Railway Vehicle Dynamics, Multibody Systems, and Bifurcation Analysis

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Introduction

DLR's Vehicle System Dynamics group has a long tradition in the computational analysis of railway vehicles' dynamics. The group's main focus used to be on the development of software tools for the efficient analysis of general mechanical systems with a special emphasis on the dynamics of ground vehicles like automobiles, railway vehicles and aircraft landing gears. The respective research activities culminated in the general multibody system software SIMPACK which provides a highly specific and sophisticated wheel–rail module for the dynamic analysis of all types of railway vehicles. The palette starts with tramways or streetcars and goes via freight cars up to high speed passenger cars – single cars as well as complete trains under arbitrary track conditions are analysable.
Nowadays, the group's main research topics have changed a bit. Concerning railway vehicles, these are devoted now to modern concepts of mechatronic wheelsets resulting in independently rotating wheels actively steered by a torque–control algorithm. This concept has already been implemented and verified on a test roller rig. Also, the modelling of elastic wheelsets and rails extends conventional simulation models of wheel–rail systems in order to reach mid–frequency behaviour (up to ca. 500Hz). Besides a more exact analysis of high velocity running behaviour, the possibility of simulating more precisely wear and corrugation of the tracks and polygonisation of the wheels or the noise emission of the wheel–rail interface is one of the final aims of this project. A third major topic is about the crosswind stability of railway vehicles. And, finally, some of the recently developed effective methods of computational bifurcation analysis are enhanced especially with respect to a robust applicability to mechanical systems being of industrial relevance – like railway vehicles.
After a short introduction of simulation models of arbitrary railway vehicles basing on a multibody system (MBS) approach and allowing for a virtual design process, the major part of this paper is about the bifurcation analysis of railway vehicles. Here, the application on 'realistic', i.e. complex and sophisticated simulation models is a fundamental concern.

Virtual design of railway vehicles

The MBS approach is a powerful and widely used method for the computational analysis and design of a railway vehicle's dynamic behaviour while running on arbitrary tracks under arbitrary manoeuvres. As multibody system, the vehicle's major structural parts are modelled as rigid or elastic bodies – e.g. the car body, the bogies, and the wheelsets – interconnected by massless force elements and joints, see Fig. 1. Due to the relative motion of the bodies, force elements such as springs and dampers generate applied forces and torques. Contrarily, joints give rise to constraint forces by constraining the relative motion of the bodies. Modelling primary (wheelset – bogie) and secondary (bogie – car body) suspensions, the complete spectrum of linear and nonlinear force elements might come into operation. Typical modelling tasks comprise leaf and flexi–coil springs, air–springs, and damper elements with rubber bearings.

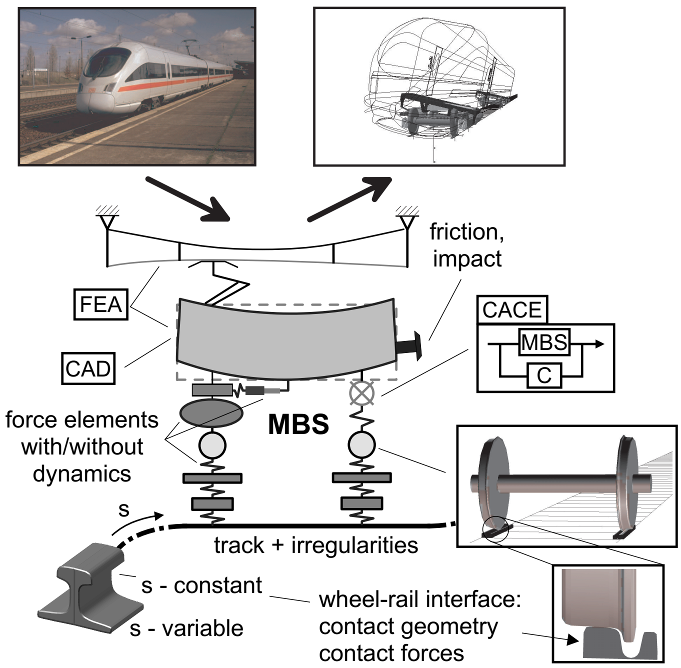

Figure 1: Generic simulation model of a railway vehicle. (Photograph: C. Splittberger, Internet: `http://mercurio.iet.unipi.it`.)

To avoid the time consuming and error–prone task of compiling the mathematical model by hand, different professional software packages based on the MBS approach are commercially available. They provide the engineer not only with software tools for the model set up but also allow to apply a wide range of different methods for an extensive analysis of the automatically generated system equations in a way optimized for the specific modelling and for the simulation task. In what follows, the general multibody simulation tool SIMPACK is outlined which is equipped with an comprehensive wheel–rail module, see [1, 2].

Concerning modern design concepts, certain simulation tasks and requirements may go beyond the scope of the typical MBS approach. In this case, bi–directional interfaces to established and widely–used CAE (Computer Aided Engineering) software allow to reproduce specific phenomena or to follow particular design principles. In Fig. 1 the more important of such interfaces are summarised, too.

To accomplish the growing demand for a reduction of the total vehicle weight by applying lightweight structures or within the realm of a comfort analysis, it might be necessary to take the structural flexibility of some particular vehicle components into account. This is done usually by means of an interface to FEA (Finite Element Analysis) software. The procedure is in principle that first a certain number of mode shapes of the bodies to be modelled elastically are pre–calculated by the FEA software. Then, within the subsequent nonlinear multibody simulations, these mode shapes can be used to superimpose the rigid body motions with small elastic deformations.

The interface to CAD (Computer Aided Design) software facilitates and accelerates the MBS setup while reducing the risk of modelling errors by the possibility of integrating directly graphical as well as physical CAD data into the MBS model.

Active tilting technique is an example for the inclusion of electronically controlled elements into a railway vehicle. The necessary control algorithms are designed usually by means of suitable CACE (Computer Aided Control Engineering) tools such as MATLAB/Simulink. Efficient interfaces to those software tools allow an integrated design approach following mechatronic principles.

In contrary to the modelling elements described up to now, one of the characteristics of a railway vehicle is its guidance along the track. A comprehensive vehicle design requires the simulation of different manoeuvres on arbitrary tracks, usually with stochastic irregularities superimposed. These irregularities are taken either directly from measuring data or are defined as a stochastic process via its power spectral density. And finally the usual assumption of the rails' profiles being constant along the track has to be abandoned for the simulation of vehicles running through a switch.

The second characteristic of railway vehicles heavily influencing their dynamic running behaviour is the contact between steel wheel and steel rail with their profile cross sections being the decisive factor. For adequate simulation results the strong nonlinearity of the contact geometry as well as of the contact mechanics has to be taken into account. To give an idea of at least the geometric nonlinearities, figure 2 shows the cross sections of the common wheel–rail profile combination *S1002/UIC60–ORE* with the potential points of contact being interconnected. To smooth the discontinuities inherent in the straight forward rigid contact model revealed in this figure, too, a quasi–elastic contact model is introduced in [3]; the smoothed wheel's lateral location of the point of contact is an indispensable prerequisite for an efficient simulation of railway vehicles.

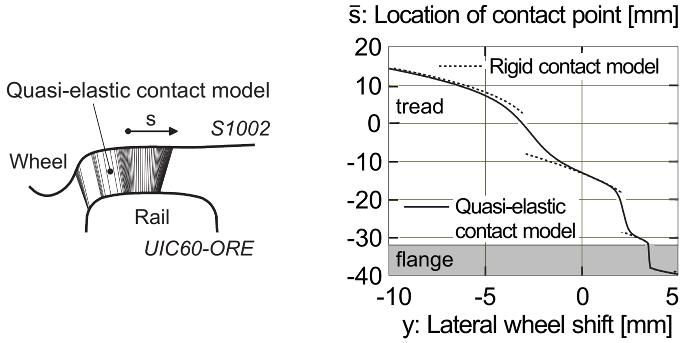

Figure 2: Profile combination *S1002/UIC60–ORE*: *Left*: Cross–sections of wheel and rail incl. potential points of contact. *Right*: Contact point location $\bar{s} = \bar{s}(y)$; y is the lateral shift between wheel and rail.

Bifurcation analysis

A prominent feature of nonlinear dynamical systems is the possible dependence of their long–time behaviour on the initial state. This means that one and the same system may show a multiplicity of quantitatively *and* qualitatively totally different behaviour patterns of its steady state, i.e. for system time $t \to \infty$, even though all the system's parameters remain unchanged. If possible transient processes are neglected, stationary, periodic, quasi–periodic, and/or chaotic behaviour has to be expected. A computer aided method for the examination of nonlinear dynamical systems with respect to the influence of one or more system parameters on existence and shape of these potential behaviour patterns is *numerical bifurcation analysis*, see e.g. [4] for an extensive description. This analysis can be performed either by means of a simple parameter variation over time integrations or by means of the more sophisticated direct method of *path–following* or *continuation*. The principle of path–following combines methods for the direct computation of a system's steady state (up to now, robust algorithms exist only for stationary and periodic solutions) with the continuation of one–dimensional curves in higher dimensional spaces.

A crucial aspect within the comprehensive computational design of a railway vehicle's running dynamics concerns its long–time behaviour while running on an ideally straight track. Starting from an (initial) disturbance, the vehicle's motion relative to the track is evaluated after all transient activity has died away, i.e. only its steady state is regarded. Of particular

interest is the influence of certain system parameters like the vehicle's velocity or the coefficient(s) of a suspension element.
Varying for example the vehicle's velocity v, a typical scenario given as bifurcation diagram in figure 3 is as follows: As long as the velocities are small or moderate every initial disturbance decays down to a stationary equilibrium rapidly. As soon as the so–called critical velocity is reached, the vehicle's long–time behaviour changes abruptly and radically and an initial disturbance may result in the so–called hunting or limit cycle motion, a periodically oscillating motion of the complete vehicle relative to the track that has to be avoided in everyday operations. Detailed analyses have shown, see e.g. [5], that there even exists a velocity range $v_{\mathrm{nlin}} \leq v \leq v_{\mathrm{lin}}$, where the final steady state depends qualitatively on the initial disturbance. Thus, for one and the same velocity of this range just as a stationary steady state a periodic steady state (hunting) has to be expected as well. The velocity v_{lin} characterises a Hopf bifurcation, whereas for $v = v_{\mathrm{nlin}}$ a saddle node bifurcation is found.

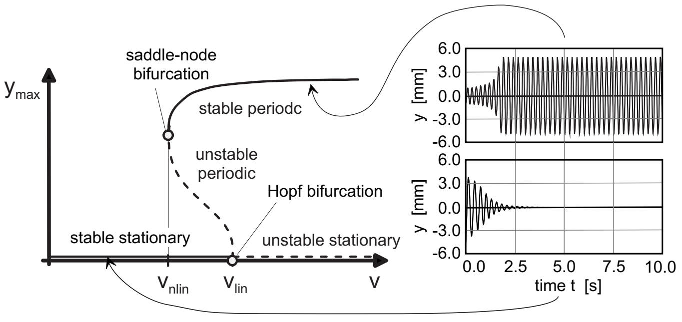

Figure 3: Typical bifurcation diagram of a wheel–rail–system. The bifurcation parameter is the velocity v along the track, $y_{\max}$ is the maximum relative lateral displacement of a wheelset.

Computational bifurcation analysis is an ideal software tool for examining this kind of dynamic behaviour. A more detailed description of a software environment for the continuation based bifurcation analysis of arbitrary mechanical systems developed in recent years at DLR can be found in [6, 7]. There, a special emphasis is placed on the application on realistic and therefore necessarily complex simulation models of arbitrary railway vehicles. Algorithms and analyses are restricted to stationary and periodic behaviour, i.e. to the range of technical–industrial relevance within railway vehicle dynamics. Three major topics are of primary interest: The integration of the bifurcation software Path into the software package Simpack for the simulation of general mechanical systems; the direct calculation of periodic solutions (limit cycles); and the handling of differential algebraic equations (DAEs). The following section concentrates on the second topic; the third topic, i.e. the continuation and the direct computation of limit cycles of DAEs is addressed rather scarecly in literature, see [8] for a different approach.

Direct computation of limit cycle solutions

Under a theoretical as well as under an algorithmic perspective the direct calculation of periodic solutions is by far the most difficult and costly part of every bifurcation analysis. For equations of motion given as an autonomous system of Ordinary Differential Equations (ODE), $\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y})$, $\mathbf{y} \in \mathbb{R}^n$, the task can be formulated as a Boundary Value Problem (BVP) with the initial time $t_0 = 0$, the unknown initial states $\mathbf{s} := \mathbf{y}(t_0)$ and the also unknown period T_{P}:

$$\begin{aligned} \dot{\mathbf{y}} &= \mathbf{f}(\mathbf{y}) && (1) \\ \mathbf{0} &= \mathbf{y}(T_{\mathrm{P}}, \mathbf{s}) - \mathbf{s} \, . && (2) \end{aligned}$$

Described below is the *Poincaré Map Method*, a special kind of single shooting method implemented in the software tool Path to solve such BVPs, see [9].
A Poincaré plane of the equations of motion (1) is a $(n-1)$–dimensional hyperplane in the n–dimensional state space being transversal to the flow $\boldsymbol{\varphi}(t, \mathbf{s})$ of the system, see figure 4. Starting from a point $\mathbf{s}$ near a periodic solution (i.e. near a closed trajectory in state space) and being located on a suitable Poincaré plane Σ of (1), $\mathbf{s} \in \Sigma$, the flow $\boldsymbol{\varphi}(t, \mathbf{s})$ will hit Σ for the first time in the same direction again after the return time T_{R}, $\boldsymbol{\varphi}(T_{\mathrm{R}}, \mathbf{s}) \in \Sigma$. Then, via the time discrete Poincaré map $\mathbf{P} : \mathbf{s} \to \boldsymbol{\varphi}(T_{\mathrm{R}}, \mathbf{s})$, the *residual map* $\mathbf{Q}(\mathbf{s}) : \mathbf{s} \to \mathbf{q}$ with

$$\mathbf{Q}(\mathbf{s}) := \mathbf{P}(\mathbf{s}) - \mathbf{s} = \boldsymbol{\varphi}(T_{\mathrm{R}}, \mathbf{s}) - \mathbf{s} \, , \mathbf{Q}(\mathbf{s}) : \Sigma \to \Sigma \quad (3)$$

can be defined. A fixed point $\mathbf{s}_{\mathrm{p}}$ of the Poincaré map $\mathbf{P}(\mathbf{s})$ representing a periodic solution of system (1) results now as zero of this residual map:

$$\mathbf{Q}(\mathbf{s}_{\mathrm{p}}) = \boldsymbol{\varphi}(T_{\mathrm{P}}, \mathbf{s}_{\mathrm{p}}) - \mathbf{s}_{\mathrm{p}} = \mathbf{0} \, . \quad (4)$$

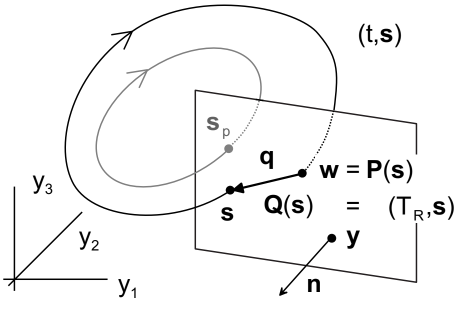

Figure 4: Poincaré plane, Poincaré map, and residual map of a dynamical system's flow $\boldsymbol{\varphi}(t, \mathbf{s})$ in $\mathbb{R}^3$.

One advantage of this method is that it can be enhanced on systems of DAEs $\mathbf{f}(\dot{\mathbf{y}}, \mathbf{y}) = \mathbf{0}$ via the concept of a *reduced* Poincaré map quite easily, see also [8] for a more detailed description.

Thus, to compute a periodic solution directly, the system of nonlinear equations (4) has to be solved. This is done as usually by a Newton iteration of the type $\mathbf{Q}_\mathbf{s}^j \cdot \Delta\mathbf{s}^j = -\mathbf{Q}^j$ with the increment $\Delta\mathbf{s}^j = \mathbf{s}^{j+1} - \mathbf{s}^j$ and the Jacobian matrix $\mathbf{Q}_\mathbf{s} = \partial\mathbf{Q}/\partial\mathbf{s}$. In principle, this gradient can be approximated column by column via finite differences; but for numerical reasons, as shown in [6] this approach leads easily to non–convergence of the iteration in case of larger or more complex MBS – like simulation models of complete railway vehicles taking into account the complete nonlinearities of the wheel–rail interface.

As also proven and demonstrated in [6], a more reliable and more efficient way for the computation of the Jacobian $\mathbf{Q}_\mathbf{s}$ is to combine the outlined shooting method with an integrated sensitivity analysis of the system equations. The basic procedure is to generate the Jacobian $\mathbf{Q}_\mathbf{s}$ analytically on the base of the sensitivity matrix $\mathbf{S}(T_\mathrm{R}, \mathbf{s}) := \partial\mathbf{y}(T_\mathrm{R}, \mathbf{s})/\partial\mathbf{s}$. On the other hand, differentiation of the initial value problem built from the system equations (1) and the initial condition $\mathbf{y}(t = 0) = \mathbf{s}$ with respect to the unknown initial states $\mathbf{s}$ yields the *Variational (Differential) Equations* (VDE, with $\mathbf{S}(t, \mathbf{s}) := \partial\mathbf{y}(t, \mathbf{s})/\partial\mathbf{s}$),

$$\dot{\mathbf{S}} = \frac{\partial\mathbf{f}}{\partial\mathbf{y}}\big(\mathbf{y}(t, \mathbf{s})\big) \cdot \mathbf{S}\ , \quad \mathbf{S}(t_0 = 0) = \mathbf{I}\ , \tag{5}$$

a set of altogether $n \cdot n$ differential equations in the unknown sensitivities $S_{i,j} := \partial y_i/\partial s_j$, $i, j = 1, \ldots, n$. Consequently, besides solving the system equations (1) (the nominal system) for the flow $\boldsymbol{\varphi}(t = T_\mathrm{R}, \mathbf{s})$, this method requires the n^2 VDEs (5) (the sensitivity system) to be solved for the sensitivity matrix $\mathbf{S}(t = T_\mathrm{R}, \mathbf{s})$ additionally.

The basic principle now is the synchronous integration of the nominal and the sensitivity system by means of the code DAGSL, see [6], a derivative of the famous DAE–solver DASSL, see [10]. Following roughly the algorithm described in [11], in every single time step t_m DAGSL first calculates the discrete solution $\mathbf{y}_m$ of the nominal system (1) by a BDF–approach (BDF–Backward Differentiation Formula) and a modified Newton iteration as usually. Immediately after convergence of the iterates $\mathbf{y}_m^q$, the n independent, discrete sensitivity vectors $\mathcal{S}_{i,m} = \partial\mathbf{y}_m/\partial s_i, i = 1, \ldots, n$ follow from an analogous BDF–discretisation of the variational equations (5) basing on the current solution $(\mathbf{y}_m, \dot{\mathbf{y}}_m)$ in a subsequent, sequential (or parallel) loop. Due to the linearity of (5) this means merely the additional solution of n systems of linear equations. It must be emphasised that the VDEs do not have to be defined by the user (or the surrounding MBS–algorithm generating the equations of motion) but are derived internally only on a purely numerical base by the DAGSL–algorithm itself. Let TOL be the error tolerances applied for the integration of the nominal system. Then, compared to the former finite differences approach, the approximation error of the Jacobian $\mathbf{Q}_\mathbf{s}$ is reduced from $\varepsilon_\mathrm{Q} = \mathcal{O}(\sqrt{TOL})$ down to $\varepsilon_\mathrm{Q} = \mathcal{O}(TOL)$ by this algorithm.

Application on a passenger car

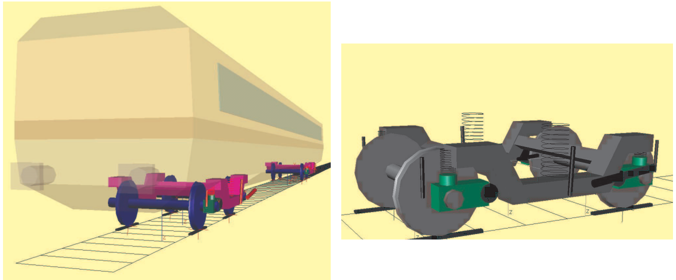

Figure 5: Simulation model of an *Avmz*–passenger car with Fiat 0270 bogies.

The software environment for bifurcation analysis of arbitrary mechanical systems mentioned above has been applied on a couple of simulation models of railway vehicles showing different degrees of complexity. In what follows, some results of the bifurcation analyses performed for a 1st class *Avmz* coach's model given in figure 5 are presented. The modelling is according to the benchmark description [12]. The complexity of this model is typical for models used within industry for the extensive computational analysis and design of a railway vehicle's running behaviour.

The mechanical model of the passenger car consists of altogether 15 rigid bodies. The suspension system comprises flexicoil springs with nearly parallel dampers, yaw and lateral dampers, as well as stiff lateral bump stops. All the springs are modelled with constant stiffness while each damper shows an only piecewise linear (thus nonlinear) force–velocity–characteristic with a serial stiffness superimposed; hence, the eigendynamics of these damper elements have to be considered, too. The state space of the vehicle model is described by altogether 114 position, velocity, and algebraic coordinates. Hence, a total of 9576 equations has to be integrated for limit cycle calculations (i.e. the nominal system (1) and the variational equations (5)).

Some results of the respective bifurcation analysis are displayed in figure 6. The dynamic long–time behaviour of the vehicle depending on the velocity v as varied system parameter is represented in the bifurcation diagram to the left by the maximum lateral deviation y of the leading wheelset with respect to the track. The stability of the limit cycle solutions can be evaluated by the evolution of the complex Floquet–multipliers (the eigenvalues of the monodromy matrix

$\mathbf{M} \equiv \mathbf{S}(T_\mathrm{P}, \mathbf{s}_\mathrm{p}))$ given to the right of figure 6: A solution is stable if and only if the moduli of all these multipliers (besides one) are less than one.

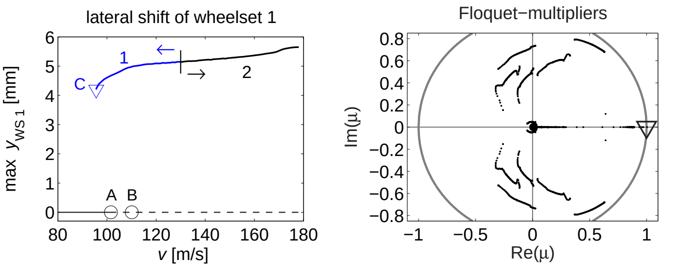

Figure 6: Bifurcation analysis of the *Avmz* passenger car. *Left:* Numerically computed bifurcation diagram. *Right:* Floquet–multipliers of the limit cycle solutions.

The stationary solutions are continued from nearly zero velocity up to the first Hopf bifurcation A, characterised by the velocity $v_\mathrm{lin} = v_\mathsf{A} = 101.62\,\mathrm{m/s}$, and beyond, where a second Hopf bifurcation B is detected. Up to now, it is not possible to continue the unstable limit cycle solutions branching off from Hopf bifurcation A. Therefore, the first step to continue periodic attractors is to generate an initial estimation with the help of a conventional, external time integration. Here, this was done for a velocity $v = 130.0\,\mathrm{m/s}$. For decreasing velocities, the path of stable periodic solutions ends in the saddle–node bifurcation C at $v_\mathrm{nlin} = v_C = 95.62\,\mathrm{m/s}$. This kind of bifurcation is indicated by the Floquet–multipliers with one of them crossing the stability limit, i.e. the unit circle, along the real axis. Since below this limiting velocity no oscillations occur, it also represents the critical velocity for this type of vehicle. For increasing velocities, the amplitude of the limit cycle oscillations grows rather slowly due to the nonlinear geometry of wheel and rail profiles and the increasing intensity of the contact between the flanges of the wheels and the rails.

Conclusions

The paper gives a short overview of the MBS–modelling of general railway vehicles and the bifurcation analysis of the resulting simulation models. Described partially is a software environment for the continuation based bifurcation analysis of complex mechanical systems that has been developed at DLR in recent years. As a result of this software project, bifurcation analysis is now available as an additional analysis tool of a software package for the simulation of mechanical systems – including railway vehicles. The application on a passenger car's 'complete' simulation model proves the applicability of the developed software on detailed, realistic and therefore necessarily complex models being of industrial relevance. Though throughout the paper a particular emphasis is laid on wheel–rail systems, the software environment's potential range of application is of course not restricted to this specific case. Decoupled form the MBS–code, the bifurcation algorithms can be applied on general dynamical systems just as well.

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