Time Interference Alignment via Delay Offset for Long Delay Networks

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Abstract— Time Interference Alignment is a flavor of Interference Alignment that increases the network capacity by suitably staggering the transmission delays of the senders. In this work the analysis of the existing literature is generalized and the focus is on the computation of the degrees of freedom for networks with randomly placed users in a n-dimensional Euclidean space. In the basic case without coordination among the transmitters analytical expressions of the sum degrees of freedom (dof) can be derived. If the transmit delays are coordinated, in 20% of the cases time Interference Alignment yields additional dof with respect to orthogonal access schemes. The potential capacity improvements for satellite networks are also investigated.

I. INTRODUCTION

The concept of Interference Alignment (IA) has aroused quite significant interest in the recent past for its ability to achieve a first order approximation of the multiuser communication capacity in some important network configurations [1], [2]. The core idea is to describe the signal as an element of a suitable space and divide this set into a desired subspace (where the intended signal should lie) and confine all interference into an interference subspace. In principle, the desired subspace is interference free and thus the capacity can grow as the Signal to Noise Ratio (SNR) increases. It can be shown that the number of dof of the network (by other words, the scaling coefficient of the sum rate)[1] may be significantly beyond 1, i.e., the number of dof when the resources are orthogonally allocated, as for instance in TDMA [1].

Most studies on IA focus on MIMO based systems, since the transmitted and received signals are obviously complex vectors with as many components as the number of antennas [3]–[7]. Interference Alignment with MIMO has achieved some degree of maturity, with results on the capacity and feasibility of IA [1], [2], [6], [8], on signal processing for IA [3]–[5] or even testbeds [7], [9].

Another interesting but not as much studied approach is time based IA via delay offset (here named simply time IA for the sake of brevity) [1], [10]–[13], where long propagation delays are exploited. Such scenario can be relevant for instance in satellite or underwater networks, whose propagation delays

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[1] The dof for a network and a certain transmission scheme are defined as:

$$dof = \lim_{P \to +\infty} \frac{C(P)}{\log(P)}, \qquad (1)$$

where P is the transmit power and $C(P)$ is the sum rate of the network.

are intrinsically very large. The reference scenario is the K-user interference channel (see Fig. 1), where K transmitters communicate with a dedicated receiver (one per sender) and all nodes will be assumed to have just one antenna. The key necessary property for time IA is that the difference of the propagation delays between transmit-receiver pairs are comparable to or larger than the packet durations. It was already noted in the seminal paper [1] that if the difference between propagation delays is large, it is in principle possible to perform IA by overlapping the undesired transmissions, so as to have a non zero amount of time devoid of interference for the desired signal. Indeed, if $K = 3$ and each sender transmits for $\rho = 50\%$ of the time, $K\rho = 1.5$ dof are attainable under particular conditions. We shall name "time IA" this attempt to reduce the portion of time occupied by the interference through mutual coordination of the transmitters.

It is arguable that when time IA is meaningful, this method may show some advantages with respect to other types of IA, like MIMO IA. For instance, the transmitters need to know the propagation delays rather than the complex gain of a fading channel, and normally the former can be estimated quite accurately for instance by means of a GPS receiver and may not be so time varying as a fading channel. Moreover, propagation delay variations can be partly predicted if information on the user speed is available, which is not possible for many other forms of IA. Hence, it may be speculated, although yet to prove, that time IA is more robust than other implementations of IA.

By perfect IA it is meant that at every receiver the total time where interference is present is equal to the duration of a single transmission (see Fig. 1 for an example). The work of [11], [12] has shown that in $\mathbb{R}^n$ it is possible to deploy $n + 1$ pairs which achieve perfect time IA and each couple transmits $\rho = 50\%$ of the time, if transmitters are synchronized and start transmitting at the same time instant. The sum rate will then scale with $(n+1)\rho \log_2(1+\Lambda)$, where Λ is the SNR (assumed to be equal for all pairs). Note that the capacity of orthogonal access schemes would grow with $\log_2(1 + \Lambda)$. Although it is not known how many pairs can be placed in $\mathbb{R}^n$ if the transmitters are allowed to delay the beginning of their transmission, it is to be expected that more than $(n + 1)$ can be placed [11]. For example in [10] it is shown how to place 4 pairs in 2 dimensions. Thus, the sum rate gain can be particularly large but it requires very special and

regular placements of the transmitters and receivers. In real world networks, the positions of the users cannot be arbitrarily decided and hence it is not always possible to reduce the span of time taken by interference to its minimum value of ρ at every receiver. However, a question of practical relevance is to investigate how much capacity can be attained by suitably aligning the transmissions of the terminals in a network with long delays. While it will be very unlikely to approach as many as $(n+1)\rho$ dof, it is important to understand whether more than one degree of freedom can be obtained with non negligible probability.

The contribution of this work lies in an analysis of the achievable dof with time IA in an n-dimensional Euclidean space with random user positions. Section II introduces the system model and Section III investigates the attainable dof without optimizing the transmission delay, where insightful analytical results can be derived. Section IV studies the sum rate when the transmission delays are jointly optimized among the senders, and in particular the performance improvements due to this coordination are highlighted. Section V investigates the amount of additional dof of time IA for a satellite network and finally Section VI draws the conclusions.

II. SYSTEM MODEL

We will denote matrices by capital letters. $m_{i,j}$ will be the element at the i-th row and jth column of a matrix M. $\mathbb{R}^n$ shall stand for the the n-dimensional Euclidean space. Further $M \mod p$ will denote the elementwise modulo p operator of matrix M. The rectangle function or unit pulse centered around 0 and of length 1 will be expressed as $rect(x)$ and $\delta(x)$ will be used to denote Dirac's delta.

The K-user interference channel will be considered, with $K = 3$, as shown in Fig. 1, in which K transmitters communicate with K receivers. We shall assume that transmitter 1 wants to communicate with receiver 1, transmitter 2 with receiver 2 and so on. Let us consider the general case in which transmitters and receivers are placed in $\mathbb{R}^n$ and the delay between two nodes is proportional to their Euclidean distance. Note that n is generic. The propagation delay among all the nodes in the channel can be expressed in a $(K \times K)$ matrix A, where $a_{i,j}$ is the propagation delay between transmitter j and receiver i. Time will be divided into slots of length T. Transmitters will be allowed to transmit only for a time ρT in every time slot, $\frac{1}{K} < \rho \leq \frac{1}{2}$. The ratio of the transmit duration over the length of a time slot, ρ, will be called duty cycle. This framework is more general than the cases considered in literature up to now [1], [11], [12], but it also includes the canonical example of interference alignment by means of delay offsets, where transmitters are allowed to transmit for a fraction $\rho = \frac{1}{2}$ of the time. If we would choose $\rho < \frac{1}{K}$ an orthogonal resource allocation in time or frequency would outperform interference alignment. Moreover, there is no reason for choosing $\rho > 1/2$, since it is known that it is not possible to obtain more than 1/2 dof per user [1].

The elements $a_{i,j}$ shall be assumed to take values between $(0, \gamma)$ where $\gamma >> T$. Given the fact that the time allocation

scheme used is periodic with period T, it is useful to define:

$$B = \frac{A \mod T}{T}. \tag{2}$$

B will be referred to as the normalized propagation delay matrix. The elements $b_{i,j}$ take values in $[0, 1)$. Additionally transmitters will be allowed to delay the start of their transmission. We will denote by δ_i the initial transmission delay of transmitter i. Let us define a new matrix,

$$D = \frac{(A + \hat{\Delta}) \mod T}{T}, \tag{3}$$

where $\hat{\Delta}$ is a matrix whose elements, $\hat{\delta}_{i,j}$, correspond to the initial transmit delay δ_j of transmitter j, $\hat{\delta}_{i,j} = \delta_j$, $\forall i$. The matrix D together with ρ completely specify the IA network.

III. ANALYSIS FOR NON-COORDINATED TRANSMITTERS

In this section the case in which the transmit delays are not coordinated is considered. Hence transmitters do not have any knowledge of A, B or D. The initial transmission delay, Δ_i, will be assumed to be independent and uniformly distributed between 0 and T. While this transmission scheme does not attempt to perform interference alignment, analytical expressions can be derived which actually correctly predict some behaviors of a network whose transmit delays are coordinated to approximate interference alignment. We shall start with a normalized propagation delay matrix B whose elements are independently picked from a uniform distribution between 0 and 1. This may seem unrealistic at first. However, this assumption is true if we assume transmitters and receivers to be placed randomly in a "big enough" section of $\mathbb{R}^n$. Under these hypotheses, it is straightforward to infer that the elements in D are also independent and uniformly distributed between 0 and 1. Let us also recall that we work under the assumption that $1/3 \leq \rho \leq 1/2$.

Let us denote by α_i the dof attained by pair i, and as $\phi = \sum_i \alpha_i$ the sum of the dof achieved by the 3 transmitter receiver pairs. In this section we derive analytically the probability density function (pdf) of the sum dof, $f(\phi)$, of the 3-user interference channel. Because of the symmetry of the problem, $f(\alpha_i)$ does not depend on i and will be just denoted as $f(\alpha)$. Let us say, for example, concentrate on receiver 1. The dof α for this pair correspond with the ratio of time in which the signal from transmitter 1 is received at receiver 1 without any interference. For simplicity it will be assumed that the interference from transmitter 2 starts at $t = 0$, $d_{1,2} = 0$. Our first step to calculate $f(\alpha)$ will be calculating the cumulative distribution function (CDF) of α conditioned to $d_{3,1}$:

$$\mathbf{F}(\alpha|d_{3,1}) = P(\boldsymbol{\alpha} \leq \alpha|d_{3,1}). \tag{4}$$

Let us first consider the case $d_{3,1} < \frac{1}{2}$. For simplicity two auxiliary variables η and ω will be introduced. η and ω are associated with the interference free time before and after $d_{3,1}$, respectively. A negative value implies that no interference free time exists. Fig. 2 shows the three different cases which have to be taken into account depending on the value of η and ω.

The first case is depicted in the upper part of Fig. 2. In this case both η and ω are smaller than α. These conditions
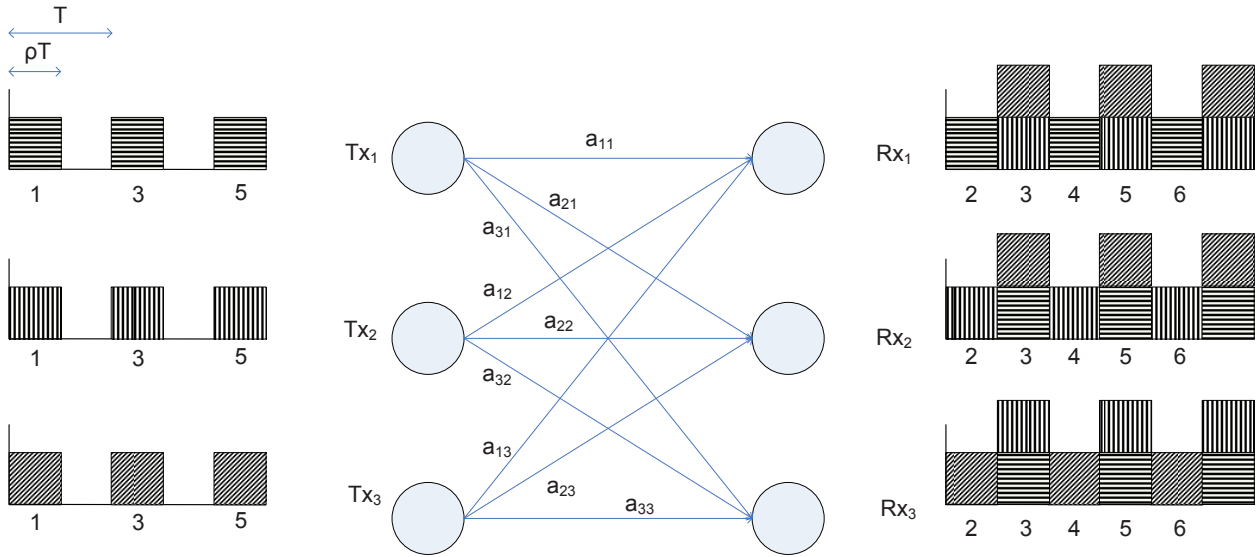
Fig. 1. The 3-user interference channel. Note that perfect IA is attained.

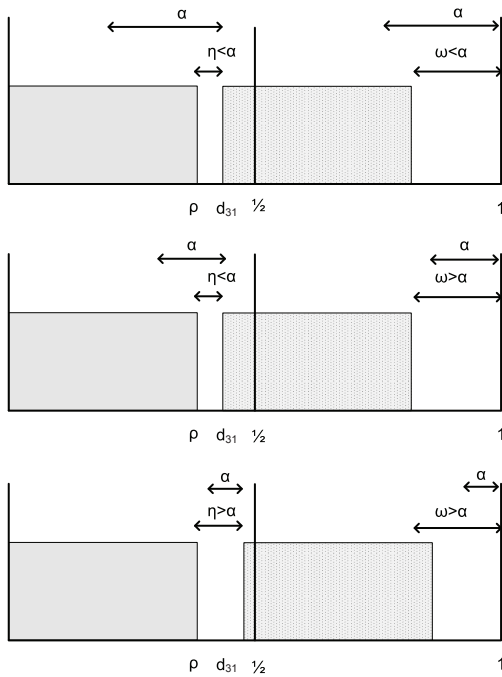Fig. 2. The three possible configurations of the intervals to calculate $f(\alpha)$. The upper part corresponds to $\eta < \alpha$ and $\omega < \alpha$. The middle part to $\eta < \alpha$ and $\omega > \alpha$ and the bottom part to $\eta > \alpha$ and $\omega > \alpha$

translate respectively into $\rho + \alpha > d_{3,1}$ and $d_{3,1} + \rho + \alpha > 1$. Both conditions can be put together as $\rho + \alpha > d_{3,1} > 1 - \rho - \alpha$. Therefore, this situation can only take place if $\rho + \alpha > 1 - \rho - \alpha$, which implies $\alpha > \frac{1}{2} - \rho$. In this case since η and ω are smaller than α no matter which values $d_{1,1}$ takes it is impossible to achieve α dof:

$$\mathbf{F}(\alpha|d_{3,1}) = 1, \alpha > \frac{1}{2} - \rho. \tag{5}$$

In the second case $\eta < \alpha$ and $\omega > \alpha$. This situation is shown in the middle part of Fig. 2. These conditions imply $\rho + \alpha < d_{3,1}$ and $d_{3,1} + \rho + \alpha < 1$. Or equivalently: $\rho + \alpha < d_{3,1} > 1 - \rho - \alpha$. Given the values of ρ considered, this second case is always possible independently from the value of α. In this case the probability of achieving at least α dof is not

zero. Recalling the assumption that $d_{i,i}$ are independent and uniformly distributed from 0 to 1:

$$\begin{aligned}\mathbf{F}(\alpha|d_{3,1}) &= P(\boldsymbol{\alpha} \le \alpha|d_{3,1}) = \\ &= P(d_{1,1} < d_{3,1} + \alpha) + P(d_{1,1} > 1 - \alpha) = \quad (6) \\ &= d_{3,1} + \alpha + \alpha = d_{3,1} + 2\alpha.\end{aligned}$$

The last case, shown in the bottom part of Fig. 2 corresponds to the situation in which $\eta > \alpha$ and $\omega > \alpha$. This corresponds to $\rho + \alpha > d_{3,1} > 1 - \rho - \alpha$. Which can only happen if $\alpha < \frac{1}{2} - \rho$. For this case the CDF of α conditioned to $d_{3,1}$ is:

$$\begin{aligned}\mathbf{F}(\alpha|d_{3,1}) &= P(\boldsymbol{\alpha} \le \alpha|d_{3,1}) = \\ &= P(d_{1,1} < \alpha) + P(d_{3,1} - \rho - \alpha < d_{1,1} < d_{3,1} - \rho) + \\ &\quad + P(d_{3,1} < d_{1,1} < d_{3,1} + \alpha) + P(d_{1,1} > 1 - \alpha) = \\ &= 4\alpha.\end{aligned}$$
$$\tag{7}$$

So far we have assumed $d_{3,1} < \frac{1}{2}$. It is easy to see that considering the case $d_{3,1} > \frac{1}{2}$ is equivalent to exchanging η and ω. By virtue of this symmetry, the CDF of α can be calculated as:

$$\begin{aligned}\mathbf{F}(\alpha) &= \int_0^1 \mathbf{F}(\alpha|d_{3,1}) f(d_{3,1}) \, \mathrm{d}d_{3,1} = \\ &= 2 \int_0^{\frac{1}{2}} \mathbf{F}(\alpha|d_{3,1}) f(d_{3,1}) \, \mathrm{d}d_{3,1}\end{aligned} \tag{8}$$

In order to solve the integral in (8) two different cases must be considered, $\alpha < \frac{1}{2} - \rho$ and $\alpha > \frac{1}{2} - \rho$. If $\alpha < \frac{1}{2} - \rho$, $\mathbf{F}(\alpha|d_{3,1})$ is given by (6) and (7):

$$\begin{aligned}&\int_0^{\frac{1}{2}} \mathbf{F}(\alpha|d_{3,1}, \alpha < \tfrac{1}{2} - \rho) f(d_{3,1}) \, \mathrm{d}d_{3,1} = \\ &= \int_0^{\rho+\alpha}(d_{3,1} + 2\alpha) f(d_{3,1}) \, \mathrm{d}d_{3,1} + \int_{\rho+\alpha}^{\frac{1}{2}}(4\alpha) f(d_{3,1}) \, \mathrm{d}d_{3,1} = \\ &= -\tfrac{3}{2}\alpha^2 - \alpha\rho + 2\alpha + \tfrac{3}{2}\rho^2, \ \alpha \le \tfrac{1}{2} - \rho\end{aligned}$$
$$\tag{9}$$

Similarly if $\alpha > \frac{1}{2} - \rho$, $\mathbf{F}(\alpha|d_{3,1})$ is given by (5) and (6):

$$\begin{aligned}&\int_0^{\frac{1}{2}} \mathbf{F}(\alpha|d_{3,1}, \alpha > \tfrac{1}{2} - \rho) f(d_{3,1}) \, \mathrm{d}d_{3,1} = \\ &= \int_0^{1-\rho-\alpha}(d_{3,1} + 2\alpha) f(d_{3,1}) \, \mathrm{d}d_{3,1} + \\ &\quad + \int_{1-\rho-\alpha}^{\frac{1}{2}} 1 f(d_{3,1}) \, \mathrm{d}d_{3,1} = \\ &= -\tfrac{3}{2}\alpha^2 - \alpha\rho + 2\alpha + \tfrac{3}{2}\rho^2, \ \alpha \le \tfrac{1}{2} - \rho\end{aligned} \tag{10}$$

Note that (9) and (10) yield the same result. From (8), (9) and (10) the CDF of α is obtained:

$$\mathbf{F}(\alpha) = -3\alpha^2 - 2\alpha\rho + 4\alpha + \rho^2, \quad \alpha \le \tfrac{1}{2} - \rho \qquad (11)$$

By taking the first derivative of (11) with respect to α it is possible to obtain the pdf of α:

$$f(\alpha) = \frac{\partial \mathbf{F}(\alpha)}{\partial \alpha} = (4 - 2\rho - 6\alpha)rect\left(\frac{\alpha - \frac{\rho}{2}}{\rho}\right) + \\ + \rho^2\delta(\alpha) + (1 - 4\rho + 4\rho^2)\delta(\alpha - \rho) \qquad (12)$$

Now that the pdf of the dof for *one* user is known, the pdf of the dof for 3 users can be calculated as the convolution of 3 single user pdfs. After some straightforward but tedious computations the pdf of ϕ is obtained:

$$f(\phi) = a^3\delta(\phi) + 3a^2b\delta(\phi - \rho) + 3ab^2\delta(\phi - 2\rho) + \\ + b^3\delta(\phi - 3\rho) + p_1(\phi)rect\left(\frac{\phi - \frac{\rho}{2}}{\rho}\right) + \\ + p_2(\phi)rect\left(\frac{\phi - \frac{3\rho}{2}}{\rho}\right) + p_3(\phi)rect\left(\frac{\phi - \frac{5\rho}{2}}{\rho}\right), \qquad (13)$$

where:

$a = \rho^2$
$b = 1 - 4\rho + 4\rho^2$
$p_1(\phi) = -6p^5 - 6p^4\phi + 12p^4 + 32p^3\phi^2 - 48p^3\phi + 6p^2\phi^3 + \\ \quad -48p^2\phi^2 + 48p^2\phi - 9p\phi^4 + 48p\phi^3 - 48p\phi^2 - \frac{9}{5}\phi^5 + \\ \quad +18\phi^4 - 48\phi^3 + 32\phi^2$
$p_2(\phi) = \frac{303}{5}p^5 + 39p^4\phi - 198p^4 - 118p^3\phi^2 + 240p^3\phi + \\ \quad +78p^3 - 12p^2\phi^3 + 204p^2\phi^2 - 426p^2\phi + 96p^2 + \\ \quad +18p\phi^4 - 96p\phi^3 + 78p\phi^2 + 96p\phi - 48p + \frac{18}{5}\phi^5 + \\ \quad -36\phi^4 + 114\phi^3 - 136\phi^2 + 48\phi$
$p_3(\phi) = -\frac{273}{5}p^5 - 33p^4\phi + 186p^4 + 86p^3\phi^2 - 192p^3\phi + \\ \quad -42p^3 + 6p^2\phi^3 - 156p^2\phi^2 + 378p^2\phi - 168p^2 + \\ \quad -9p\phi^4 + 48p\phi^3 - 30p\phi^2 - 96p\phi + 78p - \frac{9}{5}\phi^5 + \\ \quad +18\phi^4 - 66\phi^3 + 104\phi^2 - 66\phi + 12$

$$(14)$$

Fig. 3 shows the complementary cumulative distribution function (CCDF) of ϕ for $\rho = 0.5$ and $\rho = \frac{1}{3}$. The figure shows results of numerical simulations and analytical formulas. It can be observed how the analytical results and simulation results match perfectly. Note that the dirac deltas in (13) should create discontinuities in the CCDF of ϕ. For $\rho = \frac{1}{2}$ the effect of the deltas can not be appreciated because they have a very low weight. However for $\rho = \frac{1}{3}$ there is a discontinuity at $\phi = 3\rho = 1$. It can be observed how at $\phi = 3\rho = 1$ the CCDF goes from $b^3 = 1.4E - 3$ to zero.

With the results provided in (13) it is possible to calculate which is the value of ρ that maximizes the probability of exceeding some given number of dof. For example, the probability of exceeding 1 dof can be calculated as:

$$P(\phi > 1) = \int_1^{3\rho} f(\phi)\,\mathrm{d}\phi \qquad (15)$$

After some elementary calculus it can be found that the value of ρ which maximizes $P(\phi > 1)$ is $\rho_{opt} = 0.4305$. Fig. 4 shows $P(\phi > 1)$ as a function of ρ. In the figure numerical and theoretical results are shown. It can be observed how $P(\phi > 1)$ has maximum at ρ_{opt}. Moreover simulation results match quite well theoretical formulas. Note also that the probability that the 3-user interference network without cooperation exceeds 1 dof is in any case very low, around $7E - 3$. If transmitters do not coordinate at all, it is very unlikely to achieve a high capacity.
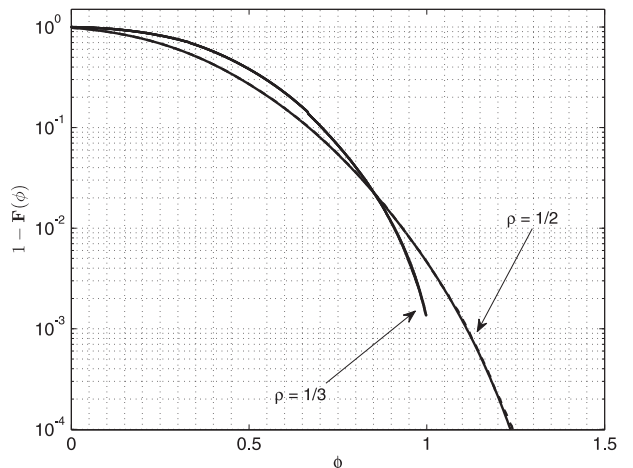

Fig. 3. CCDF of ϕ for $\rho = 0.5$, and $\rho = \frac{1}{3}$ for non-coordinated transmitters. Theoretical results are plot with discontinuous lines and simulation results with solid lines.
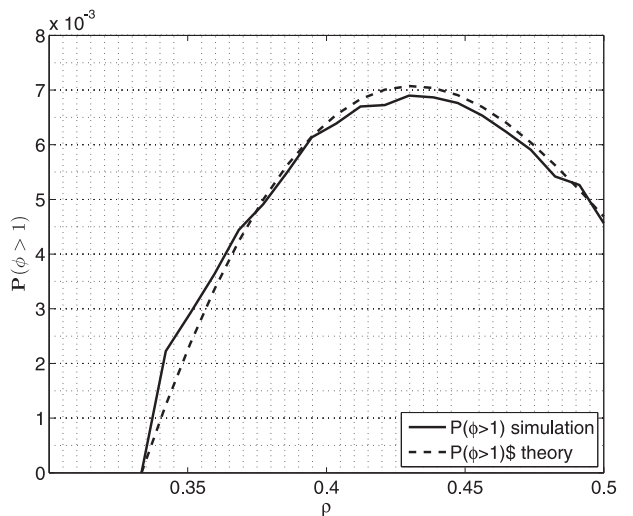

Fig. 4. Probability that the sum of dof in the network exceeds 1 against the duty cycle ρ for non-coordinated transmitters. Theoretical results are plotted with a discontinuous line and simulation results with a solid line.

IV. NUMERICAL RESULTS FOR COORDINATED TRANSMITTERS

Section III characterized the performance of a transmission scheme whose transmitters had no knowledge of the matrices A and B and did not coordinate their transmit delays δ_i. However, according to the interference alignment paradigm, the transmitters should arrange their signals so as to minimize the impact of the interference they generate. In this section all senders are assumed to have perfect knowledge of matrices A and B. The target metric has been the maximization of the sum dof ϕ, which can be achieved by two goals: first, the reduction of the amount of time that is occupied by the interference at each receiver and secondly to minimize the overlap between the desired signal and the interference. Hence, the senders arrange their transmission delays δ_i in order to maximize ϕ. This optimization is non trivial generally speaking and it has been performed numerically in a centralized fashion. Note that the purpose of our work is not to develop a distributed

algorithm for this task but rather to obtain insight into the potential capacity gains attainable by time IA. Moreover, the impact of the duty cycle ρ has been studied as well. Indeed, a larger ρ will increase both the duration of the useful signal and also of the mutual interference, hence it is non trivial to infer whether the duty cycle should approach its maximum possible value $1/2$ or it should be close to the minimum $1/3$.

Fig. 5 shows the CCDF of ϕ for $\rho \in \{1/3, 2/5, 1/2\}$. As it can be noticed, the optimization of the transmit delays has brought a significant capacity improvement. For instance, the 90-th percentile for $\rho = 1/2$ has increased from $\phi = 0.65$ to 1.05. Moreover, while the uncoordinated case would have a non negligible probability of attaining almost no dof (see Fig. 3), in the coordinated case the probability density function would be non zero only after 0.5. Another important observation is that the duty cycle plays a role especially for the right tail of the CCDF. First of all, the maximum number of dof is upper bounded by $K\rho = 3\rho$, and this can be seen in Fig. 5, since the CCDF does not exceed that limit. Furthermore, the value of ρ does impact the probability of attaining a high capacity, because for instance the curve for $\rho = 2/5$ achieves better performance than $\rho = 1/3$ and $\rho = 1/2$ for $0.5 < \phi < 1.1$. Indeed, Fig. 6 depicts the probability of obtaining more than one degree of freedom, and is the equivalent of Fig. 4 with transmit delay optimization. One can notice that apart for the minimum value $\rho = 1/3$, the coordination of the transmit delays attains more dof than an orthogonal access scheme with probability 15%. This probability is further increased to 19.5% for the optimum value of $\rho \simeq 0.42$ and thus this metric has increased by over one order of magnitude with respect to the non coordinated case. Moreover, note that the optimum value of ρ for the uncoordinated and coordinated case look very similar (0.43 and 0.42 respectively), which suggests that $\rho \simeq 0.43$ is a good duty cycle for possibly many configurations. It may be expected that further optimizations are possible with respect to these quite simple strategies and therefore further capacity improvements may be attainable.
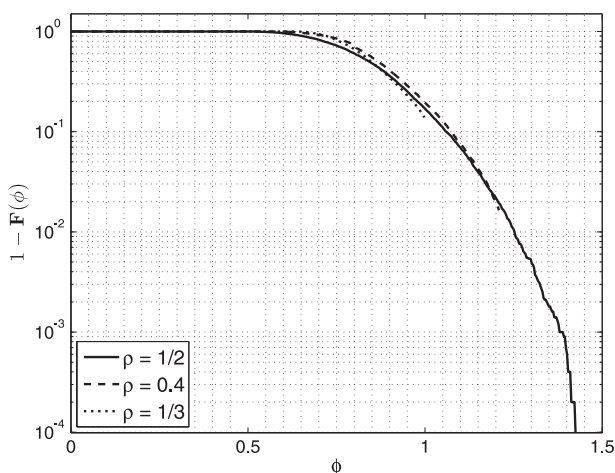

Fig. 5. CCDF of ϕ for coordinated transmitters for $\rho = 0.5$, $\rho = 0.4$ and $\rho = \frac{1}{3}$. Note the scale is same as in Fig. 3
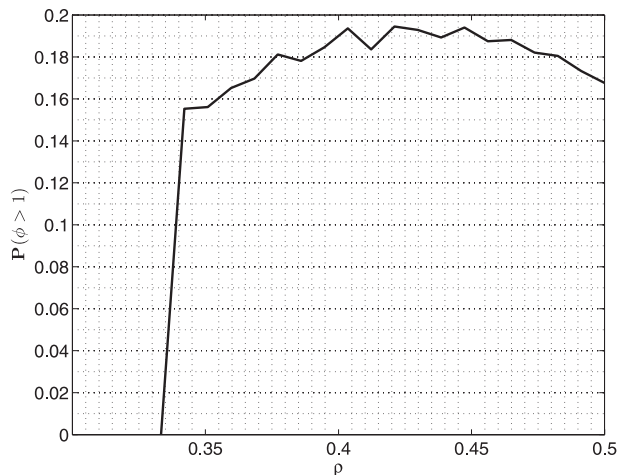

Fig. 6. Probability that the sum of dof in the network exceeds 1 against the duty cycle ρ for coordinated transmitters.

V. APPLICATIONS FOR SATELLITE NETWORKS

As stated in the introduction, one of the motivating scenarios for our study are satellite networks. These systems are distinguished by long delays and it is of interest to investigate whether time IA may bring an advantage for instance for multisatellite networks. Many reasons can be brought to have multiple satellites in the same frequency, but two are of particular relevance. First of all, the system may reuse the bandwidth very aggressively by deploying multiple satellites. Moreover, it may be more viable from an economic point of view to first deploy one satellite and launch additional systems when the revenues pick up. Hence, a first order investigation of how much capacity can be achieved by time IA can have practical importance. It should also be noticed that transmit delay coordination is in fact already in place in some TDMA standards for the return link, like DVB-RCS, which require that the signals of the users arrive at specific time instants at the satellite. These delay corrections are computed by the network gateway and therefore from a system point of view there would be almost no cost in implementing time IA for multiple satellites directed by a single gateway: this controller should compute in any case the appropriate transmit delays of the ground users, hence the difference at the gateway lies mainly in the software algorithm that computes such numbers, not in the hardware.

The analyzed scenario is still the K network, where K satellites communicate with some stations on ground in the same frequency and with partial overlap of time. In fact, this approach works equally well in both transmission directions (ground to space and viceversa) and can be applied without major differences in the forward and return link. In our study, the focus is on geostationary satellites, because a large number of communication satellites are moving along this orbit. The performance of time IA depends quite heavily on the distances between the satellites. Indeed, from the previous discussion, the larger the difference between the transmitter-receiver distances, the better our previous analysis applies. Moreover, also better performance can be in general attained, as it will be shown. However, large distances between the

satellites imply rather different orbital positions. Therefore the minimal orbital separation present today for geostationary satellites is assumed, which corresponds to $0.5°$. Three satellites at 24.5, 25 and 25.5 degrees east have been assumed. The constellation is positioned over Europe, and thus the ground stations are generated randomly but their latitude and longitude is constrained in the ranges $[35°,\ 55°]$ north and $[-10°,\ 20°]$ east, respectively. A duty cycle $\rho = 0.43$ has been adopted, since it was optimal for the previous two configurations and finally the delays are jointly optimized.

An important element here is the duration of T. According to our previous discussion, the shorter T, the more random the matrix looks like. We have evaluated some possible values of T in the range from tens to hundreds of microseconds, which imply bandwidth of tens to hundreds kHz and are still reasonable for satellite systems. Indeed, the forward link bandwidth is often in the order of 5 MHz, so these T can work for short bursts of symbols. Fig. 7 shows the CCDF of the sum dof ϕ for this configuration with $T = 25$ and $250\ \mu s$. In addition, the curve for completely random delay matrices is also depicted. It can be noticed that long T like $250\mu s$ impose a heavy loss in terms of dof, since the delay matrices B do not look random anymore but rather the delays become quite similar. For shorter T in the order of $25\mu s$, the CCDF significantly deviates only at the tail, thus randomness of the delay matrix is important especially to achieve the configurations with large number of dof.This deviation comes from the fact that the position of the 3 satellites is not random but always fixed. Note that with $T = 25\mu s$, the CCDF evaluated at $\phi = 1$ is 0.2, that is to say that the probability of outperforming TDMA is 20%, as in the previous setting. It should be noticed that in these scenarios, the number of ground users can be extremely large (it could easily be several thousands), and thus a smart scheduler could group the users in configurations that do achieve the high end of the CCDF curve. Thus, by virtue of multiuser diversity (which is truly abundant in this setting), the high performance of time IA might be routinely attained.

It is possible to relax the constraints on T by increasing the separation between satellites. However, some satellite antennas can be quite large (in the order of 1 m of aperture) and thus their mainlobe would be narrow. In this case, no mutual interference would be present by design, since the narrow lobes would suppress the other interference. Hence, satellites in far orbital slots would be more suitable for mobile receivers, whose antennas must be small and thus would suffer from inter-satellite interference. A multi-satellite constellation for mobile users has actually been rolled out in the XM-Sirius satellite radio system in order to provide satellite diversity, therefore such systems are in fact already reality and therefore the proposed scheme may be practically relevant.

VI. CONCLUSIONS

This work has studied the capacity improvements brought by time interference alignment in a more general setting than its predecessors [1], [10]–[12]. In particular, the possibility to approximate perfect interference alignment by means of delay
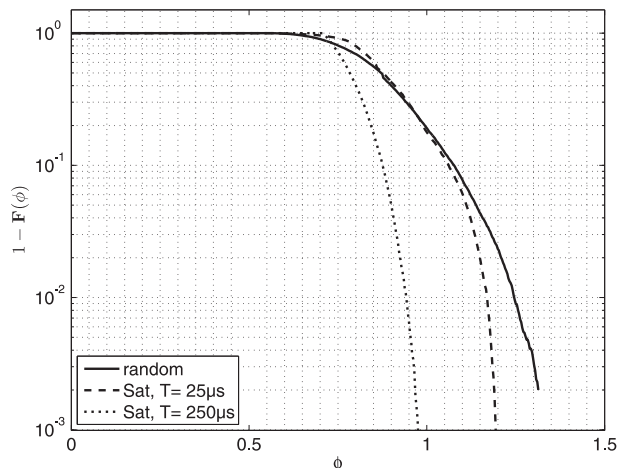

Fig. 7. CCDF of ϕ for cooperative transmitters satellite. $\rho = 0.43$ is assumed.

and duty cycle optimization has been investigated and a first-order evaluation to the satellite case has been performed as well. Analytical expressions have been derived for the dof of the non coordinated case, which yielded useful predictions for a system whose transmitters coordinate to approach interference alignment. The results show that by simple ideas there can be a non negligible capacity improvement and further optimizations are currently under investigation.

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