

Orbit Determination Error Analysis for a future Space Debris Tracking Radar

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ABSTRACT

Independent of light and weather conditions radar systems provide observations of space debris in low and medium earth orbits. In general two operational principles can be distinguished - in survey mode all objects passing through the radar instrument's field of view and above a certain size threshold are detected; subject to the tracking mode is the task to collect positional information on an already detected object.

A recently proposed radar concept balances survey and tracking performance in a novel way by combining a mechanical steerable reflector antenna with digital beam-forming techniques. The utilization of multiple digital feed elements allows illumination of a larger survey zone, compared to the relative narrow pencil beam of conventional tracking radars. Signal processing from independent digital channels preserves a high antenna gain on the reception path. It is also a prerequisite for the implementation of an advanced Track While Scan operational mode [1], [2].

During design phase an optimal combination of surveillance and tracking functionality will have to be found. As the number of simultaneously observable space debris is directly linked to the angular extension of the survey zone plus the amount of transmitted/received power, it is straightforward to define system requirements to meet an envisaged survey performance. Moreover, it is less obvious to characterize tracking data that lead to reliable and accurate orbit determination results. Variables of influence are the type of involved measurement data, their temporal and geometrical distribution as well as systematic measurement errors, e.g. measurement bias and noise, or erroneous measurement correction and force modelling. The amount of error introduced by each variable can be quantized in a so called Consider Covariance Analysis. This paper presents the outcomes of this kind of orbit determination error analysis.

Qualitative simulations for a small set of tracking scenarios serve as a starting point for the establishment of tracking requirements for a possible future space debris radar located at DLR ground station in Weilheim. The analyzed tracking scenarios are representative for three satellite missions currently operated at German Space Operation Centre (GSOC) and take into account the timeliness constraints inherited from a tracking request for refined orbit determination in case of a conjunction warning. Based on analysis findings the number of station passes, space debris can be tracked on, is investigated in more detail and an analytical formula is given for the optimization of minimum elevation angle.

Keywords: Space Debris, Radar Tracking, Orbit Determination, Consider Covariance Analysis, MODEAS

ORBIT DETERMINATION ERROR ANALYSIS

Orbit determination accuracy depends on a variety of influencing parameters. Using classical batched least-square methods or Kalman filtering covariance information are provided on the estimated state vector. The obtained covariance matrix reflects the influence of tracking geometry, number and distribution of measurement data as well as measurement type and noise. In reality orbit determination accuracy is much more deteriorated by systematic measurement errors and non-estimated force model-related parameters. The task of an orbit determination error analysis is to evaluate the error introduced by each considered parameter and to construct a more appropriate covariance matrix.

The Modular Orbit Determination Error Analysis Software (MODEAS) [3] was used to perform the presented analysis on space debris tracking requirements. The program's modular character arises from the successive execution of the following processing steps:

1. Propagation of reference orbit, State Transition Matrix and Sensitivity Matrix
2. Measurement simulation and generation of measurement related partial derivations
3. Merging simulated tracking data of all contributing observation instruments
4. Epoch state filtering, time propagation of covariance matrix, coordinate system mapping

This separation allows a quick reprocessing when only some of the input variables have changed, e.g. for a variation of measurement noise level it is only necessary to run the orbit propagation module once and repeat step two to four.

The following model parameters that either affects the dynamical motion or the measurement process are considered as sources of orbit determination error:

- c_D coefficient to account for mis-modeled atmospheric drag
- c_R coefficient to account for mis-modeled solar radiation pressure
- Deviation of the earth and sun standard gravitational parameter
- Uncertainties in tracking station location
- Measurement noise and bias
- Time tag bias
- Tropospheric correction errors
- Light-time correction errors

The mathematical concept of MODEAS, like most other orbit determination error analysis programs, fosters the idea of processing only measurement noise and variances of considered parameters rather than specific tracking data. The measurement residuals Δz are modelled as a function of errors in the vector of estimated parameters Δx (alternatively called solve-for or adjusted parameters) and of errors in the vector of considered parameters Δy [3]:

$$\Delta z = H_x \Delta x + H_y \Delta y + \nu \quad (1)$$

The vector ν introduces white Gaussian noise with zero mean value and covariance matrix $R = E[\nu\nu^t]$. Equation (1) may be stated for different measurement types at any point in time by means of linearization around reference epoch. Partial derivation of the observation measurement with respect to the estimated state vector at reference epoch may be written as [3]:

$$H_x = \frac{\partial z}{\partial x_0} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial x_0} = \frac{\partial z}{\partial x} \Phi_0 \quad (2)$$

Where Φ_0 is the State Transition Matrix for arbitrary times. The partial derivations with respect to the considered parameters are represented by [3]:

$$H_y = \frac{\partial z}{\partial y} \quad (3)$$

Mathematical expression for the missing partial derivations $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ can be found in [3], [4] and [5].

Assuming that the considered parameters exhibit zero mean values and that the linearized deviation of the estimated state $\Delta \hat{x}$ is a linear function of the measurement error $\Delta \hat{x} = F \cdot \Delta z$ one can show that [3]:

$$\Delta \hat{x} = \Delta x + S_{xy} \Delta y + F \nu \quad (4)$$

Where the Sensivity Matrix, $S_{xy} = F H_y = \frac{\partial x}{\partial z} \frac{\partial z}{\partial y}$, reflects the amount of error introduced by a change in the considered parameters. Defining the estimation error, $\varepsilon = \Delta \hat{x} - \Delta x$, the so called Consider Covariance Matrix is computed in the form [3]:

$$P_{\Delta x}^{CON} = E[\varepsilon_{\Delta x} \varepsilon_{\Delta x}^T] = P_{\Delta x}^C + S_{xy} P_{\Delta y} S_{xy}^T \quad (5)$$

Where $P_{\Delta x}^C = F R F^T$ represents the Computed Covariance Matrix, that usually introduces no significant orbit determination error.

MODEAS implements an epoch state filter, which means that the measurement partial derivatives are computed with respect to the state at a reference epoch rather than with respect to the current state (Kalman filter). To avoid a non-negative definite structure of covariance matrix the sequential measurement updating relies on Biermann's method for factorization into an upper triangular matrix U and a diagonal D matrix.

Table 1: Initial orbital elements (J2000)

	Scenario I	Scenario II	Scenario III
<i>Epoch</i>	2010/11/18 14:52:50	2011/02/15 12:00:00	2010/07/06 15:30:00
<i>Semi-major Axis</i>	6892.3 km	6834.1 km	7122.9 km
<i>Eccentricity</i>	0.001087	0.001176	0.004526
<i>Inclination</i>	97.44°	89.01°	98.29°
<i>RAAN</i>	327.62°	281.55°	14.15°
<i>Argument of Perigee</i>	70.01°	173.77°	137.78°
<i>Mean Anomaly</i>	280.12°	138.92°	298.63°

ANALYSIS SETUP

Each MODEAS analysis is dedicated to a single tracking scenario that has to be specified by an initial state vector, geocentric locations of contributing tracking stations and the tracking periods. The selection of these presets should reflect the spatial density distribution of space debris. Peaks of high density exist in circular low earth orbits (LEO) with altitudes between 800 km and 1000 km and characteristic high inclinations [6]. Timely constraints arise from tracking requests for orbit refinement of the secondary object in case of a critical conjunction warning. At the best up to 48 hours are available for a radar tracking campaign [7]. Another valid assumption can be made on the conjunction geometry of near polar orbits. Most encountering space debris move along similar high inclined circular orbits with a clear distinction in orbital plane orientation.

Considering everything mentioned the simple approach of using historic database records as initial state vectors for the space debris will be made to create a set of three example scenarios representative for a typical request for orbit refinement. Radar observations are provided by a future tracking station located at DLR Weilheim Ground Station (WHM: 47°52' N, 11°05' E). With the objective to obtain repeating groups of three station passes from successive orbital revolutions three database records from the polar orbiting spacecrafts GRACE-1, TerraSAR-X and PRISMA-TANGO have been selected. The different orbital heights of the origin satellite missions (≈ 515 km, ≈ 455 km, ≈ 745 km) lead to a rough altitude profile. The full set of osculating orbital elements is given in Tab. 1.

A single MODEAS run covering three station passes within twelve hours of the first tracking scenario was used to get a first impression on the major sources of error. Orbit determination was based on range-only tracking with noise of 10 m, bias level of 30 m and a sampling frequency of 1 Hz. Uncertainties in earth gravitational field were taken from the GRACE Gravity Model 02 (GGM02) [8]. Tab. 2 lists the variance of each considered parameter and the introduced orbit determination error in radial, tangential and normal component (RTN) at reference epoch.

For this exemplary case the greatest errors are introduced by measurement bias, uncertainties in atmospheric drag and residual errors after tropospheric correction. As the total amount of orbital information collected in less than 48 hours is only sufficient to adjust a few parameters subject to uncertainty, atmospheric drag and tropospheric correction will be added to the estimated state vector in the following analysis. Additional estimation of measurement bias seems reason-

Table 2: Exemplary Source of Error Comparison

Source of Error	Variance	Root Mean Square (RMS) Error		
		radial	tangential	normal
<i>Range Noise</i>	10.0 m	2.138 m	3.369 m	9.641 m
<i>Range Bias</i>	30.0 m	-63.532 m	-124.199 m	-398.896 m
c_D	0.5	85.700 m	554.466 m	540.888 m
c_R	0.3	1.372 m	1.829 m	0.093 m
GM_\odot	$132.7 \cdot 10^6 \text{ km}^3/\text{s}^2$	0.000 m	0.000 m	0.002 m
GM_\oplus	$398.6 \cdot 10^6 \text{ km}^3/\text{s}^2$	0.009 m	-0.039 m	-0.111 m
$C_{20} - C_{66}$	GGM02S	[-0.002 0.001] m	[-0.003 0.004] m	[-0.005 0.004] m
$S_{21} - C_{66}$	GGM02S	[-0.002 0.001] m	[-0.008 0.004] m	[-0.005 0.002] m
WHM_{east}	0.1 m	0.000 m	-0.021 m	0.157 m
WHM_{north}	0.1 m	0.013 m	0.151 m	0.065 m
$WHM_{vertical}$	0.1 m	0.005 m	0.083 m	0.179 m
<i>Tropospheric Correction</i>	0.5 %	-13.982 m	-22.517 m	-78.787 m
<i>Light-time Correction</i>	0.01%	0.014 m	-0.304 m	0.103 m
<i>Time Tag Bias</i>	1 ms	0.008 m	-7.671 m	0.491 m

able but may be impossible if more than one measurement type or more than one tracking station get involved. As a compromise bias in the order of the assumed measurement noise level will be considered.

ANALYSIS RESULTS

The first variation of parameter is carried out for the number of station passes the space debris can be tracked on in scenario I. The results of seven MODEAS runs are shown in Fig. 1 subplot a), please note the logarithmic ordinate axis scaling. By increasing the number of station passes from one (high elevated middle pass out of three successive orbital revolutions) to two (middle pass out of second triplet 12 hours later) to three (another low elevated pass of the first triplet) the orbit determination error improves by more than two and a half orders of magnitude. Slight further improvements arise if additional passes within 24 hours (4 and 6 passes) are taken into account. Extending orbit determination period up to 48 hours (8 and 11 passes) results in no more accuracy gain. Tracking scenario II and III show essentially the same dependency of orbit determination error from the number of available station passes.

Second subject to variation is the noise level for range-only, Doppler-only, and angles-only orbit determination. Subplots b) to d) of Fig. 1 depicts the error functions for each measurement type. For inordinate precise measurements and regardless of type the orbit determination accuracy is limited to approximately 8 m in tangential and 1 m in radial and normal direction. These values correspond to the error amount of the remaining unadjusted parameters, mainly due to time tag bias as can be seen from Tab. 2. With increasing measurement noise a section of linear dependency between noise level and orbit determination error becomes apparent. Transition to section of linear dependency occurs at range noise of ≈ 3 m, Doppler noise of ≈ 3 mm/s and angle noise of $\approx 1''$ respectively. A trend of error saturation may be presumed for very high measurement noise level. Finally range and Doppler measurements show a lack of sensitivity in normal component. This may be the case only for limited set of tracking data as indicated by subplot a), but also for unfavourable tracking geometries when the observation site is located close to the orbital plane and range or Doppler measurements do not provide sufficient information on the out-of-plane component.

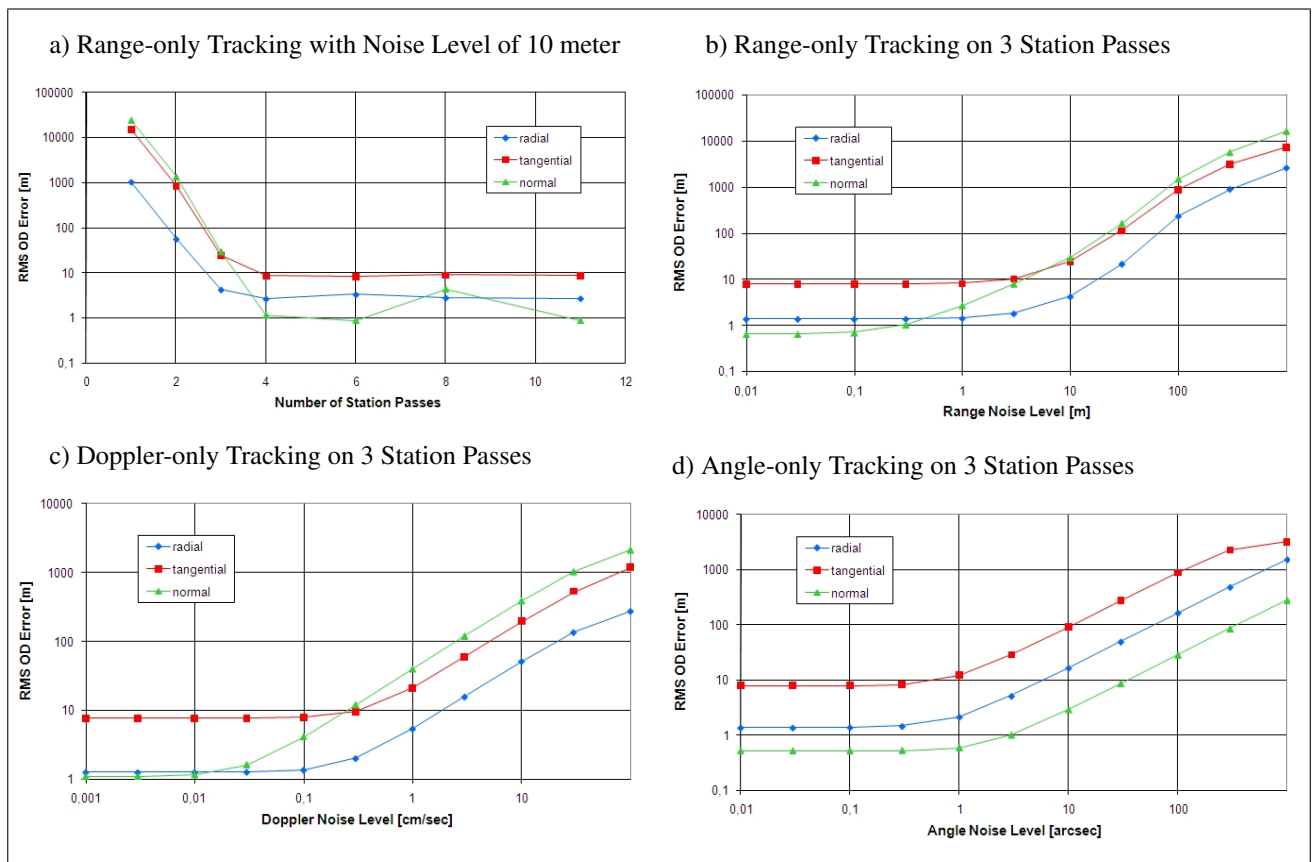


Fig. 1: Variation of Number of Station Passes and Noise Level for Tracking Scenario I

Next orbit determination accuracies obtained from combinations of different measurement types are compared with one another. Of particular interest is the question if combinations involving angle observations with its inherent complementary geometrical information act beneficial in case of short orbit determination periods. Analysis was performed with more appropriate, constant noise levels of 10 m, 1 cm/s and 100'' respectively. First noted from Fig. 2 are the apparent differences in error amount and component distribution for the three tracking scenarios. This is most likely due to the number of collected tracking data. The two station passes of scenario II are relatively low elevated, resulting in total observation duration of 898 seconds compared to 976 seconds in scenario I and 1269 seconds in scenario III.

Nevertheless some universal principles on measurement type combination can be made. Combinations involving two measurement types with the same component distribution lead to an averaged orbit determination accuracy, e.g. in tracking scenario I orbit determination from range or Doppler observations is most sensitivity to the radial component and least sensitivity to the out-of-plane component. A combined processing leads to orbit determination errors lower than from range-only processing but greater than from Doppler-only processing.

On the other hand, combinations based on complementary component distributions offer advantages even for great differences in single processing accuracy. For example, the normal component in tracking scenario III is best fixed by angle measurements. In this case angle data, despite the poor accuracy in radial and tangential accuracy, are well combined with range or Doppler measurements. A further distinction between combinations based on azimuth or elevation angles indicates that it is even more convenient to solely involve azimuth angles. For reasons of tracking geometry this is probably not the case for low elevated station passes.

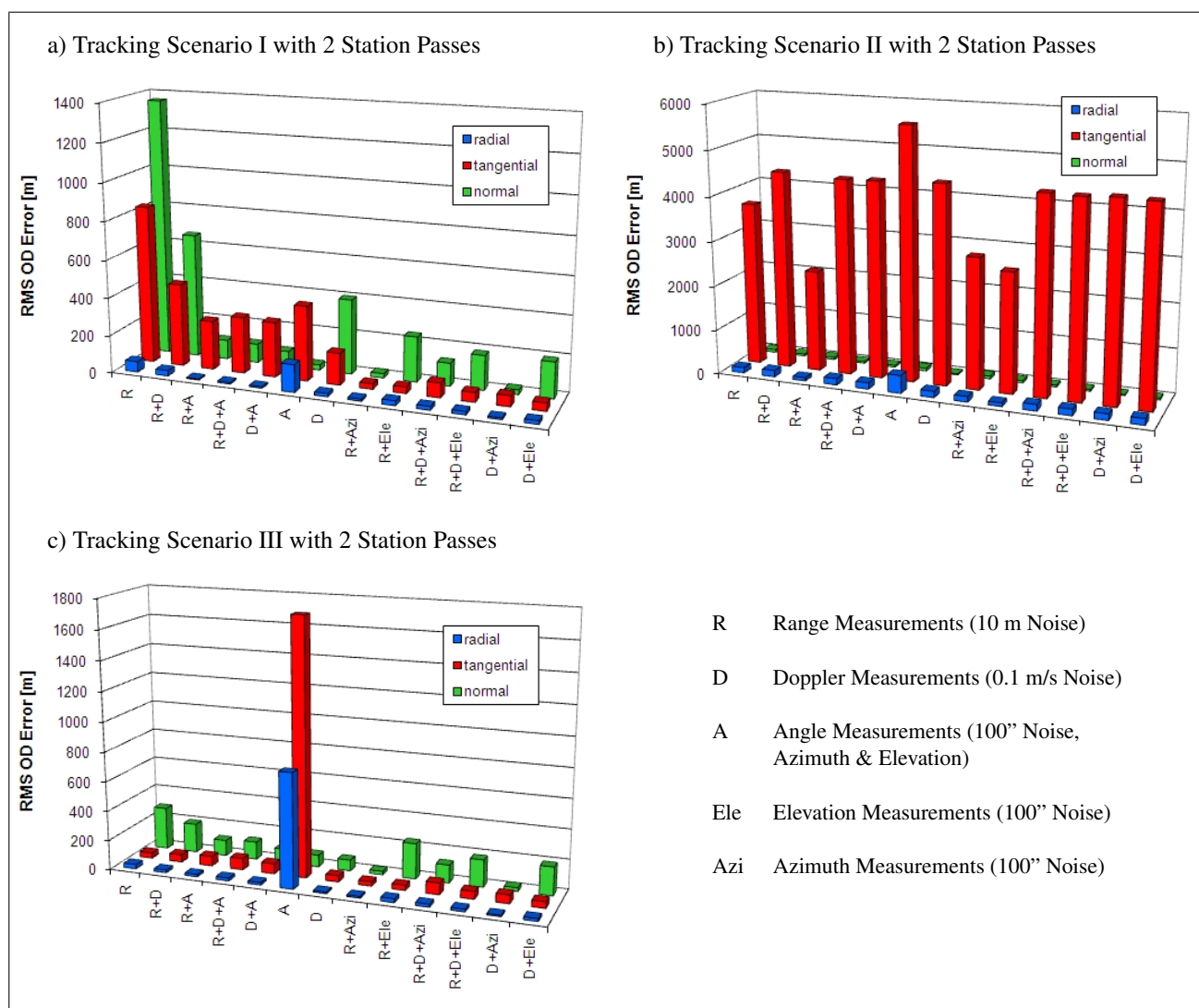


Fig. 2: Comparison of different Measurement Types and their Combinations

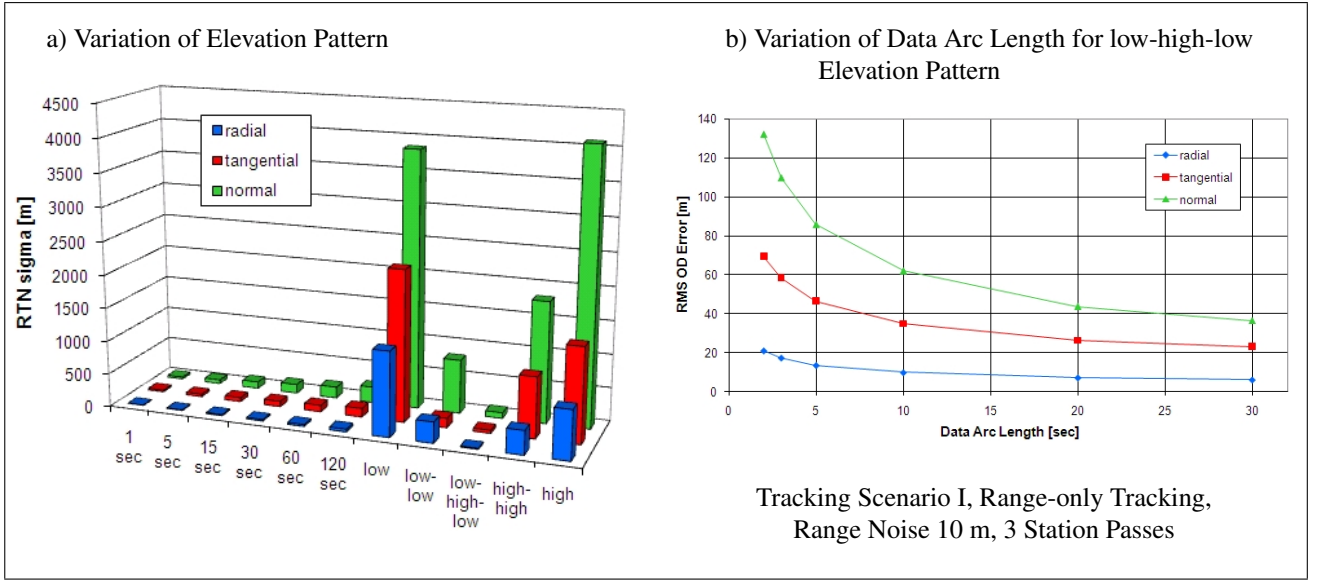


Fig. 3: Data Reduction Strategy

From an operational point of view it is interesting to evaluate the costs in terms of orbit determination accuracy for a reduced set of tracking data. A simple data reduction strategy is to extend the sampling interval. As can be seen from the left side of Fig. 3, subfigure a) this strategy leads to a modest increase in orbit determination error. Minute long separations between data points could be used as free observation time. Still tracking of the entire pass would be required, inherent with a wide range of mechanical antenna steering. Therefore alternative data reduction strategies implement periods of concentrated data collection. With a constant sampling frequency of 1 Hz five elevation patterns have been analysed and results are shown on the right side of subfigure a). In the first case five observations are made at the beginning of each pass (low), in the second case five observations are made at the beginning and at the end of each pass (low-low), and so on. Except for the low-high-low elevation pattern substantial increases in orbit determination error can be observed. Last subject to variation is the length of each data arc for the data reduction strategy following a low-high-low elevation pattern. Subfigure b) of Fig. 3 indicates a growing limitation for orbit determination improvements for increasing data arc length. Collecting five to ten measurements at each elevation section seems to be a good compromise.

OPTIMIZATION OF RE-OBSERVABILITY

It was shown that the number of station passes space debris can be tracked on is the most critical parameter for orbit determination accuracy. The majority of LEO resident objects move along near polar circular orbits with periods of 90 to 120 minutes. In this time the earth rotates around 22.5° to 30.0° . Depending on geographic latitude and minimum station elevation successive orbital revolutions can be observed. In the worst case space debris passes at zenith and maximum elevation of the prior and following pass falls below the station's minimum elevation. If again only one station pass can be tracked 12 hours later, not more than two station passes are available for a typical orbit determination period of 24 hours.

Let λ_2 represent the earth central angle between ground station location and space debris position at time of maximum elevation. λ_1 and λ_3 describe this angle at maximum elevation of the prior and following pass. In case the orbit inclination i exceeds the geographic latitude of the ground station ϕ the change in earth central angle may simply be expressed as:

$$\lambda_{1,3} = \lambda_2 \pm \cos \phi \sin i \Delta\lambda \quad (6)$$

Where $\Delta\lambda$ donates the earth rotation angle during one orbital revolution:

$$\Delta\lambda = \omega_E \cdot T_{circ} = \omega_E \cdot 2\pi \cdot \sqrt{a^3/GM} \quad (7)$$

The angular relationship that lead to the maximum elevation of the prior and following pass can be read from Fig. 4:

$$\eta + \lambda + \varepsilon + 90^\circ = 180^\circ \quad (8)$$

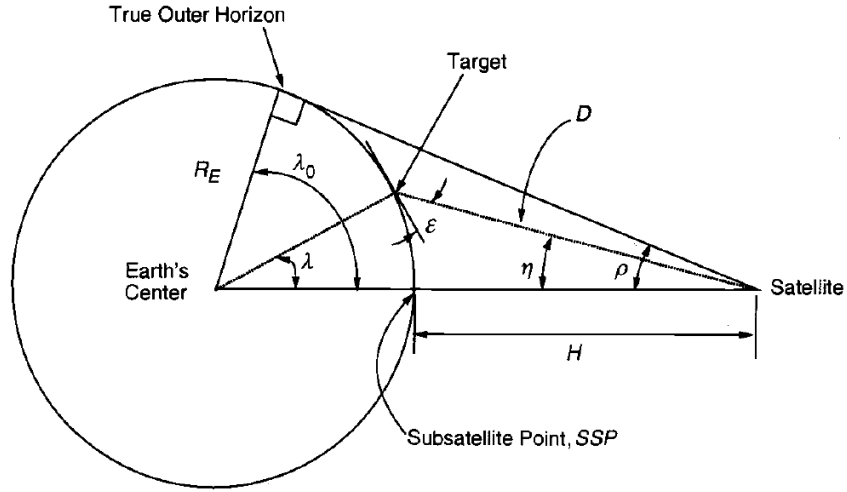


Fig. 4: Angular Relationships [9]

The unknown angle η follows from the law of sin:

$$\frac{D}{\sin \lambda} = \frac{R_E}{\sin \eta} \quad (9)$$

The slant range D is given by:

$$D = \sqrt{R_E^2 + (R_E + H)^2 - 2R_E((R_E + H) \cos \lambda)} \quad (10)$$

For a zenith passage maximum elevations equal on the prior and following pass and results are given in Fig. 5. To avoid impractical short tracking periods additional elevation margin will have to be introduced. Considering the increasing atmospheric attenuation for radio waves transmitted at low elevation it will even more challenging to realise a space debris tracking radar with optimal reobservation performance and therefore optimal orbit determination accuracy.

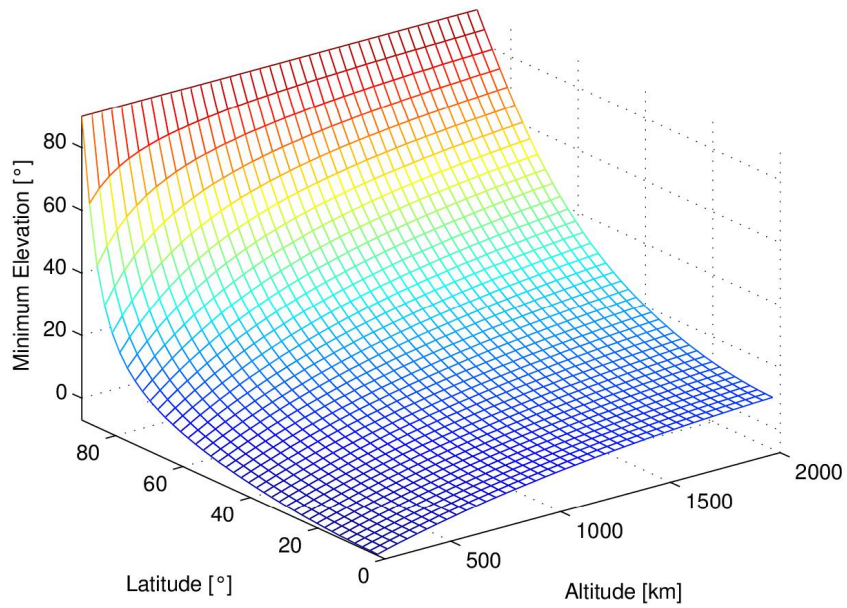


Fig. 5: Minimum Elevation for optimal Re-observation Performance of Near Polar Orbits

CONCLUSIONS

An orbit determination error analysis was performed for a future space debris tracking radar located at DLR Groundstation Weilheim. The number of station passes was identified as the most important variable of influence. Measurement type combinations including angle measurements of modest precision allowed orbit determination from two station passes with an acceptable error amount in two of the three analysed tracking scenarios. It is therefore recommended to use all station passes within 24 hours. A linear dependency between measurement noise level and orbit determination error was observed up to a point where error amount is governed by unadjusted measurement errors. Different data reduction strategies have been compared and suggest observation collection following a low-high-low elevation pattern. The created graphs will assist in the definition of system requirements and operational modes.

Considering the influence of minimum station elevation on re-observability an analytical formula is given to guarantee tracking of successive orbital revolutions for the great number of space debris in highly inclined circular orbits.

Further research may concentrate on a simulation of the entire space debris population, an analysis of time propagated covariance information, introduction of signal to noise ratios to the measurement data model and advantageous combination of measurement types. A comparison with real world data, e.g. from the Tracking and Imaging Radar (TIRA) of FHR [10], and cross-checks with other error analysis software like the Advanced Space Surveillance Simulator (AS4) developed by DEIMOS Space [11] could serve for a verification of the calculated orbit determination error.

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