A NOVEL CONTROL STRUCTURE FOR DYNAMIC INVERSION AND TRACKING TASKS

N. Bajcinca and T. Bünte

Institute of Robotics and Mechatronics
German Aerospace Center
Oberpfaffenhofen, Germany

Abstract: A new structure for model inversion and tracking control tasks is introduced. It represents a two-degree of freedom controller, which unifies the principle of feedforward exact and high-gain feedback inversion, while preserving advantages of each.

Keywords: Model inversion, tracking control, inverse disturbance observer

1. INTRODUCTION

Dynamic model inversion and tracking problems are equivalent in terms of computation of the input needed to make a plant follow a given output trajectory. The set of assumptions which must hold for the provision of exact inversion/tracking in the control sense are: exact plant modelling, input-output uniqueness, asymptotic stability and minimum-phase behavior. In practice, however different conditions like modelling errors, parameter uncertainties, exogenous disturbances, hard nonlinearities, non-minimum phase behavior, etc. make just an approximate model inversion/tracking possible. This paper introduces a new control structure, which is therefore appropriate. The structure is designated as *inverse disturbance observer* (IDOB). It represents a modification of the *disturbance observer* (DOB) structure, (Umeno and Hori, 1991). The designation 'inverse' refers to its model inversion task.

Basically, the IDOB controller is a two-degree of freedom inversion structure, which unifies the high-gain feedback and feedforward exact model inversion principles. The respective design parameters are a Q-filter in the feedback path, and an approximate inverse $\tilde{G}^{-1}$ for the plant G in the feedforward path, Section 2. While the IDOB controller inherits the basic structural properties of the standard DOB, s.a. stability conditions and sensitivity functions, the two structures implement essentially different tasks. Namely, rather than for tracking, DOB is primarily used for model regulation task targeting closed-loop input-output dynamics to some nominal one G_n.

The basic convenience with the IDOB structure is the design simplicity of its parameters $\tilde{G}^{-1}$ and Q. For instance, $\tilde{G}^{-1}$ is designed as the approximate inverse of the plant G, i.e. $G\tilde{G}^{-1} \approx 1$ within the operational frequency range. The mathematical paradigm for the design of the IDOB controller is based on modelling of non-exact (imperfect) inversion. Therefore we introduce the notion of *imperfection* in Section 3. For instance, $G = \tilde{G}(1 + W_2\Delta)$ describes the multiplicative imperfect inversion model, where W_2 weights the imperfection.

Q can be designed by using analogous techniques to those for the one-degree of freedom feedback structures, (Doyle *et al.*, 1992). In Section 3 it will be shown that for a multiplicative imperfection weighted by W_2 and a performance specification weighted by W_1 the condition for simultaneous stability and performance reads $\||W_1(1-Q)| + |W_2Q|\|_\infty < 1$. This equation represents the basic equation of IDOB and it is primarily used to shape Q. E.g. by loopshaping techniques (not presented here) based on this equation, Q can be shown to be a unity gain low-pass filter. The first summand in the latter condition $|W_1(1-Q)|$ refers to performance specifications and it requires a high bandwidth for Q. This is, however, bounded by

the second stability term $|W_2 Q|$, which typically increases for higher frequencies. Hence, the bandwidth of Q is determined by compromising between stability and performance, Section 3.
The paper is concluded by some important design remarks referring to robust inversion, realization, different IDOB structures, systems with sinusoidal inputs, and multivariable systems in Section 4.

2. INVERSE DISTURBANCE OBSERVER

2.1 Basic idea

The new control structure in its basic form is shown in Fig 1. Identify here the plant G, and the two design parameters (two degrees of freedom): $\tilde{G}^{-1}$, the approximate inverse of G, and the $Q-$filter. In addition to the standard signal designation (r reference, u control effort, y measurement, $e = r - y$ error, d output disturbance and n noise), two additional signals $\tilde{y}$ and ε are of interest for the structure in Fig 1. Note that ε can be interpreted as the 'error integral' (see below).

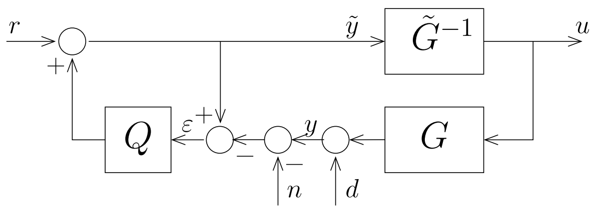

Fig. 1. IDOB control structure

The transfer functions from the reference r to the plant input u (G_{ru}) and its output y (G_{ry}) are easily obtained from Fig 1

$$G_{ru} = \frac{\tilde{G}^{-1}}{1 - Q(1 - G\tilde{G}^{-1})} \tag{1}$$

$$G_{ry} = \frac{G\tilde{G}^{-1}}{1 - Q(1 - G\tilde{G}^{-1})}. \tag{2}$$

To present the basic idea of the IDOB control structure introduce the notation:

(A):	$Q = 1$	(infinity-gain feedback)
(B):	$\tilde{G}^{-1} = G^{-1}$	(feedforward exact inversion)
(C):	$G_{ru} = G^{-1}$	(perfect inversion)
(D):	$G_{ry} = 1$	(perfect tracking).

Then (1) and (2) read directly

$$(A) \vee (B) \Rightarrow (C) \vee (D). \tag{3}$$

Note that the opposite implication does not necessarily apply.
In practice, none of the conditions (A) and (B) is realizable for all frequencies. In fact, (A) would destabilize any practical loop, so Q must ultimately roll-off for high-frequencies. Further, the feedforward exact inversion as defined in (B) fails at least for high-frequencies. Hence, the conditions (A) and (B) make sense only in the operational frequency bandwidth. Even then the two conditions must be weakened due to implementation limitations to

(A'):	$Q \approx 1$	(high-gain feedback)
(B'):	$\tilde{G}^{-1} \approx G^{-1}$	(feedforward inversion).

However it is important that though weakened the conditions (A') and (B') collaborate, that is, they contribute independently towards situations (C) and (D). For $Q \approx 1$ due to the positive feedback in the Q-loop in Fig 1 a high-gain controller results. If $\tilde{G}^{-1} \approx G^{-1}$, then exact feedforward inversion is approximately realized so the $Q-$feedback loop is almost idle. Therefore, IDOB control structurally unifies the high-gain feedback and the feedforward exact inversion principle, (3).

2.2 Stability

For internal stability the transfer function matrix from the input vector $(r,\ d,\ n)^T$ to the output vector including internal signals $(\varepsilon,\ \tilde{y},\ y)^T$ is examined, see Fig 1. The reader may easily check that

$$\begin{pmatrix} \varepsilon \\ \tilde{y} \\ y \end{pmatrix} = \frac{\begin{bmatrix} 1 - G\tilde{G}^{-1} & -1 & 1 \\ 1 & -Q & Q \\ G\tilde{G}^{-1} & 1 - Q & QG\tilde{G}^{-1} \end{bmatrix}}{1 - Q(1 - G\tilde{G}^{-1})} \begin{pmatrix} r \\ d \\ n \end{pmatrix}. \tag{4}$$

The above system is well-posed since the determinant of the transfer function matrix

$$\det = 1 - Q(1 - G\tilde{G}^{-1}) \tag{5}$$

does not identically vanish. Note that (A') implies that $\det \approx G\tilde{G}^{-1}$, and (B') $\det \approx 1$. Beyond the operational frequencies $\det \approx 1$.
Essential implications for the IDOB structure are provided by the sensitivity $S = y/d$ and complementary sensitivity $T = y/n$ functions

$$S = \frac{1 - Q}{1 - Q(1 - G\tilde{G}^{-1})} \tag{6}$$

$$T = \frac{QG\tilde{G}^{-1}}{1 - Q(1 - G\tilde{G}^{-1})}. \tag{7}$$

As usual, the two are constrained by the fundamental algebraic condition $S + T = 1$. Also $S = 1/(1 + L)$ and $T = L/(1 + L)$, where L stands for the loop transfer function

$$L = \frac{QG\tilde{G}^{-1}}{1 - Q}. \tag{8}$$

It is convenient to define also the three latter functions for the situation that corresponds to feedforward perfect inversion (condition (B))

$$\tilde{S} = 1 - Q,\ \ \tilde{T} = Q,\ \text{ and }\ \tilde{L} = \frac{Q}{1 - Q}. \tag{9}$$

One important implication of sensitivity function is related to the error $e = r - y$ dynamics, refer to equation (15). For typical applications requiring zero DC error tracking, $S(0) = 0$ must hold, implying $Q(0) = 1$. The latter condition produces always a pole at $s = 0$ in the expression [1] $1/(1 - Q)$. Hence according to (8) the pole $s = 0$, as expected, appears also in the loop transfer function L.
According to definition, the IDOB structure is internal stable if the nine transfer functions in (4) are stable. The necessary and sufficient conditions for internal stability are set by the following theorem, which is an adoption of the standard theorem on internal stability.

Theorem 1. The IDOB control loop with a filter Q and an approximate inverse $\tilde{G}^{-1}$ is internal stable iff the following two conditions hold:

(a) The transfer function $1 - Q(1 - G\tilde{G}^{-1})$ has no zeros in $\mathrm{Re}(s) \geq 0$.
(b) There is no pole-zero cancellation in $\mathrm{Re}(s) \geq 0$ in the product $\frac{QG\tilde{G}^{-1}}{1-Q}$.

Note that the expression in condition (a) corresponds to $1 + L$, and that in condition (b) to L.
Due to the pole $s = 0$ in L, the expression $QG\tilde{G}^{-1}/(1 - Q)$ includes the factor $\frac{1}{s}G\tilde{G}^{-1}$. Given that condition (b) in the above theorem prohibits pole-zero cancellations for $\mathrm{Re}(s) \geq 0$ in $\frac{1}{s}G\tilde{G}^{-1}$, the two restrictions

$$G(0) \neq 0, \;\; G(0) \neq \infty \tag{10}$$

arise. Furthermore any non-minimum-phase and unstable dynamics in G is non-invertible. In this case, a trade-off between exact inversion and stability is needed. One possible approach is to invert by $\tilde{G}^{-1}$ just the invertible (stable minimum phase) part and then design Q appropriately to guarantee stability.

G with a RHP zero: It is often the case that physical systems include right half-plane (RHP) zeros (non-minimum phase behavior). Here we confine the discussion to the case of just one RHP zero

$$G = \tilde{G}\frac{1-\lambda s}{1+\lambda s} = \tilde{G}\left(1 + \frac{-2\lambda s}{1+\lambda s}\right), \;\; \lambda > 0. \tag{11}$$

Note that this case is often met in practice. For the sake of simplicity let $Q = 1/(\tau s + 1)$. Then condition (a) in Theorem 1 reads $\tau > \lambda$. Thus if $\tilde{G}^{-1}$ is constrained to cancel the invertible dynamics of G a Q-loop with a sufficiently low bandwidth is capable of stabilizing the IDOB structure with a non-minimum-phase zero. Of course, for a large enough RHP zero ($\lambda \ll 1$) this condition is noncritical. However, the situation becomes critical if the RHP zero is slow. Then the stability condition forces the crossover at $s = 1/\tau$ be close to the RHP zero at $s = 1/\lambda$, which is very inconvenient. The closed-loop performance degrades and the phase margin substantially reduces due to the RHP zero, which is a reflection of well-known difficulties with slow non-minimum phase zeros.

G with a RHP pole: Again consider the case with one RHP pole at $s = 1/\lambda$

$$G = \tilde{G}\frac{1+\lambda s}{1-\lambda s} = \tilde{G}\left(1 + \frac{-2}{1+\lambda s}\right)^{-1}, \;\; \lambda > 0. \tag{12}$$

Then condition (a) in Theorem 1 requires an unstable filter Q with a high enough bandwidth $-\tau > \lambda$. If the RHP pole is not too high, then the latter condition is easily met. A high RHP pole, on the other hand, may drive the bandwidth of Q too high, so the complementary sensitivity function T may become too large, thus reducing stability phase margin. In addition, most probably too much noise would have already been injected into the system. This reflects the well-known difficulties with fast open-loop unstable poles.

3. ROBUSTNESS AGAINST IMPERFECTION

3.1 Imperfection

In practice, $\tilde{G}^{-1}$ can never provide exact inversion of the plant G. Here we want to set a paradigm for the analysis in case of imperfect inversion. Different models for the imperfect inversion may be defined. For the sake of simplicity we concentrate here mainly on the multiplicative imperfection [2]

$$G = \tilde{G}(1 + W_2\Delta) \tag{13}$$

where W_2 is a proper stable weighting function and $\|\Delta\|_\infty < 1$ represents a stable unstructured disk-like uncertainty. Typically, W_2 increases with frequency due to model mismatching.
Note that referring to (11) the effect of a RHP zero in G at $s = 1/\lambda$ can be included in a multiplicative imperfection with $W_2 = (-2\lambda s)/(1 + \lambda s)$. Similarly, a RHP pole at $s = 1/\lambda$ is possible to include in the imperfection model $G = \tilde{G}/(1 + W_2\Delta)$ with $W_2 = -2/(1 + \lambda s)$. In both cases this is done at the price of conservativeness.

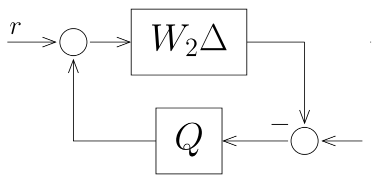

Fig. 2. Stability accounting imperfection

[1] Here stems the motivation for the denotation 'integral error' for the variable $\varepsilon = 1/(1 - Q)e$ from.

[2] Here we feel more comfortable in using the term *imperfection* rather than *uncertainty*, since generally $W_2\Delta$ models the missing dynamics in $\tilde{G}^{-1}$.

3.2 Stability

The problem explored here is: assuming that IDOB closed loop with perfect inversion (condition (B)) is stable, how big is the minimal imperfection that destabilizes the IDOB loop? The simplest answer to this question uses the small-gain theorem. Therefore check that after substitution of the model (13) in Fig 1 the IDOB structure collapses to the one shown in Fig 2. Now according to the small-gain theorem the latter structure is stable for all Δs within the disk of radius 1 if and only if the condition

$$\|W_2 Q\|_\infty < 1 \tag{14}$$

holds. Note that this condition basically represents the known robust stability condition of a one-degree of freedom feedback structure $\|W_2\tilde{T}\|_\infty < 1$.
Equation (14) sets stability constraints in the interaction between the imperfection W_2 and the design parameter Q. It has a simple and elegant geometrical interpretation in terms of the Bode plots of Q and W_2. Namely, the IDOB structure with the imperfection W_2 is internally stable iff the magnitude Bode plot of the function W_2^{-1} lies above that of Q. Recall that $|W_2|^{-1}$ rolls off toward zero at high frequencies, thus putting a limitation for the bandwidth of the filter Q. Similar interpretation can be done using Nyquist analysis. Suppose $Q = 1/(1+\tau s)$. Due to the imperfection the Nyquist plot of L is a region delineated by circles with center at $\tilde{L} = 1/\tau s$ and frequency dependent radii $|W_2\tilde{L}|$. Now if the bandwidth of Q continuously increases, so too does $\tilde{L}$. At a certain frequency the disk L gets larger and simultaneously moves away from the origin, thus reducing both the phase and magnitude margin.

3.3 Tracking

Asymptotic tracking is defined as the ability of the control loop to drive $y \to r$, that is, $e \to 0$ as $t \to \infty$. For instance, if r is a step, then due to

$$\frac{e}{r} = S(1 - G\tilde{G}^{-1}) \tag{15}$$

the function S must have a zero at $s = 0$, which is equivalent to $Q(0) = 1$. For example this is fulfilled by $Q = 1/(\tau s + 1)$.
Now consider a set of sinusoidal inputs confined within some frequency bandwidth. Let W_p weight a desired tracking response in (15) in the sense that $\|e\|_2 < \|W_p\|_\infty^{-1}\|r\|_2$. For larger $|W_p|$ the tracking performance improves. However, it is intuitive that for sufficiently large imperfections $W_2\Delta$, the performance set by W_p may get lost. It is thus important to set up the conditions for meeting the performance set by W_p in the presence of the imperfections W_2. In other words, a relationship linking imperfection W_2, performance W_p and the filter Q is searched for. Therefore, substitution of (13) into (15) yields $|SW_1\Delta| < 1, \;\forall\omega$, or equivalently

$$\|W_1 S\|_\infty < 1 \tag{16}$$

where we switch to the notation $W_1 = W_p W_2$. Due to perturbation of the sensitivity function S by the imperfection $W_2\Delta$ the latter equation reads

$$\left\| \frac{W_1\tilde{S}}{1 + W_2\Delta\tilde{T}} \right\|_\infty < 1. \tag{17}$$

This equation can be further manipulated provided that internal stability condition $\|W_2\tilde{T}\| < 1$ holds to

$$\left\| |W_1\tilde{S}| + |W_2\tilde{T}| \right\|_\infty < 1 \tag{18}$$

or finally to

$$\| |W_1(1-Q)| + |W_2 Q| \|_\infty < 1. \tag{19}$$

Equation (19) is an elegant description of simplicity and efficiency of the IDOB control structure. It represents the basic equation for the loopshaping design by Q. It can be shown that thereby a very simple Q results (loopshaping details are avoided here). Namely, for operational frequencies, where $|W_1| \gg 1 > |W_2|$ holds, the condition $Q \approx 1$ results, and for high frequencies ($|W_2| \gg 1 > |W_1|$) $Q \approx 0$ results. In general, it is easy to define such a Q, e.g. $Q = 1/(\tau s + 1)$. The only design parameter here is basically its bandwidth. Therefore, (19) should be used. Note that, for better performance the term $|W_1(1-Q)|$ in (19) requires a high bandwidth, which is however compelled by the stability term $|W_2 Q|$. Thus, the designer should meet a compromise between these two conflicting specifications.
This situation can also be given a nice geometrical interpretation in the Nyquist plane. Note that (18) can be equivalently written in the form $|W_2 W_p| + |W_2\tilde{L}| < |1 + \tilde{L}|, \;\forall\omega$. If $W_p = 0$, then the geometrical interpretation given for stability in the previous section holds. However, due to $W_p \neq 0$, the region delineated by L's, instead of just the point -1 must avoid now a disk with center at -1 and radius $|W_2 W_p|$. Thus, for a given performance W_p, the bandwidth of Q $(= 1/\tau)$ has to be narrowed.

4. MISCELLANEOUS REMARKS

4.1 Robust inversion

A drawback of exact feedforward inversion is the missing robustness. Robust inversion is defined as the accomplishment of exact inversion of a system irrespectively of plant uncertainties in form of parameter variations, un-modelled dynamics and external disturbances. The point here is to note that the high-gain feedback path with the Q-loop (condition (A')) provides robustness to the approximate inversion. This is motivated by the discussion in Section 2.1, that is, by substitution of condition (A') in (1), yielding

$$\frac{u}{r} \approx \frac{1}{G} \tag{20}$$

that is, for a given reference r any parameter changes or un-modelled dynamics of the plant G are accordingly observed at u.

Furthermore, while providing model inversion, the system response on disturbances d is minimized at operational frequencies due to $S \approx 0$. The situation $T \approx 1$ in this range is not critical, since the measurement noise n is usually high-frequent. Similarly, beyond operational frequency range, where $Q \approx 0$ and therefore $T \approx 0$, the injected noise into the feedback loop is attenuated. Again $S \approx 1$ here is uncritical since the disturbance d is usually low-frequent. In summary, for a given application, in addition to the constraints set by stability/performance condition (19), for an adequate shape of Q, the disturbance and noise information need to be taken into account, too.

4.2 IDOB vs DOB

As already noted in the introduction IDOB is a modification of the so-called *disturbance observer* (DOB) control structure shown in Fig. 3. Due to the analogous structure, some inherent structural properties of DOB are inherited by IDOB. For instance, the sensitivity functions for the two structures are of identical form. However, the two structures implement essentially different tasks. This difference is reflected by the different design and different location of G_n^{-1} (DOB) and $\tilde{G}^{-1}$ (IDOB).

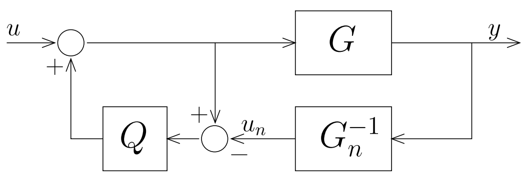

Fig. 3. DOB scheme

Due to the location of G_n^{-1} in the feedback path, the DOB is usually used for model regulation purposes, that is, it tries to impose an input-output dynamics that matches to that of G_n. This task should also hold in the presence of uncertainties and disturbances. The IDOB controller, on the contrary, tries to recover the imperfection of $\tilde{G}^{-1}$. In both cases, the respective differences between G_n and G, and G and $\tilde{G}$, are seen respectively as fictive disturbances at u_n in Fig. 3 and at y in Fig. 1. And, they both use the identical mechanism for its compensation by the $Q-$loop.

Another distinction resulting from the first one refers to the design of G_n^{-1} and $\tilde{G}^{-1}$. Usually G_n is designed to be of lower complexity than G, while $\tilde{G}$ should be designed to model G as accurate as possible. Thus, the mathematical formalism described in Section 3 is less conservative for the IDOB structure.

4.3 Resonant IDOB

Many problems involve robust tracking of sinusoidal references. Such problems are e.g. current control in electrical drives, design of active filters etc. While most of the hitherto discussion refers to tracking of slowly changing signals, the proposed structure can be easily adopted for perfect tracking of sinusoidal references. For the sake of simplicity assume $r = \sin \omega_0 t$. Then, for zero-error tracking the sensitivity function S must have a zero at $s = j\omega_0$, that is, $Q(j\omega_0) = 1$. The lowest order filter which satisfies this condition is

$$Q = \frac{2D\tau s + (1 - \omega_0^2 \tau^2)}{\tau^2 s^2 + 2D\tau s + 1} \tag{21}$$

where $0 \leq D \leq 1$ and $\omega_0 \tau < 1$. Given that $\tilde{L} = Q/(1 - Q)$ a resonance appears at the frequency ω_0 in the loop transfer function, so no matter what imperfection, zero-error tracking of the reference is guaranteed.

If $\omega_0 \tau = 1$ is chosen, then $Q(0) = 0$ and $T(0) = 0$, see (7). This may be useful if the sensors are corrupted with offset. Then the zero of T at $s = 0$ takes care of structural offset compensation.

4.4 Derivative structures

Different structures sharing same basic properties may be derived from the IDOB structure presented in Fig 1. Two such structures are shortly presented in the sequel.

If $\tilde{G}^{-1}$ is improper then for realization purposes the low-pass filter Q may be relocated as shown in Fig 4. Effectively, the first row of the matrix in (4) is multiplied by Q. Hence the conditions (C) and (D) transform to

(C'): $\quad G_{ru} = QG^{-1}$ (approximate inversion)
(D'): $\quad G_{ry} = Q$ (approximate tracking).

This is not critical if the bandwidth of Q is high enough. On the other hand, the sensitivity functions S, T and the loop transfer function L are identical to those of the basic IDOB structure. Consequently the stability and performance conditions in (14) and (19) will hold, too.

Fig 4 additionally suggests adaption of measurable states p_j of the plant G, which in $\tilde{G}^{-1}$ appear as parameters.

Other than the two structures in Figs 1 and 4, the structure in Fig 5 features an input $Q-$loop. Here G_a stands for the actuator dynamics. If $G_a = 1$, that is, it is lumped in G (as was the case with other structures), then input-output and sensitivity functions are

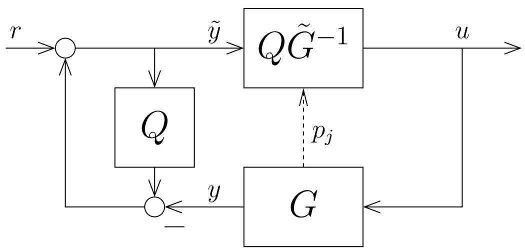

Fig. 4. IDOB derivative scheme

identical to those for the scheme in Fig 4. Otherwise, they are directly therefrom derived by the substitution $Q \to QG_a$. E.g. the high-gain condition (A') turns to

(A''): $\quad QG_a \approx 1 \quad$ (high-gain feedback).

Note that in applications using rate saturated actuators G_a this specific structure turns out to be notably efficient in providing robustness against limit cycles.

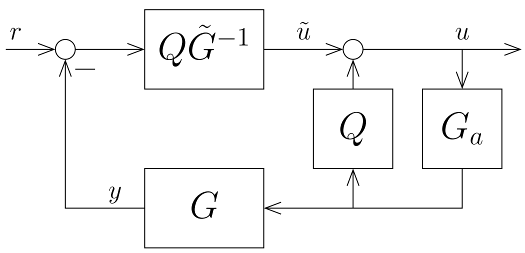

Fig. 5. IDOB derivative scheme with input $Q-$loop

4.5 Multivariable IDOB

Let G in Fig. 1 describe a MIMO system with $u \in R^k$ and $y \in R^l$ and let $\tilde{G}^{-1}$ represent a right-inverse of G (G is assumed to have full column rank). After some algebraic manipulations [3] it can be shown that for the MIMO structure

$$G_{ru} = \tilde{G}^{-1}(I - Q(I - G\tilde{G}^{-1}))^{-1} \tag{22}$$
$$G_{ry} = G\tilde{G}^{-1}(I - Q(I - G\tilde{G}^{-1}))^{-1} \tag{23}$$

whereby $I = I_{l\times l}$ is a unity matrix. Notice that the form of the latter equations is identical to the counterpart equations for the SISO structure (1) and (2). Following the same lines as for the SISO structure, it can be concluded that both degrees of freedom

(A'''): $\quad Q \approx I \quad$ (high-gain feedback)
(B'''): $\quad \tilde{G}^{-1} \approx G^{-1} \quad$ (feedforward inversion)

independently contribute to the inversion of the MIMO plant G. In particular, due to (A''') equation (22) reads

$$G_{ru} \approx G^{-1} \tag{24}$$

that is, for slowly varying inputs robust inversion w.r.t. parameter and model uncertainties applies. For the analysis of disturbance and noise rejection, the sensitivity functions S and T are computed to be

$$S = (I - Q)(I - (I - G\tilde{G}^{-1})Q)^{-1} \tag{25}$$
$$T = G\tilde{G}^{-1}Q(I - (I - G\tilde{G}^{-1})Q)^{-1}. \tag{26}$$

Again both equations posses the identical form as sensitivity functions for SISO systems in (6) and (7). In the operational frequency range, where (A''') holds, $S \approx 0$ results, and robust inversion w.r.t. external disturbances is accomplished.
For the accomplishment of the condition (A'''), the $Q-$filter may be designed as the diagonal matrix

$$Q = \begin{bmatrix} Q_1 & 0 & 0 & \dots & 0 \\ 0 & Q_2 & 0 & \dots & 0 \\ 0 & 0 & Q_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & Q_l \end{bmatrix} \tag{27}$$

whereby $Q_i,\ i = 1, 2, \cdots l$ are low-pass filters with unity gain.

5. CONCLUSION

A novel two-degree of freedom controller structure denoted as *inverse disturbance observer* (IDOB) for model inversion and tracking tasks has been introduced. The controller features a unification of the high-gain feedback and feedforward exact inversion principles. It is especially useful if the plant is not exact invertible. A mathematical paradigm has been developed for its design.
Many other two-degree of freedom structures carrying different names have been already published. It has been claimed, that all of them are actually equivalent, (Horowitz, 1963). The structure presented here is by no means an exception, however the authors believe that it provides essential advantages w.r.t. design simplicity due to its very natural structure.
The application field of the IDOB controller is very wide including motion control, force control, automotive, robotics, flight dynamics, chemical engineering, electrical drives, etc.

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[3] Here the matrix identities $(I + BA)^{-1}B = B(I + AB)^{-1}$ and $(A - BC^{-1}D)^{-1} = A^{-1} + A^{-1}B(D - CA^{-1}B)^{-1}CA^{-1}$ are used.