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Modeling of Flexible Multibody Systems Excited by Moving Loads with Application to a Robotic Portal System

Stefan Hartweg and Andreas Heckmann

Department of System Dynamics and Control
Robotics and Mechatronics Center (RMC)
German Aerospace Center (DLR)
Muenchner Strasse 20, D-82234 Wessling, Germany
[Stefan.Hartweg, Andreas.Heckmann]@DLR.de

ABSTRACT

In this paper, an approach to solve mechanical problems with moving loads and flexible bodies is presented. Therefore, the multi-disciplinary modeling language Modelica together with the DLR FlexibleBodies Library is utilized. Rather than solving the equations of motion analytically, this approach solves them numerically. While large overall motions of the flexible parts are described using the Floating Frame of Reference (FFR) method, the deformation is represented by appropriate shape functions. It is shown how a subsystem with masses and spring-damper elements can be connected to the flexible body and how the occurring forces of this moving load affect it. The equations of motion are derived in a general way, yielding a formulation compatible to e. g. a finite element models which are reduced to global shape functions. The usage of this formulation is shown for the simulation of a robotic portal system.

1 INTRODUCTION

A variety of structures in the field of engineering is subject to moving loads. These loads can result from external forces, masses or whole subsystems moving with constant or varying velocity along the structure. One of the classical examples of these structures are railway bridges. A modern example is a robotic portal system. In such a system, large parts, e. g. blades for wind energy plants or carbon-fiber parts are manufactured automatically by several robots moving along a portal structure.

For vibration analysis and control of processes which rely on high accuracy, being able to calculate a structure's reaction to these moving loads is crucial. While the analytical solution for several special cases is described by Fryba [2] and other authors, loads changing arbitrarily both in position and magnitude require numerical calculation. For an overview on many problem classes of moving load problems see [6].

This paper will illustrate how a numerical solution for the class of mechanical systems with moving loads can be obtained. It is organized as follows. In Section 2, the equations of motion for a flexible body without (2.1) and with (2.2) continuously moving attached subsystems are derived. The theory described here is used in an illustrative example in Chapter 2.3. Section 3 presents simulation results for the application of this theory to a real-world example. While this theory can be in general applied to every kind of mechanical structure, it will be shown for a system containing beams. The application considered here is a typical process within a robotic portal system including continuously moving portal systems. It is part of the DLR Center for Lightweight Construction, a research facility for the production of carbon-fibre reinforced composites.

2 MATHEMATICAL MODELING

In this chapter the governing equations of a flexible body and a connected subsystem will be derived. The first part in Section 2.1 shows the derivation as it has been published by Wallrapp in [7] and is now implemented in the DLR FlexibleBodies Library[4] using Modelica[5]. This allows to connect subsystems to a flexible body at fixed positions. The second part in Section 2.2 derives the kinematics for the general case of a moving subsystem. These kinematics are used to calculate the forces and torques acting on the flexible body. The derived equations are implemented as an extension to the FlexibleBodies library and validated using an example system.

2.1 Kinematics of the flexible body

A possible approach to derive the equations of motion for a flexible body is the Floating Frame of Reference (FFR) formulation. Every position vector of a particle of the flexible body is described with respect to a frame of reference, (*R*), as

$$\boldsymbol{r}(\boldsymbol{c},t) = \boldsymbol{r}_R(t) + \boldsymbol{c} + \boldsymbol{u}(\boldsymbol{c},t), \tag{1}$$

see Figure 1. The vector $\boldsymbol{r}_R$ describes the position of (*R*) and thereby the large overall motion of the whole body, $\boldsymbol{c}$ is the position vector in undeformed state and $\boldsymbol{u}$ describes the deformation. Variables with index $_R$ denote in the following a quantity related to the movement of the floating frame of reference.

The translational and rotational velocities $\boldsymbol{v}, \boldsymbol{\omega}$ and accelerations $\boldsymbol{a}, \boldsymbol{\alpha}$ of a mass particle of the flexible body are then derived by differentiation. This leads to the following kinematics of a frame locally fixed to the deforming body at point $\boldsymbol{c}$,

$$\begin{aligned}
\boldsymbol{v} &= \frac{d}{dt}(\boldsymbol{r}) = \boldsymbol{v}_R + \tilde{\boldsymbol{\omega}}_R(\boldsymbol{c}+\boldsymbol{u}) + \dot{\boldsymbol{u}}, && (2)\\
\boldsymbol{\omega} &= \boldsymbol{\omega}_R + \mathbf{w}(\boldsymbol{c},t), && (3)\\
\boldsymbol{a} &= \boldsymbol{a}_R + (\dot{\tilde{\boldsymbol{\omega}}}_R + \tilde{\boldsymbol{\omega}}_R\tilde{\boldsymbol{\omega}}_R)(\boldsymbol{c}+\boldsymbol{u}) + 2\tilde{\boldsymbol{\omega}}_R\dot{\boldsymbol{u}} + \ddot{\boldsymbol{u}}, && (4)\\
\boldsymbol{\alpha} &= \dot{\boldsymbol{\omega}}_R + \dot{\mathbf{w}}(\boldsymbol{c},t) + \tilde{\boldsymbol{\omega}}_R\mathbf{w}(\boldsymbol{c},t). && (5)
\end{aligned}$$

Here, $\mathbf{w}$ is the rotation at the point $\boldsymbol{c}$. The $\tilde{}$ operator on a vector $\boldsymbol{\omega}$ is a skew-symmetric matrix such that the identity $\boldsymbol{\omega} \times \boldsymbol{c} = \tilde{\boldsymbol{\omega}}\boldsymbol{c}$ holds. It is used to express the vector product using a matrix multiplication. All variables are resolved with respect to the floating frame of reference, (R).

The vectors $\boldsymbol{u}(\boldsymbol{c},t)$ and $\mathbf{w}(\boldsymbol{c},t)$ are approximated by first order Taylor expansions,

$$\begin{aligned}
\boldsymbol{u}(\boldsymbol{c},t) &= \boldsymbol{\Phi}(\boldsymbol{c})\boldsymbol{q}(t), && (6)\\
\mathbf{w}(\boldsymbol{c},t) &= \boldsymbol{\Psi}(\boldsymbol{c})\dot{\boldsymbol{q}}(t), && (7)
\end{aligned}$$

where $\boldsymbol{\Phi}(\boldsymbol{c})$ and $\boldsymbol{\Psi}(\boldsymbol{c})$ contain the values of shape functions evaluated at the point $\boldsymbol{c}$. These shape functions have to be chosen in such a way that they represent the possible deformations as well as the body's boundary conditions. $\boldsymbol{q}(t)$ is the vector of the time-dependent modal coordinates.

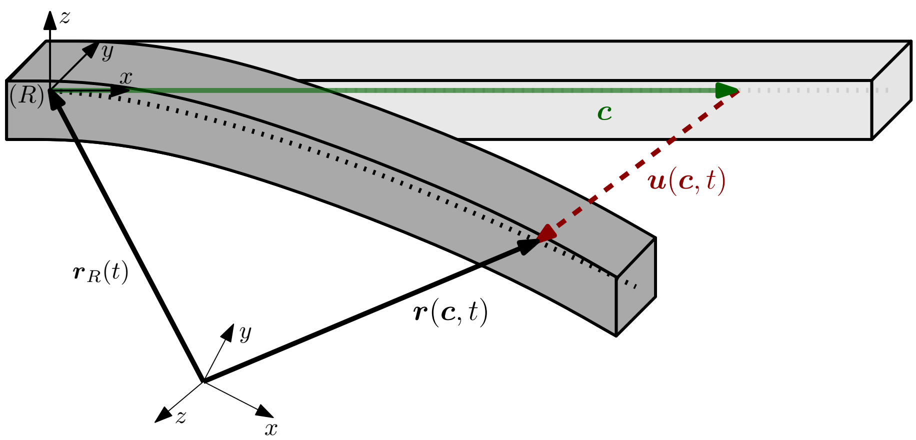

Figure 1. Vector notation for a point on a flexible body under deformation. (R) denotes the floating frame of reference.

The equations of motion can then be derived by the kinematics calculated in this chapter. This is done by utilizing the principle of virtual power. This means that the variation of the sum of the virtual power of inertia, internal and external forces has to sum up to 0,

$$\delta\boldsymbol{\Pi}_m + \delta\boldsymbol{\Pi}_i + \delta\boldsymbol{\Pi}_e = 0. \tag{8}$$

For an in-depth treatise of the principle of virtual power and a derivation of $\delta\boldsymbol{\Pi}_m, \delta\boldsymbol{\Pi}_i$ and $\delta\boldsymbol{\Pi}_e$, see [7].

Inserting the kinematic expressions into the variational terms, the following equations of motion of an unconstrained elastic body are obtained.

$$\begin{bmatrix} m\boldsymbol{I} & m\tilde{\boldsymbol{d}}_{CM}^T & \boldsymbol{C}_t^T(\boldsymbol{q}) \\ m\tilde{\boldsymbol{d}}_{CM}^T & \boldsymbol{J}(\boldsymbol{q}) & \boldsymbol{C}_r^T(\boldsymbol{q}) \\ \boldsymbol{C}_t(\boldsymbol{q}) & \boldsymbol{C}_r(\boldsymbol{q}) & \boldsymbol{M}_e \end{bmatrix} \begin{bmatrix} \boldsymbol{a}_R \\ \dot{\boldsymbol{\omega}}_R \\ \ddot{\boldsymbol{q}} \end{bmatrix} = \boldsymbol{h}_\omega - \begin{bmatrix} 0 \\ 0 \\ \boldsymbol{K}_e\boldsymbol{q} + \boldsymbol{D}_e\dot{\boldsymbol{q}} \end{bmatrix} + \boldsymbol{h}_e, \tag{9}$$

where

m	body mass
$\boldsymbol{I}$	3x3 identity matrix
$\boldsymbol{d}_{CM}$	position of center of mass
$\boldsymbol{C}_t$	elasticity-translation coupling matrix
$\boldsymbol{C}_r$	elasticity-rotation coupling matrix
$\boldsymbol{J}$	mass moment of inertia of body
$\boldsymbol{M}_e$	modal mass matrix
$\boldsymbol{K}_e$	modal stiffness matrix
$\boldsymbol{D}_e$	modal damping matrix
$\boldsymbol{h}_\omega$	gyroscopic and centripetal forces
$\boldsymbol{h}_e$	external forces

The vector of external forces $\boldsymbol{h}_e$ will be examined in more detail in the next chapter showing how systems connected to the flexible body affect it.

2.2 Kinematics of a Connected Subsystem

The Eqs. (1) to (7) hold for a mass point of the flexible body as well as for subsystems rigidly connected to it. If the subsystem can move along the flexible body, additional derivatives occur. This section will show the generalization of (1) to (7) for bodies attached to the flexible body in a non-fixed way. However, these subsystems cannot move unconstrained in space. The underlying assumption is that the subsystem slides along the surface of the deformed flexible body with neither separation nor penetration. To distinguish to the constant coordinate $\boldsymbol{c}$, the coordinate along the moving body will be denoted with $\boldsymbol{s}(t)$ in this section.

The position vector of such a subsystem can then be expressed as

$$\boldsymbol{r}^s(\boldsymbol{s}, t) = \boldsymbol{r}_R(t) + \boldsymbol{s}(t) + \boldsymbol{u}(\boldsymbol{s}(t), t). \tag{10}$$

To distinguish between the kinematics of the flexible body and the subsystem, variables associated with the latter one will be denoted by an upper index ^{s}. This leads to the kinematics of the subsystem

$$\begin{aligned} \boldsymbol{v}^s &= \frac{d}{dt}(\boldsymbol{r}^s) = \tilde{\boldsymbol{\omega}}_R(\boldsymbol{s} + \boldsymbol{u}) + \boldsymbol{v}_R + \dot{\boldsymbol{u}} \quad \underbrace{+\dot{\boldsymbol{s}}}_{\text{due to } s=s(t)}, && (11) \\ \boldsymbol{\omega}^s &= \boldsymbol{\omega}_R + \mathbf{w}(\boldsymbol{s}, t), && (12) \\ \boldsymbol{a}^s &= \boldsymbol{a}_R + (\dot{\tilde{\boldsymbol{\omega}}}_R + \tilde{\boldsymbol{\omega}}_R\tilde{\boldsymbol{\omega}}_R)(\boldsymbol{s} + \boldsymbol{u}) + 2\tilde{\boldsymbol{\omega}}_R\dot{\boldsymbol{u}} + \ddot{\boldsymbol{u}} \underbrace{+ 2\tilde{\boldsymbol{\omega}}_R\dot{\boldsymbol{s}} + \ddot{\boldsymbol{s}}}_{\text{due to } s=s(t)}, && (13) \\ \boldsymbol{\alpha}^s &= \dot{\boldsymbol{\omega}}_R + \dot{\mathbf{w}}(\boldsymbol{s}, t) + \tilde{\boldsymbol{\omega}}_R\mathbf{w}(\boldsymbol{s}, t). && (14) \end{aligned}$$

The vector $\boldsymbol{u}(t)$ is again obtained by a first order Taylor expansion,

$$\boldsymbol{u}(t) = \boldsymbol{\Phi}(\boldsymbol{s}(t))\boldsymbol{q}(t). \tag{15}$$

The $\boldsymbol{\Phi}(\boldsymbol{s}(t))$ are the known modal shape functions which only depend on $\boldsymbol{s}$ and not (explicitly) on time. Deriving this term and using the chain rule leads to the following material derivatives with respect to time:

$$\dot{\boldsymbol{u}}^s = \left(\sum_{i=1}^{3} \frac{\partial \boldsymbol{\Phi}}{\partial s_i}\frac{\partial s_i}{\partial t}\right)\boldsymbol{q} + \boldsymbol{\Phi}\dot{\boldsymbol{q}} = \underbrace{(\nabla_{\boldsymbol{s}}(\boldsymbol{\Phi}\boldsymbol{q}))}_{=:\boldsymbol{J_T}(\boldsymbol{s},\boldsymbol{q})}\dot{\boldsymbol{s}} + \boldsymbol{\Phi}\dot{\boldsymbol{q}} = \underbrace{\boldsymbol{J_T}\dot{\boldsymbol{s}}}_{\text{convective}} + \underbrace{\boldsymbol{\Phi}\dot{\boldsymbol{q}}}_{\text{local}}, \tag{16}$$

$$\ddot{\boldsymbol{u}}^s = \underbrace{[\nabla_{\boldsymbol{s}}((\nabla_{\boldsymbol{s}}(\boldsymbol{\Phi}\boldsymbol{q}))\dot{\boldsymbol{s}}) + 2\nabla_{\boldsymbol{s}}(\boldsymbol{\Phi}\dot{\boldsymbol{q}})]}_{=:\dot{\boldsymbol{J}}_{\boldsymbol{T}}(\boldsymbol{s},\dot{\boldsymbol{s}},\boldsymbol{q},\dot{\boldsymbol{q}})}\dot{\boldsymbol{s}} + \boldsymbol{J_T}\ddot{\boldsymbol{s}} + \boldsymbol{\Phi}\ddot{\boldsymbol{q}}(t), \tag{17}$$

where the matrices $\boldsymbol{J_T}(\boldsymbol{s},\boldsymbol{q})$ and $\dot{\boldsymbol{J}}_{\boldsymbol{T}}(\boldsymbol{s},\dot{\boldsymbol{s}},\boldsymbol{q},\dot{\boldsymbol{q}})$ are the Jacobian matrix of translation and its derivative form. Note that $\dot{\boldsymbol{J}}_{\boldsymbol{T}}$ is not simply calculated as a time derivative of the entries of matrix $\boldsymbol{J_T}$, but is a separate matrix.

The convective terms, this means parts containing derivatives of $\boldsymbol{s}$, can also initiate change of $\boldsymbol{u}$ with respect to time even if the vector of modal coordinates, $\boldsymbol{q}$, remains constant (cp. to the formulation of Eulerian equations and substantial derivatives in fluid dynamics, e. g. [1], p.85). Changes in $\boldsymbol{u}^s$ result in this case only from movement along the deformed shape itself rather than due to change of deformation.

For the rotational quantities it follows from a first order Taylor expansion of the angle, $\boldsymbol{\alpha}(t) = \boldsymbol{\Psi}(\boldsymbol{s}(t))\boldsymbol{q}(t)$ the derivative form

$$\mathbf{w}^s = \left(\sum_{i=1}^{3} \frac{\partial \boldsymbol{\Psi}}{\partial s_i}\frac{\partial s_i}{\partial t}\right)\boldsymbol{q} + \boldsymbol{\Psi}\dot{\boldsymbol{q}} = \underbrace{(\nabla_{\boldsymbol{s}}(\boldsymbol{\Psi}\boldsymbol{q}))}_{=:\boldsymbol{J_R}(\boldsymbol{s},\boldsymbol{q})}\dot{\boldsymbol{s}} + \boldsymbol{\Psi}\dot{\boldsymbol{q}} = \underbrace{\boldsymbol{J_R}\dot{\boldsymbol{s}}}_{\text{convective}} + \underbrace{\boldsymbol{\Psi}\dot{\boldsymbol{q}}}_{\text{local}}, \tag{18}$$

$$\dot{\mathbf{w}}^s = \underbrace{[\nabla_{\boldsymbol{s}}((\nabla_{\boldsymbol{s}}(\boldsymbol{\Psi}\boldsymbol{q}))\dot{\boldsymbol{s}}) + 2\nabla_{\boldsymbol{s}}(\boldsymbol{\Psi}\dot{\boldsymbol{q}})]}_{=:\dot{\boldsymbol{J}}_{\boldsymbol{R}}(\boldsymbol{s},\dot{\boldsymbol{s}},\boldsymbol{q},\dot{\boldsymbol{q}})}\dot{\boldsymbol{s}} + \boldsymbol{J_R}\ddot{\boldsymbol{s}} + \boldsymbol{\Psi}\ddot{\boldsymbol{q}}(t). \tag{19}$$

Note: In contrast to most literature, e. g. [7], $\boldsymbol{\Psi}$ is defined by an angle and not by the (rotational) velocity, i. e. $\mathbf{w}^s \neq \boldsymbol{\Psi}(\boldsymbol{s},t)\dot{\boldsymbol{q}}(t)$. This is only true for constant $\boldsymbol{s}$ like described in Section 2.1.

Inserting (16) to (19) into (13) and (14), the translational and rotational acceleration of an arbitrary system moving along the flexible body can be described by

$$\begin{aligned}
\boldsymbol{a}^s(\boldsymbol{s},\dot{\boldsymbol{s}},\ddot{\boldsymbol{s}},\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) =& \boldsymbol{a}_R + (\dot{\tilde{\boldsymbol{\omega}}}_R + \tilde{\boldsymbol{\omega}}_R\tilde{\boldsymbol{\omega}}_R)(\boldsymbol{s} + \boldsymbol{\Phi}(\boldsymbol{s})\boldsymbol{q}) \\
&+2\tilde{\boldsymbol{\omega}}_R(\boldsymbol{J}_T\dot{\boldsymbol{s}} + \boldsymbol{\Phi}(\boldsymbol{s})\dot{\boldsymbol{q}}) + \dot{\boldsymbol{J}}_T\dot{\boldsymbol{s}} + \boldsymbol{J}_T\ddot{\boldsymbol{s}} + \boldsymbol{\Phi}\ddot{\boldsymbol{q}} + 2\tilde{\boldsymbol{\omega}}_R\dot{\boldsymbol{s}} + \ddot{\boldsymbol{s}} \\
=& \underbrace{\boldsymbol{a}_R + (\dot{\tilde{\boldsymbol{\omega}}}_R + \tilde{\boldsymbol{\omega}}_R\tilde{\boldsymbol{\omega}}_R)(\boldsymbol{s} + \boldsymbol{\Phi}\boldsymbol{q}) + \boldsymbol{\Phi}\ddot{\boldsymbol{q}} + 2\tilde{\boldsymbol{\omega}}_R\boldsymbol{\Phi}\dot{\boldsymbol{q}}}_{\text{independent of change in s}} \\
&+ \underbrace{(\boldsymbol{J}_T + \boldsymbol{I})\ddot{\boldsymbol{s}} + [2\tilde{\omega}_R(\boldsymbol{J}_T + \boldsymbol{I}) + \dot{\boldsymbol{J}}_T]\dot{\boldsymbol{s}}}_{\text{dependent of change in s}}, \\
\boldsymbol{\alpha}^s(\boldsymbol{s},t) =& \dot{\boldsymbol{\omega}}_R(t) + \dot{\boldsymbol{J}}_R\dot{\boldsymbol{s}} + \boldsymbol{J_R}\ddot{\boldsymbol{s}} + \boldsymbol{\Psi}\ddot{\boldsymbol{q}}(t) + \tilde{\boldsymbol{\omega}}_R(\boldsymbol{J_R}\dot{\boldsymbol{s}} + \boldsymbol{\Psi}\dot{\boldsymbol{q}}).
\end{aligned} \tag{20}$$

According to [7] and taking the movement of contact forces into account, the external generalized forces acting on the flexible body can be formulated as

$$\boldsymbol{h}_e = \begin{bmatrix} m\boldsymbol{I} \\ m\tilde{\boldsymbol{d}}_{CM}(\boldsymbol{q}) \\ \boldsymbol{C}_t(\boldsymbol{q}) \end{bmatrix} \boldsymbol{g} + \oint_S \begin{bmatrix} \boldsymbol{I} \\ \tilde{\boldsymbol{d}}(\boldsymbol{q}) \\ \boldsymbol{\Phi}^T(\boldsymbol{s}) \end{bmatrix} \boldsymbol{f} dS + \oint_S \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I} \\ \boldsymbol{\Psi}^T(\boldsymbol{s}) \end{bmatrix} \boldsymbol{t} dS, \tag{21}$$

where $\oint_S$ is a surface integral over the flexible body, $\boldsymbol{d} = \boldsymbol{c} + \boldsymbol{u}$ and the other quantities are defined as in (9).

2.3 An Example System

As a simple illustrative example, (20) is utilized for a point mass moving along a deformed beam with supported boundary conditions at both ends. The beam is fixed at its frame of reference, leading to $\boldsymbol{a}_R = \boldsymbol{0}$,

$\boldsymbol{\omega}_R = \mathbf{0}$. This corresponds to the definition of a 'chord' frame configuration according to [3]. The beam is modeled as an Euler-beam which results in a one-dimensional model with the vector $\boldsymbol{s}$ being a scalar describing the position along the x-direction of the beam, see Figure 2. This means that $\ddot{s}$ is the acceleration in x-direction. Only deformation in one direction, the local z-direction is allowed. For a flexible body

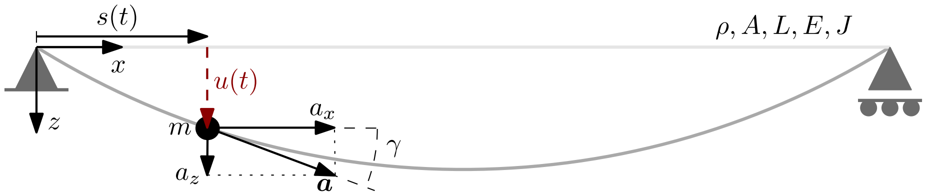

Figure 2. Description of the position of a mass attached to a beam

constrained in this way, (9) is simplified to the classical structural equation

$$\boldsymbol{M}_e\ddot{\boldsymbol{q}} + \boldsymbol{D}_e\dot{\boldsymbol{q}} + \boldsymbol{K}_e\boldsymbol{q} = \boldsymbol{f}_q, \tag{22}$$

with the vector of generalized applied forces $\boldsymbol{f}_q$.

The first step is to determine the matrices on the left side of the equation by choosing suitable shape functions. For the sake of simplicity only one mode shape function is considered in this example. A possible mode shape function of a beam with length L fulfilling the supported-supported boundary condition is

$$\Phi(x) = k\sin(\frac{\pi}{L}x), x \in [0, L], k \in \mathbb{R} \tag{23}$$

with an arbitrary factor k. This factor can be chosen in a way that the associated mass matrix turns out to be the identity matrix. This eigenmode belongs to the lowest eigenfrequency of the beam, taking the shape of exactly one half-period over the length of the beam.

The mass matrix's entries $m_{i,j}$ are a function of the shape functions. In this example there is only Φ_1, leading to

$$\begin{aligned} m_{1,1} &= \int_V \rho\Phi_1\Phi_1 dV = \int_V \rho k^2 \sin^2(\frac{\pi}{L}x)dV = k^2\rho A \int_0^L \sin^2(\frac{\pi}{L}x)dx \\ &= k^2\rho A\frac{1}{2}L. \end{aligned} \tag{24}$$

The factor k is still arbitrary, but with the demand $m_{1,1} = 1$ it turns out to $k = \sqrt{\frac{2}{\rho AL}}$. The value of this shape function at the mass position $x = s(t)$ is thus

$$\Phi(s(t)) = k\sin(\frac{\pi}{L}s(t)). \tag{25}$$

The second step towards the equations of motion is the determination of the generalized external forces acting on the beam. The rotational kinematics are not relevant because no external torque is present. To calculate the external forces, the acceleration of the attached mass in z-direction is required in order to determine its inertia forces. The translational acceleration of the mass is obtained by evaluating (20) and setting all variables associated with movement of the FFR (index $_R$) to zero:

$$a_z^s = \dot{J}_T\dot{s} + J_T\ddot{s} + \Phi(s)\ddot{q} \tag{26}$$

The other matrices in (16) and (17) are then determined as

$$\boldsymbol{J}_T = (\nabla_{\boldsymbol{s}}(\Phi q))^T = \frac{\partial}{\partial s}(\Phi q) = k\frac{\pi}{L}\cos(\frac{\pi}{L}s(t))q, \tag{27}$$

$$\dot{\boldsymbol{J}}_T = \frac{\partial}{\partial s}(\frac{\partial}{\partial s}(\Phi q)\dot{s}(t)) + 2\frac{\partial}{\partial s}(\Phi\dot{q}) = -\frac{\pi^2}{L^2}k\sin(\frac{\pi}{L}s)q\dot{s} + 2\frac{\pi}{L}k\cos(\frac{\pi}{L}s)\dot{q}. \tag{28}$$

With the acceleration a_z^s described, it is possible to specify the vector of external forces $\boldsymbol{f}_q$ in z-direction, which has the scalar form

$$f_q = C_t(q)g_0 + \Phi(s,q)f + \Psi(s,q)t = C_t(q)g_0 + \Phi(s)(-ma_z + mg_0). \tag{29}$$

Here, $C_t(q)g_0$ is the gravitational force acting on the mass of the beam, while $(-ma_z + mg_0)$ is the inertia and gravitational force of the attached body. Under the assumption that the mass moves without friction along the deformed surface of the beam, the s-acceleration of the attached body with mass m is determined by

$$a_s = \tan(\Psi q)(a_z - g_0) + \frac{f_{\text{ext}}}{m}, \tag{30}$$

with the standard gravitational acceleration g_0, external forces f_{ext} tangent to the deformed surface (here zero) and the body's mass m. Here, Ψq is the deformation angle at $s(t)$, whereas Ψ is defined as the spatial derivative of Φ, i. e. $\Psi = \frac{\partial \Phi}{\partial s}$.

The following parameters have been selected for the simulation: The beam has a rectangular cross-section area with $0.1\,\text{m}$ size, leading to an area $A = 0.01\,\text{m}^2$ and inertia $J = 8.33 * 10^{-6}\,\text{m}^4$, while the beam's length is $L = 15\,\text{m}$. The material parameters are density $\rho = 7850\,\frac{\text{kg}}{\text{m}^3}$, Young's modulus E = $2.1 * 10^{11}\,\frac{\text{N}}{\text{m}^2}$ and modal damping $D = 2\%$. The attached body has a mass of 100 kg and is initialized with $s = 2\,\text{m}$ and $\dot{s} = 0\,\frac{\text{m}}{\text{s}}$. The beam's deformation is initialized with $q = 0$ and $\dot{q} = 0$.

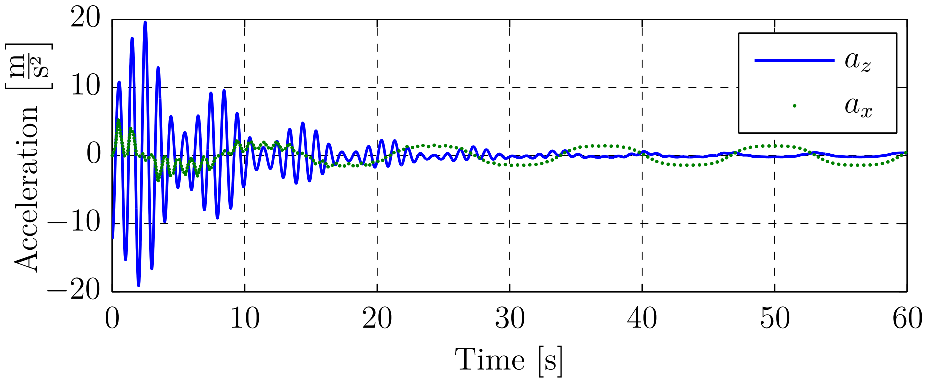

Figure 3. Acceleration of the attached body in x and z direction

Figure 3 shows the resulting accelerations of the mass in x- and z-direction. The acceleration of up to 20 $\frac{\text{m}}{\text{s}^2}$ corresponds to around two times the statical gravitational acceleration. In the long turn, the gravitational acceleration of 9.81 $\frac{\text{m}}{\text{s}^2}$ is much higher than the dynamic acceleration a_z which results mainly from the convective terms. The x-acceleration a_x changes periodically, indicating a sliding of the mass along the deformed beam. The beam's deformation changes only slightly after $t = 40\,\text{s}$ at times when the mass' acceleration reaches its limits, this means when the mass is in the center of the beam. Due to the negligible effects of structural damping at such low accelerations of the beam, the whole systems is weakly damped.

3 APPLICATION IN A ROBOTIC PORTAL SYSTEM

3.1 Robotic Portal System

As an application of the presented theory, a typical process within the robotic portal system of the DLR Center for Lightweight Production Technology is considered, see Figure 4. It is a multi-robot production facility to handle and place carbon-fibre material. Its dimensions are about 30 m in x-, 15 m in y- and 7 m in z-direction. The basis of the structure consists of three steel beams supported by concrete columns. It supports three portal robots and two industrial robots on a linear axis, enabling them to move along the middle beam. The portal robots have six, the industrial robots have seven degrees of freedom, including the linear axis.

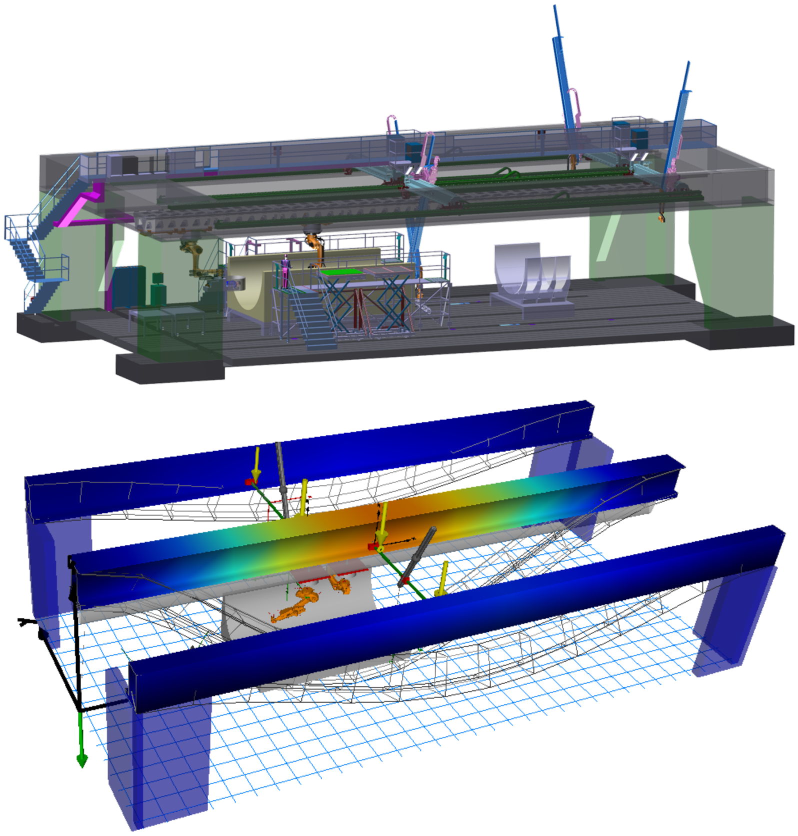

Figure 4. Robotic portal system as part of the DLR Center for Lightweight Production Technology with the dimensions of ca. 30 m x 15 m x 7 m in construction view and as model representation

Due to the high masses of the moving parts, the whole portal deforms and furthermore changes its vibrational behavior. The occurring deformation of the supporting structure reaches up to 4 mm. The acceleration of the moving parts and thereby their inertia forces are limited in a first testing phase. The long-term aim is to decrease the cycle time by increasing the accelerations, leading to higher loads on the structure and higher deformations in the facility.

3.2 Simulation Results

The structure is simplified to a multiple beam system, the robots are modeled as controlled multibody systems. The beams are modeled as Euler beams, resulting in easily obtained shape functions. The three flexible beams are modeled with five bending and one torsional mode each. The whole model has 162 degrees of freedom. Of these, 36 result from the 18 elastic states and their derivatives.

The scenario used to apply the presented modeling approach is the placement of carbon-fibre material by four cooperating robots. The robots accelerate along the structure to pick up the material and place it in a form, interrupted by time used for the process itself, this means there is no movement. In Figure 4, the resulting deformation in the structure consisting of displacement and torsion of the beam model is shown. The amplitude of the deformation is visualized by color as well as through the by factor 10000 exaggerated deformed shape, seen as a mesh in the figure. The whole simulation takes 25.9 s to simulate until $t = 120$ s.

The important stages of lifting and positioning the material are denoted in Figure 5 by (1) and (2), respectively. The process itself ends at time (3), when the material is fixed and the robot can move away again. Figure 6 shows the positioning error at the tool center point of one of the industrial robots. Dynamic movements and their inertia forces result in peaks in the error diagram. In the stages when the manipulators do not

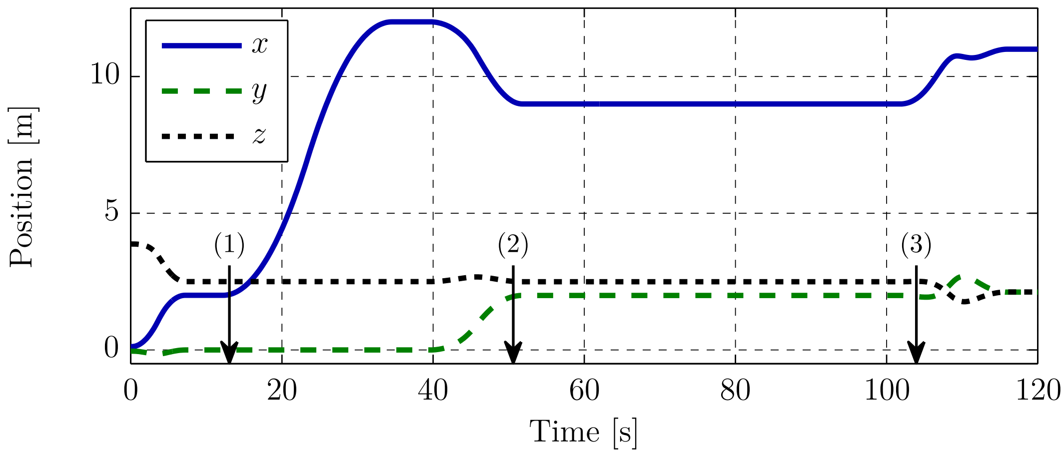

Figure 5. Position of the tool center point

move but a process takes place, the accuracy is much higher, but still non-zero due to the static deformation of the beams.

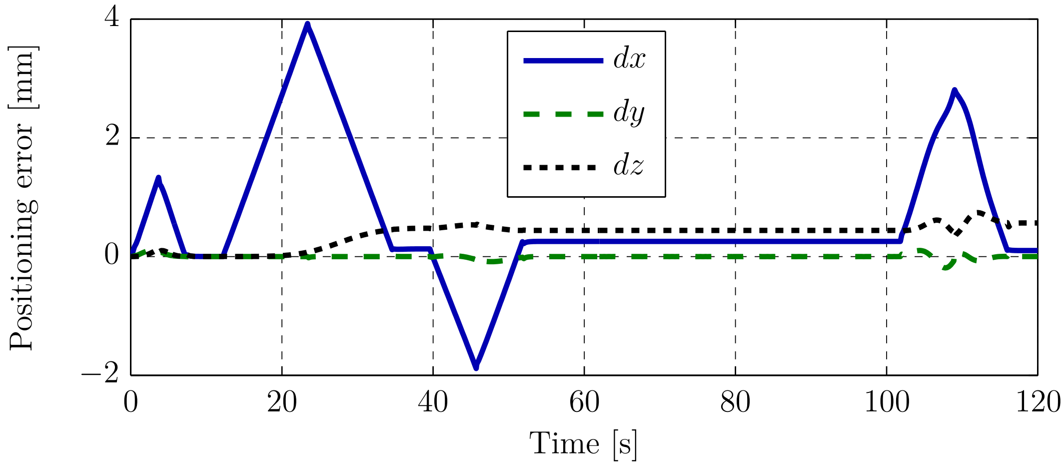

Figure 6. Difference between planned and actual trajectory of the tool center point

4 CONCLUSIONS

In this paper a formulation of a flexible multibody system with attached, moving subsystems is derived. The application of this formulation is shown in an illustrative as well as in a real-world example. The increased insight of such a system obtained by the model can be used e. g. for dimensioning of certain parts or for the design of a suitable controller. The usage is shown by analysis of simulation results. The theory will furthermore be verified against measurements of the real system in the future.

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