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INFLUENCE OF MECHANICAL YIELDING ON PREDICTIONS OF SATURATION: THE SATURATION LINE

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Abstract. It is now well accepted that the mechanical and the water retention behaviour of a soil under unsaturated conditions are coupled and, that such coupling, should be incorporated into a constitutive model for a realistic representation of soil's response. In existing models, the influence of the mechanical behaviour on the water retention is often represented by a shift of the main wetting retention curve to higher values of matric suction (the difference between pore air and pore water pressures) when the specific volume decreases. This means that any variation of total volumetric strains of compression (whether these are elastic or elasto-plastic) will result in a shift of the main wetting and drying curves to the right, when these curves are represented in the water retention plane. This shift of the main water retention curves, however, should not only influence the unsaturated stress states as often described in the literature, it should also have some impact on the saturated stress states and, more specifically, on the predictions of de-saturation (air-entry point) and saturation (air-exclusion point). From a modelling point of view, it is advantageous to represent this influence through the plastic component of volumetric strain of compression only because, in this way, a consistent representation of the mechanical behaviour for both unsaturated and saturated states can be naturally achieved. This and other advantages resulting from this singular approach are demonstrated in the paper in the context of the Glasgow Coupled Model (GCM).

1 INTRODUCTION

A major challenge of existing constitutive models for soils is the proper representation of transitions between unsaturated and saturated conditions. This challenge is intimately linked to: the incorporation of hysteresis in the water retention behaviour, the consideration of the influences between mechanical and water retention behaviour and the choice of stress state variables. As discussed in Gens [1], proper representation of these transitions is likely to be difficult in models that use net stress (excess of total stress over pore air pressure) and matric suction (difference between pore air pressure and pore water pressure) as stress state variables. This is because de-saturation during drying will not necessarily occur at zero suction and subsequent re-saturation on wetting will neither occur always at zero suction

(Lloret-Cabot, Wheeler and Sánchez [2]). The consequence of having saturated states at non-zero values of suction makes especially complicated the unification of the mechanical behaviour for unsaturated and saturated states when using net stress and matric suction as stress state variables, because net stress only reverts to the saturated effective stress tensor (the conventional stress state variable for the representation of the mechanical behaviour for saturated soils, Terzaghi [3]) when suction equals zero. This situation worsens when using a critical state framework (Roscoe and Burland [4]) as underlying formulation for the saturated conditions, because representations of elastic and plastic volumetric straining for unsaturated states will only converge to the saturated case for the single case of zero suction. These challenges are discussed in the paper in the context of the Glasgow Coupled Model, showing how to resolve them through the use of non-conventional stress state variables, proper consideration of water retention hysteresis and appropriate representation of the influences between mechanical and water retention behaviour.

2 THE GLASGOW COUPLED MODEL (GCM)

The Glasgow Coupled Model (GCM) is an elasto-plastic constitutive model to represent the mechanical and water retention behaviour in unsaturated soils. It was first presented for isotropic stress conditions by Wheeler et al. [5] and has been later extended to general stress states ([6]-[7]). The analysis presented in this paper is based on how this model represents unsaturated soil behaviour, with particular emphasis on the way the influence of mechanical behaviour on water retention is formulated within the model. It has been then considered convenient to discuss first its basic features. As the discussion is only a summary (limited to isotropic stress conditions) interested readers are referred to other publications of the authors ([8]-[10]).

2.1 State or constitutive stress variables

For the restricted range of stress states that apply in tests under isotropic stress conditions, it is sufficient to consider only the mean Bishop's stress p^* (sometimes called *average skeleton stress*, Jommi [11]) and the modified suction s^* defined as:

$$p^* = p - S_r u_w - (1 - S_r) u_a = \bar{p} + S_r s \tag{1}$$

$$s^* = n(u_a - u_w) = ns \tag{2}$$

where p is mean total stress, $\bar{p}$ is mean net stress, s is matric suction, u_a and u_w are the pore fluid pressures for air and water respectively, S_r is degree of saturation and n is porosity. The stress variables p^* and s^* are work-conjugate with volumetric strain increment $d\varepsilon_v$ and decrement of degree of saturation $-dS_r$ respectively [12]. As first suggested by Schrefler [13], Equation 1 has replaced the weighting factor χ proposed in the original Bishop's expression [14] by the degree of saturation.

2.2 Elastic behaviour

Elastic components of $d\varepsilon_v$ and $-dS_r$ are:

$$d\varepsilon_v^e = \frac{\kappa dp^*}{vp^*} \tag{3}$$

$$-dS_r^e = \frac{\kappa_s ds^*}{s^*} \tag{4}$$

where κ is the gradient of elastic swelling lines in the v: $\ln p^*$ plane (mechanical behaviour) and κ_s is the gradient of elastic scanning curves in the S_r: $\ln s^*$ plane (water retention behaviour).

2.3 Yield curves

To model isotropic stress conditions, the GCM includes three yield curves in the s^*:p^* plane (see Figure 1). A Mechanical yield curve (M) to represent mechanical behaviour and a Wetting Retention (WR) and Drying Retention (DR) yield curves to represent water retention behaviour. Plastic volumetric strains occur during yielding on M curve, whereas plastic variations of S_r occur during yielding on WR or DR curves. The M curve is the only one of the three describing mechanical yielding and this can occur during loading, wetting or drying as demonstrated in [8]. The other two (WR and DR) represent retention behaviour. Their respective equations are given by:

$$p_0^* - p^* = 0 \tag{5}$$

$$s_1^* - s^* = 0 \tag{6}$$

$$s^* - s_2^* = 0 \tag{7}$$

where p_0^*, s_1^* and s_2^* are hardening parameters defining the current positions of the M, WR and DR yield curves respectively (Figure 1).

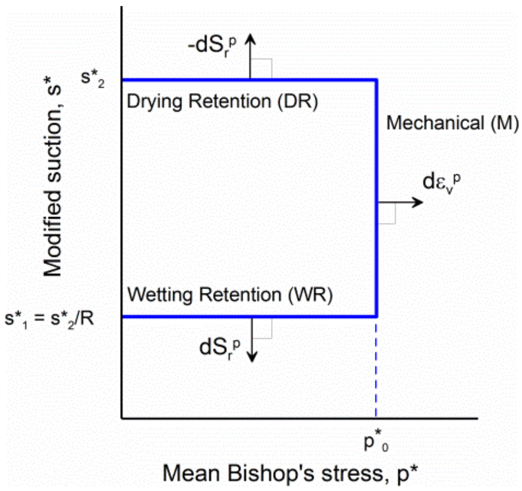


Figure 1: Yield curves for isotropic stress states (after [5])

2.4 Flow rules

Associated flow rules are assumed on all three yield curves. This means that yielding on the M curve alone corresponds to:

$$dS_r^p = 0 \qquad \text{and} \qquad d\varepsilon_v^p > 0 \tag{8}$$

Yielding on the WR curve alone corresponds to:

$$d\varepsilon_v^p = 0 \qquad \text{and} \qquad dS_r^p > 0 \tag{9}$$

and yielding on DR alone corresponds to:

$$d\varepsilon_v^p = 0 \qquad \text{and} \qquad dS_r^p < 0 \tag{10}$$

2.5 Hardening laws

The mechanical hardening law gives movements of the M curve and includes a direct component of movement caused by plastic volumetric strain (due to yielding on the M curve) and also a second (coupled) component of movement caused by any plastic changes of S_r due to yielding on WR or DR curves:

$$\frac{dp_0^*}{p_0^*} = \frac{v d\varepsilon_v^p}{\lambda - \kappa} - k_1 \frac{dS_r^p}{\lambda_s - \kappa_s} \tag{11}$$

where λ and κ are the gradients of normal compression and swelling lines respectively in the v: $\ln p^*$ plane for isotropic loading and unloading tests involving no plastic changes of S_r (such as the saturated tests), λ_s and κ_s are the gradients of main wetting/drying curves and scanning curves respectively in the plane S_r: $\ln s^*$ (see Figure 2a) for retention tests involving no plastic volumetric strains, and k_1 is a coupling parameter.

The hardening law for the water retention gives movements of the WR or DR yield curves and includes a direct component movement caused by plastic change of S_r (due to yielding on the WR or DR curve) and a second (coupled) component of movement caused by any plastic volumetric strains due to yielding on the M curve:

$$\frac{ds_1^*}{s_1^*} = \frac{ds_2^*}{s_2^*} = -\frac{dS_r^p}{\lambda_s - \kappa_s} + k_2 \frac{v d\varepsilon_v^p}{\lambda - \kappa} \tag{12}$$

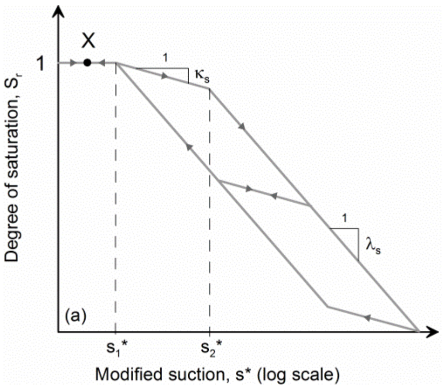
where k_2 is a second coupling parameter. Equation 12 ensures that the movements of the DR and WR yield curves are such that the ratio of s_2^* to s_1^* remains constant and equal to a soil constant R:

$$\frac{s_2^*}{s_1^*} = \mathrm{R} \tag{13}$$

The special cases of the hardening laws during yielding on only a single yield curve (M, WR or DR) are given by inserting the relevant condition from Equation 8, 9 or 10 ($dS_r^p = 0$ or $d\varepsilon_v^p = 0$) into Equations 11 and 12 [2].

Figure 2 illustrates how the GCM treats the saturated conditions. When the soil reaches

$S_r = 1$, further elastic increases of degree of saturation are prevented (Equation 4 no longer applies for decreases of s^*) and further plastic increases of degree of saturation are prevented ($d\varepsilon_v^p = dS_r^p = 0$ replaces Equation 9 for states on the WR yield curve alone). In addition, the consistency condition on the WR yield curve is removed, so that the stress state can pass beyond the WR curve [2]. Figure 2a shows water retention behaviour (with $d\varepsilon_v^p = 0$), including a saturated stress state X. Figure 2b shows the corresponding positions of the yield curves when the stresses are at X. While the soil remains saturated, the M yield curve is still operative, and Equation 11 (with $dS_r^p = 0$) reverts to the conventional Modified Cam Clay (MCC) hardening law [4], because $p^* = p'$, where p' is the saturated mean effective stress [3]. Also, while the soil remains saturated, Equation 12 (with $dS_r^p = 0$) is still used to determine coupled movements of the WR and DR curves caused by plastic (not total) volumetric strain [9]. As explained in further detail in [2], this represents changes of air-entry value caused by plastic volumetric strain.



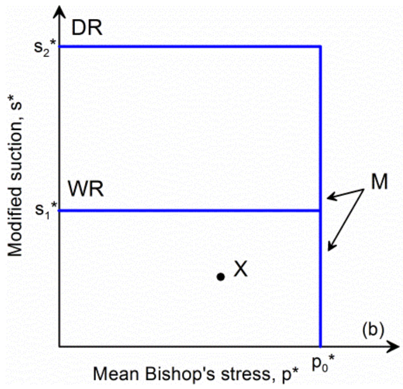


Figure 2: Modelling retention behaviour and treatment of saturated conditions [2]

3 MODELLING TRANSITIONS BETWEEN UNSATURATED AND SATURATED CONDITIONS

The GCM represents soil behaviour under both unsaturated (S_r < 1) and saturated conditions (S_r = 1). For S_r = 1, the GCM converges to the mechanical relationships of the MCC model for saturated soils because, in contrast to mean net stress, mean Bishop's stress p^* reverts to the saturated mean effective stress p' every time $S_r = 1$, even if $s \neq 0$.

Furthermore, under isotropic compression states on the M curve, as saturation progresses further, approaching S_r to 1, the GCM response for v should converge to the conventional Normal Compression Line, NCL:

$$v = \mathrm{N} - \lambda \ln p' \tag{14}$$

where λ and N are, respectively, the gradient and intercept of the saturated NCL in the v: $\ln p'$ plane. To satisfy this requirement with the formulation of the GCM it is necessary that $\kappa_s = 0$ [2]. This restriction on the value of κ_s is due to a small inconsistency in the model highlighted by Ravendiraraj [16] and is discussed in the following.

Figure 3 shows a wetting stress path ABC, followed by a loading-unloading cycle CDE (not plotted in the figure) while the soil is saturated and then a drying path EFG. If yielding on the M curve occurs during the loading-unloading cycle CDE, this will cause plastic volumetric strains while the soil is saturated (for simplicity, no plastic volumetric strains occur during either AB or FG, while the soil is unsaturated). Consequently, coupled movements of the WR and DR yield curves occur resulting in a translation of the water retention curves from the positions shown by fine continuous lines to those shown by the fine dashed lines. This means that, whereas the soil reaches a saturated state at a value of modified suction s_B^* during wetting, de-saturation occurs at a higher value of modified suction s_F^* during subsequent drying. Thus, elastic increases of S_r occur over the range of modified suction s_F^* to s_B^* during the wetting path (plastic changes of S_r also occur) but no elastic decreases of S_r occur between s_B^* and s_F^* during the drying path, which means that elastic changes of S_r have not been reversible over the range of modified suction s_B^* to s_F^*, and this contravenes a basic tenet of elastic behaviour [2]. A simple way to overcome this problem is by assuming $\kappa_s = 0$ and this is the assumption adopted for the remainder of the paper.

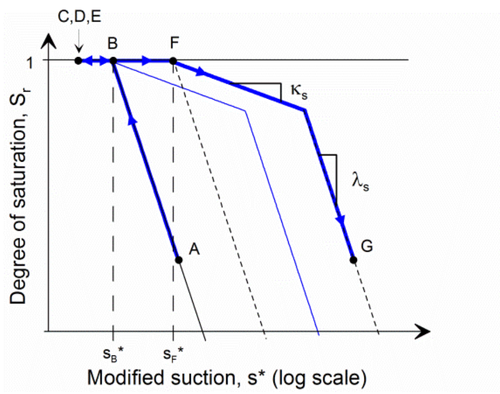


Figure 3: Demonstration of irreversible elastic changes of S_r if $\kappa_s > 0$ [2]

3.1 Isotropic normal compression planar surfaces for v and S_r

To show how the GCM handles the transition from unsaturated to saturated states, it is convenient to introduce first the work of Lloret [15] on the model predictions for isotropic stress states at the intersection of the mechanical (M) and wetting retention (WR) yield curves. This work shows that any stress paths involving mechanical yielding (occurrence of plastic compression) and wetting retention yielding (occurrence of plastic increases of S_r) correspond to this intersection of M and WR curves; and that, for these isotropic normal compression (NC) states at the intersection of M and WR curves, the GCM predicts unique expressions for specific volume v and degree of saturation S_r. The form of these unique expressions corresponds to two planar surfaces when plotted, respectively, in the v: $\ln p^*$: $\ln s^*$ and S_r: $\ln p^*$: $\ln s^*$ spaces. Their respective expressions are given by:

$$v = N^* - \lambda^* \ln p^* + k_1^* \ln s^* \tag{15}$$

$$S_r = \Omega^* - \lambda_s^* \ln s^* + k_2^* \ln p^* \tag{16}$$

where λ^*, k_1^*, λ_s^*, k_2^*, N^* and Ω^* are soil constants (see Appendix).

Lloret-Cabot, Wheeler and Sánchez [2] show that for the unsaturated NC planar surfaces for v (Equation 15) to converge to the saturated NCL (Equation 14) at $S_r = 1$, it is necessary that $\kappa_s = 0$.

3.2 Transitions from unsaturated to saturated conditions: the saturation line

With $\kappa_s = 0$, Figure 4a shows a three-dimensional view (in $v\!:\ln p^*:\ln s^*$ space) of the unsaturated isotropic NC planar surface for v (Equation 15) and the saturated isotropic NCL. The intersection of the two surfaces defines a "saturation line" (Figure 4a) corresponding to the transition from unsaturated to saturated conditions. Equivalent surfaces for S_r are illustrated in Figure 4b (in S_r: $\ln p^*:\ln s^*$ space), where the same saturation line is observed.

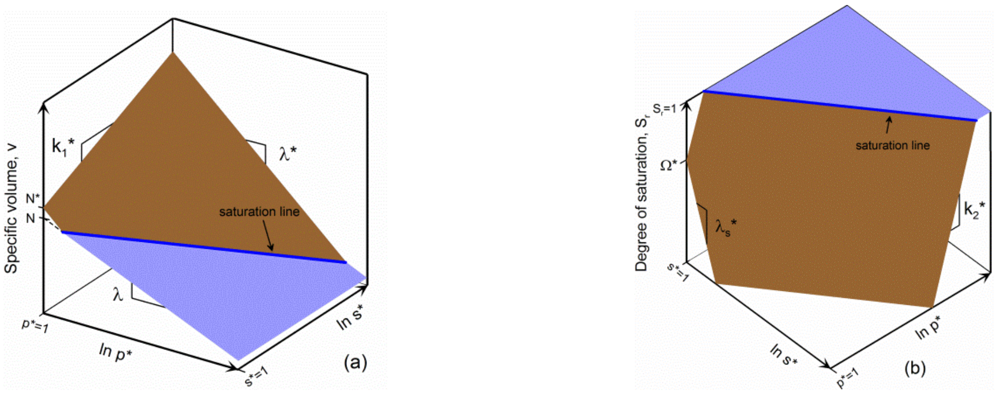


Figure 4: Isotropic NC planar surfaces for unsaturated and saturated conditions: (a) for v; (b) for S_r [2]

Lloret-Cabot et al. [2] demonstrate that the following expression for the saturation line shown in Figures 4a and 4b is obtained by using Equation 16 with $S_r = 1$ (and assuming $\kappa_s = 0$):

$$\ln s^* = \frac{\Omega^* - 1}{\lambda_s^*} + k_2 \ln p^* \tag{17}$$

Equation 17 represents the pairs of s^* and p^* at which transitions from unsaturated to saturated conditions will occur if the stress state is isotropic and at the intersection between M and WR yield curves. According to [2], with $\kappa_s = 0$, transitions from unsaturated to saturated conditions can only occur whilst on the WR yield curve, but it is not necessary for the stress state to be on the M yield curve or for the stress state to be isotropic at the point of transition from unsaturated to saturated conditions. Given that changes of p^* do not produce elastic changes of S_r, it is straightforward to derive a more general expression for transition from unsaturated to saturated conditions, applicable to any isotropic, including those not on the M yield curve:

$$s^* = \exp\left(\frac{\Omega^* - 1}{\lambda_s^*}\right)\left(p_0^*\right)^{k_2} \tag{18}$$

Equation 18 defines the expression for the saturation line, corresponding to transition from unsaturated to saturated conditions (sometimes known as the air-exclusion point), and is illustrated in Figure 5 (in both a log-log plot and a linear plot). Equation 18 and Figure 5 show that the saturation value of s^* is uniquely dependent on the position of the M yield surface (i.e. the value of p_0^*) [2].

3.3 Transitions from saturated to unsaturated conditions: the de-saturation line

Lloret-Cabot, Wheeler and Sánchez [2] demonstrate that transitions in the reverse direction, from saturated to unsaturated conditions, must occur on the DR yield surface if $\kappa_s = 0$, but it is not necessary for the stress state at the point of de-saturation to be on the M surface. This transition from saturated to unsaturated conditions occurs on a "de-saturation line" defined by:

$$s^* = \mathrm{R}\exp\left(\frac{\Omega^*-1}{\lambda_s^*}\right)\left(p_0^*\right)^{k_2} \tag{19}$$

Figure 5 illustrates that the GCM includes the influences of both water retention hysteresis and plastic volumetric straining on transitions between saturated and unsaturated conditions. The difference between the saturation and de-saturation values of s^* (at the same value of p_0^*) shows the influence of retention hysteresis, whereas the variation of both saturation and de-saturation values of s^* with p_0^* shows the influence of plastic (not total) volumetric strains on air-exclusion and air-entry points [2].

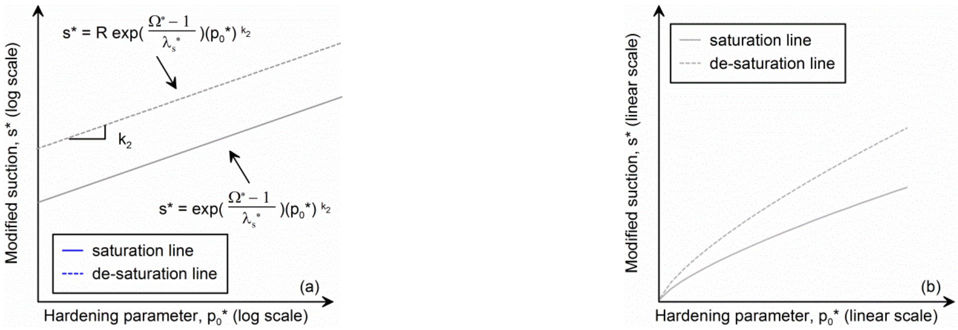


Figure 5: Predicted saturation and de-saturation lines [2]

4 CONCLUSIONS

- The Glasgow Coupled Model (GCM) predicts that isotropic normal compression states in isotropic stress paths involving plastic volumetric strains and plastic increases of S_r correspond to points at the intersection of M and WR yield curves. For these states, the model predicts unique unsaturated isotropic normal compression planar surfaces for v and for S_r (in v: $\ln p^* : \ln s^*$ and S_r: $\ln p^* : \ln s^*$ spaces, respectively).
- The GCM represents consistently the transitions between saturated and unsaturated states, including the influence of retention hysteresis and the effect of plastic

volumetric strains on water retention behaviour. The GCM gives unique expressions to predict saturation and de-saturation conditions (air-exclusion and air-entry points respectively), in the form of two unique straight lines in the $\ln s^* : \ln p_0^*$ plane.

APPENDIX

For cases with $\kappa_s = 0$, the gradients of the isotropic normal compression surface for v are given by:

$$\lambda^* = \frac{\lambda - k_1 k_2 \kappa}{1 - k_1 k_2} \tag{A1}$$

$$k_1^* = k_1 \frac{(\lambda - \kappa)}{(1 - k_1 k_2)} \tag{A2}$$

Similarly, the gradients for the isotropic normal compression surface for S_r are given by:

$$\lambda_s^* = \frac{\lambda_s}{1 - k_1 k_2} \tag{A3}$$

$$k_2^* = k_2 \frac{\lambda_s}{(1 - k_1 k_2)} \tag{A4}$$

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