On the Dynamics of Conversation*

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Abstract

There is a longstanding debate in the literature about *static* versus *dynamic* approaches to meaning and conversation. A formal result due to van Benthem (1986, 1996) is often thought to be important for understanding what, conceptually speaking, is at issue in the debate. We introduce the concept of a *conversation system*, and we use it to clarify the import of van Benthem's result. We then distinguish two classes of conversation systems, corresponding to two concepts of staticness. The first class, the *strongly* static conversation systems, corresponds to a generalization of the class of systems that van Benthem's result concerns. The second class, the *weakly* static conversation systems, corresponds to a broader class, one permitting a certain commonly recognized form of context sensitivity. In the vein of van Benthem's result, we supply representation theorems which independently characterize these two varieties of conversation system. We observe that some canonically dynamic semantic systems correspond to weakly static conversation systems. We close by discussing some hazards that arise in trying to bring our formal results to bear on natural language phenomena, and on the debate about whether the compositional semantics for natural language should take a dynamic shape.

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1 Introduction

Consider two rough but familiar shapes that a theory of meaning for a fragment of natural language might take. One shape is *truth-conditional* and *propositional*. It consists substantially in a compositional assignment of truth-conditions to sentences of the fragment. (Or more abstractly, in a compositional mapping from sentences of the fragment to sets of points of evaluation in a specified model.) The truth-conditions of (or the proposition expressed by) a sentence reflect the informational content it encodes; and the meaning of a sentence is more or less identified with this informational content.[1] The legacy of this approach traces back at least to Frege and the early Wittgenstein. Modern paradigms of it include Lewis (1970) and Montague (1973).

The second shape is *dynamic*. Sentence meanings are not truth-conditions, not items of informational content. Rather, the meaning of a sentence is like an instruction for changing a state of information. Sentences are compositionally assigned functions which take a state of information as input and return an updated state of information. These *update functions*, or *context change potentials* (CCPs), are their meanings. The meaning of a sentence tokened in context directly acts on the state of the conversation, itself understood as a state of information. This approach traces back to the pathbreaking work of Karttunen (1969), Stalnaker (1975), Kamp (1981), and Heim (1982). From the perspective of this dynamic approach, the truth-conditional approach gets called *static*.

What, if anything, is at issue in the choice between these two shapes that a theory of meaning might take? How do we decide whether a natural language like English, or some fragment of it, should receive a static or a dynamic semantics?

From a big picture point of view, it seems obvious what to say. The thing to do is just go ahead and attempt to build the best theories in each style that we can dream up. We then compare the resulting theories for overall empirical adequacy and explanatory power, using the usual sort of criteria we use to evaluate any scientific theory.[2] The situation here is no different from other cases in which we have two competing theoretical explanations for the same phenomena.

While we endorse this general attitude toward the matter, we are interested in

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[1] For elaboration on the "more or less", see the discussions of the distinction between compositional semantic value and content in Dummett (1973), Lewis (1980), Stanley (1997), Ninan (2010), Rabern (2012), Yalcin (2014).

[2] See Schlenker (2009), Rothschild (2011), Lewis (2012, 2014), Yalcin (2015) for some recent work broadly in this vein.

examining a certain more direct line of response to the question, one we find in the literature (van Benthem 1986, 1996; Groenendijk and Stokhof 1991b; Dever 2006; von Fintel and Gillies 2007; van Eijck and Visser 2010). This response involves giving an abstract formal characterization of the properties which mark the main difference between the static approach and the dynamic approach. Once we have such a characterization, we can try to assess whether natural languages like English are best modeled as having or lacking these properties; and this in turn is a means of addressing, or at least beginning to address, the question whether the language is best modeled with a static or dynamic semantics.

On its face, this kind of response can seem puzzling. What do we even mean when we ask what the "main difference" is between a static approach and a dynamic approach? Didn't we already explain that difference at the outset? We said that a static approach compositionally assigns sentences functions from points of evaluation to truth values, whereas a dynamic approach compositionally assigns sentences context change potentials. What more could there be to say about the formal difference between these approaches?

The more that might be said concerns a question which we can, to a rough first approximation, put as follows:

> When can a dynamic semantics be "reduced" to a static semantics? Under what conditions does a given dynamic semantics admit of "straightforward translation" into a static semantics, so that everything which was achieved dynamically is instead achieved statically? And under what conditions is this *not* possible?

The background thought here is this: if what you achieve with your dynamic semantics could also be achieved by a straightforwardly static counterpart of that semantics, then there is some sense in which your dynamic semantics is not *interestingly* dynamic. It is only superficially dynamic, or dynamic in notation; it lacks "real dynamics" (as von Fintel and Gillies 2007 put it). The dynamic approach is thought to make possible varieties of explanation not achievable along static lines. But if your dynamic semantics does not actually exploit this additional power, then (it seems) it might just as well have been formulated statically. It isn't dynamic in any deep sense.

Of course, the question now becomes what it amounts to, formally speaking, for a semantics which is articulated dynamically to admit of "straightforward translation" or "reduction" to a static framework. And to approach this question, we need to give

a precise characterization of what a "static framework" for thinking about meaning and communication is supposed to be. There are a variety of imaginable approaches one could take to this question. But there now prevails in the literature a relatively standard way of thinking about the matter, which owes in different ways to the model of conversational update defended by Stalnaker (1978) and to a formal result due to van Benthem (1986). So we will start with that approach.[3]

2   The received idea of staticness

Before proceeding, we should separate two senses of 'context' relevant for understanding what follows. One notion of context corresponds to the idea of *the state of the conversation* at a given stage of the discourse. This is sometimes described as the *informational context* of the discourse. On one common way of thinking about it, the state of a conversation corresponds to the body of information that is mutually taken for granted by the interlocutors at that point in the discourse (Stalnaker (1978, 2002, 2014)). Communication happens by way of updating this shared body of information. This is the sense of 'context' in play in 'context change potential': a CCP is an operation on contexts understood as conversational states. The second notion of context is the notion familiar especially from Kaplan (1989). In this sense of 'context', the context is the concrete world, place, and time (centered world) of the conversation. A context in this second sense generally determines a context in the first sense (a conversational state), since one feature of any concrete discourse situation is the state of the conversation in that situation. In what follows we use c as a variable over conversational states and k as a variable over contexts in Kaplan's sense—centered worlds understood as locations of discourse.

Now we expect any compositional semantics, dynamic or static, to tell us how the meaning of a complex is expression is determined as a function of the meanings of its parts. A dynamic semantics will do this, but it does more besides: it tells us also how each sentence, when tokened unembedded, characteristically serves to update the state of the conversation. A static compositional semantics by itself does not encode this kind of information about conversational dynamics. In the most basic form of static setting, each sentence is ultimately associated with a proposition. To get from this to the characteristic sort of update the sentence imposes upon

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[3]For a different but related take on these issues, see Bonnay and Westerstahl (2014). They analyze the static/dynamic divide from the perspective of logical consequence. Their discussion includes a comparison of their results to some of those reported here.

the state of the conversation, we need something further. We need some sort of bridge principle which maps the static meaning of the sentence to a context change potential for the sentence. Only assuming some such principle can we meaningfully compare a static semantics to a dynamic semantics—at least from the point of view of conversational dynamics.

Stalnaker (1978), in some of the earliest work formalizing the notion of a conversational state, framed one natural bridge principle of this kind. On his picture, propositions are modeled as sets of possible worlds. A conversational state is understood to be the set of worlds in the intersection of the propositions which are common ground among the interlocutors—what Stalnaker calls a *context set*. Stalnaker assumes a static truth-conditional semantics, wherein a sentence ϕ is semantically associated with a proposition $[\![\phi]\!]^k$ relative to a context k. Stalnaker's bridge principle was designed for declarative fragments of natural language, and was intended to model the characteristic dynamic effect of assertion. The bridge principle he framed was very simple: the update of a successful assertion of ϕ in k on a conversational state c proceeds by intersecting the proposition $[\![\phi]\!]^k$ with c:[4]

STALNAKER ASSERTION RULE. $c[\phi^k] := c \cap [\![\phi]\!]^k$

The characteristic effect of asserting a proposition is the elimination of possibilities incompatible with it from the common ground. The package of a static mapping from sentences to truth-conditions together with Stalnaker's assertion rule provides us with one textbook example of a static picture of conversation. A key feature of this picture of conversational update is that it embraces what we can call PROPOSITIONALITY:

PROPOSITIONALITY. Conversational update is always just a matter of adding a proposition to the conversational state.

At least as concerns declarative fragments of language, static approaches to meaning and communication are often taken to assume PROPOSITIONALITY. Indeed, PROPOSITIONALITY is usually taken to be a key part of what makes any given approach to conversational update static. (Of course, one might embrace PROPOSITIONALITY without embracing Stalnaker's particular conceptions of propositions and conversational states. We return to this below.)

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[4]The CCP of a sentence ϕ is $[\phi]$, and its argument is placed to left (postfix notation): thus the result of updating a context c with ϕ is $c[\phi]$.

Recent discussions of the static/dynamic distinction have in essence taken a version of Stalnaker's assertion rule as a starting point for thinking about what it is for a system to be static. Here is the basic idea. First, set context sensitivity aside, and consider a fragment of language for which we have a mapping from sentences ϕ directly into propositions $[\![\phi]\!]$. Consider the following Stalnaker-inspired definition of CCPs for this fragment:

SIMPLE INTERSECTIVE RULE. $c[\phi] := c \cap [\![\phi]\!]$

The basic thought in the literature is that whenever you can take all the CCPs of a language fragment and re-express them in terms of a static mapping from sentences to propositions together with the above definition lifting each proposition to an intersective dynamic update, you have a fragment of language that can be considered static—even if it is superficially dynamic (in the sense of being stated as compositional assignment of CCPs to sentences).

There are some difficulties with this way of thinking about what staticness amounts to. But before we get into them, let us make the relevant concepts here more precise. The formal notion of staticness at issue here is best formulated at what we will call the *conversation systems* level of description:

Def 1. A *conversation system* is a triple $\langle L, C, \cdot[\cdot] \rangle$, where L is a set of sentences, C is a set of conversational states, and $\cdot[\cdot]$ is an *update function* from L to a set of context change potentials (unary operations) on C (i.e., $\cdot[\cdot] : L \to (C \to C)$).

A conversation system is simply mapping from sentences to operations on conversational states.[5] Note that the above definition presupposes nothing about the structure of conversational states and nothing about the structure of the language in question. Any dynamic semantics will determine a conversation system. But the reverse is not the case, as a conversational system abstracts from compositional details. If propositions and informational contexts are both modeled as sets of the same type (as in Stalnaker's framework), then any mapping from sentences to propositions, paired with a bridge principle like the simple intersective assertion rule, will also determine a conversation system.

Any conversation system determines what we call a *state system*:

Def 2. A *state system* is a pair of a set C of conversational states and a set O of unary operations (context change potentials) o on C (so that $o : C \to C$ for every $o \in O$).

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[5]This notion is equivalent to the computational notion of a deterministic labelled state transition system, and can be pictured as a directed graph with labeled arrows.

A state system is simply a set together with some operations on the set. Given conversation system $\langle L, C, \cdot[\cdot]\rangle$, the corresponding state system is just the pair $\langle C, O\rangle$, where $o \in O$ iff $o = [s]$ for some $s \in L$. (When we speak of the state system *of* a conversation system, or of a conversation system *having* a certain state system, this is the relation we have in mind.) Isolating the state system of a conversation system lets us focus on just the CCPs and their dynamics, abstracting from whether or not multiple sentences of the conversation system correspond to the same CCP.

Now the idea of a conversation system wherein (i) every sentence ϕ is associated with a truth-condition (set of points) $[\![\phi]\!]$, (ii) every conversational state is modeled by a set of points, and (iii) update always works in accord with the simple intersective rule, is exactly the idea of a conversation system which has an *intersective* state system:

Def 3. A state system $\langle C, O\rangle$ is *intersective* just in case $C \subseteq \mathcal{P}(W)$ for some set W, and there exists some set $P \subseteq \mathcal{P}(W)$ such that for all $o \in O$, there exists $p \in P$ such that for any $c \in C$, $co = c \cap p$. (A conversation system is *intersective* just in case its state system is intersective.)[6]

Putting it in our terminology, the leading formal idea of staticness we find in the literature is the idea of an intersective conversation system. A dynamic semantics which determines an intersective conversation system is said not to be "genuinely dynamic" (von Fintel and Gillies (2007)); it is "not really dynamic after all" because "it is equivalent with a static semantics with a globally defined notion of update" (Groenendijk and Stokhof, 1991b, 57). We will sometimes refer to this formal notion as "the received idea of staticness", but we do not mean to suggest that it is the only notion of staticness in the literature. It is just the one that seems to us to have had the most influence in formal discussions of the static/dynamic divide.

Aside from its simple intuitive appeal, one key reason this particular formal notion of staticness has attracted attention is the fact that it admits of a nice technical characterization. A well-known result due to van Benthem (1986, 1989) isolates a pair of formal properties that characterize the intersective conversation systems when the space of conversational states forms a powerset:

Fact 1. (van Benthem) A state system $\langle \mathcal{P}(W), O\rangle$ is intersective iff for all $c, c' \in \mathcal{P}(W)$ and $o \in O$,

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[6]We use postfix notation in connection with the operations of a state system, since we are thinking of them as context change potentials. (Thus co is the result of applying o to c; coo' is the result of applying o' to co; etc.)

Eliminativity. $co \cup c = c$

Finite distributivity. $(c \cup c')o = co \cup c'o$

Commenting on this result, Groenendijk and Stokhof (1991b) write that "a truly dynamic semantics will assign at least to some sentences updates which lack at least one of the properties of distributivity and eliminativity" (57). Dever (2006), von Fintel and Gillies (2007), and others pick up on this idea.

3 Situating the received idea of staticness

The received idea of staticness—the idea of an intersective conversation system—is an interesting one, and one useful for making the debate about static versus dynamic approaches to meaning and communication more precise. But its exact relevance for the large-scale question whether a theory of meaning for natural language should take a dynamic shape is much less direct than it may appear. There are various hazards applying this formal concept, and van Benthem's result, to the static/dynamic debate. Here are three of them.

I. *The conversation systems level of description abstracts from compositional semantic structure.*

Perhaps the most important point to emphasize is that the received formal idea of staticness is at the conversation systems level of description. This notion is at a high level of abstraction, one encoding primarily the conversational dynamics of the language. A conversation system for a language fragment is generally insufficient to determine facts about the compositional semantics of the fragment by itself. Without additional assumptions, little can be concluded about the structure of a semantics for language given only its conversation system.

One might try to use the notion of an intersective conversation system to define a notion of staticness at the level of compositional semantics. While that may be a worthwhile project, we will not explore the possibilities here. The main point we want to emphasize is that the received notion of staticness, while it is widely taken to bear on the question whether the compositional semantics for a fragment of language should take an explicitly dynamic shape (as a compositional assignment of CCPs), is really at some remove from issues about compositional semantics proper; it is not at the compositional semantic level.

(Thus while we opened the paper with a question about how to settle what sort of shape a theory of meaning for a given fragment of language should have, it turns out that the received formal concept of staticness does not really directly bear on that issue. Again, the received notion of staticness applies to conversation systems, not to compositional semantic theories. This paper is focused on notions of staticness/dynamicness that can be framed at the conversation systems level of description. We think this level of description is of interest in its own right, and that formalizing notions at this level may be a useful prelude to any attempt to formalize notions of static and dynamic at the compositional semantic level.)

II. *The received concept of staticness incorporates* INSENSITIVITY, *an idea that most theorists in the broadly static tradition would reject.*

The inspiration for the notion of an intersective conversation system is the SIMPLE INTERSECTIVE RULE, which was in turn inspired by the STALNAKER ASSERTION RULE. Both of these rules incorporate the idea of PROPOSITION-ALITY, an idea static approaches stereotypically embrace and dynamic approaches stereotypically transcend. But the SIMPLE INTERSECTIVE RULE also incorporates a second idea which is not built into STALNAKER'S ASSERTION RULE. This is an idea we can call INSENSITIVITY:

> INSENSITIVITY. Conversational update is always insensitive to the input conversational state.

Dynamic semantic approaches are often explicitly designed with an eye towards rejecting INSENSITIVITY. In many dynamic systems, the way a given sentence updates a conversation may depend on special features of the input conversational state.[7] Sensitivity to the flux of discourse is a hallmark of dynamic approaches. The rejection of INSENSITIVITY certainly is one component of dynamic pictures of communication.

However, it is important to be clear that one can embrace a stereotypically static approach to meaning and communication without accepting INSENSI-TIVITY. Notably, STALNAKER'S ASSERTION RULE can be accepted while INSENSITIVITY is rejected. The Stalnaker rule allows that sentences might be

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[7]For example, on the dynamic system of Heim (1982), the way that a sentence containing a free variable updates the conversational state depends on whether that variable is a discourse referent according to that state.

sensitive to context (in Kaplan's sense); and as noted above, any context k determines a conversational state c_k. Thus Stalnaker's assertion rule allows for the possibility that the proposition $[\![\phi]\!]^k$ which is to be intersected with the context set c_k can be such as to vary as a function of c_k. Indeed, Stalnaker was explicitly concerned to exploit this very possibility in early work; see Stalnaker (1975, 1978).

What this highlights is that the received formal notion of staticness is a rather austere one, one that does not actually encompass what might be considered canonical examples of static approaches to meaning and communication. The realm of application of this technical notion is thus more circumscribed than it may appear. Perhaps it is right to say, in the spirit of Groenendijk and Stokhof (1991b) and von Fintel and Gillies (2007), that the intersectivity of a conversation system is *sufficient* for staticness. What is much less clear is whether we should claim it is also *necessary* for staticness. On the contrary: if we want to count a theorist like Stalnaker in the static camp, then we should not say this.

One might think that these observations reveal that the notion of an intersective conversation system is not after all tracking an interesting concept of staticness. But this is not so. The kind of conversation system behind the received picture of staticness corresponds to a conceptually important class. Crudely speaking, it is the vanilla of conversation systems—it corresponds to one of the simplest, most boring, least surprising shapes communication could take. It frames a very natural starting point for thinking about declarative fragments of language. It is illuminating to formalize this class and find characterizing properties for it in the style of van Benthem. Other kinds of conversation system can be usefully characterized in terms of the way that they depart from this basic kind of conversation system.

So we should not abandon the concept of staticness behind the received picture. Instead we should seek to characterize a second concept, one that picks out the class of conversation systems that embrace PROPOSITIONALITY but not necessarily INSENSITIVITY. Such a formal notion would pick out a "weaker" notion of staticness, one that would encompass those theorists who, like Stalnaker, employ a static truth-conditional semantics wherein the proposition expressed by a sentence may be a function of the state of the conversation.

Indeed, we will formalize two concepts of staticness below. One concept picks

out the class of conversation systems that embody both PROPOSITIONALITY and INSENSITIVITY. We will call these systems *strongly* static. The other concept picks out the class of conversation systems that embody PROPOSITIONALITY but not necessarily INSENSITIVITY. We will call these systems *weakly* static.

III. *The target concept of staticness behind the received picture is more general than the concept of an intersective conversation system.*

Worries about INSENSITIVITY aside, there is another kind of concern one could have about the generality of the notion of an intersective conversation system.

First, if we identify the static conversation systems with the intersective conversation systems, we are building into staticness structural assumptions that seem intuitively irrelevant. One of these assumptions is the assumption that propositions can be modeled as unstructured sets. Another is the idea that conversational states can be represented as sets of the same type as propositions. If we think of PROPOSITIONALITY as the centerpiece of a static picture of conversation, then these assumptions seem quite orthogonal to the concept of staticness we are trying to model. Perhaps one thinks that sentences meanings are structured propositions, that conversational states are sets of structured propositions, and that assertion involves adding the structured proposition expressed to the conversational state. Such a picture seems to fit the stereotype of a static picture of conversation perfectly well. Nothing about it seems particularly dynamic. But such a picture is not well-modeled by an intersective conversation system.

Relatedly, suppose we assume nothing in particular about the structure of propositions, and we think of conversational states as sets of propositions. Take conversational update now to be a matter of putting the proposition expressed into the stock of propositions already in the context. That is, "adding a proposition to the context" is just unioning the context with the singleton containing the proposition. The class of state systems where update works in this way can be defined as follows:

Def 4. A state system $\langle C, O \rangle$ is *incremental* just in case for some set P, $C \subseteq \mathcal{P}(P)$, and for all $o \in O$, there exists $p \in P$ such that $co = c \cup \{p\}$ for any $c \in C$. (A conversation system is *incremental* just in case its state system is incremental.)

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If one wanted an intuitively static conversation system for some fragment of language, but also wanted to model propositions as structured objects, one might reach for an incremental system (or something like it). But obviously, incremental systems are not intersective.

Summarizing, a limitation that comes with identifying the static conversation systems with the intersective systems is that this requires assumptions about the structure of propositions, about the structure of conversational states, and about the nature of conversational update that are orthogonal to the aspect of conversation dynamics that we are trying to capture with a formal concept of staticness. Again, if we think of PROPOSITIONALITY as at the heart of the notion of staticness we are trying to define, then we can easily see that nothing about PROPOSITIONALITY *per se* requires unstructured propositions, or Stalnaker's particular intersective conception of assertion.

In the sections to follow, we will address the issues raised by (II) and (III) above, in reverse order. (We touch on (I) in the conclusion.) We will start by defining a concept of staticness at the conversation systems level that incorporates both PROPOSITIONALITY and INSENSITIVITY, but which applies to a broader class than just the intersective conversation systems. We will call this class of systems *strongly* static. We will then provide a representation theorem for this class of systems. This result will give us a reasonably strong formal handle on the intuitive ideas expressed by PROPOSITIONALITY and INSENSITIVITY, and one that escapes the concerns just mentioned in (III).

Next we define the class of conversation systems that embody PROPOSITIONAL-ITY but not necessarily INSENSITIVITY. These we call *weakly* static. We supply a representation theorem for this class of systems as well. This addresses the issue raised in (II).

Having defined these two classes of conversation systems and supplied them with representation theorems, we then raise the question: how does one tell, in connection with any given fragment of natural language, whether it is well-modeled by a strongly static (or weakly static) conversation system? Here we merely want to stress that while the representation theorems are useful here, applying them in practice can be less than straightforward. We review some of the pitfalls that arise in trying to bring these abstract formal results to bear on the analysis of natural language.

4 Strong staticness characterized

Our first objective is address the issue raised in (III) above. We want to pick out a natural class of systems that respect both PROPOSITIONALITY and INSENSITIVITY. We observed that the class of intersective conversation systems is a less than ideal choice here, as it leaves out many conversation systems that have both of these properties. Notably, it leaves out incremental systems. The CCPs in incremental systems serve to add something to the conversational state; they do not eliminate elements from the state, as in intersective systems. What we should like to find is a more abstract perspective, one on which both intersective and incremental systems will count as static.

To get there, it is useful to think about isomorphisms between state systems:

Def 5. State systems $\langle C, O \rangle$ and $\langle C', O' \rangle$ are *isomorphic* just in case there are bijections $f : C \to C'$ and $g : O \to O'$ such that $f(co) = f(c)g(o)$ for all $c \in C$ and $o \in O$.

Isomorphic systems have conversational dynamics with the same structure. Thus if we count the intersective systems as static, it is very natural to count any system that is isomorphic to an intersective system as static as well.

Indeed, we will define *strongly static* in terms of isomorphism to an intersective system:

Def 6. A state system is *strongly static* iff it is isomorphic to some intersective state system. (And a conversation system is *strongly static* iff its state system is strongly static.)

It turns out that this notion does supply a concept of a staticness according to which both the intersective and incremental systems count as static. Obviously, any intersective system is strongly static. Less obviously, any incremental system is also strongly static:

Fact 2. If a conversation system is incremental, then it is strongly static.

(See the appendix for proof.)

We suggested above that the target concept of staticness behind the received picture is more general than the concept of an intersective conversation system. Our suggestion is that the target concept is the concept of strong staticness just formal-

ized. We will take this formal notion to capture the relevant class of conversation systems obeying both PROPOSITIONALITY and INSENSITIVITY.[8]

Above (section 2) we noted that a widely discussed result due to van Benthem shows that the intersective systems correspond exactly the eliminative, finitely distributive systems. Now we have abstracted away from the intersective systems and fixed on the more general class of strongly static systems. In this context, van Benthem's result supplies us with a sufficient, but not a necessary, condition for strong staticness. That is to say: eliminativity and finite distributivity are jointly sufficient for strong staticness, but neither of these properties are necessary for strong staticness. One simple way to see this is to note that van Benthem's result requires the set of conversational states to have Boolean structure, whereas there are strongly static conversation systems which do not have Boolean structure (see Figure 1 for a simple example).[9]
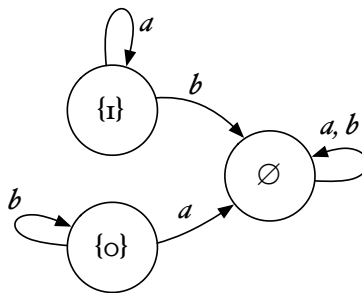

Figure 1: A strongly static state system whose cardinality is not a power of 2 (and hence which lacks Boolean structure). The arrows represent the operations on the states, the circles information states themselves. Thus, the graph depicts a state system $\langle C, O \rangle$ where $C = \{\{1\}, \{0\}, \emptyset\}$, $O = \{a, b\}$, a, the operation of intersection by $\{1\}$, and b, the operation of intersecting by $\{0\}$.

It is natural to seek both necessary and sufficient conditions for strong staticness.

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[8]Of course, there may be other interesting ways of formalizing these intuitive ideas. In that connection, one might ask why we do not instead define the strongly static systems as those isomorphic to some incremental system. The reason is that this would be a less ecumenical notion than ours, since some intersective systems are not isomorphic to any incremental system:

Fact 3. Not every strongly static system is isomorphic to some incremental system.

(See the appendix for proof.)

[9]A slightly more general result of Veltman (1996), discussed in the appendix, also gives sufficient but not necessary conditions for a state system to be static.

Just as van Benthem sought and isolated characterizing properties for the class of intersective systems, we can seek characterizing properties for the more general class of strongly static systems. We can ask:

> What general properties of a state system indicate whether or not it is a strongly static state system?

The main formal result of this paper is a representation theorem answering this question. A state system is strongly static if and only if it has the properties of *idempotence* and *commutativity*:

Fact 4 (Static representation). A state system $\langle C, O \rangle$ is strongly static iff for all $o, o' \in O$ and $c \in C$,

Idempotence. $coo = co$

Commutativity. $coo' = co'o$

(See appendix for proof.) This supplies an independent grip on the class of strongly static systems.

We can deploy this representation theorem as a test for strong staticness. The test can be applied straightforwardly to dynamic semantic systems in the literature. For example, consider the conversation system induced by file change semantics (Heim 1982, 1983a). We can say this system is not strongly static because it allows for violations of commutativity (e.g., sentence pairs of the form $Fx, \neg Gx$ are not commutative). Dynamic predicate logic (Groenendijk and Stokhof 1991a,b) and usual versions of update semantics (Veltman 1996) are not strongly static because their conversation systems are neither commutative nor idempotent. This formal result accords with the widely-held view that conversational update in "interestingly dynamic" systems is not generally commutative.[10]

5   Weak staticness characterized

We have addressed the issue raised in (III) of section 2 above. Let us turn now to the issue raised in (II) of that section. As we have noted, strong staticness incorporates

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[10]Non-commutativity of update should not be confused with the idea that conjunction is non-commutative. The fact that the state system of a conversation system is static (or not) entails nothing by itself about the semantics of conjunction (indeed, the language may not even contain a conjunction operator). However, if it is assumed, following Stalnaker (1974) and Heim (1983b), that an unembedded conjunction is equivalent to consecutive assertions of the two conjuncts, then staticness will require unembedded conjunctions to be commutative.

INSENSITIVITY, and that idea is one that many theorists who align themselves with a stereotypically static perspective on meaning and communication would reject. In this section, we look at the possibility of retaining PROPOSITIONALITY but dropping the requirement of INSENSITIVITY by permitting a kind of context sensitivity into the conversation system of a language: sensitivity of the proposition expressed to the conversational state itself. Many authors have explored this form of context sensitivity (for some examples, see Stalnaker 1975, 1978, 1998, 2014; Lewis 1979; Heim 1982; von Fintel 2001; Yalcin 2007; Kripke 2009; Klinedinst and Rothschild 2012). The class of conversation systems that embody PROPOSITIONALITY but not necessarily INSENSITIVITY we call *weakly* static. These are the systems where (i) each sentence serves to add a proposition to the conversational state, but (ii) at least sometimes, the proposition added is a function of the conversational state being updated.

To arrive at a formal definition of the weakly static systems, it is once again useful to begin our thinking with state systems that are Stalnakerian in shape, wherein propositions and conversational states are represented as sets of the same type. Restricting attention to these kind of systems for the moment, the weakly static systems are like the intersective systems in that the update of a conversational state can always be represented as the result of intersecting of it with a proposition. The key difference is that there is not necessarily a stable, conversational state-independent mapping from sentences to propositions in a weakly static system. At best there is a mapping from sentence-conversational state pairs into propositions.

Limiting attention to the weakly static systems that fit this Stalnakerian shape, it is clear that these systems are just the systems where every update of theconversational state yields a conversational state that is subset of the original state. That is to say, these systems are the *eliminative* systems:

Def 7. A state system $\langle C, O \rangle$ is *eliminative* just in case $C \subseteq \mathcal{P}(W)$ for some set W, and there exists some set $P \subseteq \mathcal{P}(W)$ such that for all $o \in O$ and $c \in C$, there exists $p \in P$ such that $co = c \cap p$.[11]

Now the eliminative systems are to the weakly static systems what the intersective systems are to the strongly static systems. And just as we defined strong

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[11]This definition was chosen to facilitate comparison with the definition of the intersective systems: notice that these definitions differ only in the order of the quantifiers appearing in them. A simpler but equivalent definition of eliminativity would be: a state system $\langle C, O \rangle$ is *eliminative* just in case $C \subseteq \mathcal{P}(W)$ for some set W, and for all $c \in C$ and $o \in O, co \subseteq c$.

staticness in terms of isomorphism to an intersective system, we will define the weakly static systems in terms of isomorphism to an eliminative system:

Def 8. A state system is *weakly static* iff it is isomorphic to a eliminative system. (A conversation system is *weakly static* iff its state system is weakly static.)

The rationale for identifying the weakly static systems with those isomorphic to some eliminative system, rather than with the eliminative systems, is the same as the rationale for identifying the strongly static systems with those isomorphic to some intersective system, rather than with the intersective systems. We want to abstract away from specific assumptions about how conversational states and propositions are modeled, at least insofar as these assumptions are orthogonal to the embrace of PROPOSITIONALITY.

It should be clear that any intersective system is eliminative. Thus all strongly static state systems are weakly static systems. For a toy example of a system which is weakly static but not strongly static, see Figure 2.
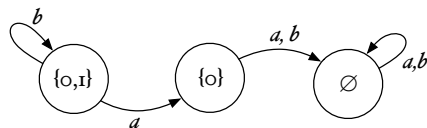

Figure 2: A strongly static state system. The system is antisymmetric (as defined below) but is neither commutative nor idempotent.

We may seek formal properties independently characterizing the class of weakly static systems, just as we sought formal properties characterizing the strongly static systems. One crucial formal property weakly static systems clearly exhibit is that they prohibit backpedaling: once the conversation moves beyond a given conversational state, it cannot return to that state by any series of operations. Formally speaking, this property is *antisymmetry*:

Def 9. A state system $\langle C, O \rangle$ is *antisymmetric* iff for all $c, c' \in C$, if c is O-reachable from c' and c' is O-reachable from c, $c = c'$ (where c' is *O-reachable* from c just in case $c = c'$ or there exists $o_1, ...o_n \in O$ such that $co_1...o_n = c'$).

Indeed, it turns out that antisymmetry is both necessary and sufficient for weak staticness:

Fact 5. A state system is weakly static just in case it is antisymmetric.

(See the appendix for proof.) Thus antisymmetry is to weak staticness as commutativity and idempotence are to strong staticness.

The class of state (conversation) systems that are weakly static is quite broad. It includes the conversation systems induced by a number of key dynamic semantic systems in the literature—notably Heim's file change semantics (Heim 1982) and Veltman's update semantics (Veltman 1996). This may come as something of a surprise—these are, after all, canonical examples of dynamic semantic systems. What we see here is that once we have a notion of staticness that is broad enough to encompass a sort of context sensitivity that most anyone in the static tradition of semantics would be happy to allow (namely, sensitivity to the conversational state), we find that the notion actually encompasses classic examples of dynamic systems.

6 Investigating conversational dynamics in natural language

If we are investigating the conversation system of some fragment of natural language, our results tell us that the system is strongly static iff it is idempotent and commutative, and that it is weakly static iff it is antisymmetric. But how do we tell in the first place whether the conversation system for a fragment of language is idempotent, commutative, or antisymmetric? (Or anything else?) Or to put the question differently: in modeling a fragment of natural language, when is best to reach for a (strongly or weakly) static conversation system?

Generally speaking, static conversation systems (weak and strong) provide a natural starting point in theorizing about arbitrary declarative fragments of natural language. These systems correspond to two simple and paradigmatic ways that communication might work. Sorting out whether certain phenomena call for a departure from a static perspective is often not straightforward matter. That is the main point of this section. To make this point, we are going to look at some *prima facie* failures of idempotence and commutativity.[12] On the face of it, these examples might be taken to recommend conversation systems for the relevant language fragments that lack one or more of these properties. But very often there are alternative ways that the data might be explained (or explained away). We are going to highlight some of those alternative ways. The observations of this section will be elementary. But

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[12] *Prima facie* failures of antisymmetry seem to us strikingly harder to come by. (But see Yablo (2014) on epistemic modals: he suggests that epistemic possibility modals can sometimes serve as devices for stepping the conversation back to a previous state.)

they will help to bring out some complications that arise in theorizing about natural language at the conversation systems level.

We are not interested in actually settling particular questions about whether this or that phenomena does or does not call for a non-static conversation system. Our purpose is only to highlight some alternative avenues of explanation, in order to help clarify the empirical bearing of our results.

A *prima facie* counterexample to idempotence would be any case wherein tokening the sequence ϕ, ϕ differs in acceptability or communicative import from ϕ; likewise a *prima facie* counterexample to commutativity would be any case where tokening the sequence ϕ, ψ differs in acceptability or communicative import from ψ, ϕ. In this section we review three ways of explaining such apparent counterexamples away. These involve (i) exploiting the distinction between phonological form and logical form; (ii) building context sensitivity into the language at the conversation systems level; and (iii) exploiting the secondary changes that speech acts make to the conversational state. We discuss each in turn.

6.1   Separating phonological form and logical form

The following discourses do not generally have the same communicative import:

(1)   a.   Bob turned around. A man walked in. He looked angry.
        b.   Bob turned around. He looked angry. A man walked in.

On the face of it, the only difference between the discourses concerns the order of the last two sentences. So this is a *prima facie* counterexample to commutativity.

But one can accommodate these data without dropping commutativity at the conversation systems level. One might instead postulate a principle governing the logical forms of sentences containing definites and indefinites in discourse, one that permits an anaphoric relationship between 'a man' and 'he' in the first discourse but which prohibits it in the second discourse. (The Novelty-Familiarity Condition proposed by Heim (1982) is an example of such a principle.) A standard way of implementing such a principle assumes that these expressions are equipped with covert indices in logical form, indices that track anaphoric relations. Given such a view, the sentences in (1) do not wear their logical forms on their sleeves. The natural understanding of (1-a) corresponds to:

(2)   Bob$_1$ turned around. [A man]$_2$ walked in. He$_2$ was tall.

Now if we wanted to commute the last two sentences in the strict sense relevant to the conversation systems level of description, we would require a discourse with the following structure:

(3) Bob_1 turned around. He_2 was tall. $[A man]_2$ walked in.

But this is just what would be disallowed by the discourse-level principle under consideration. Such a principle would preclude the well-formedness of this discourse. The natural understanding of (1-b) corresponds instead to:

(4) Bob_1 turned around. He_1 looked angry. $[A man]_2$ walked in.

On this understanding of the situation, (1-a) and (1-b) do indeed differ in communicative import, but (1-b) does not in fact involve the commutation of the last two sentences of (1-a), despite appearances; hence we have no counterexample.

The simple point is that on the theoretically relevant notion of 'sentence', distinct sentences may have the same phonological form, and this may give rise to the illusion of commutativity or idempotence failure.

6.2   Building context sensitivity into the language at the conversation systems level

There is a second way that the notion of 'sentence' relevant to the conversation systems level of description might require us to individuate sentences with something more than just surface phonological form alone. This arises in connection with context sensitivity—using 'context' now in the Kaplanian sense. Consider:

(5)   a.   *Speaker A:* I love you.
       b.   *Speaker B:* I love you.

(6)   a.   This [*pointing to the lamp*] is old.
       b.   This [*pointing to the table*] is old.

In each of these cases, the relevant (b.)-sentence obviously serves to add new information to the common ground. Superficially, these are counterexamples to idempotence. But it is intuitively clear that there is nothing especially dynamic going on here. It could plausibly be maintained that each of the sentences in these examples is serving to add a proposition to the conversational state, along familiar static lines. It is just that the propositions expressed by the respective (b.)-sentences are

different from the propositions expressed by the respective (a.)-sentences, owing to context sensitivity.

Now we have already discussed a particular kind of context sensitivity, namely, sensitivity to the conversational state. The formal concept of weak statcness captures that notion at the conversation systems level. But how should we think about the more routine sort of context sensitivity we observe in expressions like indexicals and demonstratives from a conversation systems perspective—especially if we are inclined to given their semantics broadly in the style of Kaplan (1989)?

At the conversation systems level, context sensitivity of this kind can be understood as requiring that the 'sentences' of the conversation system be modeled by what we would intuitively think of as sentence-context pairs (again, with 'context' understood in the Kaplanian sense). If we picture a conversation system as a state transition system, one version of the idea is that the labels for the transitions of the system are given by sentence-context pairs. See Figure 5 for a simple illustration of the idea in connection with example (5).
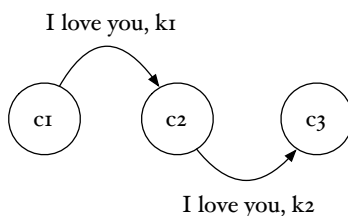
I love you, k1

c_1 c_2 c_3

I love you, k2

Figure 3: An idempotence-compatible fragment of a conversation system for example (5). $k1$ and $k2$ are worlds centered on distinct speakers and times (i.e., distinct contexts).

This helps us to see why (5) does not impugn idempotence. Idempotence does not require state c_2 to equal c_3, because the arrow connecting c_2 and c_3 does not share a common label with the arrow connecting c_1 and c_2. (5-a) does not correspond to same sentence as (5-b) at the conversation systems level.[13]

Another way to think about matters (in much the same spirit) would be to hold that context-sensitive expressions are systematically replaced by context-invariant

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[13]As an anonymous reviewer notes, since the state of the conversation is a component of any context, one theoretically can smuggle sensitivity to the conversational state into a strongly static conversation system by enriching the language of the conversation system in the style described. There is nothing in the technical notions alone here that could prevent a theorist from making this transparently bad modeling decision. We are taking it that resort to a richer conversation system language would only be motivated in the presence of context sensitivity which is not sensitivity to the conversational state.

sentences in the language of the corresponding conversation system, with the mapping determined with the help of the semantics of the language. So given a suitable context, (5-a) will correspond in the language of the conversation system to something tantamount to 'Romeo loves Juliet', and (5-b) will correspond to 'Juliet loves Romeo'. Thus again, idempotence is preserved.

The basic move here is as simple and as it is powerful: any *prima facie* counterexample to idempotence might in principle be blamed in this way on tacit context sensitivity instead. Whether or not this is a plausible move to make will of course depend on the particularities of the case.

6.3 Exploiting secondary changes to the conversational state

A third way one might maintain staticness in the face of seeming counterexamples is to appeal to the simple fact that in anything like normal discourse, the conversational state which results from the update of a sentence in that discourse is never identical to the conversational state that is updated by the CCP of the subsequent sentence in that discourse. This owes to the fact that the conversational state is undergoing constant changes, changes not owing to the CCPs of the sentences used in the discourse.

This is particularly obvious in the case of (6-a) and (6-b). There an event of ostension that occurs in between sentence tokenings, and this event will become common ground as soon as it happens in any normal conversation. As a result, the update associated with (6-b) will, strictly speaking, apply to something other than the output of the update associated with (6-a). And this entails that this discourse is not—at least without further argument—an example of a failure of idempotence.

Other examples can be explained in the same fashion. Suppose a sergeant is inspecting a cadet's uniform. He shouts, "Turn around!" The cadet obliges. The sergeant, satisfied that the cadet's shirt is properly tucked, again shouts "Turn around!" The cadet then returns to his original position. Here it is clear that the sergeant's second command is not redundant, and nor is it merely adding emphasis to the first command. Nevertheless, plausibly this involves no failure of idempotence, as the second command does not update the conversational state resulting from the update due to the first command. Rather, it updates a conversational state which incorporates (*inter alia*) the information that the first command was satisfied. If the cadet had not obliged by turning around after the first command, the sergeant's full discourse ("Turn around! Turn around!") would have amounted to one (emphasized) directive to turn 180 degrees, not a directive to turn 360 degrees. Thus the example

is not a counterexample to idempotence.

The general point here has very wide application. As Stalnaker has emphasized in many places (for example, Stalnaker 1978, 1998, 2014), the result of successfully asserting a sentence is not only to update the conversational state by adding to it the proposition expressed. It also changes the conversational state by adding to it the information that that very utterance was just made. Even if I don't accept your assertion, we will go on to take for granted that you uttered a certain sentence at a certain time, and that itself constitutes a change to the conversational state. We could call this the *secondary effect* of the speech act on the conversational state. The fact that speech acts have secondary effects on the conversational state presents an obstacle to probing idempotence and commutativity in natural language. In theorizing from a conversation systems perspective, we abstract from secondary effects. But there is no escape from secondary effects in ordinary communication. As a consequence, merely reversing the order in which sentences are uttered does not generally result in commutation in the strict sense we have in mind at the conversation systems level, because the secondary effects corresponding to the sentences induce intermediate changes to the conversational state. Likewise, merely repeating a sentence does not yield a case where the context change potential of a sentence applies to its value relative to an initial conversational state. Figure illustrates the basic difficulty in connection with commutativity.
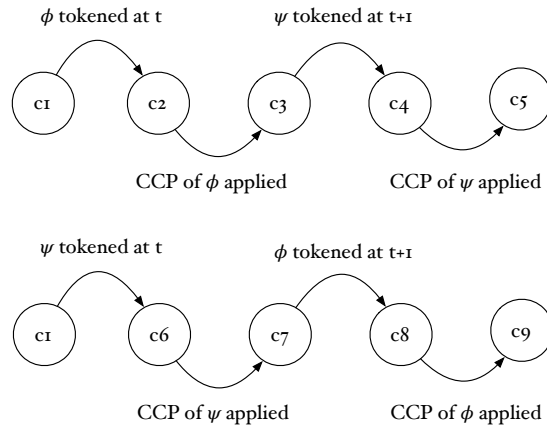

Figure 4: Merely reversing the order of sentences in ordinary natural language conversation does not result in commutation. Commutativity does not require that $c_5 = c_9$.

One might have thought that if a conversation system containing ϕ and ψ is com-

mutative, it would follow that tokening ϕ, and then ψ, against a conversational state c_1 would have to result in the same conversational state as would result from tokening ψ, and then ϕ against c_1. But in real examples, there is no way to apply two context change potentials in immediate succession, for the update effect of the CCP of a sentence is always preceded by the update corresponding to its secondary effect. Since commutativity is a claim about the equivalence of two ways of applying a pair of context change potentials in immediate succession, commutativity is not the sort of thing we can immediately observe (or fail to observe) in ordinary natural language examples. The point applies *mutatis mutandis* to idempotence.

This highlights the extent to which properties like idempotence and commutativity are at some nontrivial remove from observation. To say any given phenomena supports or undermines commutativity or idempotence is in part to make a substantive judgment about the role of these secondary effects on conversational update.

7 Closing

In closing, we want to reiterate the points made in (I) of section 3 above. In theorizing with conversation systems, we abstract from compositional semantic structure. We should thus distinguish two rough levels at which a static/dynamic distinction might be framed:

> *Conversation systems dynamicness.* The CCPs of the sentences of the language are not each equivalent to an operation which adds a proposition to the conversational state.

> *Compositional semantic dynamicness.* The compositional semantic values of sentences are identical to their CCPs.

Picking up on the received formal concept of staticness in the literature, this paper has investigated static/dynamic distinctions at the conversation systems level. The relation between staticness/dynamicness at the conversation systems level and the question whether a language fragment requires something tantamount to a dynamic compositional semantics is not straightforward, and is not something we have tried to explore here. If one is interested in whether the compositional semantics of some natural language fragment must take a dynamic shape—and it seems fair to say the debate about static versus dynamic approaches to meaning in the literature is largely about that—then one is really interested in a question about compositional

dynamicness. One of our key points has been that the received formal concept of staticness is not at this level of description, and so has less to do with semantics than one might naturally assume. Our concern in this paper has really been with the dynamics of conversation, and not with the dynamics of meaning (if such there be).

Stalnaker (2014) has said that he prefers "a dynamic pragmatics to the dynamic semantic story" (65). One could understand the work above as characterizing formal notions of dynamicness at the pragmatic level.

It seems to us that questions of compositional dynamicness remain to be formalized clearly. Just as we isolated properties of conversation systems that correspond to certain interesting features of conversational dynamics, so one might try to seek formal properties of compositional semantic systems that make them "interestingly dynamic". Perhaps the results above will prove useful in clarifying what exactly it would be for a compositional semantics to be interestingly dynamic. We suspect that one key aspect of this project will involve formalizing the notion of a *local context*, a concept essential to standard dynamic semantic explanations. Work in this direction may help to clarify further just what is conceptually at issue in the debate over dynamic versus static approaches to meaning.

A   Appendix

This appendix supplies proofs of the facts about conversation systems cited in the main text. Facts 2 and 3 concern the relation between strong staticness and incremental systems. Fact 4 is the representation theorem for strong staticness. Fact 5 characterizes the weakly static systems. Facts 1 (van Bethem) and 6 (Veltman) follow from Fact 4.

Fact 2. If a conversation system is incremental, then it is strongly static.

Proof. Suppose $\langle C, O \rangle$ is an incremental conversation system. Then for some set P, $C \subseteq \mathcal{P}(P)$, and for all $o \in O$, there exists $p \in P$ such that $co = c \cup \{p\}$ for any $c \in C$. Define $[\![\cdot]\!] : O \to P$ such that for all $o \in O : [\![o]\!] :=$ the p such that for all c, $co = c \cup p$. Define $j : O \to \mathcal{P}(P)$ as follows: $j(o) := P \backslash \{[\![o]\!]\}$ $(= \{[\![o]\!]\}^c)$. Define $h : C \to \mathcal{P}(P)$ as follows: $h(c) := c^c$. Clearly h is an injection.

Now it remains to show that $h(co) = h(c) \cap j(o)$ for all $c \in C$ and $o \in O$. Since $\langle C, O \rangle$ is incremental, $co = c \cup \{[\![o]\!]\}$. Taking the complement of each side, $(co)^c = (c \cup \{[\![o]\!]\})^c$. Distributing on the right, $(co)^c = c^c \cap \{[\![o]\!]\}^c$. Hence $h(co) = h(c) \cap j(o)$.

□

Fact 3. Not every strongly static system is isomorphic to some incremental system.

Proof. Consider an intersective conversation system $\langle C, O \rangle$ with o, o' and $o \wedge o'$ in O, such that: (i) $c(o \wedge o')o = c(o \wedge o')$; (ii) $c \neq co \neq c(o \wedge o')$. Suppose for contradiction the system is isomorphic to an incremental system $\langle C', O \rangle$. Then there exists a bijection $h : C \to C'$ and a function $j : O \to \{\{c'\} : c' \in C'\}$ such that for all $c \in C, o \in O, h(c) \cup j(o) = h(co)$. Given such a mapping, it follows that

$$h(c(o \wedge o')o) = h(c) \cup j(o \wedge o') \cup j(o)$$

From this and (i), it follows that

$$h(c) \cup j(o \wedge o') \cup j(o) = h(c) \cup j(o \wedge o')$$

Hence $j(o) \subseteq h(c)$ or $j(o) \subseteq j(o \wedge o')$. Suppose $j(o) \subseteq h(c)$. Then $h(c) = h(c) \cup j(o) = h(co)$. Since h is a bijection, $c = co$, contradicting (ii). So suppose instead $j(o) \subseteq j(o \wedge o')$. Since j is into singletons, it follows that $j(o) = j(o \wedge o')$. Hence $h(co) = h(c(o \wedge o'))$. Therefore $co = c(o \wedge o')$, contradicting (ii).

□

To state Fact 6, we define the notion of an *information lattice* after Veltman (1996):

Def 10. A quadruple $\langle V, \top, \wedge, \leq \rangle$ is an *information lattice* iff V is a set, $\top \in V$, $\wedge$ is a binary operation on V, and $\leq$ is a partial order on V such that for all $c, c' \in V$:

$\top \wedge c = c$

$c \wedge c = c$

$c \wedge c' = c' \wedge c$

$(c \wedge c') \wedge c'' = c \wedge (c' \wedge c'')$

$c \leq c'$ iff there is some c'' such that $c \wedge c'' = c'$.[14]

Then we can state Veltman's result in our terminology as follows:

Fact 6. A state system $\langle C, O \rangle$ is strongly static if there exists an information lattice, C_I, $C_I = \langle C, \top, \wedge, \leq \rangle$, such that for all $c, c' \in C$ and $o \in O$,

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[14]The specification of $\leq$ adds no structure as it is induced by $\wedge$, but we will find the explicit specification convenient below. An intuitive gloss on $c \leq c'$ would be "c' is at least as informationally strong as c".

Idempotence. $coo = co$

Persistence. If $co = c$ and $c \leq c'$ then $c'o = c'$

Strengthening. $c \leq co$

Monotony. If $c \leq c'$ then $co \leq c'o$

Fact 6 applies to a broader class of conversation systems than Fact 1 (van Benthem), since any Boolean algebra is an information lattice but not *vice versa*. Both Facts 1 and 6 straightforwardly follow from the following fact:

Fact 4 (Static representation). A state system $\langle C, O \rangle$ is strongly static iff for all $o, o' \in O$ and $c \in C$,

Idempotence. $coo = co$

Commutativity. $coo' = co'o$

Proof. Any system isomorphic to an intersective system is idempotent and commutative, since intersection is idempotent and commutative. Hence we need only show that if a state system is idempotent and commutative, then it is static.

Let $\langle C, O \rangle$ be an idempotent and commutative state system. To show $\langle C, O \rangle$ is static, it suffices to produce an injection $h : C \to \mathcal{P}(C)$ and a function $j : O \to \mathcal{P}(C)$, such that $h(co) = h(c) \cap j(o)$ for all $c \in C$ and $o \in O$.

Define $j : O \to \mathcal{P}(C)$ as follows: $j(o) := \{c \in C : co = c\}$ for all $o \in O$. Thus j takes o to the set of its fixed points.

Define $h : C \to \mathcal{P}(C)$ as follows: $h(c) := \{c' \in C : cRc'\}$, where cRc' just in case $c = c'$ or c can reach c' from operations in O (i.e., there exist $o_1 \ldots o_n \in O$ such that $co_1 \ldots o_n = c'$). Obviously, R, the relation of O-reachability, is reflexive and transitive.

Note that from commutativity, it follows that order does not affect update: successive update of c by $o_1 \ldots o_n$ is equal to successive update of c by any reordering of $o_1 \ldots o_n$, for any $c \in C$ and $o_1 \ldots o_n \in O$.

To see that h is an injection, note R is antisymmetric. Suppose cRc' and $c'Rc$, but for contradiction $c \neq c'$. Then $cu_1 \ldots u_n = c'$ and $c'v_1 \ldots v_m = c$, for some $u_1 \ldots u_n, v_1 \ldots v_m \in O$. For convenience define U as the functional composition $u_i \circ \ldots \circ u_n$ (where $f \circ g := g(f(x))$); likewise $V = v_1 \circ \ldots \circ v_m$. Hence $cU = c'$ and $c'V = c$, and hence $cUV = c$. By commutativity, $cVU = c$. Hence $cVUU = cU$. By idempotence $cVUU = cVU$, so substituting, $cVU = cU$. Substituting again, $c = c'$. Contradiction.

Now if $h(c) = h(c')$, then cRc' and $c'Rc$, since R is reflexive. By antisymmetry, $c = c'$. Hence h is an injection.

Now we show that $h(co) = h(c) \cap j(o)$ for all $c \in C$ and $o \in O$. This equivalent to showing that $\{c' \in C : coRc'\} = \{c' \in C : cRc'\} \cap \{c' \in C : c'o = c'\}$, which is equivalent to showing that for all $c, c' \in C$ and $o \in O : coRc'$ iff cRc' and $c'o = c'$.

Left-right: suppose $coRc'$. There are two possibilities: (i) $co = c'$. Then clearly cRc'. Moreover, by idempotence $coo = co$, so substituting, $c'o = c'$. (ii) For some $u_1 \ldots u_n \in O$, $cou_1 \ldots u_n = c'$; that is, $coU = c'$. By commutativity, $coU = cUo$. Hence $cUo = c'$. By idempotence $cUo = cUoo$. Substituting, it follows that $c' = c'o$. Moreover, obviously cRc'. Hence cRc' and $c'o = c'$.

Right-left: suppose that cRc' and $c'o = c'$. There are two possibilities. (i) $c = c'$. Then of course $coRc'$. (ii) $cu_1 \ldots u_n = c'$ for some operations $u_1 \ldots u_n \in O$, i.e., $cU = c'$. Then $cUo = c'o$. Substituting, $cUo = c'$. By commutativity, $coU = c'$. Hence $cRc'o$.

This yields an intersective update system isomorphic to $\langle C, O \rangle$, namely the system $\langle h[C], O' \rangle$, where $h[C]$ is the image of h under C and O' is the set of all operations $o' \colon h[C] \to h[C]$ such that for some $o \in O$ and any $c' \in h[C] : c'o' = c' \cap j(o)$.

$\square$

(A shorter proof is possible using notions from abstract algebra.)

Fact 5. A state system is weakly static just in case it is antisymmetric.

Proof. Left-to-right: any system isomorphic to a quasi-intersective system is antisymmetric, since in such systems update can only map a conversational state to itself or to a subset of itself.

Right-to-left: suppose $\langle C, O \rangle$ is antisymmetric. To show $\langle C, O \rangle$ is information-sensitive, it suffices to produce an injection $h : C \to \mathcal{P}(C)$ and a function $i : (O \times C) \to \mathcal{P}(C)$, such that $h(co) = h(c) \cap i(o, c)$ for all $c \in C$ and $o \in O$.

Let $h(c) := \{c' \in C : cRc'\}$. Let $i(o, c) := h(co)$. Obviously R is reflexive, and since $\langle C, O \rangle$ is loopless it is antisymmetric. Hence h is an injection. Now if $coRc'$, then clearly $cR'c$. Hence $h(co) \subseteq h(c)$. Hence $h(co) = h(c) \cap h(co)$. Hence $h(co) = h(c) \cap i(o, c)$.

$\square$

Valby (2015) contains additional investigation of interesting classes of conversation systems, and considers in particular systems containing both intersection and unioning operations.

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