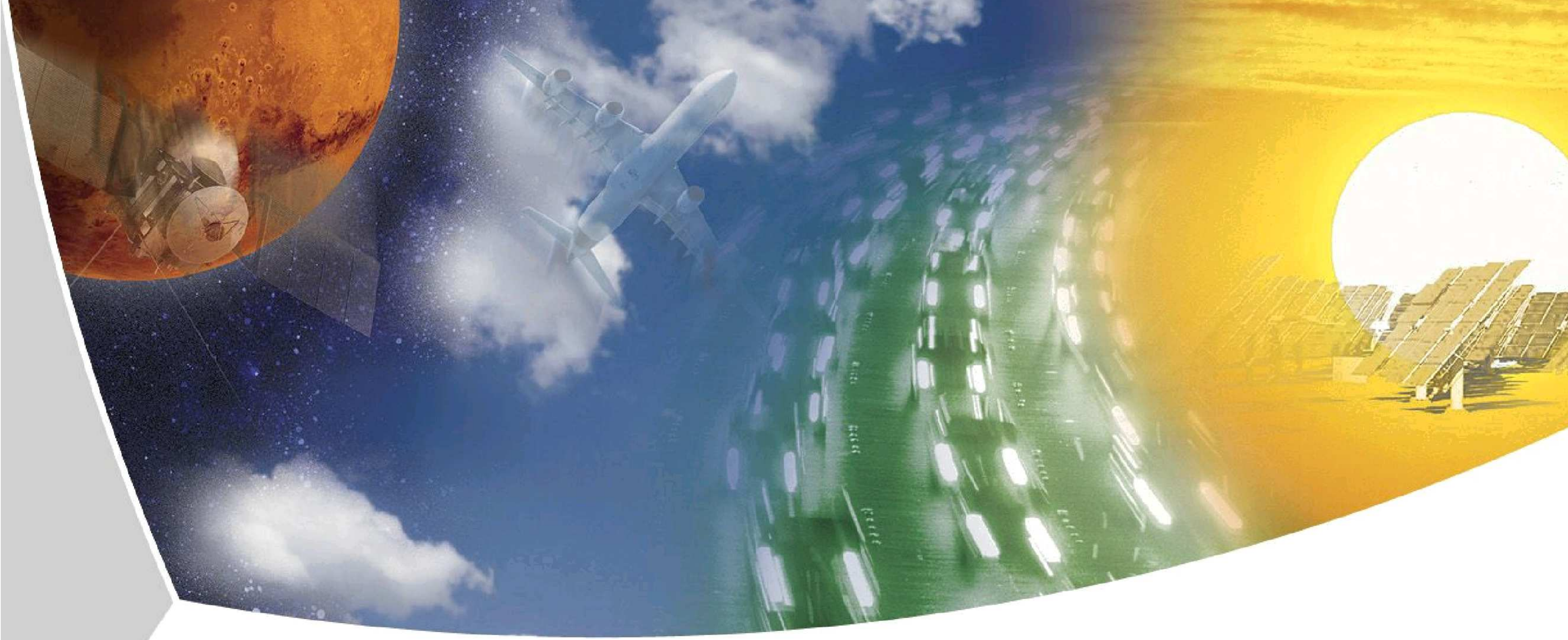

Noise sources from moving surfaces
– Therorетical background –

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Overview

- Lighthills acoustic analogy with surface sources
  - Physical interpretation of source terms
- Integration in a moving reference frame
  - Ffowcs Williams and Hawkings equation
- Thickness noise
- Loading noise

Lighthills acoustic analogy with surface sources

Permeable surface:

$$\left(\frac{\partial^2}{\partial t^2} - c^2\Delta\right)\{\rho' H(f)\} = \frac{\partial^2}{\partial x_i x_j}\{T_{ij}H(f)\}$$
$$+\frac{\partial}{\partial t}\left(\{\rho(v_i - u_i) + \rho_0 u_i\}\frac{\partial f}{\partial x_i}\delta(f)\right)$$
$$-\frac{\partial}{\partial x_i}\left(\{\rho v_i(v_j - u_j) + P_{ij}\}\frac{\partial f}{\partial x_j}\delta(f)\right)$$

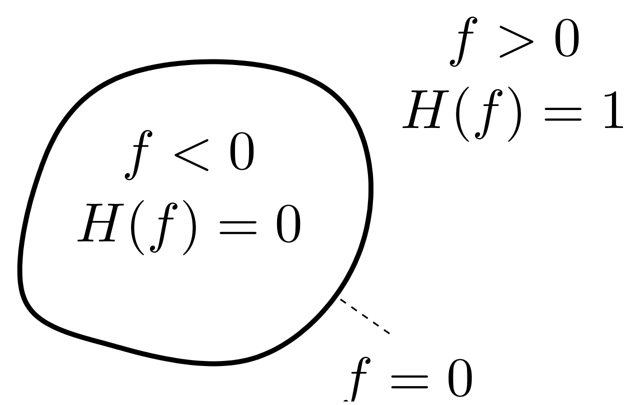
$f(\vec{x}, t)$: Auxiliary function, at the surface is $f = 0$

v_i: Velocity of the medium

u_i: Velocity of the surface $f = 0$

$P_{ij} = (p - p_0)\delta_{ij} - \tau_{ij}$

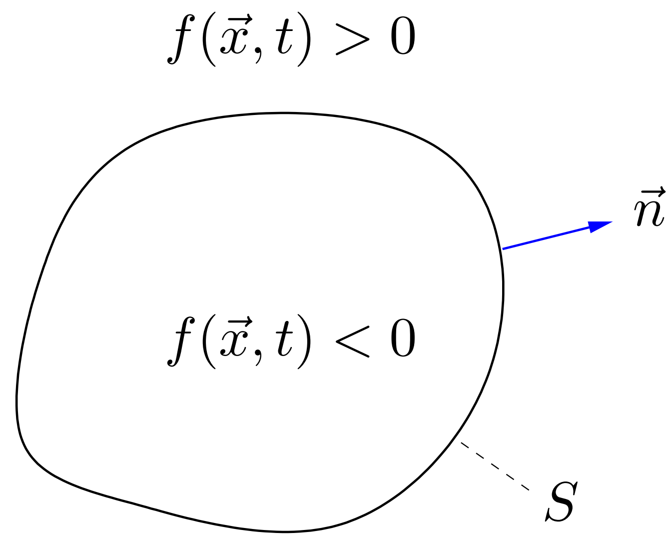
H: Heaviside function



The auxiliary function $f(\vec{x}, t)$

S: Surface $f = 0$
Surface can move and deform
$\vec{n}$: Normal vector

$$\frac{\partial f}{\partial x_i} = n_i \, |\mathrm{grad} f|$$



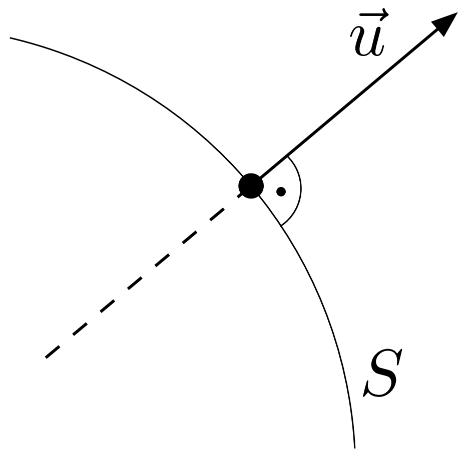
$f(\vec{x}, t)$ is not uniquely defined!

Surface distribution: $\dfrac{\partial f}{\partial x_i} \delta(f) = n_i \, |\mathrm{grad} f| \, \delta(f)$

To let f vanish after integration, the factor $|\mathrm{grad} f|$ is necessary.

Surface movement

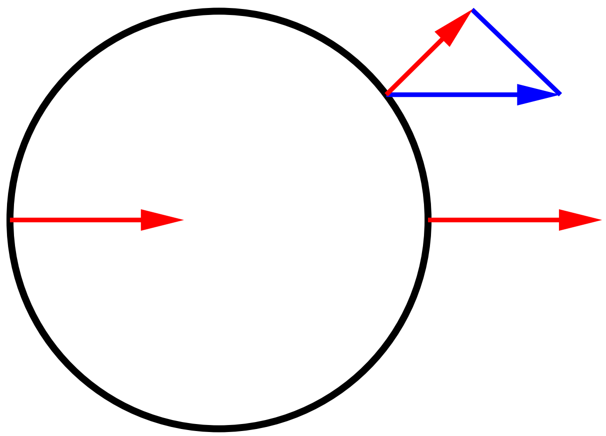
Normal velocity $\vec{u}$



$$\frac{\partial f}{\partial t} + u_i \frac{\partial f}{\partial x_i} = 0$$

No parametric description of the surface S!

Example: Sphere moving at constant speed



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Impermeable surface

S: Surface of solid body $\rightarrow$ no flow through S:

$$\left(\frac{\partial^2}{\partial t^2} - c^2\Delta\right)\{\rho' H(f)\} = \frac{\partial^2}{\partial x_i x_j}\{T_{ij}H(f)\}$$
$$+\frac{\partial}{\partial t}\{\rho_0 u_n \left|\text{grad } f\right| \delta(f)\}$$
$$-\frac{\partial}{\partial x_i}\{l_i \left|\text{grad } f\right| \delta(f)\}$$

$u_n = u_i n_i$
$l_i = P_{ij}n_j = (p - p_0)\, n_i - \tau_{ij}n_j$

$\rho_0 u_n$: Rate at which mass is displaced by the body
$-l_i$: Force per area exerted from the medium on the body

Comparison with moving point sources

Mass source:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta\right) p' = \frac{\partial}{\partial t}\left\{\rho_0\,\dot{\beta}(t)\,\delta(\vec{x} - \vec{x}_\mathrm{s}(t))\right\}$$

$\vec{x}_\mathrm{s}$: Position of the source
β: Volume displaced by the source
$\rho_0\dot{\beta}$: Rate at which mass is displaced

Momentum source:

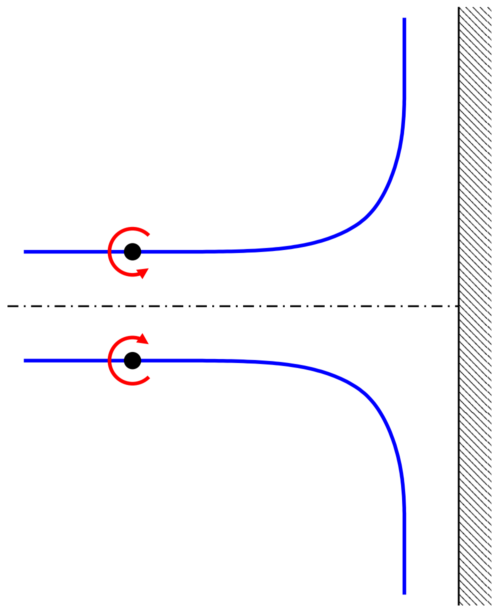
$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta\right) p' = -\frac{\partial}{\partial x_i}\,\{f_i(t)\,\delta(\vec{x} - \vec{x}_\mathrm{s}(t))\}$$

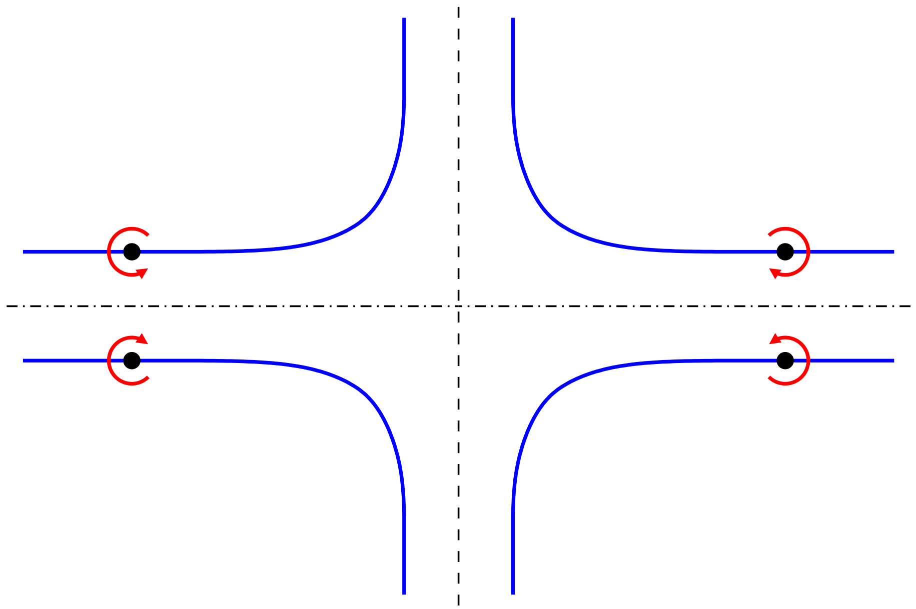
f_i: Force acting on the medium

Virtual and real physical sources

- Real physical source: Energy is transferred from the flow into acoustic perturbations
- Source terms in acoustic analogy can be considered as virtual sources
- Surface sources replace boundary conditions

Example: Ring vortex impinging on a wall





Solution of acoustic analogy with surface sources

Ffowcs Williams and Hawkings, 1969

S coincides with the surface of a rigid and impermeable body

$$4\pi c^2 \rho'(\vec{x},t) = \frac{\partial^2}{\partial x_i \partial x_j} \int\limits_{\mathbb{R}^3} \left[\frac{T_{ij}}{r\,|1-M_r|}\right]_{\tau=\tau^*} \mathrm{d}^3\vec{\eta}$$

$$+ \frac{\partial}{\partial t} \int\limits_{S} \left[\frac{\rho_0 u_n}{r\,|1-M_r|}\right]_{\tau=\tau^*} \mathrm{d}S(\vec{\eta})$$

$$- \frac{\partial}{\partial x_i} \int\limits_{S} \left[\frac{l_i}{r\,|1-M_r|}\right]_{\tau=\tau^*} \mathrm{d}S(\vec{\eta})$$

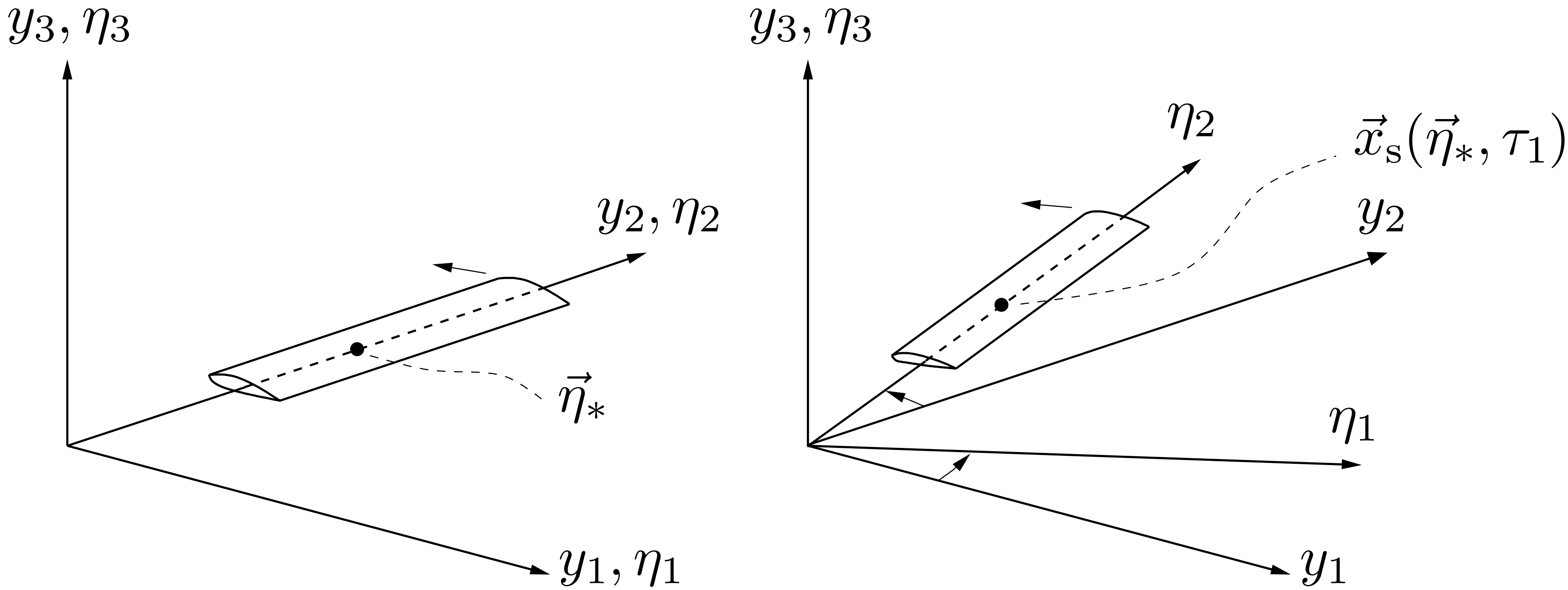
$\vec{\eta}$: Coordinate in moving reference frame

cM_r: Source velocity in the direction of the observer

τ^*: Retarded times

Body-fixed coordinate system

Example: Single rotor blade



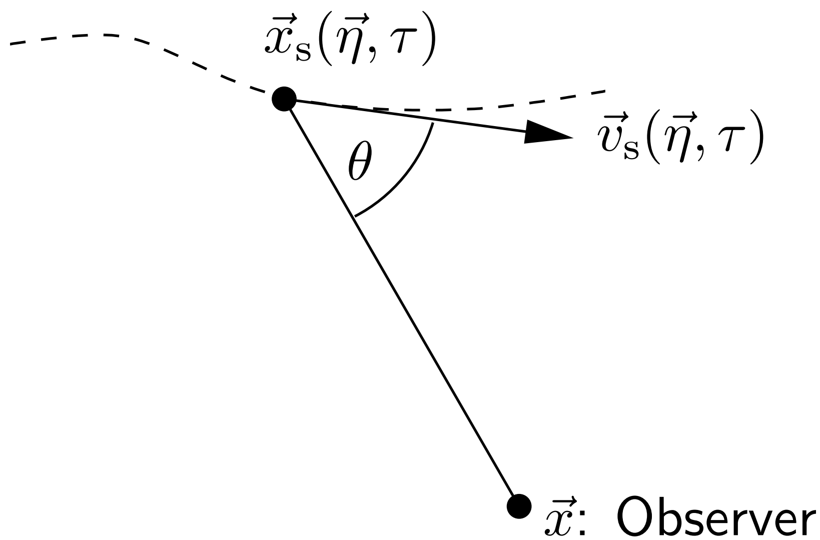
Surface S stationary in $\vec{\eta}$-system: $f = f(\vec{\eta})$

$\vec{x}_\mathrm{s}(\vec{\eta}_*, \tau_1)$: Position of point with coordinate $\vec{\eta}_*$ in $\vec{y}$-space at time τ_1

Approach does not work for flexible bodies!

Source velocity

For each coordinate in the $\vec{\eta}$-system a position $\vec{x}_\mathrm{s}(\vec{\eta},\tau)$ and a velocity $\vec{v}_\mathrm{s} = \frac{\partial \vec{x}_\mathrm{s}}{\partial \tau}$ in the $\vec{y}$-space can be defined



$$M_r(\vec{x},\vec{\eta},\tau) = \frac{|\vec{v}_\mathrm{s}|}{c}\cos\theta$$

Mach number of point with coordinate $\vec{\eta}$ in the direction of the observer

Square brackets

Summation:

$$\left[\frac{q}{r|1-M_r|}\right]_{\tau=\tau^*} = \sum_{n=1}^{N} \frac{q(\vec{\eta},\tau_n^*)}{r(\vec{x},\vec{\eta},\tau_n^*)\,|1-M_r(\vec{x},\vec{\eta},\tau_n^*)|}$$

$\tau_n^* = \tau_n^*(\vec{x},\vec{\eta},t)$ solution of

$$c\cdot(t-\tau^*) = |\vec{x}-\vec{x}_{\mathrm{s}}\,(\vec{\eta},\tau^*)\,|$$

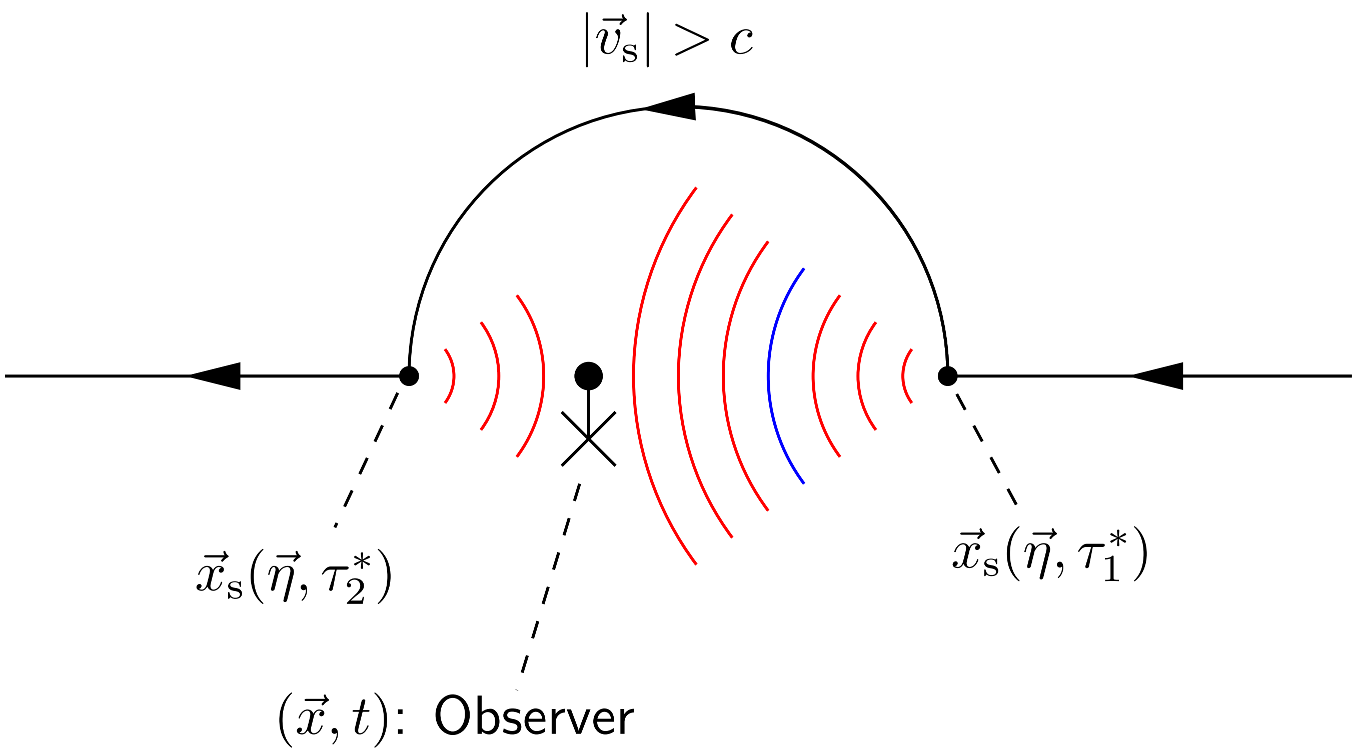
$N = N(\vec{x},\vec{\eta},t)$: Number of solutions τ_n^*
If $|\vec{v}_{\mathrm{s}}(\vec{\eta},\tau)| < c$ for all τ, then $N=1$

$$r(\vec{x},\vec{\eta},\tau) = |\vec{x}-\vec{x}_{\mathrm{s}}\,(\vec{\eta},\tau)\,|$$

Super-sonic source motion

Example: Super-sonic source point

Consider fix $\vec{\eta}$:



$$N(\vec{x}, \vec{\eta}, t) = 2$$

Sonic boom

Example:

Unaccelerated movement of source point $\vec{x}_\mathrm{s}(\vec{\eta}, \tau)$

$|\vec{v}_\mathrm{s}| > c$

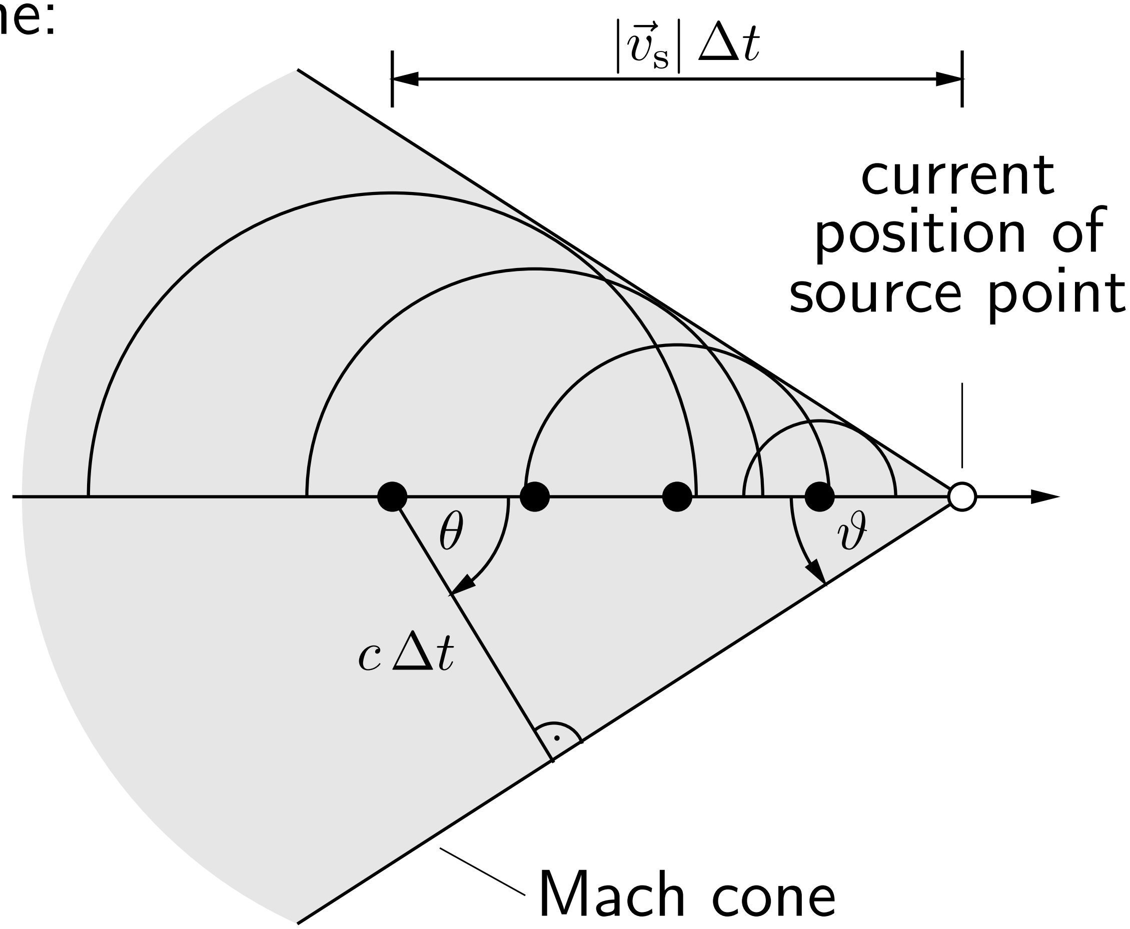
Observer on the Mach cone:

$$\cos\theta = \frac{c\,\Delta t}{|\vec{v}_\mathrm{s}|\,\Delta t} = \frac{c}{|\vec{v}_\mathrm{s}|}$$

$$M_r = \frac{|\vec{v}_\mathrm{s}|}{c}\cos\theta$$

$$\rightarrow M_r = 1$$

$\dfrac{1}{|1 - M_r|}$ is singular



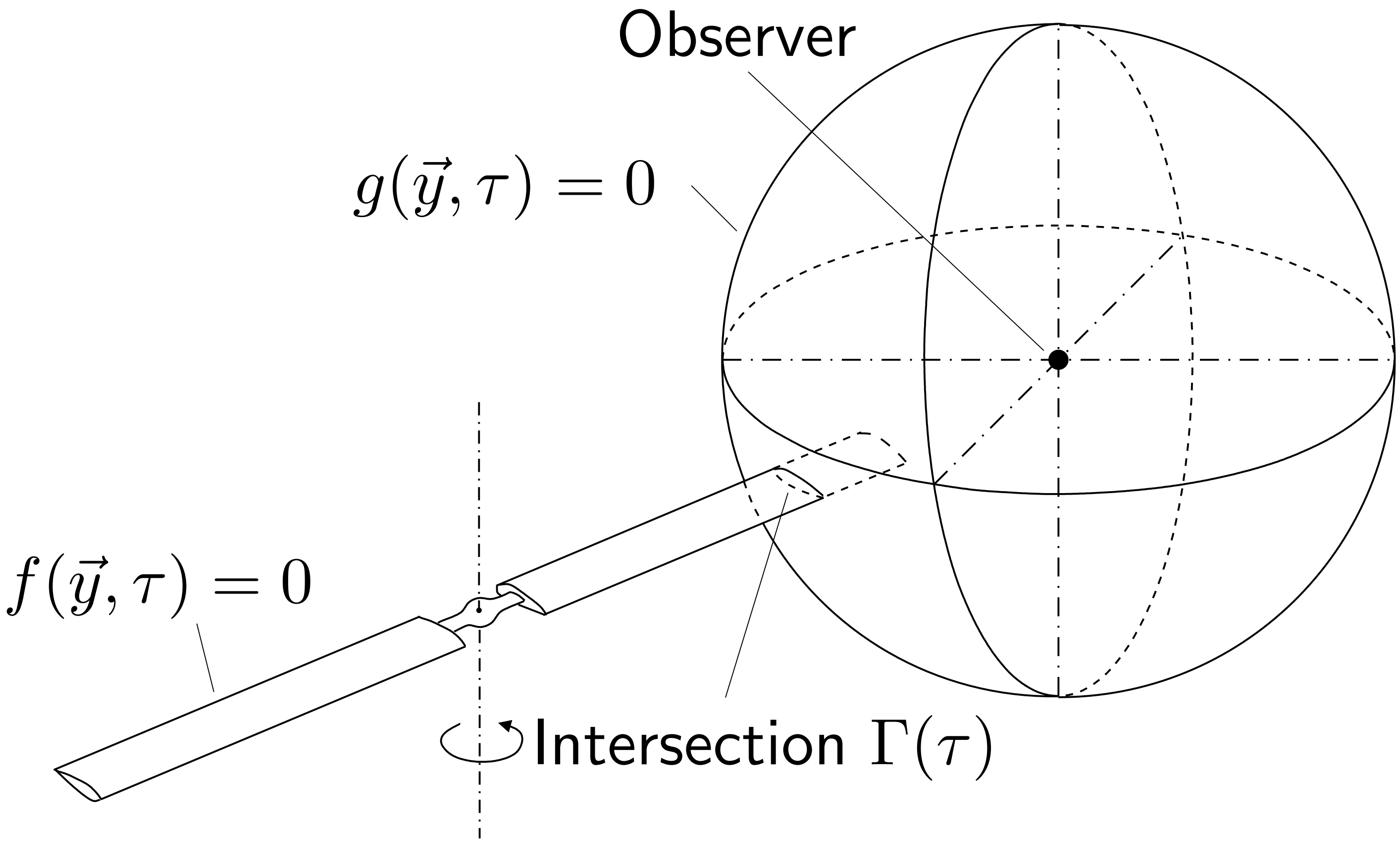
Observed source geometry

Consider all sources on the surface of a rigid body which are received simultaneously:
Observed geometry is a set of points in $\vec{y}$-space:

$$\Sigma(\vec{x}, t) = \{\vec{y} \mid \vec{y} = \vec{x}_{\mathrm{s}}(\vec{\eta}, \tau_n^*) \text{ for all } \vec{\eta} \in S \text{ and valid } \tau_n^* = \tau_n^*(\vec{x}, \vec{\eta}, t)\}$$

$\vec{\eta} \in S$ if and only if $f(\vec{\eta}) = 0$
Σ coincides with S only if the body is at rest
Σ may have a strange geometry!

Collapsing sphere



Surface Σ is set union of all lines $\Gamma(\tau)$ for all $\tau < t$

$$g(\vec{x},t,\vec{y},\tau) = t - \frac{|\vec{x}-\vec{y}|}{c} - \tau$$

Radius of sphere: $c\,(t-\tau)$

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$$- \frac{\partial}{\partial x_i} \int\limits_{S} \left[\frac{l_i}{r\,|1-M_r|} \right]_{\tau=\tau^*} \mathrm{d}S(\vec{\eta})$$

$\rho_0 u_n$: Rate at which mass is displaced by the body

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Comparison with moving point sources

Mass source:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta\right) p' = \frac{\partial}{\partial t}\left\{\rho_0\,\dot{\beta}(t)\,\delta(\vec{x} - \vec{x}_{\mathrm{s}}(t))\right\}$$

$$p'(\vec{x}, t) = \frac{\partial}{\partial t}\left[\frac{\rho_0\dot{\beta}}{4\pi r|1 - M_r|}\right]_{\tau=\tau^*} = \frac{\partial}{\partial t}\left\{\sum_{n=1}^{N}\frac{\rho_0\dot{\beta}(\tau_n^*)}{4\pi r|1 - M_r|}\right\}$$

Momentum source:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta\right) p' = -\frac{\partial}{\partial x_i}\{f_i(t)\,\delta(\vec{x} - \vec{x}_{\mathrm{s}}(t))\}$$

$$p'(\vec{x}, t) = -\frac{\partial}{\partial x_i}\left[\frac{f_i}{4\pi r|1 - M_r|}\right]_{\tau=\tau_*} = -\frac{\partial}{\partial x_i}\left\{\sum_{n=1}^{N}\frac{f_i(\tau_n^*)}{4\pi r|1 - M_r|}\right\}$$

Thickness noise

Formulation of Farassat:

Only one part of the surface sources

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta\right) p'_{\mathrm{T}} = \frac{\partial}{\partial t}\left\{\rho_0 u_n \left|\mathrm{grad} f\right| \delta(f)\right\}$$

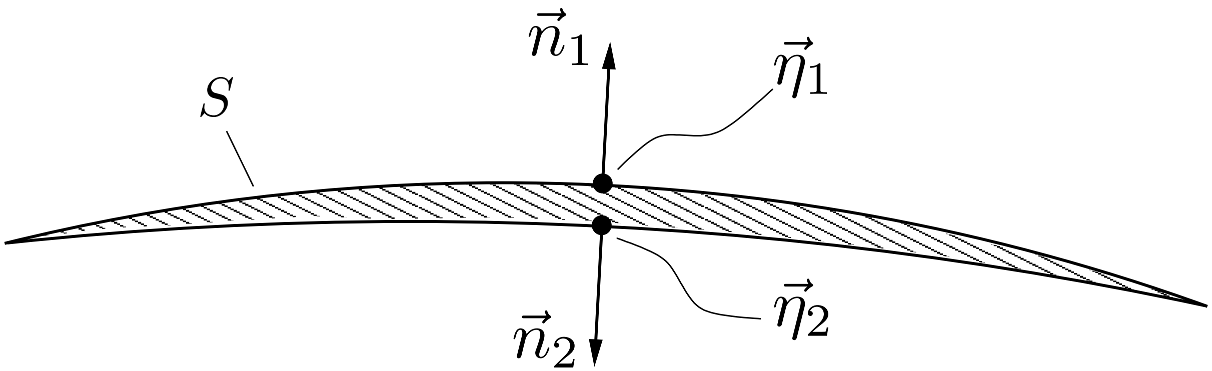
Solution:

$$4\pi p'_{\mathrm{T}}(\vec{x}, t) = \frac{\partial}{\partial t}\int\limits_S \left[\frac{\rho_0 u_n}{r|1 - M_r|}\right]_{\tau=\tau^*} \mathrm{d}S(\vec{\eta})$$

Rigid body:

$$\int\limits_S \rho_0 u_n \, \mathrm{d}S(\vec{\eta}) = 0$$

Thin body



Opposite points: $\vec{\eta}_1$ and $\vec{\eta}_2$

If $\vec{\eta}_1$ and $\vec{\eta}_2$ are close to each other

$$\tau^*(\vec{x}, \vec{\eta}_1, t) \approx \tau^*(\vec{x}, \vec{\eta}_2, t)$$

$$u_n(\vec{\eta}_1, \tau^*(\vec{\eta}_1)) \approx -u_n(\vec{\eta}_2, \tau^*(\vec{\eta}_2))$$

$$M_r(\vec{\eta}_1, \tau^*(\vec{\eta}_1)) \approx M_r(\vec{\eta}_2, \tau^*(\vec{\eta}_2))$$

Infinite thin body

It the body becomes thinner:

$$|\vec{\eta}_1 - \vec{\eta}_2| \to 0$$

and

$$\left[\frac{\rho_0 u_n}{r|1-M_r|}\right]_{\tau=\tau^*}(\vec{\eta}_1) + \left[\frac{\rho_0 u_n}{r|1-M_r|}\right]_{\tau=\tau^*}(\vec{\eta}_2) \to 0$$

The integral vanishes for infinite thin bodies!

A body without volume generates no thickness noise: $p'_{\mathrm{T}} = 0$

Loading noise

Again only one part of the surface sources:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta\right) p'_{\mathrm{L}} = -\frac{\partial}{\partial x_i}\left\{l_i \left|\mathrm{grad} f\right| \delta(f)\right\}$$

Solution:

$$4\pi p'_{\mathrm{L}}(\vec{x},t) = -\frac{\partial}{\partial x_i}\int\limits_S \left[\frac{l_i}{r|1-M_r|}\right]_{\tau=\tau^*} \mathrm{d}S(\vec{\eta})$$

Total force from the body acting on the medium:

$$F_i = \int\limits_S l_i \,\mathrm{d}S(\vec{\eta})$$

Force term with spatial derivative

Reformulation:

$$
\begin{aligned}
&4\pi p'_{\mathrm{L}}(\vec{x}, t) \\
&= -\frac{\partial}{\partial x_i} \int\limits_S \left[\frac{l_i}{r|1 - M_r|} \right]_{\tau=\tau^*} \mathrm{d}^3\vec{\eta} \\
&= \frac{\partial}{\partial t} \int\limits_S \left[\frac{l_r}{cr\,|1 - M_r|} \right]_{\tau=\tau^*} \mathrm{d}^3\vec{\eta} + \int\limits_S \left[\frac{l_r}{r^2|1 - M_r|} \right]_{\tau=\tau^*} \mathrm{d}^3\vec{\eta}
\end{aligned}
$$

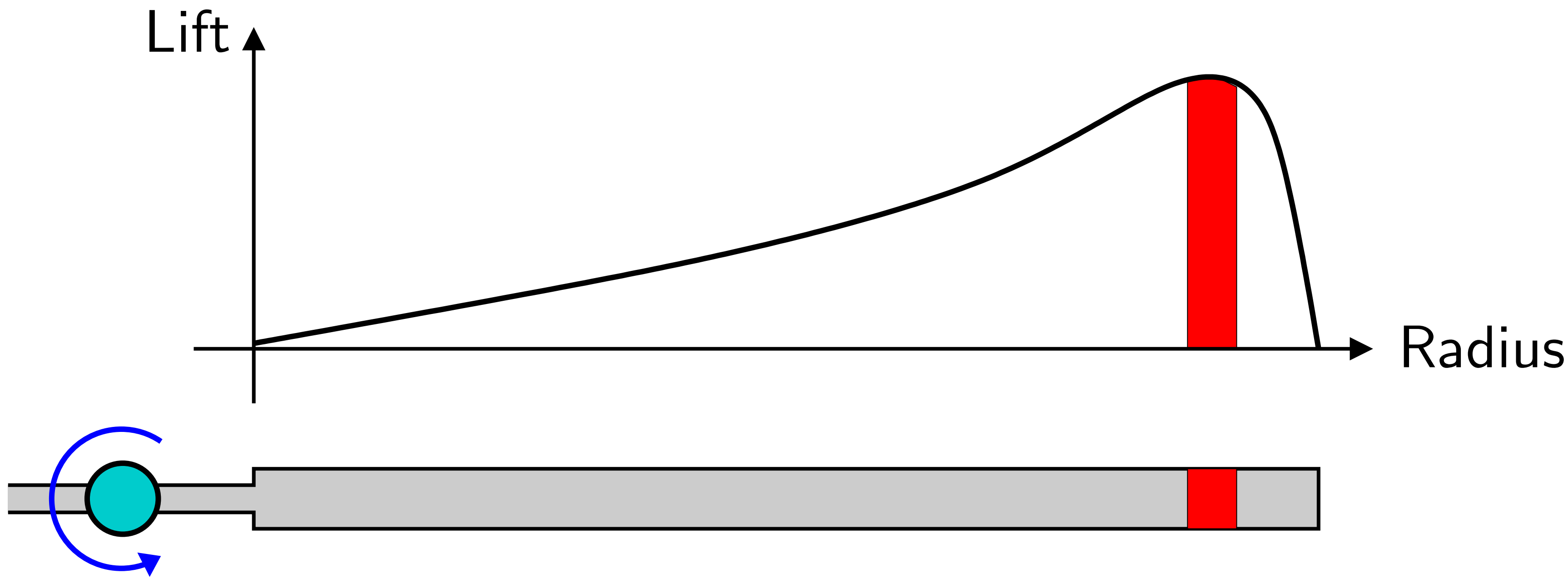
$$
l_r = l_i \left(\frac{x_i - x_{\mathrm{s},i}(\vec{\eta}, \tau)}{r} \right) = \vec{l} \left(\frac{\vec{x} - \vec{x}_{\mathrm{s}}(\vec{\eta}, \tau)}{r} \right)
$$

l_r: Component of the force l_i in the direction from the source position at $\vec{x}_{\mathrm{s}}(\vec{\eta}, \tau)$ towards the observer at $\vec{x}$

Spanwise lift distribution

Rotor blade of helicopter:

Hover flight

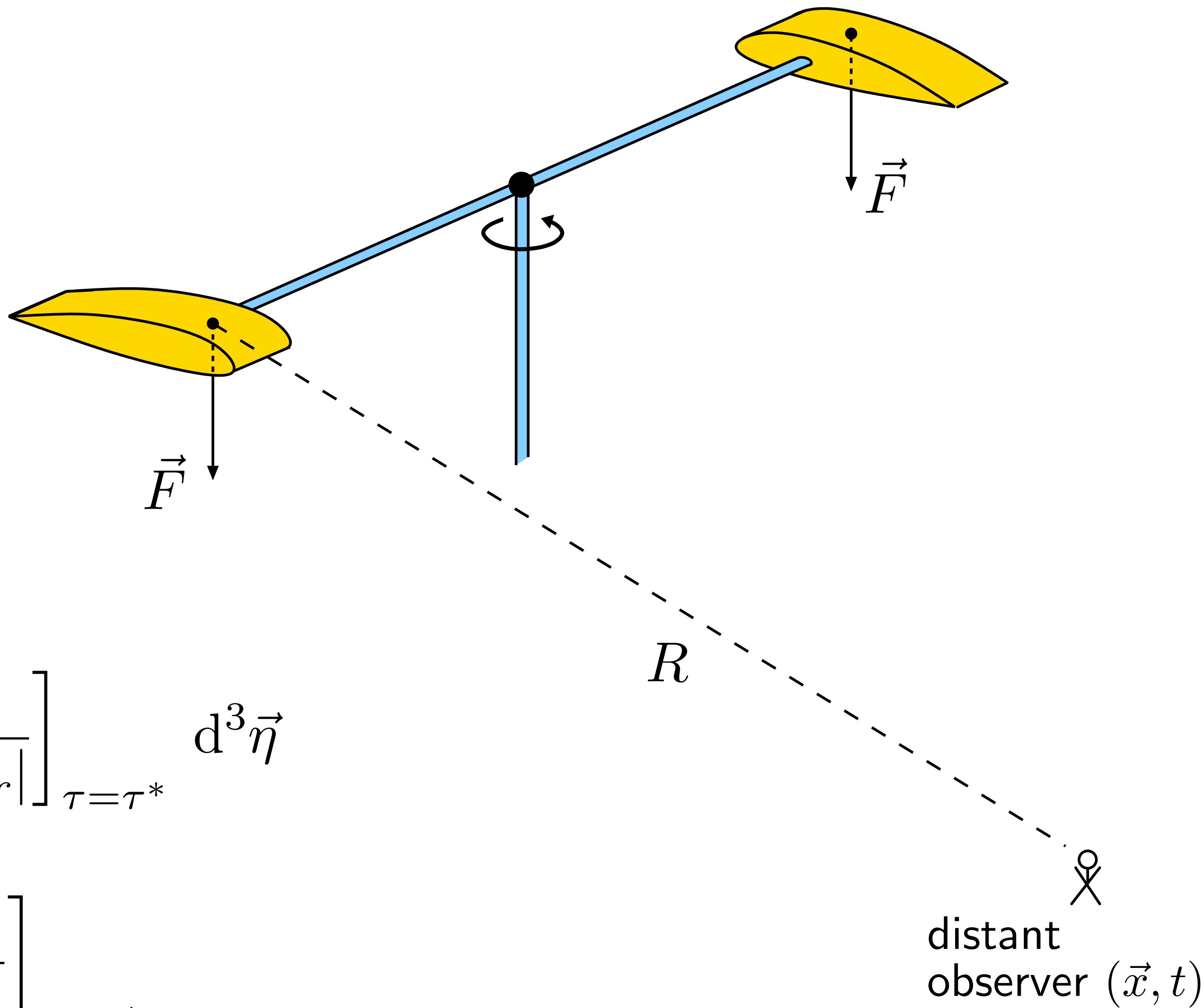


Simplified rotor

Hover flight

$\vec{F}$ force on medium

$\vec{F}$ steady



Loading noise:

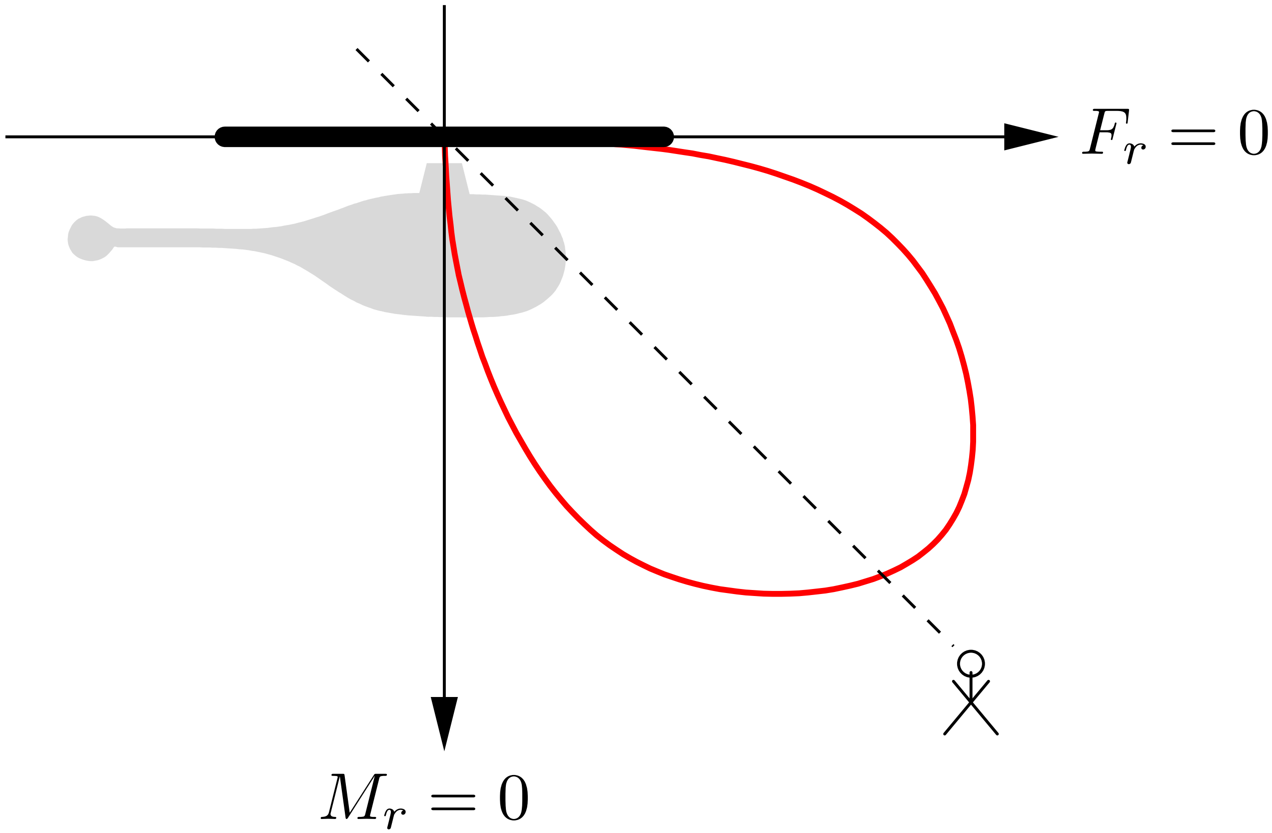
$$4\pi p'_{\mathrm{L}}(\vec{x}, t)$$

$$\approx \frac{\partial}{\partial t} \int\limits_S \left[\frac{l_r}{cr\,|1 - M_r|} \right]_{\tau=\tau^*} \mathrm{d}^3\vec{\eta}$$

$$\approx \frac{1}{c} \frac{\partial}{\partial t} \left[\frac{F_r}{R\,|1 - M_r|} \right]_{\tau=\tau^*}$$

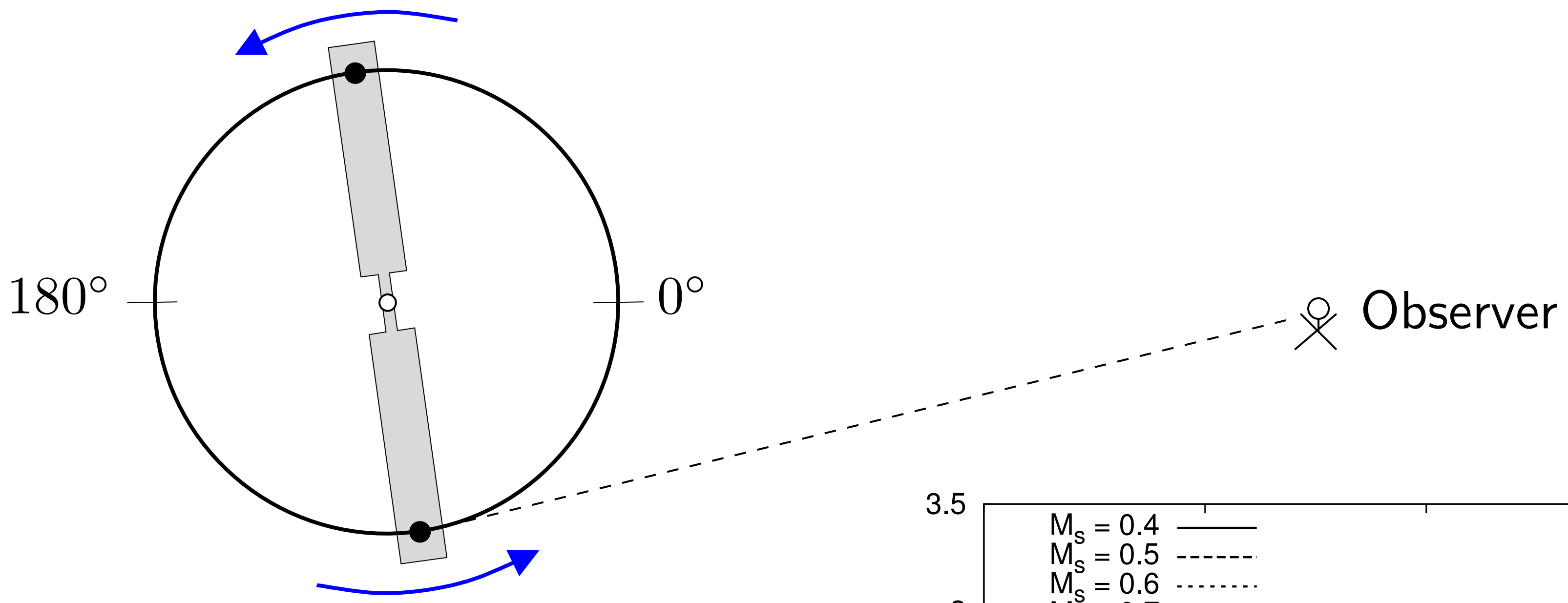
Directivity

Hover flight
Simplified rotor
Loading noise

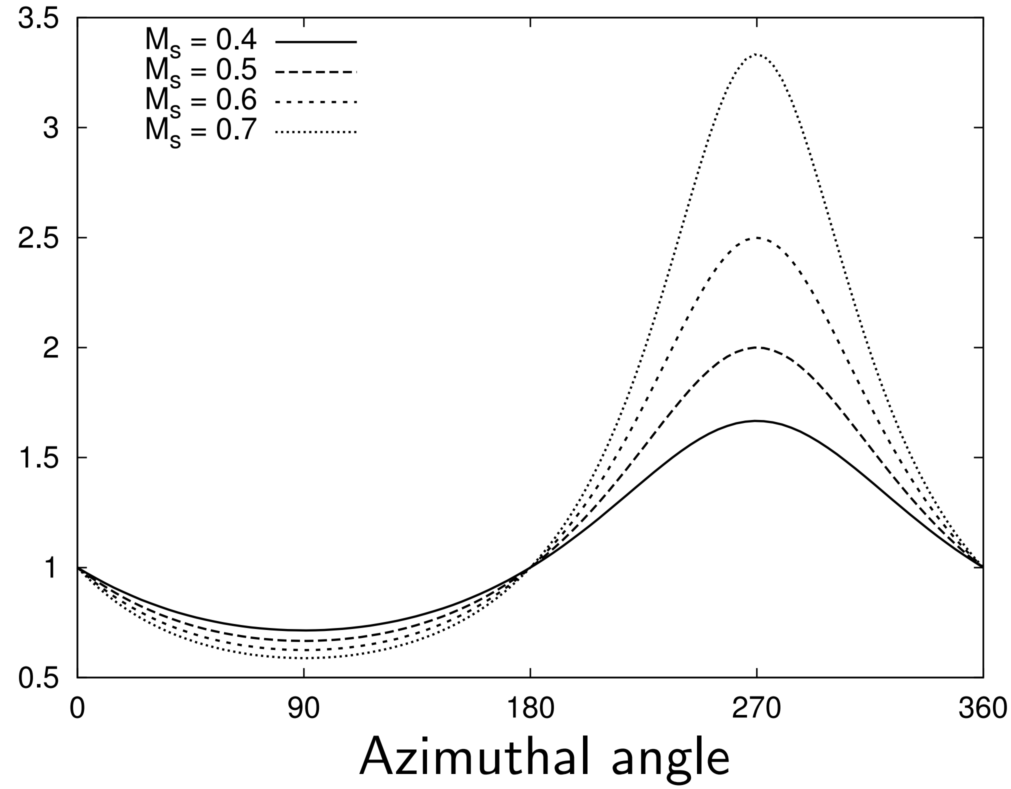


$$4\pi p'_{\mathrm{L}}(\vec{x}, t) \approx \frac{1}{c} \frac{\partial}{\partial t} \left[\frac{F_r}{R\,|1 - M_r|} \right]_{\tau=\tau^*}$$

Azimuthal variation of source strength



$$\frac{1}{|1 - M_r|}$$



Hover flight

Simplified rotor

Observer angle β

relative to rotor plane:

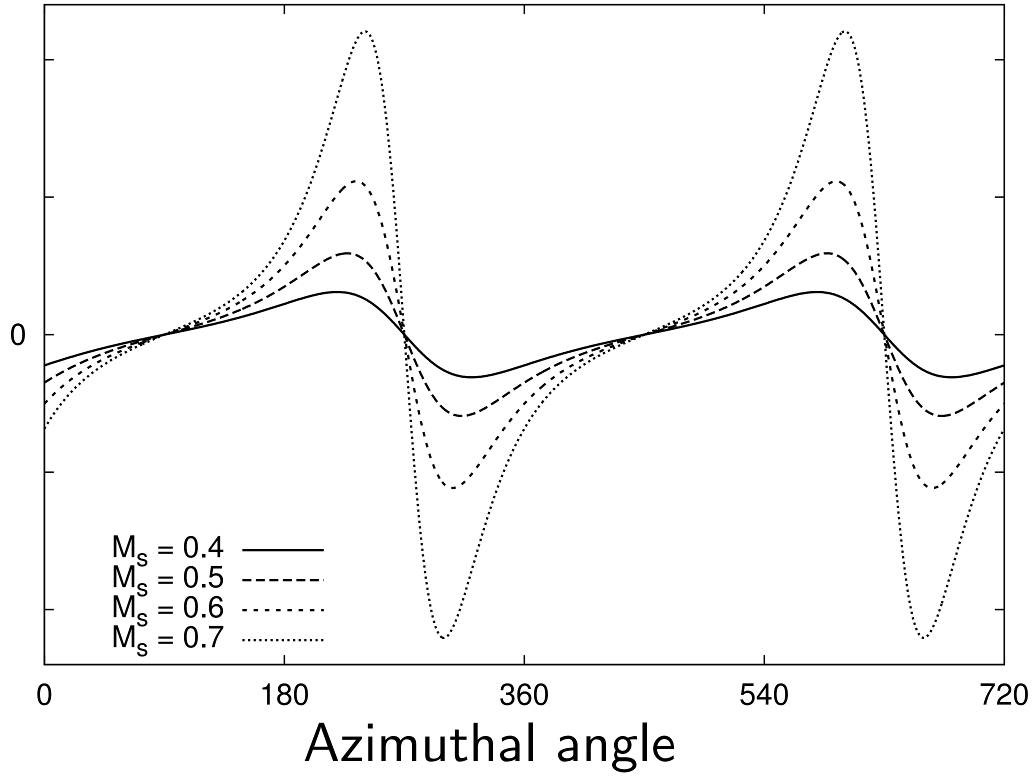
$M_\mathrm{s} = |\vec{v}_\mathrm{s}|/c \cdot \cos\beta$

Loading-noise signal

Hover flight with simplified rotor

Point source in circular motion

$$p'_L \sim \frac{\partial}{\partial t}\left(\frac{1}{|1 - M_r|}\right)$$



$\longrightarrow$ same lift at less angular velocity reduces noise

$\longrightarrow$ higher harmonics in the signal

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Concluding remarks

- A moving point source can be used as simple model for a rotor blade
- Propeller blade is analog to rotor blade
- A perfectly silent helicopter is theoretically not possible
- Lower rotor frequency $\rightarrow$ less noise
- Real helicopter in forward flight is much more complicated