UNDERSTANDING PRACTICES IN MATHEMATICS EDUCATION: STRUCTURE AND TEXT

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ABSTRACT. The practices of mathematics education can be investigated at a wide variety of levels: from the actions of individual students or teachers through classroom interactions, school structures, curriculum specifications and materials, teacher development programmes, local, national or international systems of instruction and assessment. These levels are, however, inter-related. The study of a national curriculum gains significance as we see how it impacts upon and is interpreted by teachers and students. The study of an individual's actions makes more sense when these are interpreted in light of the broader context within which the individual is situated. This article is a version of a plenary lecture presented at the Psychology of Mathematics Education conference in 2009, addressing the conference theme "In Search for Theories in Mathematics Education". In it, I trace the development of my own search for theories to address my wish to understand practices in the field and suggest some tools from linguistics, critical discourse analysis and social theory that can help to make such connections between the various levels of investigation, illustrating with a range of examples.

KEYWORDS. Bernstein; Critical Discourse Analysis; social structure; text

Introduction

In order to understand the practices of individuals it is necessary to understand how those practices and individuals relate to the social structures within which they are situated. At the same time, however, gaining insight into the way that social structures work demands attention to their operation at the level of individuals. In the last decades, especially following the 'social turn' identified by Lerman (2000), it has been widely recognised within the mathematics education research community that our field of study, mathematics education, is complex and multi-layered. The objects of research range from the behaviours of individual students in narrowly specified aspects of mathematics (or even the firing of neurons in the brain), through classroom interaction, to national and international curricular reforms or policy interventions. However, although research adopting socio-cultural perspectives tends to recognise at least some of this complexity, for example, attempting to understand individual behaviour of students or teachers as participation in a community of practice, the practical demands of research practice are such that most studies adopt (whether explicitly or not) a single level of 'zoom', focusing attention and collecting and analysing data at one level, while backgrounding others. Lerman's (1998) elaboration of the zoom metaphor suggests that those parts of the complexity that are out of focus should also be taken into account in the analysis. An alternative perspective on complexity is proposed by Paola Valero, who draws attention to a 'network' of mathematics education practices, including among others

classroom practice, academic mathematics, teacher education, policy making (Valero, 2007). Valero advocates that research should seek to understand this network by focusing simultaneously at different nodes of the network. While based on different metaphors, suggesting nested and non-hierarchical complexities respectively, the implications of Lerman's and Valero's visions are compatible: making sense of data arising from study of one node or level requires understanding of other connected nodes or levels. I would like to build on this point, suggesting that substantial analyses of different levels or of different nodes of the network need to be articulated. In order to do this, I shall use the device of reflecting on and illustrating the development of my own research interests and foci, identifying the questions that have arisen from my interest in the practices of schooling and the theoretical resources that have helped to make sense of these. Through this reflection, I propose a combination of theoretical resources that, by addressing different parts of the complexity, can help us to understand better how mathematics education functions in society for individuals and for various social groups.

In the Beginning … Mathematics, Language and Social Issues

Three fundamental themes have run through my professional career as a teacher and as a researcher: mathematics itself, language, and something that I shall call social issues. Initial interest in these themes can be linked to my family background and my educational, professional and political experience, located in particular times and places. More generally, my own practices as a teacher and a researcher can only be understood within the context of my personal history and the social structures within which this has been situated. As a young teacher in a secondary school in a multiracial area of London, problems that concerned me (beyond the everyday concerns of survival in the classroom) included such questions as:

- Why do some students, who can do mathematics successfully in class and talk fluently about it, fail to reproduce this success in written tasks and examinations?
- Why are the lower attainment groups in my school disproportionately full of students of African-Caribbean origin?
- Why do so few students seem to appreciate mathematics in the way I do?
- Why does such a lot of the teaching of my colleagues (and, let's be honest, myself) seem so boring and trivial?

Associated with these, as yet unsophisticated, questions, was the conviction that differences in students' (and teachers') achievements, interests and experiences in mathematics could not be explained solely at the level of the individual. This conviction was born of a political orientation rather than a developed theoretical rationale. However, it is my belief that such orientations lie behind our choices between theoretical frameworks. My concern with social inequalities precludes adoption of a perspective that denies or ignores the influences of social relationships and structures on individual experience and achievement. My personal search for theory has thus been shaped by a need to understand how individual and social may be connected.

Abstracting from my initial questions, I identify three core issues that shape my search for theories: the need to address the uneven distribution of knowledge and success, to understand the roles that

language may play in this and to illuminate the nature of pedagogy and how it may contribute. There are various directions that I might take in this search and there are certainly theories that are compatible with this need that I will not be addressing. In particular, theories of learning and activity based in the Vygotskian tradition offer powerful ways of understanding such connections. (See Hasan, 2002, for an account of the compatible and complementary contributions of Vygotsky, Halliday and Bernstein from psychological, linguistic and sociological perspectives respectively.) However, because many of my questions seek to address the uneven distribution of knowledge and educational success, I intend to focus here on the contributions of sociolinguistic, discursive and sociological theory to my way of understanding.

Tools for (Linguistic) Analysis of Mathematics Education Texts

As I have already indicated, one of my earliest research interests was in the nature and use of language in mathematics education, especially in the apparent mismatch between my judgement, through observation and oral discussion, of students' mathematical achievements and their performance on written tasks, in particular tasks involving reporting on the processes and outcomes of mathematical investigations. Attempting to research this interest entailed finding means of describing and judging students' mathematical writing. At that time (around 1990) most descriptions of mathematical language focused primarily on specialised mathematical notation (e.g., Ervinck, 1992; Woodrow, 1982) or vocabulary (e.g., Anghileri, 1991; Otterburn & Nicholson, 1976). However, my original observation suggested that specialised notation and vocabulary were not the principal problems encountered by my students. Rather, the students failed to explain and justify their methods and results in ways that satisfied me or the assessment criteria they were attempting to meet.

There was beginning to develop a body of research looking more broadly at the use of language in classrooms and more extended spoken and written mathematical texts (e.g., Laborde, 1990; Pimm, 1987). In particular, the title of David Pimm's book 'Speaking Mathematically: Communication in Mathematics Classrooms' pointed to three important characteristics of the language in which I was interested: it is mathematical in some sense; it is for communication, so involves some form of social engagement; and it is situated within a particular context. It is thus not only the form of the language that is significant but also the role that it plays in interactions between individuals and in the broader social context. This understanding led me to see the Systemic Functional Linguistics of Michael Halliday as an appropriate means of approaching the problem of describing students' mathematical writing. Halliday made an early contribution to the characterisation of mathematical language (Halliday, 1974), also referred to by Pimm (1987), but, more significantly, his broader social semiotic orientation and his perception of language as essentially functional provide a strong theoretical basis for understanding the role of language within social practices, while his functional grammar provides powerful tools of description (Halliday, 1985). The linguistic tools provided by Halliday have been extended to address the analysis of multimodal texts (e.g., Kress & van Leeuwen, 2001). There have also been some efforts to develop specialised tools for mathematical texts of various kinds (e.g., Alshwaikh, 2008; O'Halloran, 2005) within a social

semiotic perspective. I have written elsewhere (Morgan, 2006) about social semiotics, the analytic tools it provides and their application to research in mathematics education – with potential to apply to a wide range of texts arising within mathematics education, significantly beyond my original need to analyse students' writing. My intention here is to discuss the interaction between linguistic analysis at the level of text and analysis at higher levels to enable fuller understanding of practices.

Tools for (Discourse) Analysis of Texts in Mathematics Education Practices

To return to my original research problem, my task was not only to describe student writing but also to judge it or, rather, to understand how it was being judged. More specifically, how did the various characteristics of the texts produced by students relate to the ways in which their mathematics teachers and assessors interpreted and valued the students' mathematical achievements? The explicit assessment criteria and teachers' overt statements about their assessment practices were not at a level of delicacy sufficient to understand how they related to the characteristics of individual student texts. For example, the teachers I interviewed claimed to value diagrams as a means of mathematical communication but, in practice, paid little attention to some types of diagrams and interpreted others as evidence of lower level mathematical activity (see Morgan, 1998; Morgan & Watson, 2002).[1] Moreover, the official assessment criteria made use of terms such as "appropriate means of communication", leaving open the question of how appropriateness might be judged. As Fairclough (1992) argues, use of "appropriate" disempowers those who are not already 'insiders', able to recognise and produce forms of text that will be highly valued by those in authority.

Teachers' evaluative discussion of student texts and the student texts themselves are elements of the same practice of school mathematics. They can only be understood in relation to one another. But there are also other texts (both written and spoken) that relate to these: curriculum documents, assessment criteria, teachers' guides, professional journals, school or mathematics department policies, classroom interactions, staffroom chat over a cup of coffee – among others. The constructs, values and relationships between the various participants that are present in these texts are components of the discourse of the practice, providing possibilities for what may be said or not said and by whom. The concept of discourse that I use here is within the tradition of Foucault (e.g., 1972), consistent also, in its concern with the distribution and exercise of power, with my initial concerns about social inequality. Unlike the broad sweep of Foucault's analytic approach, however, attention to the linguistic details of texts provides a means of identifying and understanding the operation of discourse at the day-to-day micro-level of classroom interaction.

Critical Discourse Theory provides a multi-layered way of thinking about language use in context that I find helps to address the issues in which I am interested. Texts (spoken or written,

[1] The recent work of Alshwaikh (2011) in developing a social semiotic framework for analysing diagrams used in school geometry has potential to illuminate the ways such diagrams function in classroom and assessment practices.

monologic or dialogic) are considered as the linguistic elements of social events. These events are shaped by more general social practices (of which 'orders of discourse' are the language-related elements) and by social structures (systems, including linguistic systems, cultural norms and institutions that regulate what is possible at the level of practice) (Fairclough, 2003). This shaping should not, however, be seen as deterministic. Choice is a fundamental construct within Systemic Functional Linguistics, mapping out the alternatives that the producers of texts have available to them. Further, while a text may construct an 'ideal' reading position from which the author's choices appear unproblematic, it is possible for individuals to resist such normalisation (Hodge & Kress, 1993). Sensitivity to the constraints of a discourse and the possibilities for resistance orients me to ask the critical question of any text and any practice: "How might this be different?" – not in the ideal sense of "How might I prefer it to be different?" but in the practical sense of "What might be the differences if other choices had been or were to be made?"

This theoretical perspective is reflected in the methodological approach to research. While data may take a variety of forms, it is always understood as arising within a communicative interaction – a lesson, a conversation, an interview, a written text – situated within a particular configuration of social practices and structures. Analysis from a critical discourse perspective focuses at three levels: the communicative interaction itself; the discursive resources used in the interaction and the orders of discourse from which they are drawn; the social structures and socio-cultural practices within which the interaction is situated (Chouliaraki & Fairclough, 1999, p.113). It sets out to establish how the linguistic and semiotic features of social interaction are systematically related to what is going on socially.

For example, in Evans, Morgan & Tsatsaroni (2006) we presented an analysis, drawing on critical discourse analysis among other theoretical tools, of part of a transcript of a group of lower secondary school students working on a mathematical problem. This analysis, focusing on emotion in the mathematics classroom, made use of semiotic features of the students' interaction but, in order to interpret these, drew on prior analysis of the structures provided by the national education system, dominant traditional pedagogic practices and the local pedagogic practices of the particular classroom as well as an awareness of further resources available to the students from other practices, including 'everyday' discourses of the family or peer group. The structural analysis highlighted alternative interpretations – both for us as analysts and for the participants themselves - and identified possible tensions and contradictions that may be experienced by the participants within the interaction. An example of such potential for tension was the question "So how did you do it?" asked by one member of the group of another. We noted that this question could have different significance depending on the particular discursive resources drawn upon by each of the participants.

> Within the progressive pedagogy of this classroom it may call up the value placed on explaining mathematical activity and collaborating. Within a traditional pedagogy it may represent a challenge by an evaluator (in a superior position) or a request for help from a student with lower status. (p.220)

The configuration of discursive resources brought together by the participants in the interaction and their pre-existing relationships shape the way in which the question may function in the practice of the group of students: perhaps opening out collaborative exploration of alternative methods of solution; alternatively leading to a defensive response, copying of answers or a refusal to share.

Methodological Challenge: Moving from Analysis to Interpretation

One of the problems with discourse analysis (or more generally any form of qualitative research) is that you are always asking: Where does this interpretation come from? How can this reading of the text be justified? The theoretical basis of social semiotics is that any text – the raw data – may only be understood through using knowledge of the immediate context of the practice of its production and consumption (context of situation) and the broader cultural context shared by the participants in this practice (Halliday, 1978). It is through using this knowledge that the researcher may make claims about how the text functions for participants. But this raises further questions, including the practical and pragmatic question of how much knowledge of which parts of these contexts may be necessary to achieve a 'good enough' understanding. More fundamentally, how can the researcher (given sufficient information) arrive at a characterisation of these contexts? Fairclough argues that the only way this can be done is to draw upon the same 'insider' resources as the participants in the practice, but with self-conscious awareness of the common-sense assumptions of the practice (2001, p.139). An insider in a practice can interpret a text used within that practice in an authentic way, simply by virtue of being an insider. For the researcher, however, just being an insider is not enough as the assumptions, values and fundamental constructs of the practice are likely to be so naturalised that they are invisible and unquestionable. The researcher does not seek to understand the text as a participant does – to operate with it inside the practice – but to know the principles by which the text is understood and the functions it has within the practice. To achieve this it is necessary simultaneously to have insider knowledge and yet not to be in the practice at the point of analysis. It is no surprise to find that Halliday's use of context of situation and context of culture draws on ideas originating in ethnography, as ethnographic researchers struggle with similar insider-outsider dilemmas. However, unlike some ethnographic approaches, the critical discourse analyst does not attempt to withhold preconceptions, inducing theory from the data, but is likely to make explicit use of social theory to structure the analytic approach (Chouliaraki & Fairclough, 1999).

For researchers in education, insider knowledge may come from being or having been a student, teacher, teacher educator as well as a researcher, though it is important to recognise that changes over time and differences between contexts may affect the currency of such knowledge. This poses particular challenges for research across cultures, as educational practices – both at the level of the classroom and at the level of research – are not identical but have different sets of values and assumptions. Hence similar texts may not play the same role or be interpreted in similar ways. Recent years have seen a number of explorations of this issue through, for example, reanalyses of data from different theoretical perspectives (see, for example, some of the chapters in Clarke, 2001

or the 2006 ESM Special Issue 63(2) on Affect in Mathematics Education), consideration of alternative approaches to didactical design (Miyakawa & Winsløw, 2009) etc. In the recent European ReMath project,[2] the partners used a methodology of 'cross-experimentation' with technological tools (Bottino & Kynigos, 2009) that, among other aims, seeks to study the use of software in classrooms in different countries, making visible the differences that arise within different national education systems and different research traditions. In this project we have seen differences in the ways teachers and students interpret and make use of the texts offered by the technological tools as well as differences in researchers' readings of the potential didactical functionalities of these tools and of student activity in the classrooms in which they are used (Lagrange & Morgan, 2010).

We have to accept that no reading can be 'objective' as it must always be imbued with the values and assumptions of the reader. No interpretation of data is certain, as it must be constructed within a particular theoretical framework. The challenge is, on the one hand, to make the values and assumption of the framework explicit and, on the other, to find or develop a framework that matches one's fundamental philosophical and ethical beliefs and will proved insights into the issues of concern. As I indicated at the beginning, my own research concerns have from the beginning related to the distribution of knowledge and academic success, to the role that language may play in this, and to the nature of pedagogy in mathematics classrooms, so it appears natural to make use of social theory that addresses the question of how the process of distribution operates, including linguistic aspects of this, and that provides ways of characterising pedagogy. There are a number of alternatives that match one or more of these criteria and offer useful possibilities for research in mathematics education, among which the work of Bourdieu and of Bernstein stands out and is being used by increasing numbers of researchers. The articles in this Special Issue offer some examples of such programmes of research and explore in more breadth and depth what such frameworks have to offer. My intention here is just to illustrate some of the ways in which Bernstein's sociological theory has informed my own research in interaction with the discourse analytic techniques already described. In Kanes, Morgan and Tsatsaroni (this issue), analysis enabled by this interaction is further developed by incorporating perspectives drawn from Foucauldian thinking.

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[2] 'Representing Mathematics with Digital Media', funded by the European Community, Framework 6 Programme, IST–4–26751–STP. See http://remath.cti.gr. The participating research teams were: University Paris 7 Denis Diderot, Paris, France; National Kapodistrian University of Athens, Educational Technology Lab, Athens, Greece; Consiglio Nazionale delle Ricerche, Instituto Tecnologie Didattiche, Genova, Italy; MeTAH and Leibniz, IMAG, Grenoble, France; Institute of Education, University of London, London Knowledge Lab, London, UK; University of Siena, Department of Mathematics, Siena, Italy.

Tools for Interpreting Texts within Social Structures

The research into student writing and teachers' assessment practices discussed above took as its starting point the actions and interactions of individual students and teachers. Adopting a critical discourse approach necessitated drawing on other texts that provide a framework for teachers' practices, including official curriculum documents, practical guidance for teachers and teachers' professional journals, in order to know the resources available to teachers and to identify and understand the ways in which they draw on such resources to make sense of and account for their practices. It is also possible to take other starting points. More recently I have been interested in considering the guidance provided for teachers in policy and curriculum documents and the ways in which the official discourse manifested in such texts provides resources for constructing the nature of teaching, learning and assessment, of teachers and students and relationships between them.

In the United Kingdom, both the curriculum and teaching methods have been increasingly specified by legislation and by non-statutory decree of official government agencies. These are regulated by inspection and high-stakes national examinations as well as by a plethora of documentation. Yet the practice in individual classrooms still varies. This observation matches the wealth of research internationally into curriculum reform that shows the lack of uniformity of its implementation and the persistence of 'traditional' forms of pedagogy. My study of teachers assessing students' mathematical writing identified a number of different positions that teachers adopted in relation to the task of assessing and to the regulatory framework. For example, some positioned themselves as 'official' examiners making explicit use of the published assessment criteria, while others adopted a position as an advocate, imagining what the student might have been thinking and giving them the benefit of the doubt (Morgan, 1996). Working together with Anna Tsatsaroni and Steve Lerman, the origins of these positions in official and unofficial discourses of assessment were systematically identified and the resources provided by these discourses described in order to produce a model that provided a structure to classify possible positions and the relationships between them (Morgan, Tsatsaroni, & Lerman, 2002). This structure not only systematised the distinctive positions identified empirically but also predicted that such positions might occur.

One of the key concepts that informed this systematisation and my further work looking at teachers' and students' relationships to the curriculum is Bernstein's notion of recontextualisation of discourses. As they move from one sphere of activity to another, discourses are transformed – recontextualised – according to a principle, consistent with the interests of relevant agents, that "selectively appropriates, relocates, refocuses and relates" to other discourses (Bernstein, 2000, p.33). Thus the mathematics produced in the academy is transformed, drawing on theories of teaching and learning and on other educational discourses, through processes of curriculum development, professional education, production of materials for teachers, etcetera, into the mathematics that is taught in the school classroom. In the context of mathematics education there are two moments of recontextualisation that concern me: that which happens in the construction

by government agencies of official discourses of curriculum and practice and that which happens as the curriculum is deployed in the classroom.

I shall look first at an extract taken from the National Framework for Teaching Mathematics (DfES, 2001) a key document of the official discourse in England that sets out the forms of teaching expected of teachers in the lower secondary school.

> Good direct teaching is achieved by balancing different teaching strategies:
>
> - Directing and telling […][3]
> - Demonstrating and modelling […]
> - Explaining and illustrating […]
> - Questioning and discussing: questioning in ways which match the direction and pace of the lesson to ensure that all pupils take part (if needed, supported by apparatus, a calculator or a communication aid, or by an adult who translates, signs or uses symbols); using open and closed questions, skilfully framed, adjusted and targeted to make sure that pupils of all abilities are involved and contribute to discussions; asking for explanations; giving time for pupils to think before inviting an answer; listening carefully to pupils' responses and responding constructively in order to take forward their learning; challenging their assumptions and making them think...
> - Exploring and investigating […]
> - Consolidating and embedding […]
> - Reflecting and evaluating […]
> - Summarising and reminding […] (pp. 26-27)

How may we understand the description and prescription of teaching provided in this list? The headings of the list of teaching strategies succeed in incorporating aspects such as exploring, investigating, discussing, reflecting, which might be thought to belong to a 'student-centred' discourse such as those associated with constructivist-based curriculum reform. On closer examination, however, it may be seen that there are also other discourses at play. Looking in more detail at the gloss on 'questioning and discussing', it becomes clear that it privileges a 'teacher-centred' pedagogy. In Bernstein's terms, this is a pedagogy with strong framing: the teacher has explicit control of the selection, sequencing, pacing and criteria for evaluation. It is primarily the teacher who is active (questioning, asking, challenging) and student activity is controlled; the teacher must: 'ensure that all pupils take part'; 'make sure that pupils of all abilities are involved'; 'make them think'. Questioning must match the direction and pace of the lesson rather than follow the direction of student activity and interests.

As well as these discourses about teaching, the extract draws on discourses about the nature of students and learning. There is an expectation that students will participate and, in particular, will think. At the same time, however, there is a strong assumption of differences between students,

[3] Each strategy in the list is glossed by a more detailed description, omitted here except in the case of the strategy "Questioning and Discussing".

especially of deficit, as some students may need support to enable them to take part. The 'normal' student will take part as directed by the teacher; some others will need support to do so; those who refuse to take part or who participate in other ways (perhaps asking questions themselves rather than responding to teacher questions) are absent from the text.

The official discourse is thus a recontextualisation of several discourses about students, teaching and learning, with elements of constructivist and teacher-centred pedagogy, an expectation of student participation but also of student deficit. These discourses originate in part in various academic theories – a precursor of this document was accompanied by a review of educational research that sought to justify its approach (Reynolds & Muijs, 1999)[4] – as well as in everyday discourses about education, bringing these into conjunction and transforming them in the process. This raises the question of what the principles of this transformation may be – a question that I will not address here but which demands a wider consideration of whose interests may be served by the construction of such a form of pedagogy.

When considering the further process of recontextualisation of this official discourse as teachers make use of it in their classrooms, it is necessary to consider what other discourses teachers may draw on and what their interests may be as they selectively appropriate and transform the official guidance into classroom action. The discourses they draw on are likely to include a range of specialised academic and professional discourses encountered in the course of initial teacher education and further professional development activities as well as everyday discourses about education current in the community or encountered in the media. Analysing the structure of discourses that may contribute to the construction of pedagogy in the classroom provides a means of describing, understanding and, indeed, predicting the various ways that teachers may position themselves in relation to the official discourse. In the case of the National Framework, the conjunction of resources from several (not obviously compatible) discourses allows teachers to comply with the demands of the official curriculum while continuing to teach in a range of different ways. A fuller presentation of this argument is made in Morgan (2010).

My final example takes a starting point at the level of the classroom, considering the recontextualisation that occurs as teachers implement a curriculum. In this case, two experienced teachers who were undertaking a Masters course in Mathematics Education collaborated as teacher-researchers in experimentation with a new technology.[5] Their participation involved planning and executing a set of lessons using a microworld constructed using E-Slate,[6] a toolkit

[4] See (Brown, Askew, Millett, & Rhodes, 2003) for a critique of the way this research was used – a recontextualisation from the field of research into that of policy making.

[5] This work was undertaken by the TELMA (Technology Enhanced Learning in Mathematics) European Research Team, part of the Kaleidoscope Network of Excellence, funded by the European Community (IST–507838) under the Framework 6 Programme. See http://www.noe-kaleidoscope.org.

[6] E-Slate was devised by the TELMA Athens partners (NKUA-ETL). See http://e-slate.cti.gr/.

for building environments for educational exploration, designed using constructionist principles. The teachers also gathered data on students' use of the software and wrote a reflective discussion of their experience. The analysis offered here uses the data collected by the teachers and also uses their analyses and reflections as data. Both teachers have seen this analysis and agreed to its publication. The microworld, named Fraction-Slider, provides two linked forms of representation of fraction: visually as a relationship between values shown by positions on linked dynamic number lines (sliders) and, symbolically, as a rational number entered into a Logo procedure in either decimal (e.g. 0.25) or ratio (e.g. 1/4) form. The numbers entered in the Logo procedure determine the relationships between the values displayed on the sliders. In the example shown in Figure 1, as the top slider is moved, the positions of the other, dependent, sliders vary to display values respectively ½, $^1/_3$ and ¼ of the value of the top slider.

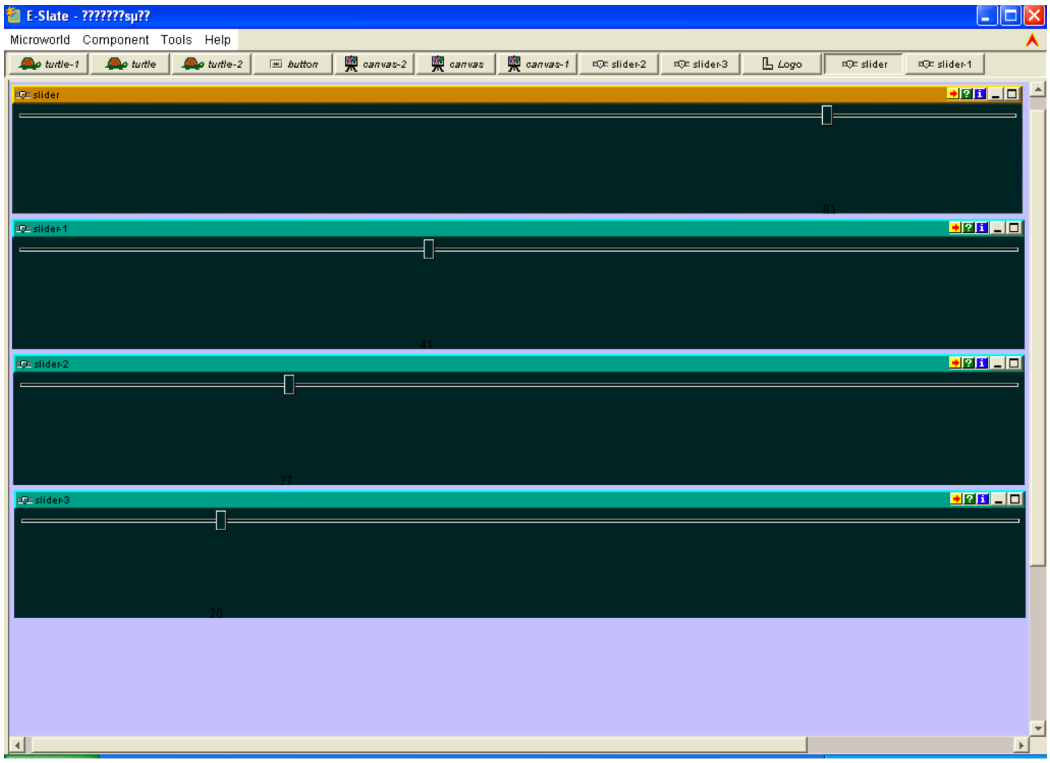

Figure 1: Screenshot of the Fraction-Slider microworld showing the top 'control' slider and three dependent sliders, representing 1/2, 1/3 and 1/4 of the value of the top slider

The teachers decided on two types of tasks making use of the microworld: first comparing and ordering fractions, then finding a fraction between two others. An outline plan was agreed for three lessons and tasks designed to use with students. However, detailed pedagogical issues were not discussed explicitly and it was left to the individual teacher-researchers to plan the conduct of their lessons. In practice, the two used similar lesson structures, starting each lesson with an interactive discussion with the whole class, followed by tasks that students engaged with individually or in pairs while the teacher provided support as needed, and finishing with further whole class interaction, reviewing what had been done during the lesson. Described in these terms,

both teachers can be said to have adopted the voice of the official pedagogic discourse of the National Framework (DfES, 2001), which recommends such a 'three-part lesson' structure. On examining the detail, however, there were significant differences between the pedagogies of the two teachers.

One important area of difference was in the strength of the control maintained by the teacher over student participation in whole class interaction. Teacher 1 adopted a position of authority over what might be considered legitimate knowledge in the classroom, with a strong asymmetry between his role and that of his students. As shown in the following extract from early in the first lesson, the teacher-student interaction followed an explicit Initiation-Response-Evaluation pattern.

Teacher 1	…which is the largest fraction out of those two, D?
Student	3/6
Teacher 1	Why do you think it might be 3/6?
Student	Because three of the … three sixes …one….[inaudible]
Teacher 1	Can anybody explain a little further, she's not wrong, I know she knows what she's talking about. E.
Student	Because three [inaudible]
Teacher 1	Right, excellent.

In contrast, Teacher 2 introduced each lesson with a discussion whose direction was not pre-determined. Initial questions were planned, but were more open in nature and concepts introduced by the students became part of the discussion. As may be seen in the following extract, Teacher 2 legitimated the students' contributions implicitly by echoing, revoicing or building on them rather than by making explicit evaluative comments.

Teacher 2	What do you think is happening here when you move the top slider? […] What do you think over here girls?
Student	I don't know. They just all seem to be moving when you move the top one along like that in a diagonal line.
Teacher 2	They're moving diagonally.
Student	They're moving proportionally, all three of them.
Teacher 2	Can you try and think about what those proportions might be? How would you try and work it out?
Student	If you move it like that
Teacher 2	Move it right over to the end

In these extracts and in their other interactions, the two teachers differed in their control over the order and nature of student contributions and the content of interactions, in the demarcation of student and teacher roles and in the means of evaluating student contributions.

Bernstein orients us to distinguish pedagogies by considering the strengths of classification and framing. We can identify clear differences between the two teachers in the strength of framing – the location of control over pacing, selection and sequencing of the material and evaluation. There were also differences in the classification – the way in which categories of knowledge were established and distinguished. From the start, Teacher 1 focused explicitly on fractions and calculations, while Teacher 2's introduction to the microworld did not even mention fractions until the word was introduced by one of the students. For more details and a fuller discussion see (Morgan, 2007).

So how may we understand these differences, given that the two teachers worked together to plan the lessons and appeared to be engaged in the same enterprise? As one of the aims of the project was to explore the role of theoretical frameworks, this is where I shall start. While the relation between espoused and implicit theory and practice is not simple, it is possible to consider at least some of the theoretical resources available to the teachers and how they appeared to deploy these in order to make sense of their practice. Through their participation in the project the two teacher-researchers had come into contact with a number of theoretical ideas, in particular, socio-cultural theory and the notion of semiotic mediation. In addition, the design of the microworld itself and recent experience in a Masters course on the role of digital technologies in mathematics learning and teaching oriented them towards the ideas of constructionism. Both teachers described their planned student activity as "exploration", appearing to draw on the discourse arising in this theoretical field. The following extracts from their written reflections after the experiment, however, suggest that the two teachers appropriated these resources in different ways.

> Teacher 1: Pupils were able to get through far more questions … This exposed pupils to far more examples and hopefully enabled pupils to think more generally … A significant number of pupils began to be able to successfully predict outcomes by rounding fractions, '96/350 is about one quarter but 34/70 is just less than a half, so that must be bigger.' … It was interesting to see how the pupils felt free to just 'try any old fraction' … [This] led to an arbitrary fraction being tried and then the denominator being 'stuck to' and the numerator being altered until successful, this technique was certainly only possible with this software … [one student] would set the main control slider to 100. 100 would then be their fractions denominator. They would then look at the two values on the two sub-sliders and chose for their numerator a number between those two numbers.

For Teacher 1, despite espousing exploration, it is "exposure" to examples which leads to generalisation. His main focus in reflecting on the outcomes is on student development of specific skills and strategies within the topic domain. The main role of the microworld is presented as facilitating acquisition of traditional forms of knowledge. This seems compatible with the discourse of the official curriculum discussed above.

> Teacher 2: Initially their talk mainly centred on the 'distance' of the sliders from one another, but some then started to talk about the movement of the sliders … What I thought was interesting about the replies was that those students who used a 'static' form

> of language ("the gap is bigger", "more space is taken up") tended to get the answer wrong, whereas those who used a more 'dynamic' language ("it moves faster", "it travels further") tended to get the answer right … Finding a fraction between 2/5 and 3/7 was hard as the fractions are so close together. This brought out a confusion about the meaning of 'in between' (does it have to be exactly in the middle?)

In contrast, Teacher 2's focus is on the language and meanings generated in interaction with the representations provided by the tool. This focus is compatible with the original framing of the aims and design of the study, presenting the microworld as a semiotic tool that may structure learning.

While the project in which they were involved privileged certain kinds of academic discourse about teaching and learning, the ways in which the two teacher-researchers planned, taught and talked about their work also drew upon the official curriculum discourse (with its recontextualisation of these and other academic discourses) and the more local discourses of teaching and learning current in their respective professional environments as well as everyday non-specialised discourses (cf. Morgan, et al., 2002). Where they appropriated resources from discourses of research, including those of explicit theory, these acquired new types of meaning as they were put to new purposes within the practice of teaching, situated within particular institutional contexts. Considering their institutional contexts suggests one reason why the teachers selected differently from the available resources. The schools differed in the socio-economic backgrounds of the student population and in their dominant forms of pedagogy. Teacher 1's school served a mixed population, located in a middle class neighbourhood but drawing students from a wider area, including many from deprived backgrounds. It was a church school with a 'traditional' ethos, including strong control of student behaviour. In contrast, Teacher 2's school had a liberal 'progressive' ethos and a predominantly well-off middle class population. While beyond the scope of such a small study, this raises questions about how social structures may influence the forms of pedagogy and hence the access to knowledge available to different groups of students. By describing and attempting to explain the paths taken by each of these teachers, it is also possible to ask how their practices might have been different. What choices were available to them and what might have enabled them to adopt different forms of pedagogy? Previous studies have also shown how mathematical and interpersonal aspects of classroom interaction may vary with student social class and gender (e.g., Atweh, Bleicher, & Cooper, 1998; Noyes, 2008; Zevenbergen, 1998). My basic political and theoretical orientation leads me to reject any suggestion that this differentiation is either an inevitable result of the class or gender characteristics of the students or the result of personal prejudice or bias on the part of the teachers. In order to avoid such conclusions, I argue that it is necessary to consider the affordances of the structures (linguistic systems, cultural norms, institutional regulation etc.) within which classroom practices are situated: what practices are normalised by these structures, what alternatives do they make possible and what forms of action might be necessary in order for a teacher and students to adopt different mathematical and interactional practices?

Conclusion

In narrating this partial account of my personal search for theory I have not had space to do full justice to the various sources that have influenced my thinking. In particular, the work of Bernstein offers a much more powerful and systematic set of concepts than the little I have touched on. Nevertheless, I hope I have provided a flavour of the resources I draw upon in order to address my research concerns, the reasons for choosing them and their scope. The multiplicity of theoretical tools, from linguistics and semiotics, discourse theory and sociology, matches the multi-layered conception of the field of study, incorporating individuals engaged in interactions within social practices and structures. My examples have taken as their starting points data arising in interactions occurring at different nodes within the educational enterprise: in classrooms, in assessment practices, in curriculum policy. In each case, the data take the form of a text or texts belonging to (i.e., originating in and/or being used in) one or more of these nodes. These texts are described at the level of the interaction using linguistic tools, but are connected to the practices within which the participants in the interaction are involved through analysis at the level of discourse. The interpretation of the significance of the texts involves consideration of the social structures that make these kinds of text possible. There are of course many other possible starting points that could contribute to understanding the complexity of practices within mathematics education. My contention is that, whatever the starting point, it is necessary both to look within – at the level of the text itself – to understand how the interaction operates locally, and to look beyond – at the discourses it draws on and the structures within which it is situated – in order to understand how it arose and how various participants may use it in different ways. The examples thus illustrate not only the course of my own engagement in research but also some possible ways of addressing the complexity, incorporating different levels of 'zoom' (Lerman, 1998) and linking different nodes in the network of practices (Valero, 2007).

Finally, to return to my original research concerns: the uneven distribution of knowledge and academic success, the role of language in this and the nature of pedagogy, my personal search for theory has brought me to social semiotics and systemic functional linguistics, critical discourse theory and Bernstein's notions of recontextualisation and pedagogic discourse. Of course a critical factor in the choice of these theoretical resources lies in their fitness for purpose – their ability to address research questions. Importantly, however, they are also compatible with my fundamental beliefs and political values. In particular, they allow me to maintain and work with my conviction that differences between students and between teachers cannot be explained solely at the level of the individual but must be understood as constructed within social structures and practices. They also allow me to challenge those social structures and practices and the inequities they give rise to by considering in a systematic way the question of how things might be different if different choices were made.

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