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31/10/2008

1. History of the study of galaxy clusters

[Bibl.: Biviano A., 2000, astro-ph/0010409v1]

→ 1784: Charles Messier notes Virgo cluster in catalogue of nebulae.

→ 1785: Wilhelm Herschel notes Coma cluster in catalogue of nebulae. Later other clusters: Ursa Major, Hydra.

→ 1864: John Herschel, General catalogue of nebulae. Fornax, Pisces.

→ 1877: Edouard Stephan discovers Stephan's quintet (compact group)

→ 1908: Max Wolf, photographic work on nebulae in Perseus and Coma clusters.

→ 1920: Shapley (galactic) and Curtis (extragalactic) debate on nature of nebulae (Kant 1755!, Herschel 1785!).

→ 1922-23 Edwin Hubble: nebulae are extragalactic objects (Cepheids in M31)

→ 1927 Lundmark: clusters can form by galaxy-galaxy encounters

→ 1931 Hubble & Humason: velocity dispersion of clusters.

→ 1936 Hubble, Zwicky: distribution of clusters looks uniform: no superclusters?

→ 1936 Hubble: suggests morphology-density relation.

→ 1937 Zwicky: estimates mass of Coma cluster using virial theorem. Dark Matter. Also suggests gravitational lensing as method to measure cluster mass!

→ 1941 Holmberg: first "N-body" simulation of galaxy encounter (with light bulbs)

→ 1942 Zwicky: spatial distribution of galaxies in clusters

→ 1943 Chandrasekhar: dynamical friction.

→ 1952 Zwicky detects intracluster light in Coma

→ 1953-1958, de Vaucouleurs: evidence of super-clustering

→ 1957 Zwicky suggests modification of general relativity as an alternative to dark matter in clusters

→ 1957-1967: Herzog, Wild & Zwicky catalogue of ∼10000 clusters

→ 1958 Abell' catalogue of clusters: 2712 clusters. Statistically homogeneous. 85% complete for the richest 1682 clusters. Starts study of clusters as a population of objects.

→ 1959 Limber speculates that clusters contain hot gas

→ 1959 Large et al. detect Coma C radio source.

→ 1960 Minkowski: clusters around radio sources (3C295 $\Longrightarrow z \sim 0.45$ cluster)

→ 1960 van den Bergh: morphology density relation in Virgo and Ursa Major

→ 1960 von Hoerner: first numerical N-body simulation

→ 1961 van Albada: clusters can be formed by amplification of density fluctuations

→ 1962 Abell's list of 17 superclusters.

→ 1964 Hénon simulation of dynamical mixing in cluster

→ 1964 Matthews et al.: cD galaxies

→ 1965-1974 Dark matter hypothesis becomes accepted

→ 1966 Byram et al.: first detection of M87 in X-rays (rocket experiment)

→ 1966 Felten et al.: Coma must have hot gas at $T > 10^7$ K.

→ 1967 Lynden-Bell: violent relaxation

→ 1970-1972 Sunyaev-Zeldovich (SZ) effect predicted

→ 1971 UHURU X-ray satellite. Coma.

→ 1972 Rood: kinematical segregation due to dynamical friction.

→ 1972 Gunn & Gott: ram-pressure stripping of galaxy gas in clusters

→ 1973-1977 Lea et al., Silk, Cowie & Binney, Fabian & Nulsen: cooling flow.

→ 1976 Schechter Luminosity Function of galaxies

→ 1976 ARIEL V satellite. Detection of 7 KeV iron line

→ 1976 White: N-body simulation of dynamical friction in clusters

→ 1976 Ostriker & Tremaine: galactic cannibalism

→ 1976 Cavaliere & Fusco-Femiano: β-model for cluster hot gas distribution

→ 1978 Butcher & Oemler effect (higher fraction of blue galaxies in higher z clusters)

→ 1978 EINSTEIN X-ray satellite

→ 1989-1993 first measures of cluster mass using weak gravitational lensing

→ 1990 ROSAT X-ray satellite

→ 1991 first reliable measurements of SZ effect

→ 1993 Boehringer: cavities in X-rays

→ 1999 XMM-Newton X-ray satellite & CHANDRA X-ray satellite

→ 2001-2002 Boehringer et al., Tamura et al., Peterson et al.: XMM and CHANDRA data inconsistent with standard cooling flow model.

→ 2005 SUZAKU X-ray satellite

→ 2000-2008 Churazov et al., Fabian et al., McNamara et al....: evidence of interaction between AGN and cooling flows (X-ray cavities + radio lobes)

31/10/2008

2. Optical observations of galaxy clusters

2.1 Optical catalogs

[Bibl.: S:2.1, S:2.2, V:II.A, R:546]

→ Optical: $3 \times 10^{14} \lesssim \nu \lesssim 10^{15}$ Hz, $3000 \lesssim \lambda \lesssim 10000$ Å

→ Abell (1958) and Zwicky et al. (1961-1968). Both used Palomar Observatory Sky Survey (POSS) plates (Northern sky). Southern Abell catalogue (Abell et al. 1989)

→ Criteria for membership. Surface number density enhancement + angular scale of enhancement. Or: number of galaxies + size. Completeness out to a limiting redshift.

→ Abell criteria: 1) at least 50 galaxies with m in the range m_3 to m_3+2 (magnitude of the 3rd brightest galaxy); 2) galaxies contained within a (Abell) radius of $1.7/z$ arcmin or $2.1/h_{70}$ Mpc; redshift $0.02 \leq z \leq 0.2$. 2712 clusters of which 1672 satisfying selection criteria. ($H_0 = 70h_{70}$ km/s/Mpc)

→ Zwicky criteria: 1) boundary of the cluster contour where density is twice the local background density; 2) at least 50 galaxies within boundary with m in the range m_1 to m_1+2 (magnitude of the brightest galaxy); Less strict criteria than Abell's: 10000 clusters.

→ Abell estimated redshift from magnitude of the tenth-brightest galaxy. Now spectroscopic redshift. Sarazin et al. (1986) for 500 Abell clusters.

→ In more modern cluster catalog, color is used as a criterion. Cluster galaxies are typically redder than field galaxies at the same redshift.

→ Automated catalogs of clusters. The Edinburgh/Durham Southern Galaxy Catalogue (EDSGC): 737 clusters (Lumsden et al. 1992). APM Galaxy survey: 1000 clusters (Maddox et al. 1990)

→ Problems with optical identification: projection effects. Solution: X-ray selection.

→ Observation in red or infra-red bands and photometric redshift techniques allow to select clusters out to redshift $z \sim 1.3$

2.2 Richness

[Bibl.: S:2.3, V:II.A.1]

→ It is a measure of the number of galaxies in a cluster. It is a statistical measure of the population of a cluster.

→ Definition should be independent of distance of the cluster.

→ Zwicky et al. Richness: number of galaxies within the cluster boundary minus the expected number of background galaxies. Definition dependent on distance! Larger area of cluster considered for more nearby clusters.

→ Abell: richness definition nearly independent of distance. Number of galaxies within the Abell radius. Richness classes (5 if > 300, 1 if $50 - 79$). e.g. Coma in richness class 2.

→ Postman et al. (1996) define richness as the number of galaxies more luminous than $L_* \sim 10^{11} L_\odot$. Correlates with Abell richness, with large scatter.

2.3 Luminosity function

[Bibl.: S:2.3, V:II.A.1]

→ Differential Luminosity Function (LF): $n(L)$ number of galaxies with luminosity between L and $L + dL$. Integral LF $N(L) = \int_L^\infty n(L) dL$. $n(L) = -dN/dL$.

→ Schechter (1976) LF:

$$n(L)dL = N_*(L/L_*)^{-\alpha} \exp(-L/L_*) d(L/L_*), \qquad (2.2)$$

where L_* is a characteristic luminosity and α is the faint-end slope. Good fit to cluster galaxies if BCG/cD galaxies ($L \sim 10L_*$) are excluded.

→ Typical values $\alpha \sim 1.2 - 1.3$ and $L_* \sim 10^{11} L_\odot$.

→ The integral Schechter LF is $N(L) = N_* \Gamma(1 - \alpha, L/L_*)$, where $\Gamma(a, x) = \int_x^\infty t^{a-1} \exp(-t) dt$ is the incomplete gamma function. Total number of galaxies diverges, but total luminosity is finite ($1 < \alpha < 2$): $L_{tot} = N_* \Gamma(2 - \alpha) L_*$ where $\Gamma(a) = \int_0^\infty t^{a-1} \exp(-t) dt$ is the gamma function.

→ Schechter: power law of index α at the faint end. Exponential cut-off at $L > L_*$. Break at L_*.

→ Brightest Cluster Galaxies (BCG) (including cD galaxies) are not statistically drawn from the LF. Special formation mechanism (galactic cannibalism).

2.4 Morphological classification

[Bibl.: S:2.5]

→ Different criteria of classification

→ Abell: regular → irregular (less dynamically evolved)

→ Zwicky: compact (concentrated), medium compact, open (not concentrated)

→ Bautz-Morgan: Type I (with cD), Type II (dominant BCG but not cD), Type III (no dominant galaxy)

→ Rood-Sastry: "bifurcated" classification based on distribution of 10 brightest galaxies . cD: dominated by cD (e.g. A2199) ; B: binary (Coma); L: line, 3 or more galaxies in a straight line (Perseus); C: core, 4 or more galaxies in the core (A2065); F: flattened distribution (Hercules); I: irregular (A400)

→ Struble & Rood revised Rood-Sastry classification, changing the tuning-fork diagram into a line-split diagram.

→ Other classification, based on galaxy content: spiral-rich, spiral-poor.

→ Different classifications are correlated. No clear trend with richness.

2.5 Velocity distribution of galaxies in clusters

[Bibl.: S:2.6, V:II.A.2]

→ Consider a cluster in which $\langle z \rangle$ is the mean galaxy redshift. The line-of-sight velocity of a galaxy with redshift z in the rest frame of the cluster is $v_{los} = (z - \langle z \rangle)c$

→ $p(v_{los})dv_{los}$ is the probability of finding a galaxy with velocity in the range v_{los} to $v_{los} + dv_{los}$. If Gaussian (as observed in many clusters) we have

$$p(v_{los})dv_{los} = \frac{1}{\sqrt{2\pi}\sigma_{los}} \exp[-v_{los}^2/2\sigma_{los}^2]dv_{los} \tag{2.3}$$

where σ_{los} is the line-of-sight velocity dispersion.

→ σ_{los} depends on position in the cluster. Typically decreasing for increasing projected distance $\Longrightarrow$ galaxy velocity distribution is not exactly isothermal.

→ Consider "Temperature" of galaxies T_{gal}:

$$\sigma_{los}^2 \propto \frac{k_B T_{gal}}{M_{gal}}, \tag{2.4}$$

where M_{gal} is the galaxy mass. σ_{los} independent (or only slightly dependent) on galaxy mass: no thermodynamical equilibrium. $\Longrightarrow$ two-body relaxation not advanced ($t_{2b}/t_{cross} \propto N/\ln N$). T_{gal} proportional to mass ($\Longrightarrow$ clusters are collisionless systems $\Longrightarrow$ violent relaxation). One can also define an equivalent gas temperature $T_{gal,equiv}$ that

$$\sigma_{los}^2 \propto \frac{k_B T_{gal,equiv}}{\mu m_p} \tag{2.5}$$

where μ is the mean gas particle mass in units of the proton mass $m_{\rm p}$ and k_B is the Boltzmann constant. $T_{gal,equiv}$ is comparable to the temperature of the ICM.

→ No mass significant segregation $\Longrightarrow$ dark matter more diffuse than associated with single galaxies.

→ Brightest galaxies have somewhat smaller velocity dispersion than the luminous galaxies (dynamical friction).

2.6 Spatial distribution of galaxies in clusters

[Bibl.: S:2.7]

→ Simplest models (for regular clusters): spherically symmetric. Intrinsic number density of galaxies $n(r)$; projected number density of galaxies (derive)

$$\Sigma(R) = 2\int_R^{R_{max}} \frac{n(r)rdr}{\sqrt{r^2 - R^2}}, \tag{2.6}$$

where R_{max} is the maximum radius of the cluster galaxy distribution. Here we assume $R_{max} = \infty$.

→ By deprojecting Σ one obtains the corresponding intrinsic density distribution (derive using Abel integral equation)

$$n(r) = -\frac{1}{\pi}\int_r^{\infty} \frac{d\Sigma}{dR}\frac{dR}{\sqrt{R^2 - r^2}} \tag{2.7}$$

→ A widely adopted model is the analytic King model

$$n(r) = n_0\left[1 + \left(\frac{r}{r_c}\right)^2\right]^{-3/2} \tag{2.8}$$

$$\Sigma(R) = \Sigma_0\left[1 + \left(\frac{R}{r_c}\right)^2\right]^{-1}, \tag{2.9}$$

where r_c is the core radius, $\Sigma_0 = 2n_0r_c$ and the integrated number of galaxies diverges logarithmically. $\Sigma(r_c) = \Sigma_0/2$. Typically $r_c \sim 0.2$ Mpc.

→ Most clusters are not spherically symmetric, but are highly elongated. They have intrinsic ellipticities $\epsilon \sim 0.5 - 0.7$, so clusters are more flattened on average than elliptical galaxies ($\epsilon \lesssim 0.3$). Ellipticity is $\epsilon \equiv 1 - b/a$, where a and b are major and minor axis.

→ In non-spherical clusters with a central dominant cluster, there is alignment between the cluster and the central cluster galaxy.

2.6.1 Abel integral equation

[BT]

$$f(x) = \int_x^\infty \frac{dt\, g(t)}{(t-x)^\alpha} \tag{2.10}$$

$\Longrightarrow$

$$g(x) = -\frac{sin(\alpha\pi)}{\pi}\frac{d}{dx}\int_x^\infty dt\, \frac{f(t)}{(t-x)^{1-\alpha}} \tag{2.11}$$

$$= -\frac{sin(\alpha\pi)}{\pi}\int_x^\infty \frac{df}{dt}\frac{dt}{(t-x)^{1-\alpha}} \tag{2.12}$$

2.7 Morphology of galaxies in clusters

2.7.1 BCGs and cD galaxies

[S:2.10.1]

→ Brightest Cluster Galaxies (BCGs) are dominant giant elliptical galaxies. Typically they sit at the centre of the cluster.

→ Many BCGs have double or multiple nuclei. If the two central nuclei are surrounded by a common luminous halo $\Longrightarrow$ "dumbbell" galaxy.

→ cD galaxies are special BCGs. They are dominant, central giant elliptical galaxies. They are special because they have an extended amorphous luminous halo of low surface brightness.

→ The central light profile is well fitted by a $R^{1/4}$ De Vaucouleurs law

$$I(R) = I_e \exp\left\{-7.67\left[\left(\frac{R}{R_e}\right)^{1/4} - 1\right]\right\}, \tag{2.13}$$

but the extended halo deviates from the $R^{1/4}$ (light excess in the outer regions). R_e is effective radius (=half-light radius).

→ $R^{1/4}$ law is a special case of Sérsic $R^{1/m}$ law:

$$I(R) = I_e \exp\left\{-b(m)\left[\left(\frac{R}{R_e}\right)^{1/m} - 1\right]\right\} \tag{2.14}$$

where $b(m) \sim 2m - 1/3 + 4/(405m)$. Typically $m \gtrsim 4$ for giant ellipticals.

→ The luminous halo can be considered as associated to the BCG ($\Longrightarrow$ cD) or to the cluster ($\Longrightarrow$ Intracluster Light: stars in clusters of galaxies that are not in galaxies). ICL: less than ∼10% of stars in galaxies.

→ About 20% of BCGs are cD galaxies.

→ Luminosity $\sim 10L_* \sim 10^{12}L_\odot$. Masses BCG/cD galaxies of the order of $10^{13}M_\odot$. But difficult to separate galaxy by cluster.

→ Mechanism of formation of BCG: galactic cannibalism (by dynamical friction). Not so clear how to form diffuse luminous halo.

2.7.2 Morphology-density relation & redshift evolution

[S:2.10.2]

→ Elliptical and S0 galaxies are more common than spirals in regular clusters, while the opposite is true in the field and in irregular clusters.

→ Many spirals in cluster have less cold neutral gas ($T \lesssim 1000$ K) than field spirals ("anemic" spirals).

→ Butcher-Oemler effect: higher redshift clusters contain a higher fraction of blue galaxies than lower redshift galaxies.

→ Most probable explanation: spirals in clusters lose their gas and become S0 (=lenticular). Gas might be removed by ram pressure stripping and/or thermal evaporation by cluster hot gas $\Longrightarrow$ stop to star-formation.

31/10/2008

3. X-ray observations

3.1 Detection and identification of clusters in X-rays

[S:4.1, V:II.B, R:548]

→ X-ray: $10^{16} \lesssim \nu \lesssim 10^{19}$ Hz, $0.1 \lesssim \lambda \lesssim 10$ nm,$0.1 \lesssim E = h\nu \lesssim 100$ keV

→ Clusters of galaxies are the most common X-ray sources. They are extremely luminous in X-ray $L_X = 10^{43-45}$ erg/s. X-ray emission from clusters is extended and not time-variable.

→ Detected in X-ray emission line of highly ionized metals (e.g. iron) $\Longrightarrow$ metal enriched plasma

→ Cluster X-ray emission is thermal emission from the hot ($2 \times 10^7 - 10^8$ K) gas of the intracluster medium (ICM).

→ X-rays cannot be observed by ground-based telescopes because they are efficiently absorbed by the earth's atmosphere. Need to observe from very high altitude $\Longrightarrow$ rockets (1960-1970) $\Longrightarrow$ satellites (1970-today).

→ Satellites: UHURU (1971), Einstein Observatory (1979), ROSAT (1990), XMM-Newton (1999), CHANDRA (1999), SUZAKU (2005). Increasing sensitivity, spatial (and spectral) resolution.

→ Highest resolution: spectral $E/\Delta E \sim 50$ (XMM), spatial ~ 0.5 arcsec (CHANDRA)

→ Advantage of X-ray selection of clusters: reveals bound systems; X-ray luminosity correlated with mass; X-ray emissivity proportional to gas density squared.

→ Cluster X-ray photometric samples mainly determined by area and flux limit.

3.2 X-ray luminosity function of cluster of galaxies

[S:4.2, R:556]

→ differential Luminosity Function (LF): $n(L_X)dL_X$ number of clusters with X-ray luminosity between L_X and $L_X + dL_X$. Integral LF $N(L_X) = \int_{L_X}^{\infty} n(L_X)dL_X$.

→ The cluster X-ray LF is well described by the Schechter function

$$n(L_X)dL_X = N_*(L_X/L_X^*)^{-\alpha_X} \exp(-L_X/L_X^*)d(L_X/L_X^*), \qquad (3.2)$$

where L_X^* is a characteristic luminosity and α_X is the faint-end slope.

→ L_X in some specified energy range ($E = h\nu$). For instance, 'soft' X-rays (0.5-2 keV).

→ Best-fit values $\alpha_X \sim 1.8$ and $L_* \sim 4 \times 10^{44}$ erg/s in soft X-rays 0.5-2 keV.

→ Cluster X-ray LF better determined than galaxy optical LF.

3.3 X-ray spectra of clusters

[S:4.3, V:II.B.2-3]

→ Spectral X-ray observations of clusters fundamental to determine emission mechanism

→ It is thermal emission from diffuse hot intracluster gas:

$$I_\nu \propto \exp(-h\nu/k_B T) \tag{3.3}$$

where T is the gas temperature; thermal velocity of protons $\sim \sqrt{k_B T/m_p} \sim \sigma$, where σ is the velocity dispersion of galaxies in the cluster; no low-energy photoabsorption (optically thin); emission line from heavy elements (iron).

→ Intensity I_ν in units of $\mathrm{erg\,cm^{-2}\,s^{-1}\,Hz^{-1}}$. Spectrum made by continuum + line emission.

→ The continuum is due to thermal bremsstrahlung $I_\nu \propto \exp(-h\nu/k_B T)$ if single-temperature gas. Temperature $2 \times 10^7 - 10^8\,\mathrm{K}$, so bremsstrahlung emission is truncated at $\lesssim$ 10 keV (often T is expressed in keV: $k_B T = 1keV$ if $T \simeq 1.16 \times 10^7\,\mathrm{K}$).

→ Observations in the hard X-ray ($\gtrsim$ 20 keV) can constrain the contribution from non-thermal emission (inverse Compton and synchrotron: $I_\nu \propto \nu^{-\alpha_{syn}}$)

→ The most important emission line in the cluster X-ray spectra is the "7 keV Fe line" from mainly Fe^{+24} and Fe^{+25} [e.g. Kβ line: $n = 1$ (K shell), $\Delta n = 2$ (β). Shells: K,L,... Δn: α, β,...]

→ Beside Fe 7 keV line, many lower energy lines from highly ionized Ni, Fe, C, N, O, Ne, Mg, Si, S...

→ Line equivalent width

$$EW \equiv \int \frac{I_\nu - I_{\nu,cont}}{I_{\nu,cont}} d(h\nu) \tag{3.4}$$

(in keV). Measures of EW of iron lines allows to determine metal abundance of cluster gas . Iron abundance $Fe/H \sim 10^{-5}$ ($\sim$ 0.3 solar value).

→ Also metallicity gradient: higher metallicity in the core.

3.4 Spatial distribution of X-ray emission form clusters

[S:4.4, V:II.B.1]

→ Extended X-ray emission peaked on the cluster center (on the cD or BGC if there is a central dominant galaxy)

→ β model: both the gas and the galaxy distribution are assumed to be isothermal, in hydrostatic equilibrium in the cluster gravitational potential. Galaxy distribution with isotropic velocity dispersion. The parameter β is the ratio between gas (T_{gas}) and galaxy ($T_{gal,equiv} = \mu m_p \sigma^2/k_b$), where μ is the mean gas particle mass in units of the proton mass m_p and k_B is the Boltzmann constant. Thus

$$\beta \equiv \frac{\mu m_p \sigma^2}{k_b T_{gas}}, \tag{3.5}$$

where σ is the 1-dimensional galaxy velocity dispersion. $\implies \rho_{gas} \propto \rho_{gal}^{\beta}$ (derive from hydrostatic equilibrium equation).

→ If galaxy distribution is King analytic model then

$$n_{gas}(r) = n_{gas,0} \left[1 + \left(\frac{r}{r_c} \right)^2 \right]^{-3\beta/2} \tag{3.6}$$

($\rho_{gas} = \mu m_p n_{gas}$).

→ The X-ray surface brightness is

$$S_X(R) = 2\int_R^\infty \frac{\epsilon r dr}{\sqrt{r^2 - R^2}}, \tag{3.7}$$

where $\epsilon = n_i n_e \Lambda(T)$ is the emissivity and $\Lambda(T)$ is the cooling function $\Lambda(T) \propto T^{1/2}$ for bremsstrahlung (n_i and n_e are the ion and electron number densities and $n_{gas} = n_i + n_e$). In the case of β model one obtains

$$S_X(R) \propto \left[1 + \left(\frac{R}{r_c}\right)^2\right]^{-3\beta+\frac{1}{2}}. \tag{3.8}$$

→ Best fit values for beta models of clusters $\beta \simeq 2/3$. Thus at $r \gg r_c$ $\rho_{gas} \propto r^{-2}$ while $\rho_{gal} \propto r^{-3}$ (gas distribution more extended).

→ Gas mass diverges at large radii for $\beta \leq 1$. The integrated X-ray luminosity converges for $\beta > 0.5$.

→ Morphology of X-ray emission: regular/irregular; with or without a central dominant galaxy; with or without cool core.

3.5 Thermal emission mechanisms in X-rays

[S:5.1.3, S:5.2]

3.5.1 Ionization and X-ray emission

→ $T \sim 10^8$ K, $n_e \sim 10^{-3}$ cm^{-3}

→ Assumptions: Coulomb collision timescale short $\Longrightarrow$ Maxwell-Boltzmann distribution at temperature $T_e = T_i$; low density $\Longrightarrow$ collisional excitation and deexcitation slower than radiative decay; gas is optically thin; ionization determined by electron-ion collisions (negligible ion-ion collisions); plasma in ionization equilibrium.

→ H and He fully ionized with primordial composition $X = 0.75$ and $Y = 0.25$ (H and He fraction in mass) $\Longrightarrow n_H = 6n_e/7$, $n_{\rm He} = n_e/14$, where n_H and $n_{\rm He}$ are H and He number densities $\Longrightarrow n_i \simeq 0.93 n_e$, $n = n_i + n_e \simeq 1.93 n_e$ and $\mu \simeq 0.59$ (derive). By definition, in fully ionized plasma $n_H = n_p$ (proton number density).

→ The equilibrium ionization state depends only on the electron temperature.

→ Continuum X-ray emission due to free-free (bremsstrahlung) and free-bound (recombination). Bremsstrahlung dominant.

→ Line X-ray emission due to radiative deexcitation of collisionally excited inner shell electrons $\Longrightarrow$ radiative recombination.

→ Line ratios useful to determine temperature, abundance and ionization state of the cluster gas.

→ The combined emissivity from this mechanisms can be expressed as

$$\epsilon_\nu = n_p n_e \sum_{X,i} \frac{n(X)}{n(H)} f(X^i, T_e) \Lambda_\nu (X^i, T), \tag{3.9}$$

where X^i is a ion of the generic element X, f is the ionization fraction and Λ_ν is the emission per ion at unit electron density. The bolometric emissivity can be written

$$\epsilon = \int \epsilon_\nu dh\nu = n_e n_i \Lambda(T), \tag{3.10}$$

where Λ is the radiative cooling function, strongly dependent on metallicity for $T \lesssim 3 \times 10^7$ K. (units of ϵ: $erg\, s^{-1}\, cm^{-3}$ or $W\, m^{-3}$)

→ It is useful to define the Emission Integral $EI \equiv \int n_e n_p dV$. The normalization of the spectrum depends on EI. The shape on the abundance and on $d(EI)/dT_e$.

→ The clearest evidence for thermal emission in clusters is the presence of metal lines, especially iron 7 keV line. Inverse Compton does not produce lines.

3.5.2 Thermal bremsstrahlung

→ $T \sim 10^8$ K, $n_e \sim 10^{-3}$ cm^{-3} gas with Maxwell-Boltzmann distribution $\Longrightarrow$ thermal bremsstrahlung (free-free) with emissivity

$$\epsilon_\nu \equiv \frac{dL}{d\nu dV} \propto Z^2 n_e n_i g(Z, T_e, \nu) T_e^{-1/2} \exp(-h\nu/k_B T_e), \qquad (3.11)$$

where T_e is the electron temperature, k_B is the Boltzmann constant, n_e (n_i) is the electron (ion) number density, Z is the charge of ions and g is the Gaunt factor accounting for quantum mechanical effects.

→ Bolometric bremsstrahlung emissivity, neglecting temperature and frequency dependence of g (derive)

$$\epsilon = \int \epsilon_\nu dh\nu \propto T_e^{1/2} n_e n_i. \qquad (3.12)$$

Total X-ray luminosity

$$L_X = \int \epsilon dV. \qquad (3.13)$$

31/10/2008

4. Microwave observations of clusters

4.1 Clusters in microwaves

[Bibl: S:3.5, V:II.C]

→ Millimetric/submillimetric band: $30 \lesssim \nu \lesssim 300$ GHz, $0.1 \lesssim \lambda \lesssim 10$ mm.

→ Clusters of galaxies do not emit significantly in the microwaves, but can be detected by their effect on the Cosmic Microwave Background (CMB): known as Sunyaev-Zeldovich (SZ) effect

4.2 Thermal Sunyaev-Zeldovich effect

→ CMB: black body radiation at $T_{CMB} = 2.725$ K. CMB spectrum:

$$I_\nu = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{k_\mathrm{B} T_{CMB}}\right) - 1}. \tag{4.2}$$

→ Free electrons of cluster hot gas scatter CMB photons $\Longrightarrow$ distortion of CMB spectrum. The optical depth is

$$d\tau_T = \sigma_T n_e dl, \tag{4.3}$$

where n_e is the electron number density and

$$\sigma_T = \frac{8\pi}{3} \left[\frac{e^2}{m_e c^2}\right]^2 \tag{4.4}$$

is the Thomson electron scatter cross section. For typical values $n_e \sim 10^{-3}$ and $l \sim 1Mpc \implies$

$$\tau_T = \int \sigma_T n_e dl \sim 10^{-2} \tag{4.5}$$

$\implies$ 1% probability of scattering.

→ $E_e \gg E_\gamma \implies$ inverse Compton $\implies$ distortion of CMB spectrum. Frequency change:

$$\frac{\Delta\nu}{\nu} = \frac{4k_B T_e}{m_e c^2}. \tag{4.6}$$

→ The distorted spectrum is not black body. Define brightness temperature T_b of a source as the temperature of a black body having the same intensity at a given frequency. In terms of T_b the spectral distortion is

$$\frac{\Delta T_b}{T_b} = \frac{\Delta I_\nu}{I_\nu}\frac{d\ln T_b}{d\ln I_\nu} = f(x)y, \tag{4.7}$$

where $x \equiv h\nu/k_B T_b$, the Compton y-parameter is

$$y \equiv \int \frac{k_B T_e}{m_e c^2} d\tau_T = \int \frac{k_B T_e}{m_e c^2} n_e \sigma_T dl \tag{4.8}$$

and $f(x) = [x\coth(x/2) - 4]$ if nonrelativistic.

→ In the Rayleigh-Jeans regime ($x \ll 1$)

$$T_b \equiv \frac{I_\nu c^2}{2k_B \nu^2}, \tag{4.9}$$

and $f(x) \to -2 \implies$

$$\frac{\Delta T_b}{T_b} = -2y = -\int \frac{2k_B T_e}{m_e c^2} n_e \sigma_T dl \tag{4.10}$$

→ Intensity decreases at $\nu < 218$ GHz and increases at at $\nu > 218$ GHz. Usually measurements at low frequency $\implies$ "microwave diminution"

→ In the hottest clusters relativistic corrections become important

→ SZ effect is small ($\Delta T_b/T_b \sim 10^{-3}$) $\Longrightarrow$ difficult to observe.

→ SZ effect independent of cluster distance (redshift) $\Longrightarrow$ detection of high-redshift clusters.

4.3 Observations of the Sunyaev-Zeldovich effect

→ Radio telescopes. BIMA radio telescope (30 GHz).

→ Brightness is independent of redshift. But not all high-redshift clusters can be resolved $\Longrightarrow Y$ parameter:

$$Y = \int y dA \propto \int n_e T_e dV, \tag{4.11}$$

where dA is projected cluster area element.

→ $Y \propto M_{gas} < T_e >$

4.4 Hubble constant with SZ effect

→ SZ + X-ray $\Longrightarrow$ independent estimate of distance $\Longrightarrow$ estimate of H_0 (derive). Absorption $A \propto n_e D_A \theta$, emission $E \propto n_e^2 D_A \theta$ $\Longrightarrow A^2/E \propto D_A \theta$ (θ angular size of cluster, D_A angular diameter distance).

→ Combining the SZ effect for a spherical cluster

$$\frac{\Delta T_b}{T_b} \propto T_e n_e \Delta l \sim T_e n_e D_A \theta \tag{4.12}$$

with the X-ray surface brightness is

$$S_X \propto \frac{n_e^2 T_e^{1/2} \Delta V}{D_L^2} \propto \frac{n_e^2 T_e^{1/2} \theta^3 D_A}{(1+z)^4} \tag{4.13}$$

[because $D_L = (1+z)^2 D_A$ and $\Delta V \propto \theta^3 D_A^3$], we get (derive)

$$D_A \propto \frac{(\Delta T_b)^2}{T_b^2} \frac{\theta}{S_X T_e^{3/2} (1+z)^4}. \tag{4.14}$$

→ at low redshift $D_A \simeq cz/H_0 \implies$ estimate of H_0. At higher redshift estimates of deceleration parameter q_0

4.5 Kinetic Sunyaev-Zeldovich effect

→ If a cluster is moving w.r.t. the CMB (or Hubble flow) $\implies$ additional spectral distortion of CMB.

→ If $\tau_T \ll 1 \implies$ brightness temperature change is almost independent of frequency

$$\frac{\Delta T_b}{T_b} = -\frac{v_r}{c}\tau_T, \tag{4.15}$$

where v_r is the radial component of the cluster peculiar velocity. Here $v_r > 0$ corresponds to a recession velocity exceeding that of the Hubble's law.

→ $v_r > 0 \implies$ negative temperature variation; $v_r < 0 \implies$ positive temperature variation;

→ Temperature change independent of $\nu \implies$ Planck spectrum.

→ Kinetic SZ effect much weaker than thermal SZ effect.