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Holographic complexity and fidelity susceptibility as holographic information dual to different volumes in AdS

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ABSTRACT

The holographic complexity and fidelity susceptibility have been defined as new quantities dual to different volumes in AdS. In this paper, we will use these new proposals to calculate both of these quantities for a variety of interesting deformations of AdS. We obtain the holographic complexity and fidelity susceptibility for an AdS black hole, Janus solution, a solution with cylindrical symmetry, an inhomogeneous background and a hyperscaling violating background. It is observed that the holographic complexity depends on the size of the subsystem for all these solutions and the fidelity susceptibility does not have any such dependence.

1. Introduction

The information theory deals with the ability of an observer to process relevant information, and it is important as studies done in different branches of physics seem to indicate that the laws of physics are informational theoretical processes [1,2]. It is important to know how much information is lost when an observer processes the relevant information, and it is also important to quantify this abstract concept relating to the loss of information in a process. The quantity which quantifies this concept relating to the loss of information is the entropy, and it is one of the most important quantities in information theory. As the laws of physics can be represented by informational theoretical processes, entropy has been used to analyze the behavior of physical systems ranging from condensed matter physics to gravitational physics. It may be noted that in Jacobson formalism, it is even possible to obtain the general relativity from thermodynamics [3,4]. Thus, it is possible that the geometry of spacetime is an emergent structure, and it emerges due to an information theoretical process. In the Jacobson formalism it is important to assume a certain scaling behavior of entropy to obtain general relativity, i.e., the maximum entropy of a region of space scales with its area, and this has been motivated from the physics of black holes. This is because the black holes are maximum entropy objects, and the entropy of a black hole scales with its area. The holographic principle is motivated from this observation that the maximum entropy of a region of space scales with its area [5,6]. The holographic principle states that the number of degrees of freedom in a region of space is equal to the number of degrees of freedom on the boundary surrounding that region of space. The AdS/CFT correspondence is of the most important realizations of the holographic principle [7], and it relates the supergravity solutions in AdS spacetime to the superconformal field theory on the boundary of that AdS spacetime.

It is interesting to note that the holographic principle which was initially proposed due to the scaling behavior of entropy in black holes, may also lead to a solution of the black hole information paradox. The black hole information paradox occurs due to the observation that classical information cannot get out of a black hole and black holes evaporate due to Hawking radiation. This is because it has been proposed that quantum entanglement can be used to analyze the microscopic picture of a black hole, and it is hoped that this may resolve the black hole information paradox [8,9]. The AdS/CFT correspondence, which is a concrete regularization of the holographic principle, can be used to quantify quantum entanglement in terms of the holographic entanglement entropy. The holographic entanglement entropy of a CFT is dual to the area

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of a minimal surface defined in the bulk of an asymptotical AdS spacetime. Now for a subsystem A (with its complement), γ_A can be defined as the $(d-1)$-minimal surface extended into the AdS bulk with the boundary ∂A. So, the holographic entanglement entropy for this subsystem, can be written as [10,11]

$$S_A = \frac{Area(\gamma_A)}{4G_{d+1}} \tag{1}$$

where G is the gravitational constant in the AdS spacetime.

It is important to know how much information is retained in a system, but it is also important to know, how easy is it for an observer to process this information. Just as entropy quantifies the abstract idea of loss of information, complexity quantifies the abstract idea of the difficulty to process this information, and so just like entropy, complexity is a fundamental quantity relating to information theoretical processes. As the laws of physics can be represented in terms of informational theoretical processes, it is expected that complexity can be viewed as another fundamental physical quantity, and it is expected that laws of physics should be written in terms of complexity. It is interesting to note that complexity (like entropy) has been used to study condensed matter systems [12,13] and molecular physics [14]. In fact, complexity is also important in quantum computing [15]. Complexity is also important in analysis of the physics of black holes, as it has been recently proposed that the information may not be ideally lost in a black hole, but it may be lost for all practical purposes as it would be impossible to reconstruct it from the Hawking radiation [16]. However, unlike entropy, there is no universal definition of complexity of a system, and there are different proposals to define the complexity of any systems. However, it is possible to define complexity holographically. In fact, recently holographic complexity has been defined as a quantity dual to a volume of codimension one time slice in anti-de Sitter (AdS) [17–20],

$$Complexity = \frac{V}{8\pi RG_{d+1}}, \tag{2}$$

where R and V are the radius of the curvature and the volume in the AdS bulk.

The different proposals for complexity could be related to the different possible ways to define this volume in the bulk. It is possible to define complexity as dual to the maximal volume in the AdS which ends on the time slice at the AdS boundary $V = V_{max}$ [22], and it has been demonstrated that this proposal corresponds to the fidelity susceptibility of the boundary CFT. Hence, this quantity is called fidelity susceptibility even in the bulk, and we will denote it by $\Delta\chi_F$. It is interesting to note that the fidelity susceptibility of the boundary theory can be used for analyzing the quantum phase transitions [23–25], and thus it is possible to study quantum phase transitions holographically. However, it is also possible to use a subsystem A with its complement, and define the volume as $V = V(\gamma)$. This volume is the volume enclosed by the minimal surface used to calculate the holographic entanglement entropy [21],and it can also be used to holographically define complexity that we will denote by $\Delta\mathcal{C}$. As we want to differentiate it from the case, where the maximum volume has been used to calculate the complexity of a system, we shall call it holographic complexity (this terminology follows from [21], where such a quantity is called holographic complexity). Thus, in this paper, the maximum volume of a system $V = V_{max}$ will be used to calculate the fidelity susceptibility, and the $V = V(\gamma)$ will be used to calculate the holographic complexity of such a system. As complexity is a new physical quantity and it is expected that laws of physics can be written in terms of complexity, we will use these recent proposals to calculate the holographic complexity and fidelity susceptibility for various deformed AdS solutions.

As it has been proposed that the holographic complexity and fidelity susceptibility of a boundary theory can be holographically calculated from a deformed AdS bulk solution, it would be interesting to calculate such quantities for AdS bulk solutions which have interesting boundary dual solutions. These quantities calculated in the bulk could be used to understand the behavior of the boundary field theory dual to such geometries. This is the main motivation to study such quantities for an AdS_{d+2} black hole, Janus solution, cylindrical solution, inhomogeneous backgrounds, and hyperscaling violating backgrounds. Most of these deformed AdS solutions have interesting boundary dual. In this paper, we will also mention some interesting field theories which are dual to these deformations of the AdS spacetime. Thus, it is important to analyze such quantities in the bulk to possibly understand their behavior in the boundary field theory dual to such a bulk. Furthermore, apart from having interesting boundary duals, these solutions are interesting geometric solutions. So, by calculating these quantities for these solutions, we will also try to understand certain universal features of holographic complexity and fidelity susceptibility for different deformations of the AdS geometry.

The organization of this paper is as follows: In Sec. 2, we will study the holographic quantities for an AdS_{d+2} black hole. In Sec. 3 and Sec. 4 we will examine the Janus and cylindrical solutions and we will show that the holographic complexity is different than the fidelity susceptibility, which is an opposite result as it was given in previous works. In Sec. 5 and Sec. 6, as two complementary and interesting examples, the holographic complexity and the fidelity susceptibility will be also studied for geometries with inhomogeneous and hyperscaling violating backgrounds respectively. Finally, we will conclude our main results in Sec. 7.

2. AdS black holes

In the holographic picture, an excited state in CFT on the boundary is dual to a deformation of AdS in the bulk. This deformed metric could be expressed asymptotically by an AdS geometry. Such AdS black hole can be used to holographically model superconductors [26,27]. It is important to understand the behavior of fidelity susceptibility for superconductors. In fact, the fidelity susceptibility in topological superconductors has been obtained, and this was done by solving Bogoliubov–de Gennes equations [28]. As it is possible to holographically describe superconductors using AdS black holes, it will be possible to obtain the fidelity susceptibility for such field theories which are boundary dual to AdS black holes, by calculating the fidelity susceptibility for AdS black holes. So, we will calculate the fidelity susceptibility for a AdS black hole, and this will be dual to the maximum volume. However, we will also use a subsystem, and calculate the holographic complexity for such AdS black holes.

We will use a deformed Poincaré metric for AdS_{d+2} black hole to perform this calculation, and this metric can be written as

$$ds^2 = \frac{R^2}{r^2}\left(-h(r)dt^2 + \frac{dr^2}{h(r)} + d\rho^2 + \rho^2 d\Omega_{d-1}^2\right). \tag{3}$$

By setting the metric function $h(r) = 1$, we can recover a pure AdS space-time. This metric function now gets deformed as $h(r) = 1 - mr^{d+1}$ where m is a constant. In analogy with the pure AdS background, subsystem in the bulk can be parametrized by $\rho = f(r)$. However, in this case, the function $f(r)$ does not have a closed simple form. The minimal hypersurface can be obtained by minimizing the auxiliary functional,

$$Area = \Omega_{d-1}\int dr\left(\frac{R}{r}\right)^d f(r)^{d-1}\sqrt{f'(r)^2 + \frac{1}{h(r)}}. \tag{4}$$

Here, prime denotes derivative with respect to the radial coordinate r and $\Omega_{d-1} = 2\pi^{d/2}/\Gamma(d/2)$. The appropriate boundary conditions for this system are $f(0) = r_t$ and $f'(0) = 0$, where r_t denotes the classical turning point of $f(r)$. The associated equation of motion obtained from this action can be expressed as

$$f'' + \frac{1-d}{f}\left(f'(r)^2 + \frac{1}{h(r)}\right) + \frac{f'}{2}\frac{h'}{h} = 0\,. \tag{5}$$

For a sufficiently small parameter m, we can write the solution to this equation up to first order in m as an expansion of $f(r)$ as follows

$$f(r) = f_0(r) + m f_1(r) + \mathcal{O}(m^2)\,, \tag{6}$$

where $f(r)$ is the exact solution of Eq. (5). For $h(r) \approx 1$, the initial conditions are given by

$$f_0(r) = r_t\,, \quad f_0'(r=0) = 0\,, \tag{7}$$

and hence we obtain

$$f_0(r) = \sqrt{r_t^2 - r^2}\,. \tag{8}$$

The profile of the minimal surface at leading order in m can be written as

$$m\ell^{d+1} \ll 1\,. \tag{9}$$

Thus, we can write the metric function as [38]

$$f(r) = \sqrt{r_t^2 - r^2}\left(1 + \frac{2r_t^{d+3} - r^{d+1}(r_t^2 + r^2)}{2(d+2)(r_t^2 - r^2)}m\right) + \mathcal{O}(m^2)\,, \tag{10}$$

where we assumed a regularity at $r = r_t$. The parameter r_t is a free positive constant which is related to the radius ℓ of the subsystem by

$$\ell = 2\int_0^{r_t} dr \left(\frac{r}{r_t}\right)^d \sqrt{\frac{1}{h(r)\left(1 - \left(\frac{r}{r_t}\right)^{2d}\right)}}\,. \tag{11}$$

The length of the entangled system is fixed, so that we can compute the turning point r_t to leading order in m, yielding

$$r_t = \ell\left(1 - \frac{m\ell^{d+1}}{d+1} + \mathcal{O}\left((m\ell^{d+1})^2\right)\right)\,. \tag{12}$$

The volume of codimension one spacetime enclosed by the minimal area is defined by the following integral,

$$V(\gamma) = \frac{\Omega_{d-1}R^{d+1}}{d}\int_\varepsilon^{r_t} dr\, \frac{f(r)^d}{r^{d+1}\sqrt{h(r)}}\,. \tag{13}$$

Here ϵ denotes a UV cut-off. By substituting Eq. (10) into the above equation and then by evaluating the integral, we obtain

$$\Delta V = V_{BH} - V_{AdS_{d+1}} = \frac{4a_d R^{d+1}\Omega_{d-1}}{(d+2)\,d}\left(m\ell^{d+1}\right)\,, \tag{14}$$

where the coefficients a_d are defined as

$$a_d = \begin{cases} A & d = 2n, n \in \mathcal{Z}, \\ B & d = 2n+1, n \in \mathcal{Z}\,, \end{cases} \tag{15}$$

and

$$\begin{aligned} A = \sum_{p=0}^{\frac{d}{2}}\sum_{q=0}^{\frac{d}{2}-1}\binom{\frac{d}{2}}{p}\binom{\frac{d}{2}-1}{q}\Big[&(d+2)(q+\frac{3}{2})(q-d/2)(q+\frac{1}{2})(-1)^p \\ &+ (p+\frac{1}{2})((q+1)d + 2q + \frac{3}{2})d(-1)^q\Big] \\ &\times\Big[(2p+1)(2q-d)(2q+3)(2q+1)\Big]^{-1}, \end{aligned} \tag{16}$$

$$\begin{aligned} B = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\binom{\frac{d}{2}}{p}\binom{\frac{d}{2}-1}{q}\Big[&(d+2)(q+\frac{3}{2})(q-d/2)(q+\frac{1}{2})(-1)^p \\ &+ (p+\frac{1}{2})((q+1)d + 2q + \frac{3}{2})d(-1)^q\Big] \\ &\times\Big[(2p+1)(2q-d)(2q+3)(2q+1)\Big]^{-1}, \end{aligned} \tag{17}$$

where $\binom{\frac{d}{2}}{p}$ denotes the binomial coefficient. Now, by using Eqs. (14) and (2) we can obtain the holographic complexity, which is

$$\Delta\mathcal{C} = \frac{4a_d R^d \Omega_{d-1}}{8\pi G\,(d+2)\,d}\left(m\ell^{d+1}\right)\,. \tag{18}$$

It may be noted that this expression for $\Delta\mathcal{C}$ is different from the holographic complexity calculated in [21], which was given by

$$\Delta\mathcal{C} = \frac{c_d R^d \Omega_{d-1}}{8\pi G d}(m\ell^{d+1})^2\,. \tag{19}$$

Therefore, $\Delta C \propto m\ell^{d+1}$, and not $(m\ell^{d+1})^2$ as was proposed in [21].

Now, we will calculate the fidelity susceptibility for a deformed AdS state with metric (3). To do this, we need to evaluate the $\mathrm{Vol}(\Sigma_{max})$ for the metric given by Eq. (3). Therefore, we can set $t = 0$ and consider the codimension one hypersurface. In order to compute the volume integral, we can use the expression (13) with different integral limits, i.e., change r_t by the horizon r_+ given by $h(r_+) = 0$, and $r_+ = m^{-\frac{1}{d+1}}$. Thus, the volume term can be expressed as [22],

$$\Delta\mathrm{Vol}(V_{max}) = \frac{b_d R^{d+1} V_d}{d}(m^{d/(d+1)})\,, \tag{20}$$

where $V_d = \mathrm{Vol}\{V_d : d\rho^2 + \rho^2 d\Omega_{d-1}\}$. The fidelity susceptibility can be written as

$$\Delta\chi_F(\lambda) = n_d\frac{b_d V_d}{d}(m^{d/(d+1)})\,. \tag{21}$$

This fidelity susceptibility can be used to obtain the fidelity susceptibility for the holographic superconductors. However, we have also demonstrated that it is also possible to derive other quantities dual to the volume in the bulk, and this is the holographic complexity. It may be noted that the holographic complexity depends on the size of the subsystem ℓ and as the fidelity susceptibility was calculated for the full system, no such dependence has been observed. Furthermore, as holographic entanglement entropy has been calculated for AdS black holes [29,30], and the holographic complexity is calculated using the same surface as entanglement entropy. Hence we have calculated both holographic complexity and fidelity susceptibility for AdS black holes.

3. Janus solution

It is possible to obtain a nonsupersymmetric dilatonic deformation of AdS geometry as an exact nonsingular solution of the type IIB supergravity [31]. The gauge theory dual to this solution has a different Yang–Mills coupling in each of the two halves of the boundary spacetime divided by a codimension one defect.

The structure of the boundary and the string configurations corresponding to Wilson loops for this solution have been studied [31]. This solution is called the Janus solution, and it has also been possible to study the supersymmetric Janus solution [32]. It has been demonstrated that the Janus solution has quantum level conformal symmetry, and this was done by using conformal perturbation theory to study various correlation functions [33]. The holographic entanglement entropy in the presence of a conformal interface has been recently calculated, and it was observed that for the supersymmetric Janus solution, the holographic entanglement entropy calculated from the bulk was in exact agreement with the calculations done using a CFT [34]. As the holographic complexity is calculated using the same surface as the holographic entanglement entropy, we will calculate the holographic complexity for the Janus solution. We will also calculate the fidelity susceptibility for the Janus solution, and this can be used to understand the behavior of fidelity susceptibility for a system of consisting of different Yang–Mills coupling in each of the two halves of the boundary.

It is possible to use the AdS_2 slice of the deformed AdS_3 to obtain the Janus solution. This solution is an exact solution defined using the following Euclidean bulk action,

$$S=-\frac{1}{16\pi G_N}\int d^3x\sqrt{g_E}\Big(R-\phi_{;\mu}\phi^{;\mu}+\frac{2}{R^2}\Big). \tag{22}$$

Here, ϕ is the massless bulk scalar field. The metric of the Janus solution and the profile of the dilaton field, is given by the Euclidean metric

$$ds^2=R^2(dy^2+\frac{f(y)}{z^2}(dz^2+dx^2)),\quad \phi(y)=\gamma\int_{-\infty}^{y}\frac{dy}{f(y)}+\phi_1, \tag{23}$$

$$f(y)=\frac{1}{2}\Big(1+\sqrt{1-2\gamma^2}\cosh(2y)\Big),\quad \gamma\le\frac{1}{2},\quad \phi_1=\phi(-\infty). \tag{24}$$

For this geometry, the coupling constant for the ground state $|Omega_1\rangle$ is dual to ϕ_1. The fidelity susceptibility was computed in [22], and is given by

$$\Delta\chi_F(\lambda)=\frac{cV_1}{12\pi\epsilon}, \tag{25}$$

where V_1 is the volume of the AdS_2 per unit radius and ϵ is a UV cutoff. Now, we will compute the holographic complexity for the Janus solution represented by the metric (23). The area functional for an entangled region $A=\{x\in[0,L], z=z(y)\}$ will be given by

$$Area=R^2L\int_{-y_\infty}^{y_\infty}dy\sqrt{\frac{f(y)}{z^2}\Big(1+\frac{f(y)}{z^2}z'^2\Big)},\quad z'\equiv\frac{dz}{dy}. \tag{26}$$

Moreover, the entangled length and volume are

$$\ell=2\int_0^{y_t}z'(y)dy, \tag{27}$$

$$V(\gamma)=R^3L\int_{-y_\infty}^{y_\infty}dyf(y)\int_{\epsilon}^{z(y)}\frac{dz}{z^2}, \tag{28}$$

which could be simplified by subtraction of the pure AdS portion from the AdS black hole. Thus, we can write the finite part as

$$V(\gamma)=-R^3L\int_{-y_\infty}^{y_\infty}dy\frac{f(y)}{z(y)}. \tag{29}$$

Next, we need to find $z(y)$ which minimizes the area functional (26) subject to the boundary conditions $z(0)=z_t$ and $z'(0)=0$. We can expand $z(y)=z_0+z_1y^2+z_2y^4$ in series to find its solution up to fourth order in y,

$$\begin{aligned}z(y)=z_t&-\frac{z_t}{2}\frac{\Big(1+\sqrt{1-2\gamma^2}\Big)y^2}{\sqrt{1-2\gamma^2}+1-\gamma^2}\\&-\frac{z_t}{12}\frac{\Big(-9-9\sqrt{1-2\gamma^2}+17\gamma^2+8\gamma^2\sqrt{1-2\gamma^2}\Big)y^4}{\Big(\sqrt{1-2\gamma^2}+1-\gamma^2\Big)^2}\\&+\mathcal{O}(y^6).\end{aligned} \tag{30}$$

Using this solution which is valid near the Cauchy surface $y=0$, we can evaluate the length ℓ. Now we obtain numerically that $z_t\approx \ell^{1/3}+O\left(\gamma^2\right)$. Finally, the holographic complexity is given by the following expression:

$$\begin{aligned}\Delta\mathcal{C}_A=\frac{1}{8\pi RG_{d+1}}&\Big(9.114502677\,z_t{}^{-1}+\frac{16}{9}\frac{y_\infty}{z_t}+\mathcal{O}\left(y_\infty{}^{-1}\right)\\&+\mathcal{O}(\gamma^2)\Big).\end{aligned} \tag{31}$$

It is remarkable to see that (25) and (31) are different even in the first orders. It can be noted that in the leading order of expansion,

$$\Delta\mathcal{C}_A\approx\ell^{-1/3}. \tag{32}$$

Thus, we have obtained an expression for the holographic complexity and fidelity susceptibility for Janus solution. It may be noted that as the holographic complexity is calculated for a subsystem, it depends on the size of the subsystem. However, the expression (25) is independent of the entangled length, as the fidelity susceptibility is calculated for the full system. Furthermore, the fidelity susceptibility for this solution can be used to understand the behavior of fidelity susceptibility for a system described by two different Yang–Mills coupling in each of the two halves of the boundary.

4. Cylindrical symmetry

In order to examine the properties of the holographic complexity and the fidelity susceptibility, it is important to study geometries with different types of symmetries as for example, cylindrical ones. A very interesting cylindrically symmetric solution was presented in [35], where a massless scalar field minimally coupled to gravity with cosmological constant was obtained. This solution can be understood as a generalization of the Buchdahl's solution without cosmological constant and the Levi-Civita-Λ solution without a scalar field. Cosmologically speaking, it was also showed in [35] that this solution can describe a Cyclic universe in a braneworld model. The Einstein–Rosen waves and the self-similarity hypothesis have been studied using cylindrical symmetric solution [36]. It has been observed that such solutions are reduced to part of the Minkowski spacetime with a conically singular axis if the homothetic vector is orthogonal to the cylinders of symmetry. A vortex line solution for Abelian Higgs field has also been analyzed using a cylindrical symmetric solution [37]. In this study, it was demonstrated that the mass density of the string is uniform and dual to the discontinuity of a logarithmic derivative of correlation function of the boundary scalar operator. It would be interesting to analyze the fidelity susceptibility for a cylindrical symmetric solution, as this can be used to understand the behavior of fidelity susceptibility for the Abelian Higgs field. So, now we will calculate the fidelity susceptibility for a cylindrical symmetric solution. We will also calculate the holographic complexity for such a solution.

To calculate the holographic complexity and the fidelity susceptibility for a solution with cylindrical symmetry, we will use a cylindrical analog of the AdS_4 [35]. This solution is obtained from the action of a massless scalar field ϕ in the presence of a cosmological constant, i.e., the action

$$S = -\frac{1}{16\pi G_N}\int \sqrt{-g}d^4x\Big(R - 2\Lambda + \phi_{;\mu}\phi^{;\mu}\Big). \tag{33}$$

In Weyl cylindrical coordinates $x^\mu = (t, r, \varphi, z)$, the field equation given by $R_{\mu\nu} + \Lambda g_{\mu\nu} = \phi_{;\mu}\phi_{;\nu}$ has the following exact solution for the metric and the scalar field:

$$ds^2 = dr^2 + e^{-2\sqrt{\frac{-\Lambda}{3}}r}(\xi^2 e^{-2\sqrt{-3\Lambda}r} + 1)^{2/3}(-dt^2 + d\varphi^2 + dz^2), \tag{34}$$

$$\phi = \pm\frac{2\sqrt{6}}{3}\tan^{-1}(\xi e^{-\sqrt{-3\Lambda}r}). \tag{35}$$

Here, ξ is a scalar field parameter which determines the curvature strength of the scalar field. If this parameter is complex, this solution has a naked singularity whereas if $|\xi| > 1$ it does not have any such singularity. This solution reduces to the cylindrical Levi-Civita-Lambda solution when $\phi = 0$, and it reduces to the Buchdahl solution (the solution of Einstein gravity with massless scalar field) when we set $\Lambda = 0$. Following the proposal of [22], to find the fidelity susceptibility, we need to evaluate the following integral

$$S(\xi) = \frac{R}{4\pi G_N}(2\pi L)\int_{-r_\infty}^{r_\infty} dr(1 + \xi^2 e^{6r/\ell})^{2/3}, \tag{36}$$

where we defined the AdS radius as $\ell^2\Lambda = -3$. The action (36) evaluated at $\xi = 0$ is

$$S(0) = \frac{R}{4\pi G_N}(2\pi L)\int_{-\hat{r}_\infty}^{\hat{r}_\infty} d\hat{r}, \tag{37}$$

where $\hat{r}$ is obtained from the asymptotic form of the metric in the pure AdS case when $\xi = 0$ given by $(r \to \infty)$

$$ds^2_{pure} \sim d\hat{r}^2 + e^{-2\frac{\hat{r}}{\ell}}(-dt^2 + d\varphi^2 + dz^2). \tag{38}$$

Here r_∞ is the one obtained from the asymptotic form $(r \to \infty)$ of the metric in the massive AdS case when $\xi \neq 0$, which reads

$$ds^2_{massive} \sim dr^2 + \xi^{4/3}e^{2r/\ell}(-dt^2 + d\varphi^2 + dz^2). \tag{39}$$

If we match these two metrics, we find that

$$\xi^{2/3}e^{r_\infty \ell} = e^{\mp\hat{r}_\infty \ell}, \tag{40}$$

and then we need to consider two cases depending on the signs in the above equation.

Choosing the minus sign in Eq. (40), up to the second order of ξ, the difference of integrals (36) and (37) gives us

$$S(\xi) - S(0) = \frac{RL}{G_N}\Big(2r_\infty - \ell(1 - \xi^{2/3}) + \mathcal{O}(\xi^2)\Big), \tag{41}$$

and using this expression, we can find the fidelity susceptibility which is given by

$$|\langle\Omega_2|\Omega_1\rangle| \approx e^{S(\xi)-S(0)} \approx 1 - \frac{LR}{G_N}\Big(1 - \xi^{2/3} + \mathcal{O}(\xi^2)\Big). \tag{42}$$

Choosing the plus sign in Eq. (40), following the same procedure as before, we find that the difference of the integrals is

$$S(\xi) - S(0) = \frac{RL}{G_N}\Big(-\frac{2\ell}{3}\ln\xi + \frac{\ell\xi^2}{9}\sinh\big(\frac{6r_\infty}{\ell}\big) + \mathcal{O}(\xi^4)\Big), \tag{43}$$

and then, the fidelity susceptibility becomes

$$|\langle\Omega_2|\Omega_1\rangle| \approx e^{S(\xi)-S(0)} \approx \big(1 + \frac{RL}{G_N}\frac{\ell\xi^2}{18}e^{\frac{6r_\infty}{\ell}}\big)e^{-\frac{2R\ell L}{3G_N}\ln(\xi)}. \tag{44}$$

Now, to compute the holographic complexity for the metric (34), we will suppose that the entangled region is $\tilde{A} = \{r = r(\varphi), 0 < z < L, t = 0, \varphi \in [0, \varphi_\infty]\}$, and so that the area functional is given by the following

$$Area = 2L\int_0^{\varphi_\infty} d\varphi\sqrt{f(f + r'^2)}, \tag{45}$$

where $f = e^{-2\frac{r}{\ell}}(\xi^2 e^{\frac{6r}{\ell}} + 1)^{2/3}$ and prime denotes differentiation with respect to φ. Since the functional is not a function of φ, the following first integral is a conserved quantity,

$$\frac{f^2}{\sqrt{f(f + r'^2)}} = E. \tag{46}$$

If we suppose that $r(0) = r_t$ and $r'(0) = 0$, then $E = f(r_t)$, and hence we obtain

$$r' = \pm\sqrt{f\Big(\frac{f^2}{f(r_t)^2} - 1\Big)}. \tag{47}$$

Therefore, the integral of area is minimized as follows:

$$Area = 2L\int_0^{r_t} dr\frac{f^2}{\sqrt{f(f^2 - f(r_t)^2)}}, \tag{48}$$

where r_t is obtained from

$$\ell = 2\int_0^{r_t} dr\frac{f}{\sqrt{f(f^2 - f(r_t)^2)}}. \tag{49}$$

In order to obtain the Holographic entanglement entropy, we need to solve (49) to find r_t and then replace it in (48). For the holographic complexity, we need to evaluate the following integral

$$V(\gamma) = 2L\int_0^{\varphi_\infty} d\varphi\int_{r_t}^{r(\varphi)} f dr. \tag{50}$$

The minimal surface near the AdS horizon is given approximately by the following series expression:

$$\begin{aligned} r(\varphi) = r_t &- \left(e^{-2\frac{r_t}{\ell}} - \xi^2 e^{4\frac{r_t}{\ell}}\right)\varphi^2\frac{1}{\sqrt[3]{1 + \xi^2 e^{6\frac{r_t}{\ell}}}}\ell^{-1} \\ &- 2/3\left(e^{-2\frac{r_t}{\ell}} - \xi^2 e^{4\frac{r_t}{\ell}}\right) \\ &\times\left(2e^{-2\frac{r_t}{\ell}} + \xi^2 e^{4\frac{r_t}{\ell}} + e^{10\frac{r_t}{\ell}}\xi^4 + 2\xi^6 e^{16\frac{r_t}{\ell}}\right) \\ &\times\varphi^4\left(1 + \xi^2 e^{6\frac{r_t}{\ell}}\right)^{-8/3}\ell^{-3} \\ &+ \mathcal{O}\left(\varphi^6\right). \end{aligned} \tag{51, 52}$$

We can compute ℓ using Eq. (49) and then we can find r_t from it. Finally, we evaluate the volume, and we get the following holographic complexity for entangled cylinder:

$$\Delta\mathcal{C}_A = \frac{-2\,L}{16\pi\,G_{d+1}}\Big[\left(\int\limits_0^{\phi_\infty}\left(-1+e^{2/3\,\frac{\phi^2\left(3\,\ell^2+4\,\phi^2\right)}{\ell^4}}\right)d\phi\right)+\mathcal{O}(\xi^2)\Big]. \quad (53)$$

It may be noted that the holographic complexity for the entangled cylinder again depends on the size of the system. However, the fidelity susceptibility does not have such a dependence, as it is calculated for the full system. It will be interesting to use this result to understand the behavior of the fidelity susceptibility for an Abelian Higgs field, as the Abelian Higgs field is dual to such solutions.

5. Inhomogeneous backgrounds

Another interesting way to study these holographic quantities is for example, to consider metric with different kind of background as those with inhomogeneous [38]. In fact, it has been demonstrated using this solution that the entanglement entropy for a very small subsystem obeys a property which is analogous to the first law of thermodynamics when the system is excited. It has also been demonstrated that the AdS plane waves describe simple backgrounds which are dual to anisotropically excited systems with energy fluxes [39]. An inhomogeneous background has been used to holographically calculate conductivity [40]. It has been demonstrated that the Drude-like peak and a delta function with a negative weight occur for the real part of this conductivity. Thus, it will be interesting to analyze the fidelity susceptibility and holographic complexity for such a solution, and use it to understand the behavior of such systems.

Thus, we will use the inhomogeneous backgrounds, and the metric for such a background can be written as follows [38]

$$ds^2 = \frac{R^2}{z^2}\Big[-f(z)dt^2+g(r,z)dz^2+dr^2+r^2d\Omega_{d-1}\Big], \quad (54)$$

here $g(r,z)=1+m(1+ar+br^2)z^d$ with $m \ll 1$. This geometry is dual to CFT$_d$. The entangled region is a round sphere of radius $r=\ell$. We parametrize the region by $A=\{t=0, z=z(r)\}$ and hence the area functional becomes

$$Area = \Omega_{d-1}R^d\int\left(\frac{r^{d-1}}{z^d}\right)\Big[g(r,z)z'^2+1\Big]^{1/2}dr\,, \quad (55)$$

where primes denote derivatives with respect to r. The Euler–Lagrange equation for the above area functional is given by

$$\begin{aligned}2rzz'' = \big(&-2\,dz(r)-2\,z^d m\,a\,d\,r\,z(r)+2\,z(r)\\&-2\,r^2mz^dbdz(r)-2\,dz^dmz(r)+z^dmarz(r)\big)z'^3\\&+\big(2\,z^dmz(r)\big)z'^3-2\,dz'^2r+\big(2\,z(r)\\&-4\,r^2mz^dbz(r)-2\,z^dm\,a\,r\,z(r)-2\,dz(r)\big)z'\\&-2\,dr+2\,r^2dz^dma+2\,dz^dmr+2\,r^3dz^dmb\,.\end{aligned} \quad (56)$$

Here, in order to obtain a series solution for $z(r)$, we can suppose that $m \ll 1$. In addition, we can suppose the boundary conditions are $z(0)=z_t$ and $z'(0)=0$, giving us

$$z(r) = z_t+\frac{\left(mz_t^n-1\right)r^2}{2z_t}-\frac{mz_t{}^na\left(mz_t{}^n-n-1\right)r^3}{3z_t\,(n+1)}+\mathcal{O}(r^4)\,. \quad (57)$$

The volume functions read as follows

$$V=\Omega_{d-1}R^{d+1}\int\limits_0^{r_t}r^{d-1}dr\int\limits_0^{z(r)}\frac{\sqrt{g(r,z)}}{z^{d+1}}dz. \quad (58)$$

Using the solution (57) and the approximation $m \ll 1$, the finite part of the volume functional which is obtained by subtracting the pure AdS$_{d+2}$ part from the massive one will be

$$V=-\frac{mz_t\Omega_{d-1}R^{d+1}}{3}\int\limits_0^{r_t}\sum c_nr^n(r^2-2z_t^2)^{k(n)-\frac{1}{2}}dr\,. \quad (59)$$

The fidelity susceptibility is proportional to the finite part of the following integral,

$$\begin{aligned}\mathrm{V}_{max} &= \Omega_{d-1}R^{d+1}\int\limits_0^{r_\infty}r^{d-1}dr\int\limits_0^{z_\infty}\frac{\sqrt{g(r,z)}}{z^{d+1}}dz\\&\approx\frac{mb\Omega_{d-1}R^{d+1}}{2(d+2)}r_\infty^{d+2}\ln(z_\infty)\,.\end{aligned} \quad (60)$$

Here z_∞ and r_∞ are UV cutoff values. It is important to mention that these two volumes (59) and (60) are different. Furthermore, the fidelity susceptibility does not scale with the size of the subsystem, as it is calculated for the whole system. It can be used to understand the behavior of fidelity susceptibility a certain CFT$_d$, which is dual to such a solution [38]. We have also calculated the holographic complexity of this solution, and this was done by using the same surface, which would be used to calculate the holographic entanglement entropy of this system.

6. Hyperscaling violating backgrounds

A hyperscaling geometry occurs in theories with an entropy–temperature relationship given by $S \sim T^{d/z}$, where d is the dimension of the space-time and z is known as a dynamical critical exponent. In other words, these theories have free energy scales determined by three dimensions [41–43]. The scaling behaviors of the mutual information during a process of thermalization of such solution has been studied [44]. It was demonstrated in this study that during the thermalization process, the dynamical exponent can be used to obtain the general time scaling behavior of mutual information. Furthermore, it was demonstrated that the scaling violating parameter can be employed to define an effective dimension. The DC and Hall conductivity for strange metal has also been studied holographically using such backgrounds [45]. This is because such solutions can be used to obtain linear-T resistivity and quadratic-T inverse Hall angle. Now we will analyze fidelity susceptibility for a hyperscaling violating background, and it will be important to understand the behavior of fidelity susceptibility for strange metals. We will also use a different volume to also calculate the holographic complexity for such systems.

Thus, we can start from a non-relativistic hyperscaling violating geometry, and this geometry can be described using the following metric [41–43]

$$ds^2=\frac{R^2}{r^2}\Big[-r^{\frac{-2(d-1)(z-1)}{d-1-\theta}}dt^2+r^{\frac{2\theta}{d-1-\theta}}dr^2+dx_i^2\Big]. \quad (61)$$

Here, the index $i=1,2,\ldots,d$ denotes the coordinates for the flat spatial part of the metric and z and θ are the dynamical and hyperscaling violating exponents respectively. Moreover, θ can be interpreted as the dimension of a zero-energy excitations momentum-space surface. Clearly, Lifshitz theories arise when we take $\theta=0$ and $z\neq 1$, whereas CFT are recovered with $\theta=0$ and $z=1$. For this space-time, the entangled region can be parametrized to $A=\{x_1=x_1(r), x_{2,3,\ldots,d}=L\}$, and then the area functional is given by

$$Area=L^{d-1}\int\left(\frac{R}{r}\right)^dr^d\Big[r^{\frac{2\theta}{d-1-\theta}}+(x_1)'^2\Big]^{1/2}dr \quad (62)$$

where primes denote differentiation with respect to r. The functional (62) does not depend on x_1, so that the first integral exists,

$$\frac{x_1'}{\left[r^{\frac{2\theta}{d-1-\theta}}+(x_1)'^2\right]^{1/2}}=\left(\frac{r}{r_*}\right)^d, \tag{63}$$

where $x_1'(r_*)=\infty$. The volume can be obtained by

$$V=L^{d-1}R^{d+1}\int_0^{r_*} r^{\frac{\theta}{d-1-\theta}-(d+1)}x_1(r)dr\,, \tag{64}$$

where r_* can be found from the total length of the entangled region, which is

$$\ell=2r_*^{\frac{d-1}{d-1-\theta}}\int_0^1\frac{\xi^{d+\frac{\theta}{d-1-\theta}}}{\sqrt{1-\xi^{2d}}}d\xi\,,\quad \xi=\frac{r}{r_*}\,. \tag{65}$$

From (63), we can find the following solution

$$x_1(r)=\frac{2r_*^{\frac{d-1}{d-1-\theta}}}{(d-1-\theta)\left(d\left(d-1\right)-\theta\left(d-1\right)+d-1-\theta\right)}\times\left(\frac{r}{r_*}\right)^{2dB}{}_2\mathrm{F}_1\Big(A,B;C;D\Big), \tag{66}$$

where

$$A=\frac{1}{2},$$
$$B=\frac{1}{2}\,\frac{d\left(d-1\right)-\theta\left(d-1\right)+d-1-\theta}{\left(d-1-\theta\right)d},$$
$$C=\frac{1}{2}\,\frac{2d^2-d-2d\theta+d\left(d-1\right)-\theta\left(d-1\right)-1-\theta}{\left(d-1-\theta\right)d},$$
$$D=-\left(\frac{r}{r_*}\right)^{2d}. \tag{67}$$

where ${}_2F_1(A,B;C;D)$ denotes the first hypergeometric function. By replacing (66) in (64), we find that the volume is

$$V=L^{d-1}R^{d+1}r_*^{\frac{\theta+d-1}{d-1-\theta}-(d+1)}\int_0^1\xi^{\frac{\theta}{d-1-\theta}-(d+1)}x_1(\xi)d\xi\,, \tag{68}$$

which can be simplified to the form

$$V=L^{d-1}R^{d+1}r_*^{\frac{\theta+d-1}{d-1-\theta}-(d+1)}N(d,\theta)\,. \tag{69}$$

This quantity is a number since $\ell\sim r_*^{\frac{d-1}{d-1-\theta}}$. Thus, holographic complexity becomes

$$\mathcal{C}=\frac{L^{d-1}R^d\ell^{(\theta-(d+1)(d-1-\theta))(d-1)}N(d,\theta)}{8\pi G}\,. \tag{70}$$

Finally, the fidelity susceptibility is proportional to the finite part of the following integral

$$\mathrm{V}_{max}=R^{d+1}L^d\int_\epsilon^{r_\infty}r^{\frac{\theta}{d-1-\theta}-(d+1)}dr\approx\Big(\frac{R^{d+1}L^d}{\frac{\theta}{d-1-\theta}-d}\Big)r_\infty^{\frac{\theta}{d-1-\theta}-d}. \tag{71}$$

Here, we can see that $\theta\neq\frac{d(d-1)}{d+1}$ and then the above volume expression is totally different than Eq. (69). It may be noted that no trace of Lifshitz exponent z appears in the volumes. Thus, both the fidelity susceptibility and holographic complexity will not depend on the Lifshitz exponent z. Furthermore, the holographic complexity also depends on the size of the subsystem, even for these backgrounds. However, no such dependence is observed in the fidelity susceptibility, as it is calculated for the full system.

7. Conclusion

The laws of physics can be represented in terms of the ability of an observer to process relevant information. The information theory deals with the ability of an observer to process information. It is important to know how much information is lost during such a process, and how difficult it is for an observer to process the relevant information during such a process. Just as the entropy quantifies the abstract idea of the loss of information, complexity quantifies the idea of the difficulty to process that information. It is possible to use the AdS/CFT correspondence to calculate the entanglement entropy of a field theory holographically from the bulk geometry dual to such a field theory. This is done by calculating the area in the bulk, as the area in the bulk geometry is dual to the holographic entanglement entropy of the boundary theory. Recently, it has been proposed that it is also possible to calculate the complexity of a system holographic, as it is dual to a volume in the bulk. As there are many ways to define a volume in the bulk, many different proposals for the complexity have been proposed. If the maximum volume is used, then we obtain the fidelity susceptibility $\Delta\chi_F$. However, if we use the same surface used to calculate the entanglement entropy, then we obtain a new quantity which is called the holographic complexity $\Delta\mathcal{C}$. In this paper, we calculate both these quantities for a variety of deformed AdS solutions.

We calculate it for an AdS black hole, Janus solution, a solution with cylindrical symmetry, inhomogeneous backgrounds and hyperscaling violating backgrounds. It was observed that most of these geometries are dual to interesting field theories. Thus, it was important to calculate and analyze the behavior of holographic complexity and fidelity susceptibility for such bulk geometries, as these results can be used to understand the behavior of the boundary field theory. Furthermore, these geometries where interesting deformations of AdS spacetime, and certain universal features were observed to occur in all these different geometries. It was observed that as the holographic complexity depended on the size of the subsystem, and the fidelity susceptibility did not depend on any such size. These observations did not depend on the kind of deformation of the AdS spacetime, and thus seem to be a universal feature of all such deformations. It is also expected to occur as the holographic complexity was calculated for a subsystem, so it depended on the size of the subsystem. However, as the fidelity susceptibility was calculated for the full system, it did not depend on the size of the subsystem.

It may be noted that these deformed AdS backgrounds are dual to interesting field theories, and many of these field theories have important condensed matter applications. Thus, we can use the results of this paper, to obtain the fidelity susceptibility for those field theories. In fact, many of those theories can be represented by a many-body system. The quantum mechanical Hamiltonian for such a system, can be written as $H(\lambda)=H_0+\lambda H_I$, where λ is an external excitation parameter [23–25]. It is possible to diagonalize this Hamiltonian by an appropriate set of orthonormal eigenstates $|n\rangle$ and eigenvalues $E_m(\lambda)$, $H(\lambda)\,|n(\lambda)\rangle=E_n(\lambda)\,|n(\lambda)\rangle$. Furthermore, for any two states λ and $\lambda'=\lambda+\delta\lambda$ (which are close to each other), it is also possible to define $F(\lambda,\lambda+\delta\lambda)=1-\frac{\delta\lambda^2}{2}\chi_F(\lambda)+\mathcal{O}(\delta\lambda^4)$. Now the fidelity susceptibility of this system is denoted by $\chi_F(\lambda)$ [23–25]. It is possible to estimate this quantity $\chi_F(\lambda)$ holographically as $\chi_F(\lambda)=$ *complexity* when $V=V_{max}$ [22].

We have calculated fidelity susceptibility for various bulk solutions, and these bulk solutions are dual to interesting boundary theories. Hence, the results of the paper can be used to understand the behavior of the fidelity susceptibility for those boundary theories, which are dual to the bulk solution analyzed in this paper. We would also like to comment, that at present it is not clear what quantity does holographic complexity represent in the boundary theory. It may be a new quantity, which might be closely related to the holographic entanglement entropy, as it is calculated using the same surface which is used to calculate the holographic entanglement entropy. It will be interesting to analyze the relation between the holographic complexity and holographic entanglement entropy, to understand the implications of the holographic complexity for the boundary theory. We have analyzed both holographic complexity and fidelity susceptibility for various solutions in this paper, and it would also be interesting to understand the relation between the holographic complexity and fidelity susceptibility. The latter might also lead to some understanding of the use of holographic complexity for the boundary theory. However, as both these proposals have only been recently made, it was important to apply them to various different deformed AdS solutions, and this is what we have done in this paper.

A connection has been established between the holographic entanglement entropy and the quantum phase transition in a lattice-deformed Einstein–Maxwell-Dilaton theory [46]. In fact, in this study backgrounds exhibiting metal–insulator transitions have been constructed. Furthermore, it has been demonstrated that for these backgrounds both metallic phase and insulating phase have vanishing entropy density, in zero temperature limit. It would be interesting to analyze holographic complexity and fidelity susceptibility for such backgrounds, and thus use them to study the behavior of metal–insulator transition. The holographic phase transition with dark matter sector in the AdS black hole background has also been studied [47]. It was observed that the properties of different phases of this system can be obtained from the holographic entanglement entropy for this system. It would be interesting to analyze the holographic complexity and fidelity susceptibility for such a system.

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