

Gauss-Bonnet-coupled Quintessential Inflation

Carsten van de Bruck,¹ Konstantinos Dimopoulos,² Chris Longden,¹ and Charlotte Owen²

¹*Consortium for Fundamental Physics, School of Mathematics and Statistics,
University of Sheffield, Hounsfield Road, Sheffield S3 7RH, United Kingdom*

²*Consortium for Fundamental Physics, Physics Department,
Lancaster University, Lancaster LA1 4YB, United Kingdom*

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Abstract

We study in detail a new model of quintessential inflation where the inflaton field is coupled to the Gauss-Bonnet term. This coupling ensures that the variation of the field is kept sub-Planckian, which avoids the 5th force problem as well as the lifting of the flatness of the quintessential tail in the runaway scalar potential due to radiative corrections. We find that the inflationary predictions of the model are in excellent agreement with CMB observations, while the coincidence requirement of dark energy is satisfied with natural values of the parameters, overcoming thereby the extreme fine-tuning of the cosmological constant in Λ CDM.

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I. INTRODUCTION

Cosmic inflation is widely accepted as the leading paradigm for describing the physics of the very early Universe. It entails an accelerating expansion of space in the earliest moments of the Universe's history, alleviating the infamous horizon and flatness problems of the Hot Big Bang, while also generating the near scale-invariant spectrum of initial curvature perturbations necessary to seed the anisotropies in the Cosmic Microwave Background (CMB) as probed by observational tests such as WMAP [1] and Planck [2, 3]. Another problem in cosmology, however, is the observed present accelerated expansion of space, as identified by type Ia supernova distance measurements [4] as well as CMB physics. From these, we can deduce that the Universe's present accelerated expansion is consistent with the effects of a perfect fluid (dark energy) with equation of state parameter close to -1 , that comprises some 70 % of the Universe's density [5]. While many approaches to solving these two problems in cosmology are largely independent of one another, it seems inevitable that cosmologists would try to unify explanations of these similar periods of accelerated expansion in one theory. Such an approach is what is known as quintessential inflation [6–37].

One of the main similarities between many approaches to understanding inflation and dark energy is the exploitation of a scalar field, ϕ , which has properties conducive to accelerating expansions of space. In theories which deal only with inflation, this *inflaton* field typically decays into the standard model particles once it has served its purpose. This has the advantage of simply recovering the conventional radiation and matter dominated epochs of the Hot Big Bang model, a so-called graceful exit from inflation, but leaves us with no choice but to later invoke a second new mechanism to generate dark energy. If instead, the inflaton at least partly persists following the end of inflation, it is feasible that it may eventually come to dominate the energy budget of the Universe again and give rise to dark energy.

While this idea sounds simple in principle, achieving it is theoretically challenging. For one thing, the energy scale of inflation and that of dark energy differ by more than a hundred orders of magnitude. Manually introducing an energy scale as small as that of dark energy into one's theory renders it hopelessly unstable under quantum corrections, and if one instead chooses to suppress the energy density of the inflaton by having it roll down a steep potential following inflation, one finds that a trans-Planckian excursion in field value typically occurs before the field freezes in at a particular value, spoiling the stability of the theory under UV completion. This latter approach also makes recovering the standard cosmology slightly more complicated because one needs to reheat the Universe after inflation by means other than the decay of the inflaton field. However, this is not intractable thanks to decay procedures such as instant preheating [38–40], curvaton reheating [41–43] or gravitational reheating [44, 45], which

we will also apply in this work.

In this paper we seek to study the feasibility of quintessential inflation scenarios in a modified theory of gravity in which the scalar field in the theory non-minimally couples to the Gauss-Bonnet combination of quadratic curvature scalars, $R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\rho\mu\sigma\nu}R_{\rho\mu\sigma\nu}$. Scalar-tensor theories with such a Gauss-Bonnet coupling (and to a lesser extent, vector-tensor theories [46]) have been studied extensively with applications to inflation [47–56] and dark energy [57, 58] investigated separately, as well as topics such as black hole formation [59]. This is unsurprising, as it is a well-motivated extension of General Relativity, appearing in UV theories such as string [60, 61] and braneworld-inspired [62–66] models, as well as just being a fairly natural object to consider when building gravity theories from the bottom up [67], as the simplest curvature scalar which does not add any additional propagating degrees of freedom to the theory as, say, R^2 or $R_{\mu\nu}R^{\mu\nu}$ alone do. Realisations of bouncing cosmologies have also been found in Gauss-Bonnet-containing theories [68, 69]. It is also a subset of Horndeski’s theory [70–72], guaranteeing that it has second order equations of motion and is free of instabilities.

A particular reason we are interested in the application of Gauss-Bonnet-coupled theories to quintessential inflation is because previous work [51, 52, 60] on fields with such a coupling has revealed that a common behaviour resulting from this is the impedance of motion of the coupled scalar field when the Gauss-Bonnet coupling function becomes large. This may be especially useful in overcoming the aforementioned theoretical problem where a field rolling down a quintessential potential tail after inflation becomes super-Planckian in displacement.

In section II we will begin by specifying the concrete model we will use to study these questions about the realisation of quintessential inflation in Gauss-Bonnet-coupled models. We will discuss the slow-roll inflationary dynamics and power spectra predictions in section II A, before moving on to considerations of how reheating is subsequently achieved in section II B, retrieving a Hot Big Bang universe. Then, the late-time behaviour and manifestation of dark energy from the leftover inflaton density will be described in section II C. Having shown how our model allows one to realise these three key steps in quintessential inflation, we will proceed in section III to use our results to assess the quantitative feasibility of the model by scrutinising its predictions against the observational evidence, deriving any resulting constraints on the parameter space of the model. In particular, this will involve constraints from inflation in section III A, reheating and dark energy in section III B, and lastly, local limits on modified gravity models in section III C. We then finish with some concluding remarks in section IV.

We consider natural units, where $c = \hbar = 1$ and Newton’s gravitational constant is $8\pi G = M_{\text{Pl}}^{-2}$, with $M_{\text{Pl}} = 2.43 \times 10^{18}$ GeV being the reduced Planck mass.

II. THE MODEL

Consider a theory of modified gravity in which a scalar field, ϕ , is non-minimally coupled to the Gauss-Bonnet combination of quadratic curvature scalars, $E = R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\rho\mu\sigma\nu}R_{\rho\mu\sigma\nu}$, such that

$$S = \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} [R - G(\phi)E] - \int d^4x \sqrt{-g} \left[\frac{1}{2}(\partial\phi)^2 + V(\phi) \right]. \quad (1)$$

While the Gauss-Bonnet combination is usually a total derivative that has no effect on the classical equations of motion, the coupling with the scalar field $G(\phi)$ in the above action, as long as it is a non-constant function, will allow it to play a non-trivial role. Considering the metric of a spatially flat, homogeneous and isotropic spacetime with

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \quad (2)$$

where $a(t)$ is the scale factor, the equations of motion derived from this action are

$$3M_{\text{Pl}}^2 H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi) + 12M_{\text{Pl}}^2 H^3 \dot{G}, \quad (3)$$

$$2M_{\text{Pl}}^2 \dot{H} = -\dot{\phi}^2 + 4M_{\text{Pl}}^2 H^2 (\ddot{G} - H\dot{G}) + 8M_{\text{Pl}}^2 H \dot{H} \dot{G}, \quad (4)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + 12M_{\text{Pl}}^2 H^2 G_{,\phi} (\dot{H} + H^2) = 0, \quad (5)$$

where the dot denotes a time derivative.

To achieve quintessential inflation in this model, we want the new Gauss-Bonnet effects to play a significant role at late times, while allowing for an inflationary expansion (e.g. a sufficiently flat effective potential) at early times. To this effect, we choose a fairly minimalistic coupling function of the form

$$G(\phi) = G_0 e^{-q\phi/M_{\text{Pl}}}, \quad q > 0, \quad (6)$$

where the prefactor should satisfy $G_0 M_{\text{Pl}}^2 \geq 1$ in a sub-Planckian theory. For large positive $q\phi$ values, this will be typically negligible, while for negative $q\phi$ it will quickly grow in magnitude. Thus, in a scenario in which $q\phi$ begins at large positive values and rolls down to negative values at late times, this coupling may behave as needed. The other component of the model, the potential, is consequently taken to be

$$V(\phi) = \frac{V_0}{2} \left[1 + \tanh \left(p \frac{\phi - \phi_c}{M_{\text{Pl}}} \right) \right], \quad p > 0. \quad (7)$$

We have chosen this potential as a mathematically convenient prototype for situations in which an early time plateau, favoured by Planck, as well as an exponential quintessential tail are present, while remaining agnostic as to its origin. We expect our results to qualitatively hold even if the precise form

of the potential is changed, so long as at late times there is a plateau suitable for quintessential inflation and that at early times inflation may be realised. Note also, that after inflation, the field becomes kinetically dominated and oblivious of the potential until it eventually freezes somewhere along the quintessential tail.

One could also consider scenarios in which the Gauss-Bonnet coupling is not negligible at early times, and the inflation-determining effective potential is due to a mixture of V and G [48], but for simplicity, we will not consider this in detail. Note that the value of the constant ϕ_c , under a field redefinition $\phi \rightarrow \phi + \phi_c$, can be absorbed into a rescaling of the constant $G_0 \rightarrow G_0 e^{q\phi_c}$, and so we can (and henceforth will) set it to zero without loss of generality.

A. Inflation

As the coupling to the Gauss-Bonnet term is assumed unimportant during inflation ($q\phi \gg 1$), unlike in e.g. [48] where both the potential and the GB coupling play a role in the inflationary dynamics, we can proceed to apply the usual slow-roll formalism to study inflation. We find that the slow-roll parameter $\epsilon = -\dot{H}/H^2$ is

$$\epsilon \simeq \frac{M_{\text{Pl}}^2}{2} \left(\frac{V_\phi}{V} \right)^2 = \frac{p^2}{2} \left[1 - \tanh \left(p \frac{\phi}{M_{\text{Pl}}} \right) \right]^2, \quad (8)$$

where \simeq denotes the slow-roll approximation. Inflation ends when $\epsilon = 1$, which results in

$$\phi_{\text{end}} \simeq \frac{M_{\text{Pl}}}{p} \tanh^{-1} \left(1 - \frac{\sqrt{2}}{p} \right). \quad (9)$$

The e -folding number is then found to be

$$\begin{aligned} N &= \frac{1}{M_{\text{Pl}}} \int_{\phi_{\text{end}}}^{\phi} \frac{d\phi}{\sqrt{2\epsilon}} \\ &\simeq \frac{1}{4p^2} e^{2p\phi/M_{\text{Pl}}} + \frac{\phi}{2pM_{\text{Pl}}} - \frac{1}{2p^2} \left[\frac{p}{\sqrt{2}} + \tanh^{-1} \left(1 - \frac{\sqrt{2}}{p} \right) - \frac{1}{2} \right], \end{aligned} \quad (10)$$

The first term here is dominant, and so inverting Eq. (10) we obtain the approximate initial condition for N e -folds of inflation to subsequently occur as

$$\phi(N) \approx \frac{M_{\text{Pl}}}{2p} \log 4p^2 N. \quad (11)$$

We can obtain an expression for the power spectrum in terms of N_* , the number of e-folds remaining until the end of inflation when the cosmological scales exit the horizon

$$\mathcal{P}_{\mathcal{R}} \simeq \frac{H^2}{8\pi^2\epsilon} \simeq \frac{p^2 V_0 N_*^2}{3\pi^2 M_{\text{Pl}}^4}, \quad (12)$$

where the value of N_* is typically about 60 for observable scales. We proceed further to find the spectral index

$$n_s - 1 \simeq -\frac{4p^2(1 + 8p^2 N_*)}{(1 + 4p^2 N_*)^2} \approx -\frac{2}{N_*}, \quad (13)$$

and the tensor-to-scalar-ratio

$$r \simeq 16\epsilon \simeq \frac{32p^2}{(1 + 4p^2 N_*)^2} \approx \frac{2}{p^2 N_*^2}. \quad (14)$$

B. Reheating

First, consider the post-inflationary evolution of the system, ignoring radiation. A static ($\phi = \text{constant}$) solution of the equations of motion (Eqs. (3) – (5)) with $\dot{\phi} = \ddot{\phi} = \dot{H} = 0$ exists, as one expects from previous work indicating that one function of a large Gauss-Bonnet coupling is freezing the time evolution of the inflaton [51]. One finds that the constant value the field approaches is given by

$$\phi_s/M_{\text{Pl}} \approx \frac{1}{q - 2p} \ln \alpha, \quad (15)$$

where

$$\alpha \equiv \frac{2qV_0G_0}{3pM_{\text{Pl}}^2}. \quad (16)$$

Numerically, we observe that this solution is approached in the post-inflationary regime as the Gauss-Bonnet term becomes important, at negative field values. As the field is frozen, this solution itself is an inflationary expansion with $\epsilon = 0$. This implies an unsuitability for perturbative reheating [73], as found in previous GB-coupled models [52], as no oscillatory behaviour about a potential minimum exists. Between the initial period of inflation, and this late-time accelerating expansion, however, there is an interval when the field is rolling quickly down the steep decline of the potential around $\phi = 0$. Around this point, it is feasible to implement instant preheating [38–40]

to recover a radiation-dominated epoch. For concreteness, we consider a coupling between the inflaton and a matter field χ (which is assumed to subsequently decay efficiently into radiation) of the form

$$\mathcal{L} = -\frac{1}{2}m_{\chi,0}^2\chi^2 - \frac{1}{2}g^2(\phi - \nu)^2\chi^2. \quad (17)$$

It is well known that this approach, if one takes the bare mass $m_{\chi,0}$ to be small compared to the induced mass $g|\phi - \nu|$, leads to the production of χ particles with total energy density

$$\rho_\chi = \frac{g^{5/2} |\dot{\phi}|_{\phi_{\text{ip}}}^{3/2} (\nu - \phi_{\text{ip}})}{8\pi^3}, \quad (18)$$

where ϕ_{ip} is the ϕ value at the time t_{ip} of instant preheating ($\phi_{\text{ip}} \equiv \phi(t_{\text{ip}})$). Furthermore, we choose $\nu = 0$ in the spirit of minimalism, though instead choosing a small non-zero value is not expected to significantly affect our results. In instant preheating, particle production occurs explosively around the time when the non-adiabaticity condition, $|\dot{m}_\chi| > m_\chi^2$, where $m_\chi \approx g|\phi - \nu|$, is first satisfied. We hence take this to be the time of instant preheating for this purpose, and can determine when this occurs via a numerical integration of the equations of motion (3)–(5) for a short time after the end of inflation.

For instant preheating to induce radiation domination, it is necessary that ρ_χ is greater than ρ_ϕ after instant preheating. Denoting the energy density of ϕ before and after instant preheating occurs as $\rho_{\phi,b}$ and $\rho_{\phi,a}$, respectively, we hence impose

$$\rho_\chi > \rho_{\phi,a} \quad \Rightarrow \quad \rho_\chi > \frac{1}{2}\rho_{\phi,b}, \quad (19)$$

where by energy conservation we require $\rho_{\phi,a} = \rho_{\phi,b} - \rho_\chi$.

After instant preheating, we also want the ϕ field's dynamics to be dominated by its kinetic energy density, because a potential-dominated inflaton field will quickly come to dominate again, thereby terminating the radiation-dominated epoch. As a result, we wish for the kinetic energy density of the inflaton after instant preheating to be greater than its potential. Considering that the potential remains constant¹ throughout instant preheating (i.e. $V(\phi_{\text{ip}}) = V_a = V_b$), this means we want

$$\rho_{\phi,a} - V(\phi_{\text{ip}}) > V(\phi_{\text{ip}}) \quad \Rightarrow \quad \rho_\chi < \rho_{\phi,b} - 2V(\phi_{\text{ip}}), \quad (20)$$

where again we considered energy conservation. Combining the inequalities in Eqs. (20) and (18), we hence obtain the range of suitable ρ_χ values as

$$\frac{1}{2}\rho_{\phi,b} < \rho_\chi < \rho_{\phi,b} - 2V(\phi_{\text{ip}}). \quad (21)$$

This result implies that the implementation of instant preheating will only be able to succeed when it occurs at a sufficiently kinetic-dominated moment in the evolution of the inflaton. From Eq. (21), we find the constraint

¹ Note that during instant preheating, it is purely kinetic energy density that is converted to radiation.

$$\rho_{\text{kin,b}} > 3V(\phi_{\text{ip}}). \quad (22)$$

The potential must hence be sufficiently steep that the field rolls fairly quickly after inflation. This will be discussed in more detail in section III B.

C. Behaviour in Radiation/Matter Dominated Epochs

After reheating has occurred according to the details of section II B, the Universe becomes radiation-, or eventually matter-, dominated. In such regimes, we have $H = k/t$, where k is a constant depending on the epoch in question (in particular, $k = 1/2$ for a radiation-dominated Universe and $k = 2/3$ for a matter-dominated Universe). We assume that immediately after instant preheating, the Gauss-Bonnet term is still negligible. This is desirable because if the field were to become GB-dominated, and hence freeze, immediately after reheating, its density would be too large to act as dark energy. Instead, we require that that GB coupling only becomes significant at some, as of yet unspecified, later time, when the field has rolled further down the quintessential tail to reduce its final energy density. To achieve this in a sub-Planckian field displacement, we expect to need a fairly large p value to make the quintessential tail of the potential steep enough to suppress the energy density by many orders of magnitude during this process. Ordinarily such a steep tail would not lead to accelerated expansion, hence the necessity of the Gauss-Bonnet term.

Due to the requirements set out in Eq. (21), the field will necessarily be kinetically dominated immediately after instant preheating. We can hence neglect the potential and GB terms in the equation of motion, and determine the subsequent evolution of the field via

$$\ddot{\phi} + 3H\dot{\phi} \simeq 0. \quad (23)$$

In conventional quintessential inflation [8, 74], the solution of this equation determines the late time behaviour of the field, and it is known that the field eventually freezes in at a certain value, but only after undergoing a super-Planckian displacement. This freezing behaviour is conducive to achieving late-time dark energy, but, as the field has become super-Planckian, radiative corrections mean the flatness of the potential cannot be guaranteed, the field may act as a ‘5th force’ violating the equivalence principle and the model is too sensitive to the details of UV completion to be trusted [75, 76].

In our model, however, as the field rolls to more negative values, the size of the Gauss-Bonnet coupling will grow exponentially and eventually become non-negligible. Eq. (23) is hence only valid up until a time

when the contribution of the GB coupling to the Klein-Gordon equation becomes comparable to $\ddot{\phi}$. To solve the full nonlinear ODE of Eq. (5), even when neglecting the potential, is difficult if not impossible, so here we instead resort to an approximation where immediately following instant preheating, the GB coupling is still negligible and Eq. (23) determines the evolution of the system (the kinetic-dominated regime).

This holds until a time t_{gb} , when $|\ddot{\phi}| = |12M_{\text{Pl}}^2 H^2 (\dot{H} + H^2) G_{,\phi}|$. After this, the second derivative term in the Klein-Gordon equation is instead treated as negligible while the GB contribution is accounted for (the GB-dominated regime), and the system's evolution is given by solutions to

$$3H\dot{\phi} + 12M_{\text{Pl}}^2 H^2 G_{,\phi} (\dot{H} + H^2) = 0. \quad (24)$$

The two regimes' solutions will then be “stitched” together via the boundary condition that the two solutions must agree at t_{gb} . With this method we lose the ability to finely resolve the evolution of ϕ about t_{gb} , but should still be able to determine the all-important late time behaviour with reasonable accuracy.

1. Kinetic Regime

We fix the initial conditions to solve Eq. (23) as $\phi(t_{\text{ip}}) = \phi_{\text{ip}}$ and $\dot{\phi}(t_{\text{ip}}) = \dot{\phi}_{\text{ip}} = -\sqrt{6\Omega_{\text{ip}}}(k/t_{\text{ip}})M_{\text{Pl}}$. The first initial condition merely imposes that the field begins at the value it had when instant preheating occurred, while the second initial condition uses the first Friedman equation (3) to set $\dot{\phi}$ at this time, as a function of $\Omega_{\text{ip}} \equiv \Omega_{\phi}(t_{\text{ip}})$ - the density parameter of the field at the moment of instant preheating (immediately after). We also assume that the field rolls towards negative values. The discussion in section II B, where we require that the produced radiation density dominates over the remaining field density, implies that $\Omega_{\text{ip}} \ll 1$. Solving Eq. (23) with these conditions gives

$$\phi(t) = \phi_{\text{ip}} - M_{\text{Pl}}\sqrt{6\Omega_{\text{ip}}}\left(\frac{k}{3k-1}\right)\left[1 - \left(\frac{t_{\text{ip}}}{t}\right)^{3k-1}\right]. \quad (t < t_{\text{gb}}) \quad (25)$$

2. Gauss-Bonnet Regime

As the GB-dominated equation of motion, Eq.(24), is first order, we only need the initial condition that at the time when the GB-dominated solution first becomes relevant t_{gb} , the field takes the value (which will later be determined) $\phi_{\text{gb}} = \phi(t_{\text{gb}})$. Using this, we find the solution

$$\phi(t) = \phi_{\text{gb}} + \frac{M_{\text{Pl}}}{q} \ln \left[1 + 2G_0 q^2 k^2 (1-k) e^{-q\phi_{\text{gb}}/M_{\text{Pl}}} \left(\frac{1}{t^2} - \frac{1}{t_{\text{gb}}^2} \right) \right]. \quad (26)$$

At very late times ($t \gg t_{\text{gb}}$) the field will then tend to a constant value

$$\phi(t \gg t_{\text{gb}}) = \phi_{\text{gb}} + \frac{M_{\text{Pl}}}{q} \ln \left(1 - \frac{\beta G_0 q^2 e^{-q\phi_{\text{gb}}/M_{\text{Pl}}}}{t_{\text{gb}}^2} \right) \approx \phi_{\text{gb}}, \quad (t > t_{\text{gb}}) \quad (27)$$

where $\beta = 2k^2(1-k)$. Hence $\beta = 1/4$ in the radiation dominated case and $\beta = 8/27$ for the matter dominated case. In the second approximate equality, we note that for typical parameter values, there is very little variation of the field in this regime as the second term is generally quite small. This is expected, as the principle of our model is that a large GB coupling impedes the evolution of the field so ϕ freezes almost immediately when GB becomes important.

3. *Stitching and Boundary Condition*

Having determined the evolution of the field for $t < t_{\text{gb}}$, Eq. (25), and $t > t_{\text{gb}}$, Eq. (26), we need to now determine the moment t_{gb} at which these two solutions coalesce. As discussed previously, this is when $|\ddot{\phi}| = |12M_{\text{Pl}}^2 H^2 (\dot{H} + H^2) G_{,\phi}|$. We hence substitute the function $\phi(t)$ from Eq. (25) into this condition and solve for the time at which it is first met. Doing so we obtain an equation of the form

$$At^\nu = \exp(-Bt^\mu), \quad (28)$$

which can be solved with the Lambert W function, which satisfies $x = W(x)e^{W(x)}$, as

$$t_{\text{gb}} = \left[\frac{\nu}{B\mu} W \left(\frac{B\mu}{\nu} \left(\frac{1}{A} \right)^{\frac{\mu}{\nu}} \right) \right]^{\frac{1}{\mu}}, \quad (29)$$

where

$$A = \frac{1}{2qkG_0(1-k)} \sqrt{\frac{3\Omega_{\text{ip}}}{2}} \exp \left(q\phi_{\text{ip}}/M_{\text{Pl}} - \frac{qk\sqrt{6\Omega_{\text{ip}}}}{3k-1} \right) t_{\text{ip}}^{3k-1}, \quad (30)$$

$$B = qk\sqrt{6\Omega_{\text{ip}}} \left(\frac{t_{\text{ip}}^{3k-1}}{3k-1} \right), \quad (31)$$

$$\mu = 1 - 3k, \quad (32)$$

$$\nu = 3 - 3k. \quad (33)$$

In the region where the Lambert function $W(x)$ has two branches ($-e^{-1} < x < 0$), this would imply there are two times at which the GB and second derivative contributions to the Klein-Gordon equation

are equal, but of course only the earlier time of the two solutions is valid, provided that it obeys $t_{\text{gb}} > t_{\text{ip}}$, as Eq. (25) is only valid up until the GB contribution first becomes important. Typically it is the lower (W_{-1}) branch of the function which evaluates to the relevant value, but in cases such as those where the lower branch yields $t_{\text{gb}} < t_{\text{ip}}$, as it is physically ruled out (instant preheating must happen before GB-domination else a viable late-time universe is not recovered), we instead use the principal (W_0) branch solution.

There may be parameter space in which there is no real solution of this equation, corresponding physically to there being no time of equality between these two terms in the Klein-Gordon equation. This would either imply that the GB term is already dominant at t_{ip} , or that the field remains kinetic-dominated forever. Both cases are undesirable as they do not correspond to late-time dark energy (either the field freezes due to GB too soon after inflation to reduce its density to that of dark energy, or conventional quintessential inflation is recovered as GB is irrelevant). The reality of Eq. (29) is hence an important check that we are looking at feasible models, in particular, the argument of the Lambert function must satisfy

$$\frac{B\mu}{\nu} \left(\frac{1}{A} \right)^{\frac{\mu}{\nu}} \geq -\frac{1}{e}, \quad (34)$$

to have at least one real value.

Substituting the value of t_{gb} into Eq. (25) allows us to determine ϕ_{gb} , and in turn this allows us to determine $\phi(t)$ for $t \gg t_{\text{gb}}$ by substituting that into Eq. (27). Doing so, we obtain

$$\phi_m = \phi(t \gg t_{\text{gb}}) \approx \phi_{\text{ip}} + \frac{M_{\text{Pl}}B}{q} (t_{\text{gb}}^\mu - t_{\text{ip}}^\mu) + \frac{M_{\text{Pl}}}{q} \ln \left(1 + \frac{\mu B}{2} t_{\text{gb}}^\mu \right). \quad (35)$$

In principle, then, in a matter-dominated universe, the field would eventually freeze to a value ϕ_m , given by Eq. (35). In practice, however, we find that the time at which this would start to happen, t_{gb} (defined in Eq. (29)) is very large (much greater than the age of the Universe today). This is not a problem, though, as it just means the field does not typically freeze during matter domination, instead slowly-rolling towards, but not reaching, ϕ_m . Following this, once matter domination eventually gives way to dark energy domination, the field dynamics will be determined by the matter-free equations of motion and the field will once again tend to freeze at a value ϕ_s specified by Eq. (15). This process will, however, only complete itself in the future of our present Universe when $\Omega_m \ll \Omega_{\text{DE}} \simeq 1$. At present, the field is envisaged to be in a state of slowly rolling towards ϕ_s , due to the friction of the GB coupling and smallness of $\dot{\phi}$.

D. Dark Energy Today

Having confirmed that the behaviour of the field is sensible in the matter-dominated epoch, we can proceed to estimate the value it takes on today. We could have taken an approach where we would model the post-matter-domination evolution with some differential equation and evolve the Universe in time until the present day, essentially continuing to use the time coordinate t to parametrise our position in the Universe's history. But instead, we use the known values of the present day dark energy and matter density parameters to this end. In this picture, the past state of perfect matter domination assumed in the calculations of section II C is when $\Omega_m = 1$ and $\Omega_\Lambda = 0$, the future dark energy domination which is eventually reached by this model as matter dilutes and it tends to the static solution in Eq. (15) corresponds to $\Omega_\Lambda = 1$ and $\Omega_m = 0$, while the present day values $\Omega_\Lambda \approx 0.7$ and $\Omega_m \approx 0.3$ indicate exactly where between these two limits we must presently lie. We are thus treating the density parameters as effective 'time' coordinates to specify our point in the Universe's history.

To formalise this, first note that the effective equation of state of the Universe $w = p/\rho$ and the derivative of the Hubble Parameter are related via the second Friedman equation in Eq. (4) such that

$$M_{\text{Pl}}^2 \dot{H} = -\frac{1}{2}(\rho + p) = -\frac{1}{2}(1 + w)\rho = -\frac{3}{2}(1 + w)H^2 M_{\text{Pl}}^2. \quad (36)$$

This can be used to rewrite the Klein-Gordon equation for the Gauss-Bonnet coupled field given in Eq. (5) as

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} - 6H^4(1 + 3w)G_{,\phi}M_{\text{Pl}}^2 = 0, \quad (37)$$

which, under the slow-roll approximation $\ddot{\phi} \simeq 0$, is approximated by

$$\dot{\phi} \approx 2H^3(1 + 3w)G_{,\phi}M_{\text{Pl}}^2 - \frac{V_{,\phi}}{3H}. \quad (38)$$

Substituting this into the Friedman equation in the form

$$3H^2 M_{\text{Pl}}^2 \Omega_\Lambda = \frac{1}{2}\dot{\phi}^2 + V + 12M_{\text{Pl}}^2 H^3 G_{,\phi} \dot{\phi}, \quad (39)$$

where Ω_Λ is the dark energy fraction, we obtain the approximate constraint equation

$$V + \frac{V_{,\phi}^2}{18H^2} + \left[3\Omega_\Lambda + \frac{2}{3}(7 + 3w)V_{,\phi}G_{,\phi} \right] M_{\text{Pl}}^2 H^2 + 2(1 + 3w)(13 + 3w)(M_{\text{Pl}}^2 G_{,\phi})^2 H^6 = 0, \quad (40)$$

which can be rewritten in terms of the explicit potentials of our model (assuming $p\phi \ll 0$) in the form

$$\begin{aligned}
V_0 e^{2p\phi_{\text{de}}/M_{\text{Pl}}} + \frac{2p^2 V_0^2}{9H^2 M_{\text{Pl}}^2} e^{4p\phi_{\text{de}}/M_{\text{Pl}}} + 2q^2 G_0^2 M_{\text{Pl}}^2 H^6 (1+3w)(13+3w) e^{-2q\phi_{\text{de}}/M_{\text{Pl}}} \\
+ \frac{4}{3} qp G_0 V_0 H^2 (7+3w) e^{(2p-q)\phi_{\text{de}}/M_{\text{Pl}}} - 3H^2 M_{\text{Pl}}^2 \Omega_\Lambda = 0.
\end{aligned} \tag{41}$$

Under an appropriate substitution, this can be reduced to a polynomial equation for more straightforward algebraic or numerical analysis (though as the numbers involved span many orders of magnitude, high-precision arithmetic should be used in the case of numerical evaluation). Regardless of the preferred method, though, this constraint can be solved for ϕ_{de} to identify the field value necessary to achieve a specific equation of state w , dark energy fraction Ω_Λ and expansion rate H for a given model, specified by p and q . Assuming that following matter-domination, the Universe contains only matter and the dark energy field, we have from observations that $w = w_\Lambda \Omega_\Lambda \approx -0.7$, and $H_0 \approx 10^{-60} M_{\text{Pl}}$. We also note that one can check that with the specifications $w = -1$, $\Omega_\Lambda = 1$ and $3H^2 M_{\text{Pl}}^2 = V$, representing perfect dark energy domination, solutions of Eq. (40) yield $\phi = \phi_s$ as in Eq. (15), as expected, and in this limit Eq. (38) unsurprisingly reduces to $\dot{\phi} = 0$.

Interestingly, we typically find for most parameters that the ϕ_m value calculated in the previous section is larger in magnitude (more negative) than the ϕ_{de} value obtained in the above procedure. This is consistent with the idea that ϕ_m is not in practice reached, and the maximum displacement of the field during matter domination is hence smaller in magnitude than ϕ_m , as well as ϕ_{de} , and the field simply continues to roll and eventually reach ϕ_{de} today. Alternatively, this could represent that the field *does* overshoot ϕ_{de} (though still not ϕ_m) but then turns around due to the impeding effects of the GB coupling. This latter explanation is preferred, as earlier-time solutions of Eq. (41) with $\Omega_\Lambda < 0.7$ are typically found to be larger in magnitude than the ϕ_{de} value today at $\Omega_\Lambda = 0.7$. Furthermore, ϕ_s , which is achieved at later times when $\Omega_\Lambda \rightarrow 1$, is smaller in magnitude (less negative) than ϕ_{de} today in the particular cases we have investigated in depth. These observations seem to suggest that the field is rolling ‘backwards’ during the transition between matter and dark energy domination, but this may not be true for all models; we have not excluded the possibility that some parameters may lead to the change in direction only occurring after ϕ_{de} , or not at all.

III. CONSTRAINTS

Having established the nature of inflation, reheating and dark energy in the framework of our model, we now proceed to use our results to constrain the parameter space to realistic values.

A. Constraints from Inflation

One can weakly constrain the parameter p from CMB constraints [3] via the prediction of the spectral index in Eq. (13), which imposes roughly $p \gtrsim 0.1$ when $N_* \approx 60$. The spectral index obtained is $n_s = 0.9678$, which is in excellent agreement with observations, but this is not surprising as our model is explicitly designed to have an inflationary plateau. The tensor to scalar ratio calculated from Eq. (14) is, meanwhile, compatible with current constraints [2] for all values of p (for $N_* \approx 60$ the maximum value of r is around 0.03 for p about 0.06), but is mentioned here for completeness. The main constraint arising from inflation is the normalisation of the primordial power spectrum's amplitude. Using Eq. (12) and imposing $\mathcal{P}_{\mathcal{R}} \approx 2.2 \times 10^{-9}$ [3] gives

$$V_0 \approx \frac{6.5 \times 10^{-8}}{p^2 N_*^2} M_{\text{Pl}}^4 \approx \frac{1.8 \times 10^{-11}}{p^2} M_{\text{Pl}}^4, \quad (N_* \approx 60). \quad (42)$$

The prefactor of the potential is hence set by inflation. To bring the potential energy down to the dark energy scale at late times, a large suppression of many orders of magnitude is hence necessary. This implies we will need a rather large value of p (much larger than the weak constraint the spectral index gives us). Just as a first estimate at this stage, noting that for $p\phi \ll 0$ (deeply post-inflation) the form of the potential in Eq. (7) is approximated by $V_0 \exp(2p\phi/M_{\text{Pl}})$, we can see that for a maximum field displacement of $O(M_{\text{Pl}})$, we will need $2p \approx \ln(V_0/\Lambda) \approx O(100)$ to facilitate this. We also note here that around this expected value of $p \simeq 100$, Eq. (42) implies $V_0 \simeq 10^{-15} M_{\text{Pl}}^4$, or $V_0^{1/4} \simeq 10^{14} \text{GeV}$, close to the energy scale of Grand Unification.

B. Constraints from Reheating and Dark Energy

We numerically integrate the background equations of motion during inflation for a range of models (specified by their p , q , g and G_0 values). Using these results we compute the energy density resulting from instant preheating, the behaviour of the field in matter domination, the field value today, and the far-future field value ϕ_s . We then impose the following constraints:

- Instant preheating should satisfy the conditions in Eq. (21) for a sensible choice of the perturbative coupling to matter g (i.e. $g \leq 1$).
- The field value today, ϕ_{de} , should be subplanckian ($|\phi_{\text{de}}| < M_{\text{Pl}}$) such that unknown UV physics do not strongly influence our results.

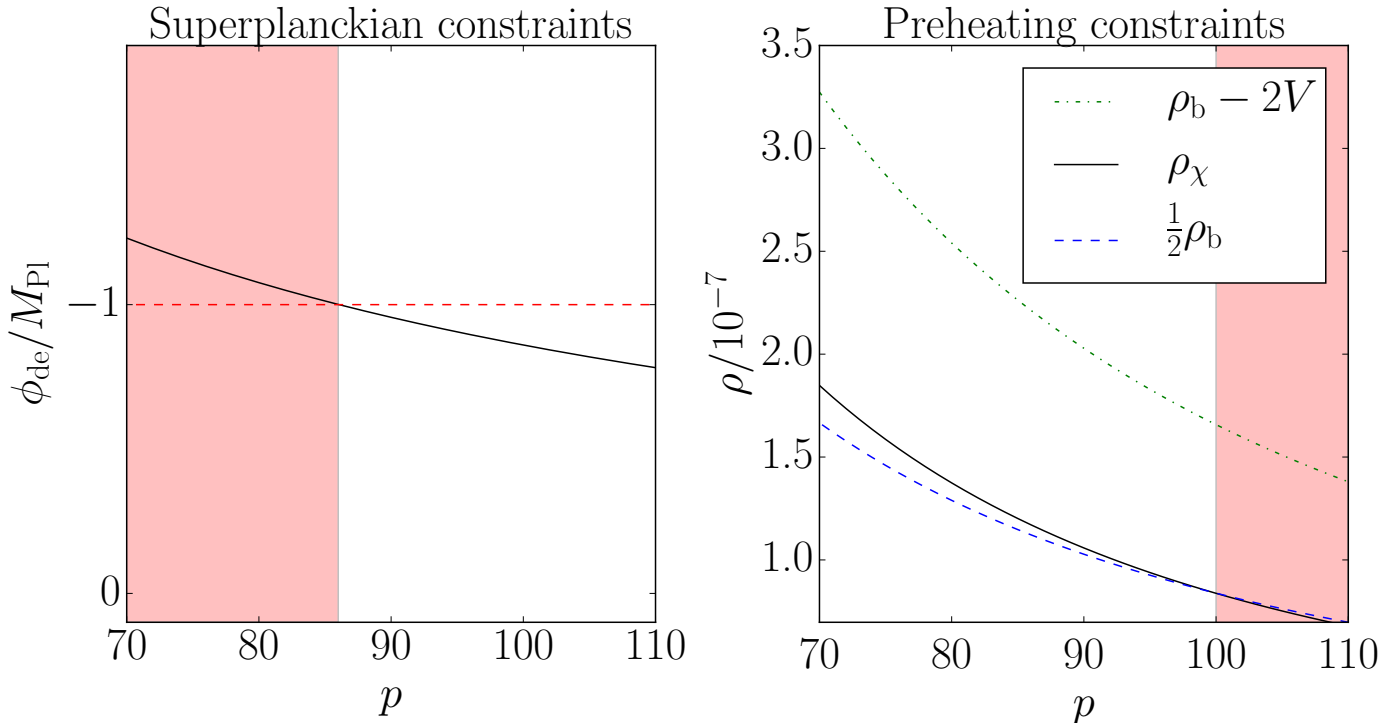


FIG. 1. The left window shows the value of ϕ_{de} computed via the methods of section IID for a range of p values when $q = 4p$, $G_0 M_{\text{Pl}}^2 = 1$ and $g = 0.8$. The shaded region on the left represents the parameter space where ϕ_{de} is super-Planckian, and hence sensitive to details of unknown UV physics, imposing a bound of $p > 86$ in this model. The right window shows the energy densities involved in the instant preheating conditions of Eq. (21). The black solid line represents ρ_χ , the density of radiation produced at instant preheating, while the blue dashed line and green dot-dashed line respectively represent the lower and upper bounds that ρ_χ must lie between. The shaded region on the right encloses the p values for which these inequalities are violated and hence imposes a bound of $p < 100$ on this model.

Examples of these constraints as a function of the parameter p for the case $q = 4p$, $G_0 M_{\text{Pl}}^2 = 1$ and $g = 0.8$ are shown in Figure 1. The former shows that we must have $p > 86$ to avoid super-Planckian field values today, while the latter shows that instant preheating may not proceed according to the requirements in Eq. (21) unless $p < 100$. The resulting allowed parameter space of $86 < p < 100$ for this case, alongside many other models with different q , G_0 and g values, is tabulated in Table I.

$G_0 M_{\text{Pl}}^2$	q/p	g	p limits
1	4	0.8	$86 < p < 100$
		0.9	$86 < p < 238$
		1	$86 < p < 507$
	8	0.8	$51 < p < 100$
		0.9	$51 < p < 207$
		1.0	$51 < p < 370$
100	4	0.8	$85 < p < 100$
		0.9	$85 < p < 238$
		1.0	$85 < p < 507$
	8	0.8	$51 < p < 72$
		0.9	$51 < p < 155$
		1.0	$51 < p < 258$

TABLE I. Table showing limits on p in the theory for various cases of the size of G_0 , q and g , due to constraints coming from sub-Planckian field displacements and instant preheating's efficacy. In each case, the lower bound on p occurs as, below this threshold, ϕ_{de} would have to undergo a super-Planckian displacement to serve as dark energy today. Similarly, the upper limits on p arise as, above these limits, the inequality in Eq. (21) is violated.

C. Constraints from Tests of Modified Gravity

As our theory is a modification of General Relativity, it has to pass local gravity tests. Some work has been done on Gauss-Bonnet mediated dark energy theories in this context [77–79], with the strongest constraint found to be

$$M_{\text{Pl}} |G_{,\phi}| \lesssim 1.6 \times 10^{20} \text{ m}^2 \approx 1.5 \times 10^{88} M_{\text{Pl}}^{-2}, \quad (43)$$

coming from constraints on the PPN parameter γ based on the Cassini spacecraft's measurements of time delay of electromagnetic signals in a gravitational field. However, the analysis was made based on an assumption that derivatives of the scalar potential V are all $O(V)$, which is strictly not applicable to our model in which $V_{,\phi}/V$ is $O(p)$, and p is necessarily large. While a more comprehensive analysis of the case $V_{,\phi}/V \gg 1$ is necessary to work out the predictions for local gravity tests in our model, we will still briefly compare our results to this constraint to gain an approximate idea of what degree of

constraint violation we may expect.

As in our model $M_{\text{Pl}}|G_{,\phi}| = qG_0e^{-q\phi/M_{\text{Pl}}}$, we can see that local constraints of this form do not strongly constrain G_0 , which only appears linearly in the constrained expression, but instead place a more stringent bound on q which appears in a highly non-linear way in the expression above. Therefore, a relatively small increase in q is more likely to violate the constraint than an increase in G_0 by an order of magnitude. The constraint in Eq. (43) can be written as

$$q \lesssim -\frac{M_{\text{Pl}}}{\phi_{\text{de}}} W\left(\frac{1.5 \times 10^{88}}{G_0 M_{\text{Pl}}^2}\right). \quad (44)$$

Taking a Planckian field displacement $|\phi_{\text{de}}| = O(M_{\text{Pl}})$ (corresponding to the lower-bound on p in parameter space) and $G_0 M_{\text{Pl}}^2 = 1$ for argument's sake, we find $q \lesssim 200$. However, in the best case scenario using the results from Table I, $q \gtrsim 340$. Similarly for models with larger p , and hence ϕ_{de} closer to zero, regardless of if we choose $q = 4p$ or $q = 8p$, we find that the constraint (44) is up to a factor of around 2 smaller than the actual q value needed. This implies that there may be some basic difficulty in finding models which obey local gravity constraints while also satisfying the necessary criteria for dark energy and instant preheating, though the q value needed is at least comparable to (i.e. less than an order of magnitude larger than) the limit, such that if the constraints are moderately weakened when allowing for $V' \gg V$, there is some chance of the model passing local tests without further modification.

As our model is largely prototypical, particularly when choosing the shape of the potential, it is also feasible that modifications such as the addition of a screening mechanism to the model may also help alleviate any tension with local tests. We do not perform a thorough assessment of this now, however, because once again we emphasise that this constraint is based on assumptions that do not strictly apply to our theory. Analysis of this matter using more fit-for-purpose constraints is left to a future work.

IV. DISCUSSION AND CONCLUSIONS

We have studied in detail a model of quintessential inflation where the inflaton field couples to the Gauss-Bonnet (GB) term. By design, the GB coupling is negligible at early times so inflation proceeds under standard slow-roll. Hence, we have considered a scalar potential, which features an inflationary plateau, as favoured by the latest CMB observations. Indeed, the scalar spectral index found, $n_s = 0.9678$, is close to the sweet spot of Planck observations, and we find a tensor amplitude considerably below the current upper bounds.

As usual in quintessential inflation, the inflationary model is non-oscillatory, so that the inflaton field

does not decay after the end of inflation, because it must survive until today to become quintessence and thereby explain dark energy observations without the need for an extremely fine-tuned cosmological constant. As the Universe must be reheated by means other than inflaton decay, we have employed the instant preheating mechanism, in which the field is coupled to some other degree of freedom, such that as the field is rapidly rolling down the so-called quintessential tail of its runaway potential, it induces massive particle production, which transforms much of the kinetic energy density of the inflaton to the newly created radiation bath of the hot big bang. Soon afterwards, the inflaton field freezes at some value with small residual energy density, which becomes important at present, playing the role of dark energy. It is important to note that, while the field is rolling down the quintessential tail, it is oblivious to the form of the scalar potential, so our choice of model only determines the value of the residual potential density, when the field freezes.

Because of the huge difference between the energy density scale of inflation and the current energy density (which is over a hundred orders of magnitude) the inflaton field typically rolls over super-Planckian distances in field space, in conventional quintessential inflation. However, this can result into a multitude of problems. Firstly, the flatness of the quintessential tail may be lifted by radiative corrections. Also, because the associated mass is so small, the quintessence field may give rise to a so-called 5th force problem, which can lead to violation of the equivalence principle. To avoid these problems, it is desirable to keep the field variation sub-Planckian. In this case, however, to bridge the huge difference between the inflation and dark energy density scales, the quintessential tail must be steep. But, if the quintessential tail is too steep, when the field becomes important today, it unfreezes and rolls down the steep potential not leading to accelerated expansion at all.

One way to overcome this problem is to make sure the scalar field remains frozen today even though the quintessential tail is steep. To this end, in this paper we have considered coupling the field with the Gauss-Bonnet term, because such coupling impedes the variation of the field even if the potential is steep². Thus, in our model the GB coupling becomes important at late times and makes sure the field freezes with sub-Planckian displacement, such that it becomes the dark energy today without the aforementioned problems.

Quintessence is motivated only if the required tuning of the model parameters is less than the extreme fine-tuning of the cosmological constant in Λ CDM. Quintessential inflation resolves one of the tuning problems of quintessence, that of its initial conditions, which are determined by the inflationary attractor. This means that the coincidence requirement (that is that the dark energy must be such that it dominates

² For a different solution to this problem see Ref. [74].

at present) is satisfied only by virtue of the choice of the model parameters (and not by initial conditions). In our model, we have four model parameters, which account for the requirements of both inflation and quintessence. For the GB coupling, shown in Eq. (6), we assume a simple exponential dependence on the inflaton, which ensures the GB coupling becomes important only at late times. The scale of the coupling is $G_0 \geq M_{\text{Pl}}^{-2}$, which agrees with our effort to stay sub-Planckian. For our scalar potential, shown in Eq. (7), the density scale is set by the COBE constraint to be $V_0^{1/4} \sim 10^{-14}$ GeV, close to the scale of grand unification. In the exponent of the GB coupling and the argument of the tanh in the scalar potential, the inflaton field is suppressed by a large mass scale M_{Pl}/q and M_{Pl}/p respectively. We considered $q \sim p$ and found that $50 \lesssim p \lesssim 500$ (cf. Table. I), which means that, in both the GB coupling and the scalar potential, the inflaton field is suppressed by the scale of grand unification $\sim 10^{16}$ GeV. Thus, we see that our model parameters are nowhere near tuned to the level of the extreme fine-tuning of the cosmological constant in Λ CDM.

In summary, we have studied quintessential inflation where a coupling between the inflaton field and the Gauss-Bonnet (GB) term allows the scenario to work, avoiding a super-Planckian variation of the field, which may otherwise be problematic. We considered a scalar potential with an inflationary plateau (favoured by CMB observations) and an exponential quintessential tail. Since the form of the potential is largely unimportant after inflation, we believe that our results are indicative for many quintessential inflation models with a GB coupling. We found that the model is successful for natural values of the model parameters, both in generating inflationary observables and also in accounting for the observed dark energy.

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- [1] C. L. Bennett *et al.* (WMAP), “Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results,” *Astrophys. J. Suppl.* **208**, 20 (2013), [arXiv:1212.5225 \[astro-ph.CO\]](#).
 - [2] P. A. R. Ade *et al.* (BICEP2, Planck), “Joint Analysis of BICEP2/*KeckArray* and *Planck* Data,” *Phys. Rev. Lett.* **114**, 101301 (2015), [arXiv:1502.00612 \[astro-ph.CO\]](#).

- [3] P. A. R. Ade *et al.* (Planck), “Planck 2015 results. XX. Constraints on inflation,” (2015), [arXiv:1502.02114 \[astro-ph.CO\]](#).
- [4] S. Perlmutter *et al.* (Supernova Cosmology Project), “Measurements of Omega and Lambda from 42 high redshift supernovae,” *Astrophys. J.* **517**, 565–586 (1999), [arXiv:astro-ph/9812133 \[astro-ph\]](#).
- [5] P. A. R. Ade *et al.* (Planck), “Planck 2015 results. XIII. Cosmological parameters,” *Astron. Astrophys.* **594**, A13 (2016), [arXiv:1502.01589 \[astro-ph.CO\]](#).
- [6] P. J. E. Peebles and A. Vilenkin, “Quintessential inflation,” *Phys. Rev.* **D59**, 063505 (1999), [arXiv:astro-ph/9810509 \[astro-ph\]](#).
- [7] Konstantinos Dimopoulos, “Towards a model of quintessential inflation,” *Frontiers in particle astrophysics and cosmology. Proceedings, 6th International Conference, EURESCO, San Feliu de Guixols, Spain, September 30-October 5, 2000*, *Nucl. Phys. Proc. Suppl.* **95**, 70–73 (2001), [,70(2000)], [arXiv:astro-ph/0012298 \[astro-ph\]](#).
- [8] Konstantinos Dimopoulos and J. W. F. Valle, “Modeling quintessential inflation,” *Astropart. Phys.* **18**, 287–306 (2002), [arXiv:astro-ph/0111417 \[astro-ph\]](#).
- [9] S. C. C. Ng, N. J. Nunes, and Francesca Rosati, “Applications of scalar attractor solutions to cosmology,” *Phys. Rev.* **D64**, 083510 (2001), [arXiv:astro-ph/0107321 \[astro-ph\]](#).
- [10] Greg Huey and James E. Lidsey, “Inflation, brane worlds and quintessence,” *Phys. Lett.* **B514**, 217–225 (2001), [arXiv:astro-ph/0104006 \[astro-ph\]](#).
- [11] A. S. Majumdar, “From brane assisted inflation to quintessence through a single scalar field,” *Phys. Rev.* **D64**, 083503 (2001), [arXiv:astro-ph/0105518 \[astro-ph\]](#).
- [12] N. J. Nunes and Edmund J. Copeland, “Tracking quintessential inflation from brane worlds,” *Phys. Rev.* **D66**, 043524 (2002), [arXiv:astro-ph/0204115 \[astro-ph\]](#).
- [13] Rogerio Rosenfeld and J. A. Frieman, “A Simple model for quintessential inflation,” *JCAP* **0509**, 003 (2005), [arXiv:astro-ph/0504191 \[astro-ph\]](#).
- [14] Rogerio Rosenfeld and Joshua A. Frieman, “Cosmic microwave background and large-scale structure constraints on a simple quintessential inflation model,” *Phys. Rev.* **D75**, 043513 (2007), [arXiv:astro-ph/0611241 \[astro-ph\]](#).
- [15] Mar Bastero-Gil, Arjun Berera, Brendan M. Jackson, and Andy Taylor, “Hybrid Quintessential Inflation,” *Phys. Lett.* **B678**, 157–163 (2009), [arXiv:0905.2937 \[hep-ph\]](#).
- [16] M. C. Bento, R. Gonzalez Felipe, and N. M. C. Santos, “Brane assisted quintessential inflation with transient acceleration,” *Phys. Rev.* **D77**, 123512 (2008), [arXiv:0801.3450 \[astro-ph\]](#).

- [17] Victor H. Cardenas, “Tachyonic quintessential inflation,” *Phys. Rev.* **D73**, 103512 (2006), [arXiv:gr-qc/0603013 \[gr-qc\]](#).
- [18] Mafalda Dias and Andrew R Liddle, “On the possibility of braneworld quintessential inflation,” *Phys. Rev.* **D81**, 083515 (2010), [arXiv:1002.3703 \[astro-ph.CO\]](#).
- [19] Richard A. Frewin and James E. Lidsey, “On identifying the present day vacuum energy with the potential driving inflation,” *Int. J. Mod. Phys.* **D2**, 323–350 (1993), [arXiv:astro-ph/9312035 \[astro-ph\]](#).
- [20] Chao-Qiang Geng, Md. Wali Hossain, R. Myrzakulov, M. Sami, and Emmanuel N. Saridakis, “Quintessential inflation with canonical and noncanonical scalar fields and Planck 2015 results,” *Phys. Rev.* **D92**, 023522 (2015), [arXiv:1502.03597 \[gr-qc\]](#).
- [21] Chao-Qiang Geng, Chung-Chi Lee, M. Sami, Emmanuel N. Saridakis, and Alexei A. Starobinsky, “Observational constraints on successful model of quintessential Inflation,” *JCAP* **1706**, 011 (2017), [arXiv:1705.01329 \[gr-qc\]](#).
- [22] Eduardo Guendelman, Emil Nissimov, and Svetlana Pacheva, “Quintessential Inflation, Unified Dark Energy and Dark Matter, and Higgs Mechanism,” (2016), [arXiv:1609.06915 \[gr-qc\]](#).
- [23] Jaume de Haro, “On the viability of quintessential inflation models from observational data,” *Gen. Rel. Grav.* **49**, 6 (2017), [arXiv:1602.07138 \[gr-qc\]](#).
- [24] Jaume Haro and Supriya Pan, “Bulk viscous quintessential inflation,” (2015), [arXiv:1512.03033 \[gr-qc\]](#).
- [25] Jaume De Haro and Llibert Aresté Saló, “Reheating constraints in quintessential inflation,” *Phys. Rev.* **D95**, 123501 (2017), [arXiv:1702.04212 \[gr-qc\]](#).
- [26] Md. Wali Hossain, R. Myrzakulov, M. Sami, and Emmanuel N. Saridakis, “Variable gravity: A suitable framework for quintessential inflation,” *Phys. Rev.* **D90**, 023512 (2014), [arXiv:1402.6661 \[gr-qc\]](#).
- [27] Md. Wali Hossain, R. Myrzakulov, M. Sami, and Emmanuel N. Saridakis, “Unification of inflation and dark energy à la quintessential inflation,” *Int. J. Mod. Phys.* **D24**, 1530014 (2015), [arXiv:1410.6100 \[gr-qc\]](#).
- [28] Ali Rida Khalifeh, “Quintessential Inflation in Mimetic Dark Matter,” (2015), [arXiv:1506.06250 \[gr-qc\]](#).
- [29] Agustin Membrilla and Mauricio Bellini, “Quintessential inflation from a variable cosmological constant in a 5D vacuum,” *Phys. Lett.* **B641**, 125–129 (2006), [arXiv:gr-qc/0606119 \[gr-qc\]](#).
- [30] Ishwaree P. Neupane, “Reconstructing a model of quintessential inflation,” *Class. Quant. Grav.* **25**, 125013 (2008), [arXiv:0706.2654 \[hep-th\]](#).
- [31] Javier Rubio and Christof Wetterich, “Emergent scale symmetry: connecting inflation and dark energy,” (2017), [arXiv:1705.00552 \[gr-qc\]](#).
- [32] M. Sami and V. Sahni, “Quintessential inflation on the brane and the relic gravity wave background,”

- Phys. Rev.* **D70**, 083513 (2004), [arXiv:hep-th/0402086 \[hep-th\]](#).
- [33] J. C. Bueno Sanchez and Konstantinos Dimopoulos, “Trapped Quintessential Inflation,” *Phys. Lett.* **B642**, 294–301 (2006), [Erratum: *Phys. Lett.*B647,526(2007)], [arXiv:hep-th/0605258 \[hep-th\]](#).
- [34] J. C. Bueno Sanchez and Konstantinos Dimopoulos, “Trapped quintessential inflation in the context of flux compactifications,” *JCAP* **0710**, 002 (2007), [arXiv:hep-th/0606223 \[hep-th\]](#).
- [35] Xiang-hua Zhai and Yi-bin Zhao, “Dynamics of quintessential inflation,” *Chin. Phys.* **15**, 2465 (2006), [arXiv:astro-ph/0511512 \[astro-ph\]](#).
- [36] Safia Ahmad, R. Myrzakulov, and M. Sami, “Relic gravitational waves from Quintessential Inflation,” (2017), [arXiv:1705.02133 \[gr-qc\]](#).
- [37] M. C. Bento, R. Gonzalez Felipe, and N. M. C. Santos, “A simple quintessential inflation model,” *Int. J. Mod. Phys.* **A24**, 1639 (2009).
- [38] Gary Felder, Lev Kofman, and Andrei Linde, “Instant preheating,” *Phys. Rev.* **D59**, 123523 (1999), [hep-ph/9812289v2](#).
- [39] Gary N. Felder, Lev Kofman, and Andrei D. Linde, “Inflation and preheating in NO models,” *Phys. Rev.* **D60**, 103505 (1999), [arXiv:hep-ph/9903350 \[hep-ph\]](#).
- [40] A. H. Campos, H. C. Reis, and R. Rosenfeld, “Preheating in quintessential inflation,” *Phys. Lett.* **B575**, 151–156 (2003), [arXiv:hep-ph/0210152 \[hep-ph\]](#).
- [41] Bo Feng and Ming-zhe Li, “Curvaton reheating in nonoscillatory inflationary models,” *Phys. Lett.* **B564**, 169–174 (2003), [arXiv:hep-ph/0212213 \[hep-ph\]](#).
- [42] Tomohiro Matsuda, “NO Curvatons or Hybrid Quintessential Inflation,” *JCAP* **0708**, 003 (2007), [arXiv:0707.1948 \[hep-ph\]](#).
- [43] J. C. Bueno Sanchez and Konstantinos Dimopoulos, “Curvaton reheating allows TeV Hubble scale in NO inflation,” *JCAP* **0711**, 007 (2007), [arXiv:0707.3967 \[hep-ph\]](#).
- [44] E. J. Chun, S. Scopel, and I. Zaballa, “Gravitational reheating in quintessential inflation,” *JCAP* **0907**, 022 (2009), [arXiv:0904.0675 \[hep-ph\]](#).
- [45] L. H. Ford, “Gravitational Particle Creation and Inflation,” *Phys. Rev.* **D35**, 2955 (1987).
- [46] A. Oliveros, “Slow-roll inflation from massive vector fields non-minimally coupled to gravity,” *Astrophys. Space Sci.* **362**, 19 (2017), [arXiv:1612.06377 \[gr-qc\]](#).
- [47] Shinji Tsujikawa, M. Sami, and Roy Maartens, “Observational constraints on braneworld inflation: The Effect of a Gauss-Bonnet term,” *Phys. Rev.* **D70**, 063525 (2004), [arXiv:astro-ph/0406078 \[astro-ph\]](#).
- [48] Zong-Kuan Guo and Dominik J. Schwarz, “Slow-roll inflation with a Gauss-Bonnet correction,” *Phys. Rev.*

- [D81](#), 123520 (2010), [arXiv:1001.1897 \[hep-th\]](#).
- [49] Peng-Xu Jiang, Jian-Wei Hu, and Zong-Kuan Guo, “Inflation coupled to a Gauss-Bonnet term,” *Phys. Rev.* **D88**, 123508 (2013), [arXiv:1310.5579 \[hep-th\]](#).
- [50] Seoktae Koh, Bum-Hoon Lee, Wonwoo Lee, and Gansukh Tumurtushaa, “Observational constraints on slow-roll inflation coupled to a Gauss-Bonnet term,” *Phys. Rev.* **D90**, 063527 (2014), [arXiv:1404.6096 \[gr-qc\]](#).
- [51] Carsten van de Bruck and Chris Longden, “Higgs Inflation with a Gauss-Bonnet term in the Jordan Frame,” *Phys. Rev.* **D93**, 063519 (2016), [arXiv:1512.04768 \[hep-ph\]](#).
- [52] Carsten van de Bruck, Konstantinos Dimopoulos, and Chris Longden, “Reheating in Gauss-Bonnet-coupled inflation,” *Phys. Rev.* **D94**, 023506 (2016), [arXiv:1605.06350 \[astro-ph.CO\]](#).
- [53] Seoktae Koh, Bum-Hoon Lee, and Gansukh Tumurtushaa, “Reconstruction of the Scalar Field Potential in Inflationary Models with a Gauss-Bonnet term,” (2016), [arXiv:1610.04360 \[gr-qc\]](#).
- [54] Jose Mathew and S. Shankaranarayanan, “Low scale Higgs inflation with Gauss–Bonnet coupling,” *Astropart. Phys.* **84**, 1–7 (2016), [arXiv:1602.00411 \[astro-ph.CO\]](#).
- [55] Nobuchika Okada and Satomi Okada, “Simple inflationary models in Gauss-Bonnet brane-world cosmology,” (2014), [arXiv:1412.8466 \[hep-ph\]](#).
- [56] Igor V. Fomin and Sergey V. Chervon, “A new approach to exact solutions construction in scalar cosmology with a Gauss-Bonnet term,” (2017), [arXiv:1704.07786 \[gr-qc\]](#).
- [57] Eric V. Linder, “ Λ is Coming: Parametrizing Freezing Fields,” *Astropart. Phys.* **91**, 11–14 (2017), [arXiv:1701.01445 \[astro-ph.CO\]](#).
- [58] O. Santillan, “General aspects of Gauss-Bonnet models without potential in dimension four,” (2017), [arXiv:1703.01713 \[gr-qc\]](#).
- [59] Thomas P. Sotiriou and Shuang-Yong Zhou, “Black hole hair in generalized scalar-tensor gravity,” *Phys. Rev. Lett.* **112**, 251102 (2014), [arXiv:1312.3622 \[gr-qc\]](#).
- [60] Shinsuke Kawai, Masa-aki Sakagami, and Jiro Soda, “Instability of one loop superstring cosmology,” *Phys. Lett.* **B437**, 284–290 (1998), [arXiv:gr-qc/9802033 \[gr-qc\]](#).
- [61] Shinsuke Kawai and Jiro Soda, “Evolution of fluctuations during graceful exit in string cosmology,” *Phys. Lett.* **B460**, 41–46 (1999), [arXiv:gr-qc/9903017 \[gr-qc\]](#).
- [62] Pierre Binétruy, Christos Charmousis, Stephen C Davis, and Jean-Francois Dufaux, “Avoidance of naked singularities in dilatonic brane world scenarios with a Gauss-Bonnet term,” *Phys. Lett.* **B544**, 183–191 (2002), [arXiv:hep-th/0206089 \[hep-th\]](#).

- [63] Philippe Brax and Carsten van de Bruck, “Cosmology and brane worlds: A Review,” *Class. Quant. Grav.* **20**, R201–R232 (2003), [arXiv:hep-th/0303095 \[hep-th\]](#).
- [64] Christos Charmousis and Jean-Francois Dufaux, “General Gauss-Bonnet brane cosmology,” *Class. Quant. Grav.* **19**, 4671–4682 (2002), [arXiv:hep-th/0202107 \[hep-th\]](#).
- [65] Cristiano Germani and Carlos F. Sopuerta, “String inspired brane world cosmology,” *Phys. Rev. Lett.* **88**, 231101 (2002), [arXiv:hep-th/0202060 \[hep-th\]](#).
- [66] Elias Gravanis and Steven Willison, “Israel conditions for the Gauss-Bonnet theory and the Friedmann equation on the brane universe,” *Phys. Lett.* **B562**, 118–126 (2003), [arXiv:hep-th/0209076 \[hep-th\]](#).
- [67] John F. Donoghue, “The effective field theory treatment of quantum gravity,” *Proceedings, 6th International School on Field Theory and Gravitation (ISFTG 2012)*, *AIP Conf. Proc.* **1483**, 73–94 (2012), [arXiv:1209.3511 \[gr-qc\]](#).
- [68] Kazuharu Bamba, Andrey N. Makarenko, Alexandr N. Myagky, and Sergei D. Odintsov, “Bouncing cosmology in modified Gauss-Bonnet gravity,” *Phys. Lett.* **B732**, 349–355 (2014), [arXiv:1403.3242 \[hep-th\]](#).
- [69] M. Gasperini, M. Maggiore, and G. Veneziano, “Towards a nonsingular pre - big bang cosmology,” *Nucl. Phys.* **B494**, 315–330 (1997), [arXiv:hep-th/9611039 \[hep-th\]](#).
- [70] Gregory Walter Horndeski, “Second-order scalar-tensor field equations in a four-dimensional space,” *Int. J. Theor. Phys.* **10**, 363–384 (1974).
- [71] Christos Charmousis, Edmund J. Copeland, Antonio Padilla, and Paul M. Saffin, “Self-tuning and the derivation of a class of scalar-tensor theories,” *Phys. Rev.* **D85**, 104040 (2012), [arXiv:1112.4866 \[hep-th\]](#).
- [72] Edmund J. Copeland, Antonio Padilla, and Paul M. Saffin, “The cosmology of the Fab-Four,” *JCAP* **1212**, 026 (2012), [arXiv:1208.3373 \[hep-th\]](#).
- [73] Lev Kofman, Andrei D. Linde, and Alexei A. Starobinsky, “Towards the theory of reheating after inflation,” *Phys. Rev.* **D56**, 3258–3295 (1997), [arXiv:hep-ph/9704452 \[hep-ph\]](#).
- [74] Konstantinos Dimopoulos and Charlotte Owen, “Quintessential Inflation with α -attractors,” *JCAP* **1706**, 027 (2017), [arXiv:1703.00305 \[gr-qc\]](#).
- [75] C. Wetterich, “Quintessence: The fifth force,” *Physik J.* **3N12**, 43 (2004).
- [76] Sean M. Carroll, “Quintessence and the rest of the world,” *Phys. Rev. Lett.* **81**, 3067–3070 (1998), [arXiv:astro-ph/9806099 \[astro-ph\]](#).
- [77] Thomas P. Sotiriou and Enrico Barausse, “Post-Newtonian expansion for Gauss-Bonnet gravity,” *Phys. Rev.* **D75**, 084007 (2007), [arXiv:gr-qc/0612065 \[gr-qc\]](#).

- [78] Luca Amendola, Christos Charmousis, and Stephen C. Davis, “Constraints on Gauss-Bonnet gravity in dark energy cosmologies,” *JCAP* **0612**, 020 (2006), [arXiv:hep-th/0506137 \[hep-th\]](#).
- [79] Luca Amendola, Christos Charmousis, and Stephen C. Davis, “Solar System Constraints on Gauss-Bonnet Mediated Dark Energy,” *JCAP* **0710**, 004 (2007), [arXiv:0704.0175 \[astro-ph\]](#).