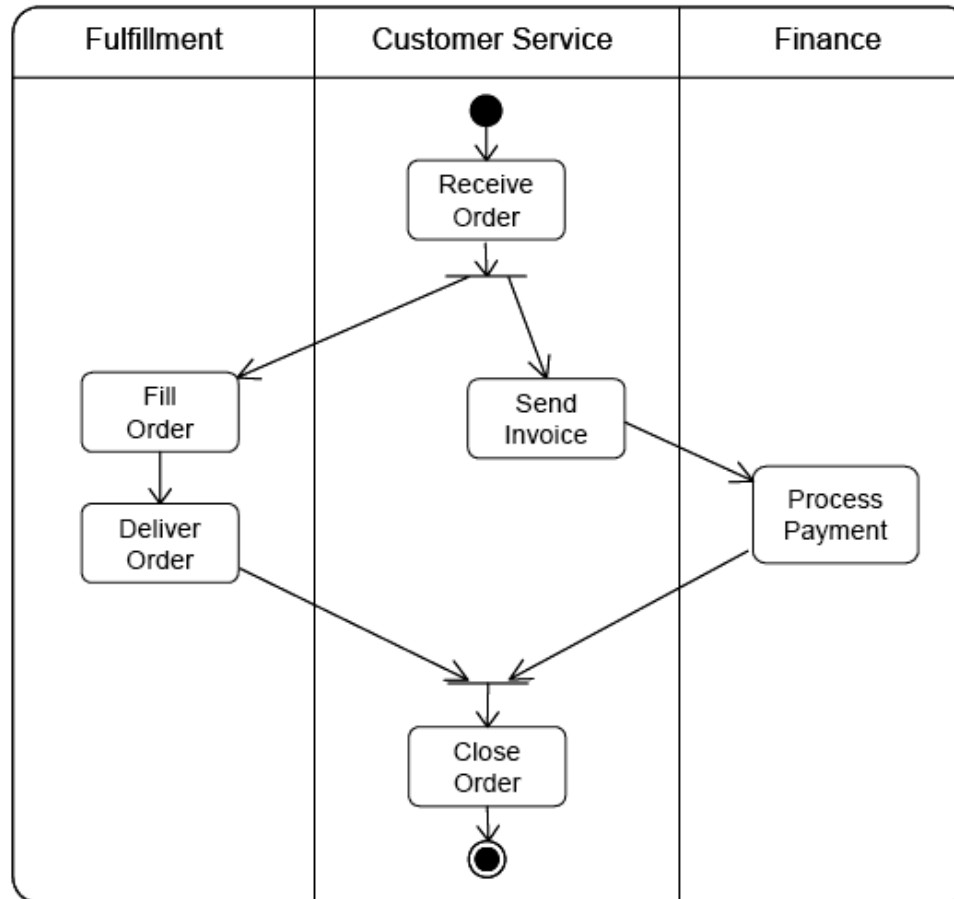
- The diagram shows the flow of actions in the system's workflow
  - once the order is received the activities split into two parallel sets of activities
  - one side fills and sends the order while the other handles the billing
  - on the Fill Order side, the method of delivery is decided conditionally.
  - depending on the condition either the Overnight Delivery activity or the Regular Delivery activity is performed.
  - finally the parallel activities combine to close the order.



(\*) taken from [http://atlas.kennesaw.edu/~dbraun/csis4650/A&D/UML\\_tutorial/activity.htm](http://atlas.kennesaw.edu/~dbraun/csis4650/A&D/UML_tutorial/activity.htm)

# SWIMLANES

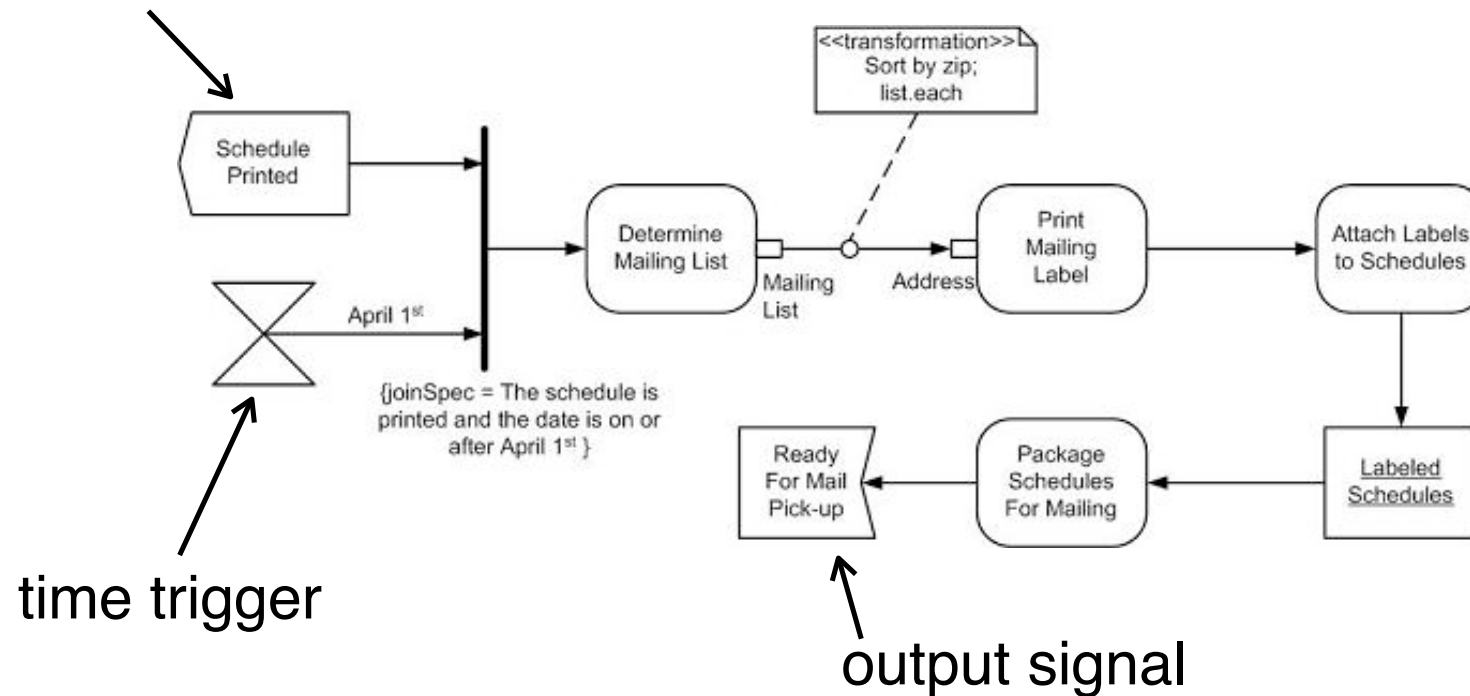
- A swimlane is a way to group activities performed by the same actor on an activity diagram or to group activities in a single thread



# SIGNALS

- *Signal Activities*
  - activities that send or receive messages (output / input signals)
- Triggers
  - temporal signals

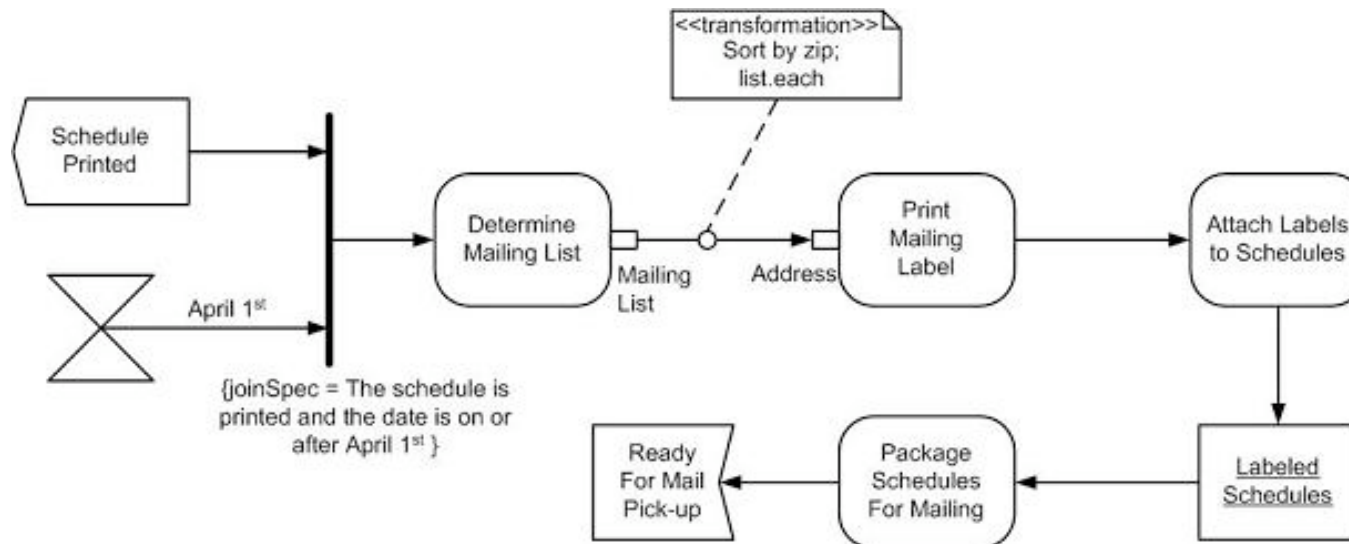
input signal





# PASSING OBJECTS

- Specifying objects passed between activities



object passed  
between activities

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