Calibrating a composite material model for analysis and design of bamboo structures

Godina, Martha [1]; Lorenzo, Rodolfo [2]
[1] martha.godina.12@ucl.ac.uk
[2] r.lorenzo@ucl.ac.uk

Department of Civil, Environmental & Geomatic Engineering, UCL, United Kingdom

Abstract

This paper proposes a methodology to develop a material model for bamboo culms to use it in a more rigorous structural analysis and design. The study presented here is part of a broader research with the aim of exploiting the mechanical properties of bamboo in lightweight structures that may transfer predominantly axial compressive forces. The methodology is based on theoretical analysis and experimental tests. Composite material theory has been adopted to describe the mathematical model that can realistically reproduce the behaviour of bamboo culms. The composite material model is linear elastic and describes the axial and flexural stiffness, and the stress distribution across the culm wall thickness. For this study a series of experimental tests of the bamboo species Moso (*Phyllostachys Pubescens*) were devised to obtain the Modulus of Elasticity (E) under axial compressive loads. Establishing suitable test methods to determine material properties is not an easy task due to the difficulty of working with a non-isotropic and variable material. Experimental tests were based on two different codified methods (JG/T 199-2007; ISO 22157-2004) with the aim of reviewing the differences in the results of small coupons and full culm specimens, as well as emphasising the issues related to the measurement of strains in a material with through-thickness gradient fibre distribution under axial compression. In order to model the variability across the culm wall, the volume fraction of the fibres was calculated by image analysis. In addition, assessment of through-thickness strain distributions of small coupons using digital image correlation (DIC) was carried out and is discussed in this paper. The validation process for the composite material model is ongoing.

Introduction

Bamboo is a fast growing and eco-friendly grass with plenty of potential to be exploited in building construction. It is one of the oldest fibrous construction materials, and has been used for many centuries as a structural element for shelters. As a structural material, bamboo has many advantages. It is a lightweight material due to its hollow shape and its fibre and matrix composition. Mechanical properties of bamboo culms in the longitudinal axis are remarkably good, its stiffness to weight ratio is better than that of steel. Moreover, it is a renewable resource that has a relatively low environmental impact in comparison with other conventional materials such as steel, concrete and timber. However, its full potential as a structural material has not been exploited, because there is very little scientific knowledge. Most of the structures built with bamboo are based on traditional structural systems such as trusses and frames that can perform very efficient in steel; however, in bamboo the combination of axial and bending stresses can trigger longitudinal splitting due to the development of circumferential stresses. Issues such as durability, splitting and connection design have not been addressed properly in bamboo design codes, and those factors can contribute to the marginal perception of bamboo as a structural material.

Design codes and testing methods for the determination of material properties are crucial. With the purpose to encourage the usage of bamboo as a structural material this study aims to devise a material model that may provide a better tool to design and analyse structures where bamboo is used as the main load-bearing material. Furthermore, experimental tests are carried out to potentially determine suitable test methods and measuring techniques to adopt in the methodology.

To model the behaviour of bamboo culms, composite material theory was adopted since bamboo can be considered as a natural composite. Composite materials are created by two or more constituents bonded together, where the synergy between them produced a superior material. The structure of a culm wall in cross-section is shown in Figure 1. The composition of the culm ground tissue consists of vascular bundles (fibres and conducting tissue) embedded in parenchyma cells (matrix). On average a culm wall consists of 52% parenchyma, 40% fibres and 8% conducting tissue (Liese 1998). In the case of high performance composites as bamboo, the unidirectional fibres provide the strength and stiffness to support loads, whereas the matrix provides the bonding within the fibres and distribute the loads between them (Daniel and Ishai 2006).

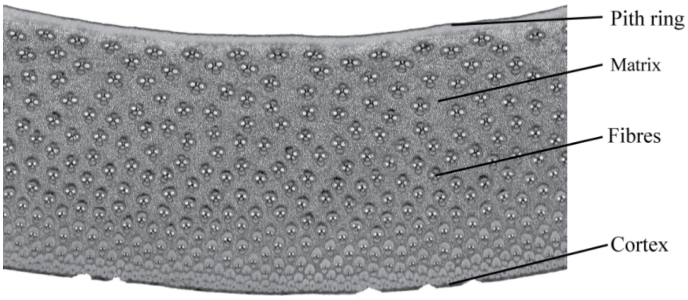


Figure 1

Methodology

The scope of this first part of the study includes the proposed composite material model and the methodology to start the validation process of the model. Experimental tests based on axial compressive loads have been carried out to assess the process to determine the axial stiffness (EA). The tests include qualitative analysis to study the influences of fibre distribution and results of different measuring techniques that can inform future design of tests methods. The methodology is illustrated in Figure 2. Compressive tests of small coupons subject to load in the linear elastic range were carried out together with analysis of volume fraction and fibre distribution of the culm wall to inform the composite material model. Relationship between volume fraction and mechanical properties such as Modulus of Elasticity was identified previously by some authors (Amada et al., 1995; Nogata et al, 1995). For this study, it was assumed that the Modulus of Elasticity (E) measured from the tests was proportional to the volume fraction of the section of the samples. The variability of stiffness across the culm wall thickness was determined based on the through-thickness gradient fibre distribution. Furthermore, experimental tests on full culm specimens under compressive load were performed in order to compare the results.

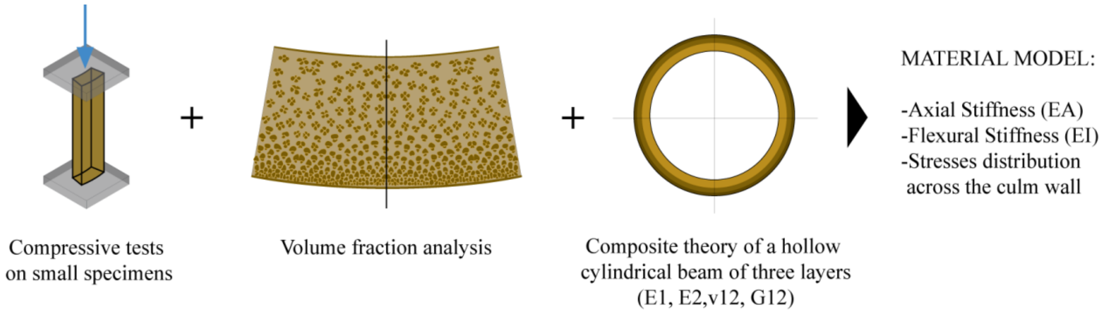


Figure 2

Theme: Architecture, Engineering and Social Housing

Methods

Experimental tests to analyse strain distributions in specimens

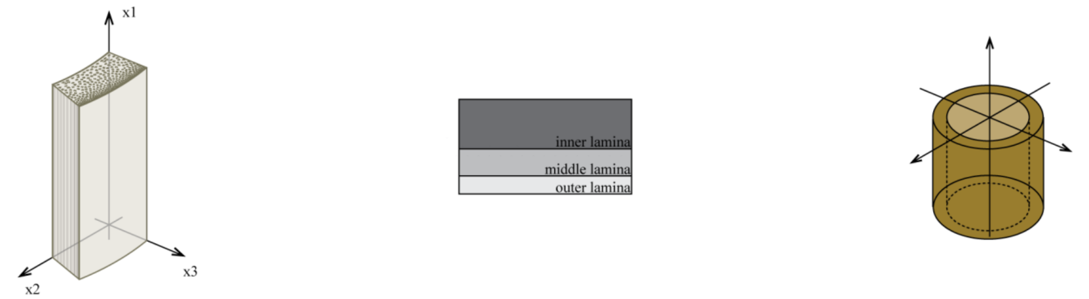
Experimental tests to understand strain distributions were carried out before analysing all the set of specimens. Comparison between two different measuring techniques was performed in small coupons. Strain gauges were attached to the inner and outer surface of one small coupon and another as prepared with a speckle pattern in the side wall in order to analyse strain distributions through-thickness by Digital Image Correlation (DIC). The images were taken with two pair of Kodak Megaplus ES 1.0 cameras with Fujinon-TV lenses, triggered to capture one image per second, and were processed in VIC-3D software. A full culm specimen was prepared with two pairs of strain gauges oriented in the longitudinal and transverse directions.

Experimental tests to determine compressive behaviour

The series of tests were devised along the length of a bamboo culm of the species Moso (*Phyllostachys Pubescens*) from bottom to top to record variability of stiffness and volume fraction. The specimens were taken from different positions along the culm, 16 samples for the small coupons and 12 for the full culm specimens were prepare and tested according to JG/T 199 (2007) and ISO 22157 (2004) respectively. Testing procedures for the small coupons specify the application of load only in the linear elastic area between 5 and 20 N/mm^2, and full culm test method loads the material until failure, where the respective Modulus of Elasticity is to be analysed between 20 and 80 percent of the ultimate load, reaching stresses up to 72 N/mm^2.

Composite material model

The mechanical properties of composite materials depend on the properties of each of its constituents, the arrangement of them, and the volume fraction of the material that provides the reinforcement. All composite materials are non-homogeneous, and depending on the properties can have different scale of anisotropy. In this study, bamboo is considered as a transversely isotropic material that is an orthotropic material where the plane transverse to the direction of the fibres behaves as an isotropic material (Daniel and Ishai, 2006). Composite materials can be analysed at different scales. Micromechanic analysis comprehends the study of each of the constituents (fibres and matrix) of the material by separate, and macromechanics consider the behaviour of the lamina or laminate as a whole. Laminas are materials with a particular arrangement of constituents. Laminates are several laminas bond together, which can be organised with different orientations and made up by different types of laminas. In macromechanical analysis, the material is assumed to be homogeneous. (Daniel and Ishai, 2006).



a)Transversely isotropic lamina b)Laminate c)Hollow cylindrical beam

Figure 3

The assumptions considered for the model were based on macromechanical analysis. The model represents the axial and flexural stiffness, as well as the stress distribution across the culm wall.

Figure 3 shows an illustrative representation of the assumptions considered that can potentially best represents and predict the behaviour of whole bamboo culms in the model. They are described as follows: a) bamboo is idealised as a transversely isotropic lamina under plane stress condition with specific material properties; b) three laminas with the same geometrical characteristics but with different mechanical properties are bonded together in order to create a laminate that represents the variation across the culm wall thickness due to fibre distribution; c) finally, to consider the cross-section geometry, the model takes the laminate properties into a hollow cylindrical beam.

In composite materials, the type of arrangement determines the type of material and therefore the respective stress-strain relationships that characterise them. The composite material model was based in Kollar & Springer (2003) concepts, developments and notations, and all the following equations are based on this.

The respective independent engineering constants are:

E_1	Modulus of Elasticity in the longitudinal direction
E_2	Modulus of Elasticity in the transverse direction
v_{12}	Poisson's ratio in the plane x1-x2
G_{12}	Shear modulus in the plane x1-x2

And the stiffness of a lamina in terms of engineering constants is:

$$[Q] = \begin{bmatrix} \frac{E_1}{D} & \frac{v_{12}E_2}{D} & 0 \\ \frac{v_{12}E_2}{D} & \frac{E_2}{D} & 0 \\ 0 & 0 & G_{12} \end{bmatrix}, D = 1 - \frac{E_2}{E_1} v_{12}^2 \tag{1}$$

Furthermore, for an orthotropic and unsymmetrical laminate, the stiffness matrix of the laminate $[A]$, $[B]$, and $[D]$and can be obtain by the following summations, in respect to the mid-plane (Figure 4) as reference plane:

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{K} \left(\bar{Q}_{ij}\right)_k (z_k - z_{k-1}) \\ B_{ij} &= \frac{1}{2} \sum_{k=1}^{K} \left(\bar{Q}_{ij}\right)_k \left(z_k^2 - z_{k-1}^2\right) \\ D_{ij} &= \frac{1}{3} \sum_{k=1}^{K} \left(\bar{Q}_{ij}\right)_k \left(z_k^3 - z_{k-1}^3\right) \end{aligned} \tag{2}$$

where k are the three laminas, z_k, z_{k-1} are the distances from the mid-plane to the surface of each of the laminas, and $\left(\bar{Q}_{ij}\right)_k$ are the stiffness matrix of each lamina.

The respective stiffness and compliance matrix for unsymmetrical orthotropic laminate are:

$$\begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix}, \begin{bmatrix} \alpha_{11} & \alpha_{12} & 0 & \beta_{11} & \beta_{12} & 0 \\ \alpha_{12} & \alpha_{22} & 0 & \beta_{12} & \beta_{22} & 0 \\ 0 & 0 & \alpha_{66} & 0 & 0 & \beta_{66} \\ \beta_{11} & \beta_{12} & 0 & \delta_{11} & \delta_{12} & 0 \\ \beta_{12} & \beta_{22} & 0 & \delta_{12} & \delta_{22} & 0 \\ 0 & 0 & \beta_{66} & 0 & 0 & \delta_{66} \end{bmatrix} \tag{3}$$

Ultimately to take into account the geometrical properties of the cross section, the model represents the material as a beam element in order to obtain the axial stiffness, and the laminate is evaluated at the neutral plane $\hat{\varrho}$ of the culm wall thickness (Figure 4).

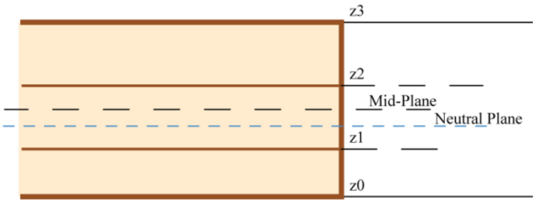


Figure 4

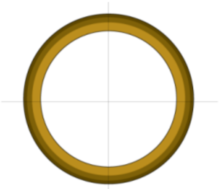


Figure 5

The material is modelled as a thin walled hollow cylindrical beam with an orthotropic and unsymmetrical layup as represented in Figure 5, with the axial and bending stiffnesses obtained as follows:

$$\widehat{EA} = 2R\pi \frac{1}{\hat{\alpha}_{11}} \tag{4}$$

$$\widehat{EI}_{yy} = \widehat{EI}_{zz} = \pi \left(R^3 \frac{1}{\alpha_{11}} + R \frac{1}{\delta_{11}} \right) \tag{5}$$

where the elements $\hat{\alpha}_{11}$, α_{11} and δ_{11} are elements from the compliance matrix and are analysed at the neutral plane $\hat{\varrho}$.

Volume Fraction

Volume fraction and fibre distribution across the culm wall were analysed for all the small coupons by image analysis in a script developed in Matlab. Volume fraction was calculated dividing the area of the fibres by the area of the whole section, and is expressed as a percentage. Figure 6 shows the image to analyse and the processed binary image. To determine the fibre distribution, the wall thickness was divided into thirty sections and for each section the volume fraction was calculated and plotted against the wall thickness position (Figure 6). The thickness of each of the three laminas and the respective volume fraction was obtained by dividing the difference between the lowest and highest value of the inside and outside layers by three, in order to obtain a relative average volume fraction for each of the three laminas (laminas division are represented as an orange dot in Figure 6). Finally, an exponential curve was fitted to the set of data, and the respective function was used to determine the thickness and volume fraction of each of the three laminas.

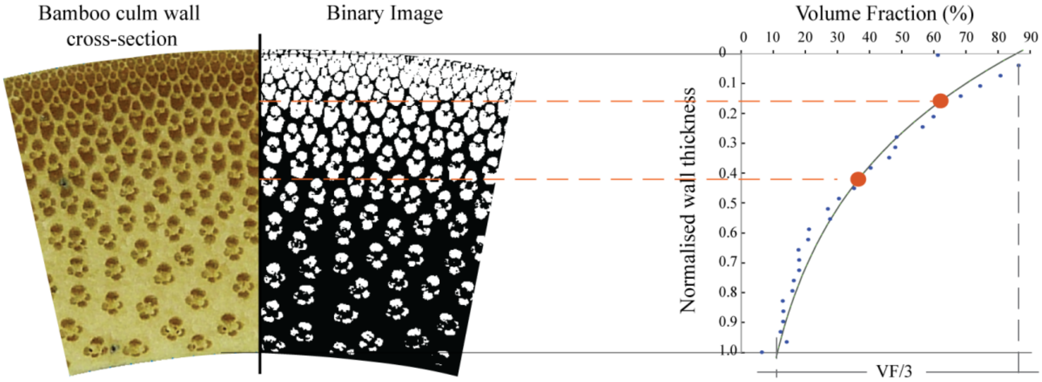


Figure 6

Results and discussion

Analysis of strains distributions

Results from the specimen prepared with strain gauges are presented in Figure 7. The stress-strain curve shows that the stiffness is higher in the inside surface. However, due to the composition of bamboo, we know that the outer surface is stiffer than the inside and therefore it is rare that the measurements recorded lower strains in the inside surface.

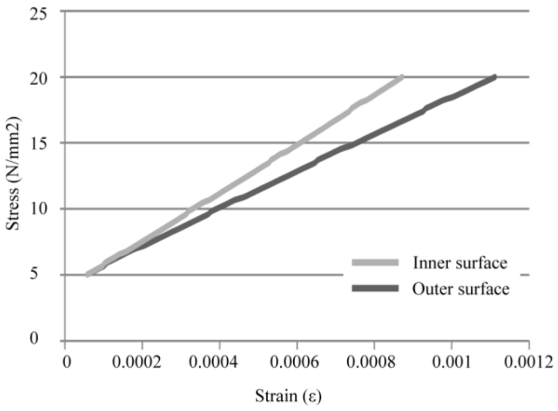


Figure 7

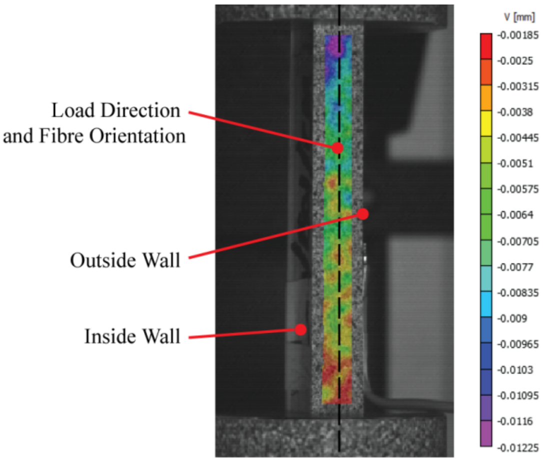


Figure 8

Some natural materials such as plant stems and quills have the structure of a cylindrical tube shell with a foam core, where this acts as a bending resistance mechanism under axial loads and prevent the ovalization of the geometry cross-section under bending moments. (L.J. Gibson et al. 1995). While the small coupons are not cylindrical tubes, the configuration of the material is similar to them. Results from DIC (Figure 8), suggests that bamboo behaves in a similar manner, as the analysed strains at the top of the specimens are higher and concentrated in the inner surface suggesting that because the material is heterogeneous the strains changes from point to point. In wood, the compressive deformation mechanism in the axial direction of low and high density woods differs. Low density wood cells collapse cell by cell until the cell ends fails and intercept the next one, while in high density woods the mechanism is local buckling (Gibson and Ashby 1997). This can explain the strains results from the inner and outer surface of the bamboo that leads to rare results. If we analyse the stiffness of the material by measuring only a small amount of area in the middle of the specimen, we may not quantified the densification of the top inner part of the specimen that is in direct contact with the steel plates.

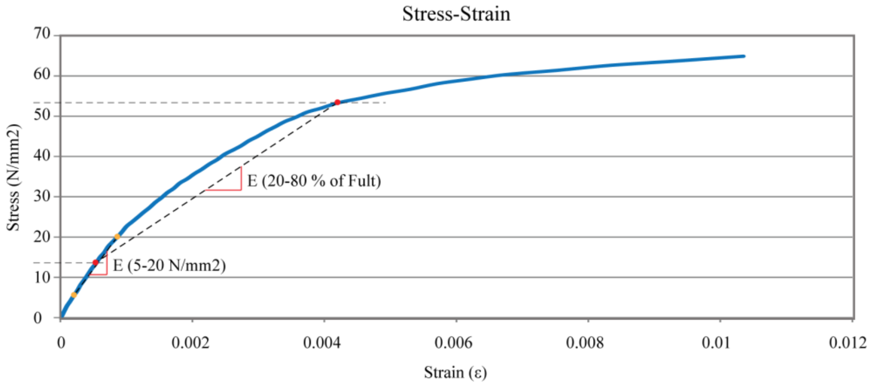


Figure 9

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The stress-strain plot of the full culm test (Figure 9) shows the non-linear behaviour of the bamboo culm. The areas for analysing the Modulus of Elasticity according to both codes is displayed, where small coupons are analysed between 5 and 20 N/mm^2 according to JG/T 199 (2007) and full culm specimens between 20 and 80 percent of the ultimate load according to ISO 22157 (2004). Due to the low range of stress applied to the small coupons, non-linear behaviour is not observed in Figure 7. Testing of small coupons at a lower stress can be useful for analysis under service loads. However if we aim to utilise the properties of bamboo to their full extent for limit state design, we need to develop new test methods to push the material further than the initial linear range.

Results of experimental tests of small and full culm specimens

Comparison of Modulus of Elasticity (E) between small and full culm specimens is shown in Figure 10 and Table 1. In order to compare results, full culm specimens were analysed and plotted in the same range of stress than the small coupons which corresponds only to the linear behaviour of the material. Strains from small coupons were obtained from the strain gauges attached in the outside wall, as the full culm specimens were provided with strain gauges in the same manner. Is important to mention that Modulus of Elasticity of full culms analysed according to ISO 22157 (2004), resulted in lower values that range between 11.0 and 16.0 kN/mm^2. This is because the stresses are beyond the linear behaviour of the material (Figure 9). As mentioned in the previous section, DIC analysis shows the nonlinear distribution of strains due to the fibre distribution across the culm wall. This is only possible to measure in a small coupon where the side culm wall is exposed. It should be noticed that the results corresponds to the strains measured in the outside surface of the samples, and variations through-thickness are expected.

The moisture content of all the specimens was 8.75±0.40. Modulus of Elasticity calculated at 12% moisture content for small coupons (JG/T 199-2007) was only 4% lower. Difference among results that arise from having a small coupon could be due to free side edges, curvature of the specimen, and end effects such as friction. However, in general results showed a similar pattern in both cases where the elastic properties tends to increase towards the top of the culm.

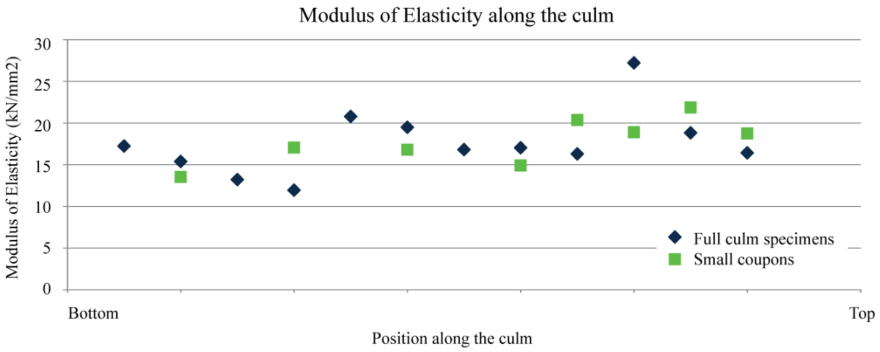

Figure 10

Modulus of Elasticity along the length of the culm (kN/mm^2)

	Bottom										Top
Full Culm Specimens	16.90	15.06	12.88	11.62	20.45	19.17	16.48	16.70	15.97	26.89	18.50	16.09
Small Coupons	-	13.17	-	16.59	-	16.43	-	14.49	19.78	18.14	21.26	18.44

Table 1

Volume fraction of each of the small coupons was calculated. An increase of volume fraction from bottom to top along the length of the culm is displayed in Figure 11, which is consistent with the increase of elastic modulus. Relationship between volume fraction and Modulus of Elasticity is shown in Figure 12. This relationship is important due to the implications that it could have in obtaining the mechanical properties of the whole culm based on image analysis of the culm cross-section. Although, this would require a sufficiently amount of tests and statistical processes to derive the relationship of one species, the aim of this methodology is to obtain the relationship based on small coupons from the bottom and top of the bamboo culms that are going to be used in a structure.

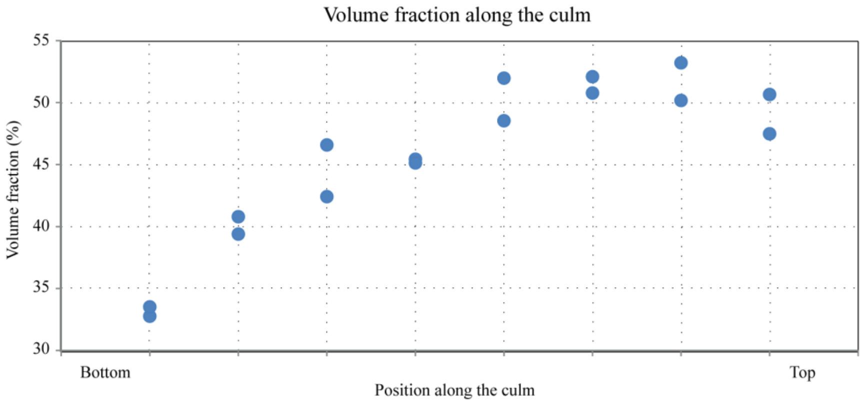

Figure 11

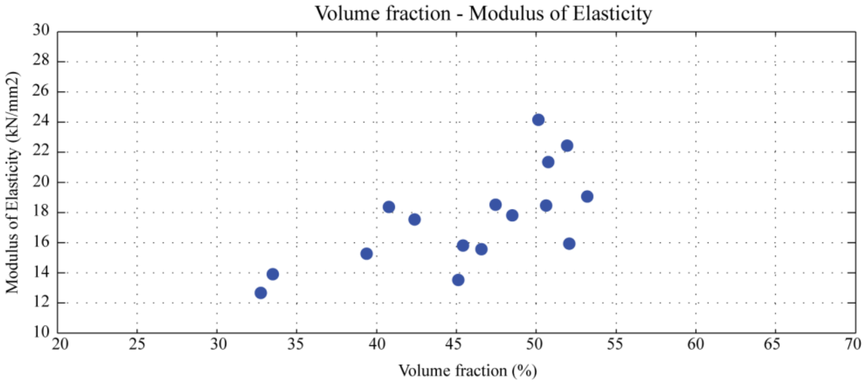

Figure 12

Conclusions

The aim of this series of tests is to enable the calibration of the proposed composite material model. The validation process of the model is ongoing. The proposed model is intended to contribute to the achievement of a better understanding of the stress distribution across the culm, as well as to provide a tool to use in structural analysis and design. The study also covers the analysis of through-thickness strain distributions that can subsequently be used to inform the design of connections. When analysing variable natural materials such as bamboo, an understanding of strain distributions and therefore strain concentrations is crucial. The influence of the nonlinear strains in the quantitative and qualitative results should be considered in test methods, together with appropriate measuring techniques to guarantee accurate results. Inherent issues related to the measuring of strains in a bamboo coupon have been discussed. This study is part of a broader research, which aims to develop a design methodology for lightweight bamboo structures, so that more efficient structural systems using whole bamboo culms can be designed. It comprises the study of the material behaviour through the analysis of different testing methods and measuring techniques to obtain physical and mechanical material properties.

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List of figures

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