OPEN ACCESS

University of BRISTOL

Corti, R., & Diambra, A. (2017). Application of a memory surface model in predicting the sand response under cyclic loading. In *Proceedings of the 8th International Offshore Site Investigation and Geotechnics Conference 2017* (Vol. 1). Society for Underwater Technology. https://doi.org/10.3723/OSIG17.384

Peer reviewed version

Link to published version (if available):
10.3723/OSIG17.384

Link to publication record in Explore Bristol Research
PDF-document

University of Bristol - Explore Bristol Research

General rights

APPLICATION OF A MEMORY SURFACE MODEL IN PREDICTING THE SAND RESPONSE UNDER CYCLIC LOADING CONDITIONS

R Corti
Arup, London, UK

A Diambra
University of Bristol, Bristol, UK

Abstract
This paper investigates the potentials of the Memory Surface Hardening model to predict the mechanical response of a Quartz Sand under drained cyclic loading conditions. The constitutive model is implemented in a kinematic hardening, bounding-surface and critical state framework. A new surface, the memory surface, is introduced to retain memory of previous stress history and to define a region of increased stiffness. The memory surface is subjected to two uncoupled hardening mechanisms linked to the experienced contractive and dilative plastic volumetric strains: the memory surface expands when the soil experiences contractive plastic volumetric strains; the memory surface contracts when the soil experiences dilative plastic volumetric strains. The model will be validated against drained cyclic triaxial test data and it will be shown that the model can simulate the magnitude of the accumulated strains for different relative densities, cyclic amplitudes and average stress ratios, while some hints for further improvement will also be provided.

1. Introduction
The soil response under cyclic loading conditions is of interest for many geotechnical structures such as road pavements, tank foundations and offshore structures. When a geotechnical structure is subjected to cyclic loading, permanent settlements and rotations are accumulated affecting its serviceability (Niemunis et al, 2005; Leblanc et al, 2009; Randolph and Gourvenec, 2011, among others). In the last years, a number of modelling strategies have been proposed to quantify the strain accumulation of soils under cyclic loading (Papadimitrou et al, 2002; Dafalias and Manzari, 2004; Taborda et al, 2014; Corti et al., 2015; Corti, 2016; Corti et al, 2016). However, most of the models are valid only for limited loading and drainage conditions, and they generally employ complex constitutive formulations.

The recent Memory Surface Hardening constituive model (Corti, 2016; Corti et al, 2016) accounts for the effect of cyclic or repeated loading conditions by postulating the existence of an evolving memory surface, which encloses a region of high stiffness where any change in the soil stress state causes limited perturbation of the soil fabric and the develoment of low levels of plastic deformations. This model has been previously validated against experimental results on silica sand samples, while the present work would like to challenge the Memory Surface Hardening model to simulate the cyclic mechanical response of a different Quartz sand, using data from the literature (Witchmann, 2005).

The Memory Surface Hardening model has been selected among others because it requires only two additional constitutive parameters to capture the cyclic loading behaviour of soils and for the fact that the model response under cyclic loading can be explained by simple geometrical mechanisms of the newly introduced memory surface.

2. The Memory Surface Hardening model
2.1 General overview of the model
The Memory Surface Hardening constitutive model is an evolution of the Severn-Trent sand model (Gajo and Muir Wood, 1999) developed in a critical state, bounding surface, kinematic hardening modelling framework. The original Severn-Trent sand model postulated the existence of two model surfaces: the yield surface (f) enclosing an elastic isotropic region and the bounding surface (f^B) which models the currently available soil strength and whose size is linked to the current value of the state parameter (Been and Jefferies, 1985). Both model surfaces bound an open wedge region in the multiaxial stress space as shown in Fig. 1.
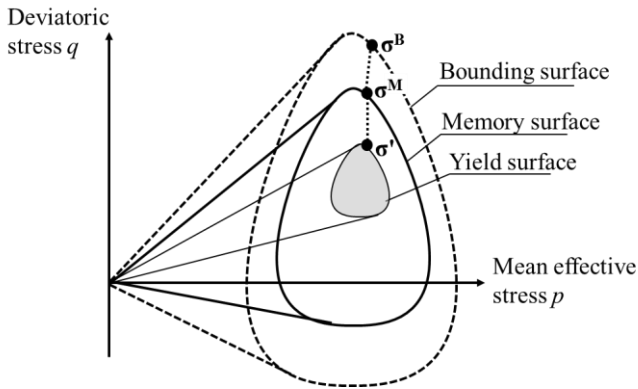
Figure 1: Schematic representation of the Memory Surface Hardening model

The memory surface (f^M, Fig. 1) is an additional model surface to track the past stress history of the soil and retain its memory (Chow et al, 2015; Corti, 2016; Corti et al, 2016). The stiffer response of the soil when loaded inside the memory surface has been modelled by modyfing the hardening modulus, which is now proportional to the distances between the current stress (σ') and its conjugates on both the memory and bounding surface, σ^M and σ^B respectively (Fig.1). The memory surface can evolve in size and rotate to include newly experienced stress states or to simulate the build up of some strong fabric but it can also shrink to mimic the loss of some memory and fabric when sheared to large strain states. This evolution has been modelled by Corti et al. (2016) with a dependency from the experienced plastic volumetric strains. It was assumed that expansion is associated with contractive plastic volumetric strains while shrinking of the surface is associated with dilative plastic volumetric strains. The rationale and mechanisms of this evolution, primarily governing the mechanical repsonse under cyclic loading, is described in the following. The full constituive formulation of the model can be found in Corti et al. (2016).

2.1 Memory surface expansion
Positive (contractive) plastic volumetric strains induce denser soil states which are generally accompanied with a more stable configuration and stronger soil fabric. Thus, it seems quite reasonable to associate such condition with an expansion of the memory surface, as shown in Figure 2. Expansion of the memory surface means also a larger distance between the current stress (σ) and its image on the memory surface (σ^M), resulting in a larger hardening modulus and, in turn, in higher soil stiffness.

Following the interpretation of Tatsuoka et al. (1997) for granular soils under cyclic loading conditions of the framework developed by Jardine (1992), the memory surface expands as a consequence of the accumulation of plastic contractive volumetric strains. The memory surface always expands ahead the current stress state and this can be thought as an evolution of the virgin reference state, represented in this case by the image stress (σ^M) on the memory surface. This ensures also that the memory surface expands to include a new stress state when the current stress lies on the memory surface boundaries (for which $\sigma=\sigma^M$), as it will be shown in the following example in Fig. 2.

The mechanism governing the memory surface expansion is now qualitatively described for a typical test available from the literature (Escribano, 2014), where a loose sample is subjected to drained cyclic triaxial conditions (Figure 2). In this test, the soil sample is initially sheared statically at constant mean pressure up to a target value B (A-B, Figure 2); then the soil is subjected to drained cyclic loading by imposing a constant cell pressure and varying the axial stress (C-D); finally the sample is sheared to failure (point E and beyond). The stress-strain response is shown in Figure 2a. At the initial pre-shearing stage, the memory surface is assumed to coincide to the yield surface and the soil behaves elastically (Figure 2b).
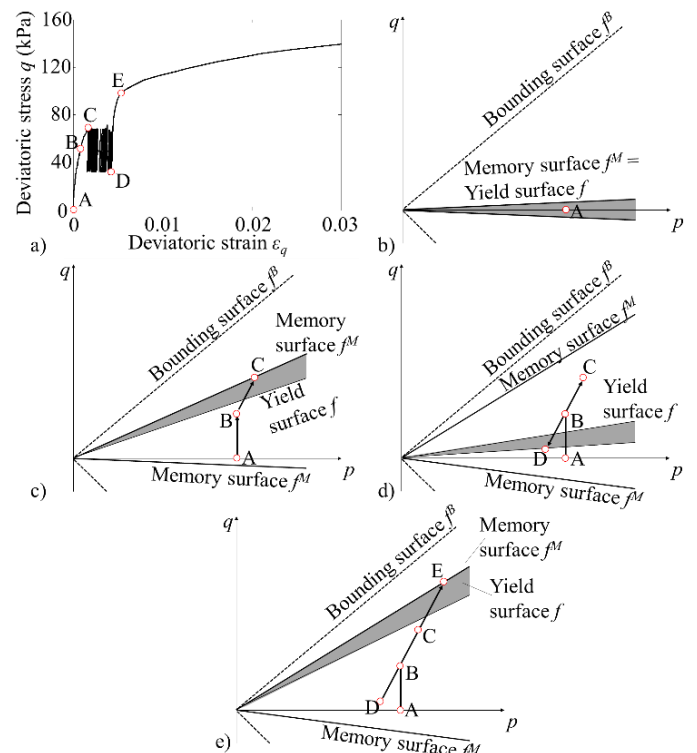

Figure 2: Memory Surface Hardening model surface evolution during a cyclic drained triaxial test followed by monotonic loading: a) stress-strain response from experiments (Escribano, 2014) b) pre-shearing initial conditions c) memory surface evolution during virgin loading d) memory surface evolution during cyclic loading conditions e) monotonic loading after cyclic loading conditions.

Then the soil is statically sheared up to the stress point C, following the stress path A-B-C; during these virgin loading conditions the memory surface f^M evolves to include the newly experienced stress states (Figure 2c). The soil is then subjected to 1500 drained loading cycles applied between the stress points C and D; as a consequence of the experienced plastic contractive volumetric strains (note the initial loose soil state), the memory surface progressively evolves and expand during cyclic loading (Figure 2d). This expansion is associated with a progressive increase of the hardening modulus and of the soil stiffness. Finally, the soil is subjected to monotonic loading. As soon as the stress state reaches the state E, representing the upper boundary of the memory surface, virgin conditions are re-established (Figure 2e) and the upper boundary of the memory surface coincides with the yield surface.

2.2 Memory surface contraction

Opposite to the memory surface contraction mechanism, it is postulated that negative (dilative) plastic volumetric strains are associated to a reduction in size of the memory surface which reproduces the loss of memory of some already experienced stress states (Figure 3). This mechanism follows the experimental evidences by Cazacliu (1996) and Tatsuoka et al. (1997) who observed a decrease in soil stiffness, compared to virgin loading conditions, if large shearing or large amplitude cyclic loading are applied. Analogously, for cyclic loading under undrained conditions, a dramatic increase in the pore water pressure development rate can be observed when the load is reversed above the Phase Transformation Line (PTL) (Ishihara et al., 1975; Georgiannou et al., 2008). Nemat-Nasser (1980) and Nemat-Nasser and Tobita (1982) related this phenomenon to the activation of different dilatancy contacts if soil experiences contraction or dilation. When the soil experiences dilation, the activation of dilative contacts allows further densification (or pore water pressure development) as soon as the stress state is reversed. Similarly to other constitutive models available in the literature where an accumulation of dilative plastic volumetric strains is related to a reduction in the plastic soil stiffness (Papadimitriou and Bouckovalas, 2002; Taborda et al., 2014), a possible "damage" (contraction) of the memory surface is introduced in the model framework by allowing a progressive reduction in the memory surface size as soon as the soil experiences plastic dilative (negative) volumetric strains.
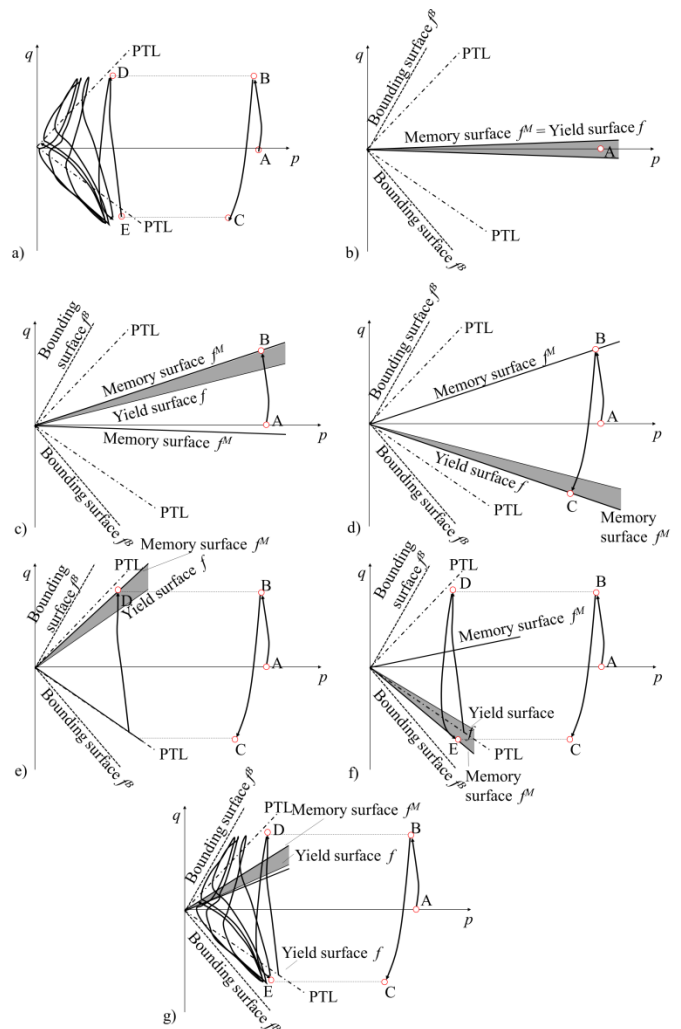

Figure 3: Memory Surface Hardening model surface evolution for the response of a dense sand subjected to undrained triaxial cyclic loading conditions (Zhang et al., 2011); a) experimental stress path b) pre-shearing model response c) first loading d) load reversal e) model response prior to stress state to cross the Phase Transformation Line (PTL) f) memory surface contraction as a consequence of the soil experiencing dilative plastic volumetric strains g) model surfaces for cyclic mobility.

The minimum size of the memory surface is the current yield surface. Both mechanisms of memory surface expansion and contraction are now employed in Figure 3 to qualitatively describe the response of a dense sand sample subjected to undrained cyclic triaxial conditions (Zhang et al., 2011).

The dense sand behaviour under undrained cyclic loading conditions is governed by the Phase Transformation Line (PTL in Figure 3), which represents the threshold beyond which the soil experiences a dramatic increase in the pore water pressure if the load is reversed.

The experimental results are shown in Figure 3a. Initially, the memory surface is assumed to coincide to the yield surface and the stress state lies inside the above mentioned surfaces (point A in Figure 3b). The soil is then subjected to shearing and the memory

surface expands to accommodate the newly experienced stress states (point B in Figure 3c); between the stress points B and C, the soil is subjected to load reversal. Initially the soil lies within the memory surface (previously tracked between the stress states A and B) and as soon as the stress state approaches the memory surface, this again expands to include the newly experienced stress states (Figure 3d). After 35 undrained loading cycles, the effective mean pressure p consistently reduces but the experienced plastic contractive volumetric strains induce a progressive increase in the memory surface size (Figure 3e). As soon as the stress state crosses the Phase Trasformation Line (PTL), the model predicts dilative plastic volumetric strains and the memory surface contraction mechanism is activated, leading now to a reduction in size of the memory surface and to a progressive reduction of the plastic soil stiffness (Figure 3f). The soil is then subjected to further loading cycles and it consequently experiences cyclic mobility (Figure 3g).

3. Model calibration
The eleven constitutive parameters affecting the soil response under monotonic loading (from G to k_d in *Table 1*) have been calibrated against six monotonic drained triaxial tests available in the literature (Wichtmann, 2005) performed at constant cell pressure and increasing the axial stress. Three tests were performed at different initial mean stresses (p_0=50, 100 and 200 kPa) and same void ratio (e_0=0.69), as shown in Figure 4a-b. Other three tests were performed at constant initial mean pressure (p_0=200 kPa) and different void ratios (e_0=0.59, 0.69 and 0.80), as presented in Figure 4c-d. The constitutive parameters governing the model cyclic response (μ and ς) have been calibrated following the procedue detailed in Corti et al. (2016). The employed values for the constitutive parameters of the Memory Surface Hardening model are shown in Table 1.
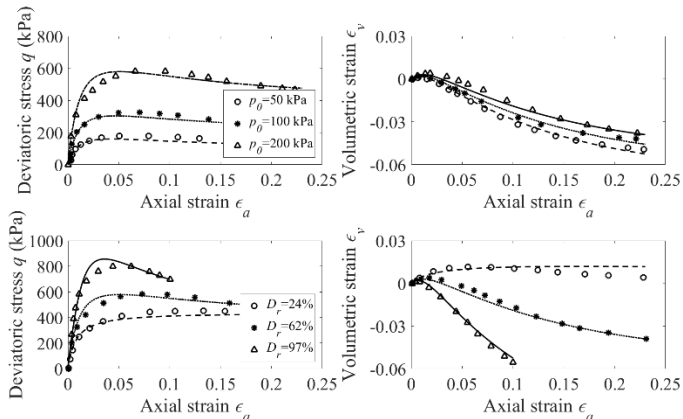
Table 1: Proposed model calibration for Quartz sand.

Parameter	Description	Quartz sand
G	Elastic Shear Modulus	$G_0/2.5$
v	Poisson's ratio	0.2
ϕ_{cv}	Friction angle at critical state conditions	31°
λ	Slope of the critical state line in the v-$\ln p$ plane	0.02
v_λ	Value of the specific volume on the critical state line at p=1 kPa	1.900
R	Ratio between the opening of the yield surface and the strength surface	0.075
β	Parameter controlling the hardening modulus	0.002
k	Parameter controlling the relationship between soil strength and state parameter	3.2
A	Flow rule multiplier	0.8
k_d	State parameter dependency of the flow rule	1
μ	New hardening constitutive parameter	13
ς	Damage rule parameter	0.0005

4. Model predictions
4.1 Effect of soil density
In this section, the model predictions are challenged against four drained cyclic triaxial tests performed at different densities and constant average stress ratio η^{ave} (defined as the average static shear stress ratio at which cyclic loading is then applied) and cyclic amplitude $\Delta\beta$. The results are shown in terms of accumulated strains ε^{acc} against the number of cycles N_{cyc}, as shown in Figure 5. The accumulated strains ε^{acc} are calculated following the work of Wichtmann (2005). The model predicts satisfactory the effect of the soil density after 250 loading cycles by simulating larger accumulated strains for soils at higher void ratios (lower soil densities). The model well reproduces the soil behaviour at the void ratio e_0=0.803, 0.715 and 0.674, while the accumulated strains are slightly overestimated when the initial soil density is e_0=0.580.

Figure 4: Model calibration for Quartz sand using experimental data from Wichtmann (2005).
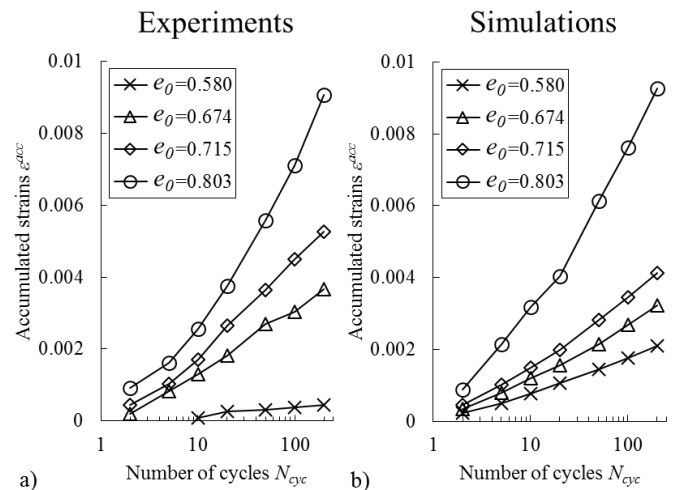

Figure 5: Effect of the soil density on the accumulation of permanent strains a) experiments b) model predictions. Experimental data after Wichtmann (2005).
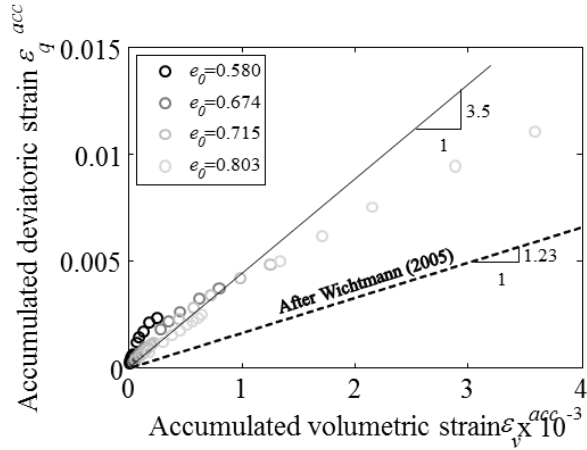
Figure 6: Direction of accumulated strains from numerical analysis.

A further analysis is provided to study the influence of the void ratio on the direction of accumulation (defined as the ratio between the accumulated deviatoric to volumetric strains $\varepsilon_q^{acc}/\varepsilon_v^{acc}$). The model confirms that the direction of accumulation, presented in Figure 6, is not affected by the soil density. However, the model overestimates the direction of accumulation by comparing the slope of the continuous line (obtained from the numerical analysis) and the dashed line (representing the experimental observations). This aspect may be improved by assuming a different the flow rule than the one adopted in Corti et al. (2016).

4.2 Average stress ratio
In this section, the Memory Surface Hardening consitutive model is challenged to capture the effect of the average stress ratio η^{ave} on the magnitude of the accumulated strains. It is experimentally observed that the magnitude of the accumulated permanent strains increases with increasing average stress ratios, as shown in
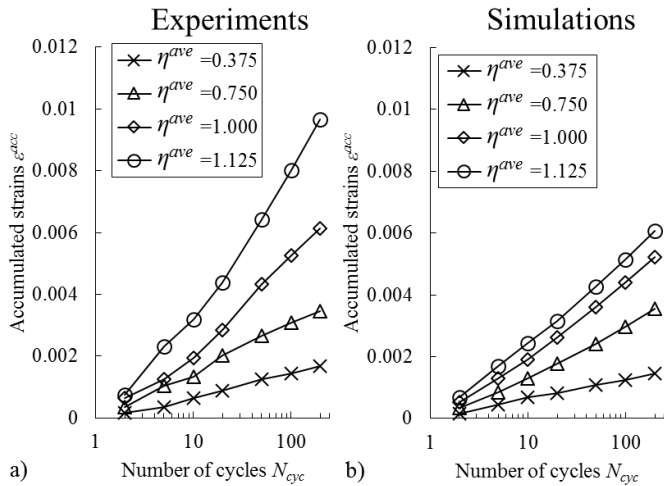

Figure 7a. This behavioural feature is well reproduced by the proposed model, as demonstrated in
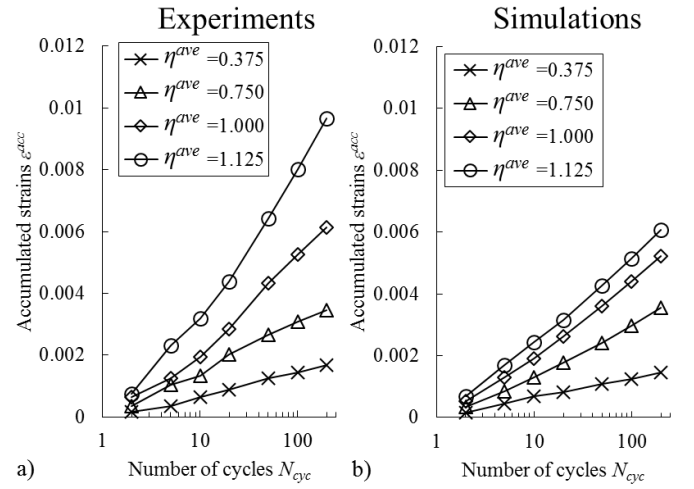

Figure 7b. For lower average stress ratios (η^{ave}=0.375, 0.75 and 1.00), the model adequately reproduces the magnitude of the accumulated strains. However, for higher average stress ratios the model slightly underestimates the magnitude of the accumulated strains (η^{ave}=1.125). A possible reason of this discrepancy between the simulations and the experimental results could be given by the evolution of the memory surface. The memory surface can be damaged (e.g. contraction of the memory surface) only if the soil experiences dilative plastic volumetric strains. However, Tatsuoka et al. (1997) stated that soil fabric is damaged by large shearing which occurs when soil experiences larger average stress ratios.
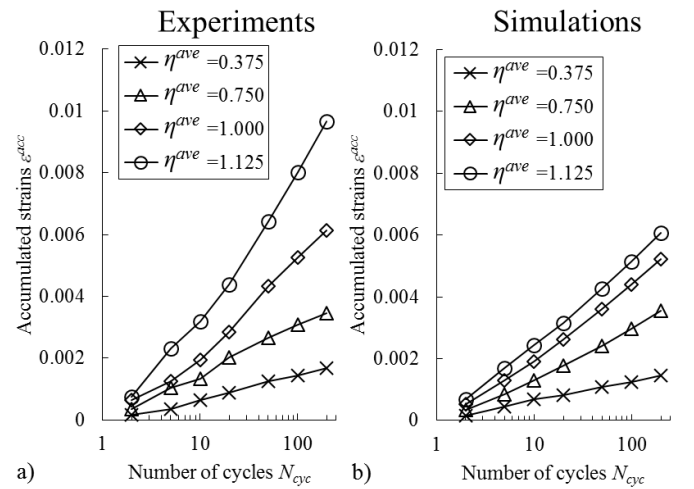

Figure 7: Effect of the average stress ratio on the accumulated strains a) experiments b) simulations. Experimental data after Wichtmann (2005).

Thus, the question whether the evolution rule of the memory proposed by Corti et al. (2016) could be improved for large average cyclic stress ratio arises.

4.3 Effect of cyclic amplitude

In this section the Memory Surface Hardening model is challenged to simulate the effect of the cyclic amplitude on the magnitude of the accumulated strains. It is experimentally observed that the accumulated strain magnitude increases if the soil is subjected to larger cyclic stress amplitudes, as illustrated in Figure 8a.

The model is able to reproduce quantitatively and qualitatively the experimental evidences as shown in Figure 8b; however, the model slightly overestimates the magnitude of the accumulated strains for Δq=68 kPa and underestimates the accumulated strains for Δq=48 kPa.
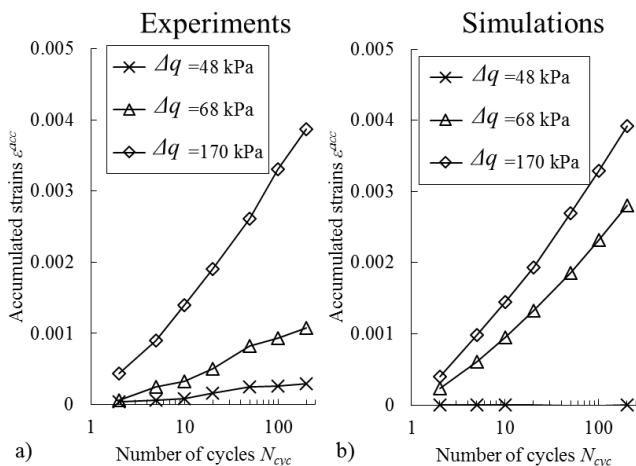

Figure 8: Strain accumulation for different cyclic amplitude a) experiments b) simulations. Experimental data after Wichtmann (2005).

5. Conclusions

The Memory Surface Hardening model has been challenged to capture the strain accumulation under drained cyclic triaxial loading for a Quartz sand. The model predictions are reasonable and generally in good agreement with experimental results for cyclic loading conditions under different densities, average stress ratios, cyclic amplitudes and stress histories. It appears that the model can capture quite well the following behavioural features:

- Progressive reduction of the accumulated strain rate with the number of cycles;
- Increase of cyclic strain accumulation with decreasing soil density;
- Increase of cyclic strain accumulation with increasing cyclic average stress ratio;
- Increase of cyclic strain accumulation with larger cyclic amplitudes.

However, the following hints for improvements have also been identified:

- More accurate prediction of the accumulated strains for low cyclic amplitudes;

- More accurate prediction of the accumulated strains for very dense materials.
- Review of the assumed flow rule controlling the direction of cyclic plastic strain accumulation.

Based on the simulations shown in this paper, the model can be implemented in a finite element analysis code to solve practical geotechnical problems. Specific attention should be given to finite element analyses involving different cyclic amplitudes. In such case it is recommended to check model calibration and predictions against cyclic triaxial tests performed using cyclic amplitudes similar to the ones applied in the finite element analysis.

6. References

Alarcon-Guzman A, Leonards GA and Chameau JL (1988) Undrained monotonic and cyclic strength of sand. *Journal of Geotechnical Engineering* 114(10): 1089-1109.

Been, K., & Jefferies, M. G. (1985, June). State parameter for sands. *International Journal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts* (Vol. 22, No. 6, pp. 198-198). Elsevier Science.

Cazacliu B (1996) Comportment des sables en petites et moyennes deformations – Realisation d'un prototype d'essai de torsion compression confinement sur cylindre creux. PhD Thesis, ECP_ENTPE, Paris (France).

Corti R, Diambra A, Nash DFT and Muir Wood D (2015). An evolving memory surface for modelling the cyclic behavior of granular soils. *Proceedings of the XV Panamerican Conference on Soil Mechanics and Geotechnical Engineering*. Buenos Aires, Argentina.

Corti R, Diambra A, Muir Wood D, Escribano DE and Nash DFT (2016). Memory Surface Hardening model for granular soils under repeated loading conditions. *Journal of Engineering Mechanics* 142(12).

Corti R (2016). Hardening memory surface constitutive model for granular soils under cyclic loading conditions. PhD Thesis, University of Bristol.

Chow SH, O'Loughlin C, Corti R, Gaudin C and Diambra A (2015). Drained cyclic capacity of plate anchors in dense sand: Experimental and theoretical observations. *Geotechnique Letters* 5(April-June): 80-85.

Escribano DE (2014). Evolution of stiffness and deformation of Hostun Sand under drained cyclic loading. PhD Thesis, University of Bristol.

Gajo, A., & Wood, M. (1999). Severn–Trent sand: a kinematic-hardening constitutive model: the q–p formulation. *Géotechnique*, 49(5), 595-614.

Georgiannou VN, Tsomokos A and Stavrou K. (2008). Monotonic and cyclic behaviour of sand under torsional loading. *Geotechnique* 58(2): 113-124.

Ishihara K, Tatsuoka F and Yasuda S (1975) Undrained deformation and liqeufaction of sand under cyclic stresses. *Soils and Foundations* 15(1): 29-44.

Jardine (1992) Some observations on the kinematic nature of soil stiffness. *Soils and Foundations* 35(5): 703-718.

Leblanc C, Houlsby GT and Byrne BW (2009). Response of stiff piles in sand to long-term cyclic lateral loading. *Geotechnique* 60(2): 79-90.

Nemat-Nasser S (1980) On the behavior of granular materials. *Soils and Foundations* 20(3): 59-73.

Nemat-Nasser S and Tobita Y (1982) A unified approach to densification and liquefaction of cohesionless sand in cyclic shearing. *Canadian Geotechnical Journal* 16(4): 659-678.

Niemunis A, Wichtmann T and Triantafyllidis T (2005). A high-cycle accumulation model for sand. Computers and Geotechnics 32(4): 245-278.

Randolph M and Gourvenec S (2011) Offshore geotechnical engineering. CRC Press.

Taborda DM, Zdravkovic L, Kontoe S and Potts DM (2014). Computational study on the modifgication of a bounding surface plasticity model for sands. *Computers and Geotechnics* 59:145-160.

Tatsuoka F, Jardine R, Lo Presti D, Di Benedetto H and Kodata T. (1997). Theme Lecture: Characterising the pre-failure deformation properties of geomaterials. *Proceedings of the International Conference on Soil Mechanics and Foundation Engineering* (4): 2129:2164.

Zhang F, Ye B and Ye G (2011) Unified description of sand behaviour. *Frontiers of Architecture and Civil Engineering in China* 5(2): 121-150.