

Teachers' attempts to integrate research-based principles into the teaching of numeracy with post-16 learners

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Abstract

This paper describes some outcomes of a nine-month design-based research study into the professional development of 24 numeracy teachers with post-16 learners. The teachers were encouraged to integrate eight research-based teaching principles into their classroom practices as they implemented a set of discussion-based mathematics learning resources. A mixed methods approach was used to evaluate the outcomes, including interviews and classroom observation. The paper examines: teachers' perceptions of the project; the reasons why they found some pedagogical principles more difficult to incorporate than others and the factors that enabled and impeded their use of the learning resources. In particular it is noted that the principles that teachers considered to be most important were not the ones that they were observed using most effectively. The paper concludes by considering the implications of the research for initial and continuing teacher education.

Introduction

In spite of its obvious importance, the teaching of adult numeracy has, until recently, been an under-researched field of inquiry (Coben et al, 2003). Part of the difficulty has been the diverse range of settings in which it is located (e.g., colleges, workplaces, prisons), and the various forms it takes within these settings (e.g., discrete subject or embedded within other subjects). Also, the very term 'numeracy' is contentious, but in this paper we take it to mean the mathematics 'necessary to function at work and in society in general' (DfEE, 1999).

Post-16 numeracy teaching in England has recently become the focus for reform and massive change, which has affected the practice of teachers working in the sector (e.g., Lucas, 2004; Gleeson et al, 2005; Avis, 2007). Since 2001, teachers working in the *Skills for Life (SfL)* [1] sector of adult basic skills have been expected to work with a standardised core curriculum divided into five levels (Entry levels 1, 2 and 3, Levels 1 and 2) [2]. Each level has a prescribed, discrete body of knowledge and level of skill, which teachers are generally expected to ‘cover’ and ‘map’ onto individualised learning plans. This has encouraged a tendency to view mathematics as a set of discrete, disconnected skills and learning as an individual activity. Recent research from Coben et al (2007) suggests that the dominant mode of teaching numeracy to adults remains one of transmission where teachers show learners procedures, break concepts down into smaller parts and demonstrate examples. The most common forms of organisation are whole class and learners working individually through worksheets. Teachers tend to ask few higher-order questions, and there is little group or collaborative work, nor use of practical resources or ICT. Evidence shows that the transmission approach does not promote robust, transferable learning that endures over time or produce knowledge and skills that can be used in non-routine situations outside the classroom (e.g., Swan, 2006; Ofsted, 2006).

We embarked on the design-research project, *Thinking through Mathematics (TTM)*, to challenge these practices (Swain & Swan, 2007). This project formed part of a larger, ongoing project commissioned by the National Research and Development Centre for adult literacy and numeracy (NRDC) called *Maths4Life* (www.ncetm.org.uk). *TTM* builds on many years of research in the Further Education

(FE) sector (Swan, 2006), and draws on the work of an earlier project commissioned by the government: *Improving Learning in Mathematics (ILM)* (DfES, 2005). ILM developed and exemplified research-based principles for teaching mathematical concepts (see below) and provided an extensive set of supporting professional development resources (including an interactive DVD illustrating different pedagogical approaches), teaching materials and computer software. The resource was sent to every FE college and post 16 provider in England. Whereas ILM focused on teaching the mainstream and more advanced mathematics classes (Levels 2 and 3), there remained a need to address the very different contexts and requirements of those teaching adult numeracy. In particular, there was a need to investigate how far the same pedagogical principles could be applied in this very different context.

The *TTM* project had two related aims. The first was to challenge teachers to examine the limitations of the dominant 'transmission' paradigm and to explore a *more 'connected'*, and *'challenging'* paradigm (Figure 1).

Figure 1 goes about here

The second aim was to help learners adopt *more active approaches towards their learning*. Many adult learners appear to view learning mathematics as something 'done to them'. Instead, *TTM* intended learners to engage in discussing and explaining ideas, challenging and teaching one another, creating and solving each other's questions and working collaboratively to share methods and results.

The research-based pedagogical principles that were incorporated into the design of *TTM* are summarised in Table 1.

Table 1 goes about here

Methodology

A design-based research methodology was employed. This approach has arisen from a desire to make research more relevant by using research-based methods to attempt to *transform* educational practices in real educational settings (Kelly, 2003; Swan, 2006; van den Akker et al, 2006). It is distinct from research that attempts to explain existing causal connections between variables and from research that attempts to understand and explain a given state of affairs; rather it considers how education may evolve to meet given standards or ideals (NCTM, 1988). This requires an interventionist and visionary approach (Bereiter, 2002). By challenging the *status quo*, it is possible to discover the difficulties and elements that resist change. This in turn helps us to understand the system more fully. It is only recently that design-based research has emerged as a recognised paradigm for the study of learning through the systematic design of teaching strategies and tools. The beginnings of this movement are often attributed to Brown (1992) and Collins (1992), though we would contend that rigorous evidence-based design and development has been around for some time under many different names and guises.

Design-based research raises important methodological issues. Firstly, the context in which the designs will be used must be taken seriously: the designs must be tested in

the target contexts with 'ordinary', busy teachers; secondly, the researcher's role may need to evolve as the research progresses (from 'hands-on' to 'fly-on-the-wall' as the design evolves to become self-sustaining); and thirdly, the research needs to account for the ways in which the intentions of the design 'mutate' in the hands of teachers. When designs are used, teachers interpret them in ways that the designer did not intend, and rather than viewing these mutations or transformations negatively, designs and theories need to evolve and try to explain them.

In this research, a substantial collection of 29 discussion-based mathematical activities were created in collaboration with teachers using an iterative process: design; trial; reflect; modify. The activities were categorised into the following 'types' that encourage distinct ways of thinking and learning (Table 2).

[Insert Table 2 about here](#)

The professional development programme involved regular meetings with the teachers over 9 months to discuss the outcomes of the classroom trials, reflect on the underlying principles, and to stimulate the creation of new activities. Throughout this process, research and design were intertwined – teaching approaches and resources were iteratively modified and developed in the light of the emerging issues and findings, and the revised versions were observed in use to generate new research findings (Swain and Swan, 2007).

This design of the professional development combined many features recommended by researchers: it was sustained over time (Cohen & Hill, 1998); it was related to the

local context in which the teacher operated (Cobb et al, 2003); it involved teachers in active and collective participation (Garet et al, 1999); it focused on developing teachers' knowledge of the content, pedagogy and the underlying principles (Hammerness et al, 2005); and it offered continuing support for teachers in translating these new ideas into everyday practice (Lee & Wiliam, 2005).

Sample

The project involved 24 teachers (6 men, 18 women) from 12 organisations in England. The sites were urban and rural, metropolitan and regional, and across different *S/L* sectors, with most of the research taking place in seven further education colleges. The majority of courses catered for more than one level of the National Qualifications Framework (NQF), and over half contained learners working between Entry Level 2 and Level 1, which provided teachers with the challenge of planning differentiated activities and different materials: only two classes were designated to be working at one specific level. The length of courses ranged between three months and nine months (September to June), and teaching sessions varied from 45 minutes to three hours. While most of the numeracy provision was discrete and stand-alone, two classes were embedded in other courses. Over one-third of the programmes were held on a 'roll-on-roll-off' basis, with learners joining and leaving the course at different points, and this caused problems of continuity.

The background, attitudes, experience and qualifications of these teachers varied considerably. Some were nominated to take part by their organisations and appeared less than fully committed to the project. Nine worked full-time and the remainder were employed on part-time or on a fractional basis. Professional experience ranged

from under a year to 29 years (mean 6.5 years). Three held a degree in mathematics as their highest mathematical qualification, while two had not achieved a mathematics qualification at GCSE / 'O' Level [3]. Eleven had gained a Level 4 subject-specific teaching qualification in numeracy [4]. These teachers also taught 24 heterogeneous classes: nineteen of them had almost exclusively white British learners, two classes were predominantly Bangladeshi and two classes had learners of mixed ethnic origin. Sixteen of the classes were predominantly female, 5 were predominantly male and 3 had a fairly equal gender balance. Learners' ages ranged from 16 to mid-60s, although one-third of the classes were composed almost exclusively of 16–19 year-olds.

Methods of data collection

Both quantitative and qualitative data were collected, which came from two main sources: teachers' views were obtained through questionnaires, interviews and unstructured oral feedback given during the professional development meetings; teachers' practices were observed first-hand by eleven researchers). In total, 49 semi-structured teacher interviews and 110 classroom observations were carried out, and each teacher was observed between 3 and 6 times. In addition to this, further, limited data was collected from learners to assist in validating the accounts. Unless otherwise stated, the data in the following sections are taken from interviews with teachers (exceptions are made explicit at the end of the data extract).

Factors affecting the impact of the professional development (PD)

Before looking more closely at teachers' adoption of the principles, we consider six factors that affected the impact of the PD. These were: teachers' alternative interpretations of the aims and terminology of the project; teachers' knowledge;

teachers' perceptions of themselves; teachers' perceptions of learners; learners' expectations of teachers; and contextual factors (college management structures, time). Each is considered briefly, below.

(a) Teachers' alternative interpretations of aims and terminology

Teachers began the professional development with different understandings of its aims. These evolved during the professional development and we frequently stopped to reflect upon them. Two common misinterpretations emerged:

'The project is mainly about developing and testing resources.'

Some teachers initially interpreted the project as essentially generating 'materials for fun activities', or simply 'adding variety' to what they saw as an otherwise dull curriculum. They saw the activities as providing 'enrichment' to existing resources that could be slotted in at appropriate points. All that was needed was technical help in referencing the new activities to the curriculum specification. This misinterpretation misses the underlying purpose— to foster different forms of mathematical reasoning in learners.

'The project is mainly about learning by discovery.'

Some teachers believed that the approaches were about 'standing back and letting the learners discover things for themselves':

It is allowing them to make discoveries for themselves rather than you writing it up on the board [...] It is their discovering, not mine; it is nothing to do with me really. I have just to keep an eye on it.

Some teachers became aware of the shortcomings of transmission methods of teaching and recognised that 'telling' was not always an effective way of helping learners to understand concepts. Perhaps in reaction to this, they moved to an extreme position of 'not telling'. In contrast, the practice we sought to promote involved teachers developing a collaborative relationship with learners; at times they would allow learners to think and reason without interruption, while at others they would intervene and challenge learners in ways that would encourage them to reconsider and reformulate their reasoning.

Further difficulties arose with the terminology used in the project. Teachers often appeared to use terms loosely or hold alternative interpretations of them. This emphasised the need for exemplification whenever general teaching principles were described. Typical confusions were between the terms 'talk' and 'discussion' and between 'mistake' and 'misconception'. Some teachers claimed that learners were holding discussions even when the teacher (or one learner) was taking the lead, dominating, showing, or telling other learners how to think. This contrasts with our own interpretation that discussion is reciprocal in nature and involves shared reasoning. Other teachers appeared to attribute all students' mistakes to misconceptions. For us, however, misconceptions were always based on reasoning; usually over-generalisations from specific domains. (An example is when someone generalises from working with whole numbers, that 'to multiply a number by 10 you always add a zero'. Although this rule works in the domain of natural numbers, it does not when this domain is enlarged to include decimals.)

(b) Teachers' knowledge

Teachers' knowledge may be categorised into three areas: *general pedagogical* (including skills in classroom organisation and management); *mathematical* (understanding the subject); and *mathematics-specific-pedagogical* (knowing 'how' mathematics is learned) Coben et al (2007) [6].

Given the variety of teachers' backgrounds, it was not surprising that their general pedagogical knowledge varied considerably. These were sometimes based on teacher's own experiences at school:

I'd been taught to sit up straight and pay attention and very much chalk talk and textbook and that's the way it had been done for me and that's what I brought with me.

Nearly all of the teachers felt that the suggested approaches challenged this knowledge, particularly with regard to the management of small group and whole class collaborative discussion. Formative assessment techniques, such as inviting learners to describe what they already knew about a topic at the beginning of a session and then building constructively on this, were new to most teachers.

There was also a wide variation in teachers' subject knowledge in mathematics. As noted above, two teachers held qualifications at a level below GCSE (Level 2), and this had an effect on their levels of confidence and conceptual understanding.

I was kind of given this, you're going to support the literacy and numeracy tutor role, it was thrown upon me, so I had my own barriers as well as these fears of teaching different types of learners. I then had to teach numeracy

which I'd never taught before. I think, 'I don't like numeracy, I don't do numeracy'.

In addition to their own personal knowledge of mathematics, teachers also need to know how learners come to understand mathematics and the teaching strategies that might facilitate this. Successful use of the discussion activities was directly related to teachers' knowledge of mathematics-specific pedagogy. Where there were gaps in this form of knowledge, opportunities were missed and learning suffered. We noted several examples where teachers appeared unfamiliar with alternative methods for performing calculations. A few teachers were unsure of the distinction between grouping and sharing in division and with the decomposition method of subtraction. Many revealed a limited knowledge of common mistakes and misconceptions, and of strategies for responding to these.

(c) Teachers' perceptions of themselves

Some of the teachers in the project did not regard themselves as authentic mathematics teachers. In some cases this was due to them having teaching backgrounds in other subjects, and they clearly felt uncomfortable when addressed as such:

We are all admin tutors, or in my case, IT. It's just that we like maths.

This perception that 'we are not maths teachers' created a sense of 'distance' and irrelevance for some teachers:

There were times that we did feel very beyond our...out of our depth ... purely because a lot of it didn't seem to apply to us, in our circumstances – we're vocational teachers first, mathematics teachers second. At that point, that was

the lowest point that I got, I thought, what am I doing here? I don't know what I'm doing here.

(d) Teachers' perceptions of learners

A few teachers had low expectations of their learners and a 'protective' attitude towards them. Many of these learners already regarded themselves as failures in mathematics and teachers, understandably, wanted to avoid further reinforcement of a poor self-image. Teachers were therefore reluctant to give learners activities that they perceived might be too demanding and intervened at the first signs of difficulty in order to 'sort them out'. One teacher, for example, appeared to believe that his learners, who were working at Entry Level 1, were unable to discuss mathematics at all, and this became a self-fulfilling prophecy as he therefore rarely gave them opportunities to do so.

These students have been doing the same thing since they were very young. They were doing "Time" when they were five years old and they are still doing "Time" now – they still haven't grasped it. If they haven't the ability to grasp "Time" then they haven't got the ability to have mature mathematical discussions.

When teachers 'held back' support until after learners had been allowed opportunities to think for themselves, they reported a considerable 'feeling of achievement':

Initially I could sense their [the learners'] frustration with their inability to understand the concept and I thought I was pushing them too far. However, after some support and guidance and discussion, both in pairs and as a group,

they began to work it out. This gave us a tremendous feeling of achievement, and it was a watershed moment.

This teacher did not expect learners to gain conceptual understanding on their own. He recognised that learners needed 'support and guidance', but he also saw the need to allow them time to think for themselves *before* intervening. When he did intervene, he *collaborated with his learners* to resolve difficulties. The achievement was shared, not imposed.

(e) Learners' expectations of teachers

Learners also come to mathematics sessions with clear expectations of the teacher, the mathematics and the ways in which they will be expected to learn. Many had previously measured success in mathematics by worksheets covered with ticks, rather than by developing their understanding. Collaborative approaches to learning conflicted with their previous experiences and they found it difficult to adjust. The following quote (from the teacher of a 'numeracy' session for ESOL learners) illustrates how many learners saw mathematics as a subject to be learned through individual practice rather than collaborative discussion:

I think a lot of my students find the approaches really, really alien, most of my students have just recently come to this country [...] they're used to sitting in a big room, teacher at the front desk: "This is what we do, copy it all down." [...] They're very "Give me a worksheet" really quite seriously, not just "I think I'd like to do a worksheet" but like "Why are you not giving us any? This is not proper. What am I learning?" I think people are feeling that quite strongly that they're not learning anything.

These learners do feel that the teacher is only there to give a method; in fact the teacher is not doing the job for which they are paid for if they do not do this. Maths classes are viewed not as places for talking; they are only places for listening, writing and pondering on your own.

Discussion-based approaches have less tangible or discernible outcomes than traditional practice-based approaches. Even when tangible outcomes exist, these are generated by groups, and learners cannot always keep individual records of them:

My students like to have work to go in their folders – at times using the approaches this wasn't always possible. I photocopied the work from posters etc to put in folders – students still didn't feel that they had learnt anything without the evidence.

Many learners wanted *physical evidence* to show they had been working, and they gained a sense of security from 'capturing' information in written form. It is almost as though productivity has displaced understanding as the primary goal for learning. For example, there were occasions where learners did not even recognise mental calculations as legitimate 'work'.

Some learners expected to be 'spoon-fed' information and became irritated when teachers asked them to discuss something with their peers. They could not understand why the teacher would not simply tell them the answer or show them the method.

As we have already stated, a few teachers initially appeared to (mis)interpret the approaches as essentially 'learning by discovery' and withdrew support completely. Instead, we were advocating a much more flexible approach, where support is offered only *after* learners had been given the chance to discuss the concepts and strategies

for themselves. Judging when to withdraw support and when to intervene are delicate pedagogical decisions – if scaffolded support is withdrawn too quickly, the learners flounder, yet if it is never withdrawn they remain unable to resolve issues without support. Some teachers told us that, although ‘holding back’ was difficult at first, learners eventually began to accept and adapt to new ways of working:

I found it very difficult at first not to intervene when the students came across a problem. You were saying yourself, they’re asking questions, “Hey you’re the teacher, you know this, you’re supposed to tell us this!” [...] “No, what do *you* think?” And to get them to continue the conversation, again, works well I think. Very pleased with it. And they don’t do that now. Now they’ll say, “he’s not going to tell us, we’ve to got work this out ourselves.

These data underlines the importance of teachers explaining to learners the purpose of the project and the approaches. Like teachers, learners need to be made aware of the reasons for working in new ways.

(f) Contextual factors (college management structures, time)

In the pre-questionnaire, almost all teachers stated that they did not teach in ways that were consistent with their personal beliefs about teaching and learning. Reasons given included the ‘corporate’ assessment-driven culture in colleges (Leonard, 2000; Hayes, 2003), the need for syllabus coverage and the requirement to ‘map’ areas of the curriculum to the ANCC (see above). The support given by their institutional managers was a key factor in determining how teachers were able to implement the teaching approaches. Whereas some managers were content to give teachers the freedom to use the approaches in the way that they were intended, others were more

prescriptive and authoritarian in their style, which undermined the chances of teachers changing their practice

As the project progressed, time for planning and preparation emerged as a recurring and important issue. Research carried out by the NRDC [5] (see also Avis, 2007) has highlighted the fact that *SfL* teachers tend to feel overworked and overburdened with bureaucracy. In order to adopt new ways of working, teachers needed to allow adequate time to absorb the session plans and guidance so that they could begin to anticipate problems, potential misconceptions, learners' questions and so on. Not every teacher did this, and occasionally observers reported that teachers had not prepared sessions adequately.

Overview of teachers' adoption of the principles

At the final project meeting, we invited teachers to write down the most important 'messages' that had arisen for them personally. They were reminded of the eight principles (Table 1) and were encouraged to add their own ideas and amplifications. We also summarised the extent to which observers considered that teachers had incorporated each principle into their teaching both 'effectively' and 'consistently' (Figure 2).

Figure 2 goes about here

Here, the term 'effectively' means that the observer considered that the principle was being used in a way that was consistent with the intentions expressed in the literature. To take one example, "Builds on knowledge that learners bring to sessions"; if a

teacher only asked one or two questions at the beginning of a session, and did not integrate and build on this knowledge, then they would not be judged as using the principle effectively. In the case of the term ‘consistently’, a teacher would need to be seen using the principle in the majority of the observed sessions.

The graph shows that the principles that teachers regarded as being most important were not the same as those that were used most effectively. In particular, teachers highlighted ‘organising co-operative small group work’, ‘exposing and discussing misconceptions’ and ‘building on prior knowledge’ as being the most important principles, and this was confirmed by their open responses:

The most important feature has been using cooperative small group work and also exposing and discussing misconceptions. The group work has helped to motivate and engage the learners and the misconceptions approach has helped me to target areas where learners require more support more effectively.

It has allowed me and my learners to explore in discussion the misconceptions, myths and barriers associated with numeracy. It helped identify why these have occurred and past experiences that have impacted their / our future learning.

According to the observers, however, the latter two principles were not used consistently and effectively. Below we discuss the possible reasons for these results, beginning with the principles that teachers incorporated easily and progressing to those they found most difficult to implement. This has direct relevance for the future PD of teachers such as these.

Principles implemented successfully by at least 50% of the teachers.

Use rich, collaborative tasks

Fifteen of the teachers were observed using this principle effectively and consistently.

'Rich' tasks are:

... accessible, yet admit further challenges; tasks which invite learners to make decisions; which involve learners in speculating, hypothesising, explaining, proving, reflecting and interpreting; which promote discussion and questioning; which encourage originality and invention; and which have an element of surprise and are enjoyable (Ahmed, 1987).

'Collaborative' tasks are designed to be used by pairs or groups rather than by individuals. This, for example, means that the resources must be physically large enough to be seen by the whole group, allow multiple entry points and incorporate many levels of challenge. In designing the activities, for this project, we sought to develop such tasks and it is therefore gratifying that observers recorded that most of the teachers used this principle consistently and effectively. At the beginning of the project, however, we became aware that not all teachers were able to discriminate between tasks that were intended to develop skills for fluency and those which were intended to develop conceptual discussion. This was particularly true when the superficial appearance of the two forms of task were similar (e.g., when both involved matching cards). For example, we would not consider an task that involves matching cards showing multiplication questions ($6 \times 3 =$; $5 \times 4 = \dots$) to cards showing answers (18, 20, ...) to be 'rich' in the above sense, as these would not encourage the discussion of concepts. An example of a rich task is where cards showing mixed multiplications and divisions ($2 \times 6 =$; $6 \times 2 =$; $6 \div 2 =$; $2 \div 6 =$) are matched to cards

showing diagrams of these operations and corresponding problems for them, followed by a discussion of the alternative meanings of the concepts. We therefore attempted to address this issue by encouraging teachers to compare superficially similar tasks in order to identify those that would provoke the intended behaviours. A few teachers found this principle difficult to implement because of the time pressures alluded to earlier. Learners too, were not always adequately introduced to such tasks, and it is our view that they need as much induction and guidance in using rich, collaborative tasks as teachers do.

Organise co-operative small group work

Once the task is appropriate, learners need to be encouraged to work on it appropriately. Twelve (one half) of the teachers were observed using small group work consistently and effectively, and only one teacher appeared to find it genuinely difficult to organise her class in this way. For some, group work was a significant change in their existing practice; previously they had attempted to meet individual learning needs by asking learners to work on separate tasks/activities - differentiating by task. Almost every teacher was pleased with the response of learners when group work was introduced:

It has been a bit of a success story for me oddly enough because I've suddenly discovered, they [the learners] love group work! They just love working together and they actually bounce off one another, like you were saying, if one of them finds a method that they can all relate to then it takes the pressure off me in a way because I can stand there till I'm blue in the face trying to explain it ... I think that's been the best part of it, just seeing them gel together as a

group and I think if it wasn't for this project I don't think it would have happened. [Oral report]

However, observers frequently reported that learners continued to work as individuals, even when they were asked to sit in groups. One possible reason for this was that teachers rarely discussed with learners their reasons for adopting small group work.

Use probing questioning to assess what learners know and how they think

By the end of the project, over half of the teachers were judged to be using probing questions consistently and effectively. The majority of the teachers were observed asking a broader range of question types, including those that were more challenging, open and diagnostic. They were also increasing 'wait times' after asking questions to allow learners more time for reflection. The following teacher (JW) offers us one example:

JW: What is the largest number that is 700 when rounded to the nearest 100?

JW waits and as learners slowly respond with 750, 699 [2 people], 754, 749,

JW lists these on the board without comment.

JW: Which should be discarded?

Learner: 699

JW: Why?

Learner: 700's are bigger. Has to be in 700's

JW: Discount another.

Learner: 754

JW: Why?

Learner: It's higher. It goes to 800

JW: Are those two OK?

Learners disagree

JW: Some say 'yes' – why?

Learner: 750 is more than 749. Therefore, its 750 that goes to the nearest 100.

JW: What are the rules of rounding?

Learner: So 750 would go up to 800.

JW: So what is the largest number that rounds to 700?

Learners: 749, 749 [all agree].

When teachers persisted in this practice, observers noted that learners became less passive and dependent on the teacher.

Principles implemented successfully by only a minority of the teachers

Build on knowledge learners bring to sessions

The teachers that used this strategy most effectively began each session by asking the class to describe what they already knew about a topic, and followed up their responses with further questions and challenges. Their aim was to treat the beginning of each session as an opportunity for *formative assessment* in order to discover what is known, what is unknown and the knowledge needed to close the gap (Black, & Wiliam, 1998).

I say "Before we start tell me what you know". I'll put that on the whiteboard and that is my starting point. So even if things are down that are not correct, it doesn't matter, we'll put them down and then discuss them. [Oral report]

This teacher reported that he often had to considerably modify or even abandon his lesson plans on the basis of what he learned in this way.

In our study, few teachers could adopt this principle as easily as this teacher. Some asked learners a few questions at the beginning of the session, but did not know how to respond constructively to their responses during the remainder of the session. This requires considerable subject-specific pedagogical knowledge coupled with a flexible lesson plan. The current emphasis, in many institutions, of insisting that teachers plan, make explicit and adhere to content objectives, clearly impedes the possibilities of such flexibility and responsiveness.

Expose and discusses common misconceptions

This objective is closely linked to the previous one. Again it requires careful listening to students and again it proved to be one of the most difficult principles to integrate into teachers' practice. Only four teachers were able to do this effectively. The design of the tasks generated many opportunities for intense discussion of misconceptions, but teachers found it very difficult to know how these discussions should be managed and resolved. The temptation to 'take over' and explain 'the correct' viewpoint before learners had been given a chance to explore the ideas proved irresistible for most.

When this happened, learners often reverted to passive behaviours, as they were unable to follow the teacher's line of reasoning. In the best implementations of this principle learners were encouraged to explore the consequences of their own

misconceptions, so that a vivid cognitive conflict/surprise would arise, leading to the realisation that new ideas needed to be accommodated.

Create connections between mathematics topics

The ANCC curriculum specification compartmentalises mathematics into discrete topics, and some teachers plan the order of their teaching according to this conceptual layout. Only eight of 24 teachers were, in our judgment, making connections between topics effectively and consistently:

I used to teach, until fairly recently, from topic to topic, I've got the core curriculum here, I know I've got to cover these topics and I'll go through them. I don't do that any more. [I now] combine all these elements together and I think it's wonderful

Some made reference and connections both to other areas of the learners' course (e.g., a cookery or financial component), and also to the world outside the classroom. We did find, however, that teachers were unwilling to tackle topics that had been placed at a 'higher' level in the curriculum than that at which they were currently working, and this resulted in an inflexible approach to teaching. Although the materials devised in the project 'forced' some links to be drawn, teachers did not tend to encourage this process themselves.

Emphasise methods rather than answers

Only eight of the 24 teachers were observed regularly encouraging learners to justify their decisions by offering reasons, rather than giving answers. For most, this was a considerable challenge to their normal practice. Sometimes, the pressure (from students) to provide short cuts proved irresistible, even when this reinforced misconceptions:

The explanations given by the tutor were always designed to help understandings but there were occasions when he would emphasise quick ways of getting answers. The clearest example of this was the “add a nought” model for multiplying by ten. He was aware of when this did not work but in some cases he felt that the learners needed a quick way of doing something.

[Observation report]

The current emphasis on instrumental, procedural teaching under time pressure seems to have produced a classroom culture diametrically opposed to a slower, deeper analysis of mathematical reasoning.

Use technology in appropriate ways

Only four of the teachers were judged to be using this principle effectively and consistently. We have little evidence of teachers using technology during the project, whether these were interactive whiteboards, computers or calculators. In total we only observed four teachers using technology: for exposition (using PowerPoint and downloaded software) or exploration (using spreadsheets and the internet). In some cases, teachers had no access to IT at the centre where they worked. For most teachers, technology was either given a low priority, or where learners were encouraged to access information using computers, this was done on an individual basis and the information did not form part of the group tasks. Although calculators were seen, they were usually used to check work rather than to facilitate or formulate concepts.

Implications for policy and teacher education and training [7]

These findings have implications for both initial teacher training and continuing professional development (CPD). It is clear from the above analysis that the design of the collaborative tasks encouraged, and had significant impact on, teachers' implementation of the learning principles. Sound task design was clearly necessary but insufficient, and we found that teachers also needed to develop pedagogical insights to be able to discriminate between classroom tasks that foster and sustain mathematical reasoning from those that simply offer practice.

We therefore advocate that the professional development of numeracy teachers should pay more attention to subject-specific pedagogical issues, such as the nature of tasks given to learners, the theories and principles that underlie the design of such tasks and the ways in which they are managed in the classroom. It should also focus on the theories, strategies and techniques of formative assessment, where teachers make effective use of higher-order questioning, and listen to, and respond flexibly, to learners' needs; tackle the issue of differentiation and the organisational consequences; and exemplify and develop the concepts of 'mathematical discussion' and 'collaborative learning' so that they have a shared meaning among teachers. Finally, it should prioritise the development of teachers' awareness that learners, like teachers, need induction and guidance in techniques of collaboration and working together.

Concluding remarks

Teachers' beliefs and practices are developed over many years and become resistant to change (Duffy & Roehler, 1986; Fullan, 1991). The project deliberately chose teachers from different backgrounds and experiences: we did not want teachers who

were all enthusiastically looking to change their practices, and who were receptive to new ideas. We set out to investigate whether a set of pedagogical principles could be integrated into the practice of a range of teachers, some of who were likely to be more conservative, some whose methods of practice were more entrenched, and some whom were sceptical about accepting new methods and ways of working. The fact that observers reported that three-quarters of the teachers had changed their general practice towards becoming more learner-centred, and over a quarter of these had introduced changes of a profound nature, is a testament to the potential of an iterative professional development model where teachers reflect on existing practice, design tasks, trial, reflect and return to discuss and refine concepts, ideas and tasks.

During the course of the project, every teacher introduced some of the 8 principles successfully into their practice. The nature of the tasks facilitated the successful implementation of collaborative small group work, and raised the quality of teachers' questioning. Thus the activities that involved 'evaluating statements' led naturally to teachers asking "Is that true? Why? Can you find a counterexample?"; classification activities led to "What is the same and what is different?"; 'multiple representations' activities led to "How do you know that this means the same as this? How could we write this is a different way?"; the 'creating questions' activities involved "How did she get that? How could we undo that procedure?" and so on.

We also recognise that some of the principles take more time to interpret, absorb and implement, and this is why it is helpful to use an iterative model of professional development in which they can explore novel ways of working with learners. There is also a need for more of us who engage educational research to take on the challenge

of *design-research*, seeking to find improved products and processes that equip 'ordinary' teachers with the means to embed research-based principles in their everyday classroom practices. Such research can have a large-scale impact, as it not only produces new insights, it also provides accessible products. The product arising from the research reported here is a ringbinder of professional development and teaching and learning activities, which is available to every teacher of adult numeracy in England (NRDC, 2007).

Notes

[1] *Skills for Life* (SfL) is a national strategy for raising standards and improving adult literacy and numeracy skills launched by the government in March 2001.

[2] In the *Skills for Life* literature much is made of the National Qualifications Framework in which certain 'levels' are supposedly meant to correspond to standards in compulsory schooling and Higher Education. In this framework Level 2 for adults is seen as the equivalent of a GCSE A*-C and Level 1 to a GCSE FD-G; Entry Level 3 corresponds to a level expected of an average 11-year-old, Entry Level 2 is compatible with standards of the average 7-year-old, and Entry Level 1 is at the level of the average 5-year-old. The authors' view is that this framework is potentially insulting, and that, as the majority of learners in the project were clearly not children, it should be regarded as a rough guide only.

[3] GCSE stands for General Certificate of Secondary Education and is the main examination that the majority of students take at the age of 16. 'O' or 'Ordinary' Level was the name of this examination that preceded GCSE.

[4] A subject-specific qualification is a teaching qualification that all Sfl teachers have to take by 2010. Level 4 is the equivalent level of first year undergraduate study.

[5] The evidence comes from emerging findings from the DfES / NRDC project, *A longitudinal study of the impact of the Skills for Life national strategy for improving adult literacy and numeracy skills on teachers and trainers*, which is due to publish its findings in 2008.

[6] We note that Shulman (1986, 1987) has made a similar classification where he distinguishes between different forms of knowledge; knowledge of mathematics, knowledge of general pedagogy, and pedagogical content knowledge specific to the general teaching of mathematics, and also to particular individual topics.

[7] We are aware that some of these issues may be addressed in the LLUK *New overarching standards for teachers, tutors and trainers in the lifelong learning sector (Application to Professional Standards, for Teachers of Mathematics (Numeracy))*, which began in the autumn of 2007.

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Figure 1: The 'Transmission' view and the 'connected', 'challenging' view (Swan, 2006)

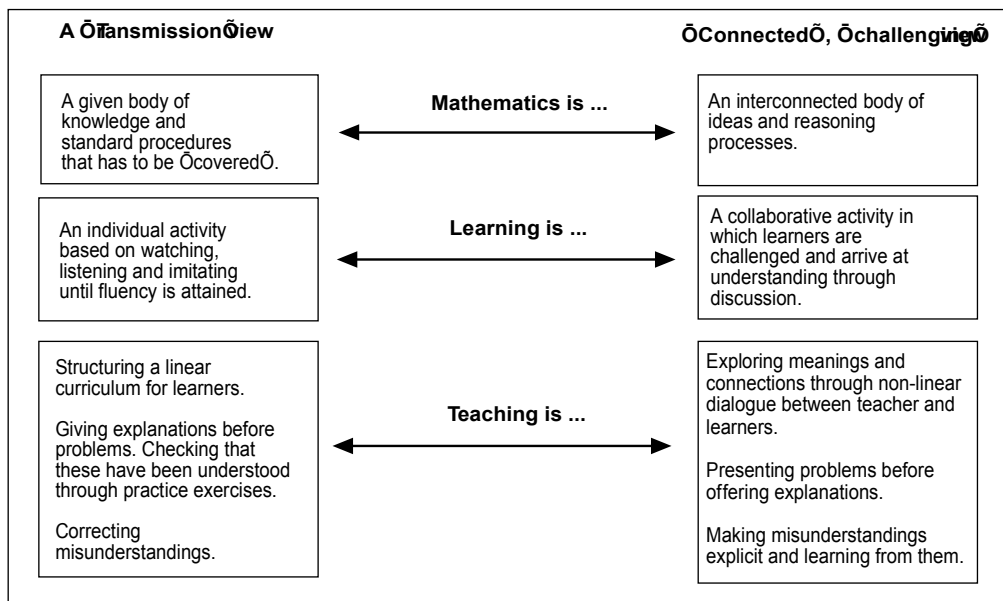


Table 1: Pedagogical principles underpinning the teaching approaches

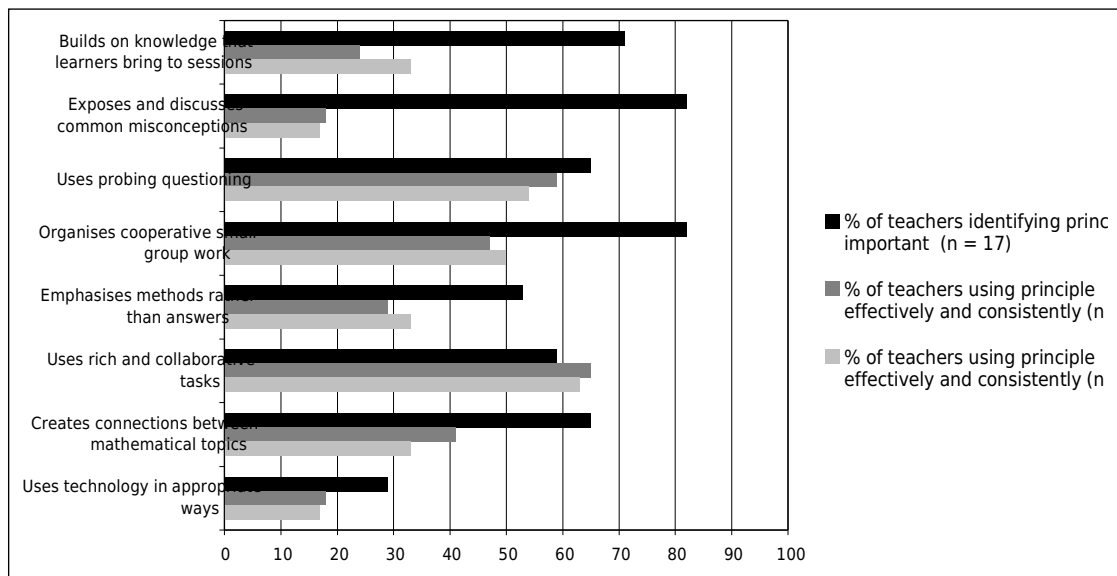
Teaching is more effective when it ...

<ul style="list-style-type: none"> • builds on the knowledge learners already have 	<p>This means developing formative assessment techniques and adapting our teaching to accommodate individual learning needs (Black & Wiliam, 1998).</p>
<ul style="list-style-type: none"> • exposes and discusses common misconceptions 	<p>Learning activities should expose current thinking, create ‘tensions’ by confronting learners with inconsistencies, and allow opportunities for resolution through discussion (Askew & Wiliam, 1995).</p>
<ul style="list-style-type: none"> • uses higher-order questions 	<p>Questioning is more effective when it promotes explanation, application and synthesis rather than mere recall (Askew & Wiliam, 1995).</p>
<ul style="list-style-type: none"> • uses cooperative small group work 	<p>Activities are more effective when they encourage critical, constructive discussion, rather than argumentation or uncritical acceptance (Mercer, 2000). Shared goals and group accountability are important (Askew & Wiliam, 1995).</p>
<ul style="list-style-type: none"> • encourages reasoning rather than ‘answer getting’ 	<p>Often, learners are more concerned with what they have ‘done’ than with what they have learned. It is better to aim for depth than for superficial ‘coverage’.</p>
<ul style="list-style-type: none"> • uses rich, collaborative tasks 	<p>The tasks we use should be accessible, be extendable, encourage decision-making, promote discussion, encourage creativity, encourage ‘what if’ and ‘what if not?’ questions (Ahmed, 1987).</p>
<ul style="list-style-type: none"> • creates connections between topics 	<p>Learners often find it difficult to generalise and transfer their learning to other topics and contexts. Related concepts (such as division, fraction and ratio) remain unconnected. Effective teachers build bridges between ideas (Askew et al., 1997).</p>
<ul style="list-style-type: none"> • uses technology 	<p>Computers and interactive whiteboards allow us to present concepts in visual dynamic and exciting ways that motivate learners.</p>

Table 2: Types of task devised with the teachers

Classifying mathematical objects	Learners devise their own classifications for mathematical objects (e.g. shapes, numbers, symbols), and/or apply classifications devised by others. In doing this, they learn to discriminate carefully and recognise the properties of objects. They also develop mathematical language and definitions.
Interpreting multiple representations	Learners work together matching cards that show alternative representations of the same mathematical idea (e.g. words, pictures, symbols). They draw links between representations and develop new mental images for concepts.
Evaluating mathematical statements	Learners are given statements and are asked to decide upon their validity. (E.g. "Max gets a 10% pay rise, Mary gets a 5% pay rise, so Max gets the bigger pay rise".) When are they true? When are they false? Learners suggest their own examples and counterexamples.
Creating and solving problems	Learners are asked to devise their own problems for other learners to solve, using given constraints. When the 'solver' becomes stuck, the problem 'creators' take on the role of teacher and explainer. These activities exemplify the 'doing' and 'undoing' processes of mathematics.
Analysing reasoning and solutions	Learners compare different methods for doing a problem, organise solutions and/ or diagnose the causes of errors in solutions. They begin to recognise that there are alternative pathways through a problem, and develop their own chains of reasoning.

Figure 2: Teachers' perceptions of the most important principles and observers' evaluations of their implementation.



Note: The middle bar shows the observational data for those 17 teachers who completed the post-questionnaires, while the lowest bar shows the observational evidence for the whole sample of 24 teachers.