Active Damping of Railway Carbody Vibrations with Piezoelements

Ondřej Vaculín, DLR, Institute of Robotics and Mechatronics
Andreas Heckmann, DLR, Institute of Robotics and Mechatronics

DLR-German Aerospace Center
Institute of Robotics and Mechatronics
Vehicle System Dynamics
PO Box 1116, 82230 Wessling, GERMANY
Phone: + 49 8153 28 3659
Fax: +49 8153 28 1850
E-mail: {Ondrej.Vaculin, Andreas.Heckmann}@DLR.de

This paper presents a methodology for the simulation and control of smart structures with piezoceramic patches by means of multibody dynamics. A theoretical background is mentioned adapting a modal multifield approach. Then a methodology for the control design is proposed. The methodology includes the optimisation of actuator placement, which is based on modal representation of the elasticity. The methodology is applied for simulation and control synthesis of an active damping for a railway carbody. The application example illustrates the implemented process chain. This procedure provides a complex development environment for the simulation, optimisation and control design of elastic structures with smart materials.

Topics/A12, A22, A25

1 INTRODUCTION

New generations of ground and air vehicles will more and more profit from lightweight structures because of economic and environmental reasons. However, the lightweight structures require additional control devices to overcome the drawback of increased structural vibrations. Several approaches for suppression of the vibration exist. The so-called smart or adaptive structures can overcome this drawback. Thin piezoceramic actuators and sensors integrated into the structure are one of the promising ways to control the structural vibrations.

The methodology for multibody simulation flexible structures and their control elements has been recently developed, [1]. It is based on a modal multifield approach, considering the coupling between displacement and electrostatic field.

With this theoretical background the multibody description of the flexible structures is extended to consider the input of the piezoelectric actuators and evaluate their sensor output. This representation enables efficient simulations of smart structures as a part of whole vehicle models including simulation scenarios such as typical guideways and manoeuvres. Thereby the design and evaluation of concepts for structural control can profit from complex multibody environment including interfaces to control design tools. The recent simulations verified the feasibility on moderately simple models, [1], however, the goal is to also control more complex systems, such as railway vehicles.

The increasing operational speeds and passenger demands require focusing on the vibration comfort of railway vehicles. Furthermore, it seems that classical bogie vehicles are being replaced by lightweight vehicles with one or two axles per vehicle, [2]. Since such future vehicles will not profit from the mechanical pre-filtration of the railway bogies, the importance of control of the railway vehicle structures and its simulation under realistic conditions will increase. First computational and experimental proposals for application of piezopatches on railway vehicles to suppress the carbody vibration have been studied, [3, 4].

2 SMART STRUCTURES

Adaptive or smart structures are mechatronic devices which allow vibration properties and responses of mechanical systems to be modified; they are particularly used to improve the performance of lightweight structures. Among the wide range of supposable physical effects and corresponding material compositions, thin piezoceramic patches integrated in the structures proved their potential as electromechanical and mechanoelectrical transducers, which can be simultaneously exploited as actuators and sensors to control the vibration of the elastic structures, [5]. The piezoceramic patches apply additional mechanical forces as actuators and generate electrical charge as sensors. The additional electrical and mechanical measures should be considered for simulating the behaviour of flexible bodies equipped with the active structures.

Since the smart structures are mechatronic systems, their design involves several engineering disciplines such as structural mechanics, electronics and control engineering. The optimisation of such a complex system is a challenging task which may be supported advantageously by multibody system (MBS) dynamics as a method of system dynamics. Moreover, the MBS approach enables an efficient simulation of complex systems composed of elastic and rigid bodies with large overall motion such as vehicles, which can be equipped with the piezopatches.

It is state-of-the-art of industrial MBS tools to incorporate the results of an appropriate finite element analysis to obtain the mechanical data of flexible bodies. This approach may not yet be applied to the data of smart structures. Although the finite element modelling of piezoelectric devices on shell elements is a field of active research, [6, 7], it is not yet introduced in an industrial finite element tool. Nevertheless, to enable the simulation of structures with shell elements, the following technique uses only purely mechanical data which are readily available.

Further, the multibody codes offer an excellent connection to computer aided control engineering (CACE) tools. These tools are brought into action during the controller design and simulation. The methods originating in control engineering and modal approach are then used to optimise the placement of the piezoceramic patches.

3 THEORY OUTLINE

Current industrial multibody tools are capable of describing the displacement field of elastic bodies based on their modal represenatation. A modal analysis of an elastic body yields discrete mode matrices for every node k, located at the position $\boldsymbol{r}_k \in \mathbb{R}^3$ which specify the displacements $\boldsymbol{\Phi}_{u,k} \in \mathbb{R}^{3,p}$ and rotations $\boldsymbol{\Psi}_{u,k} \in \mathbb{R}^{3,p}$ for all p observed modes.

In order to simulate elastic structures with piezoceramic transducers, their electromechanical and mechanoelectrical behaviour have to be considered additionally. The constitutive equation, needed to base this multifield formulation, states the linearised relationship between the mechanical strain $\boldsymbol{S}$ and stress $\boldsymbol{T}$ and the electric displacement $\boldsymbol{D}$ and electrical field strength $\boldsymbol{E}$ by defining appropriate material constants $\boldsymbol{c}$, $\boldsymbol{e}$ and $\boldsymbol{\varepsilon}$, [8]:

$$\begin{pmatrix} \boldsymbol{T} \\ \boldsymbol{D} \end{pmatrix} = \begin{pmatrix} \boldsymbol{c} & -\boldsymbol{e}^T \\ \boldsymbol{e} & \boldsymbol{\varepsilon} \end{pmatrix} \begin{pmatrix} \boldsymbol{S} \\ \boldsymbol{E} \end{pmatrix} . \tag{1}$$

The field equations are formulated by means of Jordain's principle of virtual power, [9]:

$$\begin{aligned} \int \delta \boldsymbol{v}^T \varrho \boldsymbol{a} + \delta \dot{\boldsymbol{S}}^T \underbrace{(\boldsymbol{c}\boldsymbol{S} - \boldsymbol{e}^T \boldsymbol{E})}_{\boldsymbol{T}} - \delta \dot{\boldsymbol{E}}^T \underbrace{(\boldsymbol{e}\boldsymbol{S} + \boldsymbol{\varepsilon}\boldsymbol{E})}_{\boldsymbol{D}} \ \mathrm{d}V \\ = \int \delta \boldsymbol{v}^T \boldsymbol{f}_V \ \mathrm{d}V + \int \delta \boldsymbol{v}^T \boldsymbol{f}_B - \delta\dot{\varphi} Q_\varphi \ \mathrm{d}B \ . \end{aligned} \tag{2}$$

The right hand side of equation (2) represents all external force and charge loads acting on volumes or boundaries. The variables $\boldsymbol{v}$ and $\boldsymbol{a}$ denote the absolute velocity and acceleration of a volume element; Q_φ and φ are used to name the applied charges and their electric potential. Furthermore, the dependent variables $\boldsymbol{T}$ and $\boldsymbol{D}$ are eliminated, pointing out the coupling of mechanical and electrical fields by the material description in (1).

A floating frame of reference formulation, [10], enables the superimposition of nonlinearly described, large overall motion, later on denoted by the subscript $_R$, with linearised, small elastic deformations $\boldsymbol{u}_u$. Based on the Ritz approximation, separating $\boldsymbol{u}_u(\boldsymbol{r}, t)$ in only space dependent mode shapes $\boldsymbol{\Phi}(\boldsymbol{r})$ and time dependent variables $\boldsymbol{q}(t)$, the strain tensor $\boldsymbol{S}$ can be evaluated by applying the differential displacement-strain-operator $\mathcal{L}$:

$$\begin{aligned} \boldsymbol{u}_u(\boldsymbol{r}, t) &= \boldsymbol{\Phi}_u(\boldsymbol{r}) \boldsymbol{z}_u(t) \ , \\ \boldsymbol{S} &= (\mathcal{L}\boldsymbol{\Phi}_u)\, \boldsymbol{z}_u = \boldsymbol{B}_u \boldsymbol{z}_u \ . \end{aligned} \tag{3}$$

The electric field vector $\boldsymbol{E}$ is evaluated analogously by an approximation of the scalar electric potential field φ, defining the electric mode shapes $\boldsymbol{\Phi}_\varphi$ and the patch electrode voltages $\boldsymbol{z}_\varphi$ and the negative gradient operation:

$$\begin{aligned} \varphi(\boldsymbol{r}, t) &= \boldsymbol{\Phi}_\varphi(\boldsymbol{r}) \boldsymbol{z}_\varphi(t) \ , \\ \boldsymbol{E} &= (-\nabla \boldsymbol{\Phi}_\varphi)\, \boldsymbol{z}_\varphi = \boldsymbol{B}_\varphi \boldsymbol{z}_\varphi \ . \end{aligned} \tag{4}$$

Further, the electromechanical coupling matrix $\boldsymbol{K}_{u\varphi} = \boldsymbol{K}_{\varphi u}^T$, the electric capacity matrix $\boldsymbol{K}_{\varphi\varphi}$ and the mechanical stiffness matrix $\boldsymbol{K}_{uu}$ are presentable as only volume dependent integrals:

$$\begin{aligned} \boldsymbol{K}_{uu} &= \int \boldsymbol{B}_u^T \boldsymbol{c} \boldsymbol{B}_u \ \mathrm{d}V \ , \\ \boldsymbol{K}_{u\varphi} &= \int \boldsymbol{B}_u^T \boldsymbol{e}^T \boldsymbol{B}_\varphi \ \mathrm{d}V \ , \\ \boldsymbol{K}_{\varphi\varphi} &= \int \boldsymbol{B}_\varphi^T \boldsymbol{\varepsilon} \boldsymbol{B}_\varphi \ \mathrm{d}V \ . \end{aligned} \tag{5}$$

A comparison of (2) with the classical equation of motion of unconstrained flexible multibody systems, e.g. in [10], yields the following:

$$\begin{aligned} &\underbrace{\begin{pmatrix} \boldsymbol{M}_{aa} & \boldsymbol{M}_{a\alpha} & \boldsymbol{M}_{au} \\ & \boldsymbol{M}_{\alpha\alpha} & \boldsymbol{M}_{\alpha u} \\ \text{symm.} & & \boldsymbol{M}_{uu} \end{pmatrix}}_{\boldsymbol{M}} \underbrace{\begin{pmatrix} \boldsymbol{a}_R \\ \boldsymbol{\alpha}_R \\ \ddot{\boldsymbol{z}}_u \end{pmatrix}}_{\ddot{\boldsymbol{z}}} = \\ &\begin{pmatrix} \boldsymbol{h}_a \\ \boldsymbol{h}_\alpha \\ \boldsymbol{h}_u \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\boldsymbol{K}_{uu} \boldsymbol{z}_u + \boldsymbol{K}_{u\varphi} \boldsymbol{z}_\varphi \end{pmatrix} \end{aligned} \tag{6}$$

and further, the sensor equation can be written as follows:

$$\boldsymbol{Q}_\varphi = \boldsymbol{K}_{u\varphi}^T \boldsymbol{z}_u + \boldsymbol{K}_{\varphi\varphi} \boldsymbol{z}_\varphi \ . \tag{7}$$

The mass matrix $\boldsymbol{M}$ on the left hand side of (6) is formulated as 3×3 block matrix to specify the inertia coupling between translational, angular and elastic motion acceleration terms $\boldsymbol{a}_R$, $\boldsymbol{\alpha}_R$ and $\ddot{\boldsymbol{z}}_u$. Further, $\boldsymbol{h}_a$, $\boldsymbol{h}_\alpha$ and $\boldsymbol{h}_u$ summarise all time and state dependent inertia, damping and external forces. The added product $\boldsymbol{K}_{u\varphi}\boldsymbol{z}_\varphi$ demonstrates the use of the piezopatches as structural actuators. The sensor equation (7) is needed to calculate the electric quantities, e.g. the electric charges $\boldsymbol{Q}_\varphi$, if the piezoelements are used as sensors or are parts of arbitrary electric circuits.

4 CONTROL OF SMART STRUCTURES

The controller design is connected with the selection of the patches. The proposed controller design methodology is based on the modal description of elastic bodies and the placement of the patches results from the controller gains derived for discretised elastic body.

The supposed goal for the controller design is to control vibration of one node of the elastic structure.

4.1 Transformation to the State Space Form

The transformation of the description of the elastic body with piezoelements to a state space form needed for the controller design results in a multi-input multi-output (MIMO) system:

$$\begin{array}{rcl} \dot{\boldsymbol{x}} &=& \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \quad , \\ \boldsymbol{y} &=& \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} \quad , \end{array} \tag{8}$$

where $\boldsymbol{x}$ is the state, $\boldsymbol{u}$ the input and $\boldsymbol{y}$ the output vector and $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ the system matrices as follows:

$$\begin{array}{rcl} \boldsymbol{A} &=& \begin{pmatrix} \boldsymbol{O} & \boldsymbol{I} \\ -\boldsymbol{M}_{uu}^{-1}\boldsymbol{K}_{uu} & -\boldsymbol{M}_{uu}^{-1}\boldsymbol{D}_{uu} \end{pmatrix}, \\ \boldsymbol{C} &=& \begin{pmatrix} \boldsymbol{K}_{u\varphi}^T & \boldsymbol{O} \end{pmatrix}, \\ \boldsymbol{B} &=& \begin{pmatrix} \boldsymbol{O} \\ -\boldsymbol{M}_{uu}^{-1}\boldsymbol{K}_{u\varphi} \end{pmatrix}, \\ \boldsymbol{D} &=& \begin{pmatrix} \boldsymbol{K}_{\varphi\varphi}^T \end{pmatrix}. \end{array} \tag{9}$$

The matrices $\boldsymbol{M}_{uu}$, $\boldsymbol{K}_{uu}$, $\boldsymbol{K}_{u\varphi}$ and $\boldsymbol{K}_{\varphi\varphi}$ are defined in (5) and (6), matrix $\boldsymbol{D}_{uu}$ represents the structural damping of the elastic body, matrix $\boldsymbol{I}$ is the identity matrix and matrix $\boldsymbol{O}$ is the zero matrix.

The number of inputs r and outputs m in (8) corresponds to the number of piezoelements and the number of states n is the double of the elastic degrees of freedom.

4.2 Controller Design

Traditional state feedback LQR control is proposed to be applied for the controller design of the MIMO system (8). In the first phase it is supposed that every shell element of the elastic structure is equipped with one piezopatch on both sides. Every piezopatch serves simultaneously as an actuator and a sensor. The output vector $\boldsymbol{y}$ includes output charges of the piezopatches instead of states, which are needed for the LQR design. However, one can construct a state estimate $\hat{\boldsymbol{x}}$ such that the control law retains similar closed-loop properties, [8].

The first step in the control design process is the selection of parameters of the weighting matrix $\boldsymbol{Q}$ in the LQR design cost function:

$$J = \int_0^\infty \left(\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{u}^T\boldsymbol{R}\boldsymbol{u}\right) dt \quad . \tag{10}$$

Dependent on the design goal, the $\boldsymbol{Q}$ matrix is proposed to have the block structure:

$$\boldsymbol{Q} = k_Q \begin{pmatrix} \boldsymbol{Q}_{11} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} \end{pmatrix} \quad , \tag{11}$$

where k_Q is a scalar parameter and $\boldsymbol{Q}_{11}$ is a diagonal matrix. The elements $\varphi_{u,k,i,j}$ of modal matrix $\boldsymbol{\Phi}_{u,k}$ identify the contribution of the eigenmodes on the motion of the selected node in the direction j. This information is important for the definition of the $\boldsymbol{Q}_{11}$ matrix, whitch has diagonal elements, e.g. for the z-direction:

$$q_{ii} = (\varphi_{u,k,i,z}\omega_i)^h \quad , \quad 1 \le i \le p \quad , \tag{12}$$

where p is the number of modelled eigenmodes, ω_i denotes the corresponding eigenfrequency of the i-th eigenmode and h is the power factor. The expression $x_i q_{ii} x_i$ from (10) corresponds for $h = 2$ to the local potential energy of the eigenmode i, [11].

4.3 Selection of Patches

An important feature is the efficient selection of the piezoelectric patches, which will be used for the controller of the flexible body. In the previous paper, [3], a design-by-simulation method was applied to select the important patches. Instead of that a new selection criterion is applied, which is directly based on the linear feedback gain matrix $\boldsymbol{K}$ of the LQR design:

$$\boldsymbol{u} = -\boldsymbol{K}\boldsymbol{x} \quad . \tag{13}$$

The matrix $\boldsymbol{K}$ is a r-by-n matrix, where r is the number of inputs and n is the number of states of the controlled system. Since the inputs represent the voltages applied on the piezopatches, the most important patches should have the largest norm ζ_i of the corresponding column vector in the matrix $\boldsymbol{K}$, e.g. 2-Norm:

$$\zeta_i = \left(\sum_{j=1}^{n} |k_{i,j}|^2\right)^{1/2} \quad , \quad 1 \le i \le r \quad . \tag{14}$$

In the last step, after selection of the reduced set of patches, a new LQR and observer design should be performed and the parameter k_Q from (11) should be tuned in order to exploit the patches as efficiently as possible, i.e. the controller should use the whole linear range of the piezoelement for the expected disturbances.

5 IMPLEMENTATION ISSUES

In order to implement the theory outlined above in a multibody computational environment a developer version of the multibody simulation tool SIMPACK has been chosen, [12]. The process chain begins with a finite element analysis of the considered elastic structure. The standard FE-SIMPACK interface FEMBS uses the results of a modal analysis in order to create the modal multibody representation of a flexible structure. But because the electric data are not yet available in industrial finite element tools, the capacity and coupling matrices were additionally calculated based on the purely mechanical mode shape information. The developer version of SIMPACK is extended to deal with the electromechanical and mechanoelectrical coupling terms, which are indicated in equations (6) and (7).

The outlined control approach is implemented in MATLAB/Simulink. The final system is then simulated in two packages; SIMPACK and MATLAB/Simulink are connected via an inter-process communication interface, [13].

6 SIMPLIFIED BEAM MODEL

In order to get more insight, a simplified model of a railway vehicle is selected for the first experiments, see Figure 1. The model consists of an elastic beam which has approximately the same properties (mass and eigenfrequency) as the carbody. The elastic beam is discretised in 44 beam elements and its model representation is limited to three eigenmodes. The beam is supported by linear springs and dampers which brings three additional degrees of freedom (pitch and bounce). The linearised parameters of the secondary suspension of the railway vehicle are used. The springs are excited by the stochastic railway track based on the real track Trier Karthaus – Dillingen at a velocity of 160 km/h. The simulation time is 10 seconds. The goal of the controller design is to decrease the vertical acceleration in the center of the beam. Just the first and third eigenmodes will be controlled according to the

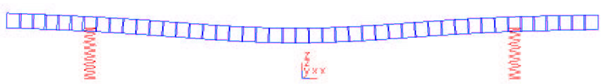

Fig. 1: Simplified beam modell

z-coordinates of the matrix $\boldsymbol{\Phi}_{u,k}$ for the corresponding node. Figure 2 presents the optimal location for a different number of patches. The first configuration in Figure 2 proposes four patches; the patches are located on both sides (collocated patches). The last configuration has 32 patches. The second configuration with eight patches has been selected for the simulation experiments. The simulation results indicate a reduction of the vertical acceleration in the center of the beam as presented in Figure 3.

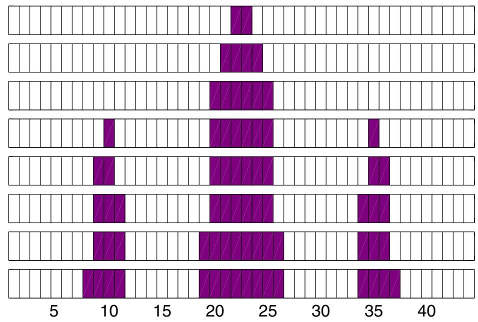

Fig. 2: Optimal location for a different number of patches

Furthemore the influence of the state estimation for the controller performance is studied. The comparison of two designs, firstly with the estimation of just three elastic degrees of freedom (six states), secondly with the estimation of three elastic degrees of freedom and two large overall body motions (pitch and bounce) is performed. The contribution of rigid body states is observed to be neglectable.

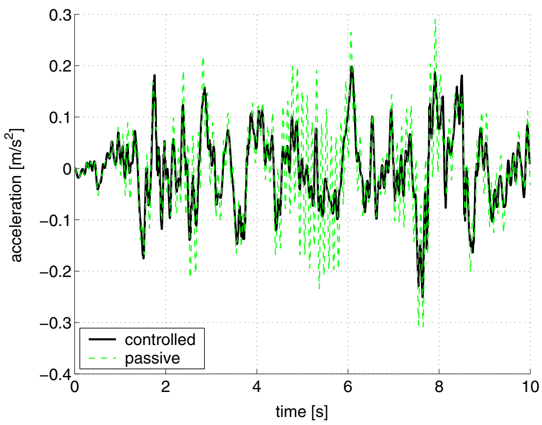

Fig. 3: Acceleration in the center of the beam

7 ACTIVE DAMPING OF A CARBODY

A verified generic model of a bogie railway vehicle is chosen to demonstrate the capabilities of the methodology presented in the previous sections. The vehicle model (Figure 4) consists of 26 rigid bodies and one elastic body, which is the vehicle carbody. The model of vehicle carbody includes the first seven eigenmodes between 9 and 20 Hz. The vehicle model

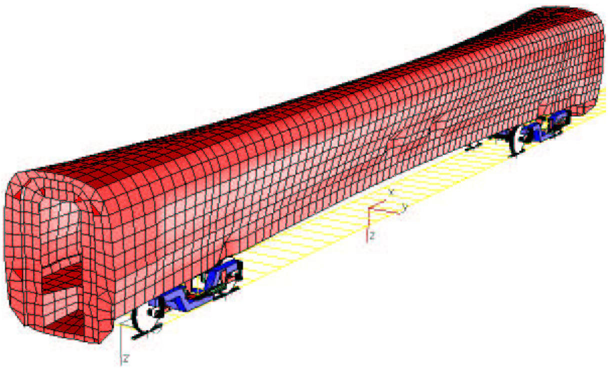

Fig. 4: SIMPACK model

has 130 states and consists of 79 force elements. The piezopatches are modelled as one user-defined force element in SIMPACK.

The piezoelements, which are 0.4 mm in width, are attached on both sides of 1170 finite elements visualised by the mesh in Figure 4. Such piezoelements provide approximately linear behaviour up to the voltage of 400 V. If higher voltages are applied, the piezoelements behave nonlinearly and expose hysteresis effects.

The simulation scenario consists of running on the same stochastic railway track as the simplified model at the same velocity of 160 km/h. The simulation time is again 10 seconds.

Originally the structure has 1170 patches, however, the number results after the reduction in 16 patches. The goal for the controller is to control the vertical acceleration in the center of the carbody.

The simulation results are presented in Figure 5. The RMS value of the vertical acceleration is decreased by 16 %. The contribution is in this case smaller as in the previous example, since the patches are of smaller size.

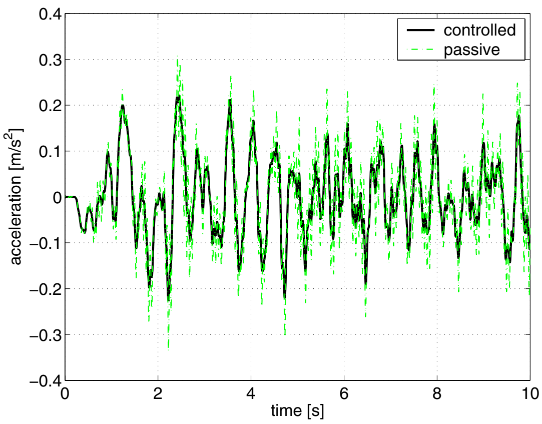

Fig. 5: Acceleration in the center of the carbody

8 CONCLUSIONS AND OPEN PROBLEMS

The presented methodology extends the classical modal description of elastic bodies in multibody systems with the effects of piezoelectricity and provides a tool which enables the development of design concepts with smart structures. In this way, the mechatronic approach may be evaluated from the very beginning of the design phase. Anyway, the performance appraisal of adaptive elements and their feasibility must be evaluated taking risks, costs, weights, complexity etc. into account. This evaluation is a challenging task which the outlined methodology is intended to support.

The presented controller design is based on the modal description of the elasticity; the finite element discretisation determines the size of the patches. The selection of the patches depends on the controller parameters in order to use the patches as efficiently as possible.

The simulation results indicate a contribution of the concept to the acceleration reduction of a railway vehicle. Future work will be focused on the extension of the methodology to control more than one node and the control of the structure exposed to the accelerations in vehicle manoeuvres such as curving.

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