
#### Abstract

In this paper, we investigated the design and optimization of a proposed RFID-enabled automated warehousing system in terms of the optimal number of storage racks and collection points that should be established in an efficient and cost-effective approach. To this aim, a fuzzy tri-criterion programming model was developed and used for obtaining trade-off decisions by measuring three conflicting objectives. These are minimization of the warehouse total cost, maximization of the warehouse capacity utilization and minimization of the travel time of products from storage racks to collection points. To reveal the alternative Paretooptimal solutions using the developed model, a new approach was developed and compared with a recently developed fuzzy approach so-called SO (Selim and Ozkarahan). A decision making algorithm was used to select the best Pareto-optimal solution and the applicability of the developed model was examined using a case-study. Research findings demonstrate that the developed model is capable of generating an optimal solution as an aid for the design of the proposed RFID-enabled automated warehousing system.


Keywords: Automated warehouse; RFID; Design; Fuzzy approach; Multi-criterion optimization.

## 1. Introduction

Warehouses are one of main components consisting of an entire supply chain network in which a warehouse receives and stores merchandising products that are often transported from suppliers to retailers. Hence, accuracy of transportation time plays an important role on the entire supply chain network, which traditionally relies on a well-organized warehouse management (Choi et al., 2013; Yeung et al., 2011). For the last decade, it has been seen a growing trend in application and implementation of automated warehouses aiming to improve the warehouse efficiency and capacity utilization, and reduce the material-handling time of warehouses. On the other hand, automation of warehouses is subject to additional costs that
need to be considered; this led to research interests in optimization of automated warehouse designs by enhancing efficiency and reducing unnecessary costs.

There are relatively a few studies in optimization of automated warehouse designs in several aspects-such as costs and capacity utilization. Lu et al. (2006) reviewed some fundamental issues, methodologies, applications and potentials of applying Radio Frequency Identification (RFID) techniques in manufacturing sectors. Van Der Berg (1999) presented a review on approaches and techniques applied for the warehouse management planning and control. Ma et al. (2015) formulated an automated warehouse as a constrained multi-objective model aimed at minimizing the scheduling quality effect and the travel distance. Huang et al. (2015) proposed a nonlinear mixed integer program under probabilistic constraints for site selection and space determination of a warehouse. The purpose of this work was to minimize the total cost of inbound and outbound transportation and the total cost of warehouse operations in a two-stage network. Lerher et al. (2013) developed a multi-objective model for analyzing the design of an automated warehouse towards the optimization of the travel time of product, the total cost of the automated warehouse and quality in the number of material handling devices. Lerher et al. (2010) also investigated the design and optimization of the automated storage and retrieval system aiming to minimize the initial investment and annual operating cost of the system using the genetic algorithm. Wang et el. (2010) presented a study of an RFID-based automated warehousing mechanism in order to address the tighter inventory control, shorter response time and greater variety of stock keeping units (SKUs), which are the most important challenges for designing future generation warehouses. Lu et al. (2006) presented a five-step deployment process aimed at developing a holistic approach for implementing RFID in manufacturing enterprises. Lerher et al. (2007) proposed a mono-objective optimization approach for seeking the cost-effective design of an automated warehouse. Ashayeri et al. (1987) developed a design model of an automated storage and retrieval system incorporating
the main influential parameters to minimize costs in investment and operation. Karasawa et al. (1980) developed a nonlinear mixed integer model aimed at minimizing the cost for an automated warehouse system.

A review of the literature reveals that there were no previous studies in applying the fuzzy multi-criterion optimization approach in the context of the warehouse design (Lerher et al., 2013), in particular for the Radio Frequency Identification (RFID)-enabled automated warehousing system. This paper addresses a contribution in developing a fuzzy tri-criterion optimization model based on a proposed RFID-enabled automated warehousing system incorporating the uncertainty in varying demand, costs and items locations. The developed model aims at simultaneously optimizing a number of conflicting criteria including minimization of the total cost, maximization of the warehouse capacity utilization and minimization of travel time of products. In other words, it aims at obtaining a trade-off that can concurrently maximizes the degree of satisfaction and minimize the degree of dissatisfaction at a time for the problem under investigation.

The remaining part of the paper proceeds as follows: In section 2 , the problem description and the model formulation are presented. In section 3, the optimization methodology is described. In section 4, it demonstrates the application and evaluation of the developed multi-criterion model using a case study. In section 5, conclusions are drawn.

## 2. Problem description and model formulation

Figure 1 illustrates the structure of the proposed RFID-enabled automated storage and retrieval racks (AS/RR) used for this study (Wang et al., 2010). The module comprises of two types of powered conveyors aligned next to one another; these are input conveyors (storage racks) and output conveyors. The entire operation of each conveyor system is controlled by a programmable logic controller that communicates with mounted sensors via a local area
network. Within the RFID-inventory management system, a chosen SKU can be released by the mechanical control system based on a number of assignment policies or rules. These rules include for example the rule of being nearest to a collection point and/or a modular arm which is free or adjacent to the chosen SKU.


Fig. 1. Structure of the proposed RFID-enabled AS/RR.

One of the main issues to be addressed in designing the proposed RFID-enabled automated warehouse include allocating the optimum number of racks and collection points with respect to three criterion functions: (1) minimization of total cost, (2) maximization of capacity utilization of the warehouse and (3) minimization of travel time of products from storage racks to collection points.

### 2.1. Notations

The following sets, parameters and decision variables were used in the formulation of the model:

Sets:
set of nominated storage racks $i \in I$ set of nominated collection points $j \in J$

K set of fixed departure gates $k \in K$

Given parameters:
$C_{i}^{r} \quad$ fixed cost required for establishing an RFID-enabled rack $i$
$C_{j}^{c} \quad$ fixed cost required for establishing a collection point $j$
$C_{i}^{t} \quad$ unit RFID tag cost per item at rack $i$
$C_{j k}^{\mathrm{T}} \quad$ unit transportation cost per meter from collection point $j$ to departure point $k$
$C_{j}^{l} \quad$ unit labor cost per hour at collection point $j$
$R_{j}^{l} \quad$ working rate (items) per laborer at collection point $j$
$N_{j}^{h}$
minimum required number of working hours for laborers $l$ at collection point j
$W \quad$ transportation capacity (units) per forklift
$\mathrm{S}_{\mathrm{i}}^{r} \quad$ maximum supply capacity (units) of rack $i$
$\mathrm{S}_{\mathrm{j}}$
$\mathrm{D}_{\mathrm{j}} \quad$ demand (units) of collection point $j$
$d_{l} \quad$ travel distance needed (m) for a pusher from its location to a selected item
$d_{3}$
travel distance (m) of a selected item from its position at an output conveyor to a collection point
$d_{j k} \quad$ travel distance (m) of a selected item from collection point $j$ to departure gate $k$
$S_{p} \quad$ speed ( $\mathrm{m} / \mathrm{s}$ ) of the moving-pusher along $d_{l}$
$S_{p p} \quad$ speed ( $\mathrm{m} / \mathrm{s}$ ) of the moving-pusher to push a selected item onto an output conveyer.
$S_{c} \quad$ speed $(\mathrm{m} / \mathrm{s})$ of the output conveyor and the spiral conveyor.
Decision variables
$q_{\mathrm{ij}} \quad$ quantity in units ordered from rack $i$ to collection point $j$
$q_{j k} \quad$ quantity in units dispatched from collection point $j$ to departure gate $k$
$x_{j} \quad$ required number of laborers at collection point $j$
$y_{i} \quad\left\{\begin{array}{l}1: \text { if rack } i \text { is opened } \\ 0: \text { otherwise }\end{array}\right.$
$\mathrm{y}_{\mathrm{j}} \quad\left\{\begin{array}{l}1: \text { if collection point } j \text { is opened } \\ 0: \text { otherwise }\end{array}\right.$

### 2.2 Formulation of the multi-criterion optimization problem

The three criteria, which include minimization of total cost, maximization of capacity utilization and minimization of travel time, are formulated as follows:

Criterion function $1\left(\mathrm{~F}_{1}\right)$

In this case, the total cost of establishing the RFID-enabled automated warehouse includes the costs of establishing RFID-enabled racks, collection points, RFID tags, transportation of products and labors in the warehouse. Thus, minimization of the total cost $F_{1}$ can be expressed below:
$\operatorname{Min} F_{1}=\sum_{i \in I} C_{i}^{r} y_{i}+\sum_{j \in J} C_{j}^{c} y_{j}+\sum_{i \in I} \sum_{j \in j} C_{i}^{t} q_{i j}+\sum_{j \in J \in} \sum_{k \in K} C_{i j}^{T}\left\lceil q_{j k} / W_{f}\right\rceil d_{j k}$
$+\sum_{j \in J} C_{j}^{l} x_{j} N_{j}^{h}$

Criterion function $2\left(\mathrm{~F}_{2}\right)$

The capacity utilization is defined as the used capacity divided by the actual capacity. Thus, maximization of capacity utilization $\mathrm{F}_{2}$ is expressed as follows:
$\operatorname{Max} F_{2}=\left(\sum_{i \in I} \frac{\left\lceil\left(C_{a}\right)-\left(C_{u}\right)\right\rceil^{2}}{\sum i}\right)^{\frac{1}{2}}$
Where $C_{a}=\sum_{i \in j} \sum_{j \in J} \frac{q_{i j}}{S_{i}^{r}}$ and $C_{u}=\frac{\sum_{i \in I} \sum_{j \in J} q_{i j}}{\sum_{i \in I} S_{i}^{r}}$, which refer to the actual (a) and used (u) capacity (C).

Criterion function 3 ( $\mathrm{F}_{3}$ )

Travel time ( tt ) of an in-store item includes, tt of a pusher from its location to an item, tt of an item from its location at the storage rack to an output conveyer and tt of an item onto a conveyer system to the collection point. Thus, minimization of travel time $\mathrm{F}_{3}$ is expressed as follows:

$$
\begin{equation*}
\operatorname{Min} F_{3}=\sum_{i \in I} \sum_{j \in J}\left(\frac{d_{1}}{S_{p}}+\frac{d_{2}}{S_{p p}}+\frac{d_{3}}{S_{c}}\right) q_{i j} \tag{3}
\end{equation*}
$$

### 2.3 Constraints

The above model was developed under the following constraints:

$$
\begin{equation*}
\sum_{i \in I} q_{i j} \leq S_{i}^{r} y_{i} \quad \forall j \in J \tag{4}
\end{equation*}
$$

$$
\begin{align*}
& \sum_{j \in J} q_{j k} \leq S_{j}^{c} y_{j}  \tag{5}\\
& \sum_{i \in I} q_{i j} \geq D_{j} \quad \forall k \in K  \tag{6}\\
& D_{j} \geq \sum_{k \in K} q_{j k} \quad \forall j \in J  \tag{7}\\
& \sum_{j \in J} q_{i j} \leq x_{j} \mathrm{R}_{j}^{1} \quad \forall j \in J  \tag{8}\\
& q_{i j}, q_{j k} \geq 0, \forall i, j, k ;  \tag{9}\\
& y_{i}, y_{j} \in\{0,1\}, \quad \forall i, j ; \tag{10}
\end{align*}
$$

Equations 4 and 5 refer to the flow balance of a product travelling from a storage rack to a collection point and from a collection point to a departure gate. Equations 6 and 7 refer to demands in quantity to be satisfied. Equation 8 determines the required number of labors at a collection point. Equations 9 and 10 limit the decision variables to binary and non-negative.

## 3. The proposed optimization methodology

### 3.1 Solution procedures

To reveal the alternative Pareto-optimal solutions using the developed model, the following procedures were used:
(1) Convert the developed model into an equivalent crisp model (shown in section 3.2).
(2) Find the upper and lower bound $(U, L)$ solution for each criterion function. This can be obtained as follows:

For upper bound solutions:

$$
\begin{aligned}
& \operatorname{Max}_{F_{1}}\left(U_{1}\right)=\sum_{i \in I} C_{i}^{r} y_{i}+\sum_{j \in J} C_{j}^{c} y_{j}+\sum_{i \in I} \sum_{j \in j} C_{i}^{t} q_{i j}+\sum_{j \in J k \in K} \sum_{i j}^{T}\left\lceil q_{j k} / W_{f}\right\rceil d_{j k} \\
& +\sum_{j \in J} C_{j}^{l} x_{j} N_{j}^{h}
\end{aligned}
$$

$\operatorname{Max} F_{2}\left(U_{2}\right)=\left(\sum_{i \in I} \frac{\left[\left(C_{a}\right)-\left(C_{u}\right)\right\rceil^{2}}{\sum i}\right)^{\frac{1}{2}}$
$\operatorname{Max} F_{3}\left(U_{3}\right)=\sum_{i \in I} \sum_{j \in I}\left(\frac{d_{1}}{S_{p}}+\frac{d_{2}}{S_{p p}}+\frac{d_{3}}{S_{c}}\right) q_{i j}$

For lower bound solutions:

$$
\begin{aligned}
& \operatorname{Min} F_{1}\left(L_{1}\right)=\sum_{i \in I} C_{i}^{r} y_{i}+\sum_{j \in J} C_{j}^{c} y_{j}+\sum_{i \in I} \sum_{j \in j} C_{i}^{t} q_{i j}+\sum_{j \in J} \sum_{k \in K} C_{i j}^{T}\left[q_{j k} / W_{f}\right\rceil d_{j k} \\
& +\sum_{j \in J} C_{j}^{l} x_{j} N_{j}^{h}
\end{aligned}
$$

$\operatorname{Min} F_{2}\left(L_{2}\right)=\left(\sum_{i \in I} \frac{\left\lceil\left(C_{a}\right)-\left(C_{u}\right)\right\rceil^{2}}{\sum i}\right)^{\frac{1}{2}}$
$\operatorname{Min} F_{3}\left(L_{3}\right)=\sum_{i \in I} \sum_{j \in J}\left(\frac{d_{1}}{S_{p}}+\frac{d_{2}}{S_{p p}}+\frac{d_{3}}{S_{c}}\right) q_{i j}$
(3) Find the respective satisfaction degree $\mu\left(x_{i}\right)$ for each criterion as follows:

$$
\mu_{1}\left(F_{1}(x)\right)= \begin{cases}1 & \text { if } F_{1}(x) \geq U_{1}  \tag{17}\\ \frac{F_{1}(x)-L_{1}}{U_{1}-L_{1}} & \text { if } L_{1} \leq F_{1}(x) \leq U_{1} \\ 0 & \text { if } F_{1}(x) \leq L_{1}\end{cases}
$$

$$
\begin{align*}
& \mu_{2}\left(F_{2}(x)\right)= \begin{cases}1 & \text { if } F_{2}(x) \geq U_{2} \\
\frac{F_{2}(x)-L_{2}}{U_{2}-L_{2}} & \text { if } L_{2} \leq F_{2}(x) \leq U_{2} \\
0 & \text { if } F_{2}(x) \leq L_{2}\end{cases}  \tag{18}\\
& \mu_{3}\left(F_{3}(x)\right)= \begin{cases}1 & \text { if } F_{3}(x) \geq U_{3} \\
\frac{F_{3}(x)-L_{3}}{U_{3}-L_{3}} & \text { if } L_{3} \leq F_{3}(x) \leq U_{3} \\
0 & \text { if } F_{3}(x) \leq L_{3}\end{cases} \tag{19}
\end{align*}
$$

(4) Transform the crisp model obtained from section 3.2 to a single criterion function using the proposed solution approaches (shown in section 3.3).
(5) Vary the weight combination set consistently for the three criteria to reveal Paretooptimal solutions. Usually, the weight combination set is allocated by decision makers based on the importance of each objective.
(6) Select the best Pareto-optimal solution using the proposed decision making algorithm.
3.2 Formulating the uncertainty

To incorporate the uncertainty in varying demand, costs and items locations, the developed tricriterion model is converted into an equivalent crisp model using the Jiménez method (Jiménez et al., 2007). Accordingly, the equivalent crisp model can be formulated as follows:

$$
\begin{align*}
& \text { Min } F_{1}=\sum_{i \in I}\left(\frac{C_{i}^{\text {rpes }}+2 C_{i}^{\text {rmos }}+C_{i}^{\text {ropt }}}{4}\right) y_{i}+\sum_{j \in J}\left(\frac{C_{j}^{\text {cpes }}+2 C_{j}^{\text {cmos }}+C_{j}^{\text {copt }}}{4}\right) y_{j}  \tag{20}\\
& +\sum_{i \in I} \sum_{j \in j}\left(\frac{C_{i}^{\text {tpes }}+2 C_{i}^{\text {tmos }}+C_{i}^{\text {topt }}}{4}\right) q_{i j}+\sum_{j \in J} \sum_{k \in K}\left(\frac{C_{i j}^{\text {Tpes }}+2 C_{i j}^{\text {Tmos }}+C_{i j}^{\text {Topt }}}{4}\right)\left\lceil q_{j k} / W_{f}\right\rceil d_{j k} \\
& +\sum_{j \in J}\left(\frac{C_{j}^{\text {lpes }}+2 C_{j}^{\text {lmos }}+C_{j}^{\text {lopt }}}{4}\right) x_{j} N_{j}^{h}
\end{align*}
$$

$$
\begin{equation*}
\operatorname{Max} F_{2}=\left(\sum_{i \in I} \frac{\left\lceil\left(C_{a}\right)-\left(C_{u}\right)\right\rceil^{2}}{\sum i}\right)^{\frac{1}{2}} \tag{21}
\end{equation*}
$$

$$
\begin{equation*}
\operatorname{Min} F_{3}=\sum_{i \in I} \sum_{j \in J}\left(\frac{d_{1}^{\text {pes }}+2 d_{1}^{\text {mos }}+d_{1}^{\text {opt }}}{4 S_{p}}+\frac{d_{2}^{\text {pes }}+2 d_{2}^{\text {mos }}+d_{2}^{\text {opt }}}{4 S_{p p}}+\frac{d_{3}^{\text {pes }}+2 d_{3}^{\text {mos }}+d_{3}^{\text {opt }}}{4 S_{c}}\right) q_{i j} \tag{22}
\end{equation*}
$$

$$
\begin{align*}
& \sum_{i \in I} q_{i j} \leq S_{i} y_{i} \quad \forall j \in J  \tag{23}\\
& \sum_{j \in J} q_{j k} \leq S_{j} y_{j} \quad \forall k \in K  \tag{24}\\
& \sum_{i \in I} q_{i j} \geq \frac{\lambda}{2} \frac{D_{j 1}+D_{j 2}}{2}+\left(1-\frac{\lambda}{2}\right) \frac{D_{j 3}+D_{j 4}}{2} \quad \forall j \in J  \tag{25}\\
& \frac{\lambda}{2} \cdot \frac{D_{j 1}+D_{j 2}}{2}+\left(1-\frac{\lambda}{2}\right) \frac{D_{j 3}+D_{j 4}}{2} \geq \sum_{k \in K} q_{j k} \quad \forall j \in J  \tag{26}\\
& \sum_{j \in J} q_{i j} \leq x_{j} \frac{\lambda}{2} \cdot \frac{x_{j 1}+x_{j 2}}{2}+\left(1-\frac{\lambda}{2}\right) \frac{x_{j 3}+x_{j 4}}{2} \mathrm{R}_{j}^{1} \quad \forall \mathrm{i} \in I  \tag{27}\\
& q_{i j}, q_{j k} \geq 0, \forall i, j, k ;  \tag{28}\\
& y_{i}, y_{j} \in\{0,1\}, \forall i, j ; \tag{29}
\end{align*}
$$

According to Jiménez's approach, it is supposed that the constraints in the model should be satisfied with a confidence value which is denoted as $\lambda$ and it is normally determined by decision makers. Also, mos, pes and opt are the three prominent points (the most likely, the most pessimistic and the most optimistic values), respectively (Jiménez et al., 2007).
3.3 Optimization approaches
3.3.1 The developed approach

With the developed approach the multi-criterion model can be transformed into a singlecriterion model which is formulated by optimizing each criterion individually. This singlecriterion model aims to minimize the scalarized differences between each criterion and its optimal value. Undesired deviations are proposed to be subtracted from the single criterion function with the aim to achieve more accurate criterion values. These values are close enough to Pareto-optimal solutions which lead to a clear insight of a compromised solution between conflicting criteria for decision makers.

The solution function $(F)$ is formulated as follows:

$$
\begin{equation*}
\operatorname{Min} F=\left(\sum_{n=1}^{3} \sum_{f=1}^{3} \vartheta_{n} \mu_{f}(x)\right)-F_{d}, \sum_{n=1}^{3} \vartheta_{n}=1 \tag{30}
\end{equation*}
$$

Set $\vartheta_{n}^{*}=\frac{\vartheta_{n} F_{n}^{*}}{F_{n}^{*}-F_{n}}$, then

$$
\begin{equation*}
F_{d}=\vartheta_{1}^{*} F_{1}+\vartheta_{2}^{*} F_{2}+\vartheta_{3}^{*} F_{3}=\frac{\vartheta_{1} F_{1}^{\bullet}}{F_{1}^{\bullet}-F_{1}} F_{1}+\frac{\vartheta_{2} F_{2}^{\bullet}}{F_{2}^{\bullet}-F_{2}} F_{Z}+\frac{\vartheta_{3} F_{3}^{\bullet}}{F_{3}^{\bullet}-F_{3}} F_{3} \tag{31}
\end{equation*}
$$

Based on the aforementioned procedures, the above criterion function can be expressed further as follows.

$$
\begin{equation*}
\operatorname{Min} F=\left(\vartheta_{1} \mu_{1}-\vartheta_{2} \mu_{2}-\vartheta_{3} \mu_{3}\right)-\left(\frac{\vartheta_{1} F_{1}^{\bullet}}{F_{1}^{\bullet}-F_{1}} F_{1}+\frac{\vartheta_{2} F_{2}^{\bullet}}{F_{2}^{\bullet}-F_{2}} F_{2}+\frac{\vartheta_{3} F_{3}^{\bullet}}{F_{3}^{\bullet}-F_{3}} F_{3}\right) \tag{32}
\end{equation*}
$$

Subject to equations 4-10.

### 3.3.2 The SO approach

In this approach, the auxiliary crisp model in section 3.2 is converted to a mono-criterion function using the following solution formula (Selim and Ozkarahan, 2008):

$$
\begin{equation*}
\operatorname{Max} \lambda(x)=\gamma \lambda_{o}+(1-\gamma) \sum_{f \in F} \theta_{f} \lambda_{f} \tag{33}
\end{equation*}
$$

Subject to:

$$
\begin{align*}
& \lambda_{o}+\lambda_{f} \leq \mu(x), \quad f=1,2,3  \tag{34}\\
& x \in F(x), \quad \lambda_{o} \text { and } \lambda \in[0,1] \tag{35}
\end{align*}
$$

In which, the value of variable $\lambda_{0}=\min \mu\{\mu(x)\}$, which indicates the minimum satisfaction degree for each criterion function. Also, $\lambda_{f}$ refers the difference between the satisfaction degree of each criterion and minimum satisfaction degree of criteria $\left(\lambda_{f}=\mu(x)-\lambda_{0}\right)$.

### 3.4 The decision making algorithm

The next step after revealing the Pareto solutions is to determine the best trade-off solution. The best Pareto optimal solution can be determined based on decision maker's preferences or by using a decision making algorithm, although there are a number of approaches which can be utilized to determine the best solution in multi-criterion problems. In this study, the technique namely TOPSIS (order preference by similarity to ideal solution) was employed for revealing the best trade-off solution. This approach can be used for selecting a solution nearest to the ideal solution, but also the farthest from the negative ideal solution (Ramesh et al., 2012).

Assume $\quad \operatorname{PR}-\left\{\mathrm{PR}_{\mathrm{op}} \mid \mathrm{o}=1,2, \ldots, \mathrm{x}\right.$ (number of pareto solutions); $\mathrm{p}=1,2, \ldots, \mathrm{y}$ (number of criteria) $\}$ refers the $x^{*} y$ decision matrix, where $P R$ is the performance rating of alternative Pareto solutions with respect to criterion function values. Thus, the normalized selection formula is presented as follows:

$$
\begin{equation*}
N P R=\frac{P R_{o p}}{\sum_{p=1}^{o} P R_{a p}} \tag{36}
\end{equation*}
$$

The amount of decision information can be measured by the entropy value as:
$E_{p}=\frac{-1}{\ln \mathrm{x}} \sum_{o=1}^{x} P R_{o p} \ln \left(P R_{o p}\right)$

The degree of divergence $\mathrm{D}_{\mathrm{p}}$ of the average intrinsic information under $p=1,2,3,4$ can be calculated as follows:

$$
\begin{equation*}
D_{p}=1-E_{p} \tag{38}
\end{equation*}
$$

The weight for each criterion function value is given by:

$$
\begin{equation*}
w_{p}=\frac{D_{p}}{\sum_{k=1}^{y} D_{k}} \tag{39}
\end{equation*}
$$

Thus, the criterion weighted normalized value is given by:

$$
\begin{equation*}
v_{o p}=w_{o} P R_{o p} \tag{40}
\end{equation*}
$$

Where, wo refers to a weight in alternatives which are normally assigned by the decision makers.

The positive ideal solution $\left(\mathrm{AT}+\right.$ ) and the negative ideal solution $\left(\mathrm{AT}^{-}\right)$are taken to generate an overall performance matrix for each Pareto solution. These values can be expressed as below:

$$
\begin{align*}
& A T^{+}=\left(\max \left(v_{o 1}\right) \max \left(v_{o 2}\right) \max \left(v_{o y}\right)\right)=\left(v_{1}^{+}, v_{2}^{+}, \ldots, v_{y}^{+}\right)  \tag{41}\\
& A T^{-}=\left(\min \left(v_{o 1}\right) \min \left(v_{o 2}\right) \min \left(v_{o y}\right)\right)=\left(v_{1}^{-}, v_{2}^{-}, \ldots, v_{y}^{-}\right)
\end{align*}
$$

A distance between alternative solutions can be measured by the n-dimensional Euclidean distance. Thus, the distance of each alternative from the positive and negative ideal solutions is given as:

$$
\begin{align*}
& D_{p}^{+}=\sqrt{\left\{\sum_{o=1}^{y}\left(v_{o p}-v_{o}^{+}\right)^{2}\right\}}, \quad p=1,2, \ldots, x  \tag{42}\\
& D_{p}^{-}=\sqrt{\left\{\sum_{o=1}^{y}\left(v_{o p}-v_{o}^{-}\right)^{2}\right\}}, \quad p=1,2, \ldots, x \tag{43}
\end{align*}
$$

The relative closeness to each of values of alternative solutions to the value of the ideal solution is expressed as follows:

$$
\begin{equation*}
r c_{p}=\frac{D_{p}^{-}}{D_{p}^{+}+D_{p}^{-}}, \quad p=1,2, \ldots, x \tag{44}
\end{equation*}
$$

Where $D_{p}^{-} \geq 0$ and $D_{p}^{+} \geq 0$, then, clearly, $r c_{p} \in[1,0]$

The trade-off solution can be selected with the maximum $r c_{p}$ or $r c_{p}$ listed in descending order. Fig. 2 shows a flowchart of the proposed optimization methodology.


Fig. 2. Flowchart of the optimization methodology.

## 4. Application and evaluation

In this section, a case study was used for examining the applicability of the developed tricriterion model and evaluating the performance of the proposed optimization methodology. A range of application data is presented in Table 1. It is assumed that (1) width, length and height of each rack are $\mathrm{W}=0.3 \mathrm{~m}, \mathrm{~L}=18 \mathrm{~m}$ and $\mathrm{H}=5 \mathrm{~m},(2)$ the distance between the start of a spiral conveyer to the end of a collection points is 2 m and (3) the pusher is located at the center of each rack. All these parameters are taken from a real-world automated warehouse design; the
prices of RFID equipment and its implementation were estimated based on the marketing prices. The optimizer of the developed tri-criterion model is LINGO ${ }^{11}$. All computational experiments were conducted on a laptop with a 2.60 GHz CPU and a 4 G memory.

Table 1. Application data used for the case study.

| $I=12$ | $\mathrm{C}_{i}^{t}=0.25 £$ | $\mathrm{~d}_{j k}=20-45 \mathrm{~m}$ | $\mathrm{~d}_{1}=0.1-4 \mathrm{~m}$ |
| :--- | :--- | :--- | :--- |
| $J=15$ | $\mathrm{C}_{j k}^{T}=0.4-0.7 £$ | $\mathrm{~S}_{\mathrm{c}}=35 \mathrm{~m} / \mathrm{s}$ | $\mathrm{d}_{2}=0.3 \mathrm{~m}$ |
| $K=2$ | $R_{j}^{l}=100$ | $W=48$ | $\mathrm{~d}_{3}=7-23 \mathrm{~m}$ |
| $\mathrm{C}_{j}^{l}=6.5-9 £$ | $S_{i}=25-35 \mathrm{~K} £$ | $\mathrm{D}_{\mathrm{j}}=6 \mathrm{~K}-9 \mathrm{~K}$ | $\mathrm{~S}_{\mathrm{p}}=1 \mathrm{~m} / \mathrm{s}$ |
| $\mathrm{C}_{\mathrm{i}}^{r}=60-90 \mathrm{~K} £$ | $S_{j}=20-29 \mathrm{~K} £$ | $C_{j}^{c}=15-18 \mathrm{~K} £$ | $\mathrm{~S}_{\mathrm{pp}}=0.8 \mathrm{~m} / \mathrm{s}$ |

### 4.1 Results and discussions

This section presents the results which were obtained based on the developed fuzzy tri-criterion model using the proposed fuzzy solution approaches for the problem previously defined. The solution steps of the developed model are described as follows:

1) Obtain the upper and lower values for each criterion function by solving them individually. The results are $\left(\left\{U_{F_{i}}, L_{F_{i}}\right\}\right)=(\{504,1,230\},\{0.66,0.94\},\{4.27,12.25\})$.
2) Find the respective satisfaction degree $\mu\left(x_{i}\right)$ for each criterion function. The satisfaction degrees are reported in Table 2.

Table 2. Result of satisfaction degree of each criterion function.

| $\mu\left(x_{1}\right)$ | 0.95 | 0.93 | 0.85 | 0.81 | 0.7 | 0.623 | 0.6 | 0.55 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mu\left(x_{2}\right)$ | 0.7 | 0.78 | 0.83 | 0.88 | 0.92 | 0.97 | 0.98 | 0.99 |
| $\mu\left(x_{3}\right)$ | 0.97 | 0.96 | 0.93 | 0.90 | 0.85 | 0.84 | 0.81 | 0.76 |

3) Convert the multi-objective crisp model to a single criterion model using (i) the developed approach by assigning weight values shown in Table 3 and (ii) the SO approach by assigning the value of $\gamma$ which is set as 0.33 by the decision makers who consider a balance in importance of each of the three criteria. The two approaches are compared by assigning different $\lambda$ levels. Table 4 shows the computational results obtained using the two approaches. Accordingly, Table 5 shows the corresponding optimum numbers of storage racks and collection points that should be established. Fig. 3 illustrates Pareto optimal fronts among the three criterion functions obtained by using the two approaches.

Table 3. Assignment of weight values for obtaining Pareto solutions using two approaches.

| \# | Criteria weights |  |  |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| $\vartheta_{1}, \theta_{1}$ | $\vartheta_{2}, \theta_{2}$ | $\vartheta_{3}, \theta_{3}$ |  |
| 1 | 1 | 0 | 0 |
| 2 | 0.9 | 0.05 | 0.05 |
| 3 | 0.8 | 0.1 | 0.1 |
| 4 | 0.7 | 0.15 | 0.15 |
| 5 | 0.6 | 0.2 | 0.2 |
| 6 | 0.5 | 0.25 | 0.25 |
| 7 | 0.4 | 0.3 | 0.3 |
| 8 | 0.3 | 0.35 | 0.35 |

Table 4. The results obtained by assigning the varying $\lambda$ values to each of the three criterion functions.

| \# | $\lambda$-level | Developed approach |  | SO approach |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Min $\mathrm{F}_{1}$ <br> (K£) | Max $\mathrm{F}_{2}$ <br> (\%) | Min $\mathrm{F}_{3}$ <br> (h) | Run time <br> (s) | Min $\mathrm{F}_{1}$ <br> (K£) | Max $F_{2}$ <br> (\%) | Min $\mathrm{F}_{3}$ <br> (h) | Run time <br> (s) |
| 1 | 0.3 | 504 | 0.66 | 4.29 | 2 | 504 | 0.66 | 4.29 | 2 |
| 2 | 0.4 | 595 | 0.71 | 5.31 | 2 | 595 | 0.71 | 5.31 | 3 |
| 3 | 0.5 | 678 | 0.78 | 6.51 | 2 | 681 | 0.78 | 6.58 | 2 |
| 4 | 0.6 | 795 | 0.84 | 7.75 | 1 | 790 | 0.84 | 7.69 | 3 |
| 5 | 0.7 | 894 | 0.89 | 8.92 | 3 | 913 | 0.89 | 9.12 | 4 |
| 6 | 0.8 | 978 | 0.92 | 10.18 | 4 | 1053 | 0.93 | 11.91 | 3 |
| 7 | 0.9 | 1064 | 0.93 | 11.97 | 4 | 969 | 0.92 | 10.33 | 4 |
| 8 | 1 | - | - | - | - | 1096 | 0.94 | 12.19 | 4 |

Table 5. The optimal numbers of storage racks and collection points that should be established.

| \# | Developed approach |  | SO approach |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Opened storage racks | Opened collection points | Opened storage racks | Opened collection points |
| 1 | 6 | 9 | 6 | 9 |
| 2 | 6 | 9 | 6 | 9 |
| 3 | 7 | 8 | 7 | 8 |
| 4 | 9 | 11 | 9 | 11 |
| 5 | 10 | 12 | 10 | 13 |
| 6 | 11 | 13 | 12 | 14 |
| 7 | 11 | 13 | 11 | 13 |
| 8 | - | - | 12 | 15 |

4) Select the best solution using the TOPSIS method, the scored values of Pareto-optimal solutions are reported in Table 6.

Table 6. Pareto-optimal solutions ranked based on scores using the TOPSIS method.

| Developed approach |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Solution | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Score | 0.245 | 0.234 | 0.266 | 0.245 | 0.2544 | 0.279 | 0.273 | - |
| SO approach |  |  |  |  |  |  |  |  |
| Solution | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Score | 0.245 | 0.234 | 0.266 | 0.245 | 0.2544 | 0.267 | 0.273 | 0.243 |

As mentioned above, Table 4 and 5 show the obtained two sets of Pareto-optimal solutions, respectively, which were obtained based on the three criterion functions to determine the
numbers of storage racks and collection points that should be established. For instance, solution 1 shown in Table 4 is obtained using the developed approach under an assignment of $\vartheta_{1}=1, \vartheta_{2}=0$ and $\vartheta_{3}=0$, it gives the minimum total cost of $504 \mathrm{~K} £$, the maximum capacity utilization of $66 \%$ and the minimum travel time for all the requested products of 4.29 h . The result shown in Table 5, the solution consists of six storage racks and nine collection points and these trade-off results are obtained based on the three criteria towards the minimization of total cost, the maximization of capacity utilization and the minimization of travel time. Nevertheless, as shown in Fig. 3, with the Pareto optimal method, it cannot generate a better overall result by gaining one best result based on one criterion function without worsening the results in the other criterion functions, although all Pareto-optimal solutions are feasible. It proves the confliction among the three criteria. For instance, an increase in the desired value of criterion two (e.g. maximization of capacity utilization) leads to an increase in the undesired value of criterion one (e.g. minimization of total cost).

It can be noted in Table 4 that by increasing the satisfaction level $\lambda$, it leads to an increase in the undesired value of the first and third criterion functions (e.g. minimization of total cost and minimization of travel time, respectively). Although it yields an increase in the desired value of the second criterion function (e.g. maximization of capacity utilization). In this case, decision makers have to spend more money to cope with the uncertainties. However, decision makers can vary weight the importance $\left(\vartheta_{n}\right.$, or $\left.\theta_{f}\right)$ of each of the three criterion functions and the satisfaction level $\lambda$ based on their preferences in order to obtain another compromised solution.

Through a comparison of the two sets of Pareto-optimal solutions shown in Table 4, the values obtained based on the three criterion functions using the developed approach are more balanced than those (of solutions 6-8) using the SO approach. The optimization run time of using the
developed approach for the eight iterations was slightly faster than the SO method. It also indicates that there is no feasible solution obtained using the developed approach when the weight for the first criterion (minimization of total cost) is set less than 0.4 . This implies that decision makers cannot ignore the importance of cost as it yields an inapplicable warehouse design. In other words, with the developed approach it gives a more realistic and balanced solution.


Fig. 3. Pareto optimal fronts among the three criterion functions obtained by the two approaches.

After obtaining a set of Pareto-optimal solutions, decision makers may determine a solution depending on their preferences or using a decision making algorithm. In this work, the TOPSIS method was employed to select the best solution. As shown in Table 6, solution 6 is chosen as the best solution as its score is the highest $\left(\boldsymbol{r} \boldsymbol{c}_{p}=0.279\right)$ with the total cost of $£ 978 \mathrm{~K}, 92 \%$
capacity utilization and the travel time of 10.18 h . Also, it requires an establishment of eleven storage racks to supply products to thirteen collection points.

## 5. Conclusions

In this research, a design of the proposed RFID-enabled automated warehousing system was studied using the multi-objective optimization approach. The work was involved in optimization of the design in terms of (1) allocating the optimal number of storage racks and collection points that should be established and (2) obtaining a trade-off decision between the negative impact of costs and the positive impact of maximization of the warehouse capacity utilization and minimization of travel time of products travelling from storage racks to collection points. To this aim, a tri-criterion programming model was developed and the model was also converted to be a fuzzy programming model for incorporating parameters in varying which include demands, costs and random locations of items in a warehouse. A two-stage solution methodology was proposed to solve the fuzzy multi-criterion optimization problem. At the first stage, the developed approach and the SO approach were used for obtaining two Pareto-optimal sets. The results, which were obtained using the two different approaches, are compared and it shows that both approaches are appropriate and efficient for the fuzzy multicriterion model; for revealing a trade-off decision among the considered criteria. Nevertheless, the developed approach has more advantages, which includes (1) the solutions gained using this approach are more balanced than using the SO approach (2) with the developed approach, the run time (s) is slightly faster than using the SO approach and (3) it gives more realistic solutions for an applicable warehouse design. In the second stage, the TOPSIS method was employed to reveal the best Pareto solution. Finally, implementation of a case study demonstrates the applicability of the developed model and the effectiveness of the proposed optimization methodology which can be useful as an aid for optimizing the design of the RFIDenabled automated warehousing system.

An interesting research study derived from this work may be a comparison between the RFIDenabled automated warehousing system and the non-RFID-enabled automated warehousing system in terms of these three criteria (e.g. minimization of total cost, maximization of capacity utilization and minimization of travel time). It was also suggested to compare the developed solution approach with the other available approaches such as e-constraint and augmented econstraint. Finally, by optimizing the developed model by a meta-heuristic algorithm may be useful for handling the large-sized problems in a reasonable time.

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