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Comparison of Methods Analyzing Bifurcation and Hunting of Complex Rail Vehicle Models

The stability assessment is an important task in the mechanical design of railway vehicles. For a detailed model of a railway passenger coach, the hunting behavior depending on the running speed, on wheel-rail contact conditions, and on different model configurations is analyzed using two different methods: The path-following method based on a direct computation of limit cycles enables an automatic computation. However, due to the direct computation, which exploits the periodicity of the solution, this method is restricted to strictly periodic behavior. In the brute-force method, an initial disturbance limited to a certain time interval is applied to the model. This method allows the analysis of the behavior independently from the type of the solution, but requires manual intervention. The comparison of the results obtained with both methods shows a good agreement and thereby the reliability of the results and the methods. [DOI: 10.1115/1.4006825]

1 Introduction and Motivation

Investigations of hunting or instability of railway vehicles possessing wheelsets with conventional solid axles is an important topic of railway vehicle dynamics. The term hunting denotes a self-excited, sustained lateral and yaw motion of wheelsets and bogies, occasionally interconnected also with large carbody motion. The lowest speed, at which the bogie hunting appears, is called critical speed. The determination of the critical speed is an important task in the development and design process of railway vehicles.

If the vehicle runs faster than the critical speed, motions excited by disturbances may not always die out, but lead to permanent hunting motions of the vehicle. Although these motions can be stable in the mathematical sense, this behavior is called "unstable running." Since large hunting motions can lead to high lateral wheel-rail forces and thereby to risk of track shift, this behavior must be avoided in regular operation. In other words, the critical speed is limiting the operational running speed of the railway vehicle.

Theoretical investigations of railway vehicle stability started with studies founded on linearized models once the safety risk due to occurrence of hunting was recognized during the mid-twentieth century. At a later date, the models used have been enhanced considering the nonlinearities in the wheel-rail contact and other elements of the system vehicle-track. Moelle and Gasch [1] and True et al. [2–6] can be mentioned if only in a representative manner. True and his co-workers investigated various aspects of nonlinear railway vehicle stability analysis under application of the continuation-based bifurcation analysis (path-following method) developed by Kaas-Petersen [7].

From many other papers related to bifurcation of railway vehicles, we can mention the works by Stichel [8], Molatefi et al. [9] and Hoffmann [10], who investigated the stability of freight wagons on straight tracks, or stability analyses on curves by Zboinski and Dusza [11–13], respectively. Investigations of the influence of track elasticity on bifurcation and critical speed have been carried out by Kaiser and Popp [14] and by Zhai and Wang [15].

The publications dealing with nonlinear stability assessment of railway vehicles often investigate simplified models; either a single wheelset [16–18], or a half-vehicle model [1–7,9,19,20], respectively. A 2-axle wagon is used in papers [8,10–13]. The modeling of wheel-rail contact is sometimes simplified; e.g. the profile shape is reduced to a cone with a nonlinear stiffness representing the flange contact [2,5,7,16–21]. Published studies often use only theoretical, design profile shapes of wheels and rails [1,4,6,9,10,12–14,22]. Bifurcation analyses dealing with worn shapes of wheel and/or rail profile are rare [8,11].

The investigated systems vehicle-track often represents only a few degrees of freedom (DOF). The system used by Zboinski and Dusza [11–13] possesses 18 DOF, Wickens [23] applies a 21 DOF-system; Kim and Seok [24] investigate a 31 DOF-system. The bifurcation analysis is seldom carried out for models with large number of DOF, using a nonlinear wheel-rail contact model and real design as well as worn profile shapes as it is presented by one of the authors in Refs. [25–27]. Considering a complex multibody vehicle/track model including variations of wheel-rail contact geometries and friction conditions in railway service, the actual behavior of a railway vehicle at the stability limit can vary significantly depending on the nonlinear properties of the actual wheel-rail contact conditions as presented in Refs. [25,27].

The stability assessment in railway rolling stock industry requires the possibility to apply the stability analysis on large, complex vehicle models under realistic operating conditions. This results in a large number of computations considering various wheel-rail conditions [27]. The present article deals with an application of bifurcation analysis for the stability assessment of a complex railway vehicle model developed in the rolling stock industry. Two methods of bifurcation analysis are presented and their performance compared.

This article is structured as follows. The next section shows the nonlinearities of the system vehicle-track and their impact on the dynamic behavior of this system. Section 3 presents the methods used for bifurcation analysis of railway vehicles and explains more in detail both methods compared. The vehicle, wheel-rail contact conditions used and the results of comparisons are presented in Secs. 4, and 5 presents the conclusions of comparisons.

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2 Nonlinear Behavior of the Vehicle-Track System

The dynamic behavior of a railway vehicle is strongly influenced by nonlinear effects. The most influential nonlinearities are related to the wheel-rail contact, which is nonlinear in several aspects: The first nonlinearity is the relation between the lateral displacement of the wheelset in the track and the difference of the rolling radii of the wheels. This relation strongly influences the frequency of the hunting motion. The second nonlinearity is the relation between the creepage in the wheel-rail contact and the tangential forces, which are responsible for the guidance of the vehicle. This nonlinear relation results from the saturation characteristics, since the tangential forces cannot exceed the maximal friction force occurring for the case of sliding.

Although the nonlinear effects of the wheel-rail contact can usually be considered as the most influential, other nonlinear effects can be caused by certain elements used in the suspension of the vehicle. Examples are:

(1) Hydraulic dampers with blow-off valves limiting the damper force, e.g., used for the yaw damping
(2) Rubber bushings with nonlinear characteristics
(3) Bump stops, i.e., elements with clearance
(4) Suspension components with friction or friction yaw damping, respectively
(5) Geometrical nonlinearities, e.g., anti-roll device with inclined links

The behavior of a nonlinear system can be characterized by its attractors. In the case of a railway vehicle, only one attractor is desirable, namely the centered stationary running of the vehicle. All other attractors including permanent hunting motions must be considered to be potentially dangerous and thereby have to be avoided in regular operation. A generic scheme of the nonlinear behavior of a wheelset is displayed in Fig. 1, showing the maximum lateral displacement depending on the running speed.

The nonlinear behavior displayed in Fig. 1 already occurs for a single wheelset, which can be considered as a self-excited oscillator. However, real railway vehicles are far more complex: Even the simplest vehicle has two wheelsets, which of course interact with each other via the suspensions and the carbody. A railway passenger coach or a locomotive usually possesses four wheelsets, articulated vehicles like motorized units can have ten or even more wheelsets. Obviously, the analysis of a system consisting of several coupled self-excited subsystems and nonlinear coupling elements is challenging. It will also turn out that this structure of the system has an impact on the type of the occurring attractors. Furthermore, as already mentioned, elements used for the suspension can have nonlinear characteristics. The analysis of such a system becomes even more complicated due to the high number of parameters, which can be varied. Such parameters are, e.g., the profiles of the wheels and the rails, which have a strong influence on the rolling radii difference, the friction coefficient occurring in the wheel-rail contact, or parameters of the suspension components. The following section presents methods used for bifurcation analysis of railway vehicles.

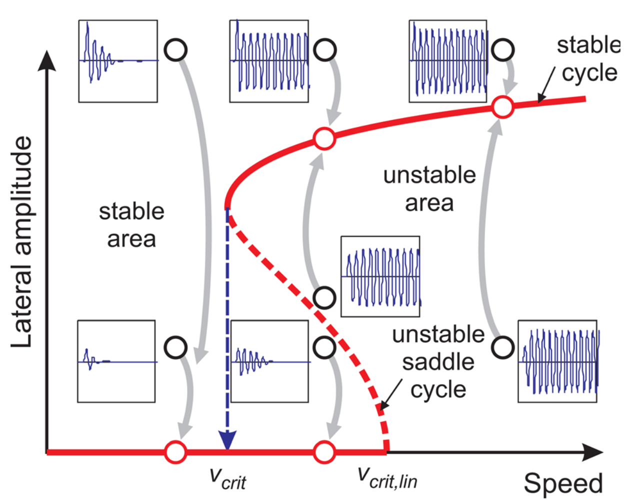

Fig. 1 Bifurcation diagram for a generic wheelset

3 Bifurcation Analysis Methods

The stability of a dynamical system is often investigated by linearizing the system equations with respect to a reference state and evaluating the eigenvalues of the linearized system. However, Fig. 1 shows that the lowest running speed v_{crit}, at which limit cycle oscillations can occur, is lower than the running speed $v_{\text{crit,lin}}$, at which the reference state loses its stability. Therefore, this stability assessment is not useful in this case.

An alternative is the application of the quasi-linearization. The basic idea of this method is to approximate the motion of the system by a periodic function with only one frequency. If the system contains a nonlinear element described by the nonlinear function $f(x)$, this function is approximated by the linear expression $k \cdot x$. Since the motion of the entire system is described by a periodic function with only one periodicity, the input x can be expressed by $x = \hat{x}\sin(\omega t + \beta)$, where $\hat{x}$ indicates an amplitude, which has to be chosen for the approximation. Then, the coefficient k is determined by an integral over a full period $0 \leq \phi \leq 2\pi$ of the input x:

$$k = \frac{1}{\pi \hat{x}} \int_0^{2\pi} f(\hat{x}\sin\phi)\sin\phi\, d\phi \tag{1}$$

In railway dynamics, this method is used for the linearization of the rolling radii function $\Delta r(y)$ depending on the lateral shift y of the wheelset, see, e.g., Ref. [28]. The resulting linear coefficient is known as the equivalent conicity. Usually, this is the only nonlinearity, which is treated that way, i.e., in most cases this method is not applied to the nonlinear relation between the creepage and the tangential forces or on other nonlinear components of the vehicle. The advantage is that the stability can be evaluated by the consideration of the eigenvalues of the linearized system. The disadvantage is that this method provides a comparatively rough approximation of the actual system behavior.

The method quasi-linearization, which uses a periodic function with only one frequency, can be extended by using a Fourier series for the approximation of the periodic motion. This leads to the Galerkin-Urabe method, which had been applied to railway vehicles and integrated into the software package MEDYNA by Moelle [29]. The time history of each DOF is approximated with a Fourier series, whereas the coefficients are determined by a minimization algorithm. The main disadvantage is the high order of the minimization problem requiring a high computational effort: If the system has n_{DOF} degrees of freedom and k harmonic waves are taken into account, the order is $n_{\text{DOF}} \cdot (2k+1)$. For a system having 20 DOFs, which is not a very high number, and for taking 5 harmonic waves into account, the algorithm has to solve a minimization problem of 220 variables.

Computations of bifurcation diagrams using commercial simulation tool are usually carried out applying numerical simulations called a brute-force method. The procedure can be a "ramping," which is simulation starting from a limit cycle at a high speed and reducing the speed very slowly until the oscillations stops [3]. An application example of a similar method is described by Stichel [8]. A run over an initial lateral disturbance is simulated at a rather high speed. The simulation continues on an undisturbed track until the oscillation of the vehicle has reached constant amplitude. The vehicle speed is then reduced and a new simulation is started with initial values from the previous simulation. This is repeated until the oscillating solution disappears.

Another variant of the brute-force method is a set of numerical simulations, in which the limit cycle oscillations occur as a result of initial conditions or lateral excitation in the initial section of track and the simulation continues on an ideal track. In all variants, it has to be checked for each result, whether all transient processes have died out and the attractor is reached. This method requires rather intensive manual work. It allows the determination of the dominating attractor with largest amplitudes. However, doubts about coexisting solutions, which were possibly not identified by this method, remain.

A more exact solution is provided by the path following method (continuation). A software tool PATH for the continuation-based bifurcation analysis has been developed at the Technical University of Denmark [7] and recently implemented into a commercial multibody simulation tool SIMPACK by Schupp [22].

While the path-following method has been used mostly with rather limited models and with non-commercial simulation computer codes, the rather straight forward method using a set of numerical simulations (brute-force method) is applicable in any multi-body simulation code including commercial tools. Both often referenced methods were compared by an example of a large nonlinear model of a railway vehicle using the simulation tool SIMPACK. The following subsections describe the compared methods more in detail.

3.1 Brute-Force Method. The application of the brute-force method requires nothing more than the existence of the solution of the equations of motion given by:

$$\dot{\mathbf{z}}(t) = \mathbf{f}(\mathbf{z}(t), t) \tag{2}$$

The basic idea is to apply a disturbance to excite the system. After the excitation has vanished, the system behavior moves towards the attractor. However, finding attractors requires interventions: It requires some experience to choose a disturbance, which will make the system reach the region of attraction of the wanted attractor, and also to choose a suitable interval of the integration process to make the transient motions die out. Whether the attractor is reached has to be checked visually from the diagrams.

The brute-force method used in the present study consists of a set of numerical simulations on ideal track, following a single lateral disturbance with a span of 10 m. The amplitude of the limit cycle or the absolute maximum displacement in case of quasi-periodic oscillations, respectively, is identified behind the transition, i.e., when the attractor is reached. A visual control is used to check if the transient motions died out. The experience shows that a simulation of a few seconds after the disturbance was usually sufficient. These numerical simulations were repeated for a set of speeds including those leading to limit cycle oscillations. At first, a large disturbance with an amplitude of 8 mm is used to identify the nonlinear critical speed. Then, a set of simulations with speed variation is repeated applying a small disturbance with an amplitude of 0.5 mm. If the solution without oscillations appears for speeds higher than the nonlinear critical speed, a set of simulations is carried out for each speed with a disturbance amplitude varying from small to large values.

A point of the unstable branch for the considered speed then lays between the value of the excitation amplitude leading to a stationary solution and the next higher amplitude leading to an oscillating solution.

The main part of the presented results has been calculated using the described procedure with manually started simulations. A semi-automatic bifurcation analysis has been developed and tested using the same procedure as described above with the simulations controlled by a steering code. This procedure allows an automatic calculation of bifurcation diagrams; however, due to many variations of possible shapes of bifurcation diagrams, manual adjustments are often required, too.

3.2 Path-Following Method. Using the path-following method, a difficulty related to the finding of an attractor is to determine whether the attractor is reached and all transient motions have died out. The version of PATH used by Schupp [22] solves this problem by using a direct calculation, which exploits the periodicity of the limit cycle.

Generally, the dynamics of the system is described by an initial value problem given by:

$$\dot{\mathbf{z}}(t) = \mathbf{f}(\mathbf{z}(t), t), \quad \mathbf{z}(t = t_0) = \mathbf{z}_0 \Rightarrow \mathbf{z} = \mathbf{z}(\mathbf{z}_0, t) \tag{3}$$

The first expressions are the equations of motion; $\mathbf{z}_0$ is the initial state. Therefore, the current state $\mathbf{z}$ of the system depends on the current time t and the initial state $\mathbf{z}_0$.

If the system performs periodic motions, its state at the beginning of a period is equal to the state after the period. If the duration of the period is T, this condition can be formulated by $\mathbf{z}(t) = \mathbf{z}(t + T)$. By applying this condition and setting $\mathbf{z}(t = 0) = \mathbf{z}_0$, the initial value problem is transformed into a boundary condition problem: A residuum function

$$\mathbf{r}(\mathbf{z}_0, T) = \mathbf{z}(\mathbf{z}_0, T) - \mathbf{z}_0 \Rightarrow \mathbf{r}(\mathbf{z}_0, T) = \mathbf{0} \tag{4}$$

depending on the initial state, $\mathbf{z}_0$ and the period T is defined. By setting the residuum function to $\mathbf{0}$ and solving this system of equations, $\mathbf{z}_0$ and T are obtained. The advantages of this direct calculation are the determination of the periodic attractor with defined accuracy and the avoidance of any transient effects. However, since this method exploits the periodicity of the solution, it can only be applied to periodic solutions. Other cases like quasi-periodic or chaotic motions cannot be handled.

Although the dynamical system can maintain its structure, its description has to be adapted for the direct calculation of the limit cycles. A railway vehicle moves along the track and covers an increasing distance, so that the longitudinal displacement $s(t)$ of the wheelsets, the bogie frames and the carbody increases permanently. Therefore, this motion cannot be periodic. To describe fluctuations of the longitudinal motion, this motion is expressed by a superposition of a reference motion with the constant running speed v_0 and a relative motion $\tilde{s}(t)$, which is the actual degree of freedom:

$$s(t) = s_0 + v_0 t + \tilde{s}(t) \Rightarrow \dot{s}(t) = v_0 + \dot{\tilde{s}}(t) \Rightarrow \ddot{s}(t) = \ddot{\tilde{s}}(t) \tag{5}$$

Using this description, the motion $\tilde{s}(t)$ can be periodic. In a similar way, the rolling motion of the wheelsets, which also increases, is formulated.

The equations of motion can depend on several parameters. In the case of a railway vehicle, an important parameter is the running speed v_0. For a given value $v_0^{(i)}$ of the running speed, the initial state $\mathbf{z}_0^{(i)}$ and the period duration $T^{(i)}$ fulfill the condition $\mathbf{r}(v_0^{(i)}, \mathbf{z}_0^{(i)}, T^{(i)}) = \mathbf{0}$. Based on the known solution $\mathbf{z}_0^{(i)}$ and $T^{(i)}$, the path-following method can determine the solutions $\mathbf{z}_0^{(i+1)}$ and $T^{(i+1)}$ for a new value $v_0^{(i+1)}$. The program PATH predicts initial guesses for $\mathbf{z}_0^{(i+1)}$ and $T^{(i+1)}$ and automatically adapts the step-length $\Delta v_0^{(i+1)} = v_0^{(i+1)} - v_0^{(i)}$ for the variation of the parameter.

4 Comparison of Bifurcation Analysis Methods

4.1 Vehicle Model. Both investigated methods of bifurcation analysis were applied on a model of a double-decker coach with 2 two-axle bogies developed in the rolling stock industry and used previously for bifurcation stability analysis by the brute-force method [28]. The model is displayed in Fig. 2.

The carbody of the vehicle is supported on the bogie frame by secondary suspension consisting of two air springs and an anti-roll device with inclined links. Lateral bump stops limit the lateral displacement between carbody and bogie frame. The dynamic

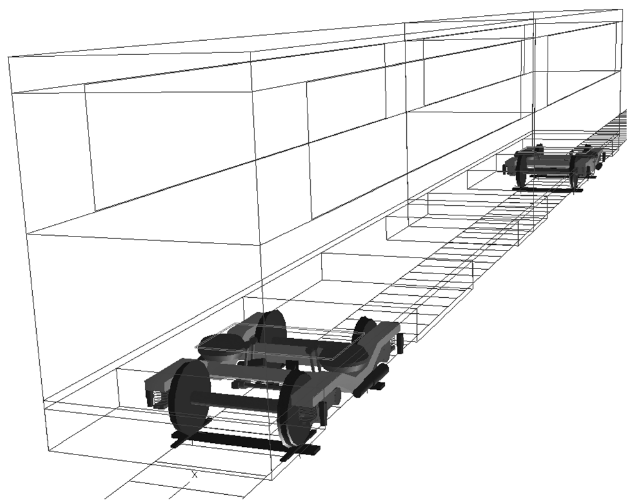

Fig. 2 Multi-body model of the double-decker coach

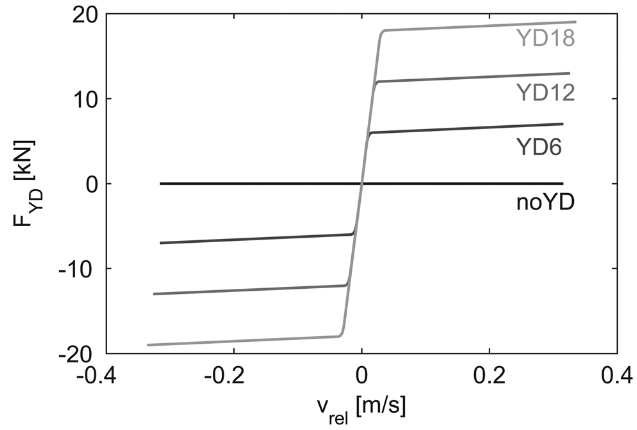

Fig. 3 Characteristics of the different yaw dampers used in the vehicle model

movements of carbody are damped by a secondary lateral damper on each bogie and in the vertical direction by orifice damping of air suspension. Two yaw dampers possess a strongly nonlinear characteristic due to a blow-off valve, which limits the damper force. Since the yaw dampers are a key component concerning the running stability of the vehicle, four variants of characteristics are studied: A configuration without any yaw dampers (noYD) and yaw dampers having a blow-off force of $F_{bo} = 6$ kN (YD6), $F_{bo} = 12$ kN (YD12) and $F_{bo} = 18$ kN (YD18). The characteristics are displayed in Fig. 3.

The wheelsets are guided against the bogie frame by axle boxes with swing arms, which are connected to the bogie frame by three-dimensional spring-damper elements representing bushings. Two helical primary springs on both sides of the axle box supports the bogie frame on the wheelsets. This three-dimensional, nonlinear vehicle model in SIMPACK possesses 72 DOFs.

To approximate certain structural deformations of the bogie frames and the carbody, these components consist of two bodies linked with a revolute joint and a stiff rotational spring. For the bogie frame, the revolute joint enables relative rotational motions around the lateral axis for an approximate description of twisting deformations. In the case of the carbody, relative rotations around the longitudinal axis are possible to approximate torsional deformations.

The wheelsets are considered as rigid bodies. Each wheelset is supported by a track element consisting of a rigid body, which can perform vertical, lateral and roll motions. The track element moves along the track together with the wheelset and is connected to the environment by linear springs and dampers with parameters according to ORE B 176 [30]. Thereby, the low-frequent dynamical behavior of the track is modeled.

The wheelset and the track element are connected by two standard SIMPACK wheel-rail force elements. These elements use a regularization for the determination of the contact point position on the profiles of wheel and rail. For the normal contact, a very stiff spring is used, which has a linear characteristic and sets the force to zero if no interpenetration of the profiles occurs. The tangential forces are calculated by the FASTSIM algorithm originally developed by Kalker. From the regularized analysis of the contact geometry, an equivalent elliptical contact patch is determined. In the present case, the friction coefficient is set to $\mu = 0.4$.

4.2 Calculation Results. The comparison of bifurcation analysis methods has been carried out for the vehicle with four differing yaw damper configurations and two variants of wheel-rail contact geometry. Both wheel-rail contact geometries (called 06A and 06B) achieve similar high levels of equivalent conicity and consequently a low critical speed. Although the equivalent conicity for the wheelset amplitude of 3 mm, which is usually considered in railway practice, is the same for both pairs of wheel and rail profiles, they possess significantly different contact conditions leading to opposite signs of the nonlinearity parameter of the contact geometry defined in Ref. [28].

The results in diagrams obtained with the brute-force method describe the highest amplitude of the lateral motion for the four wheelsets, i.e., the wheelset, at which the highest amplitude occurs, is considered. As mentioned before, the maximum amplitude is determined for the attractor, a periodic one as well as quasi-periodic one, after all transient motions have died out. These results are indicated by crosses ($\times$). In the case of the path-following method, the lateral amplitudes of all four wheelsets are displayed by lines. Generally, the thick lines indicate the leading bogie of the vehicle, the thin lines the trailing one. Black lines refer to the leading wheelset within a bogie, gray lines to the trailing one.

The comparisons of bifurcation analyses using brute-force and path-following method for wheel-rail contact geometry 06A are presented in Figs. 4–9. For the model variants noYD and YD6, a periodic attractor could only be found for the hunting of the front bogie. In this case, the front bogie performs large motions, while the motions of the rear bogie are small.

The results of the variant noYD are displayed in Fig. 4. The periodic attractor could be found in the range of $189\ \text{km/h} < v_0 < 228\ \text{km/h}$. For the variant YD6, the periodic

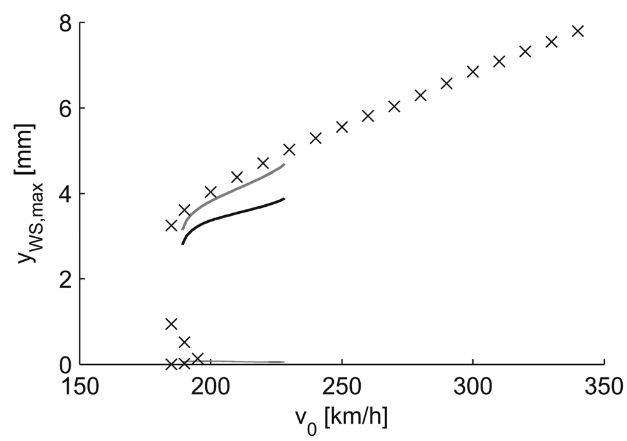

Fig. 4 Without yaw dampers (noYD), profiles 06A: Brute-force results versus path-following results for hunting of the front bogie

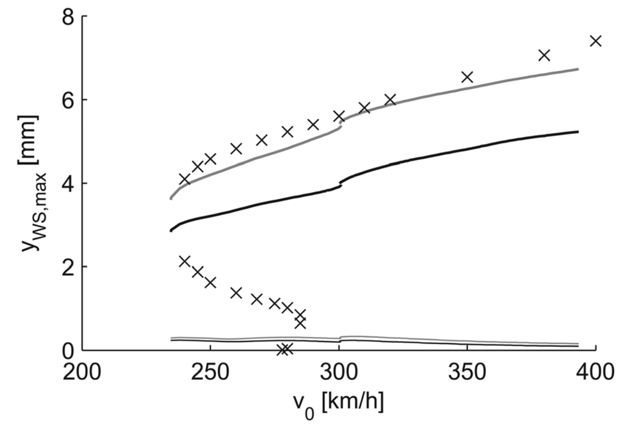

Fig. 5 Yaw dampers YD6, profiles 06A: Brute-force results versus path-following results for hunting of the front bogie

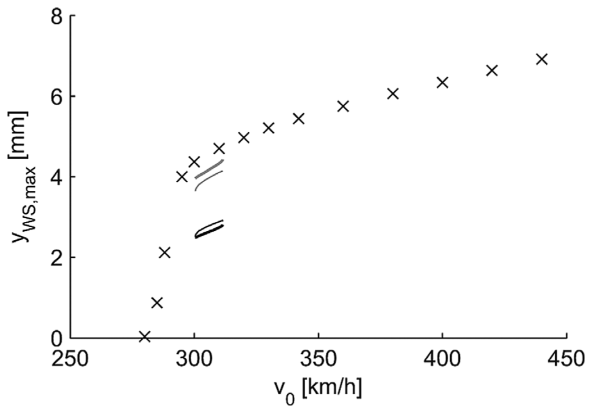

Fig. 6 Yaw dampers YD12, profiles 06A: Brute-force results versus path-following results for hunting of both bogies

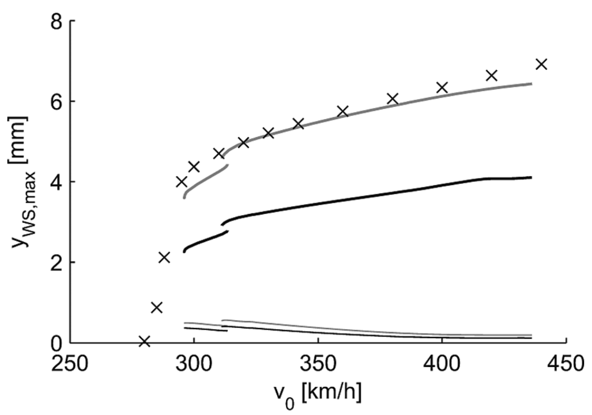

Fig. 7 Yaw dampers YD12, profiles 06A: Brute-force results versus path-following results for hunting of the front bogie

attractor exists in the range of $234\ \text{km/h} < v_0 < 394\ \text{km/h}$, as shown in Fig. 5.

For the variants YD12 and YD18, two coexisting limit cycle attractors could be found, the one describing the hunting of the front bogie, the other describing a hunting of both bogies. The results for the yaw damper YD12 are displayed in Figs. 6 and 7. Here, the periodic attractor describing the hunting of both bogies exists only in a comparatively short interval of $300\ \text{km/h} < v_0 < 312\ \text{km/h}$, while the attractor describing the hunting of the front bogie covers a wide range of $296\ \text{km/h} < v_0 < 436\ \text{km/h}$. The results for the yaw damper YD18 are displayed in Figs. 8 and 9. In this case, two isolated periodic attractors, which describe the hunting of both bogies, occur in the ranges of $297\ \text{km/h} < v_0 < 334\ \text{km/h}$ and $362\ \text{km/h} < v_0 < 389\ \text{km/h}$. Between those two ranges, quasi-periodic behavior occurs for the hunting of both bogies. The periodic attractor describing the hunting of the front bogie exists in the range of $296\ \text{km/h} < v_0 < 379\ \text{km/h}$. As the diagrams in Figs. 6–9 show, in both cases, the value for the critical speed determined by the path-following method is slightly higher than the one determined by the brute-force method.

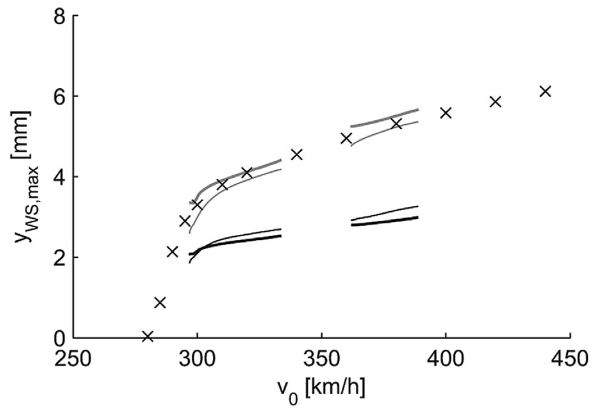

Fig. 8 Yaw dampers YD18, profiles 06A: Brute-force results versus path-following results for hunting of both bogies

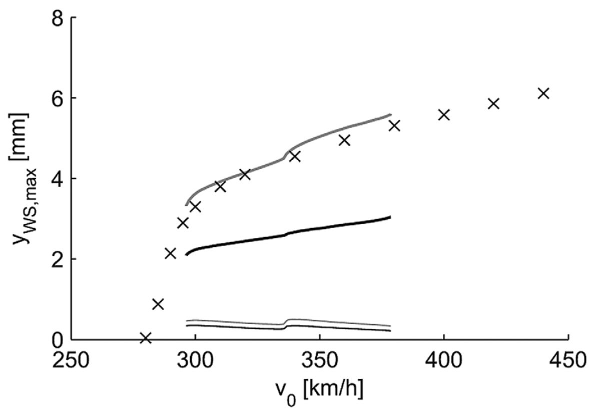

Fig. 9 Yaw dampers YD18, profiles 06A: Brute-force results versus path-following results for hunting of the front bogie

The comparisons of bifurcation analyses using brute-force and path-following method for wheel-rail contact geometry 06B are presented in Fig. 10 for the vehicle with yaw dampers YD12, as this is the only variant for which the attractors were identified using the path-following method. There is only a very short attractor at speed slightly below 200 km/h, so that the nonlinear critical speed could not be exactly identified. The brute-force method, however, shows bifurcation diagrams for all yaw damper variants as can be seen in Fig. 11.

The missing results of the path-following method can be explained by the quasi-periodic oscillations. The behavior of the vehicle without yaw dampers on the wheel-rail contact geometry 06A is exemplarily illustrated in Figs. 12–15 on phase diagrams of lateral displacement of wheelsets and carbody at vehicle speed of 250 km/h. While the behavior of the wheelsets is nearly periodic, the displacement of the carbody, although much smaller, shows quasi-periodicity. This can be seen in Fig. 15 displaying the phase diagram for the yaw motion of the carbody.

Similar behavior can be observed for the combination of the wheel-rail contact geometry 06B with the yaw damper YD12,

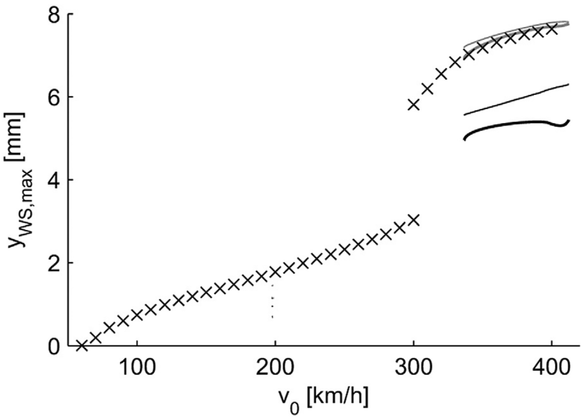

Fig. 10 Yaw dampers YD12, profiles 06B: Brute-force results versus path-following results for hunting of both bogies

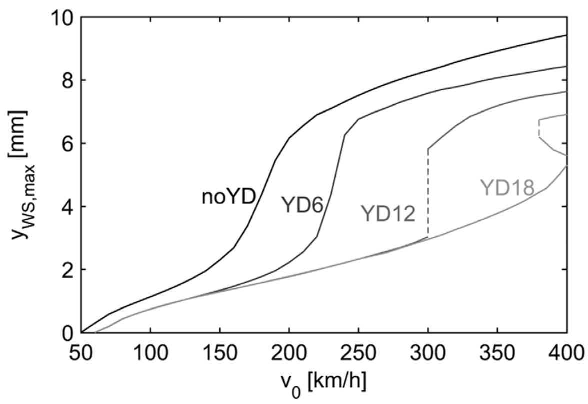

Fig. 11 All yaw damper variants, profiles 06B: Bifurcation curves obtained with the brute-force method

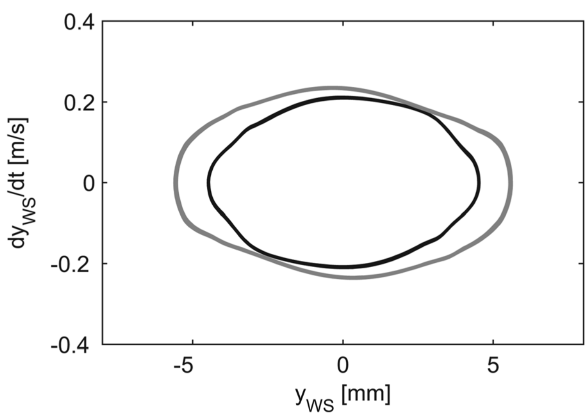

Fig. 12 Without yaw dampers (noYD), profiles 06A, $v_0 = 250$ km/h: Phase diagram for the lateral displacement of the wheelsets of the leading bogie

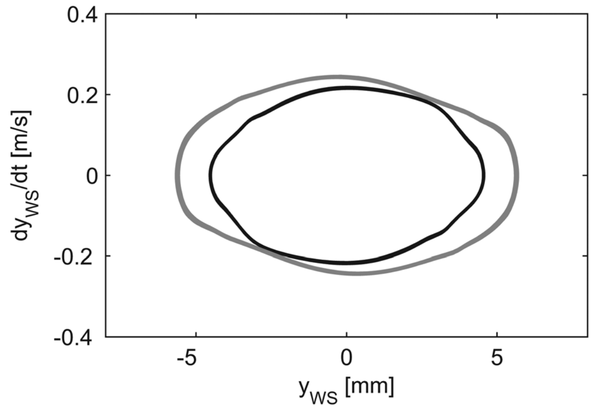

Fig. 13 Without yaw dampers (noYD), profiles 06A, $v_0 = 250$ km/h: Phase diagram for the lateral displacement of the wheelsets of the trailing bogie

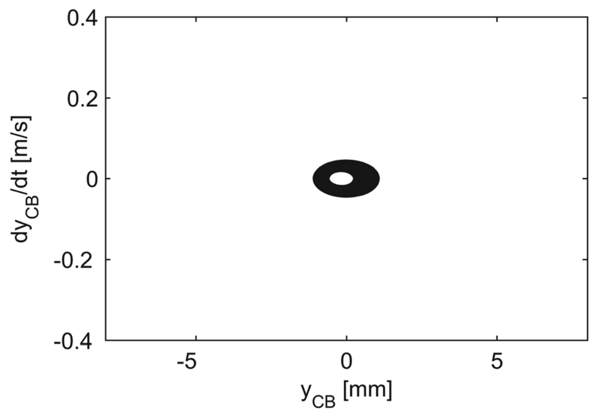

Fig. 14 Without yaw dampers (noYD), profiles 06A, $v_0 = 250$ km/h: Phase diagram for the lateral displacement of the carbody

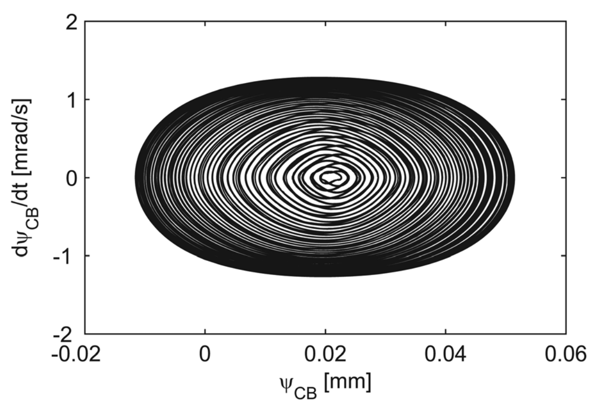

Fig. 15 Without yaw dampers (noYD), profiles 06A, $v_0 = 250$ km/h: Phase diagram for the yaw angle of the carbody

where only a very small attractor could be identified by the path-following method. Figure 16 displays the time history of lateral and yaw carbody displacement showing clear quasi-periodic behavior for the wheel-rail contact geometry 06B at a running speed of $v_0 = 250$ km/h. The phase diagrams of lateral displacement in Figs. 17–19 confirm a slightly quasi-periodic motion of wheelsets of the second bogie and clearly quasi-periodic behavior of carbody, which explains the failure of the path following method in the presented conditions.

The distinctive quasi-periodic behavior for the motions of the carbody and the nearly periodic motions of the wheelsets suggest that the carbody is excited by the motions of the bogies, whereas the frequencies of the bogies are not identical, but differ slightly. Furthermore, it is noticeable that periodic behavior for the variants noYD and YD6 combined with the wheel-rail contact geometry 06A can only be found if only one bogie is performing large oscillations and the other one only small motions. A periodic attractor describing a hunting of both bogies only occurs for the variants

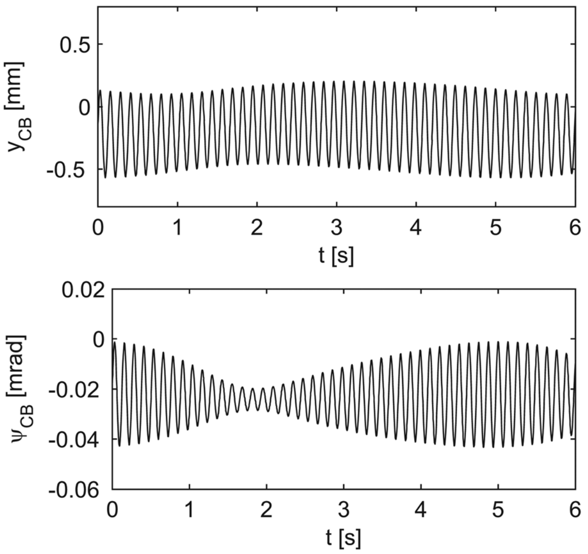

Fig. 16 Yaw dampers YD12, profiles 06B, $v_0 = 250$ km/h: Time histories for the lateral displacement y_{CB} and yaw angle ψ_{CB} of the carbody

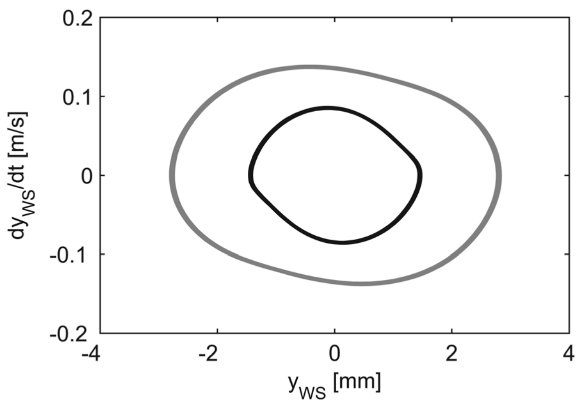

Fig. 17 Yaw dampers YD12, profiles 06B, $v_0 = 250$ km/h: Phase diagram for the lateral displacement of the wheelsets of the leading bogie

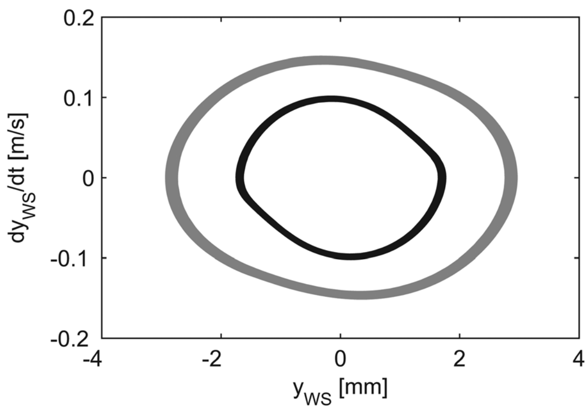

Fig. 18 Yaw dampers YD12, profiles 06B, $v_0 = 250$ km/h: Phase diagram for the lateral displacement of the wheelsets of the trailing bogie

YD12 and YD18. In these cases, the coupling between the bogies is stronger, which leads to a stronger interaction of the two bogies. Apparently, the railway vehicle consists of two subsystems, which have to be synchronized to generate periodic motions of the entire system. Each bogie consists of a frame and two wheelsets, which are connected by the comparatively stiff elements of the primary suspension, while the secondary suspension acting between the bogie frame and the carbody is softer. Therefore, each bogie can be considered as a subsystem. The mechanism of the synchronization is worth a separate study going deeper into details. The investigation described above shows, that a synchronization of two bogies can already be difficult. For more complex multi-body systems, e.g., vehicles with more than two bogies like articulated motorized units or vehicles with more complex mechanical couplings of bodies than conventional vehicles, it can be even more difficult to achieve this synchronization. Therefore, the probability of periodic oscillations of such systems can be assessed as very low.

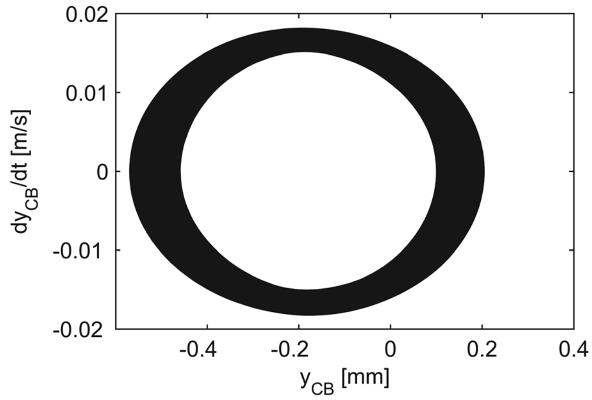

Fig. 19 Yaw dampers YD12, profiles 06B, $v_0 = 250$ km/h: Phase diagram for the lateral displacement of the carbody

5 Conclusions

For the analysis of the nonlinear hunting behavior of a railway vehicle, two methods have been compared, the brute-force method and the path-following method. The result of the comparison can be summarized as follows:

The investigation shows that the path-following method based on a direct calculation of periodic motions can be applied even to complex multi-body models, which are common in railway dynamics. Therefore, the high order of the system, in the present case 72 DOFs, is not a handicap.

The path-following method provides exact results for periodic motions. However, the quasi-periodic motions cannot be handled, since the path-following method exploits the periodicity of the motions for the direct calculation. Such quasi-periodic solutions have appeared at a conventional vehicle with two bogies due to interaction between the bogie motions via the carbody. Even wider ranges of speeds with quasi-periodic solutions can be expected at models of more complex vehicles, which would restrain the application of this method on such large models. Furthermore, the path-following method does not allow a computation of unstable attractors. The use of this method allows the automation of the calculation; however, a reliable application on complex models is not possible. The treatment of quasi-periodic motions requires new methods as, e.g., given in Ref. [31]. However, in this work, the method is applied to a comparatively small system, so the application to systems with high order like a railway vehicle is still challenging.

The brute-force method provides the shape of bifurcation diagram, although with lesser accuracy, independently of vehicle model complexity and fulfills the needs of railway engineering. It also allows an assessment of an unstable attractor. The provided solution with the largest amplitude is the most relevant solution from the point of view of industrial application. It is in a sufficient agreement with the solutions obtained by the path-following method. A reliable automation of the brute-force method is difficult; manual work is required due to a large variation of possible bifurcation diagrams.

It can be concluded that the application of bifurcation analysis for assessment of stability and hunting behavior of large railway vehicle models, as used today during the development and engineering of railway rolling stock, requires tedious and time consuming investigations; an automation as used for other kinds of multi-body simulations is hardly possible. The presented study demonstrates that a research on an automated and reliable bifurcation analysis applicable for complex railway vehicle models remains topical.

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