# A SAR Conjugate Mirror

David Hounam

German Aerospace Center, DLR,

Microwaves and Radar Institute

Oberpfaffenhofen, D-82234 Wessling, Germany

Fax: +49 8153 28 1449, E-Mail: David.Hounam@dlr.de

*Abstract*-- A radar transponder was constructed, which modifies the received signal such that its complex conjugate is returned to the radar, qualities of the conjugate mirror used in optics and acoustics. For a monostatic SAR, a perfect conjugate mirror will reflect a signal back to the radar with no phase shift due to the propagation path. The signal received by the transponder is also decorrelated from other targets, enhancing the transponder signal in the SAR image. The paper describes a transponder operated as a SAR conjugate mirror and an experiment with ERS-1, demonstrating the feasibility and characteristics. The significance for transponder design is addressed and possible applications discussed.

***Index terms*-- SAR, conjugate mirror, transponder.**

## I. INTRODUCTION

Like any conventional mirror, a passive radar reflector changes the direction of the incident radar pulse without significant alteration of the signal characteristics. A conjugate mirror (CM) not only changes the direction of the pulse but, by forming its complex conjugate, effectively reverses the phase modulation in time. Conjugate mirrors are used in lasers to increase their spectral purity and in acoustics to sharpen ultrasound medical images. Optical conjugate mirrors have been constructed using crystal lattices, utilising Brouillin scattering [1] and 3rd order harmonic mixing (four wave mixing) [2]. In acoustics piezoelectric elements have been used. In lasers and ultrasound imagers, the principle is the same. An incident wave, adulterated by phase fluctuations on the forward path, is corrected by reversing the phase modulation before it passes back down the same path. The phase modulations compensate one another, due to the summing of the conjugate waves.

The measurement quantity of remote sensing radars, the reflections of microwave signals from natural and man-made features, are characterised by single and multiple mirror effects, and no mechanism is known that would produce conjugation of the waves. However, in a radar transponder, phase shifts can be reversed, enabling a conjugate mirror to be constructed with similar potential benefits as in optics and acoustics. Also, because the reflections from the surroundings are not conjugated, all deterministic components will be suppressed relative to those of the CM. Although, here, a SAR conjugate mirror is discussed, the principles apply to all coherent radars.

## II. RADAR PHASE CONJUGATION

For a side-looking radar with a narrow antenna beam mounted on a platform moving at constant velocity and neglecting higher terms, the phase variation over time due to platform motion, i.e. the Doppler phase modulation utilised by a synthetic aperture radar, can be written as:

$$\phi_{AZ}(t) = \frac{\pi \cdot V_P{}^2 \cdot t^2}{\lambda \cdot r_o} + \frac{\pi \cdot V_P{}^2 \cdot (t + \Delta t)^2}{\lambda \cdot r_o} \qquad (1)$$

The first term represents the phase shift on the forward path of the radar signal and the second term that of the reverse path, after reflection. $V_P$ is the platform velocity, $r_0$ the range at closest approach (assumed large) and $\lambda$ the wavelength. $t = 0$ is the time of closest approach. $\Delta t$ is the time delay between the transmitted and received signals at the radar, i.e. $\approx 2 \cdot r_0 / c$.

For a space-borne radar, $V_P$ is replaced by $V_{Peff}$, the effective platform velocity, related to $V_P$ and the velocity of the beam over the ground $V_B$ [3]:

$$V_{Peff} = \sqrt{V_P \cdot V_B} \qquad (2)$$

If the left-hand phase term is reversed by conjugation, the parts of the expression compensate one another, except for a small linear phase variation corresponding to a frequency shift:

$$f_{TAZ} = -\frac{V_{Peff}^{\;2} \cdot \Delta t}{\lambda \cdot r_o} \tag{3}$$

For the ERS-1 satellite used in the experiment below, $f_{TAZ}$ is approx. -5.8 Hz. Hence, the response of a CM is to eliminate the Doppler modulation and replace it by a small frequency offset. As a consequence, no synthetic aperture can be constructed and the signal amplitude response has to be analysed differently to derive the azimuth position (see chapter IV below).

In the range direction, any range modulation to expand the pulse is reversed by conjugation. Similar to the Doppler example above, the phase or frequency shift due to the delay on the forward path is compensated by a corresponding negative shift on the reverse path. The pulse remains expanded and can be compressed with a conjugated replica of the range modulation. The position of the IRF of the compressed pulse will not be influenced by conjugation, but it will exhibit no delay-dependent phase or frequency shift.

## III. CONJUGATE MIRROR TRANSPONDER

A CM exhibiting the above characteristics can be constructed using an active transponder and including a circuit for reversing the signal spectrum (see Fig. 1). The transponder has separate antennas for receive and transmit. The receive signal with a centre frequency of 5.3 GHz is amplified in a low-noise amplifier (LNA) and mixed down to 300 MHz using a local oscillator (LO) signal at 5 GHz, i.e. below the input frequency. After filtering to remove image frequencies, the signal is amplified and mixed back up to 5.3 GHz using a local oscillator signal at 5.6 GHz, i.e. higher than the SAR frequency. A positive chirp signal emitted by the radar of the form $f_R = f_0 + k_c t^2$ will be replaced by the signal $f_R = f_0 - k_c t^2 + f_{os}$, where $k_c$ is the chirp slope and $f_{os}$ is the offset between the local oscillators. The spectrum of the SAR signal applied to the high power amplifier and, hence, transmitted back to the SAR, is thus reversed, and the phase shift on the downward signal path is negated, corresponding to the characteristics of a conjugate mirror as described above.

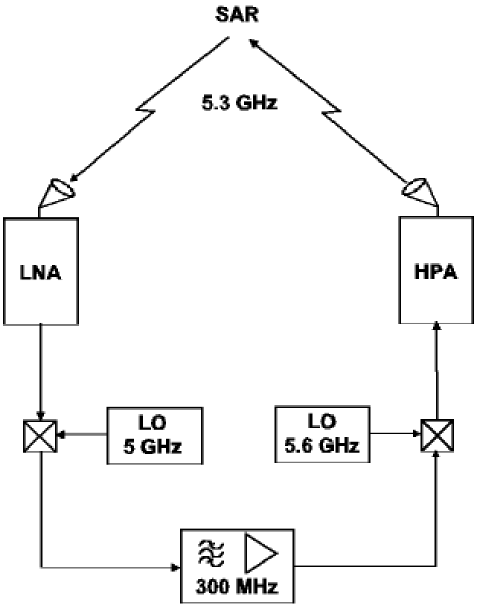

**Figure 1 Schematic of a transponder operating as a simple conjugate mirror.**

For the conjugation to be perfect, the local oscillators have to be coherent with the virtual carrier frequency of the radar signal and offset by an equal frequency difference. This couldn't be achieved in the experimental transponder. However, as shown below, the resulting frequency offset can be measured and corrected for during the processing of the data.

## IV. PROCESSING OF THE SAR SIGNAL

Because no synthetic aperture can be constructed, to detect and locate the CM transponder signal in the SAR data, a processing routine for coherent amplitude detection and peak detection has to be constructed. Below, the technique for finding the position of the azimuth peak and hence, the azimuth position is described. It is easiest to first range compress the data by correlation with the complex conjugate range modulation. For a chirp radar like ERS-1 this means compression with the reversed chirp spectrum. As the phase variation for the CM in azimuth is nominally constant, the signal is now coherently detected. Deterministic targets present in the data, such as point targets will be suppressed, due to the mismatch of the range reference function**.**

The task now is to locate the signal in the azimuth direction. This is achieved by detecting the peak signal amplitude in azimuth. For a uniformly weighted antenna pattern as for ERS-1, the amplitude of the transponder signal in azimuth will have a $\text{sinc}^2$ characteristic of the form,

$$U_{Tout} = \hat{U}_{Tin} \frac{\sin(ka_{cell})^2}{(ka_{cell})^2} \tag{4}$$

where $\hat{U}_{Tin}$ is the peak input signal voltage, $k$ is a constant and $a_{cell}$ the azimuth pixel spacing.

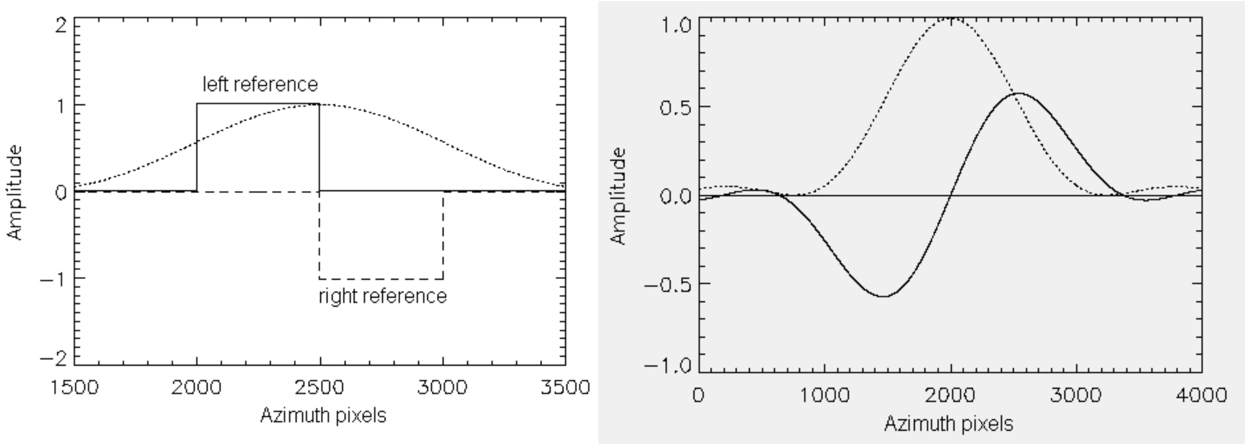

**Figure 2a Left: discriminator reference signals superimposed on the azimuth amplitude response of the SAR signal. Figure 2b Right: S-shaped response of the discriminator to a SAR signal with a $\text{sinc}^2$ amplitude response in azimuth.**

The peak detection is performed by multiplying the signal with two square-shaped reference signals, the left and right reference signals, as shown in Fig. 2a centred on the $\text{sinc}^2$ amplitude function of the coherently detected SAR signal. Integrating the combined signals, one obtains the S-shaped characteristic in Fig. 2b.

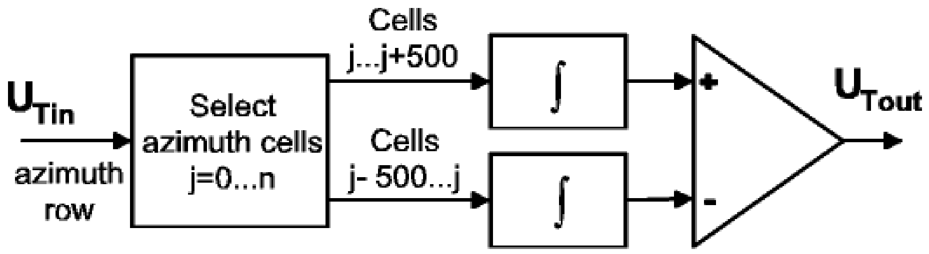

**Figure 3 Schematic of the amplitude discriminator for localising the conjugate mirror transponder signal.**

This routine is performed on the azimuth rows of the (conjugate) range compressed SAR data and can be illustrated by the discriminator circuit shown in Fig. 3. The integer j, defining the position of the reference functions as the azimuth cell number, is iterated to detect the point where the output voltage passes through 0. The width of the reference function, i.e. the number of cells integrated, was chosen to be 500 cells, corresponding approximately to the 6 dB azimuth beam width of the radar antenna, i.e. the range normally chosen for SAR processing. Note, this method of finding a maximum of an amplitude function in SAR data is often used for Doppler centroid estimation [4], which is relevant when discussing the registration accuracy.

## V. EXPERIMENTAL RESULTS

The transponder and processing method described above were used to perform an experiment with the SAR of the ERS-1 satellite. The transponder used for the experiment was set up on a balcony, i.e. close to a building and in the vicinity of many strong targets [5],[6],[7]. The transponder has different modes of operation allowing azimuth pseudo-noise encoding as well as modulation transposition (conjugate mirror), as described above.

The SAR data was first range compressed using the conjugate of the reference chirp, suppressing the surrounding deterministic targets in range. As azimuth encoding was performed by the transponder simultaneously, the signal has then to be decoded, as described in [5]. This results in further suppression of deterministic targets in azimuth. The resulting signal exhibits no Doppler modulation but an offset due to the synthesizer errors. This offset is required for coherent amplitude detection, and is found by Fourier analysis (Fig. 4). The unmodulated signal, i.e. the lack of Doppler modulation, confirms the phase conjugation. The frequency of the peak response is 284.4 Hz, is largely due to the error of the local oscillator frequencies, masking the theoretical offset (equation 3) of -5.8 Hz. The local oscillator offset is due to long-term ageing of the synthesizer oscillators and is stable for the duration of SAR data collection. It therefore has no influence on the ability to detect or process the transponder signal.

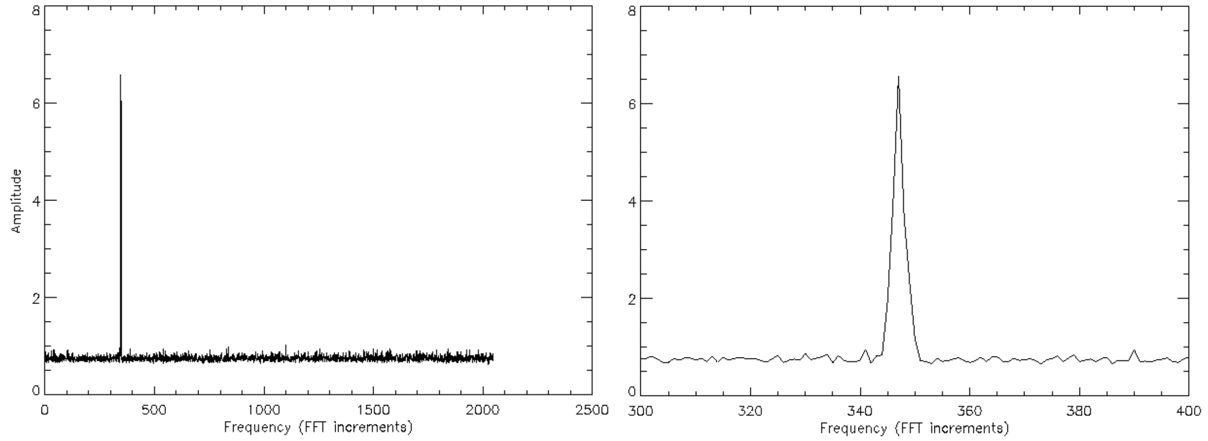

**Figure 4a Left: plot of the azimuth spectrum of the signal after range compression and decoding. showing no Doppler modulation. Figure 4b Right: expanded plot of the azimuth spectrum showing an unmodulated peak at FFT point 347 (= 284.4 Hz).**

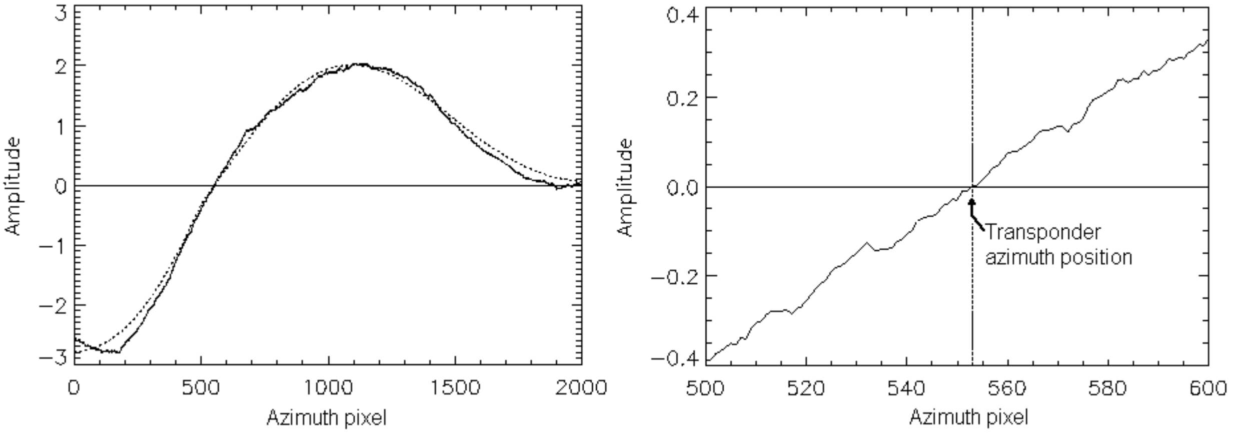

**Figure 5a Left: output from the discriminator for the experimental conjugate mirror transponder. Figure 5b Right:, expanded discriminator output showing the transponder azimuth position (zero crossing).**

The azimuth data row containing the transponder signal was searched to find its position using the coherent amplitude discriminator described above. Fig. 5a shows the resulting output function superimposed on the corresponding part of the S-curve derived as above. The characteristic shows good agreement. The position of the transponder, i.e. of the CM is at the zero crossing. Fig. 5b shows this region expanded.

The localisation accuracy is limited by the signal-to-noise ratio, largely determined by the contribution from the defocused background. At the discriminator output, the noise can be seen as a random deviation from a straight line. The standard deviation of the transponder position is 0.704 azimuth cells, or 2.88 m.

For many applications, the transponder position is needed relative to those of other features in the corresponding SAR image. The azimuth position of a target in a SAR image is directly related to the signal Doppler. Hence, if, in the SAR processor the same algorithm is used for Doppler centroid estimation as for the CM discriminator, any errors due, for instance, to asymmetry of the amplitude function, will be cancelled. Therefore, the transponder position can be accurately inserted in the SAR image.

Fig. 6 shows such the conventionally processed image with the transponder position at the centre of the circle. The transponder cannot be seen, as it is suppressed by approx. 27.6 dB in range, i.e. the range processing gain equal to the ERS-1 range time-bandwidth product. Due to the azimuth encoding, it is further suppressed by approx. 30.1 dB in azimuth, the processing gain corresponding to the code length (1023 bit).

## VI. CONCLUSIONS

The ability to detect and localise the CM transponder demonstrates that the principle of the CM works. Although SAR compression is not possible, the transponder exhibits the same amplification with respect to noise and surrounding distributed targets as conventional transponders. Deterministic targets will be suppressed relative to the CM response. This feature can be important for tagging transponders and RFID (Radio Frequency Identification) applications, due to the higher detection reliability. In this case, it can enhance the capability of the azimuth encoding transponder.

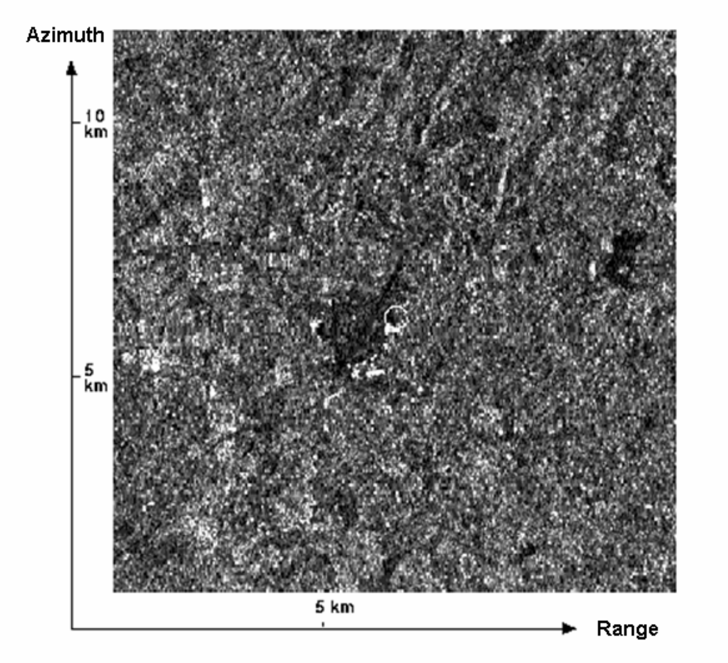

**Figure 6 Conventonally processed ERS-1 image of the surrounding area (airfield of Oberpfaffenhofen) with the (invisible) transponder position shown by a circle. Both azimuth coding as well as conjugation were in operation.**

Cancellation of phase modulation on the forward and reverse paths mean that phase fluctuations on the propagation path and within the transponder are eliminated. This feature could be utilised to construct an active target for radar calibration or as a phase stable reference for InSAR applications.

## VII. REFERENCES

[1] B. Ya. Zel'dovich, V. I. Popovichev, V. V. Ragul'skii, and F. S. Faizullov, “Connection between the wave fronts of the reflected and exciting light in stimulated Mandel'shtam-Brillouin scattering”. *Sov. Phys. JETP Lett.*, 15, 1972, pp. 109-115

[2] D. M. Pepper and A. Yariv. “Optical phase conjugation using three-wave and four-wave mixing via elastic photon scattering in transparent media”. *In R. A. Fisher, editor, Optical Phase Conjugation*,. Academic, San Diego, 1983, pp. 23-78

[3] R. K. Raney, “Considerations for SAR Image Quantification Unique to Orbital Systems”, *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 29, No. 8, Sept. 1991, pp. 754-760.

[4] S. N. Madsen, “Estimating the Doppler Centroid of SAR data“, *IEEE Transactions and Aerospace Electronic Systems*, Vol. AES-25, No. 2, March 1989

[5] D. Hounam. “Ortung und Identifizierung von Zielen in SAR-Bildern mit kodierten Transpondern“. *Deutsche Geodätische Kommission*, Reihe C, Heft Nr. 482, ISBN 3 7696 9522 4, 1998

[6] Wägel, K.-H., Hounam, D., Bauer, R., Bloetscher, H., Zink, M. , et al, “An Encoding SAR-Transponder for Target Identification”, *Proceedings IEEE, IGARSS '99*, Hamburg Germany, 1999.

[7] Hounam, D., Wägel, K., “A Technique for the Identification and Localization of SAR Targets using Encoding Transponders”. IEEE Transactions *on Geoscience and Remote Sensing*, Vol 39, 1, 2001, pp. 3-7