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Modelling and knowledge transfer in complexity science

1. Introduction

Since its inception in the late 1980s, complexity science has evolved into a well-established and highly popularized area of science (e.g., for a description of the field's history, Mitchell, 2009). The exploration of complexity has been connected to a number of foundational claims, ranging from a redefinition of the arrow of time (e.g. Davies, 2003) to the often quoted slogan, 'more is different' by Anderson (1972). Such foundational claims have generated much philosophical debate (e.g. Kauffman, 1993; Coveney and Highfield, 1995; Frigg, 2003; Bedau and Humphreys, 2008; Hooker, 2011).

However, a survey of the existing philosophical literature (e.g., for collections of philosophical works on complexity science, Gregersen, 2003; Hooker, 2011) shows that there are few works that focus on analysing the actual work of practitioners in the field.

This paper aims to address the lack of philosophical analyses of the methodology of complexity science and to provide a detailed account of one crucial aspect of the work of complexity scientists, namely the construction and transfer of complex models. The philosophical literature on modelling in the natural sciences has traditionally been focussed on the construction of these models from both empirical knowledge about a particular target system as well as the general theory that governs this system (e.g. Frigg and Hartmann, 2012; Toon, 2012). I will call this kind of model construction 'vertical construction'. In recent years, the existence of other kinds of model construction has also been highlighted, including the construction of models through the alteration of existing models (Bokulich, 2003), which I will henceforth denote as 'horizontal construction', and the transfer of models from one target system to another (e.g. Hesse, 1966; Bokulich, 2014, 2015). Some studies on the transfer of models have been based on case studies of complex models (Bokulich, 2014). However, to my knowledge, there exist no studies that provide integrated accounts of the interplay of different modelling activities in complexity science. My study here aims to provide such an account.

The paper has two main theses: (i) I will argue that all three modelling activities described above, i.e. vertical construction of models; horizontal construction of models; and the transfer of models to new target systems, can be identified in complexity science; and (ii) that the interplay of these activities is structured in a particular way. In particular, with respect to thesis (ii), I will argue that the modelling activities in complexity science can be divided into two categories: the creation of a repository of general models through large-scale horizontal modelling; and the transfer of these models to particular target systems, which can be combined with an extension of the transferred models through additional vertical constructions. This division is not just one between activities but also one between epistemic fields: the creation of the repository is mainly undertaken by computational scientists and mathematicians while the transfer and extension of models takes place in the natural and social sciences. Accordingly, this interplay between different modelling activities provides a mechanism through which knowledge is transferred between different scientific communities.

Furthermore, my identification of this division of the modelling activities in complexity science can be used to derive a methodological definition of the field of complexity science itself. Namely, the field seems to consist of a core area that can be defined methodologically as comprising those activities that contribute to the creating, cataloguing and investigation of the repository of models without fixed target systems and of a number of auxiliary areas that overlap with other disciplines and that can be methodologically defined as the use of models from the repository to investigate phenomena located in these other areas. This definition of the field of complexity science based on the crucial methodologies used by scientists that self-identify as complexity scientists (i) avoids relying on the currently not unequivocally defined term ‘complexity’ and (ii) allows for the fact that a large number of complexity scientists also have strong associations to other fields, which has contributed to the difficulties in delineating the field.

The methodological definition offered here is clearly neither the only possible definition of the field (e.g., in the context of emergence in complexity science, a division into three schools has been offered by Richardson & Cilliers, 2001) nor will it be able to offer a demarcation criterion that allows an unequivocal assignation of a given investigation or a given scientist as belonging to the field of complexity science or being a complexity scientist. Furthermore, by defining complexity science through the structure of its modelling activities, important parts of the field that could also be viewed as definitional (e.g. metaphorical descriptions, slogans and conceptual definitions) are neglected. However, a delineation of the field that includes all of these parts has so far not been possible. The methodological definition offered here can therefore be viewed as a preliminary definition: a later construction of a more comprehensive definition might find it to be a good starting point.

Furthermore, I hope to show that the methodological definition identifies a large number of activities as being part of complexity science and thereby also provides a clearer exposition of the work conducted by researchers in the field. As explained above, it also reflects the fact that there seem to be two classes of complexity scientists: those

who primarily self-identify as such (e.g. Stephen Wolfram, section 4.1) and those who profess to work with complex models but to primarily belong to an existing field of science (e.g. Michael Batty, section 4.2).

It should be noted that the transfer of models in the auxiliary regions of complexity science is not restricted to the transfer of models from the repository to a particular target system. Within these regions, models are also transferred from one target system to another. This transfer often crosses disciplinary boundaries and, in doing so, seems to have a preferred direction: in many cases, these transfers consist in the adaptation of a natural science model for a target system in the social science. Such cross-disciplinary transfers of models – in complexity science and elsewhere – have recently been investigated by several authors (e.g. Chettiparamb, 2006; Bokulich, 2014; Thebault *et al*., 2017). Since the focus of this paper is the horizontal construction of models and their subsequent transfer, I will not discuss this second transfer mechanism with the same detail. I also maintain that the creation of a repository of models and their subsequent transfer is one of the most distinguishing features of complexity science, while the transfer of models from one target system to another also takes place in other fields (e.g. Thebault et. al., 2017). However, it is clearly an important part of the methodology of complexity science and – as I will describe below – also plays a crucial role in the structuring of its auxiliary regions.

I also maintain that it is through this structuring of modelling activities and the resulting overlap with other disciplines that complexity science becomes an interdisciplinary field. As foreshadowed in the description above, I will argue that the defining methodology of complexity science includes the transfer of models into many different areas of the natural and social sciences. The adaption and use of these models could be viewed as auxiliary areas of complexity science: areas that are methodologically connected to the field as being part of the interplay of different modelling activities, but whose phenomena under investigation are traditionally part of another area of science. Accordingly, the term 'interdisciplinary' can be given a more precise meaning in this context: it denotes the fact that the methodological core area of complexity science, the stocking, investigation and cataloguing of the repository of models, is connected with many different disciplines through the transfer of these models to different target systems.

Furthermore, the cross-disciplinary transfer of models within the auxiliary regions further increases the overlap of complexity science with other fields by providing bridges to additional parts of the natural and social sciences. Given that the directionally of such cross-disciplinary transfers generally seems to be from the natural sciences to the social sciences (e.g. Thebault *et al*., 2017), the picture that seems to emerge is one in which areas of the natural sciences are primarily tied to central repository of models through the transfer of horizontally generated models, while areas of the social sciences are then added to the auxiliary regions of complexity science through the subsequent transfer of such models from a natural science target system to a social science target system. This interpretation is dependent on the classification of different branches of science as 'natural' and 'social'; in some cases, including the field of urban planning, which provides one of my major case studies (Section 4), such a classification is not easily performed. Furthermore, there are also cases in which a model has been transferred directly from the repository into a social science field (e.g., into linguistics, Lansing & Downey, 2011). As explained above, the focus of this paper is on the former mechanism; however, the cross-disciplinary transfer of models from the natural to the social sciences is clearly another important methodological feature of complexity science.

As far as I am aware, the identification and analysis of this interplay of modelling activities is a novel contribution to the philosophical discussion of complexity science.

In Section 2, I will review some relevant material on different kinds of model construction. As outlined above, I will distinguish between vertical (Section 2.1) and horizontal (Section 2.2) model construction.

In Section 3, I will discuss the transfer of models to new target systems. The framework for this discussion will be an adaptation of the analogy-based account of model transfer by Hesse (1966).

In Section 4, I will analyse the interplay of different modelling activities in complexity science. I will first introduce the kind of models that are prevalent in complexity science. I will then show that a division of modelling activities into the two categories mentioned above, i.e. the creation of a repository of models (Section 4.1); and the transfer and extension of models from this repository (Section 4.2), is supported by an analysis of both the actual interplay of models in complexity science as well as the representations of this interplay by practitioners.

2. Vertical and horizontal model construction

Models and their uses in science have been the subject of a prolific and ongoing debate in the philosophy of science (e.g., for review, Frigg and Hartmann, 2012). Much of the philosophical debate on modelling focuses exclusively on one particular kind of modelling practiced by natural scientists. According to Bokulich (2003, p. 610), the prototypical model ascribed to these scientists is a 'vertical' one: a model that has been constructed "either top-down from theory or bottom up from empirical data".

I maintain that the models used in complexity science are not exclusively vertical ones. Rather, many complex models are horizontally constructed. Bokulich (2003, p. 611) describes horizontal models as models that are not constructed vertically from theory or empirical data but are horizontal spin-offs from existing models. I do not wish to claim that horizontal and vertical model construction are the only modes of model construction in all fields of sciences. However, my analysis of complexity science shows that they seem to be the two modes of model construction that exist in this field.

In Section 2.1, I will discuss the vertical construction of models. Given the large amount of literature available on this kind of model construction, this review will be relatively brief. In Section 2.2, I will give a general account of the horizontal construction of models. I have provided extended reviews of the existing material on the construction of horizontal and vertical models in Zuchowski (2017).

2.1 Vertical model construction

Construction of vertical models Vertical model construction has been studied extensively (e.g. Toon, 2012, Cartwright, 1983). Thereby, the model is constructed from the vast amount of top-level theories that govern the processes underlying the target system by an appropriate combination of selections, idealizations and simplifications. This ‘pruning’ process is guided by bottom-level empirical knowledge about the target system; such knowledge allows the identification of the appropriate selection of laws, of suitable idealizations and of allowed simplifications. Accordingly, vertical models are constructed from both top-level theory as well as bottom-level empirical knowledge about the target system. The degree to which each level is involved in the construction can vary in each individual instance (e.g. ‘bottom-up’ vs ‘top-down’ construction). However, the resulting vertical model always constitutes a mediation between the covering theories and the existing empirical knowledge of the target system.

Explanatory function of vertical models The major epistemic role ascribed to vertical models is being informative about their target systems (e.g. Bolinska, 2013). The evaluation of a model therefore entails judging how trustworthy information gained from the model is in guiding our knowledge of, and expectations about, the target system.

Bokulich (2014) thereby distinguishes between `how actually’ and `how possibly’ explanations. `How possibly’ explanations are thereby defined as:

> “[T]hey show how a particular mechanism could, under certain circumstances which may or may not obtain, produce the effect of interest. For many complex phenomena this is a significant step forward, which can then be used as a basis from which further testable predictions can be made”.

According to Bokulich (2014, p. 324), practitioners will be willing to accept a ‘how possibly’ explanation as a ‘how actually’ explanation if sufficient evidence exits that the proposed mechanism actually operates in the model’s target system.

2.2. Horizontal model construction

Bokulich (2003, pp. 612) begins her study with a description of how models in the field of semi-classical physics are constructed: by discretizing the equations of a particular classical model. Thereby, the question of how the original model itself was constructed is unimportant; no information about its governing theory and its target system needs to be transmitted. The initial model merely provides a set of equations, which are mathematically manipulated to provide the new horizontal model. Through this process of construction, horizontal models become “part of a lineage of models with their own internal dynamics and justification” (Bokulich, 2003, p. 613). Thereby, the alterations to the model should be genuine structural changes, i.e. horizontal model construction should go beyond the mere systematic changing of a parameter in the model. While there might be cases in which the distinction between the exploration of a model’s parameter space and the horizontal construction of a new model might become blurred, the cases of horizontal modelling discussed in this paper seem to be relatively clear-cut examples of the structural alterations envisioned by Bokulich (2003).

Zuchowski (2017) identifies similar lineages of horizontal and vertical models in chaos theory. It should be noted that the fact that chaos theory and complexity science both feature horizontal modelling does not imply that chaos and complexity are conceptually similar (Section 4).

In the examples discussed by Bokulich (2003) and Zuchowski (2017), the construction of horizontal models proceeds without reference to any natural target system and no such system could be (directly) assigned to these models. I take this lack of reference to a target system to be characteristic of horizontal model construction. Therefore, I come to a different classification of two further scenarios labelled ‘horizontal model construction’ by Bokulich (2014, p. 327) and Bokulich (2015, p. 29). These studies analyse the use of models originally developed to model physical and chemical phenomena to represent the development of striped vegetation, and the use of hydrodynamic models to represent electrodynamical phenomena, respectively. The assignment of these models to their new target systems thereby rests on the identification of analogies between the phenomena (as originally discussed by Hesse, 1966). In this study, I find it useful to distinguish between these scenarios, which I consider forms of model transfer (Section 3), and those of purely mathematical construction, which proceed without reference to a target system (as discussed above), which I consider examples of horizontal model construction.

I am happy to concede that this distinction has been made for practical reasons. There is prima facie no reason to not use ‘horizontal modelling’ as a summary term that includes model transfer as an activity of model construction. Both horizontal model construction in the sense outlined above and model transfer in the sense of the scenarios described by Bokulich (2014, 2015) clearly differ from the vertical construction of models (Section 2.1). However, in the context of complexity science, I find it useful to distinguish clearly between these two activities since the demarcation between these two modelling activities also appears to be one of the natural epistemic fault lines of the field.

Investigative function of horizontal models Bokulich (2003, p. 613) assigns horizontal models an investigative function: e.g., in the case of semi-classical physics, they are a means of investigating relationships between classical and quantum theories. I agree with Bokulich (2003) that horizontal models have an investigative function rather than one of mediating between theory and data. However, I take this function to be both more general, as well as more subject-specific, than that of investigating inter-theoretical relationships. Horizontal models in chaos theory often appear to be designed with the explicit aim of gaining more information about specific other chaotic models, usually with the aim of establishing the sufficient conditions under which this model will behave chaotically (Zuchowski, 2017). In other words, they can be instrumental in establishing ‘how possibly’ explanations for a given phenomenon.

3. Model transfer and reinterpretation

In this section, I will discuss the process of model transfer. I will apply this label to any scenario in which a fully-fledged model is assigned to a new target system. The cases of model transfer discussed in the existing literature are usually scenarios of reassignment, i.e. a model that was previously used as a representation of a specific target system is now assigned to another target system.

The most influential analysis of model transfer to date is Hesse's (1966) account of the transfer of the model of the propagation of water waves to the propagation of acoustic waves and of light waves. Recently, a more historically oriented analysis of this example of model transfer has been provided by Bokulich (2015). None of these authors explicitly identifies this scenario as a case of model transfer; the focus of these two studies is on the identification of analogies between natural phenomena and models. However, I maintain that the derivation of the acoustic and electrodynamic wave model from the hydrodynamic wave model is a case of model transfer and can be analysed as such.

Hesse's (1966) account of the transfer of the wave model to different target systems is based on the identification of analogies between these systems. Hesse (1966, p. 11) provides a table of such analogies for the transfer of the 'wave model' from hydrodynamics (where it represents water waves) to acoustics (where it represents sound waves) to electrodynamics (where it represents light waves): the crest height of a water wave, the loudness of a sound wave and the brightness of a light wave are all represented by the same term in the model's formalism, i.e., the amplitude, and are therefore analogous to each other. A similar correspondence can be established between the spatial distance between water waves, the pitch of sound waves and the colour of light waves; or between the medium of propagation (water, air, 'ether').

The fact that the initial transfer of the 'wave model' involves the postulation of 'ether' as a medium for light waves shows that this mechanism should not be seen as a methodology for picking out ontological features of a phenomenon. Furthermore, Bokulich (2015, pp. 32-34) shows that the assignment of analogies is not unique: she outlines how Maxwell and Helmholtz each constructed different analogies between hydrodynamic and electrodynamic waves. The use of analogies in modelling is therefore usually interpreted as an investigative tool rather than an absolute assertion about the properties of the natural systems under investigation. Nevertheless, both Hesse (1966, e.g. p. 14) and Bokulich (2015, pp. 3-26) stress the epistemic usefulness of the transfer of a model through the identification of analogies: Hesse (1966) views this reassignment of the model as part of the development of the general theory governing the new target system while Bokulich (2015) discusses the fact that the assignment of the model allows the positing of 'how possibly' explanations for the behaviour of the new target system (Section 2.1).

Interpreting the reassignments of the wave model as a process of model transfer, it is important to note that Hesse (1966, p. 12) states that the transfer is initiated by the recognition of phenomenological analogies:

> "So far we have two sources of information to aid our construction of theories for sound and for light, namely, their observed properties and their observed analogies with water waves, and it is important to notice that both of them appeal only to descriptions of "observable" events. We may define observation statements as those descriptive statements whose truth or falsity in the face of given empirical circumstance would be agreed upon by all users of English with or without scientific training".

In other words, the process of model transfer and reinterpretation is prompted by the recognition of phenomenological similarities between water, sound and light waves. It is notable that this initial recognition does not require any 'scientific training', i.e., no knowledge of the underlying dynamics of either phenomenon is necessary to recognize the phenomenological similarities between them. While the subsequent discussion between Hesse's (1966, pp. 12-15) 'Campbellian' and 'Duhemist' makes it clear that recognizing the analogies between these phenomena requires the existence of at least a rudimentary interpretative framework (e.g. that of wave propagation), Hesse (1966) maintains that no comprehensive knowledge of the specific dynamics underlying the phenomena is required to recognize analogies on an observational level. Bokulich's (2015) historical analysis of the recognition of (different!) analogies by Maxwell and Helmholtz also supports the view that model transfer begins with a recognition of analogies between the behaviour of a model (and its currently assigned target system) and the behaviour of the target system it will eventually be transferred to. The analogies she lists (e.g. Figures 3 and 4 in her paper) are between observational properties in the Hessian sense, e.g., between the rotation visible in fluid and the lines of magnetic force (which can easily be made visible even to the untrained observer).

I will call this step in the transfer process 'phenomenological prompting'. Phenomenological prompting is the recognition that the phenomenology of the model under consideration can be reinterpreted to provide an analogous description of the phenomenology of the intended target system. While Hesse (1966) assumes that the identification of analogies will be between the phenomenologies of the models' target systems, I see no reason to not extend this process to the phenomenologies of a model and a new target system. Accordingly, phenomenological prompting can also initiate the transfer of a horizontally constructed model to a specific target system.

Both Hesse (1966, pp. 13-14) as well as Bokulich (2015, e.g. pp. 29-20) also make it clear that once the transfer has been prompted by the recognition of phenomenological analogies, scientists then proceed by finding more analogies on the dynamical level, i.e., by reinterpreting the dynamics of the existing model to fit the physics of the new target system. In the case of water, sound and light waves, Hesse (1966, p. 13-14) takes this to mean that researchers then reinterpret the wave equation, which is known to govern the dynamics of water waves, in terms of sound and light properties, i.e., they reinterpret the amplitude of this equation as loudness and brightness, respectively. From this follows the recognition that the dynamics of all three phenomena are governed by approximately the same set of laws, namely those that have this wave equation as a solution. This step is also stressed by Bokulich (2015, pp. 30), who defines the kind of analogy eventually found by Maxwell and Helmholtz as "a resemblance in the form of the equations between what are otherwise different sorts of phenomena".

I will call this step in the transfer process 'dynamical reinterpretation'. Dynamical reinterpretation is the reinterpretation of a model's equations in terms of the intended new target system. The end result of this step in the transfer process is a fully-reinterpreted model, i.e., a model for the new target system. The transfer of a model can therefore be seen as a two-step process of phenomenological prompting followed by dynamical reinterpretation. This process can be completed both for a model with pre-assigned target system (e.g., the wave model) and for a model without pre-assigned target system (i.e., a horizontally constructed model).

I do not wish to claim that these two steps are the only possible way to break down the transfer process. Furthermore, I suspect that this process might also be subject-specific and therefore only wish to claim that the two-step interpretation provides a good framework for the analysis of model transfer in complexity science. In particular, the strong emphasis by both practitioners and philosophers (e.g. Wolfram, 2002; Batty, 2005; but also, Dennett, 1991) on the recognition and matching of patterns in the phenomenology of complex systems makes this distinction a natural one. In other fields of science, a different rational reconstruction might be more suitable. However, in Section 4.2, I hope to demonstrate the merits of this analytic framework by showing that it allows me to give a clear exposition of the transfer of models in complexity science.

Explanatory function of transferred models Once the process of model transfer is completed, the transferred model is treated as a representation of its new target system.

Accordingly, the epistemic function of transferred models is the same as those of vertically constructed models (Section 2.1): to be informative about their target systems. In Section 2.1, I have analysed this function as the provision of 'how possibly'/'how actually' explanations (Bokulich, 2014).

However, I agree with Bokulich (2014) that explanations provided by transferred models are – at least initially – more likely to be treated as mere 'how possibly' explanations than those provided by vertically constructed models. This is primarily due to the fact that the transfer process is motivated by phenomenological prompting: while it is therefore guaranteed that the model is able to reproduce the desired phenomenological features, there is no guarantee that the reinterpreted dynamics of the model are representations of the mechanisms actually operating in the target system.

Accordingly, transforming a 'how possibly'-explanation into a 'how actually'explanation requires more work for transferred models than for vertically constructed ones. Bokulich (2014) maintains that this work consists in the gathering of additional empirical evidence. In Section 4.2, I will show that in complexity science there exists another method to ensure that the dynamics of a transferred model matches those of its new target system (in all relevant aspects): the subsequent vertical construction of additions and adaptions to such a model.

4. Modelling in complexity science

In this section, I will discuss the interplay of different modelling activities in complexity science. In particular, I will argue that these activities can be divided into two distinct categories: (i) the creation of a repository of models without fixed target systems through large-scale horizontal model construction; and (ii) the transfer of these models to specific target systems and their subsequent extension through vertical modelling. In Sections 4.1 and 4.2, respectively, these two aspects of modelling in complexity science will be discussed in detail. A third mechanism, which is the transfer of models from one target-system to another, also operates in the auxiliary regions of complexity science but will not be discussed in detail in this paper (Section 1).

This structure of the interplay of different modelling activities in complexity science will be demonstrated during the detailed discussions of each category. However, it is can also be discerned in the representations of modelling in complexity science by practitioners. In particular, textbook-type works aiming at given an overview of the field are usually divided into two parts: (i) one that discusses the construction and evaluation of a large number of models without fixed target systems (e.g., Wolfram, 2002, Chapter 2–5; Casti, 1992a); and (ii) one that discusses the assignment of these models to different specific target systems (e.g., Wolfram, 2002, Chapter 8–1; Casti, 1992b). Part (i) is occasionally described as introducing the 'methodology' or 'technology' of complexity science. While there is no reason to object to this labelling, a closer look at such works shows that a significant part of the methodological tools presented are actually particular models, which have been constructed by the methodologies discussed in Section 4.1.

Models in complexity science Due to the currently unresolved question of how 'complexity' should be defined, the kind of models that are seen as part of complexity science differ between different authors. In particular, some authors (e.g., Casti, 1992a) view chaos theory as a subfield of complexity science and included chaotic models in this set. In Zuchowski (2012), I argue that chaos is conceptually different from complexity theory and that chaos theory is based on different methodologies than complexity science. Furthermore, in Section 4.1, I will argue that not all models that are used by practitioners in complexity science - defined methodologically as being constituted by the characteristic interplay of modelling activities (Section 1) - also carry the label 'complex' since this label is primarily used as a phenomenological descriptor.

My methodological definition of complexity science (Section 1) leads to a straightforward formal identification of the class of complex models: this class can be defined as all models being part of the modelling activities analysed in this section. However, once the characteristic modelling activities have been analysed, it becomes clear that the models involved usually share a similar dynamical set-up, namely they tend to be many-component models with relatively simple deterministic or probabilistic dynamics. This definition includes cellular automata (CAs), agent-based models (ABMs) and network models, which also appear to be the types of models that are most robustly classified as being part of complexity science (e.g., Casti, 1992a; Wolfram, 2002; Ladyman et al., 2013; Zuchowski, 2012). For the purpose of this paper, it is not necessary to define the class of complex models more precisely. However, it should be stressed that the label 'complex' here (and within the canon of literature on complexity science) does not have the colloquial connotation of 'complicatedness', i.e., the models involved in complexity science are not models with a complicated formal structure or ones for which many parameters or variables need to be specified. In fact, it appears to be precisely the relative simplicity of these models that renders them ideal for the large-scale horizontal construction and the transfer to different target systems, i.e., for the modelling activities that constitute complexity science.

Since the dynamics of complex models are relatively simple, they can be taken to be highly explanatory (Batterman and Rice, 2014). In the framework of analysis introduced in Section 2.1, it will therefore be possible to consider the whole dynamics of a model as an explanation for a given behaviour or, framed in terms of a conditional to be transferred, to see the full dynamics as a sufficient condition for the occurrence of the phenomenon under investigation. A study focused on analysing the explanatory role of complex models would clearly need to adopt a more nuanced view. However, for the purpose of this study, which aims to be comprehensive rather than detailed, this simplified view of the explanatory function of complex models is fully sufficient.

The two case studies (Wolfram, 2002; Batty, 2005) I will use to illustrate the two modelling activities in complexity science, i.e., the creation of a repository of models, and the transfer and extension of these models, both extensively use CAs. I will therefore briefly outline this type of complex model in more detail.

CAs are discrete-time, multi-component models that operate on a grid. Each cell on the grid can assume two or more states: in a binary cellular automaton these are usually given the values 0 and 1, or 'dead' and 'alive'. At each time step, the state of a cell is updated according to a set of transition rules. These rules usually depend on the states of the neighbouring cells during the previous time step. The cells that influence the behaviour of a given cell are called the 'neighbourhood' of that cell. The number of possible rule sets to govern the behaviour of the cells depends on the size and shape of the neighbourhood and on the number of states a cell can assume.

The dynamics of CAs are therefore their rule sets. Their behaviour can be visualized through a coloration of cells according to their state, e.g., by representing 'alive' cells as black and 'dead' cells as white. Such representations allow the visual display of patterns in the phenomenologies of CAs. In the case of two-dimensional CAs, the patterns identified are two-dimensional patterns, which may change with each time step. In the case of one-dimensional CAs, the models' outputs are usually plotted on a two-dimensional spatio-temporal space. In this mode of representation, two-dimensional static patterns can be visually identified.

Modified versions of CAs can be obtained by adding 'agents', i.e., entities that occupy and can move between cells on the grid, leading to agent-based models, or allowing connections between non-adjacent cells, leading to network models. These models are not the only ones in complexity science: in fact, extensions and adaptions of these models can lead to much more complicated models and can also include combinations of these classes of models with elements from different classes of models (Section 4.2). However, I maintain that these are the classes of models which are most often involved in the large-scale horizontal construction (Section 4.1), and subsequent transfer to a target system (Section 4.1), of models that I have identified as constituting the most defining methodological mechanism of complexity science.

4.1 Creation of a repository of models

In this section, I will argue that a significant part of the modelling activities in complexity science is directed towards the creation of a repository of models without fixed target systems. This activity is best interpreted as the large-scale horizontal construction of models, i.e., the generation of many models with slightly different dynamics without reference to specific target systems**.** It should be noted that the ancestor model from which such constructions originate could be a model that has previously been used to model a given target system. In fact, the history of each class of models will likely be complicated (e.g., for an outline of the history of CAs, Batty, 2005, pp. 67-77). In this paper, I am only concerned with the large-scale horizontal construction of models as a methodology that is prevalent in complexity science (Section 1). I will identify two particular methodologies for this large-scale horizontal model construction in complexity science: rule space parsing and the use of genetic algorithms.

Rule space parsing One of the clearest examples of rule space parsing is the work by Stephen Wolfram (e.g., Wolfram, 1983a, 1984, 2002). Wolfram (2002), entitled, *A New Kind of Science*, is a particularly good illustration of the methodology of rule space parsing and the large-scale horizontal construction of models in complexity science.

The book contains an in-depth investigation of the dynamics and behaviour of CAs, AMs and network models. For example, Wolfram (2002, pp. 54-56) presents the output of all 256 of possible one-dimensional, binary, nearest-neighbours CAs. These models are constructed through the systematic generation and implementation of all possible rule sets, for which Wolfram (2002, p. 53) also develops a binary naming convention so that this process can be automated. I will call such a comprehensive construction of models 'rule space parsing'. Rule space parsing is a particular kind of horizontal model construction (Section 2.2): spin-offs from a given model are generated through systematic variation of its dynamics. In contrast to the cases of horizontal modelling found in semi-classical physics (Bokulich, 2003) and chaos theory (Zuchowski, 2016), rule space parsing can be automated and therefore leads to the construction of long lineages of models with slightly different dynamics. The parsing of the binary, one-dimensional, nearest-neighbours CAs' rule space is described as the "crucial experiment" by Wolfram (2002, e.g., p. 23); this large-scale horizontal construction of models clearly constitutes the heart of the book. In later chapters, Wolfram (2002) also constructs lineages of CAs with larger numbers of states and of agent-based models (Chapter 3; Chapter 4); of models with rules that do depend on more complicated mathematical operations in the Moore neighbourhood of a cell (Chapter 4); and of multi-dimensional CAs, including network models, whose rule sets link cells that are not spatially adjacent (Chapter 5).

Each of these lineages of models is constructed through rule space parsing, i.e., through the process of labelling all possible rules constituting the dynamics of these models, followed by an automatic generation of these dynamics on the computer. It is particularly apparent in Wolfram (2002, Chapters 2–5) that the construction of these models is accomplished without any reference to possible target systems: as a matter of fact, it is *prima facie* not obvious that any of the dynamics generated in this manner have any relation to particular natural phenomena. Other parts of Wolfram (2002) discuss possible target systems to which these models could be transferred (Chapter 8–11). However, no knowledge about these target systems or the general theories governing them is needed for the construction of these models through rule space parsing.

While Wolfram (2002) is a particularly comprehensive exercise in model construction, rule space parsing is a prevalent methodology in complexity science. For example, another long-lineage of horizontally constructed models has been spun-off from the Game of Life, a two-dimensional CA initially envisioned as a mathematical game (e.g. Gardner, 1970; Dewdney, 2008).

Accordingly, I maintain that (i) rule space parsing is a form of horizontal modelling and (ii) the use of this methodology constitutes a significant part of the modelling activity in complexity science. The result of the use of this methodology is the generation of a large number of models without specific target systems, which form part of a repository of complex models.

Using genetic algorithms In complexity science, genetic algorithms are usually discrete-time models on a discrete spatial grid whose rule sets can change according to a set of meta-rules. In particular, the meta-rule set is often one that forces the model to evolve towards a rule set that produces a particular phenomenology. Accordingly, genetic algorithms are models whose dynamics are adjusted algorithmically until the behaviour of the model fits a pre-set set of phenomenological criteria (e.g. Chambers, 1995).

A prominent example of genetic modelling in complexity science is the work by Packard and Wolfram (1985) and Langton (1990). These authors also work with the one-dimensional CAs studied by Wolfram (1983a, 1984, 2002).

In contrast to rule space parsing, genetic modelling therefore constructs a model through a targeted process of dynamical adjustments. Langton (1990) introduces the concept while working on the so-called λ-parameterization of the CA rule space. Thereby, rules which heavily favour transitions to one particular phenomenological state (called the quiescent state) s_c are assigned low λ values, while high values of λ indicate that very few configurations of the CA will lead to s_c. By employing a set of meta-rules that updates the rule set of the model at each time step with the aim of achieving particular λ values, Packard and Wolfram (1985) and Langton (1990) are able to associate specific dynamics with specific phenomenologies.

More recent studies in genetic modelling do not just use meta-rule sets that evolve the dynamics of a model towards certain phenomenologies but also update the rule sets according to meta-rules that are based on the maximization of different features, e.g., Mori et al. (1998), use a meta-rule set that relies on a parameterization of the frequency with which changes in the state of a cell lead to changes in its nearest-neighbours' states.

I maintain that genetic modelling can also be interpreted as horizontal model construction. The model eventually constructed is the final evolution of the genetic algorithm. As in the case of rule space parsing, the use of genetic algorithms is an automated way of horizontally constructing models. In contrast to rule space parsing, which generates comprehensive lineages of models that include all possible models with a given type of rule set, genetic algorithms provide a way of targeting this construction towards the generation of models that have particular phenomenologies. The lineages constructed in this way are usually not permanent: the end result of a run with a genetic algorithm is a single model, which best fulfils the criteria specified by the meta-rule set.

The generation of a model through the use of a genetic algorithm is clearly not based on the use of governing theory and empirical knowledge about a target system. This is also evident in the works cited in this section (e.g., Packard and Wolfram, 1985; Langton, 1990; Chambers, 1995; Mori et al., 1998), which do not mention any specific target systems for the models they obtain. Accordingly, I maintain (i) that the use of genetic algorithms is a way of horizontal model construction and (ii) that, like rule space parsing, it constitutes a significant part of the modelling activity in complexity science. Both the genetic algorithms and the models they create then become part of the repository.

Cataloguing and analysing the repository The methodologies used for the horizontal construction of models, rule space parsing and genetic algorithms, necessarily lead to the generation of a very large number of models. Not all of these models have phenomenologies that are judged to be interesting. Models with particularly interesting phenomenologies are usually given the label 'complex'. However, this should not indicate that only these models are used in complexity science: CAs creating random or ordered patterns (e.g., the Sierpinski triangle) have also been of interest to complexity scientists (e.g., Wolfram, 2002) and have also been transferred to specific target systems (Batty, 2005, Section 4.2). Accordingly, I maintain that the activity of bestowing different labels on the models generated through large-scale horizontal modelling is best interpreted as a cataloguing of models in the repository.

Such cataloguing is usually based on phenomenological criteria. For example, Wolfram (1983b) introduces a classification scheme for CAs, which sorts these models into four classes based on their behaviour: homogeneous (class I), periodic (class II), chaotic (class III), and complex (class IV). The scheme has been used in later works by the same author (e.g., Wolfram, 1984, 2002) and has also been widely adopted in the complexity science community (e.g., Langton, 1990; Packard and Wolfram, 1985; Mori et al., 1998).

The question of how the term 'complex' should be defined is currently unresolved (e.g., for recent philosophical reviews of this debate, Ladyman et al., 2013; Zuchowski, 2012). However, in practice, the classification has been performed with relative ease. In fact, many practitioners assume that the identification of models with complex behaviour can be performed intuitively through visual inspection (e.g., Gershenson, 2008, p. 131).

Despite the fact that the cataloguing of models in the repository appears to be mostly based on intuition, the phenomenologies of complex models have been investigated extensively. Such investigations include the computation of entropy measures (e.g., Wolfram, 1984, 2002) as well as the development and application of new statistical measures to capture the characteristics of differently classified models (e.g., Shalizi, 2001).

4.2. Transfer and extension of models

In this section, I will discuss the transfer of complex models from the repository to a given target system. My framework for the analysis of the transfer of models is the one developed in Section 3, i.e., I conceptualize the transfer process as consisting of the two steps of phenomenological prompting and dynamical reinterpretation. Furthermore, I will maintain that the transfer of models is often followed by the vertical construction of additions and adaptions to these models.

Transfer of models As described above, the division of modelling work into the large-scale horizontal construction of models and the transfer of these models to particular target systems is clearly illustrated in the structure of Wolfram (2002): the first part of the book (Chapters 1–5) presents a multitude of generic models constructed through rule space parsing, then there is an interlude in which these models are catalogued and analysed (Chapters 6–7), and in the second part of the book (Chapters 8–11) areas of application for these models are then sketched out.

The transfer of complex models into other scientific fields appears to have been most successful in those cases, in which a direct analogy can be constructed between the phenomenology of the model and the phenomenology of the systems investigated in these fields. This highlights the importance of the first step of the transfer process, phenomenological prompting: without completion of this step, the second step, a reinterpretation of the models dynamics, will not take place. The areas in which the recognition of such phenomenological similarities has been possible are fields like population dynamics, including the morphology of genetic traits, urban development, sociology, and physical morphology. In all of these areas, a reinterpretation of the discrete-time, grid-based dynamics of the relevant models has also been possible.

The adaption and use of CAs, i.e., the transfer of models from the repository of models provided by e.g., Wolfram (2002), to represent urban development can be illustrated in the cases presented by Batty (2005). The book begins with a general introduction of the models to be used in later chapters, which leaves no doubt that the use of models from complexity science, and of CAs in particular, was prompted by the recognition of similarities in the patterns visible in the phenomenologies of CAs and in the spatial pattering of cities (p. 9, pp. 108-110, p. 141, p. 90). In particular, Batty (2005, pp. 1-6) argues that the localized pattering displayed by complex (class IV) CAs (Section 4.1) can be interpreted as analogous to the localized clustering seen in the 'urban sprawl' of modern cities. It is stressed that CAs will be better able to generate these forms of urban development than models relying on the assumption of centralized planning and zoning (Chapter 1).

Batty (2005, Chapter 2) introduces a number of different CAs. Batty (2005, e.g., p. 7, p. 72) thereby extensively refers to Wolfram (2002) and the Game of Life. The models are then assigned to specific target systems according to their phenomenologies: deterministic CAs that create regular patterns are assigned to planned cities with a regular development (pp. 90), while complex CAs with probabilistic dynamics are assigned to organically grown cities (pp. 90-96). Thereby, the states of the CA are interpreted as various states of the development/residing population of a given spatial unit (e.g., p. 24, p. 69).

Once these target systems have been assigned, the dynamics of the models are then reinterpreted to fit the given scenario, i.e., the analogy is extended to the dynamics and the second step of the transfer process is concluded. In the case of regular CA development, the rule set is seen as an implementation of the rules of town layouts that govern planned cities (p. 90). In the case of probabilistic CAs, the probabilistic transition rule is interpreted as reflecting "uncertainty about the decision in question or variety in preference", i.e., the fact that in non-planned cities no deterministic rule for their development is strictly upheld.

Throughout the book, this process is repeated for various, increasingly complicated rule sets. It is important to note that these rule variations are seen as being chosen from an existing set of rules and not as being developed from the natural process through vertical modelling. This also becomes apparent in Batty (2005, pp. 110-116), where four general types of rules are listed (pattern rules, counting rules, statistic rules, voting rules). The various models in the book are then obtained – at least initially – through the reinterpretation of particular examples of these basic rule sets. This can be rephrased in the terminology of this paper as the transfer of a model from the general repository of models (Section 4.1).

Vertical construction of extensions The transfer of complex models is not the only modelling activity discernible in Batty (2005). Rather, the part of the book devoted to modelling with CAs is divided into two subparts: in the first three chapters, any models assigned to specific urban development scenarios are virtually exclusively obtained through the reinterpretation of well-known cellular automata, i.e., through the transfer of models from the repository. However, in chapter 4, these models are then developed further. For example, Batty (2005, pp. 156-162) constructs a model of urban development that is based on the interaction between the potential for development and the actual development of a given area. This construction starts with a consideration of how these two parameters interact in actual cities (pp. 156-157; Figure 41), and then proceeds to the design of a set of rules for a probabilistic CA. Batty (2005, p. 156) calls this example of model construction "modelling with extended cellular automata", which seems to be an apt description: we can interpret this process of model construction as the extension of a model, which has previously been obtained through transfer from the general repository. The construction of these extensions appears to be an example of vertical model construction rather than of mere reinterpretation.

A similar combination of model transfer followed by extension of the transferred model through vertical construction can also be seen in the second part of Batty (2005): this part begins with a general review of ABMs, including their use in other fields of science, e.g., in geography for the modelling of river systems (pp. 222-223). Similarities between the phenomenologies of the ABMs and the morphology of cities are then identified (pp. 223-240). The ABMs introduced before are then reinterpreted as settlement models, by interpreting the agents as populations (p. 241). Lastly, the existing models are extended by additions to their dynamics.

The adaption and extension of ABMs in Batty (2005) also illustrates that these extensions can be extensive and combine elements from different classes of models (Section 4): the dynamics of the ABMs is eventually extended to include an underlying morphology of resources which can be altered through the agent's actions (pp. 252-257). The interaction between the resources and the agents is governed through a feedback loop (p. 255) that includes elements from game-theoretical models.

Due to the textbook character of Batty (2005), this interplay between different modelling activities is clearly displayed in the book. A recent collection of articles in which the transfer, adaption and use of complex models in different areas of the natural and social sciences are discussed is Hooker (2011). The examples discussed in this collection range from the use of ABMs to model the natural selection of biological traits (Harms, 2011), to the use of network models to model interactions in economic exchange systems (Foster, 2011), to the use of network models to study the development of linguistic categories (Lansing and Downey, 2011). The prevalence of the transfer and adaption of complex models, as well as the interplay between such transfers and the horizontal construction of models (Section 4.1), is also evident in the research areas listed by institutes like the Santa Fe Institute or The Bristol Centre for Complexity Science, which include the use of complex models in several of the natural and social sciences. Shorter research papers often focus on either one of these activities, i.e., they either discuss the vertical construction of novel extensions to models that have already been transferred to a specific target system or they focus on identifying specific models that could be used to represent a given phenomenon (e.g., for a collection of such studies, Bandini et al., 2010). Nevertheless, the general interplay of modelling activities is still discernible in such studies. It is also apparent from a survey of such papers that the transfer and extension of models in complexity science is mostly undertaken by natural and social scientists that are interested in the exploration of particular target systems in other fields of science and would therefore profess allegiance to these fields as well.

Cross-disciplinary transfer Batty (2005) also contains examples of the second transfer mechanism operating within the auxiliary regions of complexity science, e.g., the transfer of a model with a given target system to another target system, often located in a different discipline. One such example is the use of sugarscape models, i.e., the extension of an ABM to include an underlying morphology of resources that the agents interact with in complicated ways: this class of ABMs was initially added to the repository of models at the core of complexity science by Langton (1989); it was inspired by a simple resource distribution models in biology, subsequently used to investigate the behaviour of artificial life-forms, and eventually transferred on to represent social phenomena (Batty, p. 252). This chain of transfers and adaptions also illustrates the often complicated history of models in complexity science.

Explanatory role of transferred models As outlined in Section 3, the transfer and reinterpretation of a model leads to the provision of a 'how possibly' explanation for the occurrence of the phenomenon under question. Accordingly, modelling in complexity science often begins with a 'how possible' explanation. However, this explanation is then further scaffolded by additional evidence gained during the extension of the model through vertical construction, i.e., it is shown that an extended version of the model can be constructed from direct considerations of the natural processes in questions. This provides support for a transformation of a 'how possible' explanation into a 'how actually' explanation.

The explanatory power of complex models is therefore difficult to capture: at heart, these models provide 'how possible' explanations, since their initial transfer is phenomenologically prompted. However, in the course of further modelling studies, adaptions and extensions are added that have been constructed from a-priori dynamical considerations. The more such extensions have been added to a model, the more evidence for the consideration of a model's dynamics as an 'how actually' explanation appears to exist. Accordingly, my view of modelling in complexity science is very well compatible with an account that locates the explanatory powers of models on a continuum spectrum between 'how possible' and 'how actually' explanations (e.g., Bokulich, 2014).

As described in Section 1, I view the transfer of models as part of complexity science. However, compared to the creation and maintenance of the repository (Section 4.1), these activities are best visualized as delineating auxiliary areas of the field, which overlap with other scientific fields and form the interdisciplinary part of complexity science. Similarly, practitioners engaged in this activity often view themselves as being part of both complexity science and another, primary field (e.g., Batty, 2005).

5. Conclusion

In this paper, I analysed the construction and transfer of models in complexity science. Thereby, I introduced a distinction between vertical and horizontal model construction (Section 2). Vertical models (Section 2.1) are constructed top-down or bottom-up from empirical knowledge about the target system and from general governing theory. Therefore, the existence of a specific target system is a prerequisite of vertical model construction. In contrast, horizontal model construction (Section 2.2) is the construction of a model without reference to a specific target system. It usually involves the generation of a variation of an existing model through changes of the mathematical formalism, which are usually motivated by investigative reasons.

My framework for the discussion of model transfer in complexity science is based on the account of the identification of analogies between different models by Hesse (1966). I maintained (Section 3) that the transfer of models in complexity science is prompted by a recognition of similarities between the phenomenology of a model and the phenomenology of a new target system. I named his step in the transfer process ‘phenomenological prompting’. Once these similarities have been recognized the dynamics of the model are also reinterpreted to suit the new target system. I named this step in the transfer process ‘dynamical reinterpretation’. Accordingly, my analysis is based on the assumption that the transfer of a model to a new target system can be conceptualized as a two-step process.

I then argued that all three types of modelling activity are part of the defining methodology of complexity science: vertical construction, horizontal construction and model transfer (Section 4). In particular, I argued that these modelling activities can be divided into two general categories: (i) the creation of a repository of models without specific target systems, which have been created by large-scale horizontal construction (Section 4.1), and (ii) the transfer of these models to particular target systems in the natural and (to a lesser degree) the social sciences, which can also be followed by an extension of the transferred model through vertical construction of adaptions and additions to its dynamics (Section 4.2). Additional transfers of such models to new target systems, often in the social sciences, are also frequent in complexity science but have not been the focussed on in this paper.

In category (i), I identified two prevalent methodologies of horizontal model construction: rule-space parsing and the use genetic algorithm. Rule space parsing is the systematic creation of models by an automated generation of all possible sets of dynamics for a class of models; the use of genetic algorithms is the automated optimization of the dynamics of a class of models towards the display of particular phenomenological features.

The use of these methodologies for large-scale horizontal model construction appears to be a distinguishing feature of complexity science. Their automated nature means that large numbers of models without specific target system can be constructed and evaluated, i.e., a repository of models with well analysed dynamics and phenomenologies is created through this mode of modelling.

I also analysed category (ii), the transfer of models from the repository to specific target systems and the further development of these models through vertical construction. I argued that transfer of complex models is phenomenologically prompted, i.e., that the transfer of models in complexity science follows the two-step conceptualization developed in Section 3. Furthermore, once a model's transfer is completed, i.e., once both the model's phenomenology and its dynamics have been reinterpreted, the model is often extended through the construction of additions and adaptions. The construction of such extensions is usually based on empirical knowledge about the model's new target system and on the general theory that governs its dynamics: they are therefore vertically constructed. Accordingly, the modelling of a specific target system in complexity science often involves both model transfer as well as the vertical construction of models. In addition, these models can also be transferred to new target systems in different disciplines.

The division between the two categories of modelling activities also marks an epistemic division of labour: the creation of the repository of models appears to be mostly undertaken by mathematicians and computer scientists while the transfer and extension of models is undertaken by natural and social scientists. In Section 1, Section 4.1 and Section 4.2, I have argued that this interplay of modelling activities can be seen as providing a methodological definition of complexity science, namely it delineates a methodological core area of the field – the stocking, investigation and cataloguing of the repository – and a number of auxiliary areas characterized by the transfer of models from the repository to target system in different areas of the natural and social sciences. In these areas, complexity science overlaps both methodologically and in the self-identification of the practitioners with other scientific fields. Additional auxiliary areas can also be added to complexity science through the onward transfer of models from one target system to another. This mechanism seems to be the primary way in which areas in the social sciences are methodologically connected to the core area of complexity science, while the direct transfer from the repository, which this paper has been focussed on, tends to tie in areas in the natural sciences.

Accordingly, the specific interaction of the different modelling activities in complexity science provides a mechanism for the transfer of knowledge between these different fields. In addition to the formal structure of a model, the transfer also involves gaining a 'how possibly' explanation for the new target system's phenomenology. The explanatory power of the model can be increased if additional vertical adaptions and alterations are made: the certainty of the knowledge gained through a transfer, as measured on a spectrum between 'how possible' and 'how actually' explanation, therefore depends on the given models similarity to the target system and on the possible further alterations to the model.

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