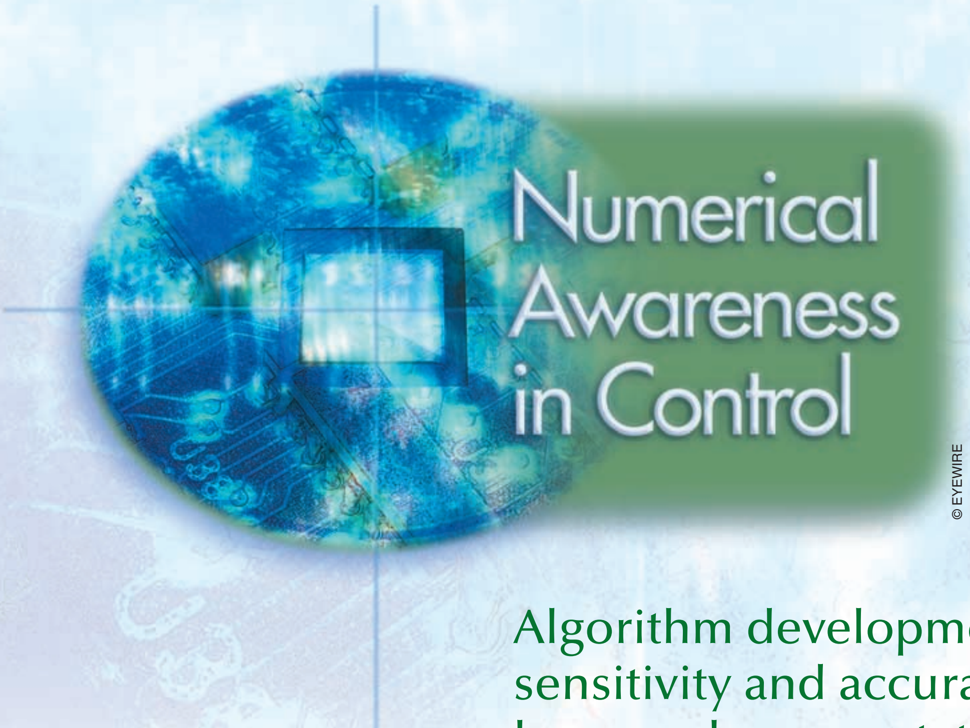
FEATURE

Numerical Awareness in Control



Algorithm development, sensitivity and accuracy issues, large-scale computations, and high-performance numerical software.

By Andras Varga

There is a continuing and growing need in the systems and control community for good algorithms and robust numerical software for increasingly challenging applications. Many of the computational problems encountered in practice are high dimensional and highly structured. Consequently, during the past several decades, the control field has been a rich source of computational problems for applied mathematicians and numerical analysts alike. It should not be surprising, then, that there exist many satisfactory algorithms that serve as the basis for robust numerical implementations of control methods. Moreover, several control design software packages exist both as commercial and free software. In view of this positive situation, the question arises: Is numerical awareness in control an issue of pressing importance?

Unfortunately, there are various signs that indicate a lack of numerical awareness in the control community. In papers dealing with methods for systems and control, we often observe a level of naivete concerning basic numerical issues related to algorithm development. For example, it is not uncommon to see improper usage of terms such as numerical stability, prob-

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lem conditioning, accuracy, and computational efficiency. In addition, unfounded claims of "numerical reliability" often appear, and methods published outside the control literature are rediscovered from time to time and occasionally republished. A more serious aspect of the lack of numerical awareness in control is blind confidence in popular black-box tools like those available in some commercial control toolboxes. For control researchers and practitioners, it may be surprising that such commercial tools, while effective on toy-size problems, can be inefficient, and sometimes unreliable, when applied to large-scale systems.

With regard to the difficulties of high dimension, there are many good methods available for solving large-scale control problems. For example, model reduction methods have been developed for reducing systems with state-space dimension $n > 10,000$ to a size that allows the application of existing control methodologies. We must also be aware, however, that there exist difficult low-order problems, even as small as $n = 2$, for which a satisfactory numerical approach is still lacking.

To judge the performance of a numerical algorithm, it is helpful to understand the requirements for satisfactory performance. Unfortunately, many algorithms used in commercially available tools are of questionable quality, and some are inappropriate for solving large-scale problems. In fact, some algorithms often fail for relatively low-order problems. However, such failures are not always caused by the underlying algorithms, but rather by their naive implementation. Therefore, to be useful for solving high-order control problems, robust numerical software must fulfill minimal quality requirements. It is perhaps not widely known that the best numerical control software is freely available and written in . . . Fortran 77! Moreover, for users of MATLAB, this software is easily accessible by means of appropriate gateways. Besides guaranteed numerical quality of the implemented algorithms, spectacular performance gains can occasionally be achieved.

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Challenges and Goals of Numerical Awareness

There are several ways to increase numerical awareness in the control community, one of which is to place more emphasis on numerical issues in the control curriculum. Ideally, the education of students would encompass examples of well-conditioned, intrinsically ill-conditioned, and artificially ill-conditioned problems. It is important that students appreciate the meaning of backward stability and realize that even the best algorithms can fail on ill-conditioned problems. Courses in numerical linear algebra must be encouraged as complementary to basic control education. An important aspect of numerical courses is the choice of an appropriate theoretical framework for describing control design algorithms. From a numerical point of view, choosing an appropriate framework is not a matter of taste but rather is often dictated by the relative sensitivities of different system representations such as state space, polynomial, or transfer function. In particular, the advantage of state-space models in numerical methods is due to their lower sensitivity as compared to polynomial and rational representations. We therefore believe that it is essential to balance the mathematical elegance of alternative system representations with attention to their intrinsic ill-conditioning.

Stimulation of research in computational control methods must be pursued. Research in algorithm development must not remain the privilege of applied mathematicians or a few control experts with numerical linear algebra skills. Developing numerical algorithms for control problems can also become an interesting activity for mathematically skilled students in control. There are many interesting computational problems waiting for algorithms to be worked out in the framework of masters-level research. Other, more difficult problems are well suited for doctoral dissertations. It is worth mentioning that the latest developments in numerical algorithms and software borrow control-related terminology such as intelligent algorithms, fault-tolerant methods, and even self-adapting numerical software. Regarding the future, we expect that numerical control methods will become increasingly adaptive and fault tolerant, relying on exploratory, intelligent, and nondeterministic search techniques supported by huge computer power such as massive parallel computation.

Stimulation of activities for implementing robust numerical software for control must be equally pursued. It is well known that developing control-relevant numerical software is concentrated around a few commercial products (Mathematica, Maple, MATLAB), while there are only a few active control groups (NICONET, INRIA, DLR) developing advanced numerical control software. The requirements for implementing generic robust numerical software is daunting and nowadays can be done only in the framework of international cooperation projects such as NICONET(Numerics in Control Network). On the other hand, most of the MATLAB control engineering software is research code written by scientists and engineers with a high-level control background but limited numerical expertise. Consequently, a cooperative agreement between Mathworks and NICONET has recently been arranged to include reliable Fortran-based routines in the Control Toolbox and replace old algorithms of dubious quality. NICONET maintains and develops SLICOT (Subroutine Library In COntrol Theory), a numerical library for control computations, which is freely available for teaching and research purposes. The NICONET project successfully involved several Ph.D. students in developing high-quality numerical control software for SLICOT.

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Establishing an information exchange forum for control-related numerical algorithms and software could be useful for popularizing good methods and software. It is important to showcase the best computational control algorithms and illustrate the best software practices for implementing such algorithms. An Internet site inside the homepage of the IEEE Control System Society's Technical Committee on Computer Aided Control Systems Design is a first attempt to gather information on control-relevant numerical methods and software (see http://www.robotic.dlr.de/control/cacsd/numerics/). Similar information is available on other links such as NICONET (http://www.win.tue.nl/niconet/) or my personal homepage (http://www.robotic.dlr.de/control/ num/).

It is unfortunate that relatively few individuals with an extensive control background are working on the development of computational methods for control. Consequently, computational control tends to be viewed as an ancillary activity in the research community. To improve this situation, the presence of numerical specialists on the editorial boards of control journals is essential for increasing interest in the publication of control-related numerical algorithms in control journals rather than in general numerical analysis journals. Computational control specialists can help assure that quality numerical papers appear in control journals, while encouraging future researchers to specialize in the development of algorithms and software in support of control technology.

Reprinting the best papers in control-related numerics as in *Numerical Linear Algebra Techniques for Systems and Control* (R.V. Patel, A.J. Laub, and P. Van Dooren, editors, IEEE Press, 1994) was a successful initiative. Since then, the development of numerical methods for control has continued to address many new domains such as descriptor systems, periodic systems, and large-scale model reduction.

Objectives and Contents of the Special Section

The objective of this special section is to increase numerical awareness in control. The contributors are leading numerical experts working on the cutting edge between applied mathematics and computational control. The four contributions address the main aspects of algorithm development for control, accuracy issues, large-scale computation, and robust and user-friendly numerical software development. The discussions should be easily accessible to control researchers and practitioners without extensive experience with numerical algorithms. Enlightening and often surprising examples are given to illustrate the main points of the discussions. The contents of the articles are sometimes critical, provocative, and speculative, while hints are given to suggest approaches to difficult computational problems.

The first feature, titled "The Basics of Developing Numerical Algorithms" by P. Van Dooren, focuses on the basics of developing algorithms for systems and control. This article identifies the general principles that lead to numerically reliable algorithms for solving a large collection of control problems. Techniques using orthogonal transformations are emphasized because they often lead to reliable numerical algorithms. This article focuses mainly on those problems for which both good algorithms and robust software implementations are available. In particular, several structured control problems are discussed, and the important role of structure-exploiting and -preserving numerically stable algorithms is emphasized.

The second article, "The Sensitivity of Computational Control Problems" by N. Higham, M. Konstantinov, V. Mehrmann, and P. Petkov, gives a comprehensive account of the main factors that contribute to the accurate and efficient numerical solution of mathematical problems such as those arising in control systems analysis and design. In simple terms, these factors are the sensitivity or conditioning of the mathematical problem under small changes in the problem data or model parameters (including the effects of round-off errors during finite precision computation), as well as the numerical stability of the computational method. Since these concepts are often confused, the first part of the article contains a comprehensive discussion of the basic concepts of numerical analysis along with a review of three factors that determine the accuracy of the results of a numerical computation, namely, the use of floating point arithmetic, the conditioning of the computational problem, and the stability of the algorithm. In the second part of the article, the authors present the sensitivity analysis of three basic problems in control theory, namely, eigenvalue assignment, optimal linear-quadratic control, and optimal H_∞-control, and they point out interesting aspects that play a role in the accurate solution of these problems.

The third article, "Solving Large-Scale Control Problems" by P. Benner, discusses approaches to solving large-scale control problems arising from high-fidelity models based on partial differential equations. By employing spatial semidiscretizations, state-space models of large state-space dimension n can result. For many of these models, standard direct methods, which are typically of computational complexity $O(n^3)$ and require the storage of dense $n \times n$ matrices, are not applicable. For such systems either parallel computing techniques can be employed in the case of dense matrices, or iterative methods, having much lower computational complexity than the direct methods, are applicable. The lower computational complexity of iterative techniques is usually achieved by permitting a certain loss of accuracy and by exploiting the sparse structure of system matrices. In this article, different approaches to two important problems, namely, linear quadratic regulator design and balanced model reduction, are discussed together with interesting applications.

Finally, the article "High-Performance Numerical Software for Control" by S. Van Huffel, V. Sima, A. Varga, S. Hammarling, and F. Delebecque concerns the development of quality software for control applications that perform efficiently and reliably on today's modern computing machines. In particular, the subroutine library SLICOT is described. This library is the main outcome of the European project NICONET. The article gives the background and motivation for the development of this library and describes its most important features. SLICOT is freely available for noncommercial use and provides many powerful user-callable routines for solving complex system analysis and control design tasks. Today SLICOT can be considered a synonym for reliable numerical algorithms, efficient computation, and robust numerical software for systems and control.

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The collection of articles in this special section spans a range of basic problems relevant to understanding and developing numerical algorithms for systems and control and to implementing them in robust numerical software. For those not familiar with numerical issues, the articles provide the opportunity to learn these basic concepts. I hope you will enjoy reading these articles as much as we enjoyed writing them!

Andras Varga (andras.varga@dlr.de) is a senior scientist at the German Aerospace Center in Oberpfaffenhofen. He received the diploma in control engineering in 1974 and the Ph.D. degree in electrical engineering in 1981, both from the University "Politechnica" of Bucharest, Romania. His main research interests are in developing reliable numerical methods and robust numerical software for computer-aided control system design (CACSD). He is a Fellow of the IEEE and past associate editor of *IEEE Transactions on Automatic Control*. He was program chair of the 1999 Symposium on CACSD and general chair of the 2000 Symposium on CACSD. Since 2000, he has been chair of the Technical Committee on CACSD within the IEEE Control Systems Society. For 2002–2003 he was a nominated member of the Board of Governors of the Control Systems Society. He received a research fellowship award from the Alexander von Humboldt Foundation. He coauthored three books, coedited one book, and published over 130 journal and conference publications. He can be contacted at the German Aerospace Center, DLR—Oberpfaffenhofen, Institute of Robotics and Mechatronics, D-82234 Wessling, Germany.