DESIGN OUT OF COMPLEXITY:

A Mathematical Theory of Design as a Universal Property of Organization

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Declaration

I, Theodore Zamenopoulos, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Abstract

Objective: One of the most challenging problems in science is to identify whether there are certain common principles that characterize how organization at a microscopic level leads to the formation of complex structures, behaviours or functions at a macroscopic level. This is the very quest of complexity science. Within this quest, there remains one special issue that has received little attention: the 'problem' or 'phenomenon' of design. In this thesis, design is perceived as a capacity that arises because of certain characteristic organizational conditions. In line with complexity research, design is studied as a 'universal' property of organization without making assumptions as to whether this is a biological, cognitive or social organization. The thesis can be summarised as follows:

- Research problem: What entails the capacity to recognize and carry out design tasks?
- Hypothesis: Design is a function of the complexity of organization.
- Objective: A mathematical theory of the organizational properties which are responsible for the capacity to recognize and carry out design tasks.

Methodology: The above hypothesis is an epistemological statement about the nature of design, but it also has a methodological dimension. Methodologically it gives emphasis to the production of knowledge 'by construction'. The study approaches this objective mathematically, using category theoretic constructions.

Results: The main results of the study are:

- It explicates the notions of organization and complexity and ascertains the basic dimensions of organizational level descriptions.
- It characterises design as a universal property of complexity and identifies the organizational conditions and principles that are responsible for the capacity to recognise and carry out design tasks.
- It develops a mathematical structure (a theory) that defines the universe within which design tasks and activities are realized.

Overall, the thesis offers a mathematical theory of design as a universal phenomenon of complexity.

Acknowledgements

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The computational models and simulations reported in Chapter 3 of the thesis were developed jointly with Katerina Alexiou: each of us was responsible for 50% of the design, implementation and experimentation with these models. OK, this is something of an understatement. For me, there would be no meaning in doing this work if not together.

Finally, the acknowledgements to my family: I thank my father for insisting that I have to explain everything with a sketch. I thank my mother for insisting that any geometrical problem can easily be resolved with a lullaby. I thank Maria for being my best friend in the world: now, as I submit this thesis she is becoming the best mother...

Table of Contents

CHAPTER 3

CHAPTER 4

CHAPTER 5

CHAPTER 6

CHAPTER 7

CHAPTER 8

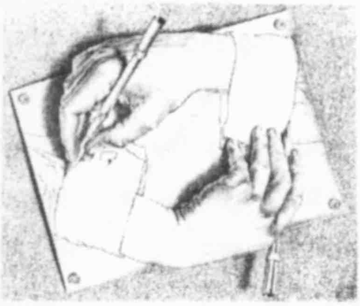
CHAPTER 9

List of Figures

List of Tables

Chapter 1

INTRODUCTION



M.C. Escher 'Drawing Hands' (1948)

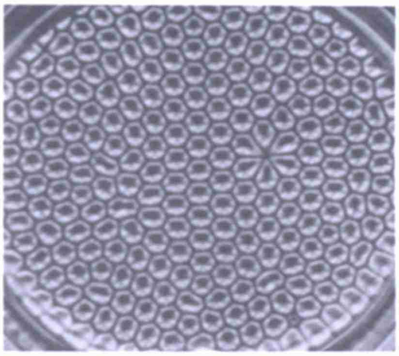
Design is customarily perceived as a ubiquitous natural phenomenon that is of fundamental importance for the creation of the world that surrounds us. Studying design is therefore of paramount importance for understanding 'artificial worlds' and the conditions that enable their conception and development. Our understanding of the meaning and role of design in nature is however quite ambiguous. On the one hand, design is associated with the creation of the artificial. This creation is often perceived as the product of a distinct way of thinking and as a signature of human intelligence which consists in being able to contemplate, generate and use 'functional' objects. As such, design is a fundamental 'mode of change' and creation, and a vital capacity of individuals and social organizations alike. It is an essential dimension of any society, manifested in its organization and governance, as well as in its products, and the abstract or physical constructs created to support its functioning. On the other hand, design is also a natural phenomenon. Nature, as an evolutionary environment of biochemical, physical or social entities, creates designers: creatures that create and recognize 'functional' artefacts. Nature creates intelligence. Nature creates *the capacity* to *recognise and carry out design tasks*. Nature is the ultimate designer.

How can we resolve this ambiguous picture of design? The drawing at the top of this Chapter is a 'summary illustration' of this research problem. My own interpretation when looking at this drawing is that Escher was concerned with the pictorial representation of design. In this picture, design (illustrated as a creative 'drawing hand') is explained as a capacity that emerges out of a given organization of hand-pencil formation. In this organization, the distinction between the designer and the design object is blurred. The designer reflects the design object, which reflects the designer. The question is not who designs what. The key is representing the organisation of the universe within which design emerges. The important thing is not who but how; how this reflection happens, how things become interrelated, what is their organization. The present study is concerned with this problem, except that the main objective is not the pictorial representation of design but the mathematical representation of the organizational conditions that are responsible for the capacity to recognize and carry out design tasks. Let us explore this research question in more detail.

1.1 THE RESEARCH QUESTION, HYPOTHESIS AND OBJECTIVES

One of the most challenging problems in science is to identify whether there are certain common principles that govern how components as diverse as atoms, cells, animals or humans 'organize themselves' and in doing so lead to the formation of macroscopic phenomena like chemical patterns, living structures, cognitive functions or social constructs. This is the very quest of complexity science.

For example, in mathematics and physics issues related with stability, diffusion, bifurcations, or synchronization of systems have been studied by looking at the underlying organization and coupling of their interacting dynamical components (e.g. Haken, 1977, 1983; Atay and Jost, 2004; Jirsa and Kelso, 2004). In chemistry, the formation of macro structures and behavioural instabilities has been studied by looking at the thermo-dynamical interaction of a system with its environment (e.g. Prigogine and Stengers, 1984). Cognitive functions have been studied by looking at the organizational structures or processes in the brain and their coupling with the external environment (e.g. Gazzaniga, 1989; Kelso, 1995). In biology, life has been studied as the product of a network of chemical reactions that has the capacity to catalyze its own reproduction; that is, as an organizational property of auto-catalytic networks (e.g. Kaufmann, 1969; Eigen and Schuster, 1979). In social science, social structures in the form of insect colonies (Bonabeau et al 1999; Eberhart et al, 2001), economic networks (Anderson et al, 1988; Arthur et al, 1997), or human settlements and cities (Portugali, 2000; Batty, 2005), have also been studied as macroscopic structures that result from certain organizational conditions at a lower level of analysis (i.e. individual behaviours, local rules of interaction, but also cognitive functions of individuals). For some examples see Figure 1.



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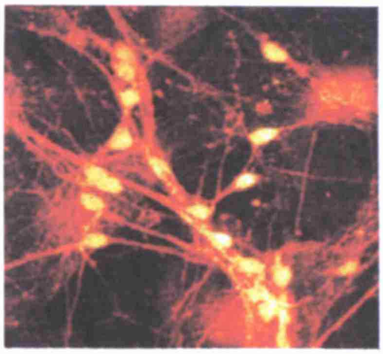

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Source: http://www.youtube.com/watch?v=ozkBd2p2piU

Figure 1: Complexity science studies the interplay between individual behaviour and the formation of macro phenomena. Clockwise from the top: the formation of a pattern in a liquid due to thermal convection, self-organised neural pathways in the brain, a 'city' created from a colony of ants, and a traditional settlement in Santorini formed without a masterplan.

In these studies, there is a common idea that the formation of structures, functions and behavioural patterns in natural and artificial systems is the effect of certain 'universal' organizational principles that govern the coupling between different dynamical components; between macro and micro structures; or between inner and outer environments. In particular, the quest for 'universality' signifies the assumption that certain underlying principles are common across different systems, including chemical, physical, biological or social systems. The term organization is used to emphasize the importance of holistic and relational explanations of phenomena. On that basis, complexity science has aimed to develop theoretical results and methodological tools for studying the origins and effects of such universal organizational principles (see for example Haken, 1983; Badii and Politi, 1997; Schuster, 2001; Boccara, 2004).

However within this research, there remains one special phenomenon or problem that has received little attention. This is the 'problem' or 'phenomenon' of design. The problem of design refers to two distinct questions: the understanding of the organizational principles that underlie the formation of design *structures* (i.e. design artefacts), and the understanding of the organizational principles that underlie the formation of design *functions* (i.e. the capacity to carry out design tasks). The former problem has been studied mainly in relation to artefacts such as cities (e.g. Portugali, 2000; Batty, 2005; Marshall, 2005) or buildings (Steadman, 2006), but the latter has for the most part been ignored. Although the two aspects of design are tightly interconnected, this study places emphasis on the second.

1.1.1 The design 'problem'

Let us introduce the 'problem' of design in more detail and contemplate the scope of design phenomena. In the most general sense, the 'phenomenon of design' arises with the formation of organisms whose survival depends on their capacity to construct or adapt their own environment. For instance, the capacity of birds to build nests, beavers to dam ponds, or humans to construct hunting tools, can be perceived as primitive examples of design tasks. This capacity can be equally distributed within a society. For instance, phenomena such as the creation of ant colonies and nests, or the creation of cities are examples where the capacity to design is distributed among a society (a collective) of agents. In these examples, organisms construct and recognize functional objects that contribute to their survival. The capacity of an organism to change or adapt its environment may be contrasted to other - logically - distinct abilities or strategies: for example, the capacity of an organism to adapt itself to environmental changes, or the capacity to migrate to a new environment (Kirsh, 1996). Overall, the phenomenon of design can be understood as the product of an evolutionary pressure that leads to the formation of organisms with the individual or collective capacity to adapt their environments for their own benefit.

Although evolutionary pressure may explain the presence of design abilities in certain organisms, or the formation of species of design artefacts as a product of 'exosomatic' adaptation, evolutionary theory in itself cannot describe what makes certain organisms - such as humans - capable of designing, or what makes certain functional objects to be recognized as design artefacts. A typical response to this quest is to assume that design requires cognition; a mind with the capacity to recognize the possibility of alternative environments and generate instantiations that fulfil the properties of these (imagined) environments. Design is therefore tied to the formation of an 'Intentional state'[1] in a mind; the representation of beliefs and desires about the world together with strategies to fulfil them. The view of design as a 'cognitive level' ability is typically followed by the hypothesis that design is a distinct way of thinking and knowing. One of the most common assumptions is that design cognition constitutes a special type of information processing system (e.g. Stiny and March, 1981; Akin, 1986; Goel and Pirolli, 1989). Many also recognize design as a knowledge intensive activity that is characterized by a special form of logical reasoning (e.g. March, 1976b; Coyne, 1988; Goel, 1988; Takeda et al, 1990; Roozenburg, 1993).

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[1] From the perspective of philosophy of mind, '*Intentional state*' is a mental state which is directed towards a state of affairs in a world (Searle, 1983). For instance, beliefs and desires are Intentional states because one has beliefs *about* something or desires *for* something in the world. It is written with a capital 'I' in order to be distinguished from the term 'intention', which is only a kind of Intentional state.

The perspective of design as a cognitive facility of an 'Intentional' mind is intuitive and convincing, but the core question remains: what are the conditions that entail the capacity to recognize and carry out design tasks? And where are these conditions situated? Do they correspond to some physical process in the brain? Do they correspond to a behavioural disposition? Do they emerge from the interaction between a cognitive agent and his/her environment? Do they grow because of social processes and interactions? The identification of design as a type of information processing system does not specify these conditions.

There are two opposed perspectives to consider when attempting to develop *a general theory of the 'design universe', a theory of the locus within which the capacity to design is realized.* The first is related to the understanding of the role of lower level, physical or sub-cognitive processes and structures in the formation of design abilities; and the second is related to the understanding of the role of social structures and processes in the formation of design thinking. One may assume that at a 'sub-cognitive level' of abstraction, there are certain organizational properties in the brain (e.g. related with the connectivity and coordination mechanisms of different functional areas of the brain) that are responsible for its capacity to recognize and carry out design tasks. Similarly, at a 'meta-cognitive level' of abstraction, one may assume that there exist certain organizational properties (e.g. certain processes and patterns of interaction and communication) that are responsible for the capacity of social systems to produce and use design artefacts. Both the 'sub-cognitive' and 'meta-cognitive' view outlined above suggest that design may be ultimately explained as the collective or macro effect of population dynamics, or similarly as an effect that emerges from the interactions of distributed entities.

In both cases, there is a universe (physical, chemical or social) whose structures, functions and behavioural patterns are determined by certain organizational conditions and no single entity or agent is able to control the overall process. In brief, seeing design as a sub-cognitive or social faculty implies the hypothesis that design is a function of the complexity of organization. In line with the assumptions of complexity research, the quest for the conditions that are responsible for the capacity to design is addressed by looking at the organizational principles behind 'design Intentionality'; the organizational principles of the universe that entails the capacity to carry out design tasks. It is the purpose of this study to develop such a theory; a theory that will be referred to as an 'organizational level theory of design'[2]. The statement that design is an organizational capacity has two functions for this study. It is a perspective on the how we can investigate the conditions that are responsible for the capacity to design, but it also defines the subject matter of the thesis.

1.1.2 Summary of the proposed research

To sum up, the preliminary assumption of this study is that there are certain qualities in the complexity of biological, cognitive or social organizations that are responsible for the capacity to recognise and carry out design tasks. Design capacity is therefore treated as the macro effect of certain organizational principles; a function of the complexity of organization. The study makes no commitment on the nature of these organizational conditions: it is assumed that the properties of 'complex organizations' that entail the capacity to design can *in principle* arise within bio-chemical, neurological, cognitive or social environments alike. It is also assumed that these organizational qualities can be investigated, understood and expressed mathematically. The overall objective of this thesis is to develop a mathematical theory of design as a universal phenomenon of complexity. The motivation for the development of a theory of design at this 'organizational level' is to establish a theoretical background for experimentally testing hypotheses regarding the locus of design (whether design is a behavioural disposition of an organism, a physical state/process in the brain or 'physical' state/process in a social environment).

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[2] Note that the quest for an organizational level theory of design has two aspects: First, it is a quest for a theory about the universe within which the capacity to design is realized. In this sense, it is a theory about the 'substrate' of design. It is a theory about the locus or environment of design capacities and not a theory about design tasks, processes and objects. Second, it is a quest for a theory about the universe within which design tasks, processes and objects are generated. In this sense, it is a meta-theory, a more abstract theory, concerned with the relation between design tasks, knowledge and objects. Chapter 7 places emphasis on the second aspect, whereas Chapter 8 places emphasis on the first.

The main research problem, hypothesis and objective of the thesis can be summarised as follows:

- **Research problem**: What are the characteristic properties of environments (whether sub-cognitive, cognitive or social) that exhibit the capacity to recognize and carry out design tasks?
- **Hypothesis**: Design is a function of the complexity of organization.
- **Objective**: The development of a theory of the 'universe of design', a theory regarding the organizational properties that are responsible for the capacity to design.

This main objective of this study can be more analytically subdivided into the following interrelated tasks:

- The first task is to mathematically explicate the notions of organization and complexity and ascertain the basic dimensions of organizational level descriptions with an emphasis on aspects pertinent to design (Chapters 5 and 6).
- The second task is to characterise design as a universal property of complexity and identify the organizational conditions and principles that are responsible for the capacity to recognise and carry out design tasks (Chapter 7).
- The third task is to develop a mathematical structure (a theory) that can define the universe within which design tasks and activities are realized, irrespective of how these are manifested in the physical world (Chapter 8).

These tasks (which also correspond to the main results of the study) are diagrammatically summarized in the following figure:

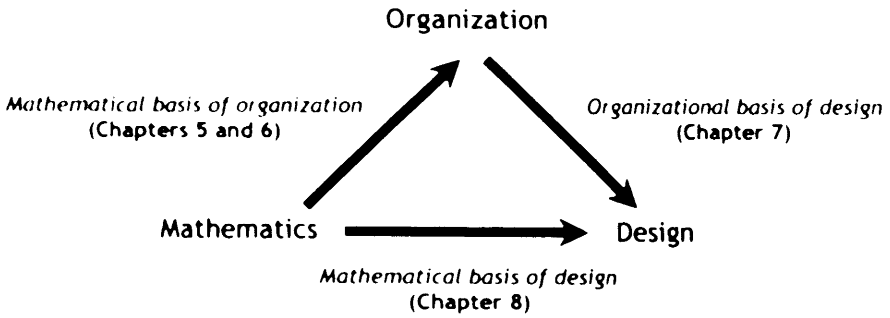

Figure 2: Summary of the main tasks of the thesis.

1.2 THE NATURE AND METHODOLOGY OF THE THESIS

The thesis sees design as a phenomenon that is fundamentally linked with the complexity of a system. This is an ontological position but also an epistemological one: it implies that the capacity to design should be explained as the result of an 'irreducible' organization. The hypothesis of the 'irreducibility' of organization means that in order to study design it is paramount to *identify* and *preserve* these organizational structures and processes that are characteristic of the capacity to carry out design tasks. The advocated epistemological stance is therefore followed with a methodological approach that puts emphasis on the production of knowledge 'by construction' of the postulated organizational principles. The realization of these constructions often takes the form of computer simulations that aim to recreate the (macro-) effects of organized structures and allow assumptions to be made about the underlying principles that generate them. Construction can also take the form of mathematical theories that explicitly focus on the analysis of the behaviour of organized structures. This is often achieved by specifying 'species' of mathematical structures that preserve certain organizational properties and then studying their 'macro' properties. In this study, category theory is perceived as a methodological approach that fulfils this purpose.

More specifically, following this methodological approach, 'knowledge about design' should be acquired through the construction of a distinct theoretical entity that turns the intuitive problem of design into an abstract, yet concrete, organizational problem. The construction of computer simulations is a typical method that can serve this purpose. In particular, computer simulations that focus on organizational structures - such as multi-agent simulations or cellular automata - seem to be well suited for studying the origins and consequences of complex organizations that are pertinent to design. The thesis includes such a computational construction (in Chapter 3) used in order to explicate the view of design as a capacity of organization. However, the main corpus of the thesis is concerned with the development of mathematical structures that aim to turn the intuitive concept of design into a mathematical object. This activity will be referred to as a 'pre-mathematical' activity (Buchi, 1989). In particular, category theory is developed here in order to serve as 'a mathematical theory of organization' and to study the intuitive concept of design as a property of organized complexity.

Overall knowledge about design in this study is derived through building or constructing the desired properties and phenomena rather than trying to decompose and analyse them. The thesis therefore seeks to build an organizational level theory of design, by proactively trying to simulate, model and mathematically realize the properties of 'design capable systems'. Rather than using existing formal structures created to model other phenomena, the thesis strives to construct appropriate mathematical structures and frameworks to fit the requirements of design phenomena as they are understood in design research.

1.3 AN OUTLINE OF THE THESIS

The thesis is structured as follows (Table 1 gives a summary of the proposed research agenda and Table 2 an outline of the thesis). Subsequent to this introductory chapter, Chapter 2 proceeds to establish the research context and position the proposed research agenda in the context of design and complexity research. The chapter also identifies and clarifies methodological and epistemological links between design research and complexity research and explains how the two areas are brought together for the purpose of the thesis. Chapter 3 presents a computational model of group design built in order to construe design as a phenomenon that emerges out of complexity, and also in order to expose and reflect on some crucial dimensions or conditions that enable design capacities. The chapter also introduces the motivation behind a mathematical exploration of these ideas. Chapter 4 discusses in detail the role and value of mathematical investigation in building the proposed theory of design, and offers an introduction to the particular language and framework (category theory) used in the thesis. Overall the first four chapters set up a research agenda in design and complexity research by identifying the research problem and hypothesis, positioning this enquiry in design and complexity research, and finally introducing the methodological approach. The following chapters are effectively concerned with the development of a mathematical theory of design at an organizational level of abstraction.

Chapters 5 and 6 both focus on the concept of organization. Chapter 5 explains the meaning of organization in complexity research, and derives some fundamental mathematical principles that underlie the description and study of organization. In Chapter 6 the meaning of organization and the basic concepts of organizational-level theories are elaborated on the basis of category theoretic constructions. The chapter proposes a mathematical theory that is able to accommodate and compare a plethora of different organizational level descriptions and therefore prepare the ground for an organizational level theory of design. Chapter 7 is concerned with describing and explaining design at an organizational level using the mathematical concepts which have been introduced. In particular, the chapter offers a mathematical definition of the task and capacity to design and articulates the fundamental organizational conditions that can explain the capacity to carry out design tasks. The chapter also compares different paradigms for the study and representation of design in order to better explicate the need for the proposed organizational-level theory. Chapter 8 revisits the results of previous chapters, and summarises the mathematical properties that uniquely distinguish the problem and capacity to design as an effect of complexity and organization. It then moves on to develop a new type of mathematical structure (i.e. a mathematical meta-theory of design) able to characterise the universe within which design tasks and capacities are expressed. The thesis concludes with an overall summary and discussion in Chapter 9.

Research problem:

- What entails the capacity to recognize and carry out design tasks?
- On what level are design-capable systems distinguished from non-design ones?
- What are the characteristic properties of environments (whether sub-cognitive, cognitive or social) that exhibit the capacity to recognize and carry out design tasks?

Research hypothesis:

Design is a universal property of complexity
(i.e. design is distinguished at an organizational level)

Objective:

A mathematical theory of the organizational properties responsible for the capacity to carry out design tasks

Method:

Pre-mathematical: Turn the intuitive concept of design into a mathematical entity.
(Postulate mathematical structures that can be associated with the capacity to design)

Table 1: Summary of the proposed research agenda.

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Table 1: Summary of the proposed research agenda.

Introduction

Chapter 1:

Introduce the research problem, hypothesis, and objective of this study

Set up the research agenda

Chapter 2:

Position the research agenda in the context of design and complexity research

Chapter 3:

Develop and clarify the meaning of the central hypothesis

Chapter 4:

Position and clarify the epistemological and methodological tools of the study

Propose a mathematical theory of organization

Chapter 5:

Examine the concept of organization in the science of complex systems

Chapter 6:

Propose a category theoretic approach to Organizational-Level theories

Propose an organizational level theory of design

Chapter 7:

Describe and explain the capacity to design as a universal property of organization

Chapter 8:

Propose a mathematical structure that entails the capacity to design

Conclusion

Chapter 9:

Sum up the aims, outcomes and contributions of the study and discuss future work

Table 2: Outline of the thesis.

Chapter 2

DESIGN AND COMPLEXITY: A RESEARCH AGENDA

The previous chapter introduced the core research question of the thesis and the general motivation behind it. The current chapter establishes a research agenda in the context of design and complexity research, and outlines the main hypotheses that drive its development. In particular, the chapter first presents the argument that design is a universal task and a capacity that arises at different levels of reality and across different domains and disciplines. The chapter then reviews how design research has approached the issue of characterising design as a distinct yet universal task and capacity. The review reveals a gap: although we have theories that distinguish design as a characteristic task or type of logical, cognitive or knowledge intensive process, we lack theories that directly specify the conditions that are responsible for the capacity to carry out design tasks. What is missing is a theory that specifies the properties of the 'universe' within which the capacity to design arises. The chapter moves on to propose a research agenda that seeks to understand design as an organizational capacity enabled by, and derived from, complexity. For that purpose in the second part of this chapter, the meaning and role of complexity and complexity research in design is explored. Finally, the chapter identifies and clarifies methodological and epistemological links between design research and complexity research and explains how the two areas are brought together for the purpose of the thesis.

2.1 DESIGN AND DESIGN RESEARCH: CHARACTERISING DESIGN AS A UNIVERSAL PHENOMENON

Design is considered as a natural activity, inherent in many of human endeavours, from architecture and engineering, to software and product design, and from service design, to organization and policy design. It is an integral component of everyday activity, as well as part of a variety of professional practices, including art, science and mathematics. With the same tenet in mind Simon (1996: 111) asserted that *"Everyone designs who devises courses of action aimed at changing existing situations into preferred ones"*. This is a regularly quoted and widely accepted definition of design activity exactly because it captures its universal character.

The study of the principles and processes of design is a very old enterprise and can be thought to include studies such as those of Archimedes, Leonardo da Vinci, Vitruvius, or Kandinski. Nonetheless, the recognition of design as a distinct and independent field formally starts in the 60's with works such as those of Asimov (1962), Archer (1965), Jones (1970) and others. The main tenet behind these studies is that design is a distinct, yet universal, task that underlies a plethora of disciplines. This also led to the recognition of design as a distinct discipline; a discipline that can be *supported* by 'universal' methods and *understood* as a natural and therefore 'universal' ability of human thinking and creation. It follows that, although there are many disciplines of design, design constitutes a distinct discipline itself insofar as it involves characteristic types of knowledge, methods/processes, and tasks that are common across domains. For a comprehensive overview of the history of design research and methodology one may consult Cross (1984 and 1993).

The study of design as a unique yet universal phenomenon or problem therefore, embraces theoretical or empirical investigations around three main questions:

- *'what designers think'*,
- *'what designers do'* ,
- *'what designers design'*

The first question concerns understanding design reasoning, the cognitive and psychological processes of designing, and the nature of design knowledge. The second question concerns understanding the nature and structure of design tasks, as well as the development of appropriate methods and practices. The third question concerns understanding the objects or products of design, including their properties, their representation and use.

The categorisation is similar to that offered by Cross (1999b) who distinguishes the themes of *design epistemology* (study of 'designerly' ways of knowing), *design praxiology* (study of practices and processes of design), and *design phenomenology* (study of the form and configuration of artefacts). These of course are not necessarily clear cut categories of investigation; many studies focus on questions that traverse these boundaries, and results and insights from one area normally influence the others (for example, what designers do is connected to the question of how designers think, and the properties of design artefacts are often dependent on the design process that produces them).

It is important to note that this categorisation of design research is made with reference to the object of investigation. We can also distinguish different studies of design according to their objective; in this case, we can categorise design research into research *about* design (research aimed to develop models and theories of design), and research *for* design (research aimed to develop methods, tools and systems to automate, support, aid or augment design). Another categorisation can be made according to the methodological approach chosen; including for example empirical or theoretical types of investigation. We will discuss some methodological issues in this chapter, but a more elaborate treatment will be offered in Chapter 4. A summary of design research is shown in Table 3.

Let us now review some of the literature looking at the epistemology, praxiology and phenomenology of design in order to establish the general background of the research and see how the question of characterising design as a universal phenomenon has been perceived and treated.

The object

- What designers think: Cognition and Knowledge representation
- What designers do: The task environment, processes and methods
- What designers design: Artefacts, functional objects and the artificial

Hypothesis: The level at which design is distinguished:

- Epistemological and Logical level
- Knowledge level
- Task (problem) level
- Information processing level
- Sub-symbolic and dynamical level

The objective

- Theory (knowledge about design)
- Methods and tools (knowledge for design)

The method

- Empirical (Setting up scientific experiments)
- Theoretical

    Mathematical (axiomatic, model building), *Computational* (computer simulations, computational support), *Logical* (discursive or formal)

Table 3: Summary of design research.

2.1.1 What designers think

Design activity is generally considered to be a manifestation of human intelligence and hence a unique cognitive ability. In effect, a very large part of design research is concerned with illuminating the nature of design thinking and knowing, and understanding what makes designers special in that respect. Such research is also tied to the notion of defining design as a distinct discipline and identifying the relation between design, art and science.

2.1.1.1 Design cognition: from problem solving to reflective practice

The view of design as a cognitive ability is linked historically with the hypothesis that human cognition can be understood as an information processing system. Craik's schema of thought was as one of the first abstractions of the information processing paradigm (Craik, 1943). However, it was the seminal work of Newell, Shaw and Simon (Newell, Shaw and Simon, 1957; Newell, Shaw and Simon, 1962; Newell and Simon, 1972) on human problem solving that paved the way to the understanding of design as a problem-solving process and to the development of cognitive explanations of design as information processing.

This view has been an article of faith for a very large number of empirical, theoretical and computational design studies. Eastman (1969a, 1970) carried out one of the first known empirical studies of designers' problem solving abilities using this paradigm. Later examples can be found in Akin (1986) and McGinnis and Ullman (1992). A general theoretical investigation of design as an information processing system can be found in Stiny and March (1981). Goel and Pirolli (1989, 1992) offer an extensive treatment of the notion of design problem space as developed within the information processing paradigm. The principles of design as problem solving also guided many different applications to computer aided design (e.g. Eastman, 1971, 1975; Weinzapfel et al, 1971; Pfefferkorn, 1975; Akin et al, 1992, Akin and Sen, 1996; Flemming et al, 1992, Flemming and Woodbury,1995).

One important question that arose with the view of design as problem solving was that of understanding the characteristic features of design problems and what makes them different from other problems. The question is tied to the quest for understanding the ability to design as a universal and at the same time unique activity. In information processing we have the distinction between "well-defined" and "ill-defined" problems (Newell, 1969). *Well-defined* problems are those for which the goal state is clearly prescribed, and only the means to arrive at the appropriate solution are unknown (so the task of problem solving is that of finding the appropriate sequence of actions to lead to the desired end state). *Ill-*

defined problems, by comparison, are those problems for which both the goal state and the means to achieve it are not given in advance. Problem solving in this sense requires heuristic techniques. Simon (1973) considered that ill-defined problems are typical of design; however this ill-definedness (if we may call it that) is not a feature intrinsic to design problems per se, but something that arises due to limited (computational) resources and the bounded cognitive ability of people (designers) to acquire and process all the necessary information for solving the problem. This led to Simon's famous notion of "satisficing" (finding "good enough" solutions) instead of "optimising" to describe the practice of human problem solving.

Many researchers however considered that the realm of design lies right within the area of problems called "wicked". The term, which is originally attributed to Churchman (1967), was used by Rittel and Webber (1973, 1984) to refer to a class of problems for which a definite formulation cannot be achieved and whose solutions are always provisional. *Wicked* problems are characterised by a special relationship between problems and solutions such that every attempt to offer a solution affects the formulation of the problem itself: 'problem understanding and problem resolution are concomitant to each other' (Rittel and Webber, 1984: 137). For various analyses and views on ill-defined and wicked problems see also Bazjanac (1974), Buchanan (1995), Coyne (2005) and Dorst (2006).

The introduction of wicked problems in design research essentially conveyed the perception that problems with social impact are intrinsically different from strictly "technical" ones; that designers are situated in and influenced by their socio-cultural environment; and, more generally, that design problems are not amenable to rational problem solving. Designers therefore operate in a particular mode of creative thinking which enables them to work with ill-defined or wicked problems.

The most seminal criticism of the view of design as information processing which placed emphasis on rational problem solving, came from Donald Schön (1983, 1988). In Schön's view, the peculiarity of design is found in reflective practice, and design ability originates from the ability to reflect on one's own actions. This "experiential learning" approach emphasises the existence of a construction cycle whereby the designer frames the design problem, generates moves towards a solution, and reflects on the outcomes of these moves (Stumpf and McDonnell, 2001). A detailed comparison of the two different paradigms of design activity is offered by Dorst and Dijkhuis (1995). See Table 4 for a summary.

	Rational Problem Solving	Reflection in Action
Designer	Information processor (in an objective reality)	Person constructing his/her reality
Design problem	Ill-defined, unstructured	Essentially unique
Design process	A rational search process	A reflective conversation
Design knowledge	Knowledge of design procedures and 'scientific' laws	Artistry of design: when to apply which procedure/piece of knowledge
Example/model	Optimisation theory, the natural sciences	Art/the social sciences

Table 4: Comparison of the rational problem solving and the reflection in action paradigms of design activity, adapted from Dorst and Dijkhuis (1995).

Most contemporary approaches recognise that the design process is a constructive endeavour which essentially involves generating and reasoning about alternative solutions as well as problems. This mutual influence between problems and solutions has also been construed and expressed as a co-evolutionary process (Maher, 1994, 2000; Dorst and Cross, 2001).

It is interesting to observe here that assumptions about what (or how) designers think are reflected in the understanding and modelling of the design process. For example, in the table above, we see that the two different views about the cognitive makeup of designers lead to different conceptualisations of the design process. For more extensive and authoritative presentations of design thinking and knowing see (Rowe, 1987; Lawson, 1997, 2004; Cross, 2006).

2.1.1.2 Design cognition: from process level to knowledge level theories

The symbol processing paradigm was tied to the idea of a general problem solver, a program or system that has a universal applicability. According to Ernst and Newell (1969), this general problem solver required a distinction between three parts: an external representation of a problem, an internal encoding of this problem, and a set of problem solving techniques. Transferred to design thinking, problem solving was tied to the idea that the design process can be represented as a general search process over a design space.

A more abstract approach for the characterisation of the design process came with the proposition of Knowledge Level formalisations. The Knowledge Level hypothesis was introduced by Newell (1982) in order to express the relation between 'knowledge' and 'symbolic representations'. The core hypothesis is that knowledge is a notion of competence that is independent from symbolic representations or any physical implementation. In particular, the Knowledge Level (KL) description lies above the Symbol level. At this level, an agent is described as having a set of objectives, a set of possible actions and a body of knowledge. According to Ossowksi and García-Serrano (1995) 'A KL-description of a system consists of the knowledge and the goals, that an observer ascribes to an "agent", in order that it can exhibit an observed behaviour by applying its knowledge according to the principle of rationality'. Knowledge level studies in design, rather than attempting to characterize design thinking in terms of a set of specific processes, instead concentrate on identifying and modelling the types of knowledge involved. This includes knowledge about the design problem or other domain requirements, knowledge about design processes and mechanisms, and knowledge about design objects (Smithers, 1996, 1998, 2002; Brazier et al, 1994, 1996, 2001b). These studies have been largely inspired by research in artificial intelligence and knowledge engineering (see also Newell, 1990; de Velde, 1993; Schreiber et al, 1994).

Finally, the view of design as a knowledge level cognitive activity has been more loosely linked with the development of theories and methods regarding the representation, decomposition and analysis of the structure of design problems (e.g. Alexander, 1963, 1964; Moore, 1970), the representation and modelling of the behaviour of design artefacts (e.g. Kalay and Carrara, 1996), and the representation and analysis of the correspondence between a problem and a solution space (Yoshikawa, 1981; Ho, 1982).

2.1.1.3 Design and creativity

A key issue in characterising design thinking is to comprehend the nature of creativity in design. While creativity is not an ability exclusively held by designers (creativity is inarguably also part of artistic and scientific activity), designers are supposed to master creative thinking. Moreover, creative solutions are highly desirable for they usually lead to better, or more successful, artefacts (products or processes). The understanding of questions like "How do creative design solutions appear?", "What strategies do designers use to generate creative solutions?" or "How we can replicate or support creative design tasks?" are very important for design research and have a significant impact on design education as well as on the development of design systems.

As creativity is a key issue in design, studies of creativity are numerous. This investigation includes empirical research, case studies and analysis of protocols, as well as investigation and development of computational models and tools (for instance see this varied set of examples Dasgupta, 1994; Roy, 1993; Cross, 1995; Akin and Akin, 1996; Candy and Edmonds, 1996; Gero, 2000; Saunders, 2002; Goldschmidt and Tatsa, 2005). For more references one may look at papers from two prominent series of conferences: the Creativity and Cognition Conference, organised by the Creativity and Cognition Research Studios, led by Ernest Edmonds and Linda Candy; and the Computational Models in Creative Design Conference, led by Gero and colleagues. It is worth noting that although most studies are predominantly focussed on creativity as an individual cognitive ability, some works also consider creativity as it appears in groups and, more generally, as an ability that is influenced or formed by social and situational parameters (see for example Fischer, 2000; Liu, 2000; Gero and Sosa, 2002; Mamykina et al, 2002;

Zamenopoulos and Alexiou, 2003a).

Design creativity has also been studied in relation to logical reasoning. March (1976) for example maintained that in contrast to scientific activity which is deductive and analytic, design activity is characterised by a kind of abductive reasoning which is essentially 'productive'. The idea of abduction, borrowed from Peirce (1934, 1958), has been considered by many to be a key to creative design (see Coyne, 1988; Goel, 1988, Roozenburg, 1993; Roozenburg and Eekels, 1995).

2.1.2 What designers do

Closely related to the question of what designers know is the question of what designers do, focussing on processes, methods and practices.

2.1.2.1 Design phases and design methods

The "design methods" exploration which spawned much of today's design research was initially focussed on describing what designers do by way of a logical structuring of design phases or tasks. The central idea was the identification and structuring of the design process into distinct, yet related, phases. For example, Asimow (1962) considered designing to proceed via an iterative analysis-synthesis-evaluation-communication cycle. Analysis, synthesis and evaluation appear in most such (prescriptive or descriptive) models of the design process and are normally considered to inform one another through a series of feedback loops. The design process typically goes on in a cyclic way until a satisfying solution for the design artefact is reached. Compare for example the models proposed by Archer (1965) and Roozenburg and Eekels (1995) in Figures 3 and 4, respectively. Design process models also typically assume that design generally moves from the more abstract (the level of ideas and needs) to the more specific (the level of detailed descriptions of the design object). Alternative diagrams can be found in Pahl and Beitz (1984) and French (1992).

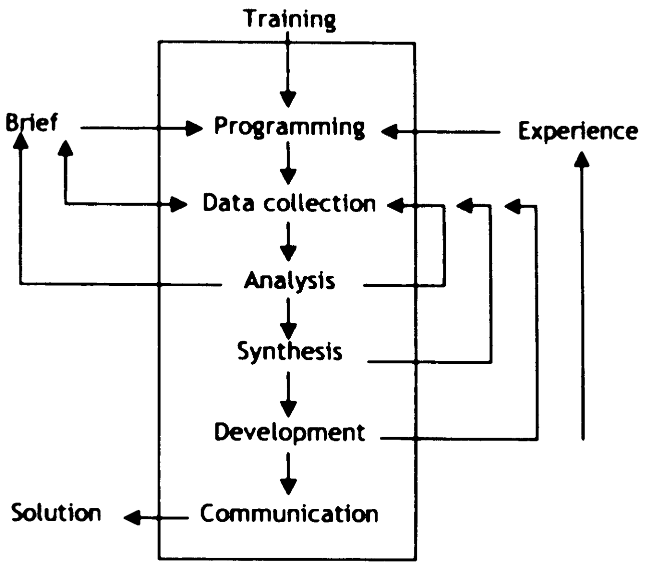


Figure 3: A breakdown of the design process by Archer (1965).

The identification of generic design tasks has also played a critical role for the understanding and development of computational design systems. For example, Chandrasekaran (1990) has identified large families of design tasks that can be carried out by different methods. More specifically, the higher level of design tasks is characterized by Propose-Critique-Modify methods and includes the following subtasks: proposing partial or complete solutions, verifying the solutions, critiquing the proposals and modifying the proposal to satisfy goals. The "propose" subtask which associates specifications with solutions thus represents the core characteristic of design systems. Different methods can realize such a subtask. Decomposition - solution composition is a method based on problem transformations to smaller and solvable problems. Case retrieval is a method that uses matching processes to identify similar successful solutions to a problem. Constraint satisfaction is a typical optimisation or constraint propagation method that incrementally satisfies constraints. Similarly, Maher (1990) has identified decomposition, case based reasoning and transformation methods as possible methods of synthesis. Other examples can be found in Marcus et al (1988).

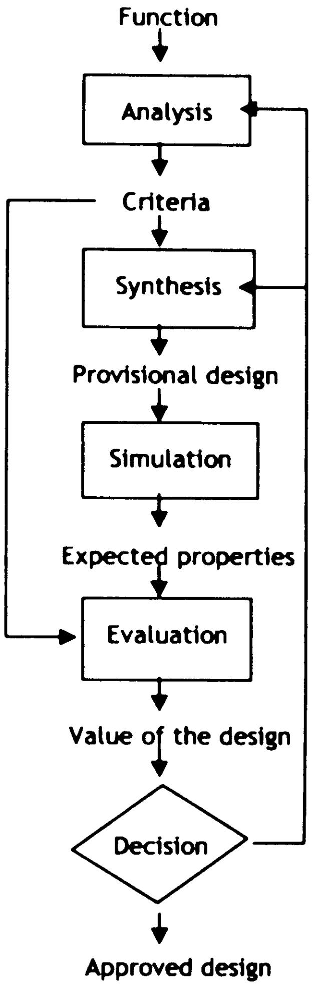


Figure 4: The basic design cycle proposed by Roozenburg and Eekels (1995).

While the early systematic approaches to design methodology have been criticised and rejected, knowledge of what happens in design (both as individual and as collective practice) is of great importance. Such research informs the development of process and methods for supporting different phases of design activity, from analysis, problem solving and management, to drawing, visualisation and communication.

2.1.2.2 Design practices: towards a social level study of design

Design praxiology has also been explored at a social level. Such studies have become the centre of attention in recent years and essentially seek to define design abilities in terms of social abilities and interactions. In a sense, as research in design thinking has moved from rational problem solving to reflective practice, so has research in design practice shifted its focus from the individual to the team. This has given rise to studies that draw upon investigations in the fields of distributed cognition, situated cognition and activity theory (e.g. Suchman, 1987; Lave, 1988; Lave and Wegner, 1991; Hutchins, 1995; Nardi, 1996; Wenger, 1998), as well as ethnography and ethnomethodology (e.g. Sacks, 1992; Taylor, 2002). Typical areas of investigation include the study and modelling of mechanisms of communication, negotiation and argumentation (e.g. Fischer at al 1996; Sonnenwald 1996; Trousse and Christiaans, 1996; Fleming, 1998; Stumpf and McDonnell, 2001); the analysis of team roles and structures (e.g. Cross and Cross, 1995; Lloyd and Deasley, 1998); the study and modelling of knowledge and cognitive processes in teams (e.g. Ball and Ormerod, 2000; Brazier et al, 2001); and the study of external representations and artefacts as a means to contextualize information and extend the cognitive abilities of designers (e.g. Bucciarelli, 1988; Perry and Sanderson, 1998; Fisher, 1999; Arias et al, 2000).

2.1.3 What designers design

The activity of design leads to design objects. As design is ubiquitous, so are its products, which include tools and artefacts, as well as buildings, software systems, services, or even human organizations. Naturally, a large part of design studies is focussed on these objects of design and their representations. Research in this case is mainly concerned with understanding the properties of designed objects, and developing processes, techniques and tools in order to analyse, categorise, configure or represent them.

2.1.3.1 Analysis and synthesis of design objects

For example, Alexander's early work in design methodology (Alexander, 1963, 1964) represents an enterprise of analysing and decomposing physical space into elements that are functionally compact and independent. These elements in a sense represent discrete subcomponents of the design problem. The proposed "pattern language" (Alexander et al, 1977) gives a set of such elementary patterns which when put together are presumed to produce different spatial configurations to fit individual circumstances and requirements.

The language of "shape grammars" is another kind of language whose development we can categorise under this type of research (Stiny, 1980a, 1990, 1991). The idea of shape grammars is essentially that of using shapes and transformation rules (an algebra of shapes) in order to compute/calculate different forms and arrange them spatially. The analysis, generation and representation of forms and layouts, particularly in the context of Computer Aided Design (CAD) systems, is a very large topic of investigation, especially in engineering design and architecture (e.g. Whitehead and Eldars, 1964; Steadman, 1970; Eastman, 1973; March, 1976a; Mitchell et al, 1976; Gips and Stiny, 1980; Krishnamurti, 1980; Earl and Johnson, 1981; Liggett, 1985; Earl, 1986; Mitchell, 1990; Cagan and Mitchell, 1993; Knight, 1994; Flemming and Chien, 1995; Chase, 1997).

Apart from production methods like shape grammars, other computational techniques, such as cellular automata and multi-agent systems (e.g. Küppers et al, 2000; Coates and Thum, 2000), or genetic and evolutionary algorithms (e.g. Frazer, 1995; Bentley, 1999; Caldas and Norford, 2002), are developed and used in order to generate and explore alternative forms. Such techniques are usually applied to graphic and physical design, but are also employed for the generation of maps and land use plans (e.g. Feng and Lin, 1999; Saarloos et al, 2005).

This kind of research also includes studies which are concerned more with the analysis rather than the synthesis of designed objects, and the development of methods and systems for modelling, representation and visualisation. Important contributions are studies like "space syntax" (Hillier and Hanson, 1984; Hillier, 1996), which are focussed on the formal analysis and modelling of the built environment, its attributes and constraints. The basic premise behind space syntax theory is that the configuration of space has social consequences and affects social life. The investigation is focussed on encoding the structure and topological relations of space using mathematical representations (basically graphs) and analysing it through the use of various metrics and measurements (like accessibility, visibility, integration, connectivity etc). Such ideas have been developed and applied for the analysis of form at the architectural, as well as the urban scale (e.g Steadman, 1983; Peponis et al, 1997, 1998; Batty, 2004; Batty and Rana, 2004; Marshall, 2005; Turner, Penn and Hillier, 2005).

Although these studies are focussed on the object of design, they also have ramifications for the way we understand design and designers. As previously mentioned, different foci of investigation sometimes reveal different perspectives about design. For example, for Stiny (2006) the uniqueness of design lies in that it involves visual thinking and a process of calculating with shapes. For space syntax, the meaning and function of designed artefacts (of built form in particular) is encoded in spatial relations, and so design is essentially a configurational enterprise. Likewise, different techniques used to generate forms, layouts and configurations are typically associated with different conceptual paradigms for design: for instance, design as optimisation (e.g. Gero, 1985; Chackrabarty, 1990), design as search (e.g. Steadman, 1970; Akin et al, 1992), design as evolution (e.g. Frazer, 1995; Bentley, 1999) or design as co-evolution (e.g. Maher, 2000).

2.1.4 What entails design: universality, complexity and organization

Up to this point, the 'problem' and 'universality' of design has been discussed as part of an investigation of the epistemology, praxiology and phenomenology of design. In particular, the uniqueness of design has been distinguished by looking at characteristic properties of the 'task environment' of design, but also at the logical, cognitive and knowledge structures involved in design tasks. The main strategy has been the development of theories of design as part of more general theories, such as for instance cognitive theories, theories of knowledge representation, logic or automata theory. Based on these studies a significant body of theoretical foundations and empirical evidence has been accumulated that has gone a long way in helping us both to understand and to support design tasks. However, although such theories are able to distinguish design on the basis of what designers think, do or design, they do not explain the *conditions* that make certain organized systems (such as the brain) *capable of designing*.

In order to explain this gap, it is instructive to note the difference between *theories of design* and *theories of the universe of design*. The development of a *theory of design* aims to form hypotheses regarding the particular conditions that distinguish the task environment of design, the cognitive processes involved, or the design artefacts produced. In a sense, a theory of design starts with an 'ontological' commitment that associates design with a social, logical, cognitive and/or artificial faculty, and concludes with a 'categorical' distinction of design. On the contrary, the development of a *theory of the universe of design* aims to form hypotheses regarding the conditions (structures or processes) that are responsible for the capacity to design. In a sense, a theory of the capacity to carry out design tasks starts with a 'categorical' premise about what distinguishes design from non design tasks, and concludes with the identification of an 'ontology' that justifies the premise.

The proposed distinction is rather artificial because both theories are ultimately theories of design, but it highlights the fact that the two kinds of theories are tied to different objectives and forms of investigation. The review of design studies offered above, reveals the lack of a 'scale free' theory that makes no assumptions about the level of analysis that is necessary in order to uniquely distinguish design phenomena. What is missing from existing design studies is a direct characterization of the properties of *the universe, or environment* within which design emerges - whether this is a sub-cognitive, cognitive, logical, or social environment. So, while design research has been predominantly concerned with the characterization of '*what is design?*' or 'what distinguishes design?' within a s cognitive or knowledge level framework, in this study the main concern is the distinct characterization of *the universe* within which the problem and capacity of design is defined - i.e. the predominant question here is 'what distinguishes a design capable organism?'.

The thesis argues that the notions of complexity and organization provide the tools to examine exactly this kind of questions. What we are after here is to uncover the complexity and organizational properties that are responsible for the capacity to recognize and carry out design tasks. The objective refers not only to an appreciation of the problem of design across different domains, but also to a theory that can uniquely characterise design as a universal, natural phenomenon that occurs because of certain organizational conditions. As the focus is on the complexity of organization there is no a-priori assumption about the level at which the phenomenon of design should be distinguished; i.e. whether design should be distinguished at a sub-cognitive, cognitive or social level. Essentially, the proposed theory aims to support the formation of a hypothesis regarding the locus of design, which then can be empirically tested. Table 5 summarizes the proposed research agenda in the context of design research (compare Table 3). In the next section we discuss complexity research, review the relationship between design and complexity, and explain how methodological and epistemological tools from complexity science can help in our quest for an *organizational-level description of design*.

The object

- The universe of design: conditions responsible for the capacity to design

Hypothesis: The level at which design is distinguished:

- Organizational level

The objective

- Theory (knowledge about design)

The method

- Theoretical (complexity science)

  Mathematical & *Computational* (Computer simulations)

Table 5: Summary of the proposed research in the context of design research (compare Table 3).

2.2 LINKING DESIGN AND COMPLEXITY: DESIGN AS A CAPACITY OF COMPLEXITY

Complexity science has a long and diverse history which can be conceptually traced back to cybernetics, general systems theory, information theory and even game theory. This history has also been built upon much older, but more domain-specific traditions, such as thermodynamics in physics, evolutionary theory in biology, abstract algebra and computability in mathematics and computer science, synthetic methods in artificial life and artificial intelligence, dialectics and social constructivism in social theory, and last, but not least, holistic approaches in philosophy.

The overarching quest in complexity science is to identify whether there are universal principles that govern how components as diverse as atoms, cells, animals, or humans, 'organize themselves' and in doing so lead to the formation of macroscopic phenomena like chemical patterns, living structures, cognitive functions or social constructs. In response to this quest, the word 'complexity' takes a phenomenological, epistemological and methodological meaning. From a *phenomenological* perspective, complexity is seen as an organizational characterization of a system (and/or its observer). For instance, complexity has been perceived as a characterization of the constraints and degrees of freedom that determine the interconnectivity of the components that build up a system (e.g. Atlan, 1974). Another view has been to define complexity as the difficulty or effort needed in order to describe the structure of a system (e.g. Bennett, 1985). Finally, complexity has also been seen as a characterization of the intrinsic capacity of a system to move from a certain random state into an ordered one (i.e. to self-organize), or more specifically to move towards a critical state between chaos and periodicity (e.g. Langton, 1990; Wolfram, 1994).

Despite the variety of the approaches to defining complexity, there exists a common *epistemological* stance. According to this stance, complex systems are taken to have structures, behaviours, or functions which cannot be deduced from (or reduced to) their constituent parts. More specifically, the core assumption is that higher level functional, behavioural and structural characteristics of systems can only be explained by looking at their organizational qualities, such as for example the interconnectivity of components, the interplay between macro properties and individual (local) behaviour, or the open interaction of an observed system with its environment (and/or observer). Such organizational qualities are assumed to have the power to describe and explain a large variety of scientific phenomena or problems.

These epistemological considerations are naturally accompanied by a certain *methodological* approach. According to this approach complex systems need to be viewed and studied as wholes; i.e. without 'breaking' their organizational properties. For this purpose, there is a number of concepts/tools customarily used in complexity research, such as for instance the graph theoretic study of networks and their dynamical formation (e.g. random graphs); the study of non-linearity, bifurcations, stability, phase transitions and coordination in spatially extended systems; and the statistical study of population dynamics based on the development of computer simulations of multi-agent systems or cellular automata (see for instance Haken, 1983; Badii and Politi, 1997; Jensen, 1998; Flake, 1998a; Schuster, 2001; Boccara, 2004). Overall, the underlying objective of complexity research is to develop theoretical results and methodological tools for the study of the origins and effects of organization. In this sense, complexity science is the science of effective organization.

The current section aims to explore the role and possible contribution of complexity science in design research (and practice). More specifically, the objective is to advocate a (new) research agenda for design research and introduce design as a new research problem for complexity science. For this purpose, the main 'directions' of complexity research are briefly reviewed in the first part, and then the existing links between complexity and design are identified in the second part. Based on this analysis, the agenda for design and complexity research is laid out in the final part.

2.2.1 A short introduction to complexity research

Complexity research is often baffled by three self-referential questions: what is complexity, what is a complex system and what is complexity science? There is no intention to give absolute answers to these questions here, but only to provide a framework in which answers can be derived. These questions are the very components of the complexity curriculum and as a result the answers should, and will, evolve together with the science itself. In the following we introduce complexity research as it is developed around three themes: the first is concerned with the description of the organization of a system; the second with the origins of organization; and the third with the implications of irreducible organization and the capacities derived by it (a summary of these themes is given in Table 6).

Descriptions of organization			**Origins** of organization	**Effects** of organization
Structural	Functional	Behavioural		
Unorganized Size Variety Order-disorder **Organized** Catastrophes Networks Scaling Fractals	**Algorithmic** Size of time/space Min description Randomness	**Deterministic** (sensitivity to initial conditions) **Aggregated** (Order by complex interactions, criticality)	Evolution Self-organizing criticality Dissipative Structures Synergetics	Autopoiesis Autocatalysis (M,R) Systems Neural Networks

Table 6: Approaches to the study of complexity as a characteristic of the organization of systems.

2.2.1.1 The description and modelling of organization

The first major subject of investigation in complexity research engages in questions such as: What is complexity? Is there a method or measurement that decides whether a system is complex? What is the appropriate language to describe the organizing principles of a system? All these questions are concerned with describing and modelling complexity. It is possible to identify three main, although at times overlapping, directions of research in this area. The first is looking at the structural characteristics of systems, the second is looking at the difficulty of building functional descriptions, and the third defines complexity by looking at the behavioural or dynamical characteristics of systems.

2.2.1.1.1 Structural complexity

It is often convenient to think of a system as a set of elements with some sort of structure. In this context, complexity has been (confusingly!) seen both as something identified with an unorganised structure (typically described by entropy measures), or as a characteristic structure which assumes a critical state between order and disorder (typically described by a power law distribution or a fractal like geometry).

Information theory has laid the theoretical background for defining complexity in terms of variety, (lack of) organization and uncertainty (Shannon, 1948). In this context, complexity is the amount of information that the organization of a structure is able to represent (or reversely, complexity is the amount of information needed in order to uniquely specify the organization of a system). A system with a large number of possible structural configurations (or states) can carry more information than a system with fewer states. As the number of possible configurations (or states) increases, more 'complex' events can be represented. Thus, complexity represents the combinatorial capacity of a system. As the combinatorial size of the system increases, more improbable events can be described. The intuitive assumption is that an improbable event contains much more information in comparison to a certain one. As a result, complexity is also linked with the probability of a given state of the system. Inspired by the quantification of entropy in thermodynamics, complexity is then defined as a function of the entropy of the system (Bar-Yam, 1997).

The view of complexity as information content (or entropy) implies that complexity is essentially a measurement of the degrees of freedom in the organization of a system. As a result, the correlations between the components of a system or, more generally, the mutual information and redundancies that constrain its organization are ignored. For this reason, this view of complexity has often been criticised because it leads to certain absurd conclusions - such as for instance that the description of a city should be less complex than the description of a random sequence of numbers (e.g. Bennett, 1985). In response to these problems, complexity has been alternatively defined as a critical state that specifies a balance between the amount of mutual information or redundancy between components of a system, and the degrees of freedom of the system (e.g. Atlan, 1974; Von Foerster, 1984).

In biology, physics, and social science, the term complexity has been used more freely to describe an emergent/characteristic property in the structure of a system. A predominant example is the characterization of the structure of a system by an exponential function $P(s) \propto s^{-\gamma}$ (Power Law) in the frequency distribution of some quantity s (such as size of components or the number of links that are incident to the components). A power law distribution implies a hierarchy: small sizes are common whereas large instances are extremely rare. This can be compared with other random distributions (such as the Poisson distribution) where such hierarchy is lost. Examples of such distributions are very common in the context of biological and socio-technical complex networks such as metabolic networks, the world wide web, communication systems, or cities (Barabási and Albert, 1999; Batty, 2005; Faloutsos et al, 1999; Goh et al, 2002; Troll and beim Graben, 1998; Zipf, 1949). The observation of self-similar or fractal geometries across scales is another example in the identification and description of complex structures (Mandelbrot, 1983).

Finally, in applied and pure mathematics, complexity has been identified with the representation and analysis of hyper structures. This term generally alludes to structures whose components have structures themselves, enabling the representation and study of issues of scaling, variety and organization in a concrete manner. Examples include Q-analysis (Atkin, 1974; Johnson, 1981, 1995), hyper graphs (Estrada and Rodriquez-Velasquez, 2005) and category theoretic treatments of complexity (Ehresmann and Vanbremeersch, 2002).

2.2.1.1.2 Functional complexity

Another convenient way to model a system is to see it as a function that generates an output given an input and certain rules. Complexity is a characterization of how difficult it is to describe (or generate) the output of a function with respect to the 'size' of the input or rules. According to this approach, the complexity in the output of a function increases as the length of the input or the length of the computation (i.e. application of rules) needed in order to generate the specific output increases.

Roughly speaking, there are two main traditions of research in this area. The first one is a branch of theoretical computer science known as time complexity theory. It is concerned with the resources (in terms of time and space) needed to describe the output of a function given an input of certain size. Complexity is therefore a measure of how long it takes, or how much memory is needed, in order to make this computation. The second tradition is concerned with the compressibility, or the required length, of a program needed to compute a function. It is generally known as algorithmic or Kolmogorov-Chaitin complexity, named after Andrei Kolmogorov and Gregory Chaitin who independently pioneered this approach (Kolmogorov, 1965; Chaitin, 1966, 1997; Li and Vitányi, 1997; Flake, 1998a). The core idea is that the length of description of a message (output) cannot greatly exceed that of the message itself. Intuitively, when the length of the description is much shorter than the message itself, then a higher compression is possible and the complexity of the message is minimal.

Based on this line of thought complexity has been also associated with randomness: in a random message no compressed version of the message is possible and as a result complexity is maximal. In reaction to this last definition, it is possible to argue that a random function can be simulated with a very simple algorithm much like fractals can be defined by a very concise recursive definition. So a random function may be very simple. In this sense, maximum complexity can be situated somewhere between randomness (or non-compressibility) and order. More information on formal measurements can be found in Bennett (1985, 1990) and Edmonds (1999).

2.2.1.1.3 Behavioural complexity

Systems are often modelled as dynamical systems. Complexity is then identified by the capacity of a system to exhibit a certain class of behaviours. There are two typical models: mean field models and spatial extended models. In mean field models, only average quantities are represented and spatial correlations are ignored. In this case, differential and recurrent equations are used as fundamental tools to describe complex behaviours. Spatially extended models on the other hand incorporate both temporal and spatial degrees of freedom. In this case, the fundamental tools for describing complexity are lattice or cellular automata, networks and multi-agent systems. It is also interesting to note that in mean field models the concern is with how simple rules can define very complex behaviour that is difficult to predict. On the other hand, in spatially extended systems the interest is more in how highly complex interactions in space and time can lead the system to some form of order.

More specifically, in mean field models recurrent equations of the form $x_{t+1}=f(x_t)$ have been pivotal in studying behaviour in terms of fixed points and patterns related to the stability of a system (when differential equations are used the behaviour is studied in terms of equilibrium points). These equations can exhibit interesting properties such as sensitivity to initial conditions. This is a characteristic behaviour found in a class of systems usually referred to as chaotic systems (Bar-Yam, 1997; Boccara, 2004; Crutchfield et al, 1986). In spatially extended systems, phase transitions are the important characteristics associated with complexity. A simple example of phase transition can be seen in random networks where *N* represents the number of nodes and *M* the number of edges. As the number of edges *M* increases in relation to the number of nodes *N*, a sudden qualitative change occurs. This qualitative change is the formation of a "giant component", an almost fully connected network (Boccara, 2004). For similar examples of phase transitions see Kauffman (1995). Likewise, the behaviour of complex systems is often characterized by a natural attraction to a critical state (Bak et al, 1988). This critical state is described by a power law distribution where small occurrences are very common and large occurrences rare. Other characteristic emergent behaviours that appear particularly in socio-economic systems include phenomena such as segregation and path dependence (Schelling, 1971; Arthur, 1989; Liebowitz and Margolis, 1995). Finally, the behaviour of complex systems has also been classified in the context of cellular automata into fixed periodic, complex, and chaotic (Wolfram, 1994) - another example where complexity appears on the edge between chaos and order.

2.2.1.2 The origins of organization

The second theme of investigation in complexity research is concerned with the development of theories of the origin of organization. A core question is to identify whether there are certain universal mechanisms and conditions that underlie the observation of 'phase transitions' to new organizational states. This quest has been widely perceived as a quest for a theory of self-organization and emergence. For example, the main principles of Darwinian evolution (e.g. Gould, 2002) and game theory (Axelrod and Hamilton, 1981; Axelrod, 1984, 1997) have been widely used in domains like biology, language or sociology as a general framework for explaining the formation of new organizational structures and functions. The theoretical study of open thermodynamic systems (Nicolis and Prigogine, 1977; Prigogine and Stengers, 1984), the theory of hypercycles (Eigen and Schuster, 1979), self-organized criticality (Bak, 1996; Jensen, 1998), and the theory of 'synergetics' (Haken, 1983) have also been perceived as general theories of self-organization.

A more specific problem regarding the origins of organization has to do with the explanation of particular organizational properties: properties that appear as 'signatures' of complexity. For instance, the observation of power law distributions in the organization of different systems is often linked to robust or resilient behaviour. For that reason power law distributions are often perceived as characteristic marks of complexity. An important theoretical problem is therefore to identify the mechanisms and conditions that explain the emergence of such distributions (e.g. Bak et al, 1987; Barabási and Albert 1999).

2.2.1.3 The effects of organization

Up to this point, the science of complexity has been identified with the description, modelling and development of explanatory theories of complexity. There are however complementary models and theories that study the implications of complexity for the functionality of a system rather than the complexity per se. More specifically, in these studies complexity is seen as a cause (rather than an effect) of the emergence of "high-level" functionalities and capacities such as cognition, autonomy, intelligence, language, life, or sociality.

For example, the study of artificial neural networks was one of the very first endeavours to understand cognition and the sophisticated functionalities of the brain, such as memory, pattern recognition, control, or creativity, as an attribute and consequence of the organization of the system. The crux of neural networks in general is that information is distributed between the nodes of the network and processed in parallel as a function of the strength of the links between the nodes. In contrast to symbolic approaches to understanding and modelling cognition, connectionist approaches assume that the high-level functionalities of the brain emerge from the interconnection and dynamic interaction of simple units (neurons) that have the ability to self-organise (Haken, 1977; Kohonen, 1984; Rumelhart and McClelland, 1986). Connectionist approaches are related to other functional or dynamical approaches in understanding and modelling cognitive systems such as those proposed in Port and Van Gelder (1995).

Moreover, autonomy (Beer 1995; Smithers, 1997; Christensen and Hooker, 2000; Collier, 2002) and abilities such as communication, language, as well as creation and assignment of meaning (Cangelosi and Parisi, 1998; Cariani, 1989; MacWhinney, 1998; Steels, 2003) have also been studied as abilities derived from complexity. Notably, such approaches take complexity not only as an innate attribute of a system, but also as an attribute obtained by the interrelation of systems with their environments.

The fundamental question of life has also been defined and studied on the basis of organisational properties of complex systems. Auto-catalytic (Kauffman, 1969, 1995), autopoietic (Maturana and Varela, 1980; Varela, 1979) and M-R systems (Rosen, 1991) are three very similar examples in which life is explained in terms of self-referential organizing principles and properties derived by the variety and distribution of complex networks. In fact, life and cognition have been in many cases studied together, exactly because they are both considered as high level abilities derived by a similar cause: complexity (Gregersen, 2003). Issues of organisational closure and self-organisation have also been associated with social systems (Luhman, 1990).

Finally, artificial life represents another very characteristic thread of research which focuses on the effects of organization and complexity for the creation of life. Computer simulations are used as a key vehicle for studying the consequences of emergence and self-organization for the identification and development of life forms (e.g. Langton, 1989; Varela and Bourgine, 1992; Emmeche, 1994). This research has also fed into the development of behaviour-based robotics (e.g. Steels and Brooks, 1993).

2.2.2 Existing links between complexity and design

Given the diversity of both design and complexity research it is natural that the relation between the two has been interpreted in a variety of ways which we can classify here under four different approaches, although there are inevitably many overlaps. Roughly complexity has been perceived 1) as a *problem* encountered in practicing design or understanding and representing design processes and products; 2) as a *characteristic* attribute of design systems and artefacts; 3) as a *methodology* and tool for designing; and 4) as a *theory* for understanding and defining design.

2.2.2.1 Complexity as a problem in design

The first approach sees complexity as a critical problem in design (whether it is the process or the product) that we need to manage and reduce. For example, complexity is associated with the difficulty of solving design problems, the combinatorial size of the search space, and the variety of the generated designs. Notably, the complexity of solving design problems is not only because these problems are often intractable, ill-defined or ill-understood, but also because they involve many different participants, with many different goals and needs. Examples of investigation include studying, measuring and managing the complexity of manufacturing, engineering and construction processes and projects, and looking at problems such as customisation, scheduling, or change management (Calinescu et al, 1998; Suh 1999; Austin et al, 2002; Eckert et al, 2004; Earl et al, 2005). Undoubtedly, the complexity of processes is tightly linked to the complexity of the product itself or the way we analyse, synthesise and represent it (Maimon and Braha, 1996; Koutamanis, 2001; Gero and Kazakov, 2003).

2.2.2.2 Complexity as a characteristic of design

The second approach sees complexity as a characteristic or attribute of design and suggests design systems can be seen as systems that exhibit complex abilities or have characteristic complex structures. For example design teams are seen and studied as complex networks with characteristic structures and rules of interaction (Klein et al, 2002; Sosa 2005). Research here also includes studies which seek to understand and model design artefacts as special instances of complex multilevel systems (Johnson, 1995, 2005) or measure and reproduce unique characteristics of designs - particularly urban forms and patterns (Batty, 2005; Marshall, 2005).

2.2.2.3 Complexity as a method in design

The third view sees complexity as a set of methods and tools both for design practice and research. This induces methods for solving design problems or methods of simulating design phenomena. For example, methodologies that have now become central in complexity research, such as evolutionary algorithms (Bentley, 1999; Frazer, 1995), or cellular automata and multi-agent systems (Rzevski, 1997; Campbell et al, 1999; Coates and Thum, 2000; Edmonds et al, 1994; Gero and Sosa, 2002; Leclercq and Juchmes, 2002; Saunders and Gero, 2002; Zamenopoulos and Alexiou, 2003b; Saarloos et al, 2005;), have been regularly used to support the creative exploration of alternatives, solve multi-objective optimization problems or evaluate design solutions, but also to model, represent, visualize and generally support complex design processes and tasks. There is a great tradition in using such techniques to model dynamical processes in cities, simulate the change of urban forms and visualize future planning scenarios (for overviews see Besussi and Cecchini, 1996; White and Engelen, 2000; Batty, 2005).

2.2.2.4 Complexity as a theory of design

Finally, the fourth approach sees complexity as a theory of design. In this sense, complexity can be seen as a set of epistemological concepts that help us approach reality and understand design processes and products: examples include the use of concepts such as self-organization, co-evolution, autopoiesis, or anticipation in the modelling of design processes (e.g. Portugali and Casakin, 2002; Zamenopoulos and Alexiou, 2007). It is also worth noting that there is a tradition in design disciplines (which might not be recorded in scientific papers) to use complexity concepts in designing practice and discourse more loosely - as an inspiration and as source for creativity and innovation.

2.2.3 Linking design and complexity in this study: design as a capacity of complexity

It can be argued that the linkage between design and complexity has mainly been brought about by the application of complexity concepts, measurements and methods in design research and practice. However, there is very little work on the theoretical understanding of design as a universal property that is *derived from the complexity* of a system.

Seeing design as a class of research problem that is fundamentally linked with complexity necessitates devising a theory of design as an organizational capacity. As we saw, the term organization generally alludes to a description of the principles of a space where certain phenomena are realized and interconnected - as opposed to a description of the principles of the phenomena themselves. This allows us to characterise the fundamental structures or processes that entail the generation of design knowledge, methods and objects, and develop a theory of design as a universal, natural phenomenon that occurs at different levels of abstraction.

2.2.3.1 Some issues concerning the view of design as a capacity of complexity

To consider design as a natural capacity of organization is a novel but potentially controversial approach for two reasons. First, the view implies that design is intelligible and explainable by natural processes, independently from the explicit recognition of an intelligent agent. This is in contrast to most design research which is dominated by the cognitive stance: most empirical studies on design thinking, computational tools or theoretical methods for design problem solving and models/simulations of design processes typically assume the authority of a cognitive design agent. Although many recent studies emphasise social aspects of design, very few works actually look at design at an abstract level, without making assumptions as to whether the capacity to design (or the capacity to recognise and address design problems) is to be found in the human mind, body, human society etc. An organizational level theory of design does not make assumptions about the physical basis or nature of design (its basic physical unit or realisation). This approach is not so contentious from a methodological point of view. Take for instance the human brain which is the best example of a design-capable system. The ability to design can be explained by the complexity and organization of the brain independently from the overall - intelligent - function of the system. A controversy however may arise as the assertion also implies an epistemological generalization: If design is a capacity derived from the organization of a system then other realizations of complex systems are possible embodiments of design-capable systems.

Second, complexity science as a science of effective organization normally assumes design as a top-down, externally imposed explanation. Complexity research is concerned with the development of theories about the origins and characteristic capacities of systems (such as life and cognition), where design is assumed as an alternative explanatory device. Design is often presented as the complete opposite of bottom-up evolution, self-organization and spontaneous order - indeed it is uncommon to consider that design is a natural characteristic explained by the organization of a system. Elaborating a ("complexity") theory of design capacity is not only a methodological and epistemological contribution to design research, but also a contribution to complexity research per se since it introduces design as a distinct and noteworthy problem.

2.2.3.2 The suggested epistemological and methodological synergy between design and complexity research

We saw at the beginning of this chapter that design research as a field starts with the recognition that there is a design discipline which has characteristic knowledge, methods and objects. The core objective of such research is to develop descriptive or prescriptive knowledge about what designers think, do, or design based on a variety of research methods (empirical, experimental, logical, computational etc). Although there are various views and definitions of design, design researchers generally agree that design is a synthetic, productive process that aims at the creation of some new (material or immaterial) artefact to satisfy goals and intentions that are themselves subjects of the process. Additionally, design is considered as a kind of research itself, carried out constructively through experimentation and through the process of developing design artefacts and processes.

Complexity research on the other hand starts with an ontological position that distinguishes and characterizes realities according to their complexity. Again, although there is a number of different ways to define complexity, there is a common understanding of complexity as an inner and distinct quality that determines and explains the properties of systems (and the emergence of significant phenomena such as cognition, life and autonomy). This position corresponds to an epistemological stance which asserts that system qualities can only be understood as the result of irreducible organization. The epistemological stance is followed with a methodological approach that focuses on knowledge creation through building or constructing desired properties and phenomena rather than trying to decompose and analyse them. This is evident in the use of generative methods, simulation, and experimentation, instead of analytic or reductionist methods.

The common emphasis on designing and constructing desired systems and properties is possibly the strongest motivation for attempting to nourish stronger links between the two fields of design and complexity. In design practice, knowledge about the design problem and how to solve it is gradually built through the process of developing the design artefact. However design practice does not generate knowledge *about* design (although it helps develop domain knowledge). If we wish to generate knowledge about design itself, we need to operate at a different, meta-level, of enquiry. The present thesis adopts the view that the (complexity) theory of design should be developed by building formal, mathematical and computational constructs. We will return to this argument in Chapter 4.

2.3 THE RESEARCH AGENDA OF THE THESIS: SUMMARY AND CONCLUSIONS

This thesis is concerned with the study of design in two intertwined ways. First, it is concerned with the study of design as a *universal phenomenon and capacity.* This alludes to the study of design as a phenomenon and capacity that arises across different disciplines and domains of human activity. The hypothesis of universality also alludes to the study of design as a phenomenon or capacity that arises at different levels of reality, for example, neurological, cognitive, or social.

Second, the thesis is concerned with the study of *design as a universal phenomenon and capacity of complexity.* This position alludes to a view of design as a universal problem and capacity that is uniquely distinguished by certain organizational characteristics whether these are realised in the neurological structure of the brain, a symbolic representation, or a social structure. On this basis, the purpose of the thesis is to develop a (mathematical) theory of design as a universal property of complexity and organization.

The motivation behind the development of such a theory is to contribute to an understanding of design as a characteristic problem that exists across different domains of human activity, such as art, science and mathematics. Such a theory also enables us to *understand* a large class of 'design-capable systems' (whether these are natural or artificial, cognitive or social) based on a characterisation of the structures or processes that entail the generation of design knowledge, methods and objects. This knowledge can in turn feed into the creation of hypotheses for studying design-capable systems empirically, and inform the development of methods and tools for *supporting* design activities. Finally, the constitution of design as a universal capacity of complexity is a double contribution to complexity research as well as to design research, and makes it possible to transfer results between the two fields.

In the next chapter, the meaning and scope of this thesis (i.e. design is a universal property of organization) is explored on the basis of a specific computational model. Subsequently, this position will be turned into a mathematical problem.

Chapter 3

DESIGN OUT OF COMPLEXITY: A MODELLING EXPERIMENT

In Chapter 2 the argument of the thesis was introduced that design can be understood and explained at an organizational level of description; as an organizational property of complexity. In this chapter a computational model of group design is built in order to develop and clarify the meaning of this central hypothesis. The presented system constitutes an example where design can be described as a property of the organization of the system, rather than as a capacity of individual designers. As discussed, the construction of formal (computational and mathematical) models of design is used as a methodological vehicle for developing the theory. The model is not a case study from which to generalise, but represents an effort to model how design emerges out of complexity and thus think about conditions of design-ability and identify key issues for further development.

3.1 THE COMPLEXITY OF GROUP DESIGNING AND THE NOTION OF COORDINATION

In Chapter 2 we saw that design research has been gradually shifting its attention away from rational problem solving to focus more on the situational and social aspects of design. This shift involves understanding not only the cognitive abilities of designers, but also how these abilities are expressed within, and formed by, the setting in which design takes place. This setting is taken to include the media and tools used in the design process (e.g. drawings, documents, CAD systems, artefacts etc), but also the social roles, structures and interactions developed among designers, and with their clients, or users. Indeed one of the reasons why social aspects of design have become the centre of attention in recent years is that design increasingly happens in groups of diverse participants, whether these are designers from the same or different disciplines, clients, or stakeholders. Additionally the development of design products (or services) is to a great extent driven by the desires and requirements of the recipients of those products, who ultimately evaluate and re-shape them to fit their practice. According to Rittel, the characterisation of design problems as wicked is crucially connected to the fact that design knowledge and decisions are "distributed among many people, in particular among those who are likely to become affected by the solution" (Rittel, 1984: 320). Rittel considered that the need for a parallel construction of design problems and solutions is derived from this inherent "symmetry of ignorance", the mutually limited capability of participants to grasp, form and resolve design problems.

Group design then appears to be a good starting place for understanding how design as a natural capacity may manifest itself at different levels of abstraction (from the individual to the social). How can we model group design situations where both the means (knowledge, resources, processes) and the ends (desires, goals) are distributed? How can a complex group design problem be solved (or modelled) so that distributed goals and desires can be "equally" met? Various approaches to such problems have been developed in the past. For example, in collective choice, the idea is to develop a collective function that needs to be optimised for the sake of 'social welfare' (see for example Arrow, 1951; Sen, 1979). Game theoretic approaches focus on individual (unilateral) decision making, where the dynamic between the involved parts determines the distribution of welfare, through processes like cooperation, bargaining or negotiation (see for example Von Neumann and Morgenstern, 1944; Nash, 1950; Rapoport, 1974). For more details about group decision making see Kleindorfer et al (1993).

It is instructive to consider that when knowledge, resources and desires are distributed, the process of reaching a common set of decisions or solutions is in effect a coordination process.

Coordination is extensively discussed in organizational theory where it is defined as *'the act of managing interdependencies between activities performed to achieve a goal'* (Malone and Crowston, 1990: 361). In the context of organizations, coordination is about resolving, ordering and managing a variety of complex interdependencies expressed in terms of resources, time, responsibilities or tasks. Coordination is also a central concept in the fields of distributed artificial intelligence and multi-agent systems. Agents are artificial individuals with limited knowledge, resources and information-processing capabilities, which in effect need to engage in different forms of collective activity or interaction, in order to improve their problem solving and goal attainment capabilities. Coordination refers to the process by which collective behaviour is achieved and might consist in ordering, synchronisation, planning, and adaptation of actions, decisions and goals (see e.g. Ferber, 1999; Ossowski, 1999). It is interesting to note that in the history of the field, two major views on modelling coordination have been expressed. The first was focussed on the development of appropriate ontologies, rules, shared agreements and conventions as a basis of coordination; the second, was focussed on local agent interactions and the idea of building coordinated group behaviour "from the bottom up". For an overview and pointers to relevant literature see also (O'Hare and Jennings, 1996; Sycara, 1998). The latter approach to coordination, which has links to artificial life and behaviour-based robotics (Steels and Brooks, 1993), has grown to be more dominant in recent years. Together with dynamical approaches to the emergence of coherent global behaviour (Di Paolo, 1999; Atay and Jost, 2004; Jirsa and Kelso, 2004), they have become seamlessly incorporated in the core curriculum of complex systems science.

In the following sections we detail first a conceptual and then a computational model of group designing as coordination between distributed agents with individual knowledge and desires.

3.2 A CONCEPTUAL MODEL OF GROUP DESIGN AS DISTRIBUTED LEARNING CONTROL

The notion of coordination as defined in multi-agent systems, artificial life and organizational theory, is used here to capture the idea that group activity and decision making are complex processes of reaching common solutions through resolving, exploiting and constructing interdependencies between distributed agents. The position of the thesis is that design is a universal yet distinct problem in the multiplicity of complex problems we encounter in our professional and everyday lives. So how can the concept of coordination become a useful vehicle for modelling group design?

The premise behind this treatment is that design in general arises in response to a particular situation: when/where there is a desire, need or an idea that something should or could be different in a certain environment, but the means to achieve such a change are not immediately known (see for instance Archer, 1965; Mitchell, 1990; Smithers, 2002). In particular, the situation can be described as an inconsistency that emerges between beliefs (or knowledge) about the past, current and future states of the environment, and the expressed desires, needs or goals regarding the states of this environment. The inconsistency, or conflicts, between beliefs and desires is quintessential in distributed design where different agents make design decisions and act on the basis of individual belief and desire systems. This premise resonates with the idea that the nature of design problems is "paradoxical" and requires an understanding of the design process as co-evolution among distributed designers' goals and solutions (Dorst, 2006).

The notion of coordination makes it possible to formalise some basic characteristics of group design thus constructed. First, it raises awareness about the importance of distribution. The distribution of decisions, actions and goals means that there is no central authority capable of controlling the process and outcome of design. So group designing is seen as a distributed control process carried out by agents with individual goals, resources and capabilities. To take this further it is important to delve into the idea that the knowledge carried by individual participants (expert or non-expert) is also distributed. In this sense, the activity of learning and constructing the knowledge necessary for understanding the design problem and its resolution is also of crucial importance. In group designing, knowledge of other agents' activities and goals is equally vital.

We can thus start to construct a conceptual understanding of group design coordination as a distributed learning control problem. Learning corresponds to a process of capturing interdependencies among design variables; while control corresponds to a process of using this knowledge to generate control actions (design solutions) that meet time-variant individual targets, despite conflicts introduced by other agents. In this context, creativity and innovation lie in the possibility of unforeseen solutions emerging through agent interaction and learning. Let us elaborate these ideas in more detail.

3.2.1 Control and distributed learning control: an introduction

The notion of control was born from cybernetics research (Wiener, 1948; Ashby, 1956) and since has been used in various scientific areas: in engineering, as well as the physical and social sciences. Control is tied to goal directed behaviour: it refers to actions taken by a system in order to reach or maintain a target state, despite endogenous or exogenous disturbances and perturbations. Control systems are systems in which a controller interacts, by way of one or more controlling variables, so as to influence the state of a controlled object (also called 'plant').

3.2.1.1 Control problems and mechanisms

Several typical ingredients of control problems can be identified (Jacobs, 1993: 1):

- the controlled objects have their own dynamics so their outputs cannot be changed instantaneously
- the range of available adjustments is usually constrained
- characteristics of the controlled object are not known with certainty
- the state of the controlled object may be affected by uncontrolled,

external and unpredictable inputs (known as exogenous variables)

- desired values for the state of the controlled object are often exogenous and unpredictable
- current values of the state of the controlled object are uncertain, and
- measurements carrying information about current states are noisy.

One of the main ideas behind adaptive control systems is feedback: the controller attempts to steer the controlled object by adjusting a controlling variable (control signal) on the basis of measuring the difference between a desired value and the actual value of a controlled variable. Exogenous sources may disturb the feedback signal, but the controller needs to achieve its objective despite the disturbance or other uncertainties about the characteristics of the control object (Figure 5).

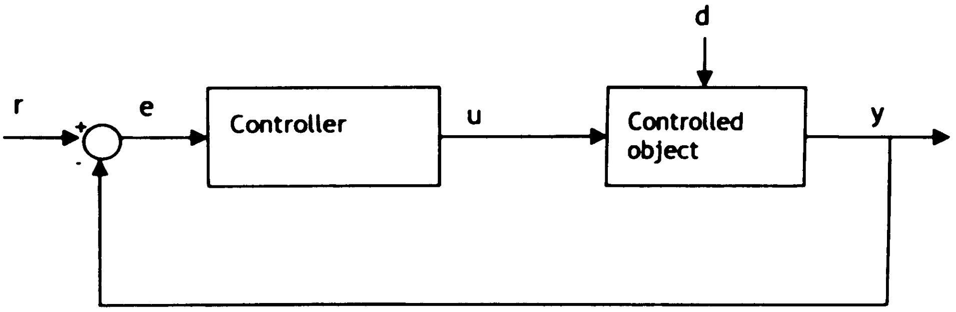


Figure 5: A simple control system using feedback. r is the reference value or target, u is the control action, y is the system output, and e is the error between actual and desired system output. d is the disturbance applied to the controlled object.

While feedback is the most used mechanism, control can also proceed through a feedforward mechanism. In this case, regulation is achieved not through the use of an error signal, but through the use of predictions performed by an internal model of the controlled object.

3.2.1.2 Adaptive learning control

We are interested here in adaptive learning control. Neural networks (NN) have been used in adaptive control because of several advantages. They are trained directly from the data and they can form generalisations (learning by experience or inductive learning); they are able to represent non-linear functions (non-linear function approximation) without the need for explicit representations of the process or problem being studied (sub-symbolic or connectionist reasoning); and they perform robustly in the presence of noise (see e.g. Kecman, 2001). Neural networks have been extensively used in adaptive control and there are many different architectures and control methods. The general idea is to train a neural network to create a model of the system to be controlled ('plant' modelling or identification phase). The trained network is then used in the control process, either directly (acting itself as a controller) or indirectly (to guide and support the control process).

For example, the 'direct inverse control' architecture uses a model of the inverse dynamics of the controlled object. This inverse model is then used as a controller, which is placed in sequence to the controlled system so that the two result in an identity mapping between the desired response (the NN inputs) and the system output (Figure 6). In other words, during the modelling phase the NN Model is trained to approximate the output-input dynamics of the controlled object. During the control phase, the same neural network is used as a controller.

As another example, the 'model reference control' architecture (Figure 7) contains two neural networks: one neural network acts as the controller, and the other acts as a model of the controlled system (the NN here is a forward model of the system). The training of the NN model is based on measuring the error (the difference) between the system output and the model output (Model Error). The architecture also contains a reference model, which provides an explicit goal for training the controller. The difference between system output and reference output (Control Error) is used to adjust the control signal.

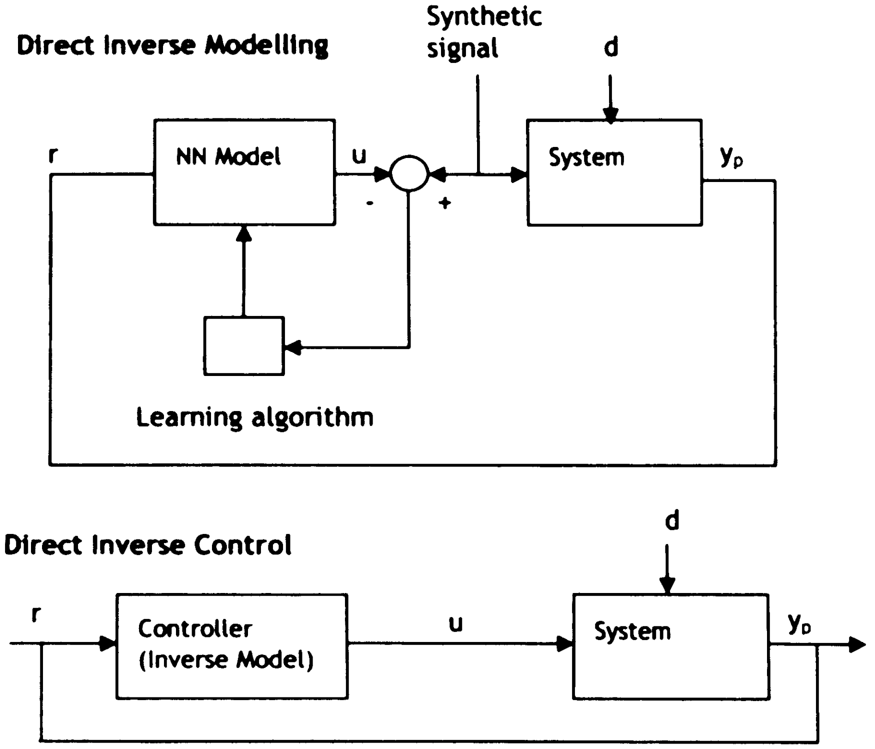


Figure 6: Direct inverse control uses an inverse model of the system. When the trained NN model acts as a controller, it is placed in sequence with the system and provides control actions u to make the system output y_p match the reference input r.

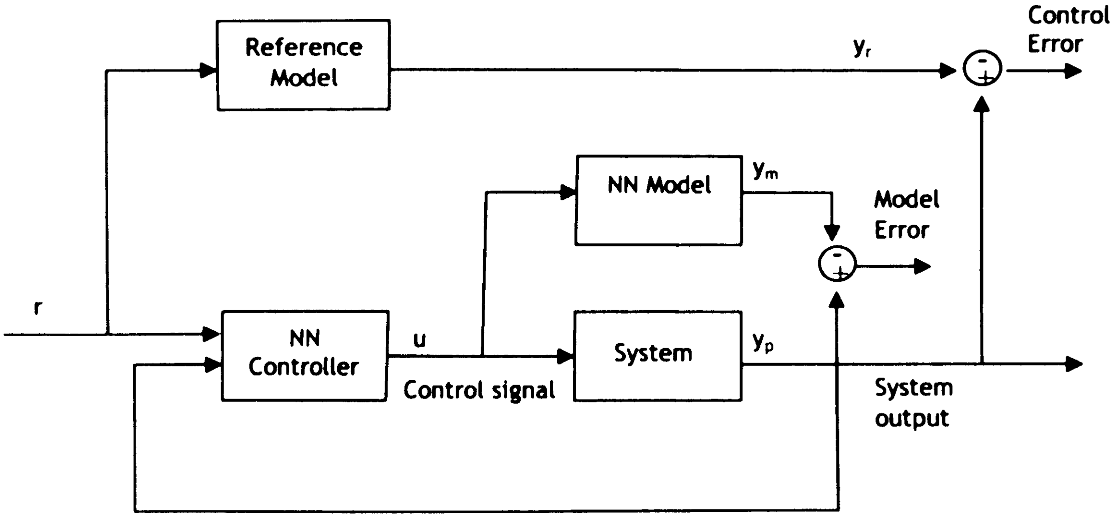


Figure 7: The model reference control architecture contains two neural networks (a forward model of the system and a controller), as well as a reference model that provides targets for training the controller.

Many control architectures use a combination of neural networks and other techniques or algorithms (for optimisation purposes for example). For an overview of neural network control see Hunt and Sbarbaro (1995), Narenda (1996) and Moscinski and Ogonowski (1995).

3.2.1.3 Why distributed learning control?

Control seems particularly suited to the constrained, goal-oriented activity of design, specifically where design problems are characterized by complexity, uncertainty and non-linearity. In the early days of design research, the conceptual category of design was identified with control and the designer was defined and studied as a controller. The design task was then defined as a process that transforms design object configurations with the objective of maintaining a principal idea or function and to satisfy certain performance criteria, despite uncertainties and exogenous disturbances. A prominent example can be found in the work of Archer (1970) who approached designing as an iterative process of generating and controlling a set of (decision) variables in order to fulfil a given set of objectives optimally. Cybernetic notions of control have also been used in the context of planning (Forrester, 1969; Ackoff, 1974; Beer, 1974; McLoughlin, 1977; Chadwick, 1978).

However the singular view of control assumes that the target of design is given, and that the designer is able to acquire and maintain all the knowledge necessary to solve the design problem. In this sense the designer is able to control the overall process single-handedly. As we discussed, especially in group design, not only are the goals distributed but so is the knowledge needed in order to understand, (re-)construct and solve design problems. So a notion of decentralised, or distributed, control is necessary for understanding the complexity of planning and design processes and how goals, knowledge and activities are organized together into a coherent whole.

3.2.2 Using the Function-Behaviour-Structure framework (FBS) to model group designing as distributed learning control

We saw in Chapter 2 that the design process is typically considered to incorporate the phases of analysis, synthesis and evaluation which inform one another through a series of feedback loops. Gero (1990, 2000) has developed a model of the design process which additionally incorporates the notions of Function, Behaviour and Structure (FBS) as connectors between the different phases. The model is motivated by the view that design is concerned with the development of specifications (or descriptions) of artefacts in order to satisfy some purpose or function. To understand these terms and identify the relevance to this experimentation it is necessary to introduce the model in more detail.

3.2.2.1 An introduction to Gero's FBS model

The FBS framework proposes a set of processes, articulated as transformations between functional, behavioural and structural classes of information, through which a *design description D* (e.g. a blueprint) is achieved. The notion of *structure S* refers to the form of the design object, or the configuration of its elements. As there are many kinds of design objects, structure may express relationships among various kinds of elements, for example artefacts, processes, attributes, or spaces. *Behaviour B* can be seen as a kind of observation or measurement applied over a particular structure and refers to the properties of an object that realise a function. Gero distinguishes two types of behaviours: *structural behaviours Bs* are those derived (directly or indirectly) from structure, and *expected behaviours Be* are those derived from function. For example, structural behaviours may describe attributes of a structure, how the structure changes in time, or may express qualitative or quantitative measures of performance. Finally, *function F* represents the purpose of the object that the structure (and resulting design description) is supposed to fulfil. Functions, behaviours and structures are connected via a set of different transformations or processes, illustrated in Figure 8.

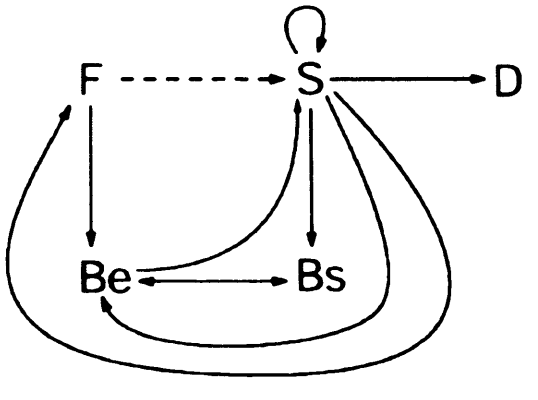


Figure 8: The FBS model proposed by Gero (1990) and Gero and Kannengiesser (2002). F=function, S=structure, Be=expected behaviour, Bs=structural behaviour and D=design description. Arrows indicate transformations between FBS. The double arrow denotes comparison, and the dashed arrow denotes an occasional transformation.

More specifically, in this framework the process by which structural behaviours Bs are deduced from structures S corresponds to *analysis*; while the process where the expected behaviours Be are used to produce structures S corresponds to *synthesis*. *Formulation* refers to a process by which a function F is mapped to expected behaviours Be; and *reformulation* refers to a process by which changes of expected behaviours Be can be produced through given structures S. Two other types of reformulation are also considered (e.g. Gero and Kannengiesser, 2002), one which refers to changes in terms of structures and one which refers to changes in terms of functions (or ranges of values of those variables). Finally, *evaluation* corresponds to a process of comparing structural and expected behaviours in order to assess whether the synthesised structures fulfil the desired function. A list of the proposed transformations is given in Table 7. The transformation from function to structure is occasional, and is considered less important as it corresponds to a kind of *catalogue lookup* rather than a constructive design process.

Analysis:	S → Bs
Formulation:	F → Be
Synthesis:	Be → S
Evaluation:	Be ↔ Bs
Reformulation type 1:	S → Be
Reformulation type 2:	S → S
Reformulation type 3:	S → F
Design Description:	S → D

Table 7: Summary of the processes involved in design as discussed by Gero and Kannengiesser (2002). Arrows represent transformations between FBS variables and the double arrow represents comparison.

3.2.2.2 Elaborating FBS into a model of distributed learning control

Gero's framework offers a way to model different processes involved in designing, explicated in terms of functional, behavioural and structural variables. The proposed ontology of processes is useful, but - as often noted by Gero's critics - the framework is biased towards an "engineering" perspective of design, particularly with regards to its conceptualisation of function. While fulfilling a particular function may be the motivation for engineering design, other design disciplines see function differently. For example, architectural design is often thought to start with an idea, or abstract concept in mind, rather than a particular function. Additionally, the term function is also customarily connected to utility (usefulness), or to the description of activities rather than purposes. For the model presented here a wider perspective of function is taken into consideration, one that accepts the notion of intentionality but also includes perceptions of function beyond the simple description of what an artefact does.

It is also necessary to adapt the FBS framework for the purpose of modelling group, rather than individual, design. The conceptual model presented here recasts the processes of analysis, synthesis, formulation, reformulation and evaluation within the context of distributed learning control.

A distributed design task is taken to involve a number of agents that act on a common world (so a direct interaction between agents and their environment - which includes other agents - is assumed). As discussed, each agent acts on the basis of individual beliefs and resources so as to achieve individual desires about the state of the world. From the discussion above, it is possible to infer that synthesis and analysis are concerned with the exploration and creation of possible structures, while reformulation and reformulation are concerned with the exploration and creation of possible functions. The activity of each agent can be construed as a dual control process (see Figure 9).

The first control activity corresponds to an analysis-synthesis-evaluation route. The objective of each agent as part of this activity, is to find suitable structures S (control actions) that can lead the behaviours Bs to follow the expected behaviour Be (derived by the function F), despite uncertainties and despite exogenous disturbances Sd produced by other agents' decisions. Each agent learns associations between proposed structures S and actual behaviours Bs and uses this knowledge in order to find suitable control actions. The evaluation of the distance or 'error' between expected and structural behaviours (Be-Bs), is also used to inform the selection of suitable control actions S.

The second activity corresponds to an evaluation-formulation-reformulation route. The objective of each agent as part of this activity, is to find a suitable function F leading to such expected behaviour Be that can satisfy a given structural configuration S, despite uncertainties and despite exogenous disturbances Fd (i.e. other agents' goals). Knowledge of associations between proposed functions F and the overall derived behaviour Be (derived through learning), as well as an evaluation of the 'error' between expected and actual behaviour of the world, are used to inform the generation of suitable targets F. Overall, each agent uses this dual control process in order to lead the common world to perform according to his or her individual desires.

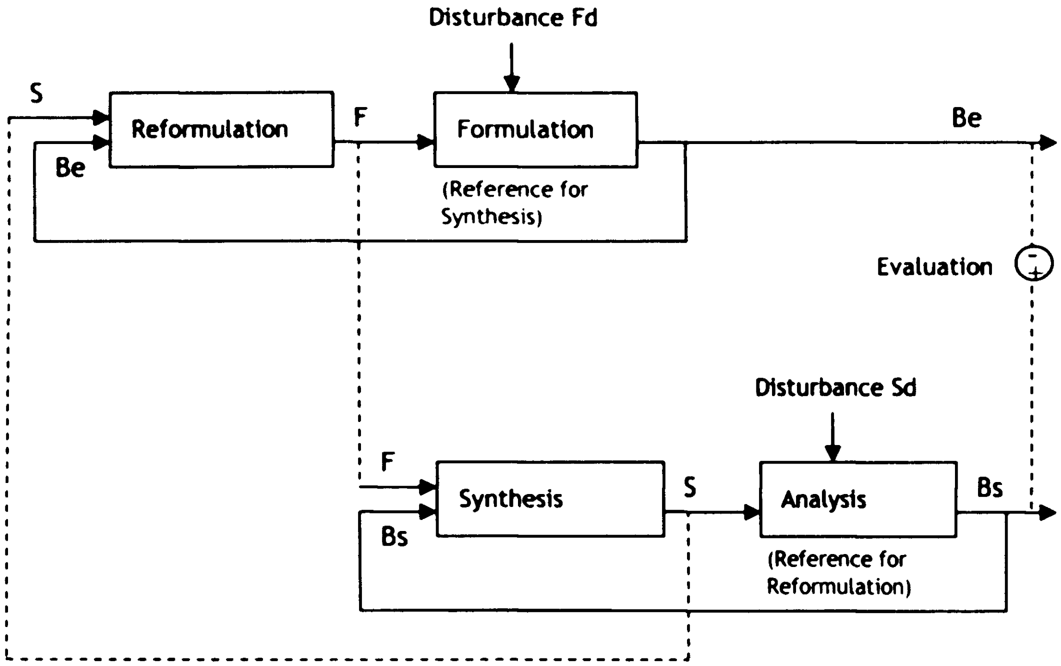


Figure 9: Coordination as a dual control process, one that corresponds to a synthesis-analysis-evaluation route and one that corresponds to a reformulation-formulation-evaluation route. The two processes co-evolve together by providing evaluations and targets for one another.

The relation between the two control processes is such that they provide targets and constraints for one another: the desired performance of the analysis-synthesis process is evaluated through the formulation-reformulation process and vice-versa. In this sense, the solution space (expressed by operations in the space of possible structures) and the problem space (expressed by operations in the space of possible functions), can be thought to co-evolve together.

The proposed model also captures the idea that each agent's desires and solutions may be conflicting and this conflict becomes explicitly part of the design process. Distributed learning control captures the idea that each agent needs to arrive at design solutions by avoiding conflicts (or disturbances) introduced in the world through the action of other agents. The process of solution generation based on learning control is a process of learning and self-adaptation of agents that leads to coordination of their distributed descriptions.

Having this conceptual model we can now explore possible computational implementations that will allow us to test the underlying hypotheses and identify issues for further development.

3.3 A COMPUTATIONAL MODEL OF GROUP DESIGN

To bring this investigation closer to the architectural and urban design domain, the example of designing an urban plan is considered here. The development of plans is a characteristic design problem: it starts with a desire to produce a change (whether it is a spatial configuration, a strategy, a policy or an agenda), but the desired change is not well-formed and the means to achieve it are not immediately clear or available to the participants. 'Plan designing' is a complex problem as it depends on distributed and often conflicting knowledge, resources and goals, and involves distributed processes which operate in different levels and at different times. Additionally, plan designing aims not only to address current needs, but also to create future opportunities (Hopkins, 2001). Hence, plan designing also crucially involves specifying novel solutions in anticipation of future situations.

3.3.1 Plan descriptions

Plan descriptions in this model are generated within a virtual reality (VR) world (a hypothetical city) and are composed by objects introduced by agents. Objects are justified on the basis of a "purpose" for the design task: initially three objects are used (three cuboids), which represent the preliminary development goals for a housing unit, a retail facility and an open space. Plan descriptions, and hence object specifications, are dynamically generated and modified through the interaction between agents (Figure 10).

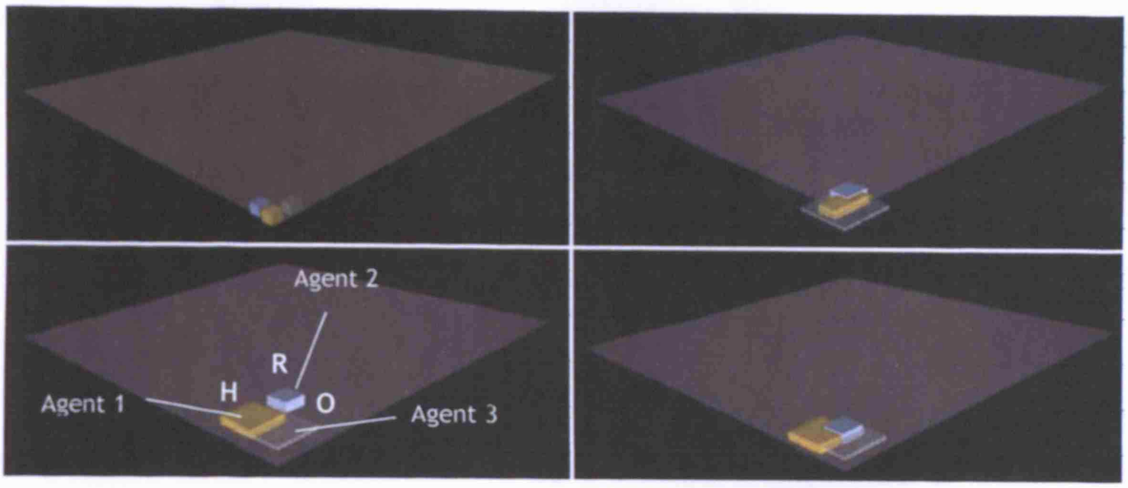


Figure 10: Evolution of plan description within a VR environment based on three objects that represent development goals for a housing unit, a retail facility and an open space. Each object is controlled by an agent.

The objects within the VR environment are built on FBS classes of information. Formally, each object is specified as an array: $A_i = [F_i, B_i, S_i]$. The overall plan description is the column matrix $P = [A_i]$ of all these objects. More specifically, functional information represents the ontology and purpose of the proposed objects expressed as land use (housing H, retail R, and open space O). Structural information specifies the elements of the proposed plan, their attributes and their relations, including the physical description of the objects and their topological relations. So, for instance, for an object A_R (retail), structural information includes location [x y], volume dimensions $[z_x\ z_y\ z_z]$ (Figure 11), but also relations with other objects such as distances between them. Behavioural information specifies the way each object reacts to changes of its state and its environment. Behaviour is a description of change of the design objects in order to reach their intended functions. For instance new land uses tend to be developed close to or far from other existing land uses in order to fulfil their functional requirements.

This formulation is of course an example rather than a strict definition of the FBS framework in the context of urban development.

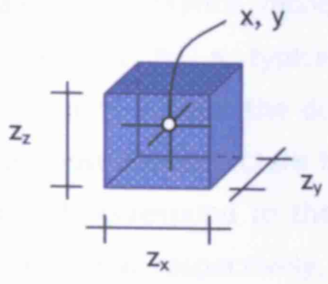


Figure 11: Example of representation of structural information of an object in the VR world, in terms of location [x y] and volume dimensions [z_x z_y z_z].

3.3.2 The control architecture of the design agents

The design problem is formulated as a coordination problem among self-interested agents (that manipulate the cuboids in the VR world) and is addressed via a distributed learning control methodology. In general, the idea is that each agent introduces and controls only a specific object of the plan (i.e. it has partial control of the overall plan). Each agent has a neural network that learns associations between FBS variables and uses this knowledge to formulate expectations and goals for the design problem. Additionally, each agent also has a control mechanism that aims to steer the overall design configuration towards individual goals by avoiding conflicts and disturbances coming from other agents. The objective for each agent is therefore two-fold: to minimize errors in modelling the environment (i.e. the difference between what the agent 'knows' about the environment and what actually happens) and to minimize control errors (i.e. the difference between the desired plan and the actual plan of the city formulated with the contribution of all agents). Even though there are several control-based formulations that might be suitable for modelling coordination problems, one alternative is presented here (for a different formulation refer to Alexiou and Zamenopoulos, 2001).

The model was developed using the MATLAB-SIMULINK (Mathworks, Inc) environment. The chosen control structure is based on the 'model reference control' architecture briefly discussed above (Figure 7). This architecture utilises two neural networks, the controller and the controlled system model (world model). It additionally contains a reference model that provides targets for training the controller. In contrast to a typical reference model control architecture however, and in order to reflect the dual control structure outlined in the conceptual model, the proposed architecture is elaborated into two control systems that work in parallel and correspond to the two processes of synthesis-analysis and formulation-reformulation respectively. The formulation component plays the role of a reference model (i.e. provides goals) for the analysis-synthesis process and the analysis component plays the role of reference model for the formulation-reformulation process.

The world is implemented as a system of three objects (cuboids) that represent three agents with different development goals. Each agent observes the interaction and behaviour of objects within this world and learns associations between FBS variables that represent decision making patterns. As will be explained later, simple artificial reasoning systems were used in order to model human reasoning in this experiment, and provide artificial agents with some initial expectations. The acquired knowledge is used to train the controller to find appropriate solutions or actions that can be used to satisfy the goals provided by the reference model. The reference model is produces time-variant targets for the controller.

More analytically, in the synthesis-analysis process (Figure 12) each agent attempts to find a suitable path of structures S that will lead the behaviours Bs to follow the reference (expected) behaviour Be, despite uncertainties and despite exogenous disturbances produced by other agents' decisions. The expected behaviour Be is defined by the reference model (formulation). The error between the behaviour Bs of the world and the predicted behaviour Bs derived by the world model (denoted as model error) is used in order to train the world model through a typical feedback control loop. Knowledge of associations between proposed structures S and actual behaviours Bs, and the observed error, are both used to inform the selection of suitable control actions S. The error between the behaviour of the world Bs and the reference behaviour Be derived by the formulation process however (denoted as control error) constitutes a performance criterion and is used to adjust the control parameters.

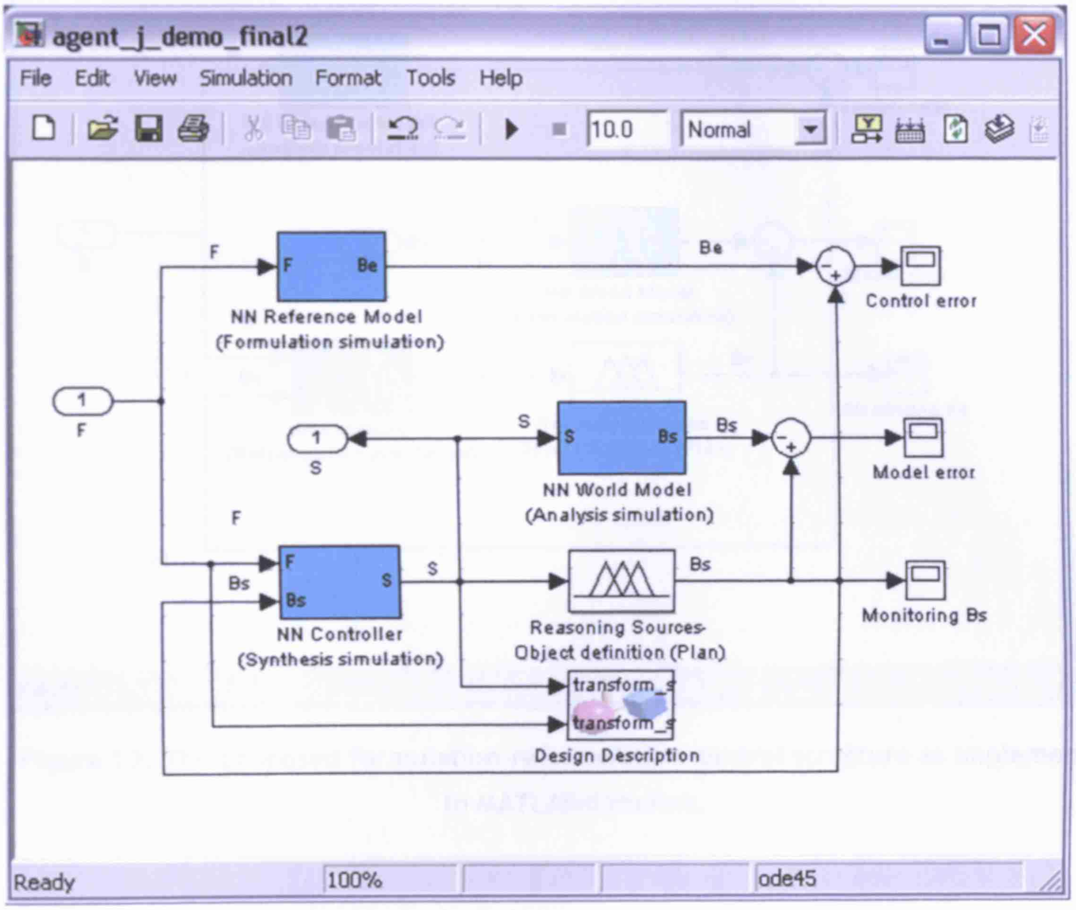


Figure 12: The proposed analysis-synthesis control structure as implemented in MATLAB/Simulink.

Similarly, in the formulation-reformulation process (Figure 13) the objective of each agent is to find the appropriate functions F that will lead to structural behaviours Bs as provided by the analysis component. The world model (associations between F and Be) is used to inform the selection of appropriate functions. Again, evaluations of errors between actual and model outputs are used to train the world model, while errors between actual and expected behaviours are used to guide the control process.

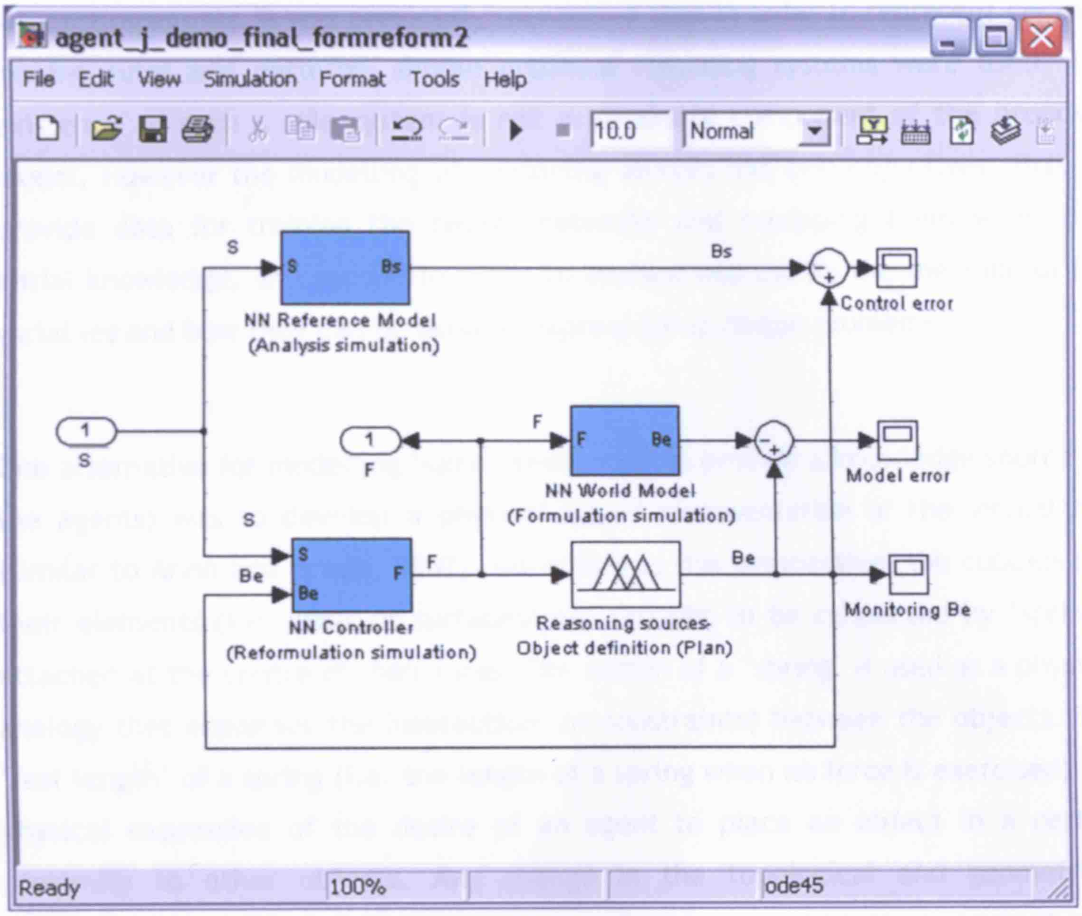


Figure 13: The proposed formulation-reformulation control structure as implemented in MATLAB/Simulink.

The overall objective of the dual control system is to make the output of the controlled system (i.e. the plan) follow the reference output asymptotically, as represented by the equation:

$$\lim_{t \to \infty} |y_p(t) - y_r(t)| < \varepsilon$$

where ε is a positive integer, $y_p(t)$ is the system output, and $y_r(t)$ is the reference signal. (Note that the reference for the synthesis-analysis circle is Be and the reference for the formulation-reformulation circle is Bs).

3.3.3 Modelling human reasoning

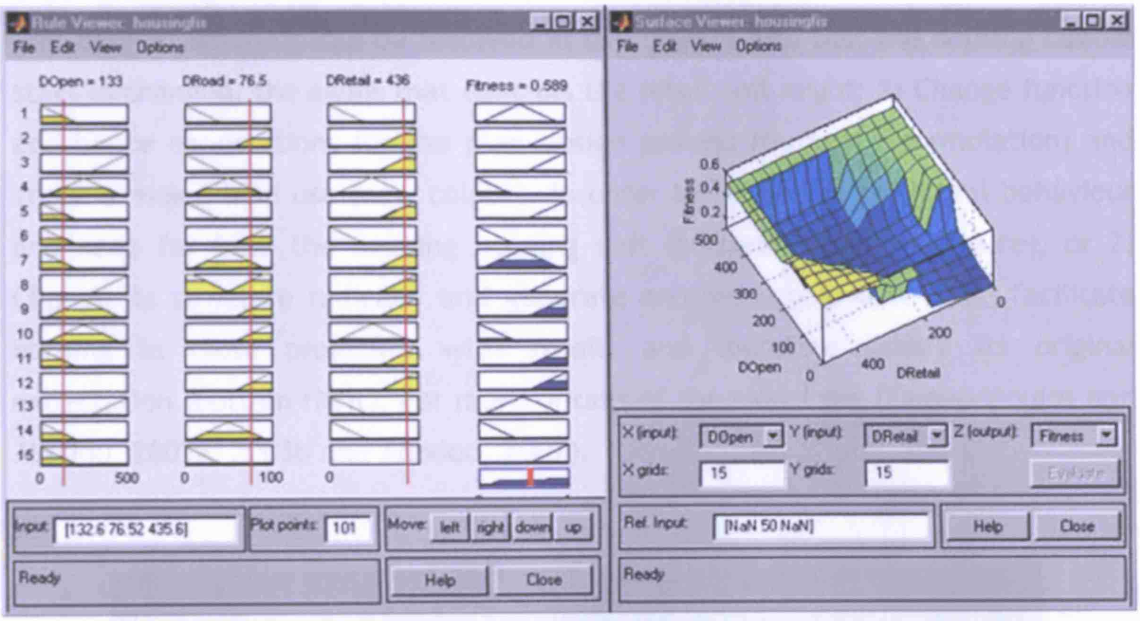
The associations between S/Bs and F/Be variables - namely the 'white box' in Figures 12 and 13 - are in principle constructed from data generated by the interacting agents. It was previously mentioned that in order to represent decision making rules and patterns, simple artificial reasoning systems were used. The existence of such a rule system is *not* a necessary component of the proposed model. However the modelling of reasoning sources had two objectives: first, to provide data for training the neural networks and equipping them with some initial knowledge, and second in order to explore and clarify the meaning of FBS variables and how they can be used to express group design problems.

One alternative for modelling human reasoning (to provide a knowledge source for the agents) was to develop a physical based representation of the virtual city (similar to Arvin and House, 2002). According to this perspective, the cuboids and their elements (i.e. walls or surfaces) are thought to be connected by 'springs' attached at the centre of their mass. The notion of a 'spring' is used as a physical analogy that expresses the interaction (or constraints) between the objects. The 'rest length' of a spring (i.e. the length of a spring when no force is exercised) is a physical expression of the desire of an agent to place an object in a certain proximity to other objects. Any change in the topological and geometrical properties of a cuboid, causes changes to all the connected components due to forces applied from the springs. So, the "moving behaviour" of the cuboids is described by a system of n equations of the following form:

$$m_j x_j'' = \sum_{i=1}^{n} k_{ij}(x_i - x_j)$$

where x_j is the displacement of cuboid j, x_i is the displacement of cuboid i, m_j is the floor area of cuboid j, k_{ij} is the interaction matrix between cuboids j and i, and x_j'' is the second derivative of the displacement of j. The interaction matrix k_{ij} defines the strength of the attractiveness between land uses (or cuboids).

The second alternative for modelling human reasoning was to use fuzzy logic inference methods. Fuzzy systems are built on the basis of fuzzy IF-THEN rules (see Figure 14), used to convey qualitative knowledge about a particular phenomenon or system (for comprehensive introductions to fuzzy logic see Kosko, 1994 and Klir and Yuan, 1995).



DOpen = 133
DRoad = 76.5
DRetail = 436
Fitness = 0.589

Input: [132.6 76.52 435.6]
Plot points: 101
Ready

X (input): DOpen
Y (input): DRetail
Z (output): Fitness
X grids: 15
Y grids: 15
Ref. Input: [NaN 50 NaN]
Ready

Figure 14: Agent reasoning as a fuzzy system.

In particular, a simple fuzzy inference system was built to model attractiveness between land uses, and represent qualitative evaluations about the fitness of a specific location based on criteria of proximity to other cuboids. This system was implemented using the Fuzzy Logic Toolbox in MATLAB. More specifically, the system has as inputs the fuzzy sets housing (H), retail (R), open space (O) and volume (Vol), and as outputs the fuzzy sets proximity to housing (ProxH), proximity to retail (ProxR), and proximity to open space (ProxO). Housing, retail and open space represent functional information, while volume expresses structural information. Proximity expresses behavioural information.

The idea of constructing such rule-based systems was to explore how to model conflict situations that may arise because of different goals or expectations for the configuration of space. Given that each agent has only partial control over the overall description, each agent needs to find the appropriate solutions (functions or structures) by avoiding or resolving conflicts and disturbances. To illustrate this in more detail we present below an imagined conflict situation. In Figure 15 the agent that controls the blue, retail unit, has developed an expectation to move close to the housing facility. In contrast, the agent that controls the yellow, housing facility has developed an expectation to move far from the retail unit. This conflict situation can be resolved in two ways. Given that the housing cuboid stays unchanged, the agent that controls the retail unit might: 1) Change function and hence expectations for the plan design process (through reformulation) and adopt a mixed land use (new colour), in order to follow the structural behaviour and keep far from the existing housing unit (bottom left in the figure), or 2) Change its structure radically and generate another cuboid that could facilitate housing in close proximity with retail, and therefore satisfy its original expectation (bottom right). For more details of the model see (Zamenopoulos and Alexiou, 2003a; 2003b and Alexiou, 2007).

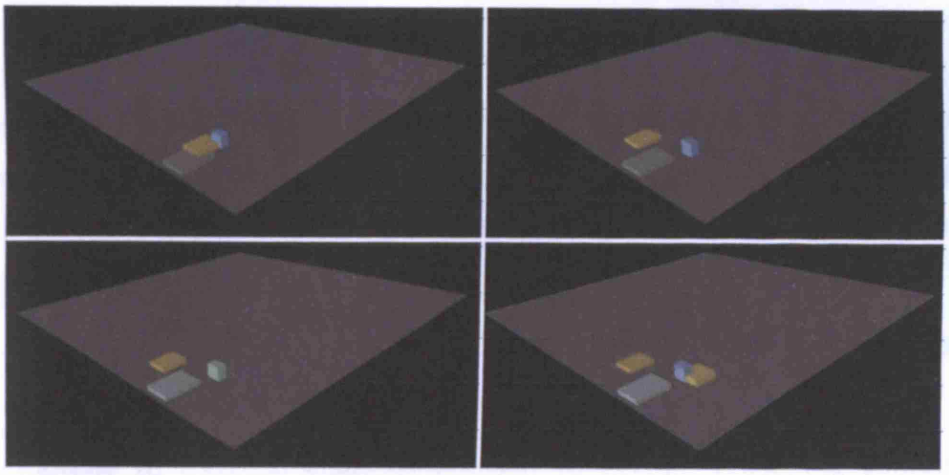


Figure 15: A conflict situation (top) and its resolution (bottom).

3.3.4 Outcomes and reflections

Although the proposed model has been implemented in the MATLAB/Simulink environment no fully fledged experiments were carried out. The model presents an interpretation of design as a coordination process that arises in a distributed system. In this interpretation, there is no real commitment regarding the nature of the distributed system. Indeed, design is presented as an organizational property explicated by a coordination process which may be equally located in a brain or a social setting. In any case, the design task, knowledge and processes are distributed in the interaction of multiple agents, and only through a process of 'self-organization' is there the creation of a design artefact. This was the core purpose of this demonstration.

Moreover, explicating the role of creativity, learning and anticipation for the identification of design as a distributed control process was also an important motivation. The creative ability of the agents is largely related to the possibility of discovering and learning novel interdependencies among FBS variables, but also the possibility of developing new goals for the individual design activity of agents. For that reason, the existence of disturbances in the control mechanism constitutes a source of variation and an opportunity for novelty for individual agents (Alexiou, 2007). In principle, creativity also depends on the construction of new FBS *variables* (as discussed for example in Figure 15). This view of creativity is in accordance with general theories of creative design such as the one presented by Gero (2000). However, for the present implementation the introduction of new FBS variables would also imply structural changes in the architecture of the neural networks. The complexity of this type of adaptation constrained the implementation of the model. So, for the individual agent only three types of leaning (i.e. adaptation) were implemented:

- Learning to anticipate the reaction of other agents and therefore build a *belief system* (implemented by the World Model Neural Network)
- Learning to develop goals in relation to certain reasoning sources and therefore build a *desire system* that does not contradict the belief system (implemented by the World Model Neural Network)
- Learning to control the reaction of other agents (implemented by the Control Neural Network)

This process of individual adaptation is effectively a realization of the co-evolution between problem and solution spaces. The problem space is related with the control mechanism in Figure 13 and the solution space with the control mechanism in Figure 12. This is a summary of the main properties or conceptual dimensions of the proposed model. For more advanced developments of this model and the notion of coordination see Alexiou (2007).

However, the task of turning this conceptual model of design into a computational simulation also revealed certain limitations regarding the way we understand and study design. In particular, by building this simulation certain questions arose that made it necessary to change the approach of this study. How can the behaviour of the proposed simulation be uniquely linked to the phenomenon or task of design? Or, to put it differently, how can the observed building configurations be uniquely linked to a process that generates design artefacts? A 'self-organization' process creates new order out of local interactions and rules, but, is this order a design artefact? Who recognizes the new order as a design artefact? Is it an external observer, or the distributed system? More generally, how is it possible to associate the underlying processes (self-organization, information processing etc) with the capacity to recognize and carry out design tasks? These questions are ultimately related with the problem of evaluating the proposed 'mechanism' of design as an hypothesis of design.

In order to expose this problem in more detail, let us consider a study where we explore the suggested view of design by focussing on the neurological basis of design activity. The objective of this hypothetical research project is to identify the organizational properties, possibly coordination mechanisms, in the brain that are associated with the execution of a design task. For this purpose and in order to minimize the noise in the data, it is important to carefully specify an archetypical design task; a task that is distinct from problem solving tasks or free-will creation (art). In particular, it may be important to identify 'milestones' in the design task - such as the *start* and the *end* of the task - in order to be able to make the mapping between the observed brain activity and the design task. Because design tasks are open-ended tasks, the milestones may be identified by the subject of the study (see Figure 16). For this project, the specification of the design task must reflect a theory about the phenomenon of design; a theory about what is design. However, although such a theory (description) of the design task may exist, the complexity of the neurological activity in the brain implies that any observed patterns may not be necessarily linked to the task. Hence, the fundamental problem remains: What type of behaviour in the brain can be uniquely linked to the proposed archetypical design task?

The problem encountered in this case is the opposite of what happens with the construction of a computer simulation of a 'design capable system'. The proposed conceptual model in this chapter reflects a theory about the nature of organizational processes that lead from an event 1 (start of a design task) to an event N (end of the design task). The start and end of the design task are a nominal attribute of an event generated by the proposed theory and are ultimately assigned by the designer of the theory. The research problem is therefore reversed. The design task - such as for instance the emergence of a design artefact - is now the phenomenon to be observed and linked with the postulated process. But, the configurations generated by the process do not necessarily correspond to a design artefact[3].

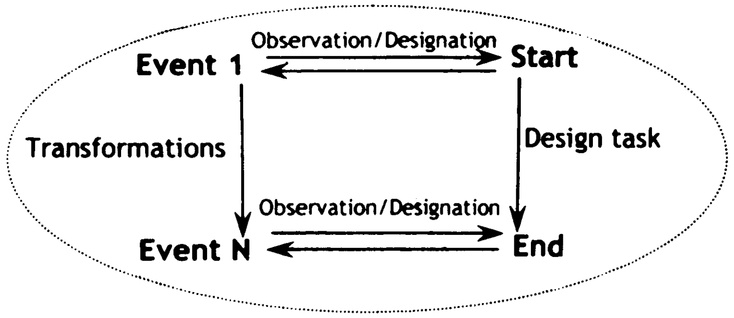


Figure 16: The identification of the underlying organizational mechanisms that are unique to design depends on the construction of a mapping between the observed activities of a design capable system (e.g. brain or computer simulation) and milestone events in a design task.

In both research problems, the 'closure' or 'commutativity' of the diagram in Figure 16 is not an intrinsic result that is derived by the theory; but it is a meta-theoretical construction. To put it simply, the recognition and mapping of a design

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[3] There is an exception: when the postulated theory is proposed as method for addressing a design task. But, in this case the design task is a task for the observer and not the observed 'system'.

task in relation to the underlying design knowledge, processes, or events are ultimately in the 'eyes of the designer/observer'. However, it is this 'meta-theory' - a theory regarding the commutativity of the diagram in Figure 16 - that is needed in order to understand the nature of design and consequently use to carry out empirical or computational experiments. The development of a theory of design that aims to explain the 'emergence' of this diagram is the main motivation of this study.

In this study it is further assumed that there must be certain properties and limitations of a mathematical nature that accompany the development of such theory of design. Making a mathematical characterisation of the capacity to recognise and carry out design tasks would be instrumental for subsequent (empirical or computational) explorations of the hypothesis that design is a universal property of organisation.

3.4 SUMMARY AND CONCLUSIONS

In this chapter a computational model of group design was presented in order to develop and clarify the meaning of the hypothesis that design is a universal property of organization. In the described system, design cannot be associated with a distinct cognitive designer (human or artificial), but may be seen as a characteristic property (attribute or ability) that depends on the organization and complexity of the system as a whole. The implementation led to the formulation of a view of design as a coordination problem where the notion of coordination is used as an abstraction and indicator of system organization. According to this view, coordination is defined both in relation to the formation of individual expectations and configurations that avoid conflicts at the micro level; and in relation to the emergence of collective (macro) behaviours and structures.

The task of turning this conceptual model of design into a computational simulation also revealed certain limitations regarding the way we understand and study design. These limitations stem from the difficulty of validating the behaviour of the model as a model of the design process and the generated configurations as a design artefact. In particular, the implementation led to the realization that there is a need for a theory that would allow us to form scientific hypotheses regarding the organizational conditions (underlying processes or structures) that are responsible for the capacity to recognize and carry out design tasks. The objective is to study the development of such a theory mathematically.

As a conclusion, it would be instructive to reflect loosely on the role of the proposed hypothesis of design in relation to predominant design theories. As discussed in Chapter 2, a number of different theoretical frameworks have been proposed in order to identify the 'phenomenon' of design. For instance, a predominant view is to perceive the notion of design as a type of thought. The view of design as thought is often associated with the hypothesis that design is an information processing or knowledge level system. Within this background, design has been perceived as a search, explorative or co-evolutionary process between a problem and a solution space. All these approaches focus on transformations identified within the diagram in Figure 16. The proposed theory by contrast aims to move the study of design into a meta-level, by focussing on the generation of this diagram. If design is a type of thought for example, then the objective for the proposed theory is the identification of the mathematical structures that explicate *the emergence of design thought*. This is the very meaning of the proposed theory; a theory that aims to explicate the formation of the diagram. The next chapter argues for the necessity and authority of mathematical thinking for the development of this theory.

Chapter 4

MATHEMATICAL AND CATEGORY THEORETIC METHODS IN DESIGN RESEARCH

In the last chapter we discussed the need to develop a *mathematical* theory of design as a universal property of organization. This chapter aims to review and explain the reasons behind this theoretical and methodological decision. For this purpose, the chapter discusses general epistemological issues regarding the acquisition of knowledge about design, and explores the role and contribution of mathematical research in design knowledge. Having established the general rationale of the approach, the last part of the chapter serves as to introduce the specific mathematical language and tools chosen for the study (i.e. category theory). Overall, the chapter closes the circle of developing the research agenda of the thesis, which started with identifying the research problem and hypothesis, positioning the enquiry within the context of design and complexity research and exploring the basic epistemological and methodological dimensions of the proposed agenda.

4.1 THE NATURE OF THE INVESTIGATION

There are many different ways of acquiring knowledge about a subject or phenomenon. Science, philosophy, mathematics, as well as design itself, are all distinct ways of creating knowledge. But, what is their difference, and how can knowledge about design be acquired?

Let us start with the first question. Scientific, philosophical, mathematical and design activity can be distinguished from one another by reference to their attitude towards reality (see Lawvere and Schanuel, 1997: 84). For the sake of this argument let us start with an assumption regarding the nature of reality, and make a gross distinction between an **objective reality**, which refers to objects that exist in a universe U, and a **subjective reality**, which refers to the existence of objects that reflect objective reality. **Reflection** is therefore a representation of an objective reality. More narrowly, reflection is an 'Intentional state' (e.g. a belief or desire) in the mind of an observer that is 'directed towards' an object in the universe U (Searle, 1983). Of course the object of reflection may well be the mind itself. Based on this crude distinction, **knowledge** is then a construction - derived by reflection - that explicates the correspondence between subjective and objective reality (for an illustration see Figure 17).

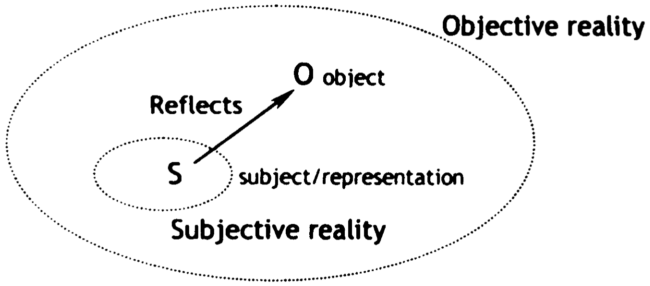

Figure 17: A distinction of reality in subjective and objective reality. The subjective reality reflects objects in the objective reality. The arrow between subject and object denotes the action of reflection.

Some basic observations regarding the nature and origins of scientific, philosophical, mathematical and design knowledge can now be made by considering their attitude towards this picture.

4.1.1 Scientific knowledge

Science is generally perceived as an effort to understand reality empirically through systematic observation. Although there is a general agreement that scientific knowledge is the product of *empirical observation*, *hypothesis generation* and *experimentation*, there is little agreement on what constitutes a scientific method. Indeed, this is one of the main problems of philosophy of science, a problem that ultimately concerns making a demarcation between scientific and non scientific knowledge. One of the widely accepted approaches perceives scientific knowledge as knowledge that is acquired through a process of empirical or theoretical falsification of a hypothesis (Popper, 1959). According to this approach, the scientific process is defined as an evolutionary problem solving process: it involves the specification of a problem setting, the creative generation of a hypothesis that addresses the identified problem, and the design and implementation of theoretical or empirical experiments that can in principle falsify the hypothesis. Through this iterative process, a theory is created from a set of hypotheses that better 'survive' the refutation from experimentation. According to this perspective, **scientific activity** alludes to the design and reflection of a subjective reality S_0 (i.e. a theory or a set of hypotheses) together with an objective reality (i.e. an experimental setting). The objective reality or experiment is set up and observed so that it is possible to falsify the capacity of a theory/hypothesis S_0 (i.e. a subjective reality) to represent the properties of an observed objective reality O. The theory is often expressed by a logical, mathematical or computational construction, while the experiment is typically carried out in a real-world controlled environment, or a simulated environment within a computer. The result of this activity is the description of the correspondence (match or mismatch) between a representation S_0 and the object O. **Scientific knowledge** is then a deductively presentable description of this correspondence (Löfgren, 2004).

Rosen (1991) represented this situation as a correspondence between a natural system (i.e. an objective reality) and a logical, formal or mathematical system (i.e. subjective reality). Scientific knowledge is thought to describe the relation between causal relations observed in a natural system (i.e. the experiment) and inferences that are derived from the logical or mathematical system (i.e. the theory or model) (Figure 18). According to this view, the relation between subjective and objective reality is determined by an *encoding activity* (observations and measurements), that associates certain states of the objective reality with logical or mathematical elements in the subjective reality; and a *decoding activity*, that associates inferences (conclusions or theorems) in a subjective reality with certain states in the objective reality.

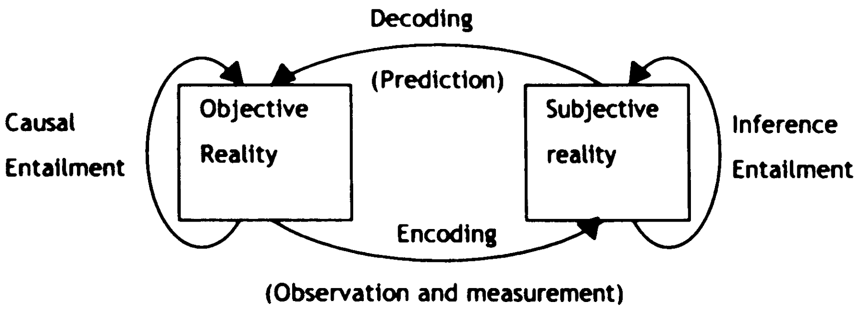

Figure 18: An abstract diagram of scientific knowledge adapted from Rosen (1991). The relation between objective and subjective reality is determined through encoding and decoding activities. In the original diagram, objective reality is referred to as a natural system and subjective reality as a formal system.

Scientific knowledge is therefore acquired from two complementary types of activity: one ***experimental*** and the other ***theoretical***. The experimental activity alludes to the design and observation of an objective reality with the purpose of identifying the correlation between a logical/mathematical/computational model and its semantics in the real world. The theoretical activity alludes to the design and reflection of logical, mathematical or computational models of an intuitive (naïve) theory about reality. In a theoretical study, the existence of an intuitive (naïve) theory is used as an objective reality that can, in principle, falsify the postulated logical, mathematical or computational constructions. The theory of computation is a typical example of theoretical work in which an intuitive concept (i.e. the concept of a computer or machine) is turned into a mathematical object. The theoretical activity is perceived as a synthetic activity that aims to answer 'why' questions and the experimental activity is perceived as an analytic activity that aims to answer 'what-if' questions (Rosen, 1986).

As a conclusion to this discussion on scientific knowledge, it is important to note the difference between theoretical and philosophical enquiry. According to Collingwood (1946: 1), 'The philosophizing mind never simply thinks about an object, it always, while thinking about an object, thinks also about its own thought about the object. Philosophy may thus be called thought of the second degree'. Philosophy is generally concerned with questions related to knowledge (epistemology), existence (ontology), thought (logic), or principles of action (ethics). **Philosophical activity** is the reflection of the relation between an objective reality O and the subject S that reflects it. Philosophical knowledge is therefore reflexive (or self-referential) in the sense that the object of the study includes the subject that reflects it (see Figure 19). According to this perspective, **philosophical knowledge** is the result of self-reflection on the object-subject relation.

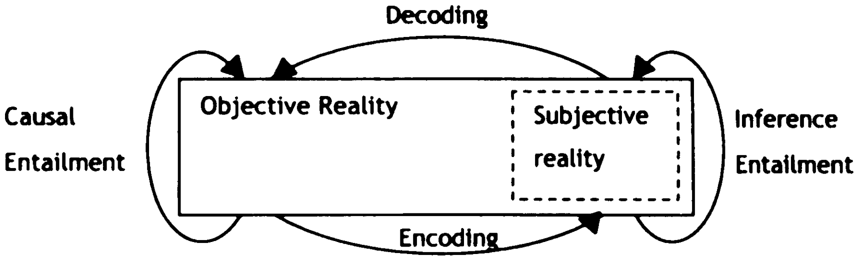

Figure 19: An abstract diagram of philosophical knowledge. In philosophical knowledge the objective reality includes the subject that reflects it.

4.1.2 Mathematical Knowledge

The nature of mathematical knowledge can be discussed independently as well as in relation to the scientific way of knowing. In general, mathematics is a field of knowledge that is concerned with the reflection of abstractions (e.g. numbers, logic, functions etc). In this sense, **mathematical activity** is generally concerned with the properties and limitations of reflection. Much of the debate on the nature and origins of mathematical knowledge concentrates on whether mathematical propositions are derived - as happens in science - from empirical observations (*'mathematical empiricism'*), or from certain 'a priori' principles that exist independently of the objective reality (*'mathematical apriorism'*). Somewhere in the middle of these two extremes lies the argument that mathematical knowledge arises from 'rudimentary knowledge acquired by perception' (i.e. empirical observation), but also by a process of historical evolution of basic mathematical principles that are held by communities of experts. For 'mathematical empiricism', mathematical entities are believed to be *created* by the human mind; for 'mathematical apriorism', mathematical entities are *discovered*; and for 'mathematical historicism' mathematical entities are *evolutionarily* created and defined. For an introduction to these ideas see for instance Kitcher (1984). Whether mathematical abstractions are created, discovered or historically defined, **mathematical knowledge** concerns the correspondence between such mathematical abstractions and the intuitive (naïve) interpretation of mathematical reality (see for instance Lawvere and Rosebrugh, 2003: 26). In a more narrow sense, mathematical knowledge is concerned with the consistency and completeness of mathematical abstractions in relation to a mathematical universe within which these mathematical abstractions are defined (Figure 20).

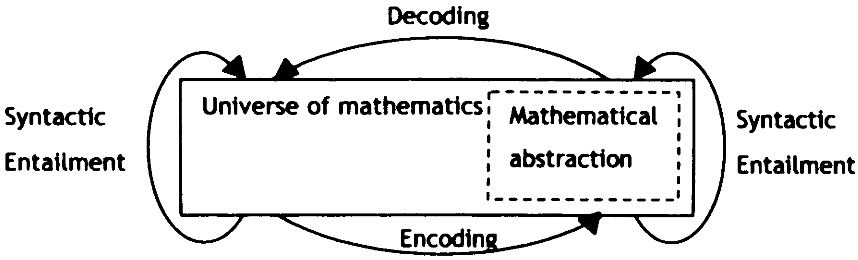

Figure 20: Abstract diagram of mathematical knowledge. Mathematical knowledge is concerned with the consistency and completeness of mathematical abstractions in relation to a mathematical universe within which these mathematical abstractions are defined.

From this discussion, it is also clear that the relation and role of mathematical constructions in science and philosophy is two fold. On the one hand, mathematical abstractions are used as a language for the representation of an objective reality, and on the other hand, they are used as a workspace for making valid inferences (that is inferences that preserve some notion of 'truth').

Both functions of mathematics play an important role in scientific research. As a language, mathematical structures are abstractions that are semantically very rich. For instance, the fields of geometry, logic, calculus or algebra are all concerned with the study of abstract entities that are applied interchangeably in many different domains, and can take a plethora of different interpretations. This is a very powerful feature for making generalizations and ultimately sharing scientific or philosophical results. Mathematical structures also have well defined syntactical properties. This attribute is a powerful feature for making valid inferences and developing abstract theories in a concrete manner. For this reason, mathematics has been widely perceived as a workspace of deductive reasoning, while mathematical thinking itself has been perceived as the 'science' or 'art' of proof. Nevertheless, mathematical thinking cannot be reduced to 'proof making'. Mathematical knowledge is not simply attained by valid proofs, but also by forming interesting and useful definitions (Mayberry, 2000; Löfgren, 2004). Mathematical thinking involves the definition and the reflection of constructions that are 'designed' to fulfil certain purposes (for example, a logical, scientific or engineering purpose).

In this sense, mathematics can be understood in relation to three different problems, or intellectual activities (a similar distinction can be found in Buchi, 1989). First, mathematical activity and knowledge are concerned with the properties of abstract objects and their deductively derived properties. This is the ***'pure' mathematical*** activity. Second, mathematics is concerned with the design of abstract objects whose properties correspond to an intuitive (naïve) theory regarding a conceptual or physical reality. This is a ***pre-mathematical*** activity that aims to translate an intuitive or empirical problem into a mathematical object through a design process. Finally, mathematical activity and knowledge is concerned with the logic or foundations of mathematics. This is a ***meta-mathematical*** problem and activity.

According to this analysis, 'design' is part of a pre-mathematical activity where a mathematician needs to develop a mathematical abstraction so that it corresponds to a certain naïve picture of an objective or mathematical reality. Because design is part of mathematical activity, in principle it should also be the subject matter of meta-mathematics.

4.1.3 Design knowledge

In the above discussion, design appears as a natural component of scientific as well as mathematical activity. Scientists ultimately design hypotheses and experiments, and mathematicians essentially design abstract mathematical structures and naïve pictures of the things that they try to understand or solve. Design alludes to the creation of both the subjective reality (i.e. models or mathematical abstractions) and the objective reality (e.g. experiments or naïve theories) motivated by a need for better fitting one to the other. So, design is a creative activity that aims to change an unsatisfactory correspondence between a subjective and objective reality to a desired one. Then, **design activity** represents the transition from a subjective reality S that can not fully reflect an objective reality O, to a new subjective reality S′ that reflects the objective reality O′. The object of **design knowledge** involves the transformations over the subjective and objective realities until a correspondence between them can be achieved (see Figure 21).

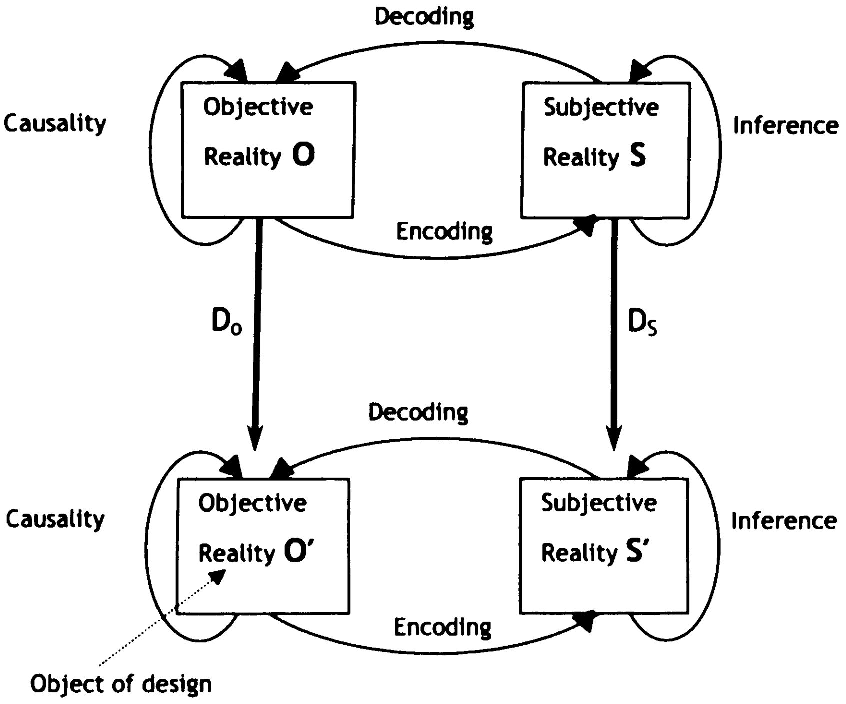

Figure 21: Abstract diagram of the design activity. Design activity represents the transition from a subjective reality S that can not fully reflect an objective reality O, to a new subjective reality S′ that reflects the objective reality O′. Design knowledge is the knowledge of the satisfactory transformations.

In design research, it is often argued that there is a 'designerly way of knowing' (see Cross, 2001). According to this argument, design is a special type of thinking and action that ultimately leads to a special type of knowing. Indeed, the diagram in Figure 21 is a crude illustration of the designerly way of knowing which may be contrasted with Figures 18, 19 and 20.

However, the view that there is a 'designerly way of knowing' may easily lead to certain fallacies regarding the nature of design research. The main fallacy is expressed as follows: Because design is a distinct way of knowing, *design knowledge can only be acquired through the practice of design*. In this sense, scientific and mathematical research has little to offer in the creation of design knowledge. This statement is erroneous. For the moment, some general thoughts will be sufficient in order to highlight the role of scientific and mathematical thinking in the creation of design knowledge.

As we saw, scientific activity involves designing and reflecting on a given objective reality in order to create and test (or falsify) appropriate models or representations of this reality. Indeed, it is customary to experiment with different (logical, computational, mathematical) constructs in order to study design activity and therefore acquire scientific knowledge about design (Figure 22).

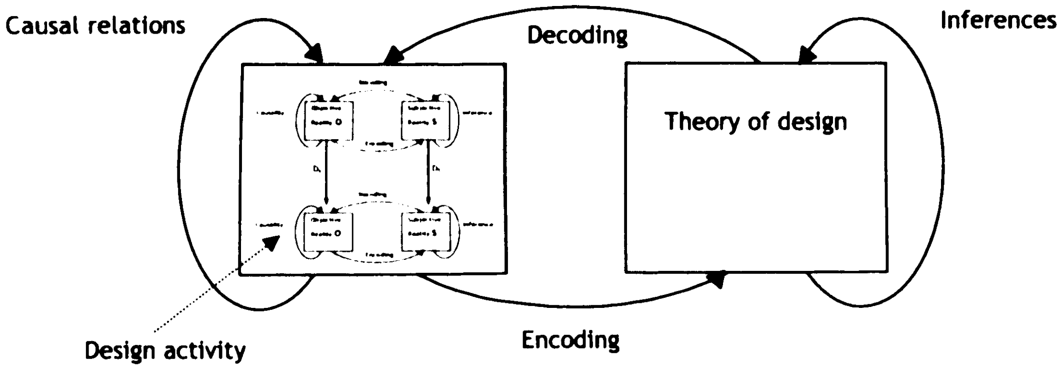

Figure 22: Scientific knowledge about design.

Can we acquire knowledge about design, through a design activity? Indeed we can. Schematically this is like fitting Figure 21 into itself - see Figure 23 below. This figure represents the idea that knowledge about design can be acquired through transformations of the objective reality (specification of an experiment) and subjective reality (our models of this world). In actual fact, this design process is part of scientific and mathematical investigation too - it is the nature of the results that is different: while the result of scientific activity is knowledge about the relation between an object and subjective reality, the result of design activity is knowledge about the transformations from one system of encodings/decodings to another.

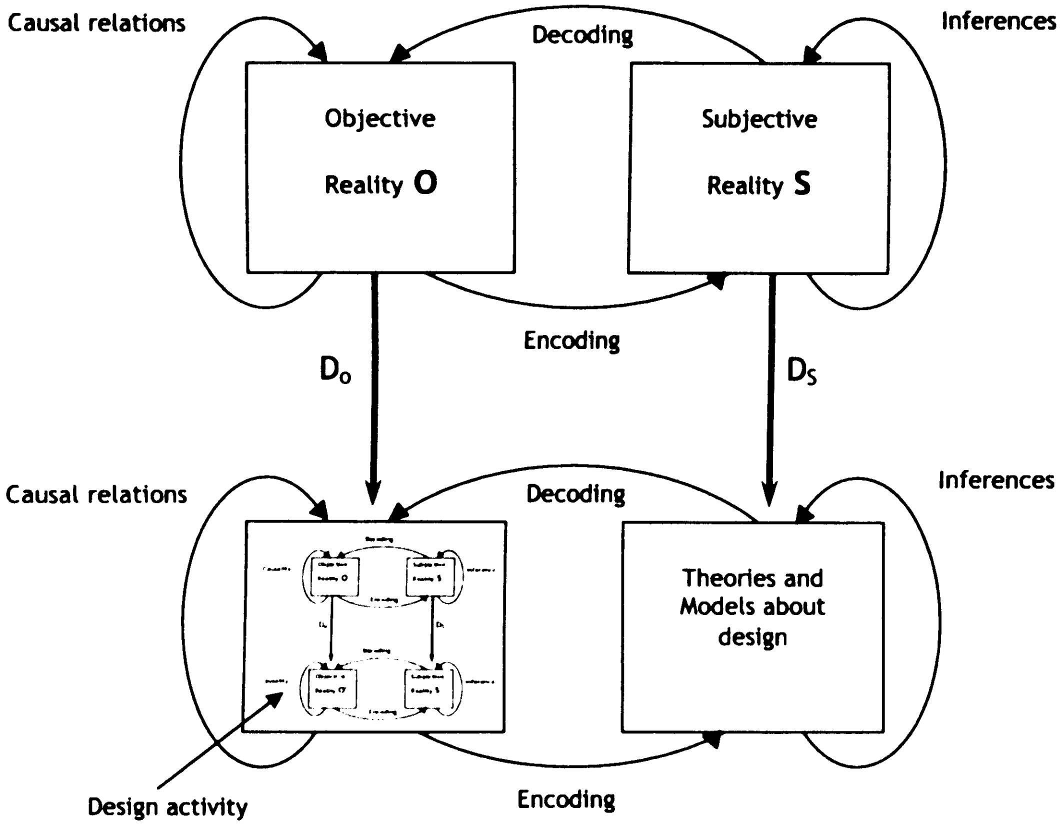

Figure 23: Design knowledge about design activity.

Moreover, it is important to understand that Figure 23 expresses an activity that happens at a different level (a higher level, a meta-level) from that of Figure 21.

We should therefore distinguish two types of knowledge: *design knowledge* or *domain knowledge in design* that is acquired through design practice and whose objective reality is the design object itself; and *knowledge about design*, that can be acquired through scientific/mathematical research, and whose objective reality is the design activity itself. In this sense, it is erroneous to assume that one can acquire theoretical knowledge about design, through knowledge of the design object. For more arguments about research and the way it contributes to design knowledge see Friedman, 2000 and Langrish, 2000.

It is also useful to distinguish two ways in which science and mathematics play a role in design research: one theoretical and the other methodological. As a theoretical contribution, the application of mathematical abstractions and scientific methods in research about design has the objective of developing descriptive (and possibly explanatory) theories about the nature and principles of design knowledge, processes and objects. As a methodological contribution, the application of mathematical abstractions and scientific methods in design practice has the objective of developing and evaluation design methods so as to automate design tasks, support design activities, and generally enhance different aspects of design. In design research, the two objectives are often intertwined. This is because the development of theories about design is often motivated by needs that actually arise in design practice and the development of design methods is often based on theories regarding what designers actually do, think or design.

4.1.4 Conclusion

Scientific knowledge of design is represented in Figure 22. This figure essentially illustrates that scientific knowledge about/for design is acquired from a process where the objective reality is a realization of design activity depicted in Figure 21. But what is a possible realization of Figure 22? How can such scientific knowledge be acquired in practice? Possible vehicles are the observation of designers at work, the analysis of what they say about their actions, or the study of their representations and their artefacts. But as we saw in Chapters 2 and 3, the object of the present thesis is the organizational principles of a 'design capable system' within which design knowledge, processes, and objects are created. For the purpose of developing a scientific theory of systems with design abilities (e.g. the designing brain, or the designing machine) it is important to uniquely identify the correspondence between design phenomena and the underlying organizational properties that generate them. In Chapter 2, it was shown that there is no such study and for that reason it is important to develop from scratch a model of this correspondence. In this sense, the thesis is a theoretical study of design. Moreover, it is an attempt to theoretically study design by developing new mathematical structures. The proposed mathematical structures aim to explicate the organizational principles that arise from a 'naïve' theory of design capable systems. In this sense, this study can be also perceived as a pre-mathematical study of design. In the next section, we focus on the role and authority of mathematical thinking in design research.

4.2 THE ROLE AND AUTHORITY OF MATHEMATICS IN DESIGN RESEARCH

In the previous section the nature of and distinction between scientific, mathematical, philosophical and designerly ways of creating knowledge were discussed. There was also the opportunity of exploring the meaning and role of design activity in all these domains. In this section, the main concern is to unravel the role and authority of mathematical abstractions and thinking for the creation of design knowledge.

The application of mathematics in design research may be perceived very broadly: from the use of statistics to analyse empirical data, to the development of formal or axiomatic theories of design knowledge, processes or objects. The present study is concerned with the application of mathematical abstractions and methods as theories of design. Therefore, the focus of the investigation in the section will be only on those design studies that explore the properties and constraints of design that are mathematical in nature.

As we saw in Chapter 2, the role and contribution of mathematics in the development of design theories has been mainly directed to three important aspects: first, the mathematical representation of *design artefacts*, second, the mathematical representation of *design knowledge* or the *design space* and finally, the logic and mathematical modelling of the *design processes* that are applied over a design space. Let us briefly consider some examples from these three areas of investigation.

4.2.1 Mathematical representations of design objects

An important application of mathematics in design is for representing the properties of individual design objects or families of design objects. For instance, the topological and geometrical properties of individual design objects have been widely represented by means of vectors, graphs, or Boolean encodings (March and Steadman, 1971; Steadman, 1983) and, more abstractly, algebraic structures (Braha and Maimon, 1998). Sets of design objects have been mathematically defined using methods such as production rules (Gips and Stiny, 1980), transformations from an 'archetypical' representation (Steadman, 1998, Steadman and Waddoups, 2000), or simply by the systematic enumeration of possibilities (Steadman, 1983).

The mathematical representation of families of design objects is often motivated by the need to study the intrinsic properties of representation that ultimately limit the space of possibilities in design. For instance, morphological or functional properties of objects have been studied in relation to their size (see for instance the special issue of Ekistics on allometry, edited by Woldenberg and Dutton, 1973). These studies identify mathematical principles - such as power law distributions - that have been *empirically* proven to hold for a general class of design objects. On the other hand general properties of design objects have also been mathematically derived on the basis of postulated principles or axioms, rather than through empirical examination (see for instance the Axiomatic Design Theory proposed by Suh, 1998, 2001).

4.2.2 Mathematical representations of design knowledge

Another application of mathematics in design is related to the representation of design knowledge. The focus of these studies is the representation of systems of design objects and the organization and interdependence of their properties. In this sense, mathematical constructions have been applied at a meta-level of abstraction where the main objective is the mathematical representation of a space within which the relation between the expression of a design problem and the expression of a design object is specified.

For instance, Alexander (1964) has proposed a graph theoretic method for decomposing design problems, which is expressed as a system of 'conflicting forces'. In particular, a system of conflicting forces is represented by a graph G(M,L), where M are misfit variables (needs or requirements), and L are dependencies between these variables. The decomposition of the graph into relatively independent systems guides the identification of 'diagrams' (or 'patterns') that effectively can resolve an identified conflict.

As opposed to Alexander's approach that focused on the decomposition of the problem space, Ho (1982a, 1982d) proposed a mathematical method that delineates and explicitly represents the relation between a problem statement and a design object. More specifically, Ho proposed the representation of the relation between objects and their features in a bicoloured graph structure which allows their systematic ordering as elements of a lattice structure. In that way Ho achieved a 'holistic' representation of the design space (March, 1982).

About the same time, a more general theory about the organization of design knowledge was proposed by Yoshikawa (1981). Since the first paper published in English in 1981 the theory has been substantially extended and elaborated (Tomiyama and Yoshikawa, 1987; Takeda et al, 1990). Similar approaches were also proposed by Kakuda and Kikuchi (2001), and Braha and Maimon (1998). In the original paper, Yoshikawa developed a formal mathematical theory of design based on three main assumptions (axioms) regarding the manipulation and properties of representations of an objective reality. The theory is based on the following definitions:

Definition 1: An *entity* is a real object that existed, exists or will exist in the future. The set of all entities is called an *entity set*.

Definition 2: The representation of an entity is called *concept of entity*.

Definition 3: A classification over the entity sets is a division of the entities into several classes. Each class is called an *abstract concept*. The set of all abstract concepts is denoted by T.

Based on these definitions, the postulated axioms are the following:

Axiom 1 (Axiom of recognition): Any *entity* can be recognized or described by its attributes and/or other abstract concepts.

Axiom 2 (Axiom of correspondence): The *entity set* and its *representations* have one-to-one correspondence.

Axiom 3 (Axiom of operation): The set of all the *abstract concepts* is a topology of the entity set S, that is (S, T) where S is the set of entities and T is a collection of subsets of S that satisfy the properties of topological space.

The core meaning of the first axiom is that an abstract concept (or subjective reality) is determined by the elements of the set of entities defined in an objective reality. To put it in other words, representations are essentially extensional descriptions of abstract concepts (Reich, 1995). The second axiom implies that the relation between representations (i.e. a subjective reality) and the content of representations (i.e. objective reality) are isomorphic. Note that this assumption will be challenged in the next chapters. The third axiom implies that abstract concepts are determined or constrained by the properties and possible operations of a topology.

Based on these assumptions (or axioms), the proposed theory aims at representing the relationship between a problem specification and a design solution. For this purpose, Yoshikawa defined two topological spaces over the same set of entities: A 'function topology' (S, T^0) and an 'attribute topology' (S,T^1). By defining these topological spaces, it is then also possible to derive the relationship between a functional specification and the attributes that satisfy a specification. Design is explicitly defined as the mapping between the function space and the attribute space.

From these axioms and definitions, Yoshikawa derives a number of theorems regarding the properties of the relation between a problem (functional) and a solution (attribute) space. The most interesting theorem for this study is a proposition regarding the 'the possibility of design'. Theorem 10 in Yoshikawa (1981) states the following: If design is possible, the identity mapping from the attribute space (S,T^0) to the function space (S,T^1) is continuous. The theorem essentially states that the necessary condition for the ability to design is that the topology of the attribute space is stronger than that of the function space; that is $T^0 \supset T^1$. One possible interpretation of this statement would be in connection to Ashby's (1956) law of requisite variety which states that 'The variety in the control system must be equal to or greater than the variety of the perturbations'. In this context, the proposition can be restated as follows: The ability to design arises when the variety of the attribute space (topology T^0) is greater than the variety of the functional space (topology T^1).

To sum up, in all these examples, mathematical abstractions are used for the representation of 'design knowledge' and the derivation of propositions (theorems) regarding the capacity to design. Design knowledge is studied by looking at the properties and limitations of the mapping between a problem formulation and a design solution.

4.2.3 Mathematical representation of design processes

Finally, mathematical abstractions have been applied for the identification of the 'problem' of design and in particular the modelling of the conditions, logical operations, or processes that lead to the creation of a design object.

In general, the 'problem' and process of design has been mathematically perceived as a mapping between two descriptive systems; a problem statement and a solution (March, 1982). For instance, design may involve a mapping between an expression of what we want to achieve and an expression of how we want to achieve it (Suh, 2001); an expression of needs and an expression of an object that satisfies the needs (Archer, 1970); the expression of functions and the expression of attributes that satisfy the functions (Yoshikawa, 1981); or the expression of conflicting forces and the configuration that resolves conflicts (Alexander, 1964, 1979). In this context, the ability to design corresponds to those characteristic conditions, logical operations, or processes that (uniquely) identify this mapping. So, how has this abstract notion of design as a mapping between two descriptive systems been defined mathematically? The mathematical identification of a mapping that is characteristic to design can take different forms.

One possible approach is to follow ***an axiomatic definition*** of this mapping. A characteristic example can be found in Suh's theory of Axiomatic Design (2001). According to this theory, design activity is realized as a mapping process between four domains; the *consumer domain*, the *functional domain*, the *physical domain* and the *process domain*. The mapping from the consumer to the functional domain is defined as *concept design*, the mapping from the functional to the physical is defined as *product design*, and the mapping from the physical to the process domain is defined as *process design*. Suh postulates two axioms for the specification a 'design mapping':

Axiom 1 (the axiom of independence): *maintain the independence of the functional requirements.*

Axiom 2 (the axiom of information): *minimize the information content of the design.*

The first axiom aims to specify a set of (good) design solutions: a set of objects that are capable of generating a set of independent functions. It requires that the functions of an object are independent, although this independence does not necessarily imply that the physical parts of an object that realize certain functions are also independent. This axiom may be perceived as a theorization of the role of design problem decomposition in the design process. For instance, it is reminiscent of Alexander's view of the role of design problem decomposition in design. According to his approach design is required to generate a set of independent problem statements, although the patterns or diagrams that resolve the identified conflict may overlap. The second axiom aims to specify the best design solution out of the set of possible design solutions. This axiom aims to provide a measure and theoretical basis of optimality in design. More specifically, the information content I_i for a given functional requirement FR_i is defined in relation to the probability of satisfying that function, that is by the equation:

$$I_i=-\log_2 P_i$$

The axiomatic approach is often linked with an ***algebraic identification*** of the mapping between a problem statement and a design solution. The main idea is that this mapping can be induced by the algebraically defined structure of design knowledge. A typical example which was already discussed in the previous section is Yoshikawa's General Design Theory. There is a clear differentiation between Axiomatic Design Theory and General Design Theory. In General Design Theory the postulated axioms are *assumptions* regarding the nature of design knowledge, while in Axiomatic Design Theory the axioms are *principles* that determine the mapping between functional specifications and design solutions. From the perspective of General Design Theory, the mapping from a functional description to attribute description is the result of the topological structure of the problem and solution space. In this sense, the possibility of identifying a design mapping is determined by this topological structure; while in Axiomatic Design Theory the mapping is determined directly by the postulated axioms.

Algebraic abstractions are also applied in the expression of a ***recursive definition*** of the design mapping. For instance, the process of design has been represented as a Markov process of averaging conflicting factors (Batty, 1974), and more generally as an information processing system that transforms specifications to design solutions. The fundamental assumption is that any goal-directed behaviour can be realized as a search process in a state space (Newell, Shaw and Simon 1957; Newell, Shaw and Simon 1962; Newell and Simon 1972). In this setting, search methods are described by 1) representations of states of knowledge, 2) operators for changing one state into another, 3) constraints in the application of operators and 4) control knowledge for deciding which operator to apply next. Different types of trial and error methods have been considered, such as generate-and-test, pattern matching, hill climbing (gradient method), means and ends analysis (heuristic search), or induction (Newell, 1970; Akin, 1986).

Finally, another approach is to follow a ***logic theoretic identification*** of design. March was the first to explicitly recognize design as a form of logical reasoning (March, 1976). In particular, design was explicitly perceived as a type of non-monotonic reasoning whose creative or synthetic aspects are linked to abduction. Since then, a number of different studies and interpretations of design as a form of logical reasoning have been developed (Coyne, 1988; Goel, 1988; Roozenburg, 1993; Takeda, 1994; Kikuchi and Nagasaka, 2003).

4.2.4 Conclusion and discussion

The purpose of the above review was to explore the way mathematical abstractions, and mathematical thinking, have been used in design research. The discussion showed how mathematics is customarily applied in order to represent design objects, design knowledge, or design processes. Additionally, different approaches to the mathematical identification of design and design problems were recognised and discussed: axiomatic, logical or algebraic.

We saw that a variety of mathematical constructions, such as vectors, sets, functions, graphs or topologies have been used to formulate formal theories of design. The importance of these studies is derived from their well-defined, but also universal, way to identify and deduce properties of design. However, although these studies constitute an important body of knowledge regarding the application of mathematical structures and methods for the representation and study of design, there are certain areas of investigation that remain uncharted. In particular, this study suggests that there are two issues (one programmatic and the other methodological) that require further attention.

The first issue is related to the scope of application of mathematical theories in design. In the last section we saw that the mathematical study of design has concentrated on the representation of design knowledge, processes and objects. However, design knowledge, processes and objects do not exist in a vacuum; they are the product of some complex socio-physical system, such as an individual agent, or a society of agents. As discussed in Chapters 2 and 3, design can be usefully studied as an emergent property linked to certain organizational processes (e.g. self-organization or coordination) that occur within, or characterise complex socio-physical systems. Mathematical studies have paid very little attention to the study of the properties of the space within which design is realized. For instance, the view of design as a mapping between a problem and a solution space concentrates on the mathematical structure of those spaces, or alternatively on the axiomatic, recursive, or logic theoretic identification of the mapping between those spaces (the 'design mapping'). In the present thesis it is suggested that the mathematical study of design can be carried out at a meta-level of investigation. In particular the idea is to study *the mathematical properties of the universe* (i.e. a complex socio-physical system) within which the 'design mapping' can be defined as an emergent entity. The motivation is therefore not to develop a mathematical theory of design, but a mathematical theory of the organization of complex systems that is responsible for their capacity to carry out design tasks.

The distinction between mathematical representations of design and mathematical representations of 'design-capable' complex systems is rather subtle. For that reason let us briefly explore a couple of other examples. In General Design Theory, Theorem 10 (presented previously) identifies the following necessary condition for design: If design is possible, the identity mapping from the attribute space (S,T^0) to the function space (S,T^1) is *continuous*. Similarly, Ho

(1982b) states that design is possible when there is an adjunction between a decoding algorithm DxS→W, and an encoding algorithm W→DxS; where D is a category of decisions, S is a category of representations of the world and W is a category of the actual world. Both statements describe a characteristic property of the design knowledge or the design process. However, at a meta-level of investigation the research problem is turned into another question: what are the mathematical conditions that describe the transition from a non-continuous mapping or non-adjunction (where design in not possible) to a continuous mapping or adjunction (where design is possible). To put it more generally, what are properties of the mathematical universe that represents the transition to mathematical structures that make design possible? This type of question is in principle an important contribution to our understanding of 'design capable systems', including natural or artificial systems.

The second issue is related to the way mathematical methods have been used to develop design theories. From the above review it is possible to distinguish two main methodological approaches. The one approach perceives mathematics as the study of formal systems, and takes an ***axiomatic*** approach. According to this approach, certain principles or assumptions about the nature of design are postulated first. Then, a number of theorems about design can be deduced from the postulated axioms. The second approach sees mathematics as a language for ***modelling*** design. According to this approach, known mathematical abstractions (such as graphs, power laws, Markov chains or topological spaces) are used to represent different aspects of design. Systematic methods can then be devised and applied over the identified mathematical space in order to identify certain properties or resolve certain problems in design. In the axiomatic approach, the mathematical study of design is understood as an endeavour to develop a formal system and deductively draw conclusions about design. For the modelling approach, the meaning of a mathematical study of design is understood as an endeavour to take a mathematical structure and give this structure an interpretation that applies to the design domain.

Between the two approaches there is third mathematical approach, which might be labelled as ***constructive***. According to this approach mathematical knowledge can be developed from the very process of constructing a new mathematical entity. This approach has rarely been used in design, but there is a plethora of examples in the history of logic and mathematics. Abstract concepts such as logic, computation, machine, or game have been turned into mathematical constructions, so knowledge can be gained by reflecting over the proposed construction.

The distinction between axiomatic, model building, and constructive approaches is again rather subtle and most of the times complementary. Their core difference must be understood in relation to whether the postulated axioms or mathematical structures are applied (i.e. take their interpretations) within a mathematical, or a design domain. Both the axiomatic and model building approaches in design research aim to identify structures that take their interpretations in the design domain. By contrast, the constructive approach aims to develop an intuitive theory of design that takes its interpretation in the mathematical domain. In this sense it takes a reverse direction in the investigation of design. While the axiomatic and model building approaches aim to propose a *design interpretation of a given mathematical theory*, in the constructive approach the aim is to develop *a mathematical interpretation of a 'naïve' (intuitive) design theory*. For this reason there is a need to work within a mathematical domain that has the expressive power to represent and study various mathematical structures.

Category theory has been widely perceived as such a general language of mathematics used in order to describe and study mathematical structures (Marquis, 1995; Hellman, 2001; Landry and Marquis, 2005; McLarty, 2005; Shapiro, 2005). After all, 'Category theory has been the standard research framework for topology, most algebra, and much functional analysis since the 1950s' (McLarty, 2005). For this purpose category theory is proposed as a vehicle for the development of this study.

4.3 CATEGORY THEORY AND CATEGORY THEORETIC THINKING

This section is an introduction to category theory which is the main mathematical language and methodological tool of this study. In particular, the section aims to introduce the main ideas of category theory, but also to explain why category theory is a useful tool in complexity and design research.

4.3.1 What is category theory

Category theory is a mathematical approach for studying properties of mathematical structures. For this reason, category theory is widely perceived as *a language* for describing and studying mathematical structures, but also as a *methodological approach* for investigating the foundations of mathematics.

Category theory is motivated by the observation that mathematical systems can be represented, unified and studied on the basis of abstract diagrams such as the following:

$$\begin{tikzcd}
 & B \arrow[dr, "g"] & \\
A \arrow[ur, "f"] \arrow[rr, "h"] & & C
\end{tikzcd}$$

Diagrams are defined by arrows, like $f:A \to B$ and $g:B \to C$, and compositions of arrows, like $g \circ f:A \to C$. Each arrow $f:A \to B$ is defined by two objects; the head of the arrow A that is called the domain object, and the tail of the arrow B that is called co-domain. Any object B can also be defined as an identity arrow $1_B:B \to B$ whose composition with an arrow f or g is cancelled (i.e. $1_B \circ f=f$ or $g \circ 1_B =g$). The specific diagram above is essentially a pictorial representation of the equation $h=g \circ f$. In category theoretic language, the above diagram is said to *commute*. A typical category theoretic study proceeds by looking at the properties of such diagrams (equations). In this sense, *category theory is a mathematical theory* that is used in order to represent mathematical structures in terms of diagrams.

Category theory requires the application of certain axioms to the compositions of arrows in a diagram: The first axiom states that the composition of arrows is associative, and the second axiom states that every object is essentially an identity arrow that can be cancelled when it is composed with another arrow. The postulation of axioms on the composition of arrows leads to the axiomatic definition of a mathematical structure called ***category***. In this sense, *category theory is an abstract mathematical structure* that holds certain mathematical properties. Any mathematical structure that satisfies the defined axioms is a model of the category. For instance, ordered sets or monoids can be seen and studied as special types of categories.

Another very dominant approach is to use *category theory as a mathematical workspace*. In this case, objects in a category are mathematical structures, and arrows are transformations between mathematical structures. For instance, objects may be defined as sets, and arrows as functions between sets. As another example, objects may be defined as sets with a certain structure (e.g. graphs, topological spaces), and arrows as structural preserving functions (e.g. homomorphisms or continuous functions). Many basic concepts in category theory (such as functors, or natural transformations that will be introduced later in this chapter) have been developed as mathematical tools in order to study the properties of mathematical objects by looking solely at their organizational characteristics. The study of mathematical objects by preserving their organizational characteristics can be perceived as part of the main epistemological and methodological objectives of complexity research. It is in this sense that category theory will be proposed as a main methodological tool for complexity research.

To sum up, up to this point category theory was introduced as a mathematical theory, mathematical structure, and mathematical approach for the study of mathematics (Barr and Wells, 1990). In the following, we will identify the main concepts behind categorical algebra.

4.3.2 Main concepts and tools of categorical algebra

In this section the main concepts of category theory are briefly reviewed. The intention is to provide a general picture of category theoretic constructions and not to provide a primer of category theory. More detailed expositions of category theoretic concepts, methods and results can be found in Goldblatt (1984), Barr and Wells (1985, 1990), Lambek and Scott (1986), Lawrere and Schanuel (1991), McLarty (1995) and Mac Lane (1998).

4.3.2.1 Categories

A 'category' is a mathematical structure. One way to describe the mathematical structure of a category is to define it axiomatically. Let us express the properties of categories we discussed above in formal terms.

Definition (Category): A category is a *graph* with two additional operations; a graph structure that consists of *objects* A, B, C..., *arrows* f, g, h..., the domain and co-domain operations:

- The *domain* operation that for each arrow f assigns an object dom(f) called the domain object, and
- The *co-domain* operation that for each arrow f assigns an object cod(f) called the co-domain object.

And two additional operations:

- The *identity* operation that for each object A assigns an identity arrow $1_A\!:\!A \to A$.
- The *composition* operation that to each pair of arrows $\langle g,f \rangle$ with dom(g)=cod(f) assigns an arrow $g \circ f$ called the composite. This situation is presented by a diagram as follows:

$$\begin{tikzcd}
 & B \arrow[dr, "g"] & \\
A \arrow[ur, "f"] \arrow[rr, "g \circ f"] & & C
\end{tikzcd}$$

The composition of arrows must satisfy two axioms:

- *The Axiom of Associativity*: The composition of arrows is associative whenever the composition is defined, that is $h \circ (g \circ f) = (h \circ g) \circ f$.
- *The Axiom of Identity*: For any arrow f:A→B, the identity arrows 1_A:A→A or 1_B:B→B act as an identity element for the operation of composition, that is $f \circ 1_A = f$ *or* $1_B \circ f = f$.

The notion of category is then defined as any interpretation that satisfies the postulated axioms.

As we discussed, there are two basic ways to work with the mathematical notion of category. One approach is to use categories as a 'mathematical workspace'. According to this approach a category is a tool for specifying species of mathematical entities on the basis of their organizational properties. For instance, the category of sets is a category whose objects are sets and whose arrows are all possible functions between sets. The category of graphs is a category whose objects are graphs and whose arrows are homomorphisms between graphs. It is easy to demonstrate that these structures satisfy the postulated axioms, because the composition of functions and the composition of homomorphisms satisfy the axioms of associativity and identity.

The second approach is to use the notion category as a 'mathematical theory' and discuss different mathematical structures as different types of category. Take for instance an ordered set $(S, \leq)$, where $\leq$ is binary relation that is reflexive (i.e. $x \leq x$ holds for all elements x in S) and transitive (i.e. if $x \leq y$ and $y \leq z$, then $x \leq z$ for all elements x,y,z of S). This structure is a category whose objects are the elements of the set S and whose arrows represent the binary relation $\leq$. It is again easy to prove that this is a category: we need to observe that reflexivity over the set S specifies the identity arrows, and transitivity ensures the composability of binary relations $\leq$ over S. In this line of thought, other mathematical structures such as monoids or groups can be perceived as categories.

4.3.2.2 Functors

A Functor is a special type of arrow (or morphism) between two categories. More specifically, a functor *F* from a category C to a category D is a graph homomorphism which preserves the identity arrows and the compositions of arrows. More formally a functor is defined as follows:

Definition (Functor): A functor F:C→D from a category C to a category D is a *graph homomorphism* defined by a pair of functions, an *object function* $F_O:C_O \rightarrow D_O$ which for each object c in C assigns an object $F_O c$ (also written $F_O(c)$) in D, and an *arrow function* $F_A:C_A \rightarrow D_A$ which for each arrow $f:c \rightarrow c'$ in C assigns an arrow $F_A(f)=F_A f$: $F_O c \rightarrow F_O c'$ in D, such that: $F_A(1_c)=1_{F_O c}$ and $F_A(g \circ f)= F_A(g) \circ F_A(f)$.

In the following, we will generally keep the convention of dropping the parenthesis for the object functor (thus writing $F_O c$ instead of $F_O(c)$) but not for the arrow functor $F_A(f)$.

A functor is an important category theoretic concept because it makes the construction of meta-level representations possible. In particular, the *category of categories* is a category whose objects are mathematical categories and whose arrows are functors. This category may be perceived as a meta-category that works as a workspace or universe of a mathematical species (categories) within which functors determine transformations from one mathematical species to another.

4.3.2.3 Naturality

Natural transformations are the category theoretic method of transforming one functor to another. A natural transformation is a structure preserving arrow between two parallel functors F,G:C→D (i.e. functors with the same domain and co-domain category). More formally, a natural transformation is defined as follows:

Definition (Natural transformation): Given two parallel functors F,G:C→D, a natural transformation $\tau{:}F\rightarrow G$ is given by a family of arrows τ_c indexed by the objects of C such that:

- For each object c in C there is an arrow $\tau_c{:}Fc\rightarrow Gc$ in D, and
- For each arrow $f{:}c\rightarrow c'$ in C the following diagram commutes in D

$$\begin{tikzcd}
c \arrow[d, "f"'] & Fc \arrow[r, "\tau_c"] \arrow[d, "Ff"'] & Gc \arrow[d, "Gf"] \\
c' & Fc' \arrow[r, "\tau_{c'}"] & Gc'
\end{tikzcd}$$

The notion of natural transformation is the main tool for constructing higher order categories. So, given two categories C and D and all possible functors *F*, *G*, *H*, *K*...:C→D, a functor category Fuc(C,D) is defined. The category of functors is a category whose objects are functors and whose arrows are natural transformations. More generally, based on these concepts n-order categories can be defined. Very roughly, a n-category is a category that consists of a collection of objects, a collection of arrows between objects, a collection of 2-arrows between arrows and so on. An introduction to n-categories can be found in Mac Lane (1998).

4.3.2.4 Universality

In philosophy, a *universal construction* is a representation of abstract properties (e.g. tallness, whiteness etc) that characterise a family of objects. The represented property is said to be a *universal property* and the objects are said to *instantiate (or participate in)* the universal property. In mathematics, universal constructions have a similar meaning. In particular, a *universal construction* is a mathematical construction (i.e. a description of the abstract property) that is characterised by the existence of a unique entity (e.g. the entity that represents the abstract property). Universal constructions in category theory are constructions (i.e. category theoretic diagrams) that are characterized by the existence of a unique arrow. More formally a universal arrow is defined as follows:

Definition (Universal arrow): Given a functor $U\colon D \to C$ and an object c in C, a universal arrow $\langle r, u\colon c \to U(\_)\rangle$ is an arrow of the form $u\colon c \to U(\_)$ with an object r in D such that the following universal property is satisfied:

For every d in D and $f\colon c \to Ud$ in C there is a unique arrow $f\colon r \to d$ such that the following diagram commutes:

$$\begin{tikzcd}
Ur \arrow[dr, "U(g)"'] & c \arrow[l, "u"'] \arrow[d, "f"] & r \arrow[d, "g"'] \\
 & Ud & d
\end{tikzcd}$$

The statement that the above diagram commutes essentially represents the equation $f=U(g)\circ u$. The universal arrow $\langle r, u\colon c \to U(\_)\rangle$ is also said to be representation of the hom-functor $H_C(c, U\_)\colon C \times D \to \mathbf{Set}$ where **Set** is the category of small sets.

Universal constructions lead to the idea of *adjoint functors* between two categories. In particular, if for every c in C there is a universal arrow $\langle r, u\colon c \to U(\_)\rangle$ then the following assignment is defined: for every c in C there is an object r in D, and for every arrow f in C there is an arrow g in D. This assignment essentially specifies a new functor $F\colon C \to D$ with the following universal properties:

- For every $f\colon c \to Ud$ in C there is a unique arrow $g\colon Fc \to d$ in D such that $f=U(g)\circ u$, and
- For every $g\colon Fc \to d$ in D there is a unique arrow $f\colon c \to Ud$ in C such that $g=\varepsilon\circ F(f)$.

Pictorially, these statements imply that the following diagrams commute:

$$\begin{tikzcd}
UFc \arrow[dr, "U(i)"'] & c \arrow[l, "u"'] \arrow[d, "f"] & Fc \arrow[d, "g"'] \arrow[dr, "F(f)"] & \\
 & Ud & d & FUd \arrow[l, "\varepsilon"]
\end{tikzcd}$$

4.3.3 Category theory for complexity and design research

In this section the objective is to explain why category theory is a useful methodological tool in mathematics and science in general but, more importantly, to explain the suitability of category theory for the purpose of this study. In his article 'Categorical manifesto', Goguen (1991) presents a number of qualities of the category theoretic approach. Let us outline some of the reasons why category theory can be usefully applied to mathematical/scientific research:

For formulating precise definitions and for developing theories: Mathematical but also scientific research often involves the formulation of definitions and theories. Category theoretic diagrams can be used as systematic presentations of 'species' of (mathematical) systems in the same way that logic theoretic constructions are employed for the formulation of formal theories. For the purpose of this study, the design of precise definitions and theories is the main incentive for applying category theory.

For exploring mathematical foundations and properties of mathematical structures: It follows from the previous point the category theory works as a (meta-) language of mathematics. Rather than focusing on individual objects, category theory can be used as a general theory of organization of 'species' of mathematical objects.

For discovering (and/or creating) general proofs: the category theoretic 'way' of working with diagrams is an alternative way of deriving properties of complex mathematical entities without 'decomposing' their organization. The main idea is that it is possible to retrieve properties of mathematical objects by looking only at their behaviour within diagrams of arrows. This methodological approach generates abstract proofs of organizational properties that might be of special interest for the development of a science of complex systems.

For discovering (and/or creating) categories of systems, and for discovering and exploring relations within and between different domains: defining categories of mathematical species often leads to formulations where comparisons and unification of concepts across different domains is possible. For example, in a later chapter we will explore the definition of the categories of machines, control systems and evolutionary systems. Creating definitions of those categories facilitates the identification of meaningful commonalities and differences between them.

From the perspective of this thesis, category theory is an appropriate mathematical tool for design research. The authority of category theory is related with the objective of developing a mathematical theory of the organizational properties of complex systems that have the capacity to carry out design tasks. According to Goguen's argument (1991: 3) '*to each species of mathematical structure, there corresponds a category whose objects have that structure, and whose morphisms preserve it*'. Mathematical entities are studied within a universe (or ecology) of mathematical entities and the properties of mathematical objects are identified in relation to the organizational properties of this universe. Category theory can then be perceived as an important mathematical approach for distinguishing the intuitive notion of design at an organizational level. It is also an important mathematical approach for relating the notion of design with other abstractions and mathematical entities such as computing, evolution or control. In the history of mathematics, different approaches have been applied in order to turn an intuitive concept into a mathematical object. For instance, logic theory and universal algebra are predominant tools for developing definitions and theories of mathematical constructions. However as discussed above, category theory has the advantage of being able to offer representations of mathematical objects in organizational terms - which is the very purpose of this research project.

4.4 SUMMARY AND CONCLUSIONS

This chapter concludes the endeavour of setting up a research agenda in design and complexity research. The chapter presented some general epistemological and methodological considerations in order to justify the decision to study the hypothesis that design is a universal property of organization by using mathematics (specifically category theory).

For that purpose, the chapter was organized in three parts. The first part offered a general epistemological discussion of the nature of scientific, mathematical and design knowledge. The discussion aimed to justify the need for a mathematical theory of design, but also to bring to the fore certain peculiarities regarding the nature of the proposed investigation. In particular, the investigation of the thesis was positioned as part of a 'pre-mathematical' activity: an activity where a 'mathematician' creates a new mathematical structure. According to this approach, the main concern is to turn the intuitive concept of design into a mathematical entity that is defined within the universe of mathematical structures.

The second part of this chapter was an exploration of mathematical research in design. The aim was to identify limitations in the existing mathematical theories of design in relation to the proposed research agenda. The study argued that mathematical studies of design have concentrated on the representation of design knowledge, processes and objects, and little emphasis has been placed on how design as a distinct type of thinking and knowing might be related to the underlying properties of the complex socio-physical systems that generate them. Moreover, it was also observed that the mathematical approach of the study is methodologically different from the axiomatic or modelling-building approaches usually adopted in design research. In specific, it was explained that the focus here is the identification of the intuitive concept of design with a distinct mathematical entity within the universe of mathematical structures.

The third part of the chapter proposed category theory as the language and methodological tool for developing such an organizational theory of design. Category theory is capable of dealing with two problems. First the problem of studying the organizational properties of complex systems that are responsible for the capacity to carry out design tasks; and second the problem of constructing definitions and theories about mathematical structures. The concepts and tools that have been introduced will be the main mathematical machinery for identifying the meaning of design and organization in the following chapters.

Chapter 5

ORGANIZATIONAL-LEVEL THEORIES

In Chapter 2, complexity research was described as the science of effective organization; that is the study of *effective processes and structures* that underlie emergent, qualitative changes in a system. The study of organization both as a process or structure is motivated by an epistemological stance that sees organization as a signature of reality with significant descriptive and explanatory power for a variety of different scientific problems. The overarching objective of complexity research is in this sense to develop a corpus of theoretical and methodological tools contributing to the development of domain-independent, organizational-level descriptions and explanations of reality. But what is organization and what are the main principles of organizational-level descriptions and explanations of reality? This chapter is concerned with the very concept of organization as it is used in complexity research and in particular with the mathematical principles that underlie the description and study of organization. The purpose is to develop a mathematical theory of organization that has the expressive power to accommodate and compare a plethora of different organizational level descriptions and ultimately set the foundations for an organizational level theory of design.

5.1 INTRODUCTION TO ORGANIZATION

The term organization generally alludes to the conditions (e.g. processes or structures) that generate, constrain and preserve an arrangement of entities into a distinct whole. The Oxford English Dictionary Online (2006) lists several general definitions of organization such as: "The development or coordination of parts in order to carry out vital functions", "the action or process of organizing, ordering, or putting into systematic form; the arrangement and coordination of parts into a systematic whole", "the condition of being or process of becoming organized", and "systematic ordering or arrangement". Although we usually tend to associate the term with human organizations, it is otherwise used to refer to a large variety of phenomena: physical, biological, social, or abstract. For example, the organization of a society can be thought to refer to the traditions, customs, or laws that generate and constrain certain social structures, behaviours and functions. The organization of the brain can be seen to refer to the network of neural interdependencies that generate patterns of neural activity and cognitive functions. We can also use the term organization for sign systems, to refer to the arrangement of symbols that generate, trigger or validate an interpretation. Similarly, we can use the term organization to refer to the arrangement of spatial entities (rooms, walls, windows) that engender and order the sequence of activities within a building and their interdependence. In each of these examples, the term organization is applied to a specific domain; the social domain, neurological, cognitive or spatial domain. In the following, the purpose will be to develop a common ground for understanding the term organization independently from its reference domain. This purpose is indeed part of a more general agenda in complexity research that is concerned with the development of a domain-independent epistemological and methodological toolkit in order to contribute to the study of the origins and consequences of complexity.

But how has the term organization arisen in the history of science? The importance of organization was explicitly recognized and rigorously defined first within the context of thermodynamics. In this context, the concept of organization is inextricably linked to the concept of entropy. The development of information theory, system theory and cybernetics reinforced the thermodynamic interpretation of organization, but also contributed to the recognition of organization as a scientific problem with epistemological significance. This research basically led to the definition and study of organization as a characterization of the order/disorder of a system. However, it was the development of complexity research that brought to the fore the concept of organization as a scientific topic with important explanatory implications. In this context, organization is defined and studied in many different ways: as a statistical, computational or algebraic quality. For instance, organization is defined as a statistical quality that characterizes the balance between ambiguity (entropy production) and redundancy (mutual information) in the description of an object (Atlan, 1974). Organization is also defined as a logical/computational quality that characterizes the balance between randomness and order (see for instance Bennett 1995, or Wolfram, 2002). Finally, organization is defined as an algebraic/relational quality that characterizes the balance between reducible and irreducible structures (Ashby, 1962). In all cases, the core issue in hand is the relation between the properties of an observed object (e.g. its structures or functions), and the properties of the conditions that generate the described object (e.g. the ambiguity, difficulty, or irreducibility of description).

The purpose of this section is to set the background for the exploration of different organizational perspectives and establish the tools for the development of mathematical descriptions of organization that have the expressive power to differentiate different levels of organization (from random to well-ordered to complex organizations).

5.1.1 The core concept of organization: conditionality

Intuitively, organization arises in a situation when there are certain conditions (i.e. processes or structures) that generate, preserve or constrain a distinct whole. According to Ashby (Ashby, 1962: 255-256) the hard core of the concept of organization is that of conditionality: 'as soon as the relation between two entities A and B becomes conditional on C's value or state then a necessary component of organization is present'. For instance, as mentioned above, the organization of a building can be associated with structural or functional conditions that specify relations between spatial entities such as rooms and walls in a floor plan layout. In this sense, organization arises when the relation or adjacency between any two spatial units A and B becomes conditional on some (third) entity C. This entity might take different interpretations. It may be perceived as a set of cultural or social rules that dictate spatial relations. Alternatively, the entity C may be perceived as a set of requirements or constraints that determine possible spatial configurations in terms of lighting, ventilation, accessibility, or visibility. Such constraints or rules may be expressed in the form of a Post productive system

(Post, 1943), a shape grammar (Gips and Stiny, 1980), or alternatively as a graph of topological relations of activities in space (March and Steadman, 1971; Steadman 1983). Whatever interpretation we choose, the entity C induces a number of possible (or desirable) building configurations that satisfy the adjacency requirement imposed by the grammar or graph.

More generally, a conditional entity C can be said to *'reflect'* (or represent) a family of objects that satisfy certain properties. From the perspective of logic, a conditional entity is a set of logical statements (or formal system) that reflects or represents a family of mathematical structures. From the perspective of computer science, a conditional entity is an effective procedure, a program, or algorithm that has the capacity to generate a family of statements in a language. From the perspective of scientific modelling, a conditional entity is an 'order parameter' that enslaves the degrees of freedom (possible expressions) of the modelled system (Haken, 1983; 1988). From the perspective of philosophy of mind, a conditional entity C is an *'Intentional state'*, that is, a mental state which is directed towards a state of affairs in a world (Searle, 1983). In other words, a conditional entity C is an entity that reflects the properties of an objective reality and on that basis specifies the interplay between the properties of 'representation' and the properties of the 'representing object'. The interplay between 'representation' and 'representing object' marks the distinction between a subjective and an objective reality[4] - as discussed in Chapter 4. In this sense, the core component of organization arises with the presence of a conditional entity that has the capacity to reflect a coupling between a subjective and an objective reality. Let us now explain this view in more detail.

5.1.1.1 The coupling between subjective and objective reality

In the above examples, organization appears to be more of an abstraction or theoretical quality, rather than a material entity. An important question is therefore whether organization is the product of an observer that interacts with an observed system, or is an intrinsic quality of the system. Ashby (1962) and von

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[4] In the distinction between subjective and objective reality there is nothing to imply that the objective reality is actually 'external' to a system. In other worlds, an Intentional state (or subjective reality) may refer to an object that is actually another Intentional state.

Foerster (1984), who studied organization and self-organization as characteristic notions of complex systems, both emphasised the role of the observer. They both argued that 'self'-organization is a misleading term as a system only increases or improves its organization by interacting and exchanging energy with another system (its environment). Maturana and Varela (1980) also maintained that living systems "cannot be understood independently of the part of the ambience with which they interact: the niche; nor can the niche be defined independently of the living system that specifies it" (ibid: 9).

In this sense the notion of organization is a property that arises with the coupling between a system and an environment and is tightly linked to the notions of emergence and complexity. For instance, Rosen's (1985, 1991) elaboration of "modelling relation" is useful for understanding the nature of organization as a conditional component that determines the coupling between subjective and objective reality. The activity of modelling relations in science refers to the formation of relations between a natural system (an attribute or object of the external world we wish to study) and a formal system (a system we create in order to model the natural system). Modelling relation refers to the encoding of a natural system into a formal one in a consistent way so as to be able to form predictions about the natural world (Figure 24). The underlying idea is that the natural world consists of a set of qualities, and linkages between qualities, that we call observables: "As such, then, a natural system from the outset embodies a mental construct (i.e. a relation established by the mind between percepts) which comprises a hypothesis or model pertaining to the organization of the external world" (Rosen, 1985: 47). Rosen associated complexity with the concept of error that appears between a system and its model, and related bifurcations (the appearance of emergent phenomena) to our capacity as modellers to produce enough independent encodings so as to fully describe a given natural system.

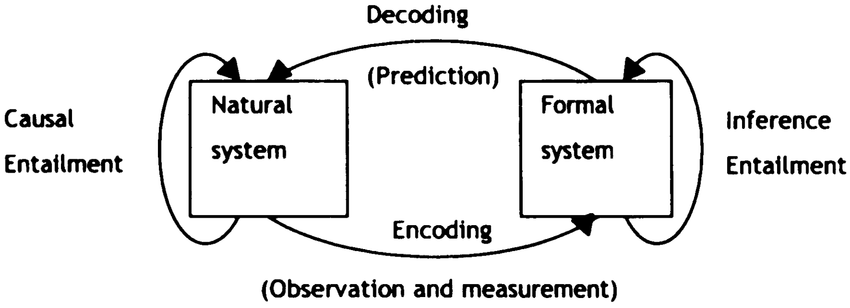


Figure 24: Rosen's diagram of modelling relation.

In the same line of thought, Casti (1986) adopts a view of "complexity as a *latent* or *implicate* property of a system, a property made explicit only through the interaction of a given system with another" (pp 146). In particular, he identifies two levels of complexity: design complexity, that is the complexity of the system in relation to the observer, and control complexity, that is the complexity of the observer relative to the system. The complexity of a system S in relation to an observing system O corresponds to the number of non-equivalent descriptions that O can generate for S.

The coupling between system and environment (or system and observer) is also often interpreted as a coupling between the macro and micro properties of a particular system. This is a typical view in statistical mechanics and thermodynamics. An explicit example can be found in the definition of emergence by Baas (1994). Baas defines emergence in relation to hierarchical organization as the creation of higher-level structures through the use of observational mechanisms. He considers three basic components: *structures* (which are the primitive entities), *observational mechanisms* for evaluating, observing and describing structures, and *interactions* among entities. His definition of emergence can be briefly described as follows: a property P is emergent at a certain level (S^2) - which is constructed from the set of primitive entities and interactions among them - if the property can be observed (and described) at this level but not at the level below it (S^1) using the same observational mechanisms. According to this definition the higher-level structure is constructed by the interaction between entities at the lower level, yet new observational mechanisms are needed in order to describe the property P. Baas also distinguishes deducible or computable emergence, from observational (non deducible) emergence: in the first case the observational mechanism is an algorithm or deductive process, and in the second the observational mechanism is a semantic meaning function or a truth function. In each case, the notion of organization is defined in relation to a coupling between scales: a coupling between an observed object and the underlying structures that generate this object.

5.1.1.2 The complexity of organization

To sum up, the notion of organization has been explained as a conditional entity that has a certain limited capacity to reflect the coupling between a subjective and an objective reality. The distinction and coupling between two systems, between a system and its environment, or between micro and macro observations, is therefore perceived as a fundamental aspect of organization. The limitation in the capacity to reflect such couplings is what determines the complexity of organization.

Hence, different qualities, or levels, in the complexity of organization can be identified by looking at the reflective capacity of the conditional entity C. For instance, a *random organization* can be assumed to arise when there is no conditional entity that reflects a distinction between subjective and objective reality. Namely, a random organization alludes to a situation when there is nothing to distinguish a system (a structure or function) from its environment. On the other hand, a strictly *ordered organization*, as is found for instance in crystals, can be considered to arise when there is a conditional entity that can completely determine the specified structures or functions. Somewhere in the middle, high level or *complex organizations*, (such as the human brain or social groups and institutions) can be considered to arise when there are certain structures or processes that cannot be completely described by a set of underlying processes or structures. It is often in this sense that these structures or processes are defined as *emergent*. The complexity of organization is then determined by the capacity of a conditional entity to effectively reflect (generate or represent) an object; a capacity that can be measured as the difficulty, length, or ambiguity of description.

5.1.2 An algebraic interpretation of organization

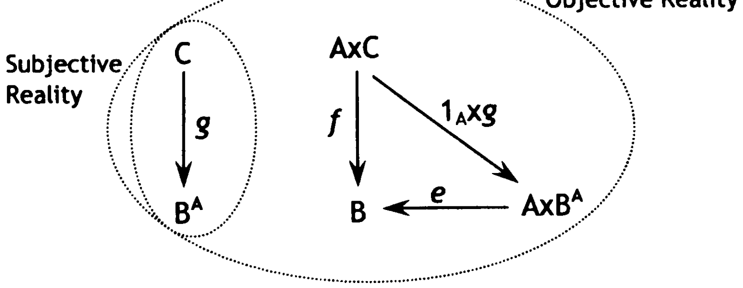
Let us now attempt to develop an algebraic interpretation of organization. From the above discussion, it seems plausible to associate the notion of conditionality with the existence of an entity (i.e. category) C that participates in a mapping from a product space AxC to B. The mapping from AxC to B expresses the idea that every relation, function or functor between A and B becomes conditional to the value or state s of C. In this setting, the value or state s in C is an intrinsic quality of the mapping f_s:AxC→B that acts as a 'program': i.e. for any a given input i defined in A the mapping generates a value or state b in B. According to this perspective, the concepts of '..."organization" and "mapping" are two ways of looking at the same thing...' (Ashby, 1962: 262).

In order to better explain this algebraic interpretation of organization it would be instructive to consider three characteristic instantiations of the mapping f:AxC→B: as a product, a sum and as an exponential operation. As a product operation Π:AxC→B, the mapping expresses the meaning of an *irreducible organization*, since the value in B uniquely depends on the relation between A and C. As a sum operation Σ:AxC→B, the mapping expresses a *reducible organization* (i.e. a non-organization), since the value in B depends on the disjoint effect of C and A. Finally, as an exponential operation e:AxC→B, the mapping express the capacity of C to reflect (deduce) all possible relations between A and B. As we saw different qualities of organization (e.g. *complex*, *ordered* or *random organizations*) can be identified by looking at the limitations of reflection. For the proposed algebraic interpretation, the study of the capacity of an entity C to reflect relations between A and B is therefore one of the basic conceptual ways for identifying the quality and complexity of organization.

In order to explain the meaning of this idea, let us briefly study the specific conditional entity C in a mapping f:AxC→B. For this purpose, the map f:AxC→B can be alternatively perceived as a family of maps f(_,c):A→B (or f_c:A→B) from a set A to a set B that are sorted or parameterized by the elements of set C. There is therefore a mapping $C \rightarrow B^A$ that parameterizes the relations between A and B; where the set B^A is the set of all possible mappings from A to B. It is possible to prove that there are certain limitations on the capacity to parameterize (i.e. reflect) all possible relations/functions from A to B. These limitations depend on the size of the sets A, B and C. The parameterization (conditioning) of all functions from A to B by C is possible if and only if the set C has a cardinality equal to, or bigger than, the cardinality of the set B^A ($C \geq B^A$). An algebraic proof - based on Cantor's diagonal argument - can be found in Lawvere and Schanuel (1997: 302). In the history of mathematics and logic, there are a plethora of related statements regarding the limitation of reflection, such as for instance Tarski's theorem of *indefinability* of truth, Church's theorem of *undecidability* of logic, Gödel's *incompleteness* theorems, or Turing's examples of *incomputable* functions. All these statements should be therefore considered as basic knowledge for the understanding of organization.

5.1.2.1 Organization as a universal property of conditionality

The existence of a conditional entity C (and therefore the concept of organization) is inextricably linked to the capacity of C to interact with an object A (i.e. with the formation of products AxC) but also the capacity of C to represent or parameterize relations from A to B (i.e. with the formation of an exponential object B^A). Formally, the formation of products and exponential objects imply the presence of certain '*universal properties*'. More specifically the core meaning of universality implies the following property: the description of a family of relations/functions/functors between A and B is conditional to an entity C if and only if there is a unique map g from C to B^A such that the following diagram on the right commutes (*i.e.* $f = e \circ (1_A x g)$):



The evaluation arrow e can be seen as a mapping that 'enslaves' (Haken 1983, 1988) or 'glues together' up to isomorphism (Ehresmann and Vanbremeersch, 1987) a family of arrows that realizes $f\text{:}A\text{x}C \rightarrow B$ (i.e. satisfies the equation $f=e\circ(1_A\text{x}g)$).

Note that this notation can be further extended to include the product of many objects A_1, $A_2,\dots,A_N$. A 'minimum' organization then arises when for at least one object A_i there is a relation from A_i to B that is conditional to the product of objects pertinent to i:

$$C_i=\prod_{j\neq i} A_j$$

More specifically, conditionality means that for any map $f\text{:}A_i\text{x}C_i \rightarrow B$ there is a unique arrow $g\text{:}C_i \rightarrow B^{A_i}$ such that the following diagram on the right commutes:

$$\begin{tikzcd}
C_i \arrow[d, "g"] & A_i\text{x}C_i \arrow[d, "f"'] \arrow[dr, "1_{A_i}\text{x}g"] & \\
B^{A_i} & B & A_i\text{x}B^{A_i} \arrow[l, "e"']
\end{tikzcd}$$

A 'maximum' organization arises when *every* object in an objective reality 'interacts' with any other object (i.e. for every object A_i there is a product $A_i\text{x}C_i$) and when *every* object C_i in a subjective reality yields the above universal construction. Formally, these conditions lead to the construction of a 'Cartesian Closed Category' and the idea that organization arises with the formation of a complementary correspondence between a subjective reality (i.e. arrows of the form $g\text{:}C \rightarrow B^A$) and an objective reality (i.e. arrows of the form $f\text{:}A\text{x}C \rightarrow B$). In category theoretic terms, this complementary relation is explicitly described by the notion of 'adjunction'. Let us now introduce the notion of adjunction more formally as the main theoretical tool for the study of organization.

5.1.2.2 The adjunction between subjective and objective reality

The above discussion aimed to show how the notion of conditionality yields a universal property. The property represents the very notion of organization; a state where the coupling between a subjective reality S and an objective reality D reach a complementary relation.

More formally, given a functor U:D→S between two categories S and D, the property of universality means that there is an object (or conditional entity) s in S with the following property: For every d in D and arrow m:s→Ud in S there is a unique arrow i:x→d in D such that the following diagram commutes (i.e. $m=U(i)\circ\eta_s$):

$$\begin{tikzcd}
Ux \arrow[dr, "U(i)"'] & s \arrow[l, "\eta_s"'] \arrow[d, "m"] & x \arrow[d, "i"'] \\
& Ud & d
\end{tikzcd}$$

The adjunction between a subjective and an objective reality means that for *each* object s in S there is a universal arrow η_s of the form η_s:s→UFs and that *for every* object d in D there is a universal arrow of the form ε_d:FUd→d such that the following diagrams commute:

$$\begin{tikzcd}
UFs \arrow[dr, "U(i)"'] & s \arrow[l, "\eta_s"'] \arrow[d, "m"] & Fs \arrow[d, "i"'] \arrow[dr, "F(m)"] & \\
& Ud & d & FUd \arrow[l, "\varepsilon_d"]
\end{tikzcd}$$

Let us try to convey an intuitive understanding of this notion. For instance, consider that the object s is an algebra of geometrical entities that determines a vocabulary of basic shapes and rules (e.g. algebra of points, lines, surfaces and volumes as defined in the context of shape grammars). The set of all possible shape configurations generated by the grammar s is denoted by Fs. Formally, the category Fs corresponds to the Kleene closure of a set s - i.e. a language s* that contains all expressions (shapes) that satisfy certain grammatical rules. The arrow $i:Fs \rightarrow d$ generates or selects a specific shape d from the set of possible expressions Fs. The arrow $m:s \rightarrow Ud$ selects the shapes and rules that underlie a specific configuration d. Hence informally, the notion of adjunction means that for every selected shape d there is a (minimum) set of rules that generates this shape; or that for every grammar s there is a shape d defined in the language Fs that satisfies this grammar. According to the above discussion, organization arises with the formation of a complementarity between grammar formation and the generation of an expression. As we will see, these theoretical constructions play important part in a number of different approaches found in complexity research.

5.2 BASIC APPROACHES TO ORGANIZATION: A SEMANTIC VIEW

Up to this point the meaning of organization has been explicated as a universal property of conditionality. Figure 25 summarizes the main terms used in order to explain the meaning of organization. This diagram is an informal presentation of the notion of complementarity that arises with the formation of universal properties between two categories S (subjective) and D (objective) and two opposite functors $U:D \rightarrow S$ and $F:S \rightarrow D$. In an informal sense, it can be said that the diagram in Figure 25 'commutes' when there is a natural bijection between arrows of the form $m:s \rightarrow Ud$ and arrows of the form $i:Fs \rightarrow d$. We will call this the 'basic diagram'.

In this figure, a *theory* is a family of objects or expressions (i.e. a category Fs) that satisfy certain conditions imposed by a conditional entity s. A *model* is the description of the properties of a specific object d generated by a theory. A *conditional entity* or *source* s is then a syntactical presentation of a theory that has the capacity to deduce the properties of all possible models. Depending with the context, these terms might take different interpretations. For instance in the context of design, *a source* may be identified with a sketch, a drawing or mental state that represents the properties of a desired objective reality. *A theory* is a family of design objects that instantiate the idea expressed by the source (i.e. the sketch, drawing or mental state). A *model* is a specification of the properties of a specific instantiation of the theory. An *object* is the design artefact (or a representation of it). The formal expression of these terms also varies according to the context. For instance, in linguistic/symbolic expressions a source is a grammar or formal system and a theory is a language. In dynamical representations, a source is a system of differential equations that determine the behaviour of a system.

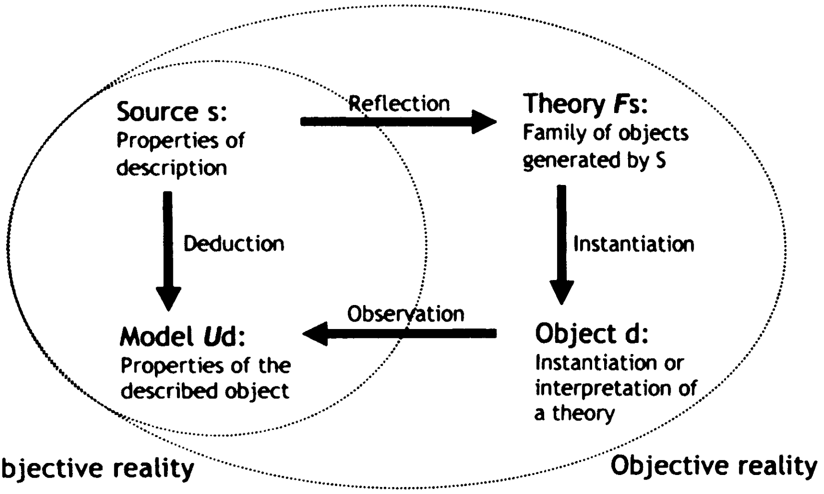


Figure 25: This is the 'basic diagram' that summarizes the essential terms that underlie any organizational level theory. Organization is defined as the capacity of a conditional entity (a 'source') to describe the properties of an object and make the diagram commute.

To give a more concrete example, consider the 'five points for a new architecture' that Le Corbusier (1985) developed during the 1920's: (1) the pilotis (2) the open plan (3) the free facade (4) the horizontal window and (5) the roof garden (Figure 26). These five points (expressed in his sketches and written work) constitute a source that describes the properties of a desired reality.

Based on this source, a theory of desired artefacts (in this case, buildings) is produced. For example, the series of sketches shown in Figure 27, illustrates a family of buildings that satisfy the five principles thus exemplifying Le Corbusier's modernist language.

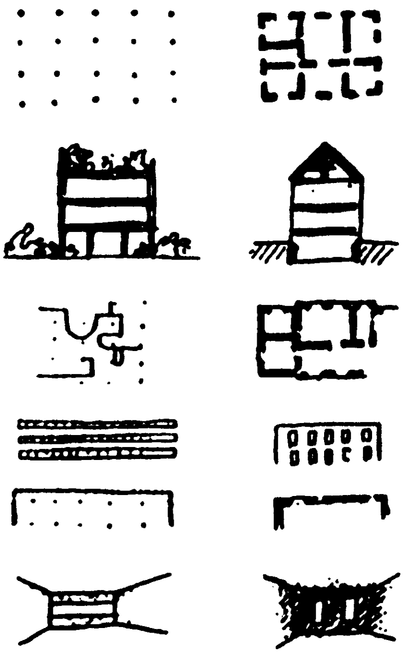


Figure 26: Le Corbusier's sketches illustrating his five points for a new architecture. Image source: http://www.en.sbi.dk/arkitektur/beredygtighed/arkitektur-og-beredygtighed-artikelsamling/de-signed-ecology

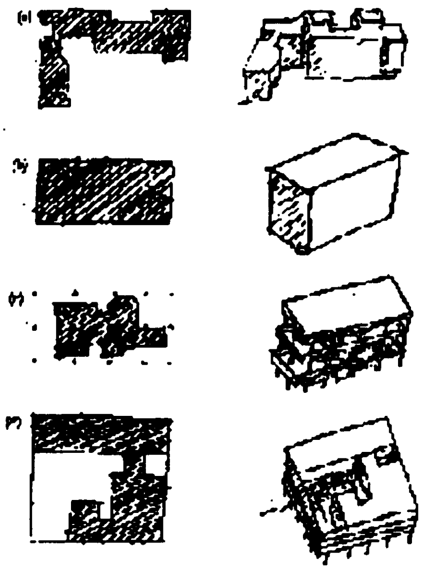


Figure 27: Exploration of the potentials of the five points by Le Corbusier: a family of buildings that exemplify the five points. Image source: http://www.geocities.com/rr17bb/LeCorbusier5.html

Each specific object that belongs to this family of buildings is described by a model. For example, the Villa Savoye, which is considered as an exemplary instantiation of Le Corbusier's theory, is in our terminology an object (Figure 28). Any description of the Villa itself (in technical drawings, text or natural language) is a model of this object.

Figure 28: The Villa Savoye. Image source: http://en.wikipedia.org/wiki/Image:Villa-Savoye.JPG

The notions of source, theory and model are also applied to science. For example, a source can be thought as a physical state or process that is responsible for certain phenomena. A source can be encoded by a scientist in a formal system (like a grammar, or a system of dynamical equations). This formal system reflects a theory about an observed reality. A model is a specification of the theory. For instance, Copernicus proposed the model that the sun orbits around the earth. The scientific process involved validating this model against the theory.

These concepts aim to bring to the fore semantic aspects of organization. Models are considered to assign specific semantics to a theory. Conversely, theories are considered to describe families of models, where each model constitutes an instance or interpretation of the theory. A theory is in correspondence with a model when it is possible to deduce the properties of the model from the theory, and respectively, it is also possible from the family of models to induce the theory. The formal specification of semantic relations in terms of theories and models has been part of mathematical logic and in particular 'model theory'

(Barwise, 1977). Model theory is concerned with the relation between a set of logical statements expressed in a language and the mathematical structures that satisfy the postulated statements. The set of logical statements forms a theory and the algebraic structures that satisfy the statements of the theory are called models. A theory is a set of 'truth conditions' for the existence of a model. The distinction between theories and models is therefore the model theoretic way to explicate the formation of semantic relations as the interplay between 'the specification of the properties of a family of objects' and 'the set of objects that satisfy the properties of a specification'.

Organization is then a quality that characterizes the correspondence between the properties of the described objects (i.e. the models, or semantics) and the properties of the description (i.e. the theory or syntax). Based on these conceptual constructs, a number of different approaches to the definition and study of organization and complexity can be distinguished. In all these approaches the common problem is how to uniquely identify the different qualities or levels of complexity in the organization of an object, by looking at the conditions that make the basic diagram in Figure 25 commute. However, each approach looks at the diagram differently. One approach focuses on ***'the ontology of organization'***. Organization is then defined and studied by looking at the structures, ambiguity or abstractions - in a family of objects - that are generated by a source in order to make the diagram commute. This approach is predominantly found in the tradition of thermodynamics and information theory. Another focuses on the ***'logic of organization'***. Organization is then defined and studied by looking at the resources needed in order to deduce the properties of an observed object and make the diagram commute. This approach has its conceptual foundations in logic, mathematics and theoretical computer science. A third approach focuses on the ***'epistemology of organization'***. Organization is then defined by looking at the capacity of a source to generate a family of independent models (independent descriptions of the properties of an object) that make the diagram commute. This approach can often be found in relational biology, cybernetics and system theory.

The basic diagram aims to propose a level of organizational analysis within which all these different approaches in complexity research can be explored. Moreover, the basic diagram also aims to emphasise aspects of organization that are more pertinent to design research. As discussed in Chapter 1, design is by and large considered to involve 'Intentionality' i.e. the capacity of a mind/object to reflect (represent) a state of affairs in the world. The need to design is seen to arise as an 'Intentional' state that expresses conflicts between a system of beliefs regarding the state of a world and a system of desires about the world. Based on this premise, research on design needs conceptual and mathematical constructions that can handle 'Intentional states' and therefore semantic relations.

For these purposes a fourth approach to organization is proposed which constitutes a ***'meta-theory of organization'***. According to this approach, organization is defined and studied by looking at the properties of a source when there is a 'distance from a critical state' in which the basic diagram commutes. In a sense, the focus is on the conditions for the generation of the diagram as a whole. This approach will be elaborated in this thesis as an alternative organizational level theory. In particular, category theory will be used in order to explicate the formation of semantic relations as a phase transition to universality. In comparison to model theory, instead of employing a formal language in order to specify the semantics (i.e the truth conditions of a set of objects) category theory uses diagrams of objects, arrows and compositions of arrows. These diagrams are formally named ***sketches***. Sketches will be used in order to lay out the proposed approach to organizational level theories. First, it is necessary to explore the conceptual foundations of all the different approaches to organization in complexity research and explain them in detail using the terminology proposed in the basic diagram.

5.2.1 The ontological nature of organization

A first approach is to define and study the organization of an objective reality by looking at *what is specified or represented by a conditional entity*; that is, by studying the content and ontology of conditionality. For instance assuming an algebraic interpretation of organization as discussed in the last section, a conditional entity C might represent relations from A to B, or structures of shape A in B, or distinctions/differentiation of entities in A on the basis of their value in B. In this sense, organization is defined and studied as a set of relations, structures, or differentiation of entities that determine the properties of an objective reality $f:AxC\rightarrow B$.

Intuitively, the basic ontological categories of organization should include not only relations and structures, but also the degrees of freedom, ambiguity or abstraction necessary for the specification of an objective reality by a conditional entity. It is instructive to consider as an example the organization of activities generated by a given layout. Let us assume a space that is divided into two sections 1 and 2 where each section must 'organize' a number of activities (letters) into the same block. The number of activities in the section 1 is N_1 and the number of activities in the section 2 is N_2. Let us also assume that there are four possible activities A, B, C, and D and no repetition of activities is allowed. When the number of activities is equally divided in the two sections, that is when $N_1=N_2$, then the variety of possible configurations of activities increases dramatically. In particular, the number of possible groupings of activities is calculated by $P=N!/N_1!N_2!$, where $N!=1x2x...(N-1)xN$. Thus, the description of the properties of an object with two activities grouped in each section will take six possible different configurations of activities (Figure 29).

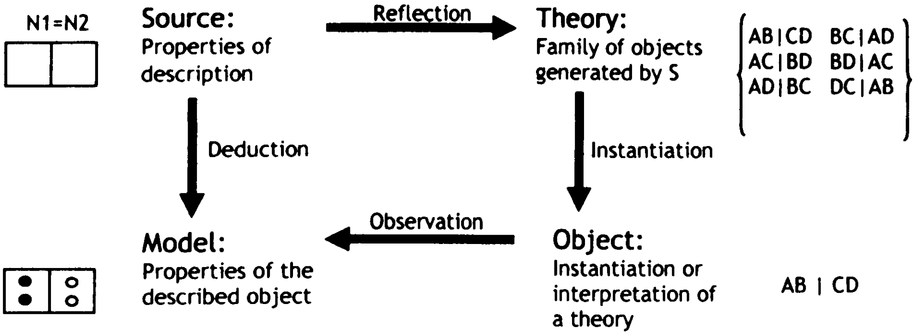


Figure 29: Given four different letters A,B,C,D to be organized in two groupings, the condition $N_1=N_2$ implies 6 different configurations of possible groupings.

Notice the consequences for the description of a configuration when all the activities are placed in one section, that is when $N_1=4$ and $N_2=0$. In this case there is only one possible realization: the configuration ABCD|∅ (Figure 30).

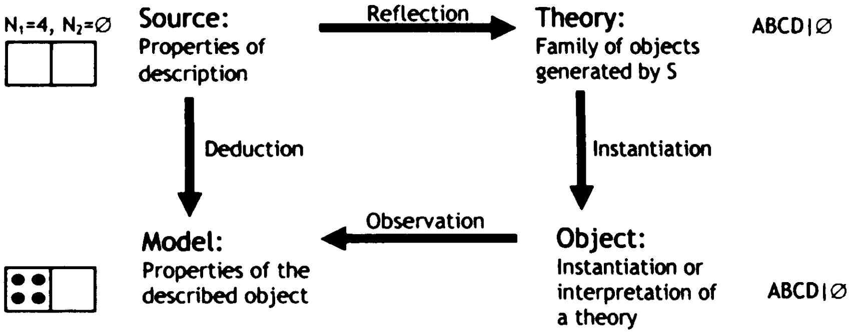


Figure 30: Given four different letters A,B,C,D to be organized in two groupings, the condition $N_1=4$ and $N_2=0$ imply 1 possible configuration.

In the first case, the condition $N_1=N_2$ generates 'ambiguity' in the description of the object. This ambiguity can be identified with the number of possible realizations that satisfy the described model. In the second case, the source expressed by the condition that $N_1=4$ and $N_2=0$ would generate only one possible description and therefore the degree of freedom over the possible groupings of activities is reduced to one.

According to this example, a conditional entity determines not only distinctions and relations between entities, but also ambiguity (variety) and repetitions. On the one hand, organization must correspond to highly autonomous entities with a high degree of ambiguity in their description. On the other hand, an organized entity should also be characterized by the formation of certain structures, patterns or repetitions, because otherwise ambiguity would be equivalent to randomness. So for instance, a living organism generated by a DNA sequence should correspond to a more complex organization than that of a crystal generated by a sequence of physical processes; and a crystal must correspond to a more complex organization than a random sequence of numbers generated by the flip of a coin.

As mentioned, the conceptual origins of this approach can be found in the study of thermodynamics, statistical physics and information theory. In these studies the notion of organization is treated as a combinatorial or statistical property of a source (or conditional entity) that generates an object or event. The core idea is that the component of organization appears with a conditional entity that balances degrees of freedom (e.g. autonomy, variety or ambiguity) with redundancy (e.g. repetitions or constraints) in the description of a family of objects. Highly organized objects are then objects derived by a conditional entity that makes the diagram in Figure 31 commute by maximizing both the degrees of freedom and the redundancy of the theory.

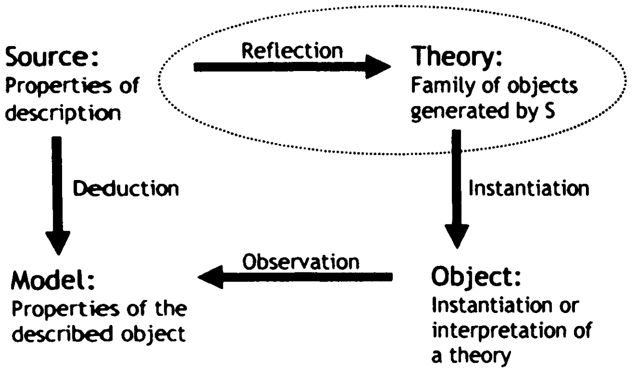


Figure 31: A view of organization that focuses on the properties of reflection necessary to uniquely identify an object. Arrows outside the dotted circle are assumed as given.

5.2.1.1 Information content and redundancy

Let us now briefly review some specific examples of organizational level descriptions that are based on these principles. One approach develops the concept of organization in relation to concepts of information content and redundancy (mutual information). Characteristic examples of this approach can be found in Von Foerster (1984) and Atlan (1974).

According to Atlan (1974) the term organization has two interrelated meanings. On the one hand, organization means *order*. Order is defined with the specification of *relations* between parts. The formation of relations between objects leads to redundancies (repetitions) in the description of an object and therefore generates abstractions (e.g. patterns) in the representation. On the other hand, organization means *autonomy*. Autonomy is defined as variety in the possible expressions of a system (i.e. the number of objects that satisfy certain properties) which implies *ambiguity* in the description. Based on these concepts, Atlan defines organization as a quality that appears as a balance between increasing variety (i.e. maximizing information content of an object) and increasing interdependencies between parts (i.e. maximizing redundancy or mutual information between parts). More specifically, the organization of an object S is defined as the information content H(S) of the object S that is derived by the sum of two terms: first, the conditional information $H(S_2/S_1)$ between any two subsystems S_1 and S_2 of S (i.e. the variety or ambiguity left in S_2 when S_1 is specified), and second, the mutual information $I(S_1;S_1^{t1})$ between the description of the component S_1 at time t and the description of S_1 at time t-dt, denoted by $S^{t\text{-}dt}$. The organization of an object S is therefore defined as the sum: $H(S)=I(S_1;\ S_1^{t-1})+\ H(S_2/S_1)$. As a summary, Figure 32 provides a set theoretic representation of the basic information theoretic terms.

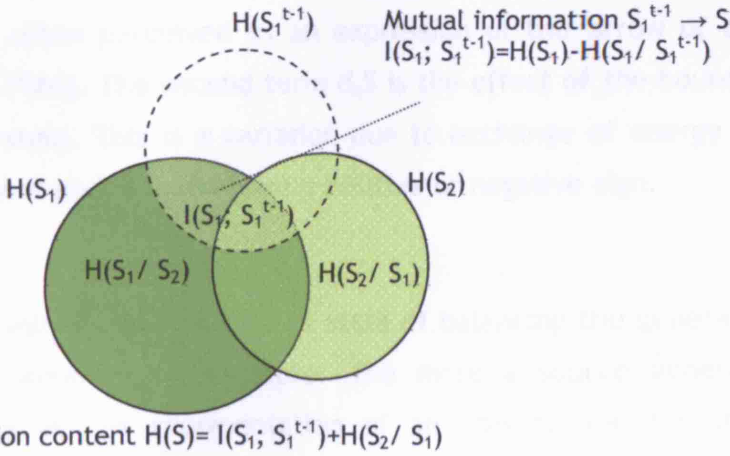


Figure 32: This is a set theoretic representation of information theoretic terms used to define organization. The variety (or ambiguity) of a component is represented by the size of the set while relations between components are represented by the intersection of sets. Organization is defined as the sum of conditional information between components of a system and mutual information between current and past states of the system.

The terms 'information content' and 'redundancy' (or 'mutual information') are used in the sense defined by Shannon and Weaver (1949). Atlan's idea is similar to Von Foerster's concept of organization. In both studies, the meaning of organization is defined in relation to Shannon's definition of Redundancy (Shannon and Weaver, 1949): that is, $H=H_{max}(1-R)$ or similarly $R=1-H/H_{max}$, where H_{max} is the information (variety) transmitted to an observer by not taking into account the constraints or relation between parts; H is the information (variety) transmitted to an observer with the imposed constraints; and R is the redundancy.

Another example can be found in the work of Prigogine. In his studies (self-) organization is studied as a spontaneous generation of structures due to the dissipation of the entropy of a system to the environment (Prigogine and Nicolis, 1967). The variation of entropy dS is defined by the sum $dS=d_iS+d_eS$ of two types of transformations. The first term d_iS is an expression of the inner changes of a system towards an attractor (i.e. towards the most probable state). The variation of the function d_iS is monotonic ($d_iS/dt\geq0$) and hence 'irreversible' in terms of its sign. In this sense, it is often perceived as an expression of the arrow of time (Prigogine and Stengers, 1984). The second term d_eS is the effect of the boundary (or environment) of a system. This is a variation due to exchange of energy and matter with an environment and it might have a positive or negative sign.

To sum up, organization alludes to a process or state of balancing the generation of autonomy and the formation of structure. The more a source generates ambiguity or uncertainty in the representation of an object, and the more constraints or patterns it imposes over an object, the more complex is the organization of the object. On the other hand, the more a source generates ambiguity with no repetitions in the description of an object, the more random is the organization of the object. This leads to an 'ontological' definition and study of organization as a quality that is determined by dual conceptual categories such as: *distinctions and relations*; *ambiguity and redundancies*; *abstraction and instantiations*. These concepts play an important role in the formation of organisational level theories and for that reason need to be more formally defined.

5.2.1.2 Conceptual 'categories' of organization

Let us now present the basic 'ontology' of organization in category theoretic language. So far we associated the notion of organization with the existence of a source and its capacity to generate a distinct whole. A source was defined as a conditional entity with the expressive capacity to represent relations between entities, determine the ambiguity or redundancy in the expression of an object, and finally represent instances of a given abstraction. In the following it is proposed to present these concepts through the concept of "sketch".

In general, sketches are mathematical constructions used to represent mathematical theories and their models. The original mathematical definition of sketches was proposed by Ehresmann (1968). However, it is important to clarify that the definitions used in this study are based on the work of Barr and Wells (1985, 1990) and Wells (1994). The formulation derived from the work of Barr and Wells is rather different from Ehresmann's approach. The interested reader may consult the paper of Bastiani and Ehresmann (1972) regarding the original formulation of sketches, and the paper of Wells (1994) regarding the relation between the two approaches. Here sketches are used because of their expressive capacity to represent structures, constraints and abstractions.

5.2.1.2.1 Distinctions and relations: Graphs

A fundamental conceptual category of an organization level description is derived by the designation of distinctions and relations. A distinction expresses the capacity of an observer, or observed system, to differentiate a system (or itself) from its environment, or a system from another system. On the other hand, relations express the capacity of an observer, or observed system, to designate (inter)-dependencies between parts, or between parts and wholes. Distinctions and relations create a space of syntactic possibilities (i.e. combinatorial possibilities), but also as space of semantic possibilities (i.e. a family of objects that preserve the designated distinctions and relations).

A sketch is a construction that determines a space of (syntactic and semantic) possibilities. The space of possibilities determined by a sketch is formally expressed by a graph G_5.

> **Definition (Graph):** A graph $G(G_0, G_1)$ is defined by a set G_0 whose elements are called *objects*, a set G_1 whose elements are called *arrows*, and two types of functions called *source* and *target*. The function *source*:$G_1 \rightarrow G_0$ assigns an object A to each arrow, and the function *target*:$G_1 \rightarrow G_0$ assigns another (but not necessarily different) object B to each arrow.
>
> In a graph G, *a path from a node A to a node C* is a sequence of arrows $\{f_1, f_2 .. f_k\}$ such that the source of f_i is the target of f_{i-1}. For instance, an arrow f:A→B and an arrow g:B→C generate a path of arrows: A→B→C.

Syntactically, a graph has the capacity of representing a number of different compositions of arrows depending on the way paths of arrows are defined. So, different paths of arrows $\{f_1, f_2 .. f_k\}$, such that the source of f_i is the target of f_{i-1}, represent different syntactic possibilities. Semantically, arrows might take a number of different interpretations as long as the basic structure (the represented paths of arrows) is preserved. An arrow f:A→B from A to B might be perceived as an action, an observation or a transformation from A to B. More specifically, an arrow might be perceived as a relation between entities A and B (e.g. a causal, spatial, scaling, logical, or semantic relation), or as a distinction between entities (e.g. separating different objects and sub-objects in a universe A based on their image in B).

Let us consider as an example, a design world constructed by objects that are distinguished in terms of behaviours Beh, structures Str and functions Func. The design process typically involves different phases, tasks, or operations, which can be expressed as arrows connecting behaviours, structures and functions. Following Gero's FBS model discussed in Chapter 3, we can consider the following processes: synthesis *syn*:Beh→Str, analysis *ana*:Str→Beh, evaluation *eva*:Func→Beh and problem reformulation *ref*:Beh→Func. A sketch of this design world is given by the following graph:

$$\begin{tikzcd}
\mathrm{Str} \arrow[r, shift left, "ana"] & \mathrm{Beh} \arrow[l, shift left, "syn"] \arrow[r, shift left, "ref"] & \mathrm{Func} \arrow[l, shift left, "eva"]
\end{tikzcd}$$

Syntactically, a graph has the capacity to represent a number of different compositions of arrows depending on the way paths of arrows are defined. For instance, one possible expression generated by the graph G_5 is the task $syn \circ ana \circ syn \circ eva : \mathrm{Beh} \rightarrow \mathrm{Str}$. Every possible composition of arrows is a specific instantiation of a design task within this space of possibilities. Semantically, graphs have the capacity to take different interpretations, assuming that the structure of the graph is preserved. For instance, arrows may be interpreted as actions over a set of structures, performance criteria, and functional descriptions, which are carried out by a design agent in order to generate alternative configurations.

5.2.1.2.2 Ambiguity and constraints: Diagrams

In any organizational level description, distinctions and relations determine another class of dual concepts: the ambiguity, autonomy, or degrees of freedom of a system and its constraints, repetitions or symmetries.

In order to demonstrate the relation between distinctions/relations and ambiguity/redundancy consider an example where we have a physical space U which can be divided into separate 'rooms' f, g, h and e. In Figure 33 the space U is first divided into f and g, and then f is divided into h and e. The number of distinctions determines the variety of possible configurations. However, the variety of possible configurations is reduced because of the existence of certain relations (symmetries) between different configurations. For instance, the configuration to the right of configuration f-g (dotted outline) can be generated from the configuration f-g simply by rotation. In this sense the variety of possibilities is restricted. Additionally, the relation f-g and h-e-g has a double meaning. On one hand, there is mutual information between f-g and h-e-g, because the h-e-g is generated by f-g. On the other hand, the relation between f-g and h-e-g also represents the generation of a new distinction that divides f into h and e; that corresponds to the introduction of a new 'vocabulary' into space U.

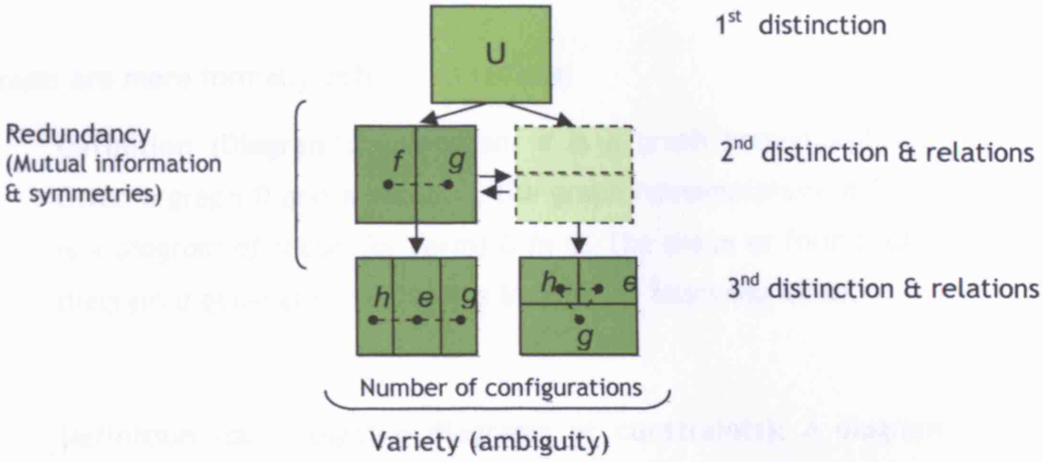


Figure 33: An example of how Variety/Redundancy is generated in a space of possibilities with the designation of new distinctions in a space U.

More generally, the organization of an objective reality generated by a source S includes the specification of the constraints that are applied over a space of possibilities. A sketch is a construction that imposes constraints within a space of possibilities (within the graph Gs of a sketch S) based on the concept of a 'diagram'. For instance, going back to the example of the previous section, the synthesis and analysis tasks can be composed in two ways (see diagrams below).

$$\begin{tikzcd}
 & Str \arrow[dr, "ana"] & \\
Beh \arrow[ur, "syn"] \arrow[rr, "T_{Beh}"'] & & Beh
\end{tikzcd}
\qquad
\begin{tikzcd}
 & Beh \arrow[dr, "syn"] & \\
Str \arrow[ur, "ana"] \arrow[rr, "T_{Str}"'] & & Str
\end{tikzcd}$$

On the left diagram, the composition of synthesis and analysis arrows are constrained by a target arrow that specifies desired behaviour (T_{Beh}), while on the right diagram, synthesis and analysis are constrained by a target structure (T_{Str}). The two diagrams express the constraints (equations) $ana \circ syn = T_{Beh}$ and

$syn \circ ana = T_{Str}$. Alternatively, the two compositions of arrows can be thought to provide a measure of deviation, or error, from desired behaviours or structures respectively.

Diagrams are more formally defined as follows:

> **Definition (Diagram):** A *diagram d* is a graph homomorphism. Given a graph D and a graph G, the graph homomorphism d:D→G is a *diagram of shape (or form)* D *in G*. The shape or form D of a diagram *d* generates constraints in G in the following sense:
>
> **Definition (commutative diagrams or constraints):** A diagram d:D→G is commutative when for any two alternative paths $\{f_1, f_2 .. f_k\}$ and $\{g_1, g_2 .. g_k\}$ in D between the objects x and y, the alternative paths in G are reduced to one path between the objects d(x) and d(y); namely the following constraint is applied: $d(f_1) \circ d(f_2) \ldots d(f_k) = d(g_1) \circ d(g_2) \ldots d(g_k)$.

A diagram is therefore a special type of arrow that specifies a form in the graph G, and whose commutative properties induce certain constraints. Constraints generate classes of equivalent paths of arrows, namely define congruence classes of arrows. In this sense, commutative diagrams specify a balance between 'degrees of freedom' (ambiguity) in Gs depending on the number of congruence classes, and 'mutual information' (redundancy) depending on the size of the class of equivalent paths of arrows. The creation of congruence classes of arrows is a fundamental aspect of observation and theory generation in science and mathematics (see for instance the generation of quotient categories in Rosen (1978) and Barr and Wells (1990: 71)).

5.2.1.2.3 Abstraction and instantiations: Limits

Finally, the organization of an objective reality is also identified with the existence of source S that determines different levels of abstraction in the complexity of an object. So, another important conceptual category of an organizational level description includes the concepts of abstraction and instantiation. The term abstraction refers to the properties that are shared by a family of entities. The entities that share this property are said to *instantiate (or participate in)* an abstraction. Entities are therefore *instances* of an abstraction. Abstraction then is the capacity of an observer or observed system to generate entities that define the properties of a *family* of entities.

Sketches are constructions that create abstractions. In category theory, universal constructions such as limits and co-limits represent this situation: diagrams specify properties in G(S), and universal arrows specify instantiations (up to isomorphism) that share the specified property. More specifically, cones (and co-cones) specify descriptions of families of arrows.

Definition (Cones): Given a diagram $d: D \to G$ of shape D, cone is a family of arrows of the form $c_k: i \to d(k)$ that is indexed by the nodes k of D where i is an object in G.

$$\begin{tikzcd}
& i \arrow[dl] \arrow[dr] & \\
d(k) & & d(k')
\end{tikzcd}$$

The dual concept of co-cone is similarly defined by reversing the direction of arrows.

A **commutative cone** is a cone $c_k: i \to d(k)$ such that for each arrow $a: k \to k'$ in D the following diagram commutes:

$$\begin{tikzcd}
& i \arrow[dl, "c_k"'] \arrow[dr, "c_{k'}"] & \\
d(k) \arrow[rr, "d(a)"'] & & d(k')
\end{tikzcd}$$

Based on this construction in G certain universal properties can be defined when for every arrow $h: \tau \to d(k)$ there is a unique arrow $f: \tau \to i$ such that the following diagram commutes:

$$\begin{tikzcd}
& \tau \arrow[ddl, "h"'] \arrow[d, "f"] \arrow[ddr, "h'"] & \\
& i \arrow[dl, "c_k"'] \arrow[dr, "c_{k'}"] & \\
d(k) \arrow[rr, "d(a)"] & & d(k')
\end{tikzcd}$$

The important characteristic of such a construction is that the specified object *i* in G is an instantiation (i.e. a solution or realization) of the properties specified by the diagram d:D→G. The inner organization of the object *i* in G is therefore an (higher level) abstraction of the properties determined by the diagram d:D→G.

5.2.1.2.4 Putting everything together: sketches

We have now identified and defined mathematically a number of fundamental concepts that underlie organizational descriptions: graphs of relations and distinctions; diagrams that determine constraints; and cones that generate abstractions. These concepts are fundamental ingredients of sketches. We can sum all this up with a formal definition of a sketch (following Wells, 1994):

> **Definition (Sketch):** A sketch S is a graph G_S together with a set D_S of diagrams, a set L_S of cones in G_S, and a set C_S of co-cones in G_S. Namely, $S=<G_S, D_S, L_S, C_S>$.

Based on the notion of a sketch, the ***category of sketches*** can also be defined. For the category of sketches, objects are sketches, and arrows are ***morphisms*** between sketches $S=<G_S,D_S,L_S,C_S>$ and $S'=<G_{S'},D_{S'},L_{S'},C_{S'}>$. More specifically the morphism is defined by a graph homomorphism h:$G_S \rightarrow G_{S'}$ such that every diagram, cone, and co-cone in S, is respectively a diagram, cone, and co-cone in S′. (Since every arrow between sketches is a graph homomorphism the proposed construction is a category). The category of sketches can be perceived as a representation of a subjective reality.

Every category C has an ***underlying sketch*** $<G_S,D_S,L_S,C_S>$. The underlying sketch of a category C is a sketch whose graph G_S is the underlying graph of the category, whose diagrams D_S are all the commutative diagrams in C, and whose cones L_S (and co-cones) are all the limits (and co-limits) in C.

This relation can be reversed: for every sketch determined by $<G_S,D_S,L_S,C_S>$ it is possible to construct a category that has as an underlying graph the graph G_S; as commutative diagrams the set D_S; and as limits and co-limits the type of cones and co-cones defined in L_S and C_S. This category, denoted as *Th*(S) or *Th*S, is defined as a ***theory of a sketch*** S. This will be discussed in more detail in Chapter 6 (see Section 6.1.3).

5.2.2 The logical nature of organization

A second approach is to study the notion of organization by looking at *what is required or how much effort is needed for a source in order to specify an objective reality*. The conceptual origins of this view of organization are situated in logic, theory of computation and mathematics. The core idea is that the component of organization arises when there is an effective process that uniquely determines the properties of an observed object, and therefore makes the following diagram in Figure 34 commute.

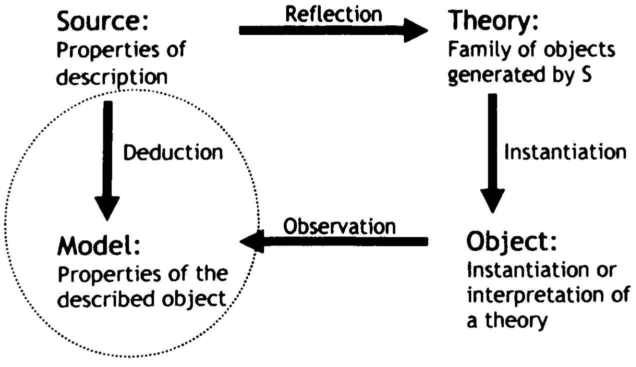
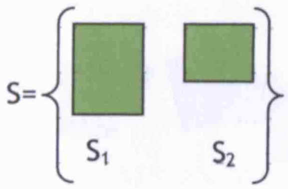


Figure 34: A view of organization that focuses on the properties of deduction that uniquely identify an object. Arrows outside the dotted circle are assumed as given.

The core assumption is that the quality and complexity of organization can be defined and studied by looking at two interrelated ideas: first, *the existence of a process* (expressed by a source) that makes the above diagram commute, and second, *the resources needed to carry out this process*. The first idea alludes to the problem of *computability*: the existence of an effective procedure (e.g. a program, an algorithm) that generates the value of a function. In the terms used in this study, the problem of computability is concerned with the existence of a source that deduces the properties of a described object, and therefore decides whether a specific model is a valid interpretation of a theory. The second idea alludes to the problem of *computational resources*: the size of the resources -such as time or space- needed in order to generate the value of a function. In the terms used in this study, the problem of computational resources is concerned with the size of the source that is necessary for generating or deciding the properties of a described object.

Let us consider a short example, for the purpose of clarifying the relation between the view of organization as an ontological property of conditionality, and the view of organization as a logical property. Let us consider an algebra of geometrical entities (algebras of points, lines, surfaces and volumes) or more specifically a vocabulary of basic configurations (or shapes) in S={S_1, S_2}.



Based on this set S, the family of possible configurations (or shapes) S* can be theoretically defined under certain operations imposed by the algebra. The set S* is restricted by certain conditions expressed by the grammar (rules of spatial relations). These restrictions generate a language of building configurations L. In this context, organization is concerned with the balance between the variety introduced by S* and the restrictions imposed by the grammar.

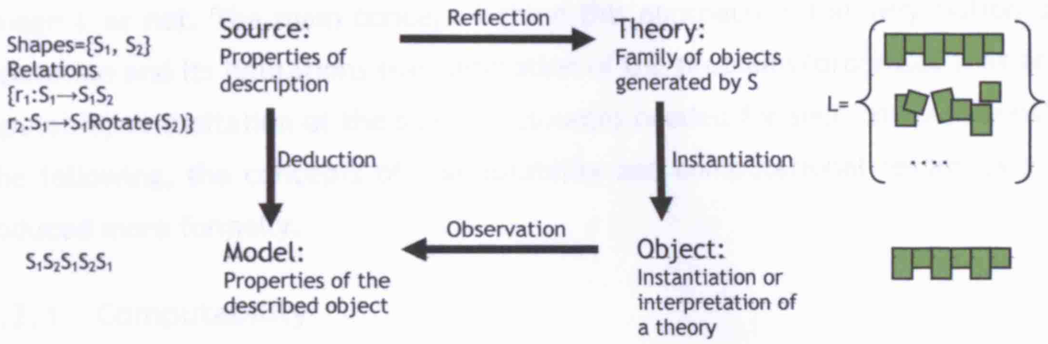
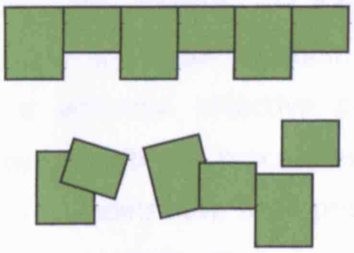


Figure 35: A theory or language L of shapes is generated by two shape rules r_1 and r_2. In the previous section, the notion of organization was defined as the capacity of a source to describe the properties of a family of objects. Here, organization is defined as the capacity of a source to describe a specific object $S_1S_2S_1S_2S_1$.

More specifically, each configuration generated by the postulated rules is equally probable and contributes to the increase of the variety of the system. For example, in the following picture the first configuration is as probable as the second.



However, the first configuration is intuitively less random (or more ordered) than the second configuration. In a representational sense, this type of order is the product of redundancy in S* (or mutual information between S_1 and S_2) that *constrains* the number of possible configurations. Redundancy is therefore a characterization of the properties of space of possible configurations S* that determines a language L and *not* a characterisation of the specific configuration. From the logical or computational perspective, the organization of a building configuration is a property of the specific possibility. Each configuration has value (true or false) depending on whether a particular possibility is realized in a certain language L or not. The main concept behind this approach is the very notion of computation and its limitations (i.e. limitation of the problems/processes that are computable, or limitation of the size of resources needed for such computations). In the following, the concepts of computability and computational resources are introduced more formally.

5.2.2.1 Computability

The assumption explored in this section is that the notion of organization implies the existence of a source with the capacity to carry out computations. The notion of computation is generally understood as an effective process: as a process (an algorithm or method) that is describable by a certain language L. Computability is then concerned with the problem of the existence of an effective process that generates the value of a function. Based on these concepts, the notion (and complexity) of organization is studied by looking at the computability of the (structural or functional) properties of an observed object.

For infinitely many computations (i.e. effective processes), the prime question is whether there is a 'universal' process that has the capacity to realize any other possible computation of a function. Indeed, the identification of such process must be a model of the very concept of computation. A model of computation is therefore a description of a universal effective process that is capable of emulating (or simulating) any describable process expressed in an appropriate language L. Many computational models have been proposed including: the *Turing Machine*, a hypothetical mechanical device that, as postulated by Turing, has the capacity to perform any possible computation; the theory of *General Recursive Functions*, a general theory of computation that constructs the value of any function based on the composition of primitive functions; *Production Systems*, a class of symbol manipulation systems proposed by Post; and *Lambda Calculus*, a function-based method of computation proposed by Church and Kleene. For a general introduction the interested reader may study Minsky (1972), Barwise (1977), Sudkamp (1997), or Boolos et al (2002).

5.2.2.1.1 Languages and machines

The distinction between computation (an effective procedure) and models of computation (a universal process) can be usefully studied in relation to the concepts of language and machine. The term machine is used in ordinary discourse in connection with mechanical artefacts. However, in philosophy, science, and mathematics, the term has been used as a metaphor to express the fundamental working principles of reality, knowledge and thought. In particular, the concept of machine has been used as conceptual metaphor for the definition of the notions of *effective procedure*, *logical process*, and *computing*. The Turing Machine is the best known model of a universal computer capable of realising any possible logical process. The study of machines includes a hierarchy of automata in which the Turing Machine is the most general. In this context, the notion of a language is a concept complementary to the concept of a machine. Languages are defined as families of possible expressions that are generated by a grammar. For instance, a grammar is defined as a formal system or syntax that generates syntactically correct expressions. On this basis, a machine is defined as an effective process that realizes a procedure expressed by a language.

A machine 'reads' a symbolic expression articulated in a given language and then generates an answer. The process realized by a machine might be interpreted both as a *generative* and as a *decision* process. In the first case, the computation carried out by a machine is a process that generates the value of a function given an expression of a problem (input string) and a program (set of rules). In the second case, computation carried out by a machine is a process that decides whether or not the expression of an object is indeed an expression that satisfies the properties of a given language L. In decision processes (or problems), the function is particularly referred to as a *characteristic* function, because it characterizes the membership of elements in a set.

5.2.2.1.2 The category of machines and languages

The existence and universality of computation can be expressed on the basis of adjoint functors. The discussed constructions are in fact generalizations of the algebraic interpretation of organization developed at the beginning of this chapter. More specifically the arrow $M{:}S\times L\rightarrow\Omega$, with the set $\Omega=\{0,1\}$, can be defined as a characteristic function that decides whether an expression taken by the set L is part of a language generated by a grammar S. The arrow $m{:}S \rightarrow \Omega^L$ is a computational model (i.e. a program or algorithm) that uniquely corresponds to this type of language. Given the generation of expressions in a language L that satisfy the syntax S, there must be a unique model of computation $m{:}S \rightarrow \Omega^L$ that makes the following diagram commute:

$$\begin{tikzcd}
S \arrow[d, "m"] & LxS \arrow[d, "M"'] \arrow[dr, "1_L x m"] & \\
\Omega^L & \Omega & Lx\Omega^L \arrow[l, "\varepsilon_\Omega"]
\end{tikzcd}$$

In this case, the characteristic function is determined by $M = \varepsilon_\Omega \circ 1_L x m$. According to this perspective, the component of organization arises when there is a universal arrow (i.e. the evaluation function ε_Ω) that constructs the characteristic function $M{:}SxL \rightarrow \Omega$. The existence of a universal arrow $\varepsilon_\Omega{:}\ \Omega^L xL \rightarrow \Omega$ expresses the capacity of a source to deduce the properties of an observed object.

More generally, let us see how this construction is explicitly linked to the diagram in Figure 34. A source S is defined as a conditional entity (a grammar or syntax) that generates a family of objects $F(S)$ that satisfy certain properties. Let the category $F(S)$ denote the family of objects that satisfy the properties expressed by a set of axioms, or grammar S. Also let M be a characteristic or generative function expressed by the functor $M{:}F(S) \rightarrow d$. According to this perspective, the component of organization arises when there is a grammar S and a universal arrow (i.e. the evaluation function ε_d) able to construct any characteristic or generative function $M{:}F(S) \rightarrow d$ as the following diagram suggests.

$$\begin{tikzcd}
UF(S) \arrow[dr, "U(M)"'] & S \arrow[l, "\eta_s"'] \arrow[d, "m"] & F(S) \arrow[d, "M"'] \arrow[dr, "F(m)"] & \\
 & U(d) & d & FU(d) \arrow[l, "\varepsilon_d"]
\end{tikzcd}$$

These constructions imply that the component of organization arises when the relation between language and machine is complementary. The complementary relation between machine and language has been more explicitly defined by Goguen (1973). Goguen showed that it is possible to define the category of machines and the category of behaviours (languages). He then identified the existence of a functor ***E***: **Mach**→**Beh**, named 'external behaviour' functor, in relation to another functor ***R***: **Beh**→**Mach**, which is its right adjoint. The functor ***R*** is therefore defined as the minimal realization of a language into a machine.

More specifically, the category of machine denoted **Mach** is defined as follows:

The objects of the category **Mach** are machines M of the form <X, S, Y, δ, λ, σ >:

$$\begin{tikzcd}
X \times S \arrow[r, "\delta"] & S \arrow[r, "\lambda"] & Y
\end{tikzcd}$$

Where X, S, Y are the input, state and output sets respectively; and δ, λ, σ are the transition, output and initial state functions respectively.

The arrows ***K***: M→M′ of the category **Mach**, correspond to the triplet of functions ***K***=<a, b, c> such that the following diagrams commute:

$$\begin{tikzcd}
X \times S \arrow[r, "\delta"] \arrow[d, "a \times b"'] & S \arrow[r, "\lambda"] \arrow[d, "b"'] & Y \arrow[d, "c"'] \\
X' \times S' \arrow[r, "\delta'"] & S' \arrow[r, "\lambda'"] & Y'
\end{tikzcd}
\qquad
\begin{tikzcd}
 & S \arrow[dd, "b"] \\
1 \arrow[ur, "\sigma"] \arrow[dr, "\sigma'"'] & \\
 & S'
\end{tikzcd}$$

It is easy to verify that these definitions form a category (the arrow K respects the associativity and identity axioms).

The category of behaviours denoted **Beh** is defined as follows:

Objects of the category **Beh** are functions of the form *f*: X^*→Y, where X and Y are sets, and X^* is the free monoid generated by X. The set X^* is therefore the set of all possible concatenations over an alphabet X. The function *f* is a characteristic function that decides whether a given expression in X^* is well-formed. For instance, Y may be the set {1, 0} and *f* the function $f{:}X^*\rightarrow\{1, 0\}$). The function *f* consequently defines a language that is a subset L in X^*.

The arrows $L{:}B\rightarrow B'$ of the category **Beh** are defined by the pair $<a^*, c>$ where c is the function $c{:}Y\rightarrow Y'$ and a^* is defined recursively on the basis of the function $a{:}X\rightarrow X'$ by $a^*(xw)=a(x)a^*(w)$ such that the following diagram commutes:

$$\begin{tikzcd}
X^* \arrow[r, "f"] \arrow[d, "a^*"'] & Y \arrow[d, "c"] \\
X^{*\prime} \arrow[r, "f'"] & Y'
\end{tikzcd}$$

The external behaviour functor *E*: **Mach**→**Beh** sends every machine $M=\lambda\delta{:}X\mathrm{x}S\rightarrow Y$ in **Mach** to a function *f* in **Beh** (that is $f=E(M)=\lambda\delta^+{:}\ X^*\rightarrow Y$), and every arrow <a, b, c> in **Mach** to an arrow *E*(<a, b, c>)=<a, c>. The minimum realization functor ***R*** is the right adjoint of the functor ***E***.

So in general, the core thesis of the computational approach to organization is that the notion of organization is identified with the existence of a source (e.g. of a brain, a society or in general a physical system) able to carry out computations that specify the structures or functions of an object (see for instance Wolfram, 2002). This thesis might take two different interpretations. On one hand, organization appears when the properties of a described object instantiate a universal machine (a universal procedure). One the other hand, organization appears when there is a minimum effective procedure (a program or algorithm) that generates or distinguishes the properties of an object when it is simulated by a universal computing machine.

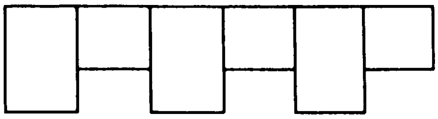
According to the first position, organization appears when the properties of a described object are equivalent to a universal machine. The intuition behind this position is that the notion of organization effectively alludes to the conditions that generate high level functions, such as cognition or life. When a physical system has the capacity to carry out universal computation then it is in principle also capable of generating structural or functional properties of any complexity. Therefore, a complex organization arises because there is a 'potential' to carry out complex functions and not because of its actual structural characteristics. This intuition has been criticised (see for instance Bennett, 1985) for its focus on possible functional properties of a described object rather than observed structural properties.

According to the second position, organization can be more generally defined and studied by looking at the existence of a minimum procedure (program or algorithm) that is necessary in order to generate the structure or function of an object. The intuition behind this position is that the notion of organization effectively alludes to the existence of a minimal algorithm/procedure (source) that describes (deduces) the properties of an observed object. The quest for a minimum procedure that deduces the properties of an observed object can be naturally interpreted as a quest for universality (a minimum source). This is the main focus of the next section.

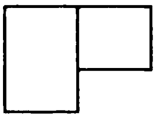
5.2.2.2 Computational resources

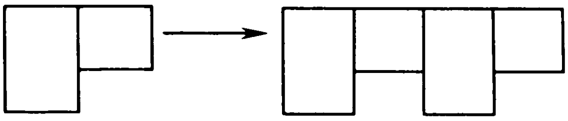
The focus in this section is the complexity of organization. In the previous section (5.2.2.1), the notion of organization was assumed to appear when there is a minimum effective procedure (a program or algorithm) that generates the properties of an object when it is simulated by a universal computing machine. In this section, the complexity of organization is identified in relation to the resources needed in order to carry out universal computations. This includes static resources, such as the size of the minimum program or algorithm, but also dynamical resources, such as the time or space needed in order to generate the structural or functional properties of an object.

Take for instance a shape configuration such as the following:

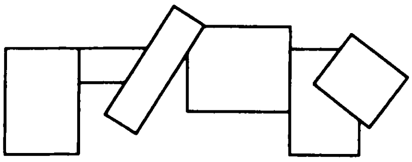


This shape is defined by a set of maximal lines (14 lines). It is also possible to generate this shape from a simple grammar, applied twice:

An initial shape 

And the rule 

The generation of the specified configurations depends on two resources. First, the size of the grammar (i.e. one rule) and second, the time (steps) required to produce or compute that desired outcome (i.e. two steps). The first corresponds to static resources (i.e. the theory needed to generate a configuration), and the second to dynamic resources (i.e. the time needed to generate a configuration given a theory). Intuitively, a more 'sophisticated' arrangement -such as the following- has a much 'bigger' grammar and possibly needs more time for its construction:



Let us focus on the size of the theory needed to generate an arrangement. On the one hand, a random shape would be a shape whose grammar is about the size of, or bigger than, the set of maximal lines. Intuitively, in this case there would be no structure in the shape that can be reduced to a few rules. On the other hand, an ordered configuration is a configuration whose minimum grammar is considerably smaller than the description of the configuration. Organization can be therefore perceived as a balance between randomness and order defined in relation to the size of the minimum grammar that generates a configuration when compared to the size of the set that defines the configuration.

5.2.3 The epistemological nature of organization

A third approach to the study of organization is looking at *the capacity of an observer to generate a family of independent models for an observed object*. This is an epistemological approach in the sense that the notion of organization is defined in relation to the qualities and types of knowledge involved in the understanding of an observed object. The core assumption is that the meaning and complexity of organization can be defined and studied by looking at the relation between a subjective and objective reality. For instance, according to Casti (1992: 22) the complexity of an observed system is a relative quality that is determined by the coupling of the observed system with an observer: the finer the capacity to discriminate details over the structure or behaviour of an observed object, the more complex the object appears in the mind of the observer. In this sense, a stone is a more complex object in the eyes of a geologist than it is in the eyes of an unskilled observer. In order to make the connection with our basic diagram, the complexity of organization for a certain objective reality is now measured by the variety of (knowledge) sources that are needed in order to deduce the properties of a described object. In other words, the quality and complexity of organization is determined by the number of independent models that make the following diagram in Figure 36 commute.

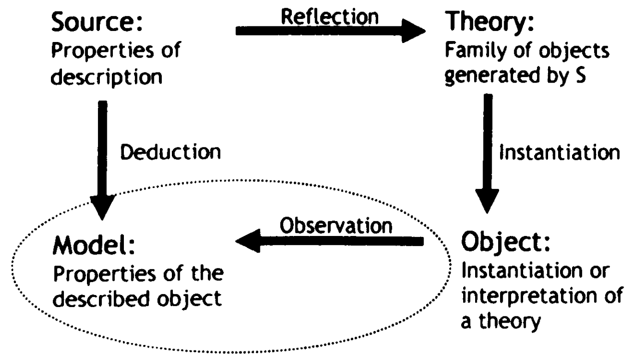


Figure 36: A view of organization that focuses on the variety of models that is necessary in order to uniquely identify an object. Arrows outside the dotted circle are assumed as given.

In the organizational approaches reviewed previously, the notion of organization was defined and studied on the basis of certain basic terms such as entropy, mutual information, or computability. Although the organizational approach described in this section has appeared in different guises, for instance in the study of emergence as a concept that is relative to a model (Cariani, 1991), there is no commonly defined methodological or conceptual basis. Nevertheless, the definition of complexity proposed by Casti (1992) and the definition of the basic units of organization by Rosen (1991) can be used here as the core ideas for elaboration of this approach.

Let us follow Casti's argument adapted to the terms developed in this chapter. Typically, a model (i.e. the description of the properties of an object) is based on a number of *observables* that can be represented as a set of mappings $F=\{f_1,f_2,\ldots f_n\}$ together with their relations (e.g. as a graph).

More specifically, the set of observables $F=\{f_1,f_2,\ldots f_n\}$ is typically split into three subsets of observables called *causes* C, *effects* E and *parameters* P (i.e. conditional entities). The description of the properties of an observed object involves the specification of the relation between causes C and effects E conditional to parameters P, written $M: CxP \rightarrow E$ as seen in Section 5.1.2. A complementary way of describing these conditional relations is to employ a family of parameterized functions $M_p: C \rightarrow E$ (or $m: P \rightarrow E^C$) for parameters $P=\{p,p' \ldots\}$. Any two models, M and M', are equivalent if and only if there is a natural isomorphism $h: M_p \rightarrow M_{p'}$ that makes the following diagram commute:

$$\begin{tikzcd}
C \arrow[r, "h_C"] \arrow[d, "M_p"'] & C \arrow[d, "M_{p'}"] \\
E \arrow[r, "h_E"] & E
\end{tikzcd}$$

In this sense, a conditional object p is isomorphic to p′ if and only if there is a natural isomorphism between M and M'. The identification of conditions p and p′ that are isomorphic, that is p-p′, split the conditional entities P into equivalence classes and determine the quotient set P/~ (i.e. a set whose elements are all the equivalence classes determined by the relation ~). Casti's core idea is that the cardinality of this set Card(P/~) is a characterization of the complexity of an object because it determines the number of equivalence classes of models.

In order to make the connection with the diagram in Figure 36 assume that a source is associated with the set of conditional entities P={p,p′...}, and the model is a description of a particular family of relations between causes and effects e^c, where e is an element of E and c is an element of C. Rephrasing Casti's core idea, the following diagram should have universal properties (a universal 'evaluation' arrow $\varepsilon:cxe^c \rightarrow e$ for a model *M*) for each equivalence class p in a category P/~.

$$\begin{tikzcd}
p \arrow[d, "m"'] & cxp \arrow[d, "M"'] \arrow[dr, "1_c xm"] & \\
e^c & e & cxe^c \arrow[l, "\varepsilon"']
\end{tikzcd}$$

Hence, the source is more precisely the quotient set P/~ whose cardinality determines the complexity of organization.

Instead of a conclusion to this section it is instructive to turn our attention to a very general but also idiosyncratic epistemological view of organization proposed by Rosen. According to Rosen (1991: 108-151) a theory of organization should not confuse the meaning of organization with the syntax of an observed system. For that purpose, the basic unit or component of organization is identified with the existence of a functional component. In his book, *Life Itself*, the notion of a functional component is an epistemological concept that is derived from the interaction of an observer with the observed system. A functional component is then identified such that when it is removed from the observed object a certain behavioural change occurs (i.e. this change is the function of the component). On this basis, organization is defined as the composition of functional components. Based on these principles, Rosen proposed category theory as a (mathematical) theory of organization where arrows or functors are called functional components and categories are pictures of organization that represent the composition of functional components. The view of category theoretic structures as theories of organization will be developed further in the next chapter.

5.2.4 The 'design' nature of organization

In all the aforementioned approaches, the notion and complexity of organization was associated with the capacity of a source (i.e. a conditional entity) to generate theories and models that make the basic diagram in Figure 25 commute. This capacity has been explicitly defined in relation to the size of a source that makes the diagram commute; the size of a theory generated by a source; or the size of independent models that are described by a source. The complexity of organization was then identified by the capacity of a source to determine the complementarity between a subjective and objective reality.

But, what is the picture of organization when the capacity of a source to make the diagram commute is not, or cannot be, well defined? All the organizational approaches recognize that a random organization is distinguished from an ordered organization by identifying this critical state of complementarity between a subjective and objective reality. All measures of the complexity of organization are given on this basis. However, none of these approaches describe the properties of a source in a state where the diagram in Figure 25 does not commute (e.g. when it contains noise or errors). In effect, none of the proposed approaches is capable of forming a meta-theory of organization. A meta-theory would be able to describe *the properties of organization* that underlie a described object, as opposed to describing *the properties of an organized/unorganized object*. The proposition is made here to study the notion and complexity of organization by looking at the capacity of a source to generate theories and models that make the diagram in Figure 25 approximate a critical state of complementarity.

But, why do we need such an alternative understanding of organization? What does it mean? Why are the three basic paradigms of organization not enough for the development of an organizational level theory of design? The short answer to these questions is two-fold. In general, the proposed theory is seen as a generalization that unifies the three predominant approaches. However, it is also developed in order to address certain limitations in the description of organization when the relation between subjective and objective reality is not clearly defined. In particular, for the three main approaches to organizational level descriptions, the notion of organization is identified by certain properties of a source (e.g. the size of the theory, or the variety of models generated by the source) that make the diagram commute. The lack of commutative properties for the diagram in Figure 25 denotes a critical state where the notion of source, conditionality - and therefore organization - does not exist. The motivation for this thesis is to look at a class of organizational level problems and descriptions where the component of organization (and the existence of a conditional entity) is "not yet" realized, but only anticipated. This is important for design. As we saw in Chapter 3 design involves distributed sources, theories, models and objects. So the complementarity between deduced models of the object and interpretations of the theory is an 'anticipated' rather than a given state. As a result it is impossible to understand the meaning of organization in design as a capacity that is derived by a given complementary relation.

In this sense, we can look at the diagram in a different way, instead of defining organization in relation to the existence or not of commutative properties; the question is to understand organization in relation to a "movement" towards the critical state of commutativity. Hence the aim is to develop an understanding of the notion of source (conditionality) when the commutative properties of the diagram constitute a reference, or attracting state, and the realization of commutative properties (the complementarity between subject and object) is lost, or distributed into a number of different sources.

So according to the proposed approach, order emerges at a critical state when a subjective reality is in complementary relation with an objective reality or, to put it differently, when the interpretations of a theory have a complementary relationship with the models that are generated by a source. In this sense, organization expresses the degree to which different realizations of a system 'play by the rules'. The complexity of organization is a characterization of the capacity of a source to construct theories that complement certain interpretations (models), or similarly the capacity of an observed system to take different interpretations/instantiations that can be explained by certain rules (theories).

From this perspective, the notion of organization relates to design in two ways:

- Organization is a product or effect of design(ing): the act of design is considered to generate some sort of organization which refers to the characteristics of an observed system or designed artifact.
- Organization is a condition or cause of design(ing): design problems and the capacity to address such problems are caused by certain organizational qualities of the design system (whether it is the human mind, a society or an artificial system).

The aim of the next chapter is to develop this approach mathematically and elaborate it in more detail.

5.3 SUMMARY AND CONCLUSIONS

In this chapter the notion and complexity of organization were defined and examined in relation to different traditions found in complexity research. Starting from an intuitive presentation of organization as conditionality (a set of processes or structures that generate, constrain and preserve a distinct whole), it was possible to elaborate a more formal understanding of the concept and its fundamental dimensions. The treatment was specifically based on the notion of reflection, and the idea that organization is associated with the existence of a source (a conditional entity) capable of reflecting the properties of an objective reality. The chapter then proposed a generalised view of basic approaches to organization based on the diagram in Figure 25, in which the notions of source, theory, model and object - and their relationships - are exemplified and summarised. In specific, organization is seen as a theoretical quality determined by the capacity of a source to reflect the properties of an object and guarantee the commutativity of the diagram.

The proposed analysis showed that the different interpretations of this basic diagram correspond to different approaches to the study of organization. More specifically, three approaches to organization were presented and elaborated, which respectively offer theories about the ontological, logical/computational and epistemological nature of organization. Finally, based on this analysis, a distinction was made between theories that describe *the properties of organization* and theories that describe *the properties of an organized object*. This implies the enterprise of defining organization not in relation to the existence of commutative properties, but in relation to a "movement" towards the critical state of commutativity. The motivation for such a (meta-) theory is to understand and reflect characteristic design situations where the component of organization (and the existence of a conditional entity or source) corresponds to an anticipated rather than a given state. A formal presentation of this fourth approach is the subject of investigation in the following chapter.

Chapter 6

A SEMANTIC META-THEORY OF ORGANIZATION: A CATEGORY THEORETIC APPROACH

In the previous chapter, the main approaches to organization were introduced and explored using certain basic concepts. In this chapter, the main concern is the development of a 'meta-theory' of the concepts introduced. The objective is multi-fold: to propose a mathematical language (or toolkit) able to handle the semantic aspects of organization and incorporate different organizational-level approaches; second, to offer a mathematical language that has the expressive power to distinguish different levels of organizational complexity; and finally to create a mathematical construction that can support the development of an organizational-level theory of design.

To this end, the meaning of organization and the basic concepts of organizational-level theories are elaborated on the basis of category theoretic constructions. In particular, the proposed category theory of organization is developed as part of a theory of sketches. The treatment is divided in two sections. In the first section, the notions of sketch and weak sketch are introduced and explained with regards to their role for the development of organizational-level descriptions. In the second section, a case is made for a notion of "weak sketch" able to express different types of organization. Weak sketches and weak theories are then introduced and elaborated as basic elements of a meta-theory of organization.

6.1 INTRODUCTION TO (WEAK) SKETCHES

In the previous chapter the meaning and complexity of organization was associated with the notion of conditionality and its ability to reflect the properties of an objective reality. Here the purpose is to turn the concept of conditionality into a mathematical construction using category theory.

The idea of using category theory as a theory of organization can be traced back to the work of Rosen (e.g. 1991). Recall the discussion in Chapter 5. According to his approach the functional activities of any system can be usefully represented by the arrows of a categorical structure. Categorical structures in this sense constitute models of complex organizations that are defined by the possible compositions of their functional components. Based on this line of thought, Rosen proposed a model of a living cell (essentially using three functional components: metabolism, repair and replication). Another elaboration of the concept of organization and complexity from a category theoretic perspective can be found in the work of Ehresmann and Vanbremeersch (1987). They developed a category theoretic model of multi-level structures in complex systems. However, their approach is different from that of Rosen. Their focus is on structural characteristics of systems. The notion of organization is expressed with the identification of structural components and their interactions that are changing in time and scale. A complex system is then modelled by a category whose objects represent the components of a system and whose arrows represent their interactions. In particular, a complex (multi-level) object arises with the existence of universal constructions (limits) that essentially 'glue together' components and their interactions into higher level entities. Changes in time are represented by functors between different categories.

In the present study, inspired by the work of Ehresmann and Vanbremeersch, the idea of a category theory of organization is developed as part of the categorical theory of sketches. However, in this study the emphasis is placed on the semantic properties of sketches. More specifically, the concept of a sketch is proposed as a mathematical interpretation of a conditional entity (a source) that has the capacity to generate, preserve or constrain a distinct whole. A sketch is therefore intended to be an expression of the properties of description (see Figure 37). It is an entity that reflects the properties of a family of objects (i.e. represents a theory of objects) and therefore has a certain capacity to deduce the properties of a described object.

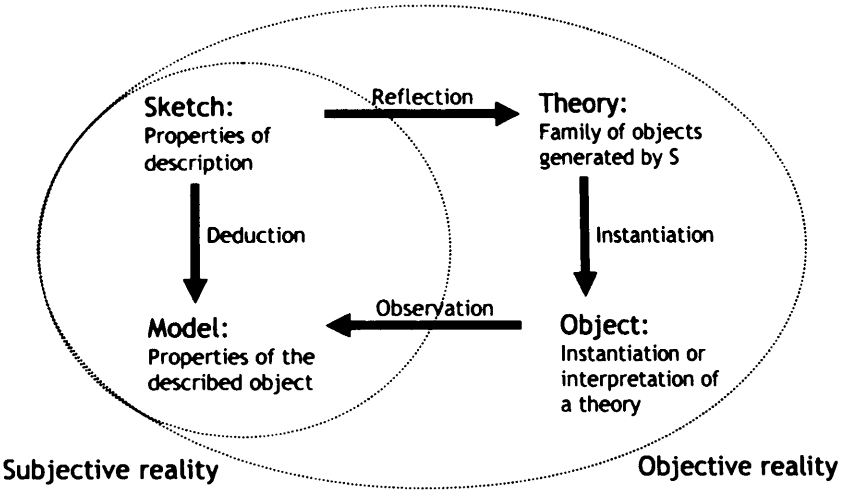


Figure 37: The component of organization arises with the existence of a sketch. A sketch is a presentation of a theory that has the capacity to deduce the properties of any object (any instantiation) of the theory. A sketch is therefore a mathematical interpretation of the notion of conditionality.

As we saw in Chapter 5 a sketch is distinguished by its capacity to represent differentiations and relations between entities; to express ambiguity and redundancy; as well as to represent abstractions in the expression of an objective reality. But let us explore the mathematical notion of sketch, and the motivation behind its use, in more detail.

A sketch in category theory is a construction that gives a symbolic representation of mathematical theories based on graph-theoretic diagrams. The category theoretic concept of sketch is a construction with only tenuous links to the concept of sketch encountered in design. A sketch is in a sense a tool for the mathematician that designs mathematical structures and needs to reflect with representations of mathematical objects. But for the purpose of this thesis, this is not a 'link' to be pursued. The category theoretic sketch is used here because it is a construction that determines the (organizational) properties of reflection; irrespective of whether it refers to a symbol system, constructed in a brain or external to it. In order to explain this understanding of the concept of sketch, it is instructive to briefly review the role of sketches in design in general, and in the work of Vinod Goel (1995) in particular.

6.1.1 Sketches as 'weak' symbol systems

In design, sketches are widely perceived and studied as a kind of drawing, or more widely, as type of representation (see for instance Goel, 1995). In the design research community, the study of sketches concerns both the description of their properties as symbol systems, as well as the understanding of their function and role in the design process. In fact, the two directions are remarkably intertwined. In Goel's work (ibid: 187), a sketch is a symbolic system -pictorial or not- that has certain properties. In his analysis, which adopts Elgin's terminology (1983), a symbol system is a representation that consists of a *scheme*, a *realm* and a *relation*. More specifically, a scheme constitutes the syntactic element of the symbol system, and is organized into tokens and types. *Tokens* are physical instances such as marks or inscriptions in a certain medium. *Types* are equivalent classes of tokens that form a language in which logical operations are defined. A realm is the object of reference for a given scheme. Relations are then the connections between a scheme and a realm. Looking at the *symbol system* as a whole, *it represents a construction that determines a correspondence between syntax and semantics*. These terms are clearly reminiscent of the terminology introduced in the previous chapter where a scheme is the source in a subjective reality, and a realm is the object in an objective reality. Therefore, a symbol system can be defined as an organizational quality: *a quality that generates and constrains the relation between a scheme and a realm*. This 'organizational nature' of a symbol system implies that it is not possible to separate the nature of the system as a physical entity (e.g. marks in paper) from its nature as a cognitive construct (e.g. mental states).

A sketch is a special type of symbol system. It is a symbol system with weak relations between tokens and types, scheme and realm, semantics and syntax. Such properties imply that a sketch -as a symbol system- is syntactically and semantically imprecise, indeterminate and ambiguous. Goel argues that mental states are essentially representations with certain structures and properties which are often equivalent to those found in sketches. He therefore claims that although cognition requires a representational medium, a symbol system, or 'a language of thought', for design problems this representational medium needs to have the structure and properties of a sketch. Sketches are then the organizing principles of thought.

6.1.2 Beyond sketches as 'weak' symbol systems

In the context of this thesis, what is considered special about sketches is that they represent the most generic symbol system; a symbol system with the minimum constraints regarding the scheme-realm relation. The generality of sketches alludes to a looser or weaker structure between a scheme, a realm and their relation. It is therefore assumed that sketches represent a generic class of symbol system that stands at the top of the hierarchy of symbol system types.

Following this line of thinking, it is possible to push the assumptions about the nature of sketches even further. Sketches can be alternatively thought of as a type of sub-symbolic or dynamical system. A sketch as a symbolic system is considered to be an organizational quality by virtue of its capacity to specify relations between syntax and semantics. A sketch as a type of sub-symbolic/dynamical system can be considered as an organizational quality by virtue of its capacity to specify the relation between the behaviour and structure of the system. For instance, for van Gelder and Port (1995) a dynamical system is described by a *phase space*, a *state space* and the *concrete system*. The phase and state space are the conceptual analogues of a scheme (types and tokens) in the computational paradigm. The *state space* is the set of all possible expressions a system, and the *phase space* is the set of abstract elements derived by (numerical) measurements over the state space. In a computational or symbol processing paradigm, expressions are derived by applying logical operations over symbols; while in the dynamical paradigm the behaviour of a system is derived by formulating differential equations over the phase space.

For the purpose of developing an organizational-level theory of organization, the thesis makes no commitment as to whether a sketch is a symbolic or sub-symbolic system. *A sketch is in general considered as an organizational quality that specifies the correspondence between theories (e.g. a scheme or phase/state space), models (a realm or concrete system), and their relations.* It should be noted that the intention in this study is clearly not to use the concept of sketch in order to address theoretical questions about sketches and drawings, but as a way to capture the meaning of organization. Let us now move to our more detailed mathematical treatment.

6.1.3 The category-theoretic notion of sketch

This section is concerned with the mathematical construct of sketch as a way to describe the relation between theories and models and offer a characterization of organization. The main objective is to introduce the idea that for each sketch S a universal property is constructed that gives a precise definition of the meaning and relation between a sketch, a theory of a sketch, and its models (Barr and Wells, 1985).

In category theoretic terms, the notion of theory is more explicitly identified with the concept of *category* (that satisfies certain properties), while the notion of model is identified with the concept of *functor*. More specifically, for each sketch determined by $<G_S, D_S, L_S, C_S>$ it is possible to construct a category that has as an underlying graph the graph G_s; as commutative diagrams the set D_s; and as limits and co-limits the type of cones and co-cones defined in L_s and C_s. This category, denoted as *Th*(S) or *Th*S, is defined as a ***theory of a sketch*** S. For a sketch S and for a theory of a sketch *Th*S there is a functor $i{:}Th\mathrm{S}\rightarrow \mathrm{C}$ that preserves the aforementioned properties of the theory. This functor is defined as a ***model or interpretation of a theory Th*S**. In the same line of thought, a *model of a sketch* S in C is a graph homomorphism $m{:}G_S\rightarrow U\mathrm{C}$ such that G_S is the graph of the sketch S; and *U*C is the underlying sketch of the category C. This definition implies that: whenever the diagram $d{:}\mathrm{D}\rightarrow G_S$ is defined in G_S, then $m\circ d{:}\mathrm{D}\rightarrow U(\mathrm{C})$ is a diagram in *U*C and a commutative diagram in C. Whenever the diagrams $l{:}\mathrm{D}\rightarrow G_S$ and $c{:}\mathrm{D}\rightarrow G_S$ are cones and co-cones in S, then they are cones and co-cones in *U*C and limit and co-limits in C.

Based on this notation, the following universal property is defined: For every sketch S and every model of a sketch m:S→ *UC* there is unique functor i:*Th*S→C for which the following diagram commutes (i.e. m= ***U(i)*** η_s):

$$\begin{tikzcd}
UThS \arrow[dr, "U(i)"'] & S \arrow[l, "\eta_s"'] \arrow[d, "m"] & ThS \arrow[d, "i"] \\
 & UC & C
\end{tikzcd}$$

This property states that there is a natural bijection between models of a sketch m:S→*UC* and interpretations of a theory i:*Th*S→C (i.e. an adjunction between a theory functor ***Th*** and the underlying functor ***U***), which is denoted as:

$$\text{Mod}(S,\ UC)\cong\text{Fun}(ThS,\ C)$$

The notion of adjunction gives a precise meaning to the interplay between theories and models generated by a sketch S. In order to intuitively explain the meaning of this construction let us briefly discuss an informal example (Figure 38).

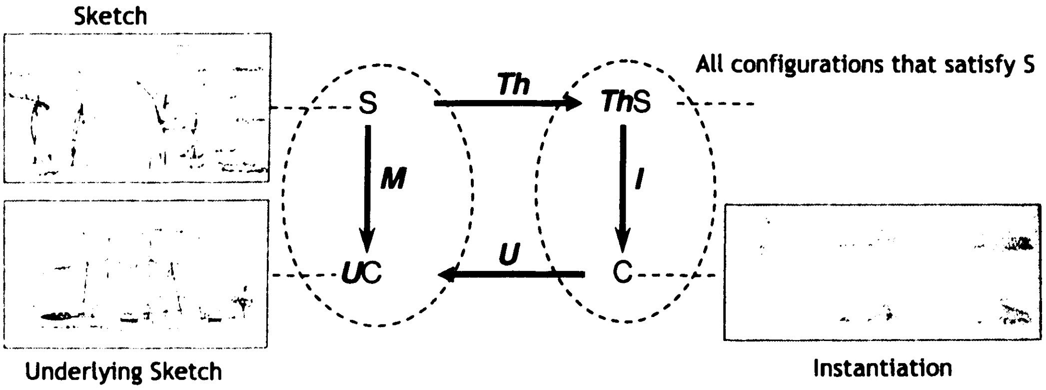


Figure 38: An informal presentation of the notion of sketches, theories, models and their relation, on the basis of the notion of adjunction. These notions are used here in connection to architectural design. S is a typical architectural sketch, *ThS* is a language that describes all possible buildings with the properties of S, C is a particular instantiation (a building), and *UC* is a description of the properties of C.

Let us imagine an architect designing a building. Let us suppose that a sketch S is a typical architectural sketch of a building. Then the category *Th*S is a theory that describes all the possible models of the sketch, namely a set of plan configurations that satisfy the properties described by the sketch. The category *U*C is the underlying sketch of the category C: a description of the properties of a specific instantiation (a building). The universal property (embodied by the adjunction) simply states that the properties *U*C of a specific building are deduced by the sketch S (i.e. they are a plausible instantiation of S) if and only if the specific configuration is derived from the theory of possible built forms *Th*S.

Another example can be given in relation to the 'design' of mathematical structures; for instance the mathematical structure of a graph (Figure 39).

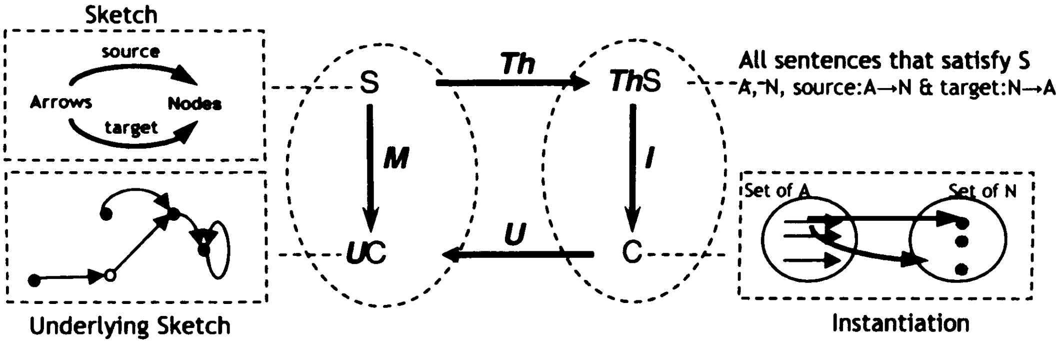
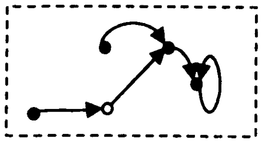


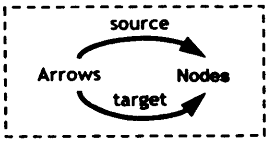
Figure 39: An informal presentation of the notion of sketches, theories, models and their relation, on the basis of the notion of adjunction. These notions are used here in connection to 'mathematical design'.

There are many different pictures of graphs that one can draw such as for instance:



A mathematician also employs a language in order to express a theory about this mathematical object. In the specific example a graph is defined by a set of arrows

A, and a set of nodes N, with two functions (the source function *source*:A→N and the target function *target*:N→A). A theory might take a number of different interpretations. That means that there are many graphs that can be drawn from the theory by specifying a particular set of arrows, set of nodes and source and target functions. The object C is one such instantiation. A sketch is then a particular kind of graph: It is an 'archetypical' graph that describes the properties of any possible instantiation of the theory. Namely, a sketch is a minimum representation of the theory. In the specific example the sketch is a graph defined by two nodes (arrows and nodes) and two arrows (source arrow and target arrow) as seen below.



In this example, the relation between theories, models and sketches is characterized by a certain 'optimality'. More specifically, the proposed sketch of a graph is the 'minimum presentation' of the mathematical theory of graphs. The proposed sketch is therefore the graph of all possible graphs. This is the very meaning of a natural bijection (adjunction) between models of sketches and the interpretations of their theories:

$$\mathrm{Mod}(S, \mathit{U}C) \cong \mathrm{Fun}(\mathit{Th}S, C)$$

It is reasonable to assume that this condition must arise only if there is a *well-defined organization*. For instance, such a situation must arise at the end of a design process; when a scientist, a mathematician or a designer effectively present the results of their efforts and prove (or demonstrate) that there is a complementary relation between subjective and objective reality. In this sense, it is also reasonable to assume that this condition must have a 'weaker form' when a notion of organization is not (yet) well-defined. This is the very meaning and motivation of a weak sketch and it is the basis for the following discussion and definitions.

6.1.4 Sketches as vehicles of organizational-level descriptions

We saw previously that the complexity of organization can be measured in terms of *ambiguity-redundancy* in the degrees of freedom of a system, or in terms of a *logical or epistemological difficulty in the description of a system*. Such characterizations basically refer to *the size of a theory* (difficulty of description) or *the variety of its models* (variety of interpretations) that make the basic diagram in Chapter 5 commute. In this section, the complexity of organization is alternatively defined in relation to *a critical state*. This critical state is explicitly specified by the complementarity between theories and models.

How is such a characterization of organization possible? The core idea is to identify the property of adjunction with a *'perfectly ordered'* organizational state. Specifically, an *'ordered'* organization implies that *all* properties of *every* possible model are known and deducible from the theory (there is no uncertainty in the system). A '*random*' organization is then identified with a situation where there is no adjunction: there is neither a sketch of a theory nor a sketch of models. Finally, a '*complex*' organizational state is identified with a situation where there is a "weak adjunction". A weak adjunction is an adjunction where the arrow $i\colon Th\mathrm{S}\to \mathrm{C}$ in the above universal construction is not required to be unique (see for instance Mac Lane, 1998: 235). A weak adjunction therefore implies that there are multiple interpretations of a theory ($i\colon Th\mathrm{S}\to \mathrm{C}$) in relation to a specific model of a sketch. In this sense, complexity is associated with a situation when there is a certain ambiguity generated by the multiplicity of possible interpretations. Informally, it is convenient to assume that the theory functor *Th* (or likewise the model *m* and interpretation *i*) 'forgets' or 'ignores' some of the information described by the sketch. It is also convenient to think that such functors introduce certain additional structure not originally included in the sketch. The core idea is summarised in the following Figure 40.

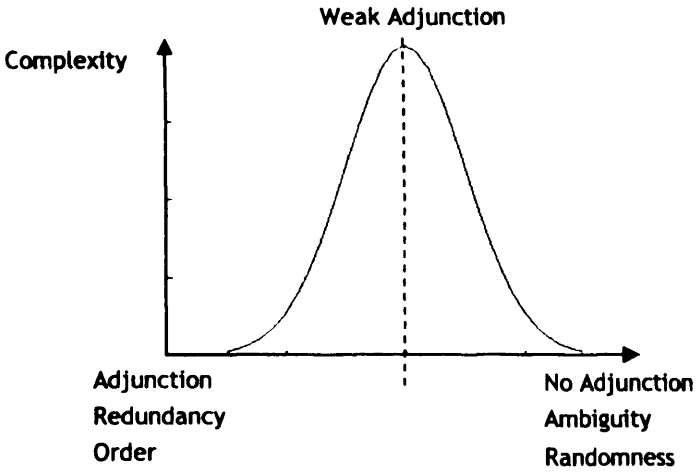


Figure 40: The complexity of organisation defined in relation to the notion of adjunction.

6.2 SKETCHES OF WEAK (NEAR) COMPLEMENTARITY

Let us now develop this last approach to organization in more detail, using the notion of weak sketch. Here, the mathematical construct of a sketch is further studied at a meta-level of abstraction. The study moves from the study of sketch as a source of a theory that specifies a family of models, to the study of sketch as a source of a theory that specifies a family of models *with a certain distance to universality*. In particular, the objective is to introduce a generalized type of sketch with the expressive power to reflect different levels of organization (ordered, random or complex) as a function of the quality (the weakness) of the adjunction between models specified by a sketch, and interpretations of the theories generated.

The core idea is to consider the existence of a type of a sketch that represents a weaker type of a theory. For that purpose, a new type of theory- a 'weak' theory- is introduced. The proposed theory aims to be general enough to describe different kinds of organization. More specifically, a weak theory is a theory that has the expressive capacity to describe the properties of its models only in relation to certain local conditions. The completeness and complementarity (i.e. adjunction) between interpretations and models for a sketch S is therefore not a universal property but a relative property distributed within a universe U. Devising a mathematical expression of the notion of proximity to complementarity and the notion of distribution would allow us to distinguish different kinds of organization and ultimately different degrees of complexity. In this sense, the introduced type of (weak) sketch is a language for developing organizational-level theories; that is, a sketch of organization.

To mathematically express these ideas, two issues are elaborated in the following sections:

- First, the identification of the basic elements - graphs, diagrams, cones and co-cones - that specify a weak sketch.
- Second, the identification of the properties of a weak theory: the properties that a generalized type of sketch is intended to capture.

6.2.1 Weak sketches

6.2.1.1 The basic elements of a weak sketch

A diagram in a weak type sketch is defined as a homomorphism $D{:}J\to G_5$ that generates uncertainty or 'noise arrows' from J to G_5. To understand this idea it is instructive to first recall that a diagram - which we can refer to here as *strict* diagram - implies that for every arrow u: $i\to j$ in a graph J, there is an arrow $D(u){:}D(i)\to D(j)$ in G_5 (see definition in 5.2.1.2.2). A diagram in a weak sketch - which we can refer to as *weak diagram* - intents to capture the idea that given an arrow u: $i\to j$, the arrow $D(u)$ is 'imperfect': so it needs to be composed -from left or right- with other arrows in G_5 in order to approach the 'target/ideal' arrow $p{:}D(i)\to D(j)$ that determines an homomorphism.

A diagram $D{:}J\to G_5$ is therefore defined as a *weak diagram* when for every arrow $u{:}i\to j$ in a graph J, there is an arrow $D(u){:}D(i)\to(\_)$ or $D(u){:}(\_)\to(\_)$ or $D(u){:}(\_)\to D(j)$ that belongs to a path of arrows p: $D(i)\to D(j)$. This can be illustrated in the following diagram:

$$\begin{tikzcd} i \arrow[d, "u"'] \\ j \end{tikzcd}
\qquad
\begin{tikzcd} D(i) \arrow[dd, dashed, "p"'] \arrow[dr, "D(u)"] & \\ & (\ldots) \arrow[dl, "e"] \\ D(j) & \end{tikzcd}
\qquad
\begin{tikzcd} D(i) \arrow[dd, dashed, "p"'] \arrow[dr, "e"] & \\ & (\ldots) \arrow[dl, "D(u)"] \\ D(j) & \end{tikzcd}
\qquad
\begin{tikzcd} D(i) \arrow[d, dashed, "p"'] \arrow[r, "e"] & (\ldots) \arrow[d, "D(u)"] \\ D(j) & (\ldots) \arrow[l, "e'"'] \end{tikzcd}$$

or

or

New conceptual opportunities are revealed when the graph structures are replaced with the underlying graphs of categories. Then, the principles of category theory make it possible to explicitly express compositions of arrows and equivalent relations between alternative paths of arrows. This is the cornerstone of *commutative diagrams*. The proposition made in this study is that the concept of commutative diagram can be extended in order to define weakly commutative diagrams and weak functors. The core idea is that for all the alternative paths of arrows between any two nodes within J (i.e. for all equivalent compositions) the diagram D:J→G_s specifies (or preserves) their equivalence in G_s - but only with some noise or ambiguity.

To demonstrate the idea, let us assume that there are two alternative paths of arrows between nodes i and k: the one-arrow path p:i→k and the two-arrow path $<u_1,u_2>$: i→k.

$$\begin{tikzcd} i \arrow[dd, "p"'] \arrow[dr, "u_1"] & \\ & j \arrow[dl, "u_1"] \\ k & \end{tikzcd}$$

The arrow $D(p)$: (_)→(_) is part of an n-arrows path $<\ldots,D(p),\ldots>:D(i)\rightarrow D(j)$ that has the same start and end nodes with the path $<<\ldots,D(u_1),\ldots>,<\ldots,D(u_2),\ldots>>:D(i)\rightarrow D(j)$. For simplicity in the notation, let us denote *$D(p)$* the path $<\ldots,D(p),\ldots>$ and *$D(u_1)$* $D(u_1)$* the path $<<\ldots,D(u_1),\ldots>,<\ldots,D(u_2),\ldots>>$.

$$\begin{tikzcd}
\mathrm{i} \arrow[dd, "p"'] \arrow[dr, "u_1"] & \\
& \mathrm{j} \arrow[dl, "u_1"] \\
\mathrm{k} &
\end{tikzcd}$$

$$\begin{tikzcd}
D(\mathrm{i}) \arrow[r, "e_l"] \arrow[d, dashed, "{}^*D(u_1)^*"', "D(u_1)^*"] & (\ldots) \arrow[d, "D(p)"] \\
D(\mathrm{j}) & (\ldots) \arrow[l, "e_R"']
\end{tikzcd}$$

The transition from J to G_s generates noise or ambiguity in the sense that the arrow $D(p)$ needs to be composed from left with the path of arrows e_L, and/or from right with the path of arrows e_R in order to form the path $p{:}D(\mathrm{i}) \rightarrow D(\mathrm{j})$. Namely, $\tau=(e_L, e_R)$ are paths of arrows such that $<e_L,D(p),e_L>{:}D(\mathrm{i}) \rightarrow D(\mathrm{j})$. These paths of arrows $\tau=(e_L, e_R)$ can be perceived as transformations $\tau{:}D(p) \rightarrow D(u_1)^*D(u_2)$. The same argument applies to $D(u_1)$ and $D(u_2)$, to define a transformation $\tau'{:}D(u_1)^*D(u_2) \rightarrow D(p)$.

More generally, we can define a *weakly commutative diagram* as a *diagram* whose alternative paths of arrows between nodes $D(\mathrm{i})$ and $D(\mathrm{k})$ are not equal (*i.e.* $u_2 \bullet u_1 = p$), but are linked via a transformation $\tau{:}D(u_2) \bullet D(u_1) \rightarrow D(p)$ - as the following diagrams suggest:

$$\begin{tikzcd}
& D(\mathrm{j}) \arrow[dr, "{}^*D(u_2)^*"] & \\
D(\mathrm{i}) \arrow[ur, "{}^*D(u_1)^*"] \arrow[rr, "{}^*D(p)^*"] & & D(\mathrm{k})
\end{tikzcd}$$

or

$$\begin{tikzcd}
& D(\mathrm{j}) \arrow[dr, "D(u_2)"] \arrow[d, phantom, "\Downarrow \tau"] & \\
D(\mathrm{i}) \arrow[ur, "D(u_1)"] \arrow[rr, "D(p)"] & {} & D(\mathrm{k})
\end{tikzcd}$$

Note that based on this line of thought, a weak functor $D{:}\mathrm{J} \rightarrow \mathrm{G}$ can be defined as a functor where compositions are preserved up to certain transformations[5] $\tau{:}D(u_2 \circ u_1) \rightarrow D(u_2) \circ D(u_1)$ and $\sigma{:}D(1_A) \rightarrow 1_{D(A)}$.

Alongside weak diagrams, a weak type of cones (and similarly of co-cones) is also

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[5] In a sense, this formulation is an expression of the intuitive idea that the whole of a system is more than the sum of its parts, or that a system is non-linear.

needed. Given a diagram D:J→G_S a weak cone is a set of arrows p_i:c→D(i) such that the following diagram is a weak (commutative) diagram:

$$\begin{tikzcd} & c \arrow[dl, "p_i"'] \arrow[dr, "p_j"] \arrow[d, phantom, "\overset{\tau}{\Rightarrow}"] & \\ D(i) \arrow[rr, "D(u)"] & {} & D(j) \end{tikzcd}$$

A weak version of the notion of product implies that this cone may have certain weak universal properties within a certain category. The induced universalities (limits and co-limits) are then defined as *critical states* with weak universal properties. More specifically, *universality* is defined in relation to a critical condition that marks a (phase) transition from a simple, non universal, cone structure to a limit (e.g. product or pullback) construction. This critical condition might be identified with the special case when the transformation τ *is* an isomorphism.

6.2.1.2 Summary definitions of a weak sketch

In summary, weaker types of diagrams, cones and co-cones are defined in order to make possible the construction of a weak theory, namely the category Th_W(S).

> **Definition (Weak diagrams):** A diagram D:J→G_S is a ***weak diagram*** when for every arrow u:i→j in a graph J, there is an arrow D(u):D(i)→(_) or D(u):(_)→(_) or D(u):(_)→D(j) that belong to a path of arrows p: D(i)→ D(j).
>
> **Definition (Weakly commutative diagrams):** A diagram D:J→G_S of shape J in G_S is a ***weak commutative diagram*** if for any pair of paths $u_n \ldots \bullet u_2 \bullet u_1$ and $p_n \ldots \bullet p_2 \bullet p_1$ between the nodes i and j within a graph J, there exists a transformation $\tau : D(u_n) \ldots \bullet D(u_2) \bullet D(u_1) \to D(p_m) \ldots \bullet D(p_2) \bullet D(p_1)$ and its opposite $\tau' : D(p_m) \ldots \bullet D(p_2) \bullet D(p_1) \to D(u_n) .. \bullet D(u_2) \bullet D(u_1)$ between two paths of arrows $D(u_n) \ldots \bullet D(u_2) \bullet D(u_1)$ and $D(p_m) \ldots \bullet D(p_2) \bullet D(p_1)$ in G_s.

So, given the shape graph J:

$$\begin{tikzcd}
& i \arrow[dr, "b"] & \\
j \arrow[ur, "a"] \arrow[rr, "c"] & & k
\end{tikzcd}$$

the following diagram in G_S is a weak diagram if there is a transformation $\tau{:}f \bullet g \to h$:

$$\begin{tikzcd}
& X \arrow[dr, "g"] \arrow[d, phantom, "\Downarrow \tau"] & \\
A \arrow[ur, "f"] \arrow[rr, "h"] & {} & B
\end{tikzcd}$$

Note, that the arrow $\tau{:}f \bullet g \to h$ is not necessarily a natural transformation. When the arrows f, g and h are functors then τ might be thought as a natural transformation.

Definition (Weakly commutative cones or weak cones): if $D{:}J \to G_S$ is a diagram of shape J in G_S, then the cone $p{:}c \to D(\_)$ is a ***weak cone*** if for any arrow $f{:}i \to j$ in J, there is a weak diagram (as seen below) where $\tau{:}D(u) \bullet p_i \to p_j$.

$$\begin{tikzcd}
& c \arrow[dl, "p_i"'] \arrow[dr, "p_j"] \arrow[d, phantom, "\Downarrow \tau"] & \\
D(i) \arrow[rr, "D(u)"] & {} & D(j)
\end{tikzcd}$$

Based on the above definitions, it is possible to define a type of limits (and co-limits) that have universal properties only in certain (critical) conditions:

> **Definition (Critical limits):** A cone p_i:c→d(_) in a category C is a critical limit cone if and only if the (natural) transformation τ:$d(h)\bullet\ p_i \rightarrow p_j$ is an isomorphism.

Having introduced these concepts the type of weak sketch is defined as follows:

> **Definition (Weak sketch):** A weak sketch $S=(G_S, wD_S, wL_S, wC_S)$ consists of a graph G_S, a set of weak diagrams D_S, a set of weak cones in G_S, and a set of weak co-cones in G_S.

6.2.2 Weak theories: theories generated by weak sketches

In this section, the aim is to identify the mathematical properties of weak theories; that is theories that are general enough to describe different types or qualities of organization. More precisely, the proposition put forward is that an ordered universe can be identified with the capacity to construct a well-formed theory for a certain objective reality; randomness can be identified with the absence of such theory; and complexity can be identified with the capacity to construct only weak theories. A weak theory can then be thought as the most general class of theories which allows us to consider well-formed and random theories as special cases.

Weak theories are defined as theories that are characterized by ambiguous descriptions and distributed (or situated) interpretations. The meaning and mathematical expression of these properties is based on two main ideas: the idea of constructing a notion of *weak adjunction*, and the idea of constructing a notion of *distance from, or transition to, universality*. These constructs will allow us to build a type of theory where different organizational qualities are represented by giving special interpretations to the theory.

In the following, the basic concepts of universality and adjunction will be introduced first. Based on the concept of adjunction, the meaning of well-formed, random, and weak theories will be elaborated. This discussion will elucidate the motivation for developing a notion of weak adjunction. The notion of weak adjunction as a vehicle for defining weak theories will then be introduced in detail, together with an explanation of its properties. Finally, based on the definition of weak theories the notion of phase transition between different organizational qualities will be presented.

6.2.2.1 Universality and adjunction: a review

First let us recall the basic concepts and facts associated with universality and adjunction. Given two categories C and D, and a functor U:D→C, the property of universality for an object s and arrow η_s:s→U_ in C requires the following property: For every d in D and arrow m:s→Ud in C there is unique arrow i:x→d in D such that $m = U(i) \circ \eta_S$. Note that from now on lower case s will be used to denote a sketch for reasons of readability (to avoid multiple parentheses).

The basic concept of adjunction between two categories C and D requires the existence of two functors F and U such that for every pair of objects s in C and d in D there is a natural bijection φ between the sets D(Fs,d) and C(s,Ud), i.e. D(Fs,d) ≅C(s,Ud). The adjunction <F,U,φ> is therefore defined when there is a bijection φ:D(Fs,d)→C(s,Ud) which is natural in d and s. The bijection implies that the following diagrams commute (i.e. $\varphi{}^\circ\theta=1_{C(s,\, Ud)}$ and $\theta{}^\circ\varphi=1_{D(Fs,\, d)}$).

$$\begin{tikzcd}
D(Fs, d) \arrow[rd, "\varphi"] & \\
C(s, Ud) \arrow[u, "\theta"] \arrow[r, "1_{C(s,\, Ud)}"] & C(s, Ud)
\end{tikzcd}
\qquad
\begin{tikzcd}
C(s, Ud) \arrow[rd, "\theta"] & \\
D(Fs, d) \arrow[u, "\varphi"] \arrow[r, "1_{D(Fs,\, d)}"] & D(Fs, d)
\end{tikzcd}$$

This construction also implies that for *each* object s in C there is a universal arrow η_S of the form η_S:s→UFs. The property of universality for *every* object s implies that for *every* arrow of the form m:s→Ud in C there is a unique arrow i:Fs→d in D such that $m = U(i) \circ \eta_S$.

$$\begin{tikzcd}
UFs \arrow[rd, "U(i)"'] & s \arrow[l, "\eta_s"'] \arrow[d, "m"] & Fs \arrow[d, "i"'] \\
& Ud & d
\end{tikzcd}$$

The dual property is applied for an object d in D where *for every* object d in D there is a universal arrow of the form $\varepsilon_d{:}FUd \rightarrow d$. In this sense the right diagram below commutes:

$$\begin{tikzcd}
UFs \arrow[rd, "U(i)"'] & s \arrow[l, "\eta_s"'] \arrow[d, "m"] & Fs \arrow[d, "i"'] \arrow[rd, "F(m)"] & \\
& Ud & d & FUd \arrow[l, "\varepsilon_d"]
\end{tikzcd}$$

Because the arrow $\eta_s{:}s \rightarrow UFs$ is universal for every object s in C it can be also induced that η_s is a component of a natural transformation $\eta{:}1_C \rightarrow UF$, as the following diagram suggests:

$$\begin{tikzcd}
UFs \arrow[d, "UF(i)"'] & s \arrow[l, "\eta_s"'] \arrow[d, "m"] & Fs \arrow[d, "i"'] \\
UFs' & s' \arrow[l, "\eta_{s'}"'] & Fs'
\end{tikzcd}$$

6.2.2.2 Well-formed, random and weak theories

The basic concept of adjunction can now be employed in order to identify the meaning of *order* or of an *ordered universe* U. An '*ordered universe*' is a universe described/realized by a well-formed theory: that is a theory where there is a natural bijection $D(Ths, d) \cong C(s, Ud)$. Following this premise, a '*random universe*' would be a universe described/realized by a theory that cannot be constructed as a natural bijection, that is D(*Th*s, d)≇C(*s*, *U*d). Between the two extreme poles there are theories that are characterized by ambiguity and distribution (instead of uniqueness and universality). '*Ambiguity*' is related with the relaxation of uniqueness within a universal construction. '*Distribution*' is related with the relaxation of the existence of universal arrows for every object s in C. The relaxation of uniqueness and universality within an adjunction <*Th*, *U*, φ> determines the meaning of a weak adjunction. A '*complex universe*' is then a universe described/realized by a theory that forms a weak adjunction.

6.2.2.3 Weak adjunction

More specifically, the concept of weak adjunction between two categories C and D requires the existence of two functors *F* and *U* with the following properties.

> **Definition (weak adjunction):** A weak adjunction between two categories C and D is defined by a tuple <*F*, *U*, φ,θ, τ_C, τ_D> where *F*:C→D and *U*:D→C are functors; the arrow τ_C is a (natural) transformation between the arrows (functors) *m*:s→*U*d and *m'*:s→*U*d; the arrow τ_D is a (natural) transformation between the arrows (functors) *i*:*F*s→d and *i'*:*F*s→d; and the arrows φ and θ make the following diagram commute naturally in s and d (i.e. $\varphi^\circ\theta = \tau_C$ and $\theta^\circ\varphi = \tau_D$):

$$\begin{tikzcd}
D(Fs, d) \arrow[dr, "\varphi"] & \\
C(s, Ud) \arrow[u, "\theta"] \arrow[r, "\tau_C"] & C(s, Ud)
\end{tikzcd}
\qquad
\begin{tikzcd}
C(s, Ud) \arrow[dr, "\theta"] & \\
D(Fs, d) \arrow[u, "\varphi"] \arrow[r, "\tau_D"] & D(Fs, d)
\end{tikzcd}$$

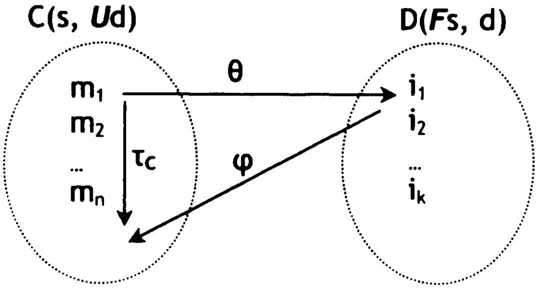
The two diagrams can be equally depicted as follows:

$$\begin{tikzcd}
C(s, \mathit{U}d) \arrow[r, "\theta"] \arrow[d, "\tau_C"'] & D(\mathit{F}s, d) \arrow[d, "\tau_D"] \arrow[dl, "\varphi"'] \\
C(s, \mathit{U}d) \arrow[r, "\theta"] & D(\mathit{F}s, d)
\end{tikzcd}$$

As usual, the condition of naturality for arrow φ (and similarly for θ) means that the following diagrams also commute for every arrow *m*:s→s′ in C and *i*:d→d′ in D:

$$\begin{tikzcd}
D(\mathit{F}s, d) \arrow[r, "\varphi"] \arrow[d, "D(\mathit{Fm}, d)"'] & C(s, \mathit{U}d) \arrow[d, "C(m, \mathit{U}d)"] \\
D(\mathit{F}s', d) \arrow[r, "\varphi"] & C(s', \mathit{U}d)
\end{tikzcd}
\qquad
\begin{tikzcd}
D(\mathit{F}s, d) \arrow[r, "\varphi"] \arrow[d, "D(\mathit{F}s, i)"'] & C(s, \mathit{U}d) \arrow[d, "C(s, \mathit{U}i)"] \\
D(\mathit{F}s, d') \arrow[r, "\varphi"] & C(s, \mathit{U}d')
\end{tikzcd}$$

Informally, this situation might be loosely visualized as a mapping between a set of models defined in C(s, *U*d) and a set of interpretations defined in D(*F*s, d) as the following diagram suggests:



Based on this definition the following special cases can be defined:

- If there is an object s in C or d in D with $\tau_C=1_{C(s,Ud)}$ or $\tau_D=1_{D(Fs,d)}$ - where $1_{C(s,Ud)}$ and $1_{D(Fs,d)}$ are identity arrows over C(s, *U*d) and D(*F*s,d) respectively - then for the objects s in C and d in D there is a universal arrow η_s:s→***UF***s and ε_d:***FU***d→d respectively.
- If for every object s in C or d in D the arrows $\tau_C=1_{C(s,\ Ud)}$ or $\tau_D=1_{D(Fs,\ d)}$ - that is, when the arrows τ_C and τ_D are identity arrows for every object in C or D - then θ and φ form a bijection that is natural in s and d (hence the tuple <***F***,***U***,φ,θ> is an adjunction).

6.2.2.4 Properties of weak adjunction

The notion of weak adjunction between two categories C and D is differentiated from the 'strong' notion of adjunction in two ways. First, the concept of weak adjunction implies that universal properties are applicable *for some* object s in C or d in D, whereas the concept of strong adjunction implies that universal properties are applicable *for every object* s in C and d in D. Second, the concept of weak adjunction implies that the very meaning of universal construction is also weakened, in the sense that the arrows *i* and *m* are not required to be unique. Let us elaborate a bit more the definition of weak adjunction in order to understand these properties more precisely.

The construction of weak adjunction between two categories C and D and two functors F:C→D and U:D→C implies the property that for some object s in C and some object d in D there is a weak universal arrow η_S:s→*UF*s for s and a weak universal arrow ε_d:*FU*d→d for d in the following sense: For some object s and arrow *m*:s→*U*d in C there is an arrow *i*:*F*s→d in D such that the following natural transformation is defined $\tau_C: m \rightarrow U(i) \circ \eta_S$.

$$\begin{tikzcd}
UFs & s \arrow[l, "\eta_s"'] \arrow[d, "m"] \arrow[dl, phantom, "\overset{\tau_C}{\Leftarrow}"] & Fs \arrow[d, "i"'] \\
 & Ud \arrow[from=1-1, "U(i)"'] & d
\end{tikzcd}$$

The dual property is applied for the object d in D. For some object d and arrow *i*:*F*s→d in D there is an arrow *m*:s→*U*d such as the following natural transformation is defined $\tau_D: \varepsilon_d \circ F(m) \rightarrow i$ (in the right diagram) :

$$\begin{tikzcd}
UFs & s \arrow[l, "\eta_s"'] \arrow[d, "m"] & Fs \arrow[d, "i"'] \arrow[dr, "F(m)"] \arrow[d, phantom, shift left=3ex, "\overset{\tau_D}{\Rightarrow}"] & \\
 & Ud \arrow[from=1-1, "U(i)"'] & d & FUd \arrow[l, "\varepsilon_d"]
\end{tikzcd}$$

6.2.2.5 Weak theories: constructions of weak adjunctions

The basic notion of universality implies the idea that there is an optimum state: given a situation described by arrows of the form $m= U(\_)\circ\eta_S$ in C there is a *unique* arrow $i{:}F(\mathrm{s})\to \mathrm{d}$ in D that satisfies the condition $m= U(\_)\circ\eta_S$ (and equivalently for an arrow $i=\varepsilon_d\circ F(\_)$ in D there is a unique arrow $m{:}\mathrm{s}\to U\mathrm{d}$). The basic notion of adjunction arises when such an optimum state exists for every object in C (and equivalently for every object in D). The family of arrows that satisfy these conditions are unique up to isomorphism.

In a weak adjunction, solutions are distributed and ambiguous. The construction of weak universality induces a *family* of arrows that participate in the same natural transformation $\tau_D{:}\varepsilon_d\circ F(m)\to i$ or $\tau_C{:}m\to U(i)\circ\eta_S$. In this sense, a congruence class of arrows $[m]{:}\mathrm{s}\to U\mathrm{d}$ and $[i]{:}F\mathrm{s}\to \mathrm{d}$ can be defined on the basis of a weak adjunction $<F, U, \varphi, \theta>$. More specifically, a family of arrows $[m]$ are all arrows m in C that construct the natural transformation $\tau_D{:}\varepsilon_d\circ F(m)\to I$; and the family of arrows $[i]$ are all the arrows i in D that construct the natural transformation $\tau_C{:}m\to \mathrm{U}(i)\circ\eta_S$. The congruent class of arrows allows us to define a quotient category C/τ_D: the objects in the category C/τ_D are the objects of C and the arrows are the congruence class of arrows $[m]$ in C (an equivalent construction is possible for the category C/τ_C). Quotient categories can be seen as an instantiation of the idea of weak theories; that is theories generated by the natural transformations $\tau_D{:}\varepsilon_d\circ F(m)\to i$ and $\tau_C{:}m\to U(i)\circ\eta_S$. More generally a weak theory is defined as follows:

> **Definition (weak theory)**: A weak theory is a category *Th*s that is constructed by a sketch s in C and a functor *Th* such that a weak adjunction $<Th,U,\varphi,\theta,\tau_C,\tau_D>$ is defined; i.e. the relation φ and θ between interpretations i of theories *Th*s in D and models m of a sketch s in C are determined by the following diagram:

$$\begin{tikzcd}
C(s, Ud) \arrow[r, "\theta"] \arrow[d, "\tau_C"'] & D(Ths, d) \arrow[d, "\tau_D"] \arrow[dl, "\varphi"'] \\
C(s, Ud) \arrow[r, "\theta"] & D(Ths, d)
\end{tikzcd}$$

'*Well formed*' and '*random*' theories can be thought as special cases of weak theories in the following sense:

- A well formed theory is constructed when for every object s in C or d in D the arrows $\tau_C=1_{C(s,\ Ud)}$ and $\tau_D=1_{D(Ths,d)}$ - that is, when the arrows τ_C and τ_D are identity arrows for every object in C or D. In this case the arrows θ and φ form a bijection that is natural in s and d, and the tuple $<Th,U,\varphi,\theta>$ is an adjunction.
- A random theory is constructed when there is no object s in C or d in D such that $\tau_C=1_{C(s,\ Ud)}$ or $\tau_D=1_{D(Ths,d)}$. In this case the adjunction $<Th,U,\varphi,\theta>$ is broken.

Weak theories are ambiguous in the sense that any interpretation i of a theory *Ths* in D has many models [*m*] in C, and any model *m* in C has many interpretations [*i*] in D. Moreover, given $\tau_C\neq1_{C(s,\ Ud)}$ or $\tau_D\neq1_{D(Ths,d)}$, the generated interpretations of a theory do not have an optimum model and vice versa. Weak theories are distributed in the sense that the complementarity of description and interpretation is fragmented. The complementary relations are relative to local conditions (situation).

6.2.2.6 Phase transitions (to universality)

The notion of weak adjunction can now be used to build a mathematical construction that describes a phase transition in the behaviour of a (mathematical) universe U. In this context, a phase transition is perceived as a transformation of the properties and degree of complementarity between descriptions (theories) and their interpretations (models). Such a transformation in the behaviour of the universe U can be more formally defined as a transformation between natural transformations:

$$\tau_D:\varepsilon_d\circ F(m)\rightarrow i \text{ to } \tau'_D:\varepsilon_{d'}\circ F(m')\rightarrow i'$$

and

$$\tau_C:m\rightarrow U(i)\circ\eta_s \text{ to } \tau'_C:m'\rightarrow U(i')\circ\eta_{s'}$$

More specifically in the last section, it was shown that the construction of weak universality generates different families of arrows $[m]$ and $[i]$, where each family of arrows is determined by a natural transformation $\tau_D:\varepsilon_d\circ F(m)\rightarrow i$ or $\tau_C:m\rightarrow U(i)\circ\eta_s$. On this basis, different quotient categories D/τ_D and C/τ_C can be defined in U. Each quotient category generated by a natural transformation τ_D or τ_C can be thought to represent a different mathematical system, with different behavioural patterns. In this sense, a phase transition is determined by the transformation of the natural transformations $\tau_D:\varepsilon_d\circ F(m)\rightarrow i$ or $\tau_C:m\rightarrow U(i)\circ\eta_s$ that leads to new types of categories in U.

Definition (phase transition): given the arrows T_s:s→s′ and T_d:d→d′ shown below,

$$\begin{tikzcd}
s \arrow[r, "T_s"] \arrow[d, "m"'] & s' \arrow[d, "m'"'] \\
Ud \arrow[r, "UT_d"'] & Ud'
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
Ths \arrow[r, "ThT_s"] \arrow[d, "i"'] & Ths' \arrow[d, "i'"'] \\
d \arrow[r, "T_d"'] & d'
\end{tikzcd}$$

phase transition is defined by the transformations

$T_C=C(T_s,UT_d):C(s,Ud)\rightarrow C(s',Ud')$ and

$T_D=D(ThT_s,T_d):D(Ths,d)\rightarrow D(Ths',d')$

that make the following diagram commute[6]:

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[6] Broken arrows here are only meant to make the diagram more readable.

$$\begin{tikzcd}
D(\mathit{Th}s, d) \arrow[rrr, dashed, "T_D"] \arrow[ddd, "\tau_D"'] \arrow[ddr, "\varphi"] & & & D(\mathit{Th}s', d') \arrow[ddd, "\tau_D'"] \arrow[ddl, "\varphi'"'] \\
& C(s, Ud) \arrow[ul, "\theta"] \arrow[r, dashed, "T_C"] \arrow[d, "\tau_C"] & C(s', Ud') \arrow[ur, "\theta'"'] \arrow[d, "\tau_C'"] & \\
& C(s, Ud) \arrow[dl, "\theta"'] \arrow[r, dashed, "T_C"] & C(s', Ud') \arrow[dr, "\theta'"] & \\
D(\mathit{Th}s, d) \arrow[rrr, dashed, "T_D"] & & & D(\mathit{Th}s', d')
\end{tikzcd}$$

The transformations T_c and T_D might be alternatively defined as functors: T_c is a functor $T_c{:}C/\tau_C \to C/\tau_C'$ from the quotient category C/τ_C to the quotient category C/τ_C'. Similarly, T_D is a functor $T_D{:}D/\tau_D \to D/\tau_D'$ from the quotient category D/τ_D to the quotient category D/τ_D'.

An alternative definition of the concept of phase transition can be formulated by setting the entities $D(\mathit{Th}s,d)$ and $C(s,Ud)$ as hom functors $D(\mathit{Th}s,d){:}C^{op}xD \to \mathbf{Set}$ and $C(s,Ud){:}C^{op}xD \to \mathbf{Set}$, and the arrows φ and θ as natural transformations $\varphi{:}D(\mathit{Th}s,d) \to C(s,Ud)$ and $\theta{:}C(s,Ud) \to D(\mathit{Th}s,d)$. So, the natural transformation φ (and similarly for θ) implies that the following diagrams must commute:

$$\begin{tikzcd}
D(\mathit{Th}s, d) \arrow[r, "\varphi"] \arrow[d, "D(Fm{,}d)"'] & C(s, Ud) \arrow[d, "C(m{,}Ud)"] \\
D(\mathit{Th}s', d) \arrow[r, "\varphi"] & C(s', Ud)
\end{tikzcd}
\qquad
\begin{tikzcd}
D(\mathit{Th}s, d) \arrow[r, "\varphi"] \arrow[d, "D(\mathit{Th}s{,}i)"'] & C(s, Ud) \arrow[d, "C(s{,}Ui)"] \\
D(\mathit{Th}s, d') \arrow[r, "\varphi"] & C(s, Ud')
\end{tikzcd}$$

Given two subcategories C_i and C_j of the category C, and two subcategories D_i and D_j of the category D, the notion of phase transition alludes to the mappings $T_C{:}C_i(\_,U\_) \to C_j(\_,U\_)$ and $T_D{:}D_i(\mathit{Th}\_,\_) \to D_j(\mathit{Th}\_,\_)$ that transform the natural transformations $\varphi_i{:}D_i(\mathit{Th}s,d) \to C_i(s,Ud)$ and $\theta_i{:}C_i(s,Ud) \to D_i(\mathit{Th}s,d)$ into the natural transformations $\varphi_j{:}D_j(\mathit{Th}s',d') \to C_j(s',Ud')$ and $\theta_i{:}C_j(s',Ud') \to D_j(\mathit{Th}s',d')$ such that the following diagrams commute:

$$\begin{tikzcd}
D_i(\mathit{Ths},d) \arrow[r, "\varphi_i"] \arrow[d, "T_D"'] & C_i(s, Ud) \arrow[d, "T_C"] \\
D_j(\mathit{Ths}',d') \arrow[r, "\varphi_j"] & C_j(s',Ud')
\end{tikzcd}
\qquad
\begin{tikzcd}
D(\mathit{Ths},d) \arrow[d, "T_D"'] & C(s, Ud) \arrow[l, "\theta_i"'] \arrow[d, "T_C"] \\
D(\mathit{Ths}',d') & C(s',Ud') \arrow[l, "\theta_j"']
\end{tikzcd}$$

Informally, this construction can be also more concisely illustrated by setting the arrows T_c, T_D and T as functors such that the following diagram commutes:

$$\begin{tikzcd}
C_i^{op} x D_i \arrow[r, bend left, "D_i(\mathit{Ths},d)"{name=A}] \arrow[r, bend right, "C_i(s,\ Ud)"'{name=B}] \arrow[d, "<T_c\,,T_D>"'] \arrow[from=A, to=B, phantom, "\varphi_i \Downarrow"] & \mathbf{Set} \arrow[d, "T"] \\
C_j^{op} x D_j \arrow[r, bend left, "D_j(\mathit{Ths}',d')"{name=C}] \arrow[r, bend right, "C_j(s',Ud')"'{name=E}] \arrow[from=C, to=E, phantom, "\varphi_j \Downarrow"] & \mathbf{Set}
\end{tikzcd}$$

Going back to the first definition of phase transition, a special case in this construction is defined when $\tau_D' = 1_{D(Th\_,\ \_)}$. In this case, the diagram specifies a special phase transition: *a phase transition to universality*. A phase transition to universality can be understood as a *critical transition*, and the universal construction <***Th***,***U***,φ,θ> as a *critical state*. The notion of criticality is applied here in the sense that the relation (distance) between any pair of phase transitions T_C:C(s,*U*d)→C(s$_u$,*U*d$_U$) and T'_C:C(s',*U*d')→C(s$_u$,*U*d$_U$) to universality can be factored by a transformation *d*:C(s,*U*d)→C(s',*U*d').

Definition (relative distance to universality): A model defined in C(s,*U*d) is far from universality in relation to a model defined in C(s',*U*d') if there is an arrow *d*:C(s,*U*d)→C(s',*U*d') that makes the following diagram commute (i.e. $T'_C = d \circ T_C$):

$$\begin{tikzcd}
C(s,\ Ud) \arrow[d, "d"'] \arrow[dr, "T'_C"] & C(s_u,\ Ud_U) \arrow[r, "\theta"] \arrow[d, "1_{C(s,\ Ud)}"] & D(\mathit{Ths}_u,d_U) \arrow[dl, "\varphi"'] \arrow[d, "1_{D(Ths,\ d)}"] \\
C(s',\ Ud') \arrow[r, "T_C"] & C(s_u,\ Ud_U) \arrow[r, "\theta"] & D(\mathit{Ths}_u,d_U)
\end{tikzcd}$$

6.3 SUMMARY AND CONLUSIONS

This chapter was concerned with the development of a mathematical structure for expressing the meaning and qualities of organization (reviewed in Chapter 5). In particular, the chapter exemplified the basic meaning and complexity of organization as part of a theory of sketches. The first part of the study introduced the notion of sketch and weak sketch, explaining its origins and relation to design. Then, the category theoretic concept of sketch was proposed as a mathematical interpretation of a conditional entity (a source) that has the capacity to generate, preserve or constrain a distinct whole. In particular, the natural bijection between models of a sketch and interpretations of a theory - namely the adjunction between a theory functor and its underlying functor- offered a precise interpretation of the relation between a sketch S, a theory of a sketch *Th*S, and its models. This investigation led to the suggestion that the notion of adjunction can be used to express different types of organization: an *ordered* organization is identified with a situation where there is an adjunction: *all* properties of *every* possible model are known and deducible from the theory (there is no uncertainty in the system). A *random* organization is identified with a situation where there is no adjunction. Finally, a *complex* organization is identified with a situation where there is a "weak adjunction".

This created the need to introduce a generalized type of sketch that has the expressive power to reflect different levels of organization as a function of the quality (the weakness) of the adjunction between models specified by a sketch, and interpretations of the generated theories. The second part developed the mathematical interpretation of the notion of "weak sketch", first, by laying out the basic elements (graphs, diagrams, cones and co-cones) that specify this construction, and second, by elaborating the properties of a "weak theory".

Weak theories are theories that are characterized by ambiguity and distribution (instead of uniqueness and universality). Weak theories are ambiguous in the sense that any interpretation of a theory *Th*s has many models, and any model m has many interpretations. Moreover, the generated interpretations of a theory do not have an optimum model and vice versa. Weak theories are distributed in the sense that the complementarity of description and interpretation (i.e. the properties of weak universality) is fragmented to different objects. In this sense, the complementary relations (weak universal properties) are relative to local conditions.

The contribution of the proposed mathematical construction is that it enables us to distinguish different levels of organizational complexity, but it also provides the basic tools for developing an organizational-level theory of design. We discussed in the previous chapter that a kind of meta-level theory was needed in order to express situations where organization constitutes an attracting or critical state, and the complementarity between subject and object is distributed. The notions of weak sketch, weak adjunction, and weak theory allow us to construct such a notion of organization (through a definition of phase transition to complementarity) in precise mathematical terms. In the next chapter, the proposed definitions and mathematical constructions will be explicitly used for the purpose of identifying the meaning and properties of design as an organizational property.

Chapter 7

AN ORGANIZATIONAL-LEVEL THEORY OF DESIGN

Chapters 5 and 6 introduced the fundamental mathematical language and concepts necessary in order not only to understand and study organization and complexity, but also to construct organizational level descriptions. This chapter is concerned with describing and explaining design at an organizational level using these concepts. For this purpose, the first part of the chapter is dedicated to the mathematical definition of the task and capacity to design by explicating the type of theories, models and sketches involved in the design process. The second part of the chapter discusses limitations in current paradigms for the study and representation of design, thus further motivating the need for the proposed organizational-level theory. The last part of the chapter identifies and articulates the fundamental organizational conditions that can explain the capacity to carry out design tasks.

7.1 DESIGN AS A UNIVERSAL PROPERTY OF ORGANIZATION

Before laying out the basic definitions that are the kernel of this chapter, it is useful to recollect some of the arguments already made in this thesis and re-draw the context of this investigation. In Chapters 2 and 4 different approaches to the understanding of design were discussed. This discussion led to the identification of some principal research questions: What is the basis or locus of the capacity to recognise and carry out design tasks? What are the fundamental structures or processes that underlie the generation of design knowledge, processes and objects? Or, more generally, what are the organizational properties that enable design capacities to emerge?

It was argued that devising an organizational level description of design provides a way to uniquely characterise design as a universal, natural phenomenon of organization. The mathematical framework developed in Chapters 5 and 6 can now be used to offer a precise mathematical interpretation of the task and capacity to design at an organizational level of description. Here, design Intentionality is discussed in order to make the link between design and the proposed theory of organization. In the first part of this section basic notions and problems related with Intentionality are introduced. In the second part, the concept, task and capacity to design is mathematically defined in terms of the type of theories, models and sketches involved in the design process. In the third part, these definitions are examined in relation to other fundamental concepts such as computation, evolution and control. The comparison aims to determine the uniqueness and universality of the design paradigm in the realm of complexity research.

7.1.1 Design Intentionality

The premise behind this treatment is that design arises in response to a certain 'problematic situation': when there is a desire, need or idea to construct a change in a certain environment, but the precise *means* and *ends* of this construction are not given. Although, this premise is commonly held in design research (see for instance Archer, 1965; Mitchell, 1990; Smithers, 2002), it can nevertheless take a number of different interpretations.

The above situation is often interpreted in relation to an 'information processing system' (i.e. a cognitive designer) that faces a special type of problem solving task (e.g. Goel and Pirolli, 1989). According to this perspective, a problematic situation arises with a problem statement or 'task environment' that an information processing system needs to address. The hypothesis implies a distinction between an external environment that sets the problem, and an internal environment that represents the task environment (*the problem space*). *The problem space* is then a representation of a set of possible states, a set of 'legal' operations, as well as an evaluation function or stopping criteria for the problem solving task (e.g. Ernst and Newell, 1969; Newell and Simon, 1972). The peculiarity of the design task is that *the means* (i.e. the representation of the problem space and the possible operations over the problem space), as well as the *ends* (i.e. the evaluation function or the stopping criteria) are not given in the task environment but are part of the design process (Simon, 1973; Goel and Pirolli, 1989, 1992).

Another interpretation postulates that 'a design situation' arises from the reflective activity of a designer over an objective reality - i.e. a professional, social or operational environment that includes humans, tools, external representations, artefacts etc (Schön, 1983). According to this paradigm, design is not a characteristic of the task environment (i.e. of a problem statement), but a characteristic of the coupling or interplay between a subject (i.e. a designer) and an objective reality (i.e. the environment). A design situation then arises not as a problem, but as a mental state of a 'reflective' agent. This view essentially conveys the idea that cognitive functions are not simply the product of an information processing system (an isolated mind), but are rather formed from the coupling between the mind, the body and its environment. In cognitive science, this gave rise to approaches like embodied cognition, where the emphasis is on the contribution of the physical body (e.g. Varela, Thompson and Rosch, 1991; Clark, 1997), and situated or ecological cognition, where the emphasis is on the role of the environment (e.g. Gibson, 1979; Suchman, 1987; Clancey, 1997). In design research in particular, a number of different approaches have been proposed: for example, design has been approached as a hermeneutic act (Snodgrass and Coyne, 2006); as a constructive act of a situated cognitive agent (e.g. Gero, 1998a); or as a self-organization process that involves the interaction between internal and external representations (Portugali and Casakin, 2002). A more abstract approach sees design as a co-evolutionary process between a problem and a solution space (e.g. Maher et al, 1996; Dorst and Cross, 2001). In this latter example, the ontological status of the problem and solution spaces is not specified explicitly. For instance, the distinction between a problem and a solution space can be defined as a distinction between a task environment and the mental state of an agent, or as a distinction between two separate mental states that have a distinct logical status.

In the following, the premise that design arises in response to a particular situation is approached in relation to the capacity of an 'organism' to have 'Intentionality' or 'Intentional states'. Although the term 'Intentionality' has its origins in the writings of medieval Scholastic philosophers, the contemporary 'technical' meaning of the term is reintroduced in philosophy of mind by Brentano in 1874 (republished in 1995). According to Brentano, Intentionality aims to describe the very essence of the mind: the capacity to represent or reflect objects or states of affairs in the world - either existing or non-existing. The capacity of a mind to represent an object or state of affairs in the world is 'Intentional' in the sense that mental states are 'semantically evaluable' and therefore 'refer to something', or they are 'about something' (Fodor, 1995).

In philosophy of mind it is generally assumed that there are two archetypical, logically distinct 'Intentional states'. The first refers to the capacity of a mind to hold *beliefs* about 'what the properties of a certain reality are', and the second refers to the capacity of mind to hold *desires* about 'what the properties of a certain reality ought to be'. Although it is analytically convenient to distinguish different types of 'Intentional states', 'Intentionality' is generally perceived as a holistic or emergent property of the mind that is derived by a network of causally related Intentional states - i.e. a network of beliefs, desires or intentions. In the following, the word 'Intentionality' will be written with a capital 'I' in order to be distinguished from the term 'intention' which is one of the possible Intentional states. Based on these terms, and in accordance with our premise about the nature of design, a design situation can be seen as an Intentional state that 'emerges' out of *inconsistencies* between beliefs held about the past, current, and future states of the world, and desires regarding the state of the world. However, in order to explicate this general intuition it is important to introduce a common theoretical background on Intentionality.

For this purpose, some general issues about the relation between Intentionality, mind, semantics and the brain will be discussed. The discussion starts with Brentano's core thesis that Intentionality is the mark of the mind. According to his position, Intentionality is the necessary and sufficient condition of mental states and therefore it is what distinguishes mental from physical phenomena. In contemporary philosophy of mind this claim is challenged in two ways. First, it is argued that there are mental states that are not Intentional (i.e. Intentionality is not a necessary condition of the mind), and second that there are Intentional states that are not mental states (i.e. Intentionality is not a sufficient condition of the mind). These objections naturally lead to ontological questions: do Intentional states correspond to physical processes or events? Are these physical processes or events in direct correspondence with the neuro-chemical processes and events occurring in the brain? Clarifying these objections and other core issues about the ontology of Intentionality is an opportunity to establish a common understanding on the proposed Intentional interpretation of 'design situations'.

7.1.1.1 The scope and structure of Intentionality

A first objection to Brentano's thesis is that mental states are not always Intentional. There are mental states that are not 'semantically evaluable' and therefore they are not directed to an object or state of affairs in the world. Typical examples are pain, general anxiety or elation. Although these are mental states, they are not reflections of a specific object or state of affairs in the word. In contrast, beliefs, desires, hopes or intentions are 'Intentional states' in the sense that they refer to something; the belief that 'it is raining' or the desire 'to stay dry' reflect a specific state of affairs in the world. In this description of Intentionality, there is a distinction between a 'mental state' and the objects at which it is directed or is about (Searle, 1983). More specifically, the Intentionality of an organism is determined by two components: the *'representational content'* or *'Intentional object'* that describes the properties of an object or state of affairs in the world (e.g. 'it is raining' or 'stay dry'), and the *'attitude'* or *'psychological mode'* that determines the type of Intentional state (e.g. belief, desire or intention). This structure is often denoted as $F(s)$; where F is the attitude and s is the representational content. Alternatively, Fodor (1975, 1987) defines the attitude of an Intentional state as a binary relation F between an organism O and its mental states/mental representations s; that is $F(O,s)$. In this notation, s is a token, a syntactical physical entity, associated with an Intentional object (e.g. 'it is raining').

According to Searle (1983), an attitude *F* (e.g. beliefs or desires) expresses the underlying assumptions regarding the relative independence of the world from the mind. These assumptions determine a *'direction of fit'*. More specifically, an attitude *F* expresses 'a direction of fit' in the sense that any mismatch between the representational content (e.g. 'it is raining', or 'stay dry') and the world is resolved by either changing the mind in relation to the world, or changing the world in relation to the mind. Beliefs and desires are thus two archetypical Intentional states that express two opposite 'directions of fit'. A *'mind-to-world'* direction of fit assumes that an Intentional object represents an independently existing world and therefore any mismatch between representation and the world must be followed by an adaptation of the representational content. A belief is either true or false in the sense that the correspondence between representational content and the world is evaluated against an independently existing world. A *'world-to-mind'* direction of fit assumes that an intentional object exists independently from the world and any mismatch between the representational content and the world must be followed by an adaptation of the world. A desire cannot be said to be true or false. A desire is satisfied or fulfilled in the sense that the correspondence between representational content and the world is realized when there is a change in the world in relation to the mind. In both cases, the representational content s of an intentional state determines the *'conditions of satisfaction'* of a certain Intentional state. Based on these terms, Intentional states are therefore defined as mental states that have certain conditions of satisfaction s with a certain 'direction of fit' *F* in relation to an object or state of affairs in the world.

7.1.1.2 The semantic content of Intentionality

The second objection to Brentano's core thesis is that although Intentionality is indeed a property of the mind, it is also a property of non-mental or physical objects. For instance, drawings, artefacts, sentences, or more generally expressions in a language are 'semantically evaluable' (they refer to an object or state of affairs in the world) and therefore they are Intentional constructions. But, drawings, artefacts or sentences are not mental states. The confusion is even greater when natural objects are perceived as 'natural signs' that hold information about other events (e.g. Millikan 2004). For instance, clouds can be perceived as natural signs that represent the possibility of rain. In this sense, both physical and mental states are distinguished for their semantic capacities and therefore for their Intentionality.

As a first response to these problems, a distinction is often made between 'intrinsic' and 'derived' Intentionality (e.g. Searle, 1983). Mental phenomena are distinguished for their 'intrinsic Intentionality', while physical phenomena (such as language) are distinguished for their 'derived Intentionality'. Derived Intentionality is a type of Intentionality that is imposed in a physical object by the 'Intentional' states of a mind. According to this view, the formation of semantic relations is part of a theory of mind and a special form of Intentionality (Searle, 1983; Fodor, 1987). For Fodor (1987), Intentionality is ultimately a linguistic problem which may involve an 'inner' language -'a language of thought'- but also any other 'outer' language. He starts with the assumption that mental states (i.e. the physical structure of the brain and its representational content) are expressed by a language and a symbol processing system. From this perspective, the distinction between the Intentionality of the brain and the Intentionality of any symbol system is reduced to the same problem. As he put it (Fodor, 1987: vi): 'It would be therefore no great surprise if the theory of mind and theory of symbols were some day to converge'.

For Searle (1983) the formation of semantics involves two layers of Intentionality. First, there is an Intentional state to be expressed by an object (e.g. the desire to stay dry may be expressed by a drawing or sentence) and second, there is an 'intention' to express that Intentional state. Meaning is therefore acquired because of the intention of the mind to 'associate' an Intentional state with a non-mental or physical expression. This association is realized because the intention to express an Intentional state holds the same conditions of satisfaction with the expressed Intentional state.

To explicate how an Intentional state leads to the formation of semantic relations we can turn into the notions of theory, model and sketch. The universal property that determines the relation between theories and models leas to an understanding of Intentional states at two layers. First, there is an Intentional state that is intended to be expressed in a certain language D. This Intentional state is explicated by the model $M{:}s \rightarrow Ud$ and has certain conditions of satisfaction s. This state essentially corresponds to the first layer of Searle's Intentionality which he calls 'sincerity condition' (Searle, 1983: 160-179).

Second, there is an Intentional state (in particular an *intention*) that is realized by the functor $F{:}S \rightarrow D$ between two categories S and D. S corresponds to the subjective reality and D to the objective reality. The functor F explicates the intention to express something in a language D given certain conditions of satisfaction expressed by the sketch s in S. This essentially corresponds to the second layer of Intentionality which Searle calls 'meaning intentions' (Searle, 1983: 160-179). A representation of the semantic properties of Intentional states is given in Figure 41.

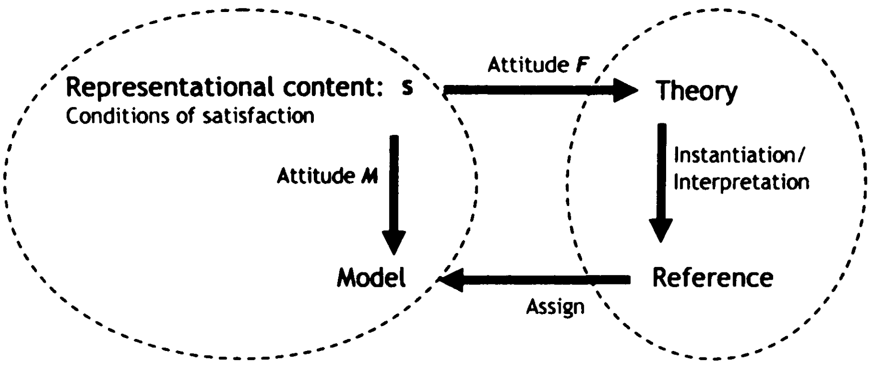

Figure 41: A schematic representation of the semantic properties of Intentional states. The category theoretic notion of sketch s is the representational content (or condition of satisfaction) of an Intentional state. The category generated by a sketch is the inner or outer language (a theory) used in order to express a specific Intentional state M.

7.1.1.3 Realizations of Intentionality

Up to this point, the term Intentionality has been discussed as a special mental capacity; a capacity that is inextricably related with the formation of semantic relations. But, what is the locus, body, or universe of Intentionality? How is Intentionality realized? Is the brain the only physical realization of an Intentional mind? Do social entities have Intentionality? In order to respond to this question, the 'problem' of Intentionality must be placed in relation to the 'mind-body problem'.

The mind-body problem concerns the relationship between mental states or processes and physical events. In contemporary philosophy and science, it is commonly held that mental phenomena are somehow linked to physical phenomena, although the exact nature of this relation is not clear. There are four main approaches regarding this relation (e.g. Harman, 1989; Chalmers, 2002).

One approach, *behaviourism*, associates mental states and processes with the 'behavioural dispositions' or behavioural tendencies generated by the underlying physical processes of an organism. According to this view, the mind is a special aspect of the behaviour of a physical system (e.g. Ryle 1949, Dennett 1987).

The second approach, *identity theory*, associates mental states and processes with physical states and processes. More specifically, identity theory claims that mental states are essentially physical states of the brain of an organism (e.g. Place, 1956; Smart, 1959).

The third approach, *functionalist theory*, attributes mental states or processes to physical states or processes whose behavioural dispositions are distinguished for their functional role. A predominant interpretation of this approach postulates that mental states are functions of a computational machine (e.g. Putnam, 1973). The view of mental states as functional entities of a computational machine helped explain how the same mental states might take multiple physical realizations. So, for instance, animals and humans may both have a similar Intentional state, but different neurological structures.

Finally, a number of alternative studies on the relation between mind and body have brought to the fore the *emergentist* hypothesis that mental states are higher-order/emergent properties of lower level (typically physical) states and process (e.g. for a review see McLaughlin, 1992, 1997; Horgan, 1993). According to this approach, Intentional states are emergent qualities that cannot be deduced from the principles of the components found at a lower level of abstraction. This relation (between higher order emergent properties and lower level states and processes) is specified in different ways: as ontological, logical or epistemological relation (see Chapter 5; Alexiou, 2007). The emergentist view of the mind has been closely linked with the concept of 'supervenience', first introduced in the context of philosophy of mind by Davidson (1970). Supervenience is a dependence relationship between a set of properties A and a set of properties B that describe a certain world: the set of properties A *supervenes* the set of proprieties B if and only if it holds that *'whenever the set of properties A cannot distinguish two different worlds, then that set of properties B cannot distinguish these two worlds either'*. According to this view, the mind supervenes the physical states or processes of a brain: i.e. although, the same mental states may have a different physical realization, identical physical states or processes specify identical mental phenomena.

This short review shows that there is a large body of studies that support the conventional wisdom that the mind, and Intentional states in particular, can have many different realizations (e.g. Kim, 1992). Intentionality can be manifested in different physical mediums, such as sounds or drawings (although one can always argue that this is a derived Intentionality as discussed above). Human or animal activities produce artefacts that are also Intentional. Artefacts are 'Intentional constructions' in the sense that they are physical realizations of the beliefs and desires of an Intentional mind. However, artefacts are not always the product of an individual organism. Social constructions such as ant nests, human cities or nations are artefacts that express the beliefs and desires of a society. These artefacts are obviously the products of some Intentional mind although this Intentionality cannot be attributed to a specific brain physiology.

The possibility of collectively attributing Intentionality to a physical entity entails an Intentional state that cannot be broken down to individual Intentional states[7].

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[7] In the same sense that Individual Intentionality is often studied as a holistic state that cannot be

It is a rather new form of Intentionality that arises as the aggregate effect of Intentional organisms. In this sense, Intentionality can be said to be embodied in certain organizational structures and processes that are not necessarily realized in the physical structure of a brain. For the purpose of this study, the term Intentionality or Intentional state will be used for any 'organism' whose physical or social realization has the capacity to hold representations of an 'objective' reality. In this sense, this distinction between 'representation' and the 'representing object' marks the distinction between a subjective and an objective reality. The ontological status of these realities is not specified, but the logical properties of this coupling will be the main focus of the next sections.

7.1.2 Organizational level description of design Intentionality

The concepts of weak theory and model can be now used in order to explicate the view of design as an Intentional state at an organizational level of description. First let us re-define the notion of 'design situation' outlined at the beginning of this section. *A problematic (or design) situation arises when certain desires about the world generate expressions of theories or models that do not follow the correspondence between theories and models as this is established by one's belief system.* In this context, a sketch S is the conditions of satisfaction of a subjective reality (i.e. an Intentional state) that produces an - often incomplete, inconsistent or unsatisfied - expression of an objective reality.

However, a sketch of a design situation s_D is an Intentional state of a particular kind. It is an Intentional state with a 'two-directional freedom of fit' (recall Searle's notion of 'direction of fit'). On the one hand, a sketch of a design situation expresses the conditions of satisfaction of a desire. A desire is *fulfilled* only when there is a theory of an objective reality that satisfies its models. On the other hand, a sketch s_D of a design situation expresses the conditions of satisfaction of a belief. A belief becomes *true (or false)* only when there is a model that validates the theory. Namely, a sketch s_D of a design situation is a particular type of Intentional state that arises when there is mismatch between

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broken down to individual attitudes (beliefs, desires, intentions etc).

subjective and objective reality, and both the subjective and objective realities need to be adapted. So, a ***problematic or design situation*** *arises when there is an underlying sketch* s_D *that generates a mismatch between an objective and a subjective reality that can only be satisfied (i.e. fulfilled and validated) with the formation of new theories as well as new models.* We can think of a mismatch as inconsistency (not only truth statements are generated), and/or incompleteness (not all the truth statements are generated), which requires changes both in terms of the belief and the desire system of an organism. To put it differently, a design situation arises when the following diagram in Figure 42 **does not commute**.

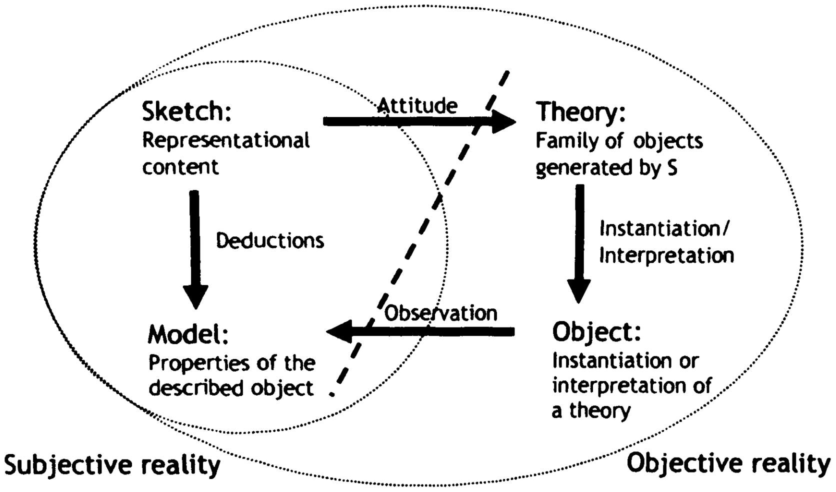


Figure 42: A sketch as an 'Intentional state'. A design situation arises when there is a 'two-directional freedom of fit' requiring both the adaptation of subjective and objective reality.

Let us consider some examples of how a design situation may arise at different levels of abstraction.

Example 1: A typical example of this situation can be described in terms of the design process or practice. The design process is often assumed to start with the expression of a design brief. Take for instance a design brief that expresses a 'desire for a building configuration that exploits natural resources in order to create the viable environmental conditions during winter and summer'. This expression is a sketch of an objective reality: it is an 'Intentional state' that represents a reality of naturally ventilated/heated buildings. A problematic situation arises when the available theory represented by the Intentional state s_D deduces that the desired properties (i.e. certain environmental conditions) cannot be achieved with known models of building configurations; and/or the known models of building configurations do not have the desired property of being able to exploit natural resources. Within this situation, the development of a theory of building configurations together with the development of models that would validate the theory is paramount. Consider Felix Trombe's design of a solar house as a motivating example (Ashley, 1983). One can imagine that his design involved the creation of a new theory represented by s_D: *in order to naturally warm the air in a building, one should find a way to circulate the cold air over a naturally warmed surface*. It also involved the development of a model: *a double wall whose internal surface is built from a material that can act as a thermal mass and the external is made of glass*. The formation of the double wall induces certain properties (a belief theory about the thermal behaviour of the wall). According to this theory *the sunlight passes through the glazed surface and is absorbed by the internal wall. Heat can then be transferred by conduction, or by channelling the hot air through heat-distributing vents*. The essential feature of design is this capacity to develop a model (the double wall) in anticipation of certain properties (the thermal behaviour of the wall) that correspond to the formed theory (heating through circulation of cold air over a naturally warmed surface).

Example 2: Another example of a design situation can be possibly found in the neurological activity of the brain. This is an uncharted area of research but we can focus here on art creation as an example. According to Zeki (1999), art (in the form of painting or sculpture) is a creative activity inextricably related to vision. In his book, Zeki argues that 'vision is an essentially active search for essentials', or constancies. In particular, vision is defined in relation to two complementary areas of brain activity: the activity of 'seeing' (located in the primary visual cortex) and the activity of 'understanding' (located in the surrounding area of the primary visual cortex). Art creation then is explained as an extension of the function of the visual brain. We can thus say that art creation is defined as a search for complementarities between seeing and interpreting (see Figure 43). In this sense, a design situation can be identified with a sketch (a pattern of brain activation) that represents incomplete or inconsistent mappings between the observed objects and their interpretations.

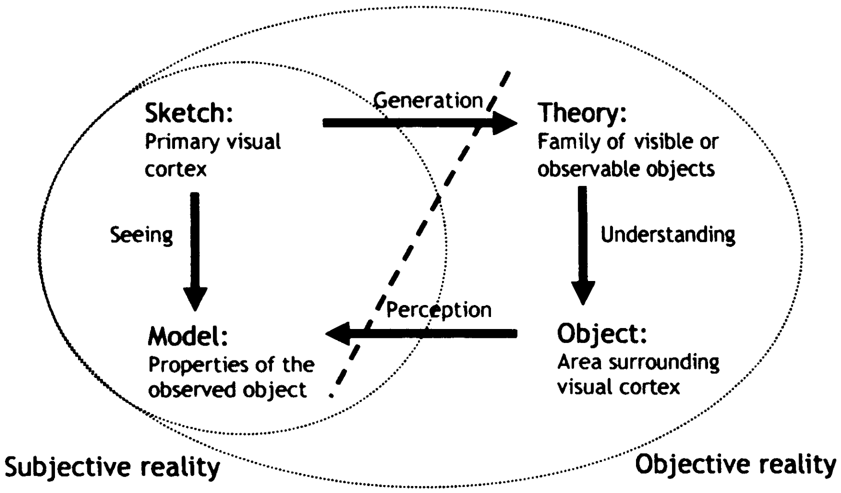


Figure 43: An example of the correspondence between two areas of the 'designing brain': seeing and understanding. A design situation arises when such a correspondence is broken or incomplete.

Example 3: A less obvious example can be found in social or socio-technical systems. According to Portugali (2006, 2007), human societies and their artefacts (e.g. cities) are distinguished by two main characteristics: an *information (or cognitive) content*, which comprises a set of conceptual, political, ethical or cultural attitudes; and also a *material content*, which corresponds to the structures that support and bound them. The former can be perceived as a subjective reality created by the society of cognitive agents, and the latter as an objective reality that is realized through the conceptual, spatial or physical artefacts produced by the society. Cities are in this sense formed by the interplay between individual behaviour and spatial infrastructure, and more specifically by the interplay between internal representations of cognitive agents and the external environment (Portugali, 2000, 2004). In this sense, we can perceive cities as large artefacts that express the beliefs and desires of a society. The Intentionality of these artefacts is not the product of an individual cognitive agent but an aggregate effect of the underlying organizational dynamics. A design situation within a social-technical system such as a city, can be thought to arise when the interpretations of its material content (the political, technological or physical infrastructure of the city) do not follow the models of social action or social structure derived by its cognitive content (the ideology, or principles held by its groups and individuals). This situation is expressed with the appearance of social groups/agents (such as political parties, social movements, religious or professional lobbies) that represent changes over existing models of political and social action, and advocate changes in the material infrastructure of society. In this sense, human societies form 'design systems' that encompass Intentional states (sketches) with a two directional freedom of fit between subjective and objective reality. A sketch (in specific the universal properties of a sketch) express an 'order parameter' that enslaves the set of possible cognitive/physical structures and their coupling. A design situation implies the existence of conflicts (Figure 44).

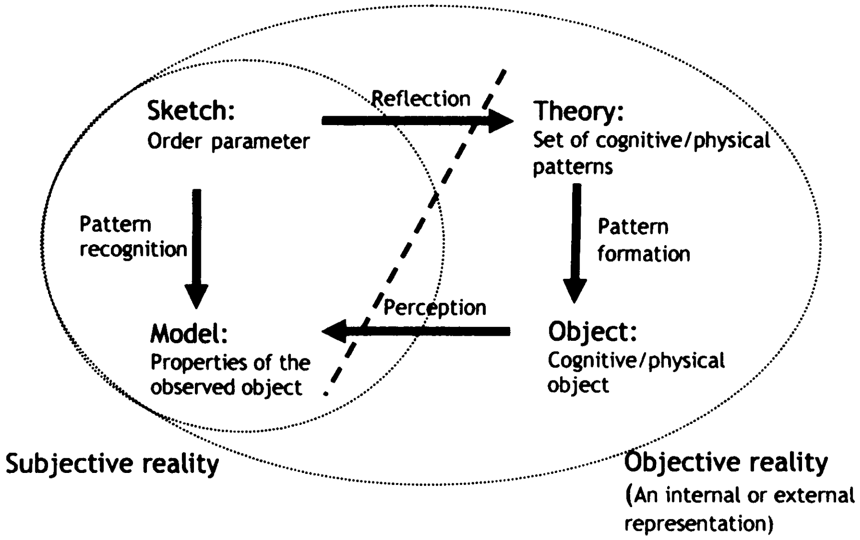


Figure 44: Example of a design situation in a social-technical system. In this situation, a sketch is an order parameter that determines the coupling between the formation and the recognition of cognitive or physical patterns within a society. A design situation arises when the political and physical structures of a society do not follow existing models of social action.

Example 4: A more formal interpretation of a design situation can be found in the system presented in Chapter 3. A problematic situation arises because of the initial inability of each agent to achieve their individual desires and goals, given the theories and models that they already maintain. In other words, what drives design is the realisation that the desires and goals of each agent cannot be achieved given their current theory about the behaviour of other agents and the specific models or instantiations of the world. More specifically, consider a number of n agents $A=\{a_1, a_2...a_n\}$ that generate artefacts in a social space P. Each agent holds a dynamically variable set of rules $R_i=\{r_1, r_2...r_k\}$. For each agent a_i, the set of rules R_i generates a family of possible configurations or shapes: a shape language denoted as **Shape(R_i)**. From the language **Shape(R_i)** a shape d is generated starting with the observation of a specific configuration and the application of certain rules. Conversely, for each agent, an observed shape d has certain underlying rules. So, each agent a_j has the capacity to *generate* or *interpret* a shape d in **Shape(R)**. A design situation arises when 'the selection of shapes' from the language of shapes **Shape(R)** cannot be deduced/generated by selected rules from the set $R=\Sigma R_i=\{r_1, r_2...r_k\}$.

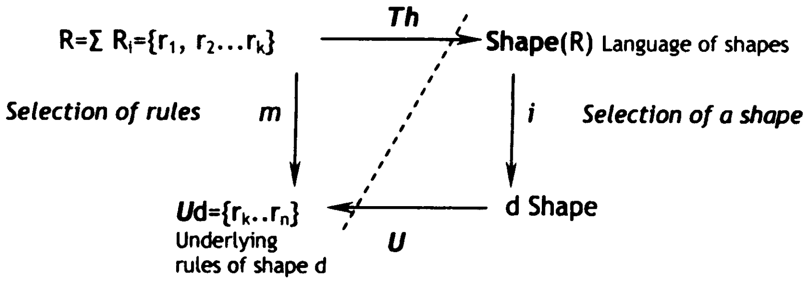


Figure 45: An example of a design situation that arises when there is a lack of complementarity between the application of syntactic rules and the interpretation of a language.

7.1.2.1 The design situation

More formally, given a sketch s, the theory functor *Th* determines a category *Th*s that constitutes a theory of plausible or desirable interpretations of a sketch. The functor *U* determines a category *U*d that constitutes the set of properties of a specific interpretation of a theory *Th*s in C. Models *m*:s→*U*d are those interpretations of the sketch that satisfy these properties. Hence, in category theoretic terms, a problematic situation appears when the developed theory *Th*s of desired objects/processes yields interpretations in C (i.e. $i\colon Ths \to d$) whose underlying properties *U*d cannot be (uniquely) derived by the sketch s (i.e. $m\colon s \to Ud$). The need to design therefore arises when the theory and model functors contain -so to speak- ambiguity, noise or errors, and hence there is no natural bijection between C(s,*U*d) and D(*Th*s,d). The following diagram is an informal illustration of this situation.

$$\begin{tikzcd}
s \arrow[r, "Th"] \arrow[d, dashed, "m"'] & Ths \arrow[d, dashed, "i"] \arrow[dl, dash, dashed] \\
Ud & C \arrow[l, "U"]
\end{tikzcd}$$

More precisely, a design situation is defined when there is weak adjunction (no unique correspondence):

$$\mathrm{Mod}(s,Ud) \ncong \mathrm{Fun}(Ths,d)$$

The weak adjunction between theories and models explains why the means to achieve a design change are not given as discussed at the beginning of this section.

7.1.2.2 The task and the capacity to design

In response to a problematic situation, the 'task' of design is identified with the generation of theories and models of sketches that bring beliefs and desires into correspondence. In all the aforementioned examples, ***the task of design** implies the existence of a sketch s that marks the transition from models with incomplete, inconsistent or unfulfilled theories, to models with complete, consistent and satisfactory theories*. In this sense, the need to design collapses when there is a sketch that matches the subjective with the objective reality in both directions: from world to mind and from mind to world. The definition expresses another generally accepted premise regarding the nature of design. For instance, according to Smithers (2002) at the core of design(-ing) lies an apparent paradox: designing has to do with arriving at a solution to a problem which is not a-priori specified. In other words, although design is driven by a need or goal, this goal is actually constructed by the very process of design. According to this view, the task of design involves the need to synthesise requirements and concepts regarding the desired objects, as well as to synthesise plans of action regarding the design process (i.e. to sketch objects and processes and to develop theories of such objects and processes). Additionally, the task of design also involves the need to generate specific interpretations of the synthesised requirements and plans into specific objects or actions (i.e. to synthesise models and give specific interpretations to the developed theories). In category theoretic terms, the core 'task' of design is to bring into complementary relation the generation of a sketch s whose models are expected to satisfy any instantiation of the theory $\mathit{Th}s$, and the generation of a theory $\mathit{Th}s$ whose interpretations are expected to be models of the sketch s.

What surfaces from this discussion is that the peculiarity of design lies in the fact that sketches, models, and theories are developed in some way independently from each other. More precisely, the postulation of a weak adjunction implies that the construction of theories and the construction of models from a sketch also lead to certain expressions of desired objects/processes that are not uniquely derived from the specified sketch. In this sense, design necessitates the (paradoxical) capacity of generating theories and models of a sketch in preparation of their adjunction, i.e. before such correspondence is constructed. In other words, design requires the capacity to generate sketches *in anticipation of* the theory-model adjunction. The main results from this treatment can now be formally summarized.

7.1.3 Formal definitions

7.1.3.1 Definition - Design situation:

A design situation is defined as an Intentional state s with a 'two directional freedom of fit' - in a subjective $C(\_,U\_)$ and objective reality $D(\mathit{Th}\_,\_)$ - whose models (i.e. $m:s \rightarrow Ud$) in $C(s,U\_)$ and interpretations (i.e. $i:\mathit{Th}s \rightarrow d$) in $D(\mathit{Th}\_,d)$ are not complementary. Namely, a design situation arises when there is a weak adjunction $<\mathit{Th},U,\varphi,\theta,\tau_C,\tau_D>$ natural in s and d that makes the following diagram commute:

$$\begin{tikzcd}
C(s, Ud) \arrow[r, "\theta"] \arrow[d, "\tau_C \neq 1_C"'] & D(Ths, d) \arrow[d, "\tau_D \neq 1_D"] \arrow[dl, "\varphi"'] \\
C(s, Ud) \arrow[r, "\theta"'] & D(Ths, d)
\end{tikzcd}$$

Given a design situation, the 'problem' of design or the 'design task' is to establish an adjunction $Mod(S,Ud) \cong Fun(ThS,d)$ by transforming both the subjective and objective reality.

7.1.3.2 Definition - The design task:

The problem or task of design is identified with the existence of a phase transition to universality; that is the existence of transformations $T_C = C(T_s, UT_d) : C(s,Ud) \rightarrow C(s_U, Ud_U)$ and $T_D = D(ThT_s, T_d) : D(Ths,d) \rightarrow D(Ths_U, d_U)$ such that the following diagrams commute:

$$\begin{tikzcd}
s \arrow[r, "T_s"] \arrow[d, "m"'] & s_U \arrow[d, "m'"] \\
Ud \arrow[r, "UT_d"] & Ud_U
\end{tikzcd}
\qquad
\begin{tikzcd}
Ths \arrow[r, "ThT_s"] \arrow[d, "i"'] & Ths_U \arrow[d, "i'"] \\
d \arrow[r, "T_d"] & d_U
\end{tikzcd}$$

$$\begin{tikzcd}
D(Ths, d) \arrow[rrr, dashed, "T_D"] \arrow[ddd, "\tau_D"'] \arrow[ddr, "\varphi"] & & & D(Ths_U, d_U) \arrow[ddd, "1_{D(Ths,d)}"] \arrow[ddl, "\varphi'"'] \\
& C(s, Ud) \arrow[ul, "\theta"'] \arrow[r, dashed, "T_C"] \arrow[d, "\tau_C"] & C(s_U, Ud_U) \arrow[ur, "\theta'"'] \arrow[d, "1_{C(s,Ud)}"'] & \\
& C(s, Ud) \arrow[dl, "\theta"'] \arrow[r, dashed, "T_C"] & C(s_U, Ud_U) \arrow[dr, "\theta'"] & \\
D(Ths, d) \arrow[rrr, dashed, "T_D"'] & & & D(Ths_U, d_U)
\end{tikzcd}$$

Based on this definition of a design task, it is plausible to assume that given an 'intentional state' s_D, the ***capacity to recognise and carry out design tasks*** implies the existence of transformations $T_S : s_D \rightarrow s_u$ in a subjective reality $C(\_, U\_)$

and transformations $T_d: d_D \to d_U$ in an objective realty $D(Th\_,\_)$ that are *components of a phase transition to universality* $T=<T_C,T_D>$. The transformation $T_S: s_D \to s_u$ is a component of the transformation $C(T_s,UT_d)=T_C: C(s,Ud) \to C(s_U,Ud_U)$ and similarly the transformation $T_d: d_D \to d_U$ is a component of the transformation $D(ThT_{s,}T_d)=T_D: D(Ths,d) \to D(Ths_U,d_U)$. It is informative to think that the transformation $T_S: s_D \to s_u$ has a 'mind-to-world' direction of fit, in the sense that any mismatch between a subjective $C(\_,U\_)$ and objective reality $D(Th\_,\_)$ is addressed by changing the properties of the sketch. Similarly, the transformation $T_d: d_D \to d_U$ has a 'world-to-mind' direction of fit because any mismatch between the subjective and objective reality is addressed by changing the properties of the object. As discussed in Chapter 6, the transformations $T=<T_C,T_D>$ can be alternatively defined as functors within a subjective and an objective reality respectively. Hence, $T_c: C/\tau_C \to C/1_{C(s,Ud)}$ is a functor from a quotient category C/τ_C to the quotient category $C/1_{C(s,Ud)}$, and $T_D: D/\tau_D \to D/1_{D(Ths,d)}$ is a functor from a quotient category D/τ_D to the quotient category $D/1_{D(Ths,d)}$. The first functor reflects a 'mind-to-world' direction of fit, and the second a 'world-to-mind' direction of fit.

Let us now assume that the theory *Ths* generated by a sketch s specifies a hom-functor $D(Ths,d): C^{op}xD \to \mathbf{Set}$ that is naturally isomorphic with the functor $C(s,Ud): C^{op}xD \to \mathbf{Set}$; so $C(s,Ud) \cong D(Ths,d)$. In category theoretic literature, the theory *Ths* is said to be a *representing object* for the hom-functor $C(s,Ud): C^{op}xD \to \mathbf{Set}$. Similarly, the underlying sketch *Ud* of an entity d is said to be a *representing object* for the hom functor $D(Ths,d): D \to \mathbf{Set}$. But, what can be said for hom-functors $D(Ths,d)$ and $C(s,Ud)$ that are not naturally isomorphic, that is when $C(s,Ud) \ncong D(Ths,d)$? In this case, the notion of an anticipatory representation is defined. The theory Ths_D generated by a sketch s_D is called an ***anticipatory representation*** of models in $C(s_U,Ud_U)$ *if and only if the sketch (intentional state)* s_D *has models* m_D *and interpretations* i_D *that are components of a phase transition to universality* $T=<T_C,T_D>$. The underlying sketch *Ud* is called an anticipatory representation of interpretations in $D(Ths_U,d_U)$. Based on this definition, ***the capacity to recognize and carry out design tasks*** is now identified as the capacity to hold anticipatory representations/interpretations of universality.

7.1.3.3 Definition - The capacity to design:

The capacity to design is the capacity of an Intentional state s_D to hold models $T_C{:}s_D \to s_U$ and generate interpretations $ThT_C{:}Ths_D \to Ths_U$ that are components of a phase transition to universality (i.e. that satisfy the diagrams in definition 7.1.3.2). The capacity to design is therefore the capacity of a sketch s_D *to* generate theories Ths_D of a subjective reality s_D and sketches Ud_D of an object d_D that are anticipatory representations of a universal construction $C(s_U,Ud_U) \cong D(Ths_U,d_U)$.

The models m_D and theories Ths_D that are derived from an Intentional state s_D are specified *in relation to a phase transition to universality.* Such anticipatory representations are clearly distinct from 'ordinary' representations where models and theories are specified *in relation to a universal construction.* More specifically, the meaning of anticipatory representation implies that a model $m_D{:}s_D \to Ud_D$ and interpretation $i_D{:}Ths_D \to d_D$ are defined in 'preparation' of a universal $C(s_U,Ud_U) \cong D(Ths_U,d_U)$; that is when the domain of both transformations T_c and T_D

$$\begin{tikzcd}
s \arrow[r, "T_s"] \arrow[d, "m"'] & s_U \arrow[d, "m'"'] \\
Ud \arrow[r, "UT_d"] & Ud_U
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
Ths \arrow[r, "ThT_s"] \arrow[d, "i"'] & Ths_U \arrow[d, "i'"'] \\
d \arrow[r, "T_d"] & d_U
\end{tikzcd}$$

has co-domain or target domains that form a part of a universal construction $C(s_U,Ud_U) \cong D(Ths_U,d_U)$.

The adjunction $C(s_U,Ud_U) \cong D(Ths_U,d_U)$ uniquely characterizes a design situation s_D in the sense that every model of s_D and interpretation of theories Ths_D are constrained by the specified transition to universality (i.e. diagrams of 7.1.3.2). However the reverse is not true. The models of the sketch s_D are not universal representations of $D(Ths_U,d_U)$. It is in this sense that the adjunction $C(s_U,Ud_U) \cong D(Ths_U,d_U)$ is perceived as an 'emergent' state of s_D. This is a crucial point for understanding the suitability of various abstractions for representing or expressing the problem of design.

7.2 COMPARING PARADIGMS FOR A THEORY OF DESIGN

The thrust of the enquiry up to this point was to identify what is the design task and what sort of capacities it entails. In this section, the main concern is the identification of the consequences of these definitions for the development of a theory that explain the capacity to carry out design tasks. For this purpose, the notions of computing, evolution and control are studied as paradigms for a theory of design. In particular, the main issue is whether design tasks and capacities can be explained as part of a theory of computation, evolution or control. Is a design task computable? Is the design task a control problem? Is the design task an evolutionary problem that adheres to the principles of natural selection?

In other words, the scope and objective of design theory will be seen in relation to the objectives of the theories of computation, evolution or control. Generally speaking, the theory of computation is concerned with the identification of universal processes that can lead to an *effective description of organization*; the theory of evolution is concerned with the identification of a universal process that explains *the generation of organization;* while the theory of control is concerned with *the preservation of organization*. So, what would be the objective and scope of a design theory starting from these ideas?

7.2.1 Computational theory of design

The term machine is used in ordinary discourse in connection to mechanical artefacts. However, in philosophy, science, and mathematics, the term has been used as a metaphor to express the fundamental working principles of reality, knowledge and thought. In particular, the concept of machine has been used for the definition of abstract notions such as *effective procedure*, *logical process* and *computing*. The best known example is the Turing Machine which was invented as a model of a universal computer capable of realising any possible logical process. For an introduction to machines the interested reader might consult Minsky (1972).

The machine abstraction has been an important epistemological basis for design, particularly due to the hypothesis that thought can be best modelled as an information processing machine. As discussed in Chapter 2, the work of Newell, Shaw and Simon on human problem solving (Newell et al, 1957; Newell and Simon, 1972) paved the way for the development of a cognitive understanding of design as information processing and its formalisation as a search process. This work has been influential for a very large number of empirical, theoretical or computationally-driven design studies (e.g. Eastman, 1970; Stiny and March, 1981; Akin, 1986; Goel and Pirolli, 1989).

The advent of computing machines and the information processing paradigm have also been instrumental for the development of research in Artificial Intelligence. The rapid growth of Artificial Intelligence has naturally generated questions about the role of the machine paradigm in understanding intelligence in general (e.g. Dreyfus, 1972; Searle, 1980), and design intelligence in particular (e.g. Cross, 1999a; Liddament, 2000). Nevertheless, the relation between design intelligence and machines has been investigated at various levels of abstraction: at the level of information processing, the level of design tasks or the Knowledge Level - although the latter is not necessarily a machine abstraction in itself (Brazier et al, 1994; Brazier et al, 2001; Brown and Chadrasekaran, 1989; Chandrasekaran, 1990; Goel and Pirolli, 1989; Smithers, 1998).

The main focus in this section is to discuss the implications (as well as limitations) of developing a computational or machine-based theory of the capacity to hold anticipatory representation to universality. For this purpose, let us assume that an Intentional state s is realized by a computing machine **Mach** with the capacity to compute (or realize) an objective reality expressed by a language **Lang** (or behaviour **Beh**). In Chapter 5 we introduced the category of machines and the category of behaviours (languages), and the existence of the functors

E:**Mach**→**Beh** ('external behaviour' functor) and *R*:**Beh**→**Mach** ('realisation' functor) (Goguen, 1973). The machine view of the world is therefore defined on the basis of a complementary relation between a language (that generates imperative or permissive statements about a world), and a machine (that evaluates the statements, and produces an answer about whether a problem is solved or not). The specification of a machine and the specification of the behaviour of the machine have therefore an optimum correspondence represented by the adjunction between the functors ***E*** and ***R***. The notion of machine in design theory can be more specifically thought as an abstraction of the problem space. A machine is a representation of a theory that determines and evaluates the principles of a specific model (a design solution) expressed in a certain language. Similarly, a language can be abstractly considered as the solution space, where the generation of alternatives is defined (see for instance Stiny and March, 1981). On this basis, design is a search process for the best possible model that satisfies the constraint (adjunction) between the functors ***E*** and ***R*** as the diagram in Figure 46 suggests:

$$\begin{tikzcd}
\textbf{Mach} & REM \arrow[dr, "R(i)"'] & M \arrow[l, "\eta_M"'] \arrow[d, "m"] & & EM \arrow[d, "i"'] \arrow[dr, "E(m)"] & & \textbf{Beh} \\
& & RL & & L & ERL \arrow[l, "\varepsilon_L"] &
\end{tikzcd}$$

Figure 46: Universal properties of computation expressed in the relation between machine and language (behaviour), where E:Mach→Beh is the 'external behaviour' functor and R:Beh→Mach is the realization functor. For any model m of a machine M there is a unique arrow i such that the left diagram commutes. Similarly for any arrow i there is an arrow m such that the right diagram commutes.

In this setting, the complementary relation between machine and language (or adjunction between the functors ***E*** and ***R***) plays the role of 'a constraint'. However, for the purpose of developing a theory of design as 'a transition to universality' the complementary relation between machine and language must represent an 'emergent' or 'critical' state. To pursue this objective there are two possible directions of research. One direction is to focus on the development of machine-language representations that allow ambiguity, noise and multiple interpretations in their correspondence. This direction of research can be found in the work of Goel (1995). The development of weak sketches in Chapter 6 can also be seen as part of this endeavour. A second direction is to assume the existence of an ecosystem of languages and machines placed within an evolutionary environment. In this environment, the complementary correspondence between machines and languages is an 'emergent' state derived from a process of natural selection. This direction is reviewed in the following section.

7.2.2 Evolutionary theory of design

Based on the machine paradigm, a plethora of different abstractions have been developed, which in general either refine or introduce more structure (restrictions) to the machine formalism. For example in evolutionary explanations, the objects of evolution are machines *M* together with their languages. According to the evolutionary perspective, the world is explained by a process of natural selection. Given a process that copies (which small variations) a population of machines *M* together with their languages, selection is taking place on the basis of the correspondence between languages and machines: only languages that are readable (acceptable) by the machine *M* may possibly survive and reproduce. Similarly only languages that are realised by machines survive and therefore reproduce. In this way, more complex machines and languages can be defined starting from simple ones - under constraints illustrated in Figure 47 (for details see Chapter 5).

The concept of evolution has also been used in design in various ways, for example as an analogy for understanding the form and function of design artefacts (Steadman, 1979), or as a methodology for solving optimisation problems and generating novel forms (Frazer, 1995; Rosenman, 1996; Bentley, 1999). The evolutionary paradigm has been a useful abstraction for design. That is mainly because theories (languages) may co-evolve together with models (machines), and this fits well with the view of design as co-evolution of problem and solution space

(Maher, 2000; Dorst and Cross, 2001).

$$\begin{tikzcd}
X \mathrm{x} S \arrow[r, "\delta"] \arrow[d, "a \mathrm{x} b"'] & S \arrow[r, "\lambda"] \arrow[d, "b"'] & Y \arrow[d, "c"'] \\
X' \mathrm{x} S' \arrow[r, "\delta'"] & S' \arrow[r, "\lambda'"] & Y'
\end{tikzcd}$$

Diagram 1

$$\begin{tikzcd}
 & S \arrow[d, "b"] \\
1 \arrow[ur, "\sigma"] \arrow[dr, "\sigma'"'] & \\
 & S'
\end{tikzcd}$$

Diagram 2

$$\begin{tikzcd}
X^{*} \arrow[r, "f"] \arrow[d, "a^{*}"'] & Y \arrow[d, "c"'] \\
X^{*\prime} \arrow[r, "f'"] & Y'
\end{tikzcd}$$

Diagram 3

Figure 47: Constraints in the category of machines and languages within which evolutionary processes take place. Diagrams 1 and 2 present morphisms between machines of the form $M=<X, S, Y, \delta, \lambda, \sigma>$, where X, S, Y are the input, state and output sets respectively; and δ, λ, σ are the transition function, output function and initial state function respectively. Diagram 3 represents morphisms between languages L of the form *f*: $X^{*} \rightarrow Y$ where X and Y are sets and X^{*} is the set of all possible concatenations over an alphabet X.

However, the main difficulty arises in the explanation of functionality, or more precisely the explanation of functionality in anticipatory terms. In evolutionary explanations the *function* of a system is generally conceived to be a behaviour selected for its correspondence with a machine *M*. The correspondence between languages and machines (problems and solutions) drives the very meaning of natural selection; any mismatch is thrown outside the evolutionary process. However, as we saw, design is motivated by this very mismatch, and the process of design involves anticipating the correspondence between languages and machines, rather than using it as a constraint. In this sense, a goal oriented or purposeful process must be also represented. This is the very object of control theory.

7.2.3 Control theory of design

The concept of control, derived from the theory of Cybernetics, has also been used as a paradigm for the understanding and description of design. Cybernetics is generally perceived as the 'science of effective organization' (Beer, 1974: 13). According to Ashby (1956: 3) 'cybernetics envisages a set of possibilities much wider than the actual, and then asks why the particular case should conform to its usual particular restriction'. Within this context, the conceptual paradigm of control aims to describe and explain how a particular structure is formed out of a given organization (i.e. the conditions that determine a family of possible structures). In particular, control is concerned with the development of a theory of variety or entropy reduction.

It is often noted that in the first cybernetic studies there was a bias towards top-down theories of (purposeful) variety reduction. The reduction of variety was described and explained on the basis of a clear organizational distinction between a controller and a controlled system where goals were embedded in the controlling system itself. In later studies, a bottom-up approach to variety reduction was developed which brought into play the issue of spontaneous variety (or entropy) reduction. This led to questions regarding the plausibility of self-organization (e.g. Von Foerster, 1984), the meaning of self-observation and self-steering where a controller acts within the same system that it aims to control (e.g. Baumgartner, 1986), and generally to the recognition of a complementary relation between autonomy and control (Goguen and Varela, 1979). In a broad sense then, control theory is concerned with the explanation of the purposeful or spontaneous generation of structures such that a given identity (or a vital performance criterion for survival) is preserved. The identity of a system is identified with the organization of the system. Hence the capacity to control alludes to the capacity to act within, or upon, a system so as to preserve its organization under changing circumstances (Geyer and van der Zouwen, 1986: 215).

From this discussion, it follows that the 'problem' of control arises within a situation where there is a specific controller-controlled system or system-environment distinction and consequently a specific organization (identity) that needs to be preserved. Following Varela's argument (1979), the designation of a distinction between a system and its environment is tied up with the formation of a complementary relation between the description of the organization of a system (i.e. a sketch s or a machine M), and the description of the structures that satisfy this organizational description (a theory *Th*s or language L that is acceptable from machine M). Following this argument, the meaning of organizational invariance is alternatively defined as the invariance over the complementary relation between machines and languages.

More formally, Goguen and Varela (1979) identify the very meaning of this complementary relation with the category theoretic concept of adjunction. Control is then associated with the capacity to generate models (i.e. variations of parameters) of a machine M that preserve the natural bijection $\mathbf{Mach}(\mathbf{M},\boldsymbol{R}\mathbf{L})\cong\mathbf{Lang}(\boldsymbol{E}\mathbf{M},\mathbf{L})$. Assuming a machine M, the capacity to control is identified with the existence of the arrow $\mathit{Cont}{:}\mathit{R}\mathrm{L}\rightarrow \mathrm{H}(\mathrm{M},\mathit{R}\mathrm{L})$ which selects models m within H(M, *R*L) that satisfy the adjunction $\mathrm{Mod}(\mathrm{S},\mathit{U}(\mathrm{C}))\cong\mathrm{Fun}(\mathit{Th}(\mathrm{S}),\mathrm{C})$ (i.e. satisfy the equation $m = R(i)\eta$ derived from the universal construction in Figure 46). The following diagram is therefore the 'archetypical presentation' of control systems (for more details see Zamenopoulos and Alexiou, 2007):

$$\begin{tikzcd}
\mathit{REM} \arrow[dr, dashed, "R(i)"] & & \\
\mathrm{M} \arrow[u, dashed, "\eta"] \arrow[r, "m"{name=mm}] & \mathit{R}\mathrm{L} \arrow[r, "\mathit{Cont}"] & \mathrm{H}(\mathrm{M}, \mathit{R}\mathrm{L}) \arrow[to=mm, dashed, bend right]
\end{tikzcd}$$

In design research, cybernetic theories of design have been, by and large, coupled with a control theoretic understanding and identification of design. Especially in the early days of design research, the conceptual category of design was identified with control and the designer was defined and studied as a controller. The design task was then defined as a process that transforms design object configurations with the objective of maintaining a principal idea or function, and to satisfy certain performance criteria, despite uncertainties and exogenous disturbances. There are several theoretical contributions in design research that explicitly or implicitly aim to explain the capacity to design as a control capacity. For instance, Archer (1970) developed a model of design activity as a control problem. In particular, he considered that designing is an iterative process of generating and controlling a set of (decision) variables in order to optimally fulfil a given set of objectives. Another less obvious example can be found in Yoshikawa's General Design Theory (1981) which builds on set and topology theory. In particular, the theory is based on certain assumptions and definitions about the nature of designed objects (their structural attributes and their functions), which are followed by certain theorems about the nature and possibility to design. As we saw in Chapter 4, design is defined as a mapping from a function space to an attribute space. This implies that design is seen as an algorithmic process of finding a specific structure in the attribute space that preserves certain topological relations in the function space (See for instance Reich, 1995: 9). An important point in Yoshikawa's theory is provided in Theorem 10 which states that the capacity to design is possible if the attribute space is topologically stronger (i.e. it has greater variety) than the function space. This is strongly reminiscent of the Law of Requisite Variety, and implicitly suggests that design ability is comparable to controllability.

However, the formulation of control as discussed above can help us understand certain limitations when the concept is used as a paradigm for design. The problem of control was defined as a problem of preservation or maintenance of a particular organization based on the existence of a complementary relation between machine and languages; while the problem of design was defined as a problem of creating a particular organization, when the complementary relation is broken. In other words, when the idea of control is transferred to design it implies that performance criteria or criteria for the termination of a design task are given in advance; and hence are not part of the design process itself as it is commonly accepted in contemporary design research. Based on this analysis therefore, design surfaces as a paradigm that needs to be differentiated from the notion of control.

7.2.4 Conclusions: the problem of universality

In this section, the notions of computing, evolution and control were studied as paradigms for developing a theory of design. In particular, the objective was to discuss problems related with the development of a theory of design as 'a transition to universality' using these paradigms. Looking at the theory of computation (exemplified by the concept of machine), it was observed that the complementary relation between machine and language plays the role of 'a constraint'. To develop a machine theory of design, it is necessary to represent this complementary relation as an 'emergent' or 'critical' state (instead of a constraint). To achieve this, one direction is to focus on the development of machine-language representations that allow ambiguity, noise and multiple interpretations. This direction was elaborated in Chapter 6. A second direction is to assume that there is ecosystem of languages and machines. In this ecosystem, the existence of the complementary relation between theories and machines is an 'emergent' state derived from a process of natural selection. However, an evolutionary theory of design generally lacks a representation of an intentional state which operates as a drive for design tasks. Control structures are important organizational characterisations that explicate the meaning of purposeful behaviour. However, much of the focus in control theories of design is placed on organizational invariance, the boundary conditions that preserve an organism or system (i.e. on preservation rather than generation of the machine-language complementarity).

Overall, given our definitions, the paradigms of computation, evolution and control cannot be seen as universal theories of design: i.e. as theories that explain the anticipation of phase transition to universality. However, this does not mean that they are not useful abstractions in order to describe, represent and realise processes that are crucial components of design. The main conclusion is that we need to (move to a meta-level and) study the complementary relation between machine and language as a critical state; as a state that is an emergent property of anticipation. This is the very purpose of the next section: the development of a theory that explains the conditions that are responsible for the capacity to hold an anticipatory representation of the machine-language complementarity.

7.3 CONDITIONS FOR THE CAPACITY TO DESIGN

The main thesis derived from the present enquiry is that the capacity to recognize and carry out design tasks implies the capacity of an 'Intentional state' (or sketch s_D) to construct anticipatory representations of a phase transition to universality. But what are the mathematical conditions that are responsible for the capacity of an Intentional state to hold anticipatory representations? What are the organizational conditions that are responsible for the capacity to carry out design tasks? The next sections aim to address these questions, and more specifically, to identify the mathematical structures that underlie the construction of an anticipatory representation of universality. To begin with, the meaning and role of anticipation is examined. This general introduction is followed by an investigation of the role of anticipation in design and design research. Finally, a theory of anticipation is developed by generalising Rosen's (M, R) model (Rosen, 1991).

7.3.1 The anticipatory problem

Anticipation is generally associated with the ability of looking ahead (or looking forward), but it also refers to an action or decision that is taken in preparation for some future event. The Cambridge Dictionary Online (2006) defines anticipation as follows: 'to imagine or expect that something will happen, sometimes taking action in preparation for it happening'. The Oxford English Dictionary (2006) lists several definitions of anticipation which link it with the idea of possessing or realising something in advance (actually or virtually); taking action that meets beforehand, provides for, or precludes the action of another; but also with á priori knowledge, precognition, preconception, and expectation. Similarly, many different conceptions and definitions of anticipation are found in domains as diverse as philosophy, psychology, cognitive science, biology, and computer science.

Preoccupation with the concept of anticipation has its origins in philosophy, but also psychology and cognitive science (e.g. James, 1890; Tolman, 1932; Piaget, 1954; Kelly, 1955; Neisser, 1975). Anticipation is an appealing but also difficult idea, as it seems to violate fundamental principles of time, causality, or construction of abstractions. This is because it implies circularity: how can future states of the world affect present time, or how can the effect of an action determine the action in advance of its realisation? As we will see, this problem is normally resolved by assuming a capacity to construct an implicit or explicit representation of future states, or effects, before the actual realisation of the action that produces them. However, anticipation is not only a time related problem. In the history of philosophy and logic, an abstraction - such as for instance the idea of 'whiteness' - is often believed to be constructed from specific examples (i.e. different whites perceived). From this perspective, anticipation seems to imply the paradoxical capacity to generate abstract ideas without having specific instantiations of these ideas.

So, while the core definition of anticipation seems to be generally accepted, the assumptions about the nature of anticipation vary substantially. For example, anticipation has been defined as a particular kind of dynamics (e.g. Dubois, 1998), as a particular pattern of causal relations (e.g. Burgers, 1975; Rosen, 1985), or as a particular relationship between wholes and parts (e.g. Van de Vijver, 2000).

In general, there are two different research directions for considering the issue of anticipation. The first focuses on anticipation as a characteristic property of certain classes of systems (cognitive, social or physical) and aims to define anticipation, as well as to identify and describe the conditions that make a system anticipatory (e.g. von Glaserfeld, 1998; Riegler, 2001; Leydesdorff, 2005). The second direction of research is focussed on the question of how anticipatory behaviour can be achieved computationally, or generally be used for developing more effective systems (e.g. Butz et al, 2003a).

Here we will mainly examine the predominant approaches of Rosen and Dubois: Rosen is considered to be the 'father' of anticipatory systems since his treatment introduced the concept in relation to the study, modelling, and control of complex systems, while Dubois reintroduced the concept in the scientific community in recent years. We will also briefly explore anticipation in relation to the concept of agency in Cognitive Science and Artificial Intelligence. In any case, we will distil some important arguments for the understanding of anticipation in the context of design, and highlight links with other related concepts such as expectation or autonomy.

7.3.1.1 Rosen's view of anticipation

Rosen's concept of anticipation, which has biological roots, is tied with the reinstatement of the 'lost' cause of Aristotelian logic: the *final cause*. Briefly, the old Aristotelian categories of causation (called material cause, formal cause, efficient cause and final cause), constituted four distinct ways of answering 'why-questions'. Material cause refers to matter, the primitive substances or components by which something is constructed; while formal cause refers to the form which these take, the shape or structure of something. Efficient cause refers to the agent or producer of an entity; and final cause refers to the end, the purpose, or function for which the entity comes into existence. We can take the simple example of a hut. The material cause of the hut is the stuff used to build it: straw, clay, wood etc. Formal cause is the shape of the hut and its structural characteristics. Efficient cause refers to the agent that transformed the materials into the specific form, the builder or designer. Efficient cause may also refer to the procedure, method, or principles by which the hut became realised. Final cause refers to the function that the hut serves, for example to provide shelter from the sun or rain. According to Rosen, explanations of the fourth kind have been excluded from science because they have been taken as a direct violation of the traditional notion of causality. How can the function of the hut, which only becomes realised after the hut is built, become a cause of its creation? Yet he suggested that finality was the only kind of explanation that could be offered to anticipatory behaviour, which is manifested at all levels of biological organisation, from the molecular level up to the human level.

There are two aspects in Rosen's work worth exploring. The first has to do with his definition of anticipatory systems which profoundly influenced the contemporary understanding of anticipation, and the second has to do with his formal elaboration of a certain class of systems called metabolism-repair (M, R)-systems. The elaboration of (M, R)-systems is considered to be useful here because it offers a formal treatment of anticipation as a systemic capacity. Transferring this to design, we have both a formal expression of anticipation that we can extend for our purposes, but also a methodology that we can use to study design.

7.3.1.1.1 Anticipatory systems

In Rosen's view, anticipation is coupled with the ability of a system to contain a model of itself and/or its environment. This ability enables the system to act not only according to its history, but also in response to possible or future states of the world. He gives some examples of systems where the existence of an internal model (whether 'wired-in' or constructed) allows the expression of future states to guide present action. In one such example he writes: '...if I am walking in the woods, and I see a bear appear on the path ahead of me, I will immediately tend to vacate the premises. Why? I would argue: because I can foresee a variety of unpleasant consequences arising from failing to do so.' (Rosen, 1985: 7). People customarily construct models which allow them to predict future situations, or consequences of future events, and on this basis to change their present course of action. But the ability to anticipate can also be found in 'lower levels' of (biological) organisation 'where there is no question of learning or of consciousness' (ibid).

Rosen's characterisation of anticipatory systems is built on the coupling between a dynamical system *S* (running in real time) and another dynamical system *M* which is a model of *S*. The idea is that this model can 'go faster' than real time and therefore predict future states: 'by looking at the state of *M* at time *T*, we get information about the state that *S* will be in at some time later than *T*' (ibid: 12). This prediction is then used to perform an action at present time. For Rosen, the idea of building and employing predictive models was a fundamental aspect of science in general. His definition of anticipatory systems with its associated premises was hence also intended as a framework for understanding, modelling and controlling (complex) systems.

7.3.1.1.2 The (M, R) model

An example that epitomises anticipatory ability is the metabolism-repair (M, R) model. The model aims to explain cell behaviour independently from material substance by adopting functional components as the basic units of analysis. An (M, R)-system is characterised by two fundamental biological qualities or functional components: a metabolic component, which can be represented as a set of mappings that convert inputs from the environment to outputs, and a repair component, which maintains and reconstitutes the metabolic activity. We will delve into the details of this model and the notation used in Rosen (1985, 1991), as these will be utilised to develop an anticipatory description of design in section 7.3.3.

More formally then, let f:A→B denote a *functional* component of a system that transforms an input A to an output B. In particular f is a metabolic element that takes part in a metabolic network. This transformation or mapping f belongs to a larger set of physically realisable metabolisms that the cell can display denoted by H(A,B). In other words, H(A,B) is the set of all possible transformations from A to B. Any process that can generate copies of f must have its range in H(A,B): if φ_f is the repair component, then this will be a mapping into this set. Hence the simplest (M, R)-system is given by:

$$\underbrace{A \xrightarrow{f} B}_{\text{metabolism}} \; \underbrace{\xrightarrow{\varphi_f} H(A,B)}_{\text{repair}}$$

Rosen showed that this formalism already contains what is needed for the operation of another decisive genetic component: a replication mechanism. That is, the replication mechanism can be naturally derived by the mappings of metabolism and repair and represented by a mapping β_b:H(A,B)→H(B, H(A,B)). The mathematical conditions for the existence of this mapping are nicely explained in Letelier et al (2006). An abstract (M, R)-system can therefore be expressed by the following diagram:

$$\underbrace{A \xrightarrow{f} B}_{\text{metabolism}} \; \underbrace{\xrightarrow{\varphi_f} H(A,B)}_{\text{repair}} \; \underbrace{\xrightarrow{\beta_b} H(B, H(A,B))}_{\text{replication}}$$

This is the quintessence of a relational model in that the property of replication is entailed by the metabolism and repair mappings (or functions); it is constructed on the basis of these mappings alone, from the organisation of the system, and independently from any particular realisation of the living cell.

Let us briefly uncover the explanatory attributes of this diagram. For the simple mapping f:A→B, the question 'why B' can only have two answers: in terms of material cause ('because a' - the value of A) and in terms of efficient cause ('because f'). But B has no explanation in terms of final cause - there is nothing to explain B by its effects in this diagram, or to endow it with a function (any finalistic answers have to pertain to the external environment of the system). On the contrary, f and a can only be explained in finalistic terms: because of their function in the diagram, that is to entail B. By extending the original mapping with the repair component the function f is efficiently entailed by φ_f:

$$\begin{tikzcd}
A \arrow[r, "f", ""{name=F}] & B \arrow[r, "\varphi_f"] & H(A,B) \arrow[to=F, dashed, bend right]
\end{tikzcd}$$

Furthermore, by adding the replication mechanism the mapping φ_f can also be entailed; but it is the fact that B can entail β_b that eventually does the trick. In this last diagram everything is efficiently entailed and all final cause answers are found within the system. Note that only the initial input A originates from the environment.

$$\begin{tikzcd}
A \arrow[r, "f", ""{name=F}] & B \arrow[r, "\varphi_f", ""{name=P}] \arrow[rr, phantom, ""{name=Bs}] & H(A,B) \arrow[r, "\beta_b", ""'{name=G}] \arrow[to=F, dashed, bend right] & H(B, H(A,B)) \arrow[to=P, dashed, bend right]
\end{tikzcd}$$

$$\begin{tikzcd}
B \arrow[r, dashed, bend right, ""] & \beta_b
\end{tikzcd}$$

This for Rosen also exemplified the essence of an organism, a system closed to efficient causation, which is characteristically non computable (contains non-simulable models). For a more comprehensive mathematical treatment of (M, R)-systems see Letelier et al (2006). It is worth noting that Rosen's notion of (M, R)-systems bears comparison with the notion of autopoiesis proposed by Maturana and Varela (1980). Autopoiesis is another characterisation of living systems, which is also made specifically in terms of a circular, self-referential, type of organisation - although the concept of anticipation is not explicitly taken into consideration. For discussions on the relation between autopoietic and (M, R)-systems see (Letelier et al, 2003; Nomura, 2002).

7.3.1.2 Dubois's view of anticipatory systems

Another prominent approach to research in anticipatory systems is advocated by Dubois (Dubois, 1997; Dubois, 1998; Dubois, 2000). Starting from a divergent position from that of Rosen, he suggests that anticipation is not a characteristic of biological systems alone (a trait of life), but is fundamentally present in all physical systems. In particular, he asserts that Rosen's notion constitutes a special form of anticipation as it is founded on model-based prediction ('weak' anticipation). He additionally discusses a formulation where anticipation as change of current state according to initial, as well as final conditions, is achieved at a system level ('strong' anticipation). His alternative interpretation is based on the concept of incursion (implicit recursion) by which future state is *computed* in a self-referential manner.

More specifically, Dubois describes Rosen's anticipatory system S as a set of differential equations as follows (M denotes the predictive model):

$$\Delta S/\Delta t = [S(t+\Delta t) - S(t)]/\Delta t = F[S(t), M(t+\Delta t)]$$

$$\Delta M/\Delta t = [M(t+\Delta t) - M(t)]/\Delta t = G[M(t)]$$

His alternative proposition, where the future state of the system S <u>and</u> the model M at time t+Δt are a function of this system S at time t and of the model M at a later time step t+Δt (Dubois, 1998: 5), is written as follows:

$$\Delta S/\Delta t= [S(t+\Delta t) - S(t)]/\Delta t = F[S(t), M(t+\Delta t)]$$

$$\Delta M/\Delta t= [M(t+\Delta t) - M(t)]/\Delta t = G[S(t), M(t+\Delta t)]$$

Dubois uses the concepts of incursion and hyperincursion (incursion with multiple solutions) as a method to investigate and develop a series of formal models, ranging from control of feedback and chaotic systems, to the generation of fractals from incursive automata and digital wave equations.

Dubois's view in fact summarises the main discussion points in research relevant to anticipatory systems: whether anticipation is a unique characteristic of biological systems or extends to all complex systems (biological, natural, and artificial), and whether anticipation can be realised computationally. The question about computation has its roots in Rosen's argument that there is a parallel between natural languages and organisms, in the sense that they both possess semantic models of entailment that cannot be encapsulated in a syntactic formalisation, a formal system, or a machine (Rosen, 1991: 247). Thus, although the existence of some sort of internal model is commonly agreed to be necessary, some consider that purely syntactic representations are sufficient for producing anticipatory behaviour, while others consider that a semantic dimension is necessary. For a discussion of computability, language and anticipation see Ekdahl (2000). On the other hand, most researchers prefer to focus on the question of how anticipation can be realised computationally, and what mechanisms or structures are more appropriate for achieving this. For instance, while some consider that anticipatory behaviour can be modelled using reactive (rule based or stimulus-response type) mechanisms, others consider that the internal model should necessarily involve functions such as expectation formation or learning (see for example the discussion by Castelfranchi, 2005). For a review and classification of anticipatory mechanisms see Butz et al (2003b). For a more comprehensive view of the work in the field one may consult the Computing Anticipatory Systems (CASYS) Conference Proceedings, that appear annually since 1997 and are edited by Dubois, as well as the book edited by Butz et al (2003a).

7.3.1.3 Anticipation and agency in cognitive science and artificial intelligence

Anticipation is often considered to be a distinguishing attribute of agency (Ekdahl et al, 1995; Ekdahl, 2000; Christensen and Hooker, 2000; Castelfranchi, 2005). Many researchers within Artificial Intelligence, Artificial Life, and Cognitive Science, see that high-level capabilities such as autonomy, intelligence or sociability can only be explained and constructed as the result of the dynamic, mutually constructive interaction between agent and environment (e.g. Beer, 1995; Port and Van Gelder, 1995; Smithers, 1997). Anticipation is also considered as such a high-level ability, which comes as a result not only of agent and environment properties, but also of the properties and dynamics of their interaction (e.g Pfeifer, 1995; Christensen and Hooker, 2000). Taking this view seems to be important for formalising, and possibly constructing, systems that are able to produce the laws of their own operation. This is essentially the problem of formalising and designing self-referential systems. Rosen's (M, R) formalism, as well as Maturana and Varela's (1980) definition of autopoietic systems, offer a way to describe organisms as systems whose fabrication process is entailed within them. For associated views of autonomy as self-governance, and arguments on the relationships and differences with concepts of cybernetics and autopoiesis, see (Steels, 1995; Smithers, 1997; Collier, 2002).

Before we conclude this section it is also useful to note that anticipation is considered as an important concept not only in relation to issues of individual agent intelligence and cognition, but also in relation to issues of coordination and cooperation in groups of agents. The ability to form expectations about the future, or 'look ahead' in time and space so as to forecast future conflicts and advantageous opportunities, has proven to be particularly useful in improving collaboration and social utility, as well as avoiding or resolving conflicts (Davidsson, 1997; El hadouaj et al, 2000; Veloso et al, 1999).

7.3.1.4 Summary and discussion

From all the works we reviewed, anticipation surfaces as an important aspect for understanding, modelling, or even constructing reality, and it is also strongly related to scientific investigation per se. In place of a summary it is important to reiterate that the discussion of anticipatory systems is closely linked to the general philosophical and technical problem of self-reference. The problem of self-reference is often discussed in set theoretic terms, which we will explain here in a non-technical way.

According to set theory, any reality is perceived and changed in (finite) stages. For instance, in Dubois' notation, stages are represented by an index of time. Alternatively, stages can be thought as levels of abstraction from 0 to N where every level has all the necessary information (let us say components) to completely define entities at the next level. It is often assumed that an entity must be defined uniquely by these components which are defined at previous stages or levels of abstraction. Note that the difference between different stages, or levels of abstraction, can also be seen as a discrepancy between system and environment. Now, the general question of anticipation is the possibility to construct an entity using not only components at previous states or lower levels of abstraction, but also entities from higher (later) levels of abstraction. This is a typical situation in design and especially in creative design. It alludes to the problem of specifying design solutions together with the components of the design artefact. So for instance, an architect often decides the general layout of a building (the building as a whole) while also resolving the configuration of the internal spaces (the components of the building). In set theory and computer science these issues have been formally expressed in the theory of Hypersets (Aczel, 1988). But, as Dubois (1998: 8) argues, '... self-referential systems have today no theoretical well-established framework'. So the development of anticipatory models and representations remains an open question. Let us now explore in more detail how anticipation relates to design.

7.3.2 The meaning and role of anticipation in design

The meaning and role of anticipation in design can be discussed from different perspectives. From a social perspective, design objects can be interpreted as anticipatory actions in preparation for a certain future situation. Designers often play this special social role: they instantiate ideas that fulfil needs or resolve problems in reference to a future state of the world. For instance in product design, devices such as video mobile phones or MP3 players are developed in anticipation of certain needs not previously expressed (or possibly only expressed with the introduction of the product!). Similarly, the design of socio-technical systems, such as buildings or urban areas, is specified by looking at the expected lifestyle of future users. The vision of a design science as a 'comprehensive anticipatory science' suggested by Buckminster Fuller is an example in this direction (see Meller, 1970).

Anticipation can also be seen as a characteristic of the design process. As we discussed previously with reference to Smithers's (2002) observations, the process of design does not start with a de facto problem, but it essentially involves the generation of design solutions in anticipation of a correspondence between a design solution and the design problem or goal. Moreover, during the design process small changes to parts of a design artefact may result in large effects for the artefact as a whole. In engineering, social, or artistic design such effects may be desirable, but may also be costly or even damaging. It is therefore important to anticipate these effects in order to drive the design process towards solutions that avoid or augment them.

Characteristic reasoning patterns in design also entail an anticipatory capacity. For instance, abduction can be seen to involve describing the structure of a hypothetical device based on the expected behaviour of this device. The generation of such a description implies the capacity to see forward, namely to observe that the realization of such a description would produce the expected properties. Additionally, as we will see, reasoning about the structure of objects that satisfy certain functions (functional reasoning), or reasoning about the possible functions carried by objects (affordance reasoning), can also be seen to involve some sort of anticipation.

Finally, anticipation is discussed in relation to design as a general capacity or attribute of human intelligence (see Nadin, 2000). But let us now investigate in more detail how different design studies have incorporated different notions or aspects of anticipation.

7.3.2.1 Anticipation of emergent designs

One of the most fundamental topics in design research is creative design, which is often linked with the phenomenon of emergence. Is there a notion of anticipation related to this fundamental aspect of design? The answer is yes, although at first sight the relation seems to be a negative one. Emergence is commonly associated with a spontaneous discovery of some new attribute (form, structure, or function) of the design description or artefact, which has not been expected or anticipated. However, this view of emergence is increasingly being challenged within the design community.

For example, looking at visual emergence in design, Oxman (2002) puts forward a view of 'anticipated emergence' that contradicts the traditional definition. She suggests that the emergence of shapes is due to a process of resolution of shape ambiguities that relies both on perception, as well as cognition, and the ability to 'think with shapes'. According to her approach, emergence is therefore not accidental but it is canalised by high-level cognitive schemata, which guide the resolution of shape ambiguities.

Knight (2003) also examines the link between emergence and unpredictability and talks about the classification of emergent shapes in shape grammars into three classes: anticipated, possible and unanticipated. According to her view, anticipated emergence constitutes a key to analysis applications of shape grammars where the emergence of shapes is carefully predicted. Possible emergence involves the formation of conjectures about the emergence of shapes from (again intentionally) applied rules. Finally, unanticipated emergence, which plays an important role in conceptual and creative stages of design, involves the generation of shapes that are not premeditated in any way. Interestingly, Knight highlights that the classification of shapes into the aforementioned categories is 'relative to the knowledge and eye of the author or user of a grammar' (ibid: 135).

Likewise, starting from a classical example of visual emergence, where four squares are placed together so that a fifth square is produced from them, Brown (1998) notes that the appearance of the new shape is something that occurs even if we do have prior knowledge of the phenomenon: that is, emergence can be expected. On this basis he distinguishes between two types of emergence: the first, *directly identifiable emergence*, occurs when the identification of a new property can be traced back in the existing knowledge of a person, whereas the second, *indirectly identifiable emergence*, is linked to a discovery process of setting apart a property as interesting and hence worth remembering and classifying. Brown further suggests that identification of an emergent property comes about by way of analogical and/or functional reasoning, the latter being concerned with the use, or purpose, of a design artefact.

7.3.2.2 Anticipation in design agents

Working along similar lines, Gero (1998a; 2003) links the notion of emergence to that of situatedness, proposing that what one 'sees' is affected by both the situation he or she is in (what is 'out there') and the previous knowledge available, which guides what one is 'looking for'. A fundamental characteristic of his approach is the notion of constructive memory. The driving idea is that memory is not only constructed by experience, but it is also re-interpreted and re-constructed in the light of the present situation. The concepts of situatedness and constructive memory are used for the development of a model of designing (Gero and Kannengiesser, 2004, 2006). The model considers situation as something that incorporates three different kinds of environments, the external world, the interpreted world and the expected world, which are linked to one another through the processes of interpretation, focussing and action. Notably, in this framework the formation of expectations is considered fundamental for both the formation of internal representations and the construction of memories. The differentiation between external, interpreted and expected world is tellingly reminiscent of the most classical and fundamental perception of anticipation, which considers that a model of the external world (or environment) constructed within a system (here design agent) could be used to form expectations about future changes and guide current action.

The ability of agents to interpret and act in the external world by constructing internal representations of this world based on memories, experiences and expectations, is generally associated with the ability for reflective reasoning (or the notion of reflection in action advocated by Schön, 1983). Reflective reasoning is important not only for individual agents but plays also a significant role in the context of distributed design. For example, Brazier et al (2001: 137) argue that in distributed design where multiple agents need to combine their efforts to achieve a design solution, agents should be endowed with the ability to reason reflectively about additional aspects, such as the knowledge and experience of other agents, their expected actions and results, as well as the types and content of interactions.

Another example, where the focus on distributed design brings about the need for expectation formation, is found in Grecu and Brown (2000). The idea of anticipation is again put forward (although the word expectation is used instead) in relation to the necessity for agents to learn and evaluate advantages and disadvantages of a decision by predicting conflict situations or potential future goals and functions. The role of expectations is to compensate for restrictions related to time and information, which make it difficult to establish causal relationships between conditions that underline an event or action, and its results (ibid: 656-657). The authors hence suggest the use of two types of expectations to guide agent learning: expectations that act as substitutes for precondition information, and expectations that act as tools for inferring the effects of decisions.

It is worth noting that the model presented in Chapter 3 discusses design as a knowledge construction process that involves learning to anticipate other agents' reactions, but also learning to create new goals; a process that supports the co-evolution of problem and solution spaces. For more on this see also Zamenopoulos and Alexiou (2003a) where the notion of anticipation has been referred to as the 'memory of the future' (pp 193).

7.3.2.3 Summary and discussion

Anticipation appears as an important characteristic of design, whether it refers to understanding design as an anticipatory science, or to the understanding of the cognitive processes that partake in the perception and generation of artefacts. It is also decisively linked to phenomena such as creativity and emergence. However, the typical association of anticipation with design as a cognitive process generally indicates a focus on individual design agents as the unit of analysis. In this sense, design abilities are fully embedded and embodied in a human or artificial design agent. In the following treatment, we will try to explicate the conditions responsible for the capacity to design without commitment to the level (cognitive, social, etc) within which these are realised.

7.3.3 Towards an anticipatory theory of design

In this section, the meaning and role of anticipation in design is more specifically studied in relation to Rosen's (M, R) model. In particular, the objective is to identify the mathematical conditions that underlie the construction of an anticipatory representation.

This is an important objective for the understanding of the capacity to carry out design tasks. According to the proposed definitions at the beginning of this chapter, the capacity to construct representations of a phase transition to universality is a necessary and sufficient condition for the capacity to carry out design tasks. Indeed, according to these definitions, the capacity to construct representations of a phase transition to universality was explicitly expressed as a capacity to form anticipatory representations. It is the objective of this last section to show how 'the construction of representations of a phase transition to universality' is connected to Rosen's view of anticipatory representations, and on that basis to develop an (organizational-level) theory of the capacity to carry out design tasks.

The main theoretical and methodological contribution of Rosen's proposed (M, R) system can be summarized in two points. Firstly, it offers an abstract mathematical structure for identifying anticipatory systems (and living systems). Secondly, it proposes a resolution of the infinite regress that occurs in self-referential systems by specifying the conditions for operational closure (closure to efficient causes). As we saw, a system is closed to efficient causes if and only if every arrow is specified within the system. For the purpose of this study these are important theoretical and methodological results as they can help explain design as the capacity to hold an 'anticipatory representation' of universality.

In the sections that follow, the objective is to explicate the meaning of anticipation in the (M, R) model and show how this connects to design. In the second section, the objective is to explicate the main conditions that are responsible for the capacity to anticipate. Finally, in the third section, the objective is to generalize the main results and explicitly state the organizational conditions that entail the capacity to carry out design tasks.

7.3.3.1 Relating the notion of anticipation in the (M, R) model to design

Let us have another look at the (M, R) formalism, this time explicitly in relation to design. In order to avoid the cumbersome notation, but also for reasons that will be apparent in the next section, we assume that Rosen's results about the set B can generally be applied to the set A. On this basis, anticipation is defined as: *the capacity -embedded in* A- *to represent an arrow F:*A→B *(i.e. efficient cause) in preparation for certain effects in* B (von Glasersfeld, 1998).

$$A \xrightarrow{?F} B$$

Rosen's (M, R) system effectively offers a mathematical interpretation of this view of anticipation and an explanatory framework of how this anticipatory capacity is possible. But, what does the expression 'in preparation for' mean precisely? And moreover, how does this view of anticipation relate to the definition of anticipatory representations given at the beginning of this chapter?

In order to address these questions, let us make the following assumptions. Let us assign the letter A to denote a language that generates a set of expressions (or Intentional states) s regarding the properties of an objective reality. The arrow *F* is then a functor that generates a family of objects $F \circ s$ that satisfy the properties expressed by s. Let us also define an opposite functor $(\_)^F$. Given an object d, the expression d^F is then the -minimal- description of the properties of a specific object d. For example, the language A might generate expressions regarding the connectivity and topological relations between activities in a building. The language B might generate descriptions of geometric objects and their relations (e.g. floor plan configurations). Then the arrow *F* translates topological relations into a possible geometry. Consider now two more arrows. Given a theory of an objective reality represented by an Intentional state s, it is possible to deduce the properties of an object d^F (i.e. $m{:}s \to d^F$), and respectively, from a family of models, $F \circ s$, it is possible to induce an object d that satisfies the deduced properties d^F (i.e. $i{:}F \circ s \to d$) as the following diagram suggests:

$$\begin{tikzcd}
 & \text{Language A} & \text{Language B} & \\
\text{An expression s} & s \arrow[r, "F^\circ"] \arrow[d, "m"] & F \circ s \arrow[d, "i"] & \text{A family of objects that satisfy the expression s} \\
\text{An expression of the properties of d} & d^F & d \arrow[l, "(\_)^F"'] & \text{A design object } d
\end{tikzcd}$$

The arrow *F* is determined so that if the description of an object d is derived by a family of possible objects $F \circ s$ (via the arrow $F \circ s \to d$), then the properties of the specific configuration d can be deduced by an expression s in A (via $s \to d^F$). Namely, the functor *F* is constructed by a process where for each arrow $F \circ s \to d$, there is unique arrow $s \to d^F$. This situation is formally expressed by the bijection $A(s,d^F) \cong B(F \circ s,d)$.

The expression 'in preparation for' then more precisely means that the expressions s (or d^F) in a language A have the capacity to represent the bijection $A(s,d^F) \cong B(F \circ s,d)$. For that purpose, Rosen's key idea is that any expression s in A is a representation of a function φ_f:B→H(A,B). For any object d in B, the function φ_f:B→H(A,B) is then a representation of a functor from A to B that satisfies the bijection A(s,d^F)≅B(F∘s,d). The dotted arrows in the following diagram are meant to depict this idea:

$$\begin{tikzcd} A \arrow[r, "F"] \arrow[r, bend right=60, dashed] & B \arrow[r, "\varphi_f"] & H(A,B) \arrow[ll, bend right=60, dashed] \end{tikzcd}$$

This view of anticipation can be perceived in relation to the definition presented in section 7.1.3.2. The arrow φ_f:B→H(A,B) effectively 'bounds' or drives the system in the bijection A(s,d^F)≅B(F∘s,d). More specifically, the arrow φ_f:B→H(A,B) realizes a transition to a universal state where for every arrow F∘s→d there is a unique arrow s→d^F (i.e. A(s,d^F)≅B(F∘s,d)). Rosen observes that this representation is mathematically possible under certain conditions. The purpose of the next sections is to clarify these ideas and the underlying mathematical conditions. The precise mathematical justification of these conditions can be found in Letelier et al (2006). So, let us start with the general meaning of the proposed conditions and how they relate to design.

7.3.3.2 An interpretation of Rosen's main results on anticipation

Rosen's main idea is that *the capacity to anticipate is possible if and only if there is an embedding of the set* A *into the set of arrows* φ_f:B→H(A,B). The set of arrows φ_f:B→H(A,B) is denoted H(B,H(A,B)) and the embedding by the arrow e:A→H(B,H(A,B)). The embedding e:A→H(B,H(A,B)) explicates the idea that the elements of the set A are anticipatory representations of the universal state A(s,d^F)≅B(F∘s,d). The mathematical conditions that specify the embedding e:A→H(B,H(A,B)) are therefore the mathematical conditions that explain the construction of anticipatory representations.

In order to explain these conditions, note that the set of arrows H(B,H(A,B)) in the embedding e:A→H(B,H(A,B)) is the 'opposite' set of arrows from the set H(H(A,B),B). This letter set is an important set because it holds a special type of arrows; the *evaluation arrows*. Assuming that A and B are sets, an evaluation arrow ε_S is a function ε_S:H(A,B)→B that gives for every function *F* in H(A,B) the value of *F* at s in B, that is:

$$\begin{tikzcd}
\varepsilon_S\colon H(A,B) \arrow[r] \arrow[d, dash, dotted] & B \arrow[d, dash, dotted] \\
F \arrow[r, mapsto] & \varepsilon_S(F)=F\circ s=d
\end{tikzcd}$$

More generally, the evaluation arrow is a universal arrow ε_S:$d^S\circ s$→d such that the following diagram commutes:

$$\begin{tikzcd}
H(A,B) & F \arrow[d, "m"'] & B & F\circ s \arrow[d, "i"'] \arrow[dr, "F\circ m"] & \\
& d^S & & d & d^S\circ s \arrow[l, "\varepsilon_S"']
\end{tikzcd}$$

Every object s in A is therefore also a 'generator' or a 'program' that produces an object d in B for a given *F* in H(A,B). More importantly, there is a unique correspondence between objects s of A and the universal arrow ε_S:H(A,B)→B. If A and B are sets, then there is a one-to-one correspondence between the elements of the set A and the evaluation arrows ε_S:H(A,B)→B. Based on this observation, Rosen's main result is that *the embedding e*:A→H(B,H(A,B)) *is defined when the evaluation function* ε_S:H(A,B)→B in H(H(A,B),B) *is invertible (one-to one)*. In that way, the elements of the set A 'hold information' about the function *F* only by looking at the value of *F* at s. Based on this information, every element s of A determines a function *F* in preparation of a certain value d in B such that $A(s,d^F)\cong B(F\circ s,d)$ (i.e such that $F\circ s=d$ for $s=d^F$). This is very meaning of anticipation. Rosen's condition can be given a weaker interpretation: *the embedding e*:A→H(B,H(A,B)) *is defined when for each arrow* $F\circ s$→d *there is an opposite universal arrow a*:d→$d^S\circ s$ *that makes the following diagram commute.*

$$\begin{tikzcd}
H(A,B) & F \arrow[d, "m"'] & B & F\circ s \arrow[d, "i"'] \arrow[dr, "F\circ m"] & \\
& d^S & & d \arrow[r, "a"] & d^S\circ s
\end{tikzcd}$$

In this weaker version, the arrow $a{:}d\rightarrow d^S\circ s$ is not *necessarily* the inverse of the evaluation arrow $\varepsilon_S{:}d^S\circ s\rightarrow d$. The postulated isomorphism $d^S\circ s\cong d$ is therefore only a special case. Nevertheless, the arrow $a{:}d\rightarrow d^S\circ s$ holds information about the evaluation function $\varepsilon_s{:}d^S\circ s\rightarrow d$ by specifying the relation between the arrows m, i and a as suggested by the above diagram (i.e. $F\circ m=a\circ i$). This condition can be alternatively discussed directly within the categories A and B. For this purpose recall that the adjunction $A(s,d^F)\cong B(F\circ s,d)$ implies that for each arrow $F\circ s\rightarrow d$ there is an Intentional state s that has the capacity to 'deduce' the properties of the object d; that is $s\rightarrow d^F$. Note also that the object d^F (also denoted $F\rightarrow d$) can be perceived as an object of the set H(H(A,B),B). Rosen's proposed condition can then be interpreted as a request for the existence of the 'opposite' object F^d (or $d\rightarrow F$) in H(B,H(A,B)): *The embedding* $e{:}A\rightarrow H(B,H(A,B))$ *is defined when for each arrow* $F\circ s\rightarrow d$ *there is an arrow* $m{:}s\rightarrow F^d$ *that makes the following diagram commute*:

$$\begin{tikzcd}
s \arrow[d, "m"'] & F\circ s \arrow[d, "i"'] \arrow[dr, "F\circ m"] & \\
F^d & d \arrow[r, "a"] & F\circ F^d
\end{tikzcd}$$

For the purpose of making the connection with design, let us briefly examine the nature of these arrows (the evaluation arrow and its opposite arrows) in relation to deductive and abductive reasoning (for a discussion of abduction in design see March, 1976; Roozenburg, 1993). Deduction simply implies that for any expression s in A there is a rule $F\rightarrow d$ (i.e. d^F) such that the result d can be deduced from the application of the condition F and the rule $F\rightarrow d$. So, the evaluation arrow $\varepsilon=ded{:}F\circ d^F\rightarrow d$ that appears in the universal constructions (see Figure 48 below) represents the very meaning of deduction. The existence of the opposite arrow

$abd\colon d \to F\circ d^F$ can be interpreted as an abduction; in the sense that certain conditions F and the rules $F \to d$ can be derived from an (anticipated) configuration d (see Figure 49 below).

Given $F \to d$ (i.e d^F)
And F
Deduce
d

$$\begin{tikzcd} s \arrow[d, "m"'] & F\circ s \arrow[d, "i"] \arrow[dr, "F\circ m"] & \\ d^F & d & F\circ d^F \arrow[l, "ded"'] \end{tikzcd}$$

Figure 48: The relation between deduction and the evaluation arrow in universal constructions.

Given d

Generate
F, $F \to d$ (i.e d^F)

$$\begin{tikzcd} s \arrow[d, "m"'] & F\circ s \arrow[d, "i"] \arrow[dr, "F\circ m"] & \\ d^F & d \arrow[r, "abd"] & F\circ d^F \end{tikzcd}$$

Figure 49: The relation between abduction and universal constructions.

The condition formulated by Rosen interestingly suggests that an anticipatory capacity is necessary for abductive reasoning, and this directly impacts on our understanding of design. Anticipation can be perceived as a generic capacity that underlies the realization of various reasoning patterns in design. To explore this more, let us assume that A is a language that specifies a theory of possible behaviours of a device, and B is a language that specifies its structural components. Anticipation is the capacity to specify the structural components in preparation for a specific behaviour. If A specifies a desired behaviour (or function) then the above reasoning pattern is in fact functional reasoning. If on the other hand A specifies a theory of devices, and B a theory of functions (or a theory of potential functions derived by user-device interaction), then the above reasoning pattern is affordance-based reasoning. For a more detailed discussion of functional and affordance-based reasoning in design see Brown and Blessing (2005) and Maier and Fadel (2002). Note that in general, the (anticipatory) specification of a model of a theory - in preparation for a certain effect in B - may be seen to refer to the description of an artefact (its structure, behaviour, function), or, equally, to the description of the process employed in order to complete a design task.

7.3.3.3 Design as the capacity for 'world-to-mind' and 'mind-to-world' adaptation

Up to this point the emphasis was on the identification of design with an abstract structure that is closed to efficient causes. Yet, there is still something missing. At the beginning of this chapter, the capacity to design was identified with the capacity to hold anticipatory representations of universality with a two-directional degree of freedom, including 'world-to-mind' adaptations where there are transformations over an objective reality d_D to d_U, but also 'mind-to-world' changes where there are transformations from intentional state s_D to an intentional state s_U. If we go back to Rosen's original diagram (Figure 50) we can see that expressions s in A remain un-entailed: namely, changes of expressions in A are still to be dictated by the 'structure' of the environment E. It is clear that an additional closure condition needs to be formulated; this is the entailment of A.

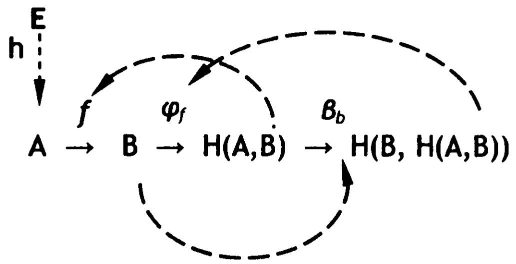


Figure 50: In the (M, R) model the set A remains un-entailed (i.e. specified by the environment E of the system).

The proposition put forward here is to postulate the existence of two parallel structures realized by the embedding e_A:A→H(B,H(A,B)) and the embedding e_B:B→H(H(A,B),A) as the following diagrams suggest (Figure 51).

$$\begin{tikzcd}
H(B,H(A,B)) & A \arrow[l, "e_A"'] \arrow[r, "F"] & B \arrow[dl, "\varphi_f"] \\
& H(A,B) \arrow[ul, "\beta_b"] &
\end{tikzcd}
\qquad
\begin{tikzcd}
A \arrow[r, "F"] & H(H(A,B),A) & B \arrow[l, dashed, "e_A"'] \arrow[dl, "\varphi_f"] \\
& H(A,B) \arrow[ul, "\beta_b"] &
\end{tikzcd}$$

Figure 51: Conditions for the capacity to carry out design tasks.

These conditions can be generally inferred and explained by supposing that Rosen's results about B are also applied to A. Given this assumption, any intentional state s_D in A represents an arrow φ_f in H(B,H(A,B)), and any object d in B represents an arrow β_b in H(H(A,B),A). The embedding e_A:A→H(B,H(A,B)) implies the capacity of an intentional state s in A to represent a functor *F* (and therefore specify an objective reality d in B) in preparation of certain intended properties in B (Figure 52).

$$\begin{tikzcd}
A \arrow[r, "F"] \arrow[rr, dashed, bend right, "e_A"'] & B \arrow[r, "\varphi_f"] & H(A,B) \arrow[ll, dashed, bend right]
\end{tikzcd}$$

Figure 52: Illustration of the conditions that explicate the capacity to change an objective reality in B in preparation of a certain intentional state s in A.

The embedding e_B:B→H(H(A,B),A) implies the capacity of an object d in B to represent a functor B_b (and therefore specify an intentional state s in A) in preparation of certain properties in B (Figure 53).

$$\begin{tikzcd}
H(A,B) \arrow[r, "\beta_b"] \arrow[rr, dashed, bend right] & A \arrow[r, "F"] & B \arrow[ll, dashed, bend right, "e_B"']
\end{tikzcd}$$

Figure 53: An illustration of the conditions that explicate the capacity to change an intentional state s in A in preparation of a certain objective reality in B.

7.3.3.4 Summary and discussion

As a summary consider that a functor F is in principle a (weak) theory functor ***Th***. Similarly, an object d^S in A represents the underlying properties of the object d in B. The capacity to design then refers to the capacity of an Intentional state s to hold models and interpretations that mark a transition to universal models $\eta_s{:}s_U \rightarrow \mathit{UThs}_U$. The core meaning of 'world to mind' adaptation refers to the existence of a Intentional state s that represents the transition from a weak theory Ths_w in B to a functor Th_U in H(A,B) such that $A(s_U,\mathit{U}d_U) \cong B(\mathit{Th}s_U,d_U)$. The core meaning of 'mind to world' adaptation refers to the specification of an objective reality that represents a transition from a weak theory function Th_D in H(A,B) to an Intentional state s_U in A such that $A(s_U,\mathit{U}d_U) \cong B(\mathit{Th}s_U,d_U)$.

This 'universe' specified by an objective reality B and a subjective reality A has the capacity to represent and carry out design tasks if and only if the embeddings $e_A{:}A \rightarrow H(B,H(A,B))$ and $e_B{:}B \rightarrow H(H(A,B),A)$ exist (i.e. the universe has a structure described by diagrams in Figure 51). The embeddings $e_A{:}A \rightarrow H(B,H(A,B))$ and $e_B{:}B \rightarrow H(H(A,B),A)$ are then defined when for each Intentional state s and each arrow $\mathit{Th}s \rightarrow d$ there is an (opposite) universal arrow $a{:}d \rightarrow \mathit{ThU}d$ such that the following diagram commutes:

$$\begin{tikzcd} s \arrow[d, "m"] & \mathit{Th}s \arrow[d, "i"'] \arrow[dr, "\mathit{Th}(m)"] & \\ \mathit{U}d & d \arrow[r, "a"] & \mathit{ThU}d \end{tikzcd}$$

The treatment so far offers some important results. First, the proposed diagram encodes the conditions that are responsible for the capacity to carry out design tasks as presented at the beginning of this chapter. This gives a characterization of design using the paradigm of anticipation which will be used in the next chapter in order to build a mathematical description of the 'design universe' (i.e. of the space within which design is carried out). Second, the treatment allows us to understand more about the meaning and role of anticipation in design. The use of abstract structures, and the general methodological approach, provides a precise basis for discussing the role of anticipation in design at different levels. This includes the study of design as a distinct reasoning pattern, but also as a distinct epistemological concept comparable to that of machine, evolution, or control.

7.4 SUMMARY AND DISCUSSION

This chapter was concerned with describing and explaining design at an organizational level using the category theoretic concepts and constructions previously introduced. The first part of the chapter was focussed on design Intentionality and the mathematical definition of the task and capacity to design. This was achieved by explicating the type of theories, models and sketches involved in the design process. In particular, a sketch of a design situation was defined as a two-directional Intentional state which includes both world-to-mind adaptations as well as mind-to-world changes. The gist of the treatment was that design arises when there is a weak adjunction between theories and models, and therefore the task of design as an Intentional state involves building anticipatory representations of this adjunction (or transition to universality).

The chapter also included a discussion of the limitations of current paradigms for the study and representation of design. More specifically, the concepts of computation, evolution and control were examined as alternative theories of the capacity to design. Using the mathematical notation established in this thesis, it was shown that these paradigms assume the existence of complementarity between theories and models and therefore cannot entail the capacity to form anticipatory representations. This further motivated the need for a meta-level theory of design able to determine the organizational conditions that explain design capacity as a capacity to form anticipatory representations of a phase transition to universality.

The third part of the chapter identified and introduced the fundamental organizational conditions that can explain the capacity to carry out design tasks. The key issue in this respect was the identification of the mathematical structures that underlie the construction of anticipatory representations of universality. To that end, the concept of anticipation as defined by Rosen was used as a critical conceptual and methodological tool. In specific, using and extending Rosen's results, the study explicated the mathematical conditions that underlie the description of design capacity as an anticipatory representation of universality.

The current treatment opens a discussion of the possibility to uniquely identify design on the basis of an abstract structure - just like Rosen did for living and anticipatory systems. But why is this endeavour useful? The answer is in fact three-fold. First, the anticipatory view of design enables us to identify and address the limitations of other paradigms used in design research. It is hoped that the proposed view can be particularly useful in understanding the capabilities and limitations of computer-based realisations, which largely assume the machine paradigm. Second, the methodological approach facilitates the study, comparison, and possibly unification of different perspectives on design. In particular, the proposed formalism delineates the conditions for identifying design as a distinct task and capacity, without committing to a specific interpretation of the processes, knowledge or entities involved in design. Finally, the proposed abstraction can also be used as an explanatory tool. The study makes a step towards a theory of design which is developed by looking at the question of 'how design emerges' aiming to define the conditions that explain the capacity to design.

Chapter 8

SKETCHING THE UNIVERSE OF DESIGN

The main assumption behind the thesis is that a number of environments such as the human brain, the mind, or society – each considered as a 'place' where design abilities are developed - have in common certain characteristics that determine their capacity to appreciate, represent and carry out design tasks and which are mathematical in nature. In this sense, the 'universe of design' is thought of and treated as a mathematical universe that underlies the physical realization of any system within which design tasks and activities are realized. This chapter aims to develop a mathematical presentation - a 'sketch' - of this universe.

This endeavour must be perceived in relation to the previous chapter. The previous chapter focused on the identification of the mathematical properties and conditions that uniquely distinguish the problem and capacity to design as an effect of complexity and organization. For the purpose of this chapter, the identified properties and conditions make up the underlying principles of the proposed type of mathematical structure. The main purpose of this chapter is therefore to design a new type (or 'species') of mathematical structure able to satisfy the postulated properties and therefore explicate the mathematical nature of the design universe.

8.1 AN ORGANIZATIONAL LEVEL THEORY OF DESIGN: OVERVIEW

This section is a brief overview of the proposed *organizational level theory of design*. The proposed theory sets out the organizational principles that are responsible for the capacity to recognize and carry out design tasks. The main premise is that the organization of certain environments - including the neurological, symbolic or social organization of cognitive agents - must have certain properties of a mathematical nature that determine their capacity to appreciate, represent and carry out design tasks. In this sense, although it is assumed that there must be a physical basis to the capacity to recognize and carry out design tasks, there is no commitment as to whether this is a neurological, cognitive or social capacity. The terms 'universe' and 'complex system' are used interchangeably to imply any plausible physical realization of design abilities. The aim of the overview is to clarify the underlying premises of this theory and present the rationale behind the proposed mathematical theory of the *design universe*.

8.1.1 Sketches of the universe of design

The precise meaning and representation of organization in this thesis is perceived in relation to the mathematical concept of a sketch. This idea was elaborated thoroughly in Chapters 5 and 6 where a *sketch* was introduced *as a symbolic or sub-symbolic system that specifies the organizing principles of the universe within which certain entities are generated.*

For the sake of the argument, let us reiterate the gross distinction between objective and subjective reality that is important for this notion. According to Lawvere and Schanuel (1997: 84), it is possible to make the distinction between an *objective reality* that incorporates objects that exist in a given universe U, and a *subjective reality* that incorporates objects in a universe U with the special property to reflect objective reality. *Reflection* represents in this sense the 'attitude' (e.g. beliefs or desires) of the subjective reality towards an object in U. A *sketch is a subjective reality (e.g. a symbolic representation, neurological or societal structure) that reflects the properties of a family of objects and their models.* A sketch is therefore perceived as a construction that specifies the interplay between the properties of the described objects (all models of an objective reality) and the properties of description (the theory of an objective reality). For instance, in the context of architectural design a sketch is a depiction of the architectural principles, or architectural 'language/theory', of a desired building (a subjective reality), within which the geometrical properties of a future-real building (an objective reality) are specified. The mathematical concept of sketch is a conceptual analogue of this idea.

A mathematical sketch specifies the principles of a mathematical theory within which all possible models of the theory are defined. For the purpose of this study, the notion of sketch aims to express the organizational principles of a universe within which design tasks are defined. The main focus of the proposed treatment is then the identification of the type of sketch that has the expressive power to describe the organizational qualities that are responsible for the reflection (i.e. recognition, representation and execution) of design tasks.

8.1.2 The organizational approach

The question of identifying the type of sketch that underlies a design universe is effectively an enquiry regarding the nature and role of linguistic complementarity (i.e. between descriptions and interpretations) or dynamical complementarity (i.e. between structures and behaviours) of a system. This incorporates a number of relevant questions like for instance: What is the relation between the generated behaviour (i.e. the formation of a pattern of activities observed within a network) and the formation of structures (i.e. network organization) that generate and constrain an observed behaviour? Or, what is the relation between generating interpretations (i.e. semantics) of a system, and generating descriptions (i.e. syntax or grammar) of the system?

The distinction between behaviours and structures, or between interpretations and descriptions, is mathematically treated as a distinction between *models* and *theories of a universe*. A sketch is then defined as an 'archetypical' model that represents the principles of a theory (and therefore the principles of all models that satisfy the theory). So, what are the particular characteristics of a sketch of a design universe? More specifically, what is the special relation between a theory and models of a design universe, and what type of sketch (i.e. organizational principles) induces such a relation?

For addressing the above questions, there are two important issues to be discussed. The first issue is concerned with the *limitations* of linguistic or dynamical complementarity. The core question is whether there is a complementary relation between models and theories (i.e. between behaviour and structures, or between interpretations and descriptions). A *complementary relation* is a special but very important relation: it is a relation between two structures that co-determine an irreducible unity (like for example the relation between a 'key and a keyhole'). Such a relationship implies that the properties of models are uniquely deduced by the theory while the theory is uniquely induced by the set of models. In mathematical terms, this relation is treated as an adjunction.

The second issue is concerned with *the role* of such complementary relations. The core question is whether a complementary relation between theory and models is *a constraint that controls* the formation of models and theories; whether it is a product and *emergent state that drives* the formation of models/theories; or whether it is a *critical state that determines* a phase transition in the behaviour of a system. In the following, let us briefly explore the organizational qualities of different types of universes by looking at the role of complementary relations.

8.1.3 The role of complementarity in a design universe

Complementary relations play a fundamental role in the natural world, as well as in mathematical and artificial worlds. They express logical principles that underlie the understanding of a plethora of problems such as for instance: computation, information flow, biological replication, and scaling of multi-level complex organizations. Let us therefore recall the role of complementarity as it appears in different types of universes that are characterized by different types of capacities.

In *a machine universe*, the relation between patterns (or interpretations) and rules (or syntax) must be a complementary one: rules generate the patterns and the patterns are manifestations of the rules. In a machine world, a language generates expressions (i.e. a theory) that are interpreted by a certain machine (i.e. a model); while the expressions that are readable by a certain machine determine the type of language that generates such expressions. A machine theory describes/explains the capacity to make valid inferences given such complementary relations. A sketch of a machine universe is therefore a type of sketch that induces a complementary relation (an adjunction between models and interpretations of theories, or between semantics and syntax).

In an *evolutionary universe*, complementary relations are also pivotal. Complementary relations effectively control replication and constrain the selection process. In an evolutionary environment, complementary relations enable the transition of information when for instance a type of molecule plays the role of a mould or template for the production of others. This template-mediated replication is the origin of fitness: the fitness of an organism is the result of the capacity to find complementary relations (and therefore reproduce) when is situated in a certain environment. In this sense, complementary relations work as templates, but also as constraints that control the selection process. For a discussion on the role of complementary relations in evolutionary dynamics see for instance Michod (1999: 19). A sketch of an evolutionary universe is therefore a type of sketch where complementary constructions represent constraints that drive natural selection.

Similarly, in a *cybernetic universe*, complementary relations are the very meaning of organizational invariance that a control process aims to preserve. A sketch of a control universe is a type of sketch where complementary relations represent the boundary conditions for the survival of an organism.

In *a design universe,* the principle of complementarity between theories and their models has a fundamentally different function. The ‘phenomenon’ of design arises within a universe U where complementary relations are anticipated emergent states. This means that within a design universe the unity between theories and their models is not a universal principle, but a critical state that determines the boundaries and direction in the transition between two qualitatively different worlds: a transition from a non-deductive to a deductively presentable universe. Take as an example the 'phenomenon' of design in human societies. In this context, the need to design arises when social values lead to 'theories' *about* the principles of a society that cannot justify the occurred 'models' of social action (including patterns of organization, institutions, processes etc). Similarly, from the perspective of a distinct cognitive agent, the 'problem' and 'phenomenon' of design arises when beliefs and desires lead to theories *about* the environment of the agent that do not correspond to the description of the environment itself. In these situations, *the problem or task of design* is identified with the representation of a unity between theories and models as an attracting or *anticipatory state*. Note that the unity between theories and models is not a specific state, but rather a *type* of attracting state determined by the satisfaction of the condition of complementarity. In response to this situation, *the capacity to design* is determined by the ability to construct theories in relation to models that lead to complementarity. A sketch of a design universe represents the formation of an anticipated complementary relation. In conclusion, the complementarity between theories and models is expressed in relation to a *weakly defined* unity between theories and models and (in opposition to the machine and evolutionary universes) plays the role of a critical (rather than a universal) state.

8.1.4 Some subtleties: sketches of design and design sketches

There are some subtleties around the use of the notion of sketch that is essential to clarify. It is important to distinguish between the *object of this study* (i.e. the notion of sketch as an Intentional state, as a 'design sketch') and the *language/method of the study* (i.e. the development of a mathematical sketch of the design universe). The object of the study is the organizational qualities of complex systems and particularly the specification of a symbolic or sub-symbolic 'sketch' that describes and explains the capacity to design. This should not be confused with the language and method of the study which is the 'category theoretic concept of sketch'. The category theoretic concept of sketch is (by its very nature) a symbolic mathematical system with certain structure and properties. It follows from the analysis in Chapters 5, 6 and 7 that the mathematical properties of the category theoretic sketch do not have the expressive power to capture the properties of the universe of design. For this purpose, a weaker type of sketch was developed in Chapter 6. The current chapter takes the proposed type of sketch and uses it to define the category of design.

Following these remarks, it is also important to make a distinction between two levels of description: the *description of the objects described in the design universe U* and *the description of the design universe U*. The former refers to descriptions of the properties of 'objects' generated by the 'subject' ('a designer'). The latter refers to descriptions generated by a 'researcher of design' and concerns the properties of the 'subject' of a design study.

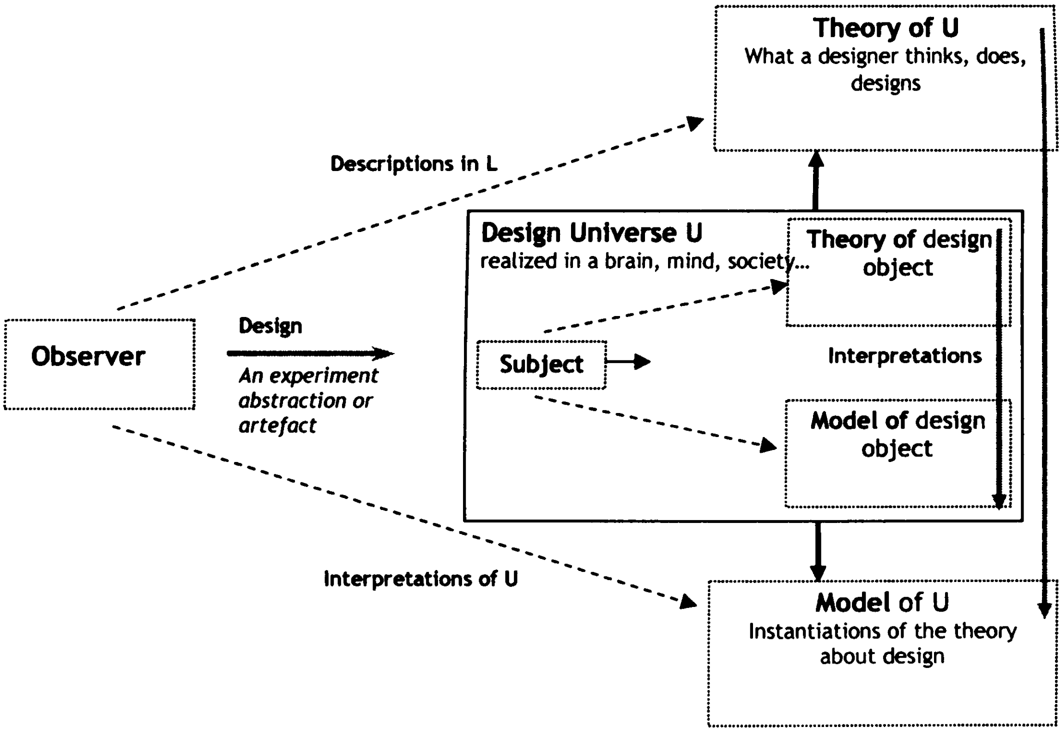


Figure 54: This is an illustration of the relation between *a sketch of a design universe* (outer Observer-Theory-Model triangle) and a *sketch of a design object* (inner Subject-Theory-Model triangle). *A sketch of the design universe* aims to device theories and models about a designing mind, brain, society...etc, while the *sketch of a design object* aims to device theories and models about a design object.

More specifically, *descriptions of the objects described in a universe* U are expressions realized by a cognitive, social or artificial 'designer' that lead to the formation of a theory about a designed object in U (the inner triangle in Figure 54). The *descriptions of the design universe U* are expressions in a meta- language L (i.e. the category theoretic language) that lead to the formation of a theory about the subject and/or object of design (the outer triangle in Figure 54). To connect this discussion to Chapter 4, the inner triangle refers to domain knowledge generated by a designerly way of thinking and leads to the formation of a theory about the domain of design (such as architecture or engineering). The outer triangle refers to knowledge generated by the 'designing' of a universe within which design objects and processes are realised and leads to the formation of a theory about design.

Based on this distinction (between an object-language U and meta-language L) several issues are brought to the fore. In a design universe, the *properties of objects in U* are assumed not to be fully describable within U; that is, the linguistic or dynamical complementarity of the language U is weak and defined only as a critical state. Therefore, the meta-language L needs to have the expressive power to capture the properties of a universe with non-describable objects. The proposed mathematical sketch S of a design universe is therefore required to describe a mathematical structure (i.e. a type of category ***Th***S) whose 'deductive properties' (i.e. category theoretic structure) arise only as a critical (non-universal) state.

Another question emerges in the context of design research by the distinction between *sketches of design* (i.e. sketches *of* the universe of design U including sketches of design thinking, design knowledge or design-led societies) and *design sketches* (or sketches expressed *in* U, including sketches of design artefacts and processes). In the literature, it is often assumed that there is some sort of correspondence between the two. For instance, Goel (1995) argues that there is a correspondence between the symbol system of the designer's mind states and the representation of a drawing. Likewise, Lawson (2004) assumes that there is a correspondence between design knowledge and the structure of a design representation that is made in a drawing. Again the same type of problem arises: that is, should the properties of representations (e.g. the symbolic system of a designer's mind) share the same properties with the 'intentional object' of representation (e.g. an amorphous and ambiguous sketch of a design object)?

As a concluding note, it is useful to point out that this chapter is focussed on the development of a meta-language L aimed at capturing the properties of the design universe; whereas Chapters 6 and 7 were focussed on the properties of the theories, models and sketches generated in a design universe. The principles of the *universe of design* are identified with a category theoretic structure that is fully constructed only in relation to a critical state. The proposed (weaker) category theoretic structure - that is explicitly laid out in the following section - constitutes the language for *a* mathematical *theory of design*. So, in the following chapter the three terms (universe of design, category of design and theory of design) are used interchangeably.

8.2 THE ORGANIZATIONAL PRINCIPLES OF THE UNIVERSE OF DESIGN

In this section, the focus of attention is turned into the principles of the (mathematical) universe U within which the task and capacity of design are defined. The intention is to develop a mathematical language and theory that has the expressive power to describe the organizational properties of the universe of design. To do this it is necessary to understand what is required for designing such a mathematical structure. The following treatment offers a rather informal presentation of the mathematical principles of the design universe in preparation for the formal presentation in the last section of the chapter. The treatment is divided in two parts. The first part draws the general properties that uniquely characterise the universe of design. The second part introduces the underlying principles of mathematical structures that explain the capacity of the universe U to represent and carry out design tasks.

8.2.1 General properties of the universe of design

As a consequence of the definitions of the task and capacity of design (in Chapter 7), it can be argued that the universe of design has three fundamental general properties: it is distributed, it is anticipatory and it is reflexive. Each of these properties is considered in detail below.

8.2.1.1 The universe of design is distributed

The main premise of this study is that design arises in a situation where theories cannot fully describe/manifest the properties of their models. This situation can be thought as a result of inherent limitations of reflection. The problem of reflection has been intensively studied in logic and mathematics mainly by looking at the consequences of self-reference and 'reflexivity of reflection' (Bartlett, 1992). There are many equivalent results in the studies of Cantor, Godel, Turing or Tarksi regarding such limitations of reflection. For the purpose of this study these results will be seen as a consequence of the 'incompleteness of description' (Löfgren, 2004): *there is no language that has its interpretations completely described by the language itself (i.e. we can not describe a universe in the universe itself).*

The limitations of reflection imply that in a design universe there is no universal theory of design. In order to understand this statement it is instructive the ask whether there is a universal model $m{:}s \rightarrow Ud_D$ of a sketch S that is a component in a phase transition to universality

$T_C=C(T_s,UT_d){:}C(s,Ud) \rightarrow C(s_U,Ud_U)$ and $T_D=D(ThT_s,T_d){:}D(Ths,d) \rightarrow D(Ths_U,d_U)$

such that the following diagrams commute:

$$\begin{tikzcd}
s \arrow[r, "T_s"] \arrow[d, "m"'] & s_U \arrow[d, "m'"'] \\
Ud \arrow[r, "UT_d"] & Ud_U
\end{tikzcd}
\qquad
\begin{tikzcd}
Ths \arrow[r, "ThT_s"] \arrow[d, "i"'] & Ths_U \arrow[d, "i'"'] \\
d \arrow[r, "T_d"] & d_U
\end{tikzcd}$$

$$\begin{tikzcd}
D(\mathit{Th}s, d) \arrow[rrr, dashed, "T_D"] \arrow[ddd, "\tau_D"'] \arrow[ddr, "\varphi"] & & & D(\mathit{Th}s_U, d_U) \arrow[ddd, "1_{D(Ths,d)}"] \arrow[ddl, "\varphi'"'] \\
& C(s, Ud) \arrow[ul, "\theta"'] \arrow[r, dashed, "T_C"] \arrow[d, "\tau_C"] & C(s_U, Ud_U) \arrow[urr, "\theta'"'] \arrow[d, "1_{C(s,Ud)}"'] & \\
& C(s, Ud) \arrow[dl, "\theta"] \arrow[r, dashed, "T_C"] & C(s_U, Ud_U) \arrow[drr, "\theta'"] & \\
D(\mathit{Th}s, d) \arrow[rrr, dashed, "T_D"'] & & & D(\mathit{Th}s_U, d_U)
\end{tikzcd}$$

It is easy to prove that there is no such sketch s that induces a universal theory *Th*s; in other words, there is *no universal* phase transition to universality. This statement is a consequence of the very concept of design and the inherent limitations of reflection. If the design task was fully describable in a universe U (and therefore there was a universal model of design tasks *m*:s→*UTh*s) then the theory of design *Th*s would be isomorphic to the universal theory *Th*s$_U$. But then this contradicts the main premise behind the definition of a design task which demands that the task of design should arise when there is 'within a weak adjunction'. The realm and task of design is therefore not fully describable within the universe U. In this sense, it is said that *the realm and task of design is distributed*. This property was more formally described in Chapter 6 where *distribution* has been linked with the 'relaxation' of the requirement to have a universal arrow *for every* sketch s in C (see Chapter 6 section 6.2.2.2). The type of knowledge that is pertinent to design is therefore by its very nature fragmented or distributed to (in principle) infinitely many theory-model approximations.

8.2.1.2 The universe of design is anticipatory

The distribution of design knowledge implies that the universe of design is open; with no clear boundaries between subjective and objective reality. We have discussed in Chapter 7 that design capacity is tied to the capacity of forming anticipatory representations of a phase transition to universality (where subjective and objective reality become complementary). The mathematical structure of the universe of design in this sense should be such that the complementarity between descriptions and interpretations is a critical rather than a universal construction. This universe would then collapse to a typical deductive structure when the critical state is achieved.

For that purpose, the core idea is to leave theories and models in U to be constructed to some degree independently from one another towards a critical state of complementarity. More specifically, the independence between the formation of descriptions and the generation of interpretations means that the expression of a theory and the expression of the properties of models involve certain 'residual' entities or 'meta'-entities. Similarly, a broken dynamical complementarity would imply that a structure S within a universe U contains sub-structures that generate behaviour patterns in U, but also 'residual' or 'redundant' sub-structures with no effects on the produced patterns. These entities are 'naturally' derived as an error, noise (or as ambiguity) because of the lack of complementarity between theories and the described properties of the models. The construction of a *'weak sketch'* presented in Chapter 6 is a realization of this idea.

In these examples the identified 'residuals' play a double role. First, they are involved in the expression of theories and the properties of their models (as noise or ambiguity); and second, they are implicit measures of distance from an *anticipated* theory-model complementarity. In this latter sense, the proposed mathematical universe U has an 'anticipated' deductive (category theoretic) structure defined only as a critical (instead of universal) condition.

8.2.1.3 The universe of design is reflexive

The *objects reflected by a design universe are often the reflections themselves.* As argued in Chapter 7, the task of design is ultimately concerned with the complementarity between subjective and objective reality and therefore with the construction of reflections that realize this complementary relation. In particular, given a functor *F* to represent the reflection of an objective reality B from an objective reality A, the capacity to carry out design tasks was identified by two conditions: the embedding e_A:A→H(B,H(A,B)) and the embedding e_B:B→H(H(A,B),A). Pictorially these conditions have been illustrated by the two (informal) diagrams where the reflection (i.e. functor) *F* appears as the very object of the design activity (Figure 55).

$$\begin{tikzcd}
A \arrow[r, "F"{name=F}] \arrow[rr, phantom, ""{name=X}] & B \arrow[r, "\varphi_f"{name=P}] & H(A,B) \arrow[to=F, dashed, bend right=50] \\
\arrow[from=1-1, to=P, dashed, bend right=50, "e_A"']
\end{tikzcd}$$

$$\begin{tikzcd}
H(A,B) \arrow[r, "B_b"{name=G}] & A \arrow[r, "F"{name=F}] & B \arrow[to=G, dashed, bend right=50, "e_B"'] \\
\arrow[from=1-1, to=F, dashed, bend right=50]
\end{tikzcd}$$

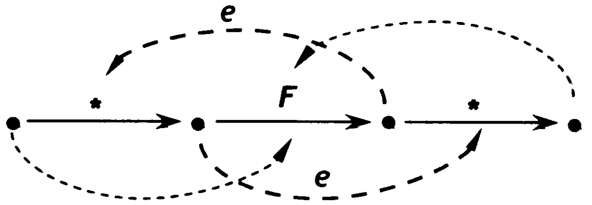
Figure 55: Fundamental diagrams of the design universe. They depict the conditions that are responsible for the capacity to design

So, what form should the proposed mathematical structure of the design universe take so that it can incorporate reflexion? Let us assume that the objects of a (subjective and objective) reality are represented by nodes of a directed graph, and the reflection of the object A within an object B is represented by an arrow A→B. In this setting, objects are primitive entities in a universe U, and reflections are defined by a source and a target object. The object A plays the role of a subjective reality that is reflected in B and the object B plays the role of an objective reality that reflects A. As shown above in the fundamental diagram (Figure 55), in a design universe the objects of reflections are reflections themselves (e.g. H(A,B)). As a result, a design universe includes reflections of reflections (e.g. represented by the dotted arrows e_A and e_B). In this sense the universe of design as illustrated by this figure cannot be perceived as a typical graph structure.

The intention is therefore to 'sketch' a type of mathematical universe where nodes (i.e. objects) are embedded into arrows (i.e. reflections) in the way the figure suggests. For that purpose, and in order to motivate the formal definitions in the latter part of this chapter, we will use informal pictures (schemata) of the design universe. A ***schema*** will be defined as a directed graph with some additional structure that includes arrows between nodes and arrows. So, a schema consists of a set of nodes N; a set of arrows A; functions t:A→N and s:A→N that assign a target and a source node for each arrow; a function t′:A→A that assigns a target arrow to each arrow; and a function e that assigns an arrow to each node (which is not necessarily the identity arrow), as shown in the diagram:

$$\begin{tikzcd} A \arrow[loop left, "t'"] \arrow[r, shift left, "{s,t}"] & N \arrow[l, shift left, "e"] \end{tikzcd}$$

For instance, a schema of the two conditions in Figure 55 is the following:



8.2.2 The principles of a mathematical universe of design

It is now possible to summarise the general properties of the universe of design. The design universe U is characterized by the capacity to manifest a phase transition to universality. A phase transition to universality implies the generation of a complementary relation between the object and the subject of description; i.e. between the properties of the universe U and properties of the described objects in U. However, the complementarity of this distinction is defined as a critical, non-universal state. Moreover, the universe of design U is characterized by distributed, anticipatory and reflexive properties. The main objective of this chapter is to 'sketch' these properties mathematically and therefore specify the mathematical structure of the universe of design. For this purpose, the core idea is to devise[8] a mathematical universe within which the complementarity between descriptions and interpretations is a *critical* (rather than universal) construction. Once in this critical state, the proposed mathematical structure should become a deductive structure (i.e. a typical category theoretic structure).

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[8] As explained in Chapter 4, this is a pre-mathematical study that aims at the design of new mathematical structures. It is therefore more accurate to claim that in this work 'I devise' a mathematical structure rather than 'I postulate the existence' of a mathematical universe.

On this basis, the principles of this mathematical universe U can be explored by looking first at 'neighbourhoods' where the universe U is 'near' a theory-model universality, and then continuing to consider the most general case when the universe U is 'far' from any theory-model universality. This is done in this section rather informally, using a number of schemata (informal pictures). The present treatment is a preparation for constructing a category theoretic sketch of the universe of design.

8.2.2.1 Near to universal constructions

We start with the observation that a sketch s of a mathematical universe U that generates the natural bijection

$$A(s,\mathit{U}d)\cong B(\mathit{Th}s,d)$$

implies that the universe U has a category theoretic structure (i.e. ***Th*s** is a category). The universe U is therefore described (or manifested) in terms of subjective/objective *objects* C_0, *reflections* C_1, *compositions* of reflections C_2, and more generally *complexes* C_n as follows:

The *category of objects* C_0: This is a category generated by one object in U and one identity arrow (reflexion).

$$\begin{tikzcd} O \arrow[loop left] \end{tikzcd}$$

The *category of reflections* C_1: This is a category generated by two objects A and B and one arrow f:A→B between a subjective and objective reality.

$$\begin{tikzcd} A \arrow[r] & B \end{tikzcd}$$

The *category of compositions* C_2: This is the category with three objects A, B and C and one arrow that form the following triangle:

$$\begin{tikzcd} & B \arrow[dr, "g"] & \\ A \arrow[ur, "f"] \arrow[rr, "h"] & & C \end{tikzcd}$$

This idea can be generalized into higher order structures; that is the category of n composites.

The *category of complexes or n composites* C_n: This is a category with n+1 objects and one arrow which forms the following diagram:

$$\begin{tikzcd} & {} \arrow[r] & \cdots\ n \arrow[dr] & \\ 1 \arrow[ur] \arrow[rrr] & & & n+1 \end{tikzcd}$$

A universe U is therefore defined by ***objects*** (described as a 0-type-arrow with one 'face' object), ***reflections*** (described as a 1-type-arrow with two 'face' objects) and ***'knowledge' structures*** (described as n-type-arrow with n+1 'face' objects). In near complementarity, these terms (objects, reflections and complexes) are related with a well-formed order. More specifically, when the limitations of reflection are assumed to play *no* role in the reflection itself (i.e. when descriptions and interpretations are complementary) the universe of design is specified by the following category theoretic principles:

Principle 1

The category of objects C_0 in U corresponds to a special type of reflection C_1 (i.e. the identity arrow) described by an embedding $e:C_0 \to C_1$.

Principle 2

The category of reflections C_1 in U corresponds to a composition of reflections C_2 described by the embedding $e':C_1 \to C_2$. Moreover, the category of reflections is a universal construction of objects C_0 described by the isomorphism $C_1 \cong C_0 X C_0$ (or $xC_0:C_0 \to C_1$).

Principle 3

The category of complexes C_n corresponds to a composition of reflections C_{n+1} described by the embedding $e^n:C_n \to C_{n+1}$. Moreover, the complex C_n is a universal construction of reflections C_1 described by an isomorphism $C_n \cong C_1 X C_1 \ldots X C_1$ (or $xC_1 : C_n \rightarrow C_{n+1}$).

This order between objects, arrows (reflections), and compositions of arrows underlie the generation of a category theoretic structure. A precise mathematical sketch of this category theoretic structure is presented in Barr and Wells (Barr and Wells, 1990: 231). When near to universality, a design universe U is therefore described in terms of objects, reflections and compositions of reflections that generate well-formed category theoretic structures (i.e. well-formed theories) as a critical construction. It is sometimes instructive to think of this structure as a logical one: the phase transition to universality can be then thought as the critical formation of a well-defined deductive structure with objects C_0 as formulas, reflections C_1 as deductions, and compositions of reflections (knowledge) C_2 as rules of inference. But, this deductive type of structure is generated in U when design is already finished and the universe of design collapses into an optimization universe. What is the type of order that characterizes a design universe U in 'neighbourhoods' where the object and subject of description in U are *not* constrained by a complementary relation?

8.2.2.2 Far from universal constructions

In this section we are concerned with the structure of the category *Th*s that is generated by a weak sketch. According to the definitions given in Chapters 6 and 7, a sketch s of a mathematical universe U that generates a weak adjunction $<\mathit{Th}, \mathit{U}, \varphi, \theta, \tau_C, \tau_D>$

$$\begin{tikzcd}
C(s, \mathit{U}d) \arrow[r, "\theta"] \arrow[d, "\tau_C"'] & D(\mathit{Th}s, d) \arrow[d, "\tau_D"] \arrow[dl, "\varphi"'] \\
C(s, \mathit{U}d) \arrow[r, "\theta"] & D(\mathit{Th}s, d)
\end{tikzcd}$$

implies that the structure of the category *Th*s is a weaker category theoretic structure. Moreover, the properties of objects, reflections and compositions of reflections in a design universe U (i.e. category *Th*s) should satisfy the properties of objects, arrows and compositions of arrows that are induced by the embedding $e_A{:}A \to H(B,H(A,B))$ and the embedding $e_B{:}B \to H(H(A,B),A)$ as the following informal diagrams in Figure 56 suggest:

$$\begin{tikzcd}
A \arrow[r, "F"] \arrow[rr, dashed, bend right, "e_A"'] & B \arrow[r, "\varphi_f"] & H(A,B) \arrow[ll, dashed, bend right]
\end{tikzcd}$$

$$\begin{tikzcd}
H(A,B) \arrow[r, "B_b"] \arrow[rr, dashed, bend right] & A \arrow[r, "F"] & B \arrow[ll, dashed, bend right, "e_B"']
\end{tikzcd}$$

Figure 56: Fundamental diagrams of the design universe. The diagrams depict the properties of compositions of arrows induced by the embeddings $e_A{:}A \to H(B,H(A,B))$ and $e_B{:}B \to H(H(A,B),A)$.

More specifically, when the limitations of reflection are assumed to play a fundamental role in the reflection itself (i.e. when descriptions and interpretations are far from complementarity), then the relation $R(C_n,C_{n+1})$ is defined by the following principles:

8.2.2.2.1 Schema 1

The category of objects C_0 in U *is a special type of reflection* C_1 *described by the embedding* $e{:}C_0 \to C_1$. *This embedding is defined such that for each object* Q in U *there is a reflection * that is composable with a reflection* $g{:}Q \to \bullet$ *of the object* Q, *or/and with a reflection* $f{:}\bullet \to Q$ *into the object* Q. The following schema informally illustrates the idea:

$$\begin{tikzcd}
\bullet \arrow[r, "*"] \arrow[rr, dashed, bend left, "e"] \arrow[rr, dashed, bend right] & \bullet \arrow[r, "f"] \arrow[rr, dashed, bend left] \arrow[rr, dashed, bend right] & Q \arrow[r, "g"] \arrow[rr, dashed, bend left] \arrow[rr, dashed, bend right, "e"'] & \bullet \arrow[r, "*"] & \bullet
\end{tikzcd}$$

Figure 57: Schema 1a.

A more concise expression of the same principle is given by the following schema:

$$\begin{tikzcd}
\bullet \arrow[r, "*"] \arrow[rr, dashed, bend right] & \bullet \arrow[r, "f"] \arrow[rr, dashed, bend right, "e"'] & \bullet \arrow[r, "*"] \arrow[ll, dashed, bend right, "e"'] & \bullet \arrow[ll, dashed, bend right]
\end{tikzcd}$$

Figure 58: Schema 1b.

The stated principle aims to capture the idea that objects and reflections are anticipatory constructions. Schemata 1a and 1b (Figures 57 and 58above) offer a presentation of the principles that underlie the definition of objects, arrows and compositions of arrows in the following diagram (Figure 59). The diagram in this figure is a representation of the conditions that are responsible for the capacity to hold an anticipatory representation to universality as discussed in Chapter 7.

$$\begin{tikzcd}
H(A,B) \arrow[r, "\beta_b"] \arrow[rr, dashed, bend right] & A \arrow[r, "F"] \arrow[rr, dashed, bend right, "e_A"'] & B \arrow[r, "\varphi_f"] \arrow[ll, dashed, bend right, "e_B"'] & H(A,B) \arrow[ll, dashed, bend right]
\end{tikzcd}$$

Figure 59: The intended interpretation of Schema 1a and 1b.

According to this diagram, an object A is a reflection φ_f that is defined in preparation of certain properties in B. Similarly, an object B is a reflection β_b that is defined in preparation of a certain properties in A. More generally, Schema 1 states that any object Q in the category *Ths* is an arrow that is defined in preparation of certain properties in the domain or co-domain object of arrows $f{:}\bullet \rightarrow Q$ or $f{:}Q \rightarrow \bullet$ respectively (i.e. an arrow *f* that also has the object Q as a co-domain or domain object). Similarly, a reflection *f* can be perceived as an anticipated composition of reflections. Schema 1 states that each reflection *f* in the category *Ths* is specified by (an anticipated) composition of reflections f^* and $^*f$ that is constructed by the domain and co-domain objects of *f*. In this context, the notion of anticipation means that the composition of reflections f^* and $^*f$

represents an (hypothetical or potential) composite of arrows in preparation of an arrow *f*.

8.2.2.2.2 Schema 2

The relation of the category of objects C_0 and the *category* of *reflections* C_1 *described by the embedding* $e{:}C_0 \rightarrow C_1$ *is reflexive*, as the following schema illustrates:

$$\begin{tikzcd}
 & Q \arrow[d, dashed, "e"] \arrow[ddr, "g"] & \\
 & * & \\
\bullet \arrow[uur, "f"] & & \bullet \arrow[ll]
\end{tikzcd}$$

Figure 60: Schema 2.

This principle states that each object (and reflection) in the category *Ths* is both a domain and a co-domain object. This principle aims to capture the idea that objects and reflections are defined reflexively. The existence of an object Q in U is determined by the composition of reflections *f* and *g* over Q and, similarly, the existence of the reflections *f* and *g* is determined by their composition on the object Q. An interpretation of Schema 2 is given in Figure 61 below:

$$\begin{tikzcd}
A \arrow[rr, "F", ""{name=F, below}] & & B \arrow[dl, "\varphi_f"] \\
 & H(A,B) \arrow[ul, "\beta_b"] \arrow[to=F, dashed] &
\end{tikzcd}$$

Figure 61: The intended interpretation of Schema 2.

8.2.2.2.3 Schema 3

The category of reflections C_1 *in* U *is an anticipatory composite* C_2 *described by the embedding* $e'{:}C_1 \rightarrow C_2$ (Figure 62). More generally, the category of compositions of reflections C_n is an anticipatory composite in C_{n+1} described by the embedding $e'{:}C_n \rightarrow C_{n+1}$.

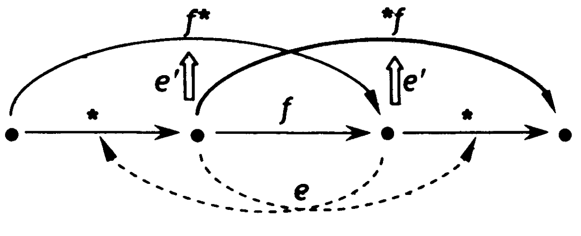


Figure 62: Schema 3.

An *anticipatory composite* is defined as a composition (i.e. f^* or $^*f$) of a reflection arrow f with its domain or co-domain object. This principle makes it possible to express structures and theories in U with a certain ambiguity and distance from the 'perfect' category theoretic structure. In a well-formed category this composition corresponds to the axiom of identity which states that the composition of an arrow f with its domain and co-domain object gives the same arrow. Based on Schema 1, the domain and co-domain objects are not (necessarily) identity arrows. So in the postulated category of design, the composition of an arrow f with its domain or co-domain arrow is not necessarily the same arrow, but more generally an anticipated construction f^* or $^*f$. This principle also aims to describe a structure with weak associative composition of arrows. In this structure, the composition between two reflections f and g is determined by the domain and co-domain objects: i.e. $^*g^*f^*$. Then, the composability of two reflections f *and* g, and the associativity of their composition (i.e. $g(fh)=(gf)h$) are only defined as critical states.

8.3 A SKETCH OF A MATHEMATICAL THEORY OF DESIGN

In this last section, the mathematical concept of sketch $S=(G_S, wD_S, wL_S)$ is used in order to 'design' a new mathematical structure. The new mathematical structure is designed to capture the organizing principles that were identified in the previous sections as those responsible for the capacity to carry out design tasks. For this purpose, we need to specify the kind of graphs, diagrams and cones that determine the sketch of a design universe $S=(G_S, wD_S, wL_S)$.

8.3.1 The underlying graph G_s of a design universe

First, the graph G_s of the sketch S that underlies the universe of design is introduced. The graph G_S has four nodes: C_0, C_1, C_2, C_3. These nodes are intended to be interpreted as objects, reflections, compositions $C_1 x_{C_0} C_1$ and complexes (i.e. compositions of compositions) $C_1 x_{C_0} C_1 x_{C_0} C_1$ respectively. It is important to be careful with this notation. The symbols C_0, C_1, C_2 and C_3 simply denote the nodes of the graph G_S. So for instance, the symbol C_2 is not a product structure $C_1 x_{C_0} C_1$, but a node in a graph G_S whose interpretation in a category *Th*G_S is a product of arrows. In addition to these nodes, the graph G_S has the following arrows:

e: $C_0 \rightarrow C_1$, and e': $C_1 \rightarrow C_2$

d_0, d_1: $C_1 \rightarrow C_0$

p_1, p_2, c: $C_2 \rightarrow C_1$

a: $C_1 \rightarrow C_1$

These arrows constitute the arrows of the graph G_S whose interpretation in a category *Th*G_S is intended to be the following:

The arrow e: $C_0 \rightarrow C_1$ is an embedding of objects to arrows and the arrow e': $C_1 \rightarrow C_2$ is an embedding of arrows to composition of arrows.

The arrows d_0, d_1: $C_1 \rightarrow C_0$ are the domain and co-domain objects of an arrow.

Arrow p_1:$C_2 \rightarrow C_1$ is a projection (selection) of the first arrow from a product of arrows $C_1 x_{C_0} C_1$ and arrow p_2:$C_2 \rightarrow C_1$ is a projection (selection) of the second arrow from a product of arrows $C_1 x_{C_0} C_1$. The arrow c:$C_2 \rightarrow C_1$ determines the composition of the two arrows into a new arrow.

The arrow a: $C_1 \rightarrow C_1$ is representation of a (natural) transformation from one arrow to another.

The syntactical properties of these arrows are defined by the underlying diagrams of the design universe.

8.3.2 The underlying diagrams wD_s of a design universe

The diagrams of a design sketch are weak diagrams. In particular, the proposed diagrams are a generalization (or relaxation) of the category theoretic diagrams presented by Barr and Wells (Barr and Wells, 1990: 231). Their main difference is their weak commutative properties that imply the introduction of certain 'noise' in the definition.

The following diagrams describe the main relations between arrows p_1, p_2, p_3, $\langle p_1,p_2\rangle$, $\langle p_2, p_3\rangle$, $\langle p_1, c\rangle$, $\langle c, p_3\rangle$, $\langle id, e\circ d_1\rangle$, $\langle e\circ d_0, id\rangle$ in G_5:

$$\begin{tikzcd}
& C_3 \arrow[dl, "p_2"'] \arrow[d, "{\langle p_2, p_3\rangle}"'] \arrow[dr, "p_3"] & \\
C_1 & C_2 \arrow[l, "p_1"'] \arrow[r, "p_2"] & C_1
\end{tikzcd}$$

$$\begin{tikzcd}
& C_3 \arrow[dl, "p_1"'] \arrow[d, "{\langle p_1, p_2\rangle}"'] \arrow[dr, "p_2"] & \\
C_1 & C_2 \arrow[l, "p_1"'] \arrow[r, "p_2"] & C_1
\end{tikzcd}$$

$$\begin{tikzcd}
C_2 \arrow[d, "c"'] & C_3 \arrow[l, "{\langle p_1, p_2\rangle}"'] \arrow[d, "{\langle c, p_3\rangle}"'] \arrow[dr, "p_3"] & \\
C_1 & C_2 \arrow[l, "p_1"'] \arrow[r, "p_2"] & C_1
\end{tikzcd}$$

$$\begin{tikzcd}
& C_3 \arrow[dl, "p_1"'] \arrow[d, "{\langle p_1, c\rangle}"] \arrow[r, "{\langle p_2, p_3\rangle}"] & C_2 \arrow[d, "c"] \\
C_1 & C_2 \arrow[l, "p_1"'] \arrow[r, "p_2"] & C_1
\end{tikzcd}$$

$$\begin{tikzcd}
C_0 \arrow[d, "e"'] & C_1 \arrow[l, "d_1"'] \arrow[d, "{\langle id, e\circ d_1\rangle}"'] \arrow[dr, "id"] & \\
C_1 & C_2 \arrow[l, "p_1"'] \arrow[r, "p_2"] & C_1
\end{tikzcd}$$

$$\begin{tikzcd}
& C_1 \arrow[dl, "id"'] \arrow[d, "{\langle e\circ d_0, id\rangle}"] \arrow[r, "d_0"] & C_0 \arrow[d, "e"] \\
C_1 & C_2 \arrow[l, "p_1"'] \arrow[r, "p_2"] & C_1
\end{tikzcd}$$

Based on the above definitions, it is now possible to introduce two diagrams that describe the relaxation of the identity and associativity axioms in category theoretic structures (Mac Lane, 1998: 1). The first diagram is a generalization of the identity axiom. The diagram is effectively a description of Schema 3 which states that the composition of an arrow *f* with its domain or co-domain arrows is not necessarily the same arrow, but more generally an anticipated construction f^* or $^*f$. This condition is described by the following diagram:

$$\begin{tikzcd}
C_1 \arrow[r, "{\langle id,\ e\circ d_0\rangle}"] \arrow[dr, "a"'] & C_2 \arrow[d, "c"] & C_1 \arrow[l, "{\langle e\circ d_1, id\rangle}"'] \arrow[dl, "a"] \\
& C_1 &
\end{tikzcd}$$

In order to explain this diagram, let us first describe the arrows $\langle id,\ e\circ d_0\rangle$ and $\langle e\circ d_1,\ id\rangle$. The arrow $\langle id, e\circ d_0\rangle$ is intended to specify a composite of two arrows $f\circ g$. The first argument $id{:}C_1\rightarrow C_1$ specifies an arrow f. The second argument $e\circ d_0{:}C_1\rightarrow C_0\rightarrow C_1$ specifies the arrow g which is the embedding of the domain object of f. In typical category theoretic structures this arrow g is the identity arrow of the domain object (i.e. $f\circ 1_{Domf}$). A similar reading can be applied for arrow $\langle e\circ d_1, id\rangle$ which intents to specify a composite $h\circ f$. The arrow $c{:}C_2\rightarrow C_1$ represents the composition of arrow f with g from the right, and with h from the left. In a typical category theoretic structure, when we follow the path $\langle id,\ e\circ d_0\rangle$, c (or the path $\langle e\circ d_1, id\rangle$, c) the composition from right (or left) is cancelled: i.e $f\circ 1_{Domf}=f$ (or $1_{Codf}\circ\ f=f$). In this case, the arrow $a{:}C_1\rightarrow\ C_1$ is the arrow $id{:}C_1\rightarrow\ C_1$ (and the Principle 1 in Section 8.2.2.1 of this chapter is realized). The above diagram represents the more general case where following the paths $\langle id,\ e\circ d_0\rangle, c$ or $\langle e\circ d_1, id\rangle, c$ may result in a new composite arrow.

The second diagram is a generalization of the associativity axiom[9].

$$\begin{tikzcd}
C_3 \arrow[rr, "{\langle p_1, c\rangle}"] \arrow[d, "{\langle c, p_3\rangle}"'] & & C_2 \arrow[d, "c"] \\
C_2 \arrow[r, "c"] & C_1 \arrow[r, "a"] & C_1
\end{tikzcd}$$

---

[9] Note certain similarities with relaxed monoidal categories. A relaxed monoidal category is a category C with compositions x that are associative up to natural isomorphism - i.e. $k{:}ax(bxc)\cong(axb)xc$ - and with an identity element e that is the left and right unit of compositions but only up to a natural isomorphism - i.e. $\lambda{:}exa\cong a$ and $\mu{:}axe\cong a$- (MacLane, 1998: 162).

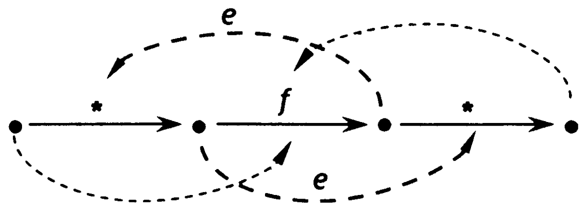
Given a composite of three arrows C_3 (say $a \circ b \circ c$), the arrow $<p_1,c>$ expresses the composition of the last two arrows of C_3 ($b \circ c$) and the projection of the first one. The arrow $<c,p_3>$ expresses the composition of the first two arrows of C_3 ($a \circ b$) and the projection of the third. So, the path $<p_1,c>$, c expresses a sequence of compositions of arrows $a \circ (b \circ c)$, and the path $<c,p_3>$, c expresses a sequence of compositions of arrows $(a \circ b) \circ c$. The arrow a is again a natural transformation from an arrow that is the result of the composition $a \circ (b \circ c)$, to another that is the result of the composition $(a \circ b) \circ c$. A special case would be that a is an isomorphism.

In conclusion, the two proposed diagrams describe the relaxation of the identity and associativity axioms in category theoretic structures. The objective is to describe a mathematical structure that has the typical category theoretic structure only as a special case. This is in accordance with the 'desired properties' of the universe of design presented in Section 8.2 of this chapter. In the next section, the cones that were just introduced will express the anticipatory and reflexive characteristics of this 'relaxed' category.

8.3.3 The underlying cones wL_s of a design universe

The cones of a design universe formally capture the idea that reflections C_1 and compositions C_2 in a design universe are anticipatory and reflexive. The following cones aim to explicitly describe the properties that were informally presented by Schemata 1,2 and 3.

Schema 1 (which is repeated here for clarity):



is described by the following cones:

$$\begin{tikzcd}
C_1 \arrow[rr, "d_1"] \arrow[dd, "d_0"'] & & C_0 \\
& C_0 \arrow[ul, "e"] & \\
C_0 \arrow[rr, "e"] & & C_1 \arrow[uu, "d_0"'] \arrow[ul, "d_1"']
\end{tikzcd}$$

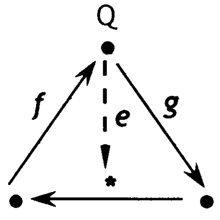
AND

$$\begin{tikzcd}
C_1 \arrow[rr, "d_1"] \arrow[dd, "d_0"'] & & C_0 \arrow[dd, "e"] \\
& C_0 \arrow[ul, "e"] & \\
C_0 & & C_1 \arrow[ll, "d_1"'] \arrow[ul, "d_1"']
\end{tikzcd}$$

The cones above specify that every arrow C_1 in the category of design (seen at the top left corner) with a domain $d_0:C_1 \to C_0$ and co-domain object $d_0:C_1 \to C_0$ satisfy certain properties depicted in Schema 1. Let us name this arrow f in accordance with the above Schema 1. In the left cone, the domain object C_0 of f is embedded via $e:C_0 \to C_1$ in another arrow C_1 (bottom right corner). Let us name this arrow * . The arrow * has as a domain object the co-domain object of the arrow f and as co-domain object an object C_0 (centre) which is a representation of the arrow f. The same 'reading' can be applied to the right cone.

These constructions are weak cones in the sense that their interpretation in the category of design would generate a universal construction as a critical state. In particular, the universal construction determines that every object C_1 in this category is the product of its domain and co-domain objects (in this case the two proposed diagrams in the previous section specify a category theoretic structure where the identity and associative laws hold). So, the notion of critically implies that when such a universal construction is defined then Principle 2 in Section 8.2.2.1 of this chapter is realized.

Schema 2 (which is repeated here for clarity):



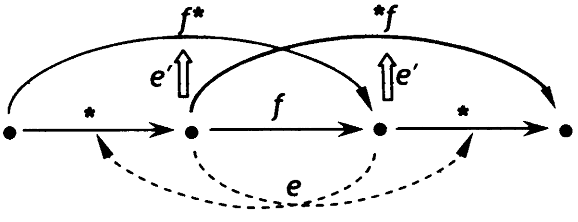
is described by the following cone:

$$\begin{tikzcd}
C_2 \arrow[rr, "p_2"] \arrow[dd, "p_1"'] & & C_1 \arrow[dd, "d_1"'] \arrow[dr, "d_0"] & \\
& & & C_0 \\
C_1 \arrow[rr, "d_0"] \arrow[dr, "d_1"'] & & C_0 \arrow[dr, "e"] & \\
& C_0 & & C_1 \arrow[uu, "d_1"'] \arrow[ll, "d_0"]
\end{tikzcd}$$

This cone describes the properties of composability of two arrows over an object. The paths of arrows p_2, d_1 and p_1, d_0 (that form the rectangular part of the cone) specify how two arrows (i.e. f and g) are composed over an object Q. The embedding $e{:}C_0 \rightarrow C_1$ of the object Q in an arrow * then specifies the circular (reflexive) structure of the design universe.

This construction is a weak cone in the sense that its interpretation in the category of design would generate a universal construction as a critical state. In particular, the universal construction determines that every composition of arrows C_2 in this category is the product of arrows C_1 (in this case the two proposed diagrams in the previous section specify a category theoretic structure where the identity and associative laws hold). So, the notion of critically implies that when such a universal construction is defined, then Principle 3 in Section 8.2.2.1 of this chapter is realized.

Finally, Schema 3 (which is repeated here for clarity):



is described by the following cones:

$$\begin{tikzcd}
& & C_1 \arrow[dl, "d_0"'] \arrow[dd, "e'"] \arrow[ddrr, "id"] & & \\
& C_0 \arrow[dl, "e"'] & & & \\
C_1 \arrow[d, "d_1"'] & & C_2 \arrow[ll, "p_1"'] \arrow[rr, "p_2"] \arrow[d, "c"] & & C_1 \arrow[d, "d_0"] \\
C_0 & & C_1 \arrow[ll, "d_1"'] \arrow[rr, "d_0"] & & C_0
\end{tikzcd}$$

AND

$$\begin{tikzcd}
& & C_1 \arrow[ddll, "id"'] \arrow[dd, "e'"] \arrow[dr, "d_1"] & & \\
& & & C_0 \arrow[dr, "e"] & \\
C_1 \arrow[d, "d_1"'] & & C_2 \arrow[ll, "p_1"'] \arrow[rr, "p_2"] \arrow[d, "c"] & & C_1 \arrow[d, "d_0"] \\
C_0 & & C_1 \arrow[ll, "d_1"'] \arrow[rr, "d_0"] & & C_0
\end{tikzcd}$$

These cones specify the properties that characterize the composition of arrows in the category of design. Let us concentrate on the left cone that specifies the properties of the composition of an arrow *f* with an (anticipated) arrow or place-holder arrow * that 'sits' on the right of *f* in Schema 3 (i.e. **f*). The cone specifies that every arrow *f* (see C_1 at the top) has a domain object d_0 that holds a representation of an arrow * (via the embedding $e{:}C_0 \rightarrow C_1$) which is composable with *f* to the right. The lower rectangular part of the cone specifies the composability of arrows. The same 'reading' should be applied to the cone on the right.

8.4 SUMMARY AND CONCLUSIONS

The aim of the current chapter was to develop a mathematical structure able to express the nature of the universe within which design tasks and design activities are realized. For this purpose, the intuitive concept of design was effectively turned into a mathematical object. The hypothesis driving this study is that this universe may take different interpretations, but that there are certain properties of a mathematical nature which characterise it and distinguish it from other types of universe. The study merged together the results from previous chapters in order to inform the development of the desired mathematical structure. The treatment was divided in three parts.

The first part recapitulated the main points of the proposed organizational approach to defining and explaining the task and capacity to design. In particular, this part offered a review of how the notion of organization is understood and expressed in the study through the notion of sketch. The argument was also reaffirmed that the universe of design is a special kind of universe that involves a special type of sketch. The question of identifying the type of sketch that underlies a design universe was discussed as part of an enquiry regarding the nature and role of linguistic or dynamical complementarity between theories and models of a system. Within a design universe the complementarity between theories and their models is not a universal principle, but a critical, anticipated state that determines a boundary and a transition from a non-deductive to a deductive universe. Complementarity is crucially different in other types of universes where the preservation or control of complementarity is the main focus. A sketch of a design universe is therefore an embodiment of the capacity to form anticipatory representations of the complementary relation between theories and models.

The second part of the chapter offered a rather informal presentation of the mathematical principles of the design universe in preparation for the more formal definitions. More precisely, the study offered an exposition of three general properties of the design universe: the properties of distribution, anticipation and reflexivity. These were then used in order to formulate some basic principles and requirements for the proposed mathematical structure. In particular, the principles of the design universe U were explored by looking first at 'neighbourhoods' where the universe U is near a theory-model universality, and then continuing to consider the most general case when the universe U is far from any theory-model universality. Based on this analysis, three main pictures (Schemata) of the universe of design were identified. More specifically, the postulated schemata are informal representations of the properties discussed in Chapter 7.

Finally, the third part of the chapter presented formal definitions of the proposed mathematical structure, which incorporate the principles outlined in the second part. In particular, the study offered definitions of the types of graph, diagrams and cones that make up the category theoretic sketch of design. Although the proposed sketch-based presentation of mathematical structures might look complicated it has also certain important advantages. In order to understand these advantages it is important to see the general picture. The general motivation behind this endeavour is not only to offer a notation that describes the principles of the universe of design but also to introduce a method that positions the postulated structures of design within the universe of mathematical structures. This last section is the final result which concludes the thesis.

Chapter 9

SUMMARY AND CONCLUSIONS

This last chapter reaffirms the main hypotheses and objectives of the thesis, reviews how these objectives were met, and discusses the results and contributions of the study. It also offers a discussion of possible future enquiries and developments springing from the arguments and findings presented.

9.1 HYPOTHESES AND OBJECTIVES OF THE THESIS

This study is concerned with the understanding of design as a 'natural' capacity of complex organizations which arise within a variety of environments including bio-chemical, neurological, cognitive or social environments. Design is therefore treated as a universal phenomenon (biological, cognitive or social) that arises because of certain organizational conditions. The objective of the thesis, as stated at the introduction, was the development of a mathematical theory of design that identifies and describes these unique conditions or qualities that are responsible for the capacity to recognise and carry out design tasks. To define and explain design in such terms it was also necessary to develop a suitable mathematical language and method. The main objective was thus divided into the following interrelated sub-aims:

- To explicate the notions of organization and complexity and ascertain the basic dimensions of organizational level descriptions by emphasising aspects pertinent to design.
- To characterise design as a universal property of complexity and identify the organizational conditions and principles that are responsible for the capacity to recognise and carry out design tasks.
- To develop a mathematical structure (a theory) that can define the universe within which design tasks and activities are realized.

9.2 REALISATION OF THE OBJECTIVES

The thesis can be roughly divided into two parts. The first part, consisting of Chapters 1, 2, 3 and 4, establishes the research agenda of the thesis. In specific, it introduces the general research problem and approach of this study; it positions the thesis in the general context of design and complexity research; and finally, explores and explains the basic dimensions of the postulated hypothesis.

More specifically, Chapters 1 and 2 establish the research question and purpose of the thesis, and introduce the hypothesis that design is a distinct capacity and type of phenomenon that can be understood and studied at an organizational level of description. In Chapter 2 more specifically, the argument is unfolded that although design research has grappled with the issue of understanding design as a universal phenomenon with characteristic processes, knowledge and objects, there is no theory that characterises the organizational basis of the capacity to recognise and carry out design tasks. Chapter 2 hence identifies a research agenda that seeks to understand design as an organizational capacity enabled by, and derived from, complexity. This argument is of epistemological but also methodological importance. From an epistemological perspective, the argument implies an understanding of design as a universal problem or phenomenon that is uniquely distinguished by the organization of certain underlying natural processes/structures - independently from their realization in a social, logical, or physical realm. From a methodological perspective, the argument implies a constructive approach to research, which consists in creating and building formal constructs in order to synthesise the desired design properties and phenomena.

As part of this investigation, Chapter 3 presents a computational model built in order to construe design as a phenomenon that emerges out of complexity. The model is also used in order to expose and reflect on some crucial dimensions or conditions that enable design capacities. The chapter specifically approaches group design as a problem of coordination between distributed agents with diverse knowledge and goals. Starting from a conceptual model of coordination as distributed learning control, the chapter moves on to present details of the computational modelling experiments. As a result of this investigation, coordination is defined both in relation to the formation of individual expectations and configurations that avoid conflicts at the micro level; and in relation to the emergence of collective (macro) behaviours and structures. Additionally, creativity, learning and anticipation are identified as crucial dimensions that enable the emergence of collective design solutions. The chapter concludes with a discussion of the problem of evaluating the proposed 'mechanism' as an hypothesis of design, and introduces the motivation behind a mathematical exploration of the exposed ideas.

Finally, Chapter 4 discusses in detail the role and value of a mathematical investigation in building the proposed theory of design, and offers an introduction to the particular language and framework (category theory) used. More specifically, the chapter is organized in three parts. The first part offers a general epistemological discussion of the nature of scientific, mathematical and design knowledge, and positions this thesis as part of a 'pre-mathematical' activity: an activity where a 'mathematician' creates a new mathematical structure. According to this approach, the main concern is to turn the intuitive concept of design into a mathematical entity. The second part of this chapter is an exploration of mathematical research in design. The study argues that mathematical studies of design have concentrated on the representation of design knowledge, processes and objects, and little emphasis has been placed on how design as a distinct type of thinking and knowing might be related to the underlying properties of the complex socio-physical systems that generate design knowledge, processes and objects. The third part of the chapter, introduces category theory as a language and methodological tool for developing such an organizational theory of design. The study argues that category theory is capable of dealing with two problems: the problem of studying the organizational properties of complex systems, and the problem of constructing definitions and theories about mathematical structures. The concepts and tools introduced constitute the main mathematical machinery for identifying the meaning of design and organization in the following chapters.

The second part, consisting of Chapters 5, 6, 7 and 8, is focussed on the development of the mathematical theory of design. There are three strands to this investigation: first, defining the concept of organization and the basic characteristics of organizational level theories and descriptions; second, developing the mathematical principles of an organizational level theory of design; and third, developing a mathematical structure able to embody these principles.

More specifically, Chapters 5 and 6 focus on the concept of organization. Chapter 5 explains the meaning of organization in complexity research, and derives some fundamental mathematical principles that underlie the description and study of organization. In particular, the chapter emphasises semantic aspects of organization in order to reflect issues more pertinent to design research. Starting from an intuitive presentation of organization as conditionality (a set of processes or structures that generate, constrain and preserve a distinct whole), the chapter reaches a more formal understanding of the concept as a universal property. In particular, the study introduces the notions of universality and adjunction, and the idea that organization is associated with the existence of a source (a conditional entity) capable of reflecting the properties of an objective reality. The chapter then proposes a generalised (semantic) view of basic approaches to organization based on a diagram, in which the notions of source, theory, model and object - and their relationships - are exemplified and summarised. According to this view, organization is seen as a theoretical quality determined by the capacity of a source to reflect the properties of an object and guarantee the commutativity of the diagram. The analysis further shows that different interpretations of this basic diagram correspond to different approaches to the study of organization, and identifies a need to consider the task of defining organization not in relation to the existence of commutative properties, but in relation to a "movement" towards the critical state of commutativity. The motivation for such a (meta-) theory is to understand and reflect characteristic design situations.

In Chapter 6 the meaning of organization and the basic concepts of organizational level theories are elaborated on the basis of category theoretic constructions. The chapter proposes a mathematical framework that is able to accommodate and compare a plethora of different organizational level descriptions and prepare the ground for an organizational level theory of design. In particular, the proposed category theory of organization is developed as part of a theory of sketches. The treatment is divided in two sections. In the first section, the notions of sketch and weak sketch are introduced and explained with regards to their role for the development of organizational-level descriptions. The study first discusses these notions more generally explaining their origins and relation to design. Then, the category theoretic concept of sketch is proposed as a mathematical interpretation of a conditional entity (a source) that has the capacity to generate, preserve or constrain a distinct whole. In the second section, a case is made for a notion of weak sketch able to express different types of organization. The basic tool is the concept of adjunction between models of a sketch and interpretations of the theories generated by the sketch. The proposed type of sketch reflects different levels of organization as a function of the quality (the weakness) of this adjunction. Weak sketches are then introduced and elaborated as basic elements of a meta-theory of organization. The study includes defining the basic elements (graphs, diagrams, cones and co-cones) that specify this construction, and elaborating the properties of the theories that are generated by the proposed sketch (weak theories).

Chapter 7 is concerned with describing and explaining design at an organizational level using the mathematical concepts introduced. The chapter is divided in three parts. The first part grapples with the question of how we can define the design situation (the situation within which design arises), the design task, and the design capacity at an organizational level of abstraction. In this part, the notion of design Intentionality is explicated and linked to the proposed theory of organization. The thesis proposes that a design situation can be understood as a two-directional Intentional state which includes both world-to-mind adaptations as well as mind-to-world changes. According to the proposed treatment, design tasks arise when there is a weak adjunction between theories and models. The capacity to design is defined as the capacity of an Intentional state to hold anticipatory representations of a phase transition to universality (i.e. transition to an adjunction). Formal definitions of these terms are offered in category theoretic language. The second part of the chapter makes a comparison of different paradigms for the study and representation of design in order to better explicate the need for the proposed organizational-level theory. The paradigms of machine (computation), evolution and control are explored in detail. In particular, the chapter discusses problems related with the development of a theory of design as 'a phase transition to universality' using these paradigms. The comparison shows that in all these paradigms, the world is explained through a process whereby new entities are built out of old ones without reference to future anticipated states. In these paradigms the role of complementarity (adjunction) is a universal constraint, rather than an emergent state of the world. This observation motivates the need for extending the scope of these paradigms, by incorporating the notion of anticipatory representations. The third and final part of the chapter is concerned precisely with identifying the mathematical conditions that are responsible for the capacity of an Intentional state to hold anticipatory representations. The investigation includes an introduction to anticipation and explication of its meaning and role in design and design research. Subsequently the study builds on and generalises Rosen's (M, R) model in order to offer a description of the mathematical structures that underlie the construction of an anticipatory representation of universality.

Finally, Chapter 8 revisits the results of previous chapters, and summarises the mathematical properties that uniquely distinguish the problem and capacity to design as an effect of complexity and organization. It then moves on to develop a new type of mathematical structure (i.e. a category theory of design) able to characterise the universe within which design tasks and capacities are expressed. The new structure, expressed in category theoretic terms, defines the universe of design as a sketch that embodies the capacity to form anticipatory representations of the complementary relation between theories and models. As per the requirements expressed in the previous chapters, within this design universe the complementarity between theories and their models is a critical, anticipated state that determines a boundary and a transition to universality.

9.3 REFLECTIONS ON RESULTS, CONTRIBUTIONS AND FUTURE DEVELOPMENTS

The main result of the study is the construction of a mathematical theory of design as a universal capacity of complexity. This result is the product of a number of findings which can be summarized as follows:

- The thesis reviews design research and identifies a gap in our knowledge of design: that is, a theory that can characterise design as a universal, natural phenomenon that occurs at different levels of abstraction. This introduces the view of design as a universal phenomenon derived from complexity (Chapter 2).
- The thesis identifies critical conditions and dimensions of design as a universal phenomenon, and introduces a research agenda that utilises mathematical language in order to identify the principles of design-capable complexity (Chapters 3 and 4).
- The thesis develops a mathematical framework for understanding organization and complexity, and classifying different organizational level theories according to their assumptions about complexity and focus of investigation (Chapters 5 and 6).
- The thesis proposes a semantic theory of organization that can encompass the Intentional character of design (Chapters 5 and 6).
- The thesis explicates the notion of design Intentionality and links it to complexity research (Chapter 7).
- The thesis reveals and explains mathematically the limitations of existing fundamental categories of systems (computational, evolutionary, or cybernetic systems) to express design phenomena (Chapter 7).
- The study identifies and defines the mathematical principles of organization that are responsible for the capacity to recognize and carry out design tasks (Chapter 7).
- The thesis develops a theory that defines the mathematical universe within which design task and capacities are realized (Chapter 8).

The contribution of this theory is that it proposes an understanding of design as a characteristic phenomenon, problem, or question that exists across different domains of human activity, such as art, science and mathematics. Such a theory also enables us to understand a large class of design-capable systems (whether these are natural or artificial, cognitive or social, physical or conceptual) based on a characterisation of the fundamental structures or processes that entail the generation of design knowledge, methods and objects. The proposed view can be also particularly useful in understanding the capabilities and limitations of various realizations of design-capable systems. The knowledge generated can in turn feed into the creation of hypotheses for studying design-capable systems empirically, and inform the development of new methods and tools to model and support design activities. Finally, the constitution of design as a universal capacity of complexity is a double contribution to complexity research as well as to design research, and makes it possible to transfer results between the two fields. The proposed framework for defining organization and organizational level theories not only unifies and clarifies the various approaches and their objectives, but also helps place the problem of design within the realm of complexity research and vice versa. In this sense, the study is a demonstration of how an epistemological and methodological synergy can be achieved, between complexity, mathematics and design.

What kind of research can spring from this investigation in the future? As mentioned, the proposed theory can inform empirical analysis of design-capable systems. Future investigations may thus focus on different biological, cognitive or social systems, and attempt to derive different types of organizational descriptions. For example, it is possible to use the category theoretic constructions developed in the thesis in order to study the organization of brain activities that accompany design tasks (i.e. to study design at a neurological level), or to study the organization of design functions and activities developed in social systems (i.e. to study design at a social level). Another route of future investigation is to use the theory in order to explore the possibility of creating artificial design-capable systems. The thesis already offers a critical examination of various approaches that have been used in the past, particularly the computational approach. Is it possible to construct artificial design systems that go beyond the computational approach? What (new?) elements can support such an endeavour (material, digital, analogue, quantum)? This is of course a very large and ambitious research programme and, very importantly, one that can only be achieved with work that cuts across research domains and disciplines.

Appendix

PUBLICATIONS DERIVING FROM THIS RESEARCH

Journal Papers

Alexiou, K. and Zamenopoulos, T. (2008) 'Design as a social process: a complex systems perspective', *Futures*, 40(6): 586-595.

Zamenopoulos, T. and Alexiou, K. (2007) 'Rethinking the cybernetic basis of design: the concepts of control and organisation', *Kybernetes*, 36(9/10): 1570-1589.

Zamenopoulos, T. and Alexiou, K. (2007) 'Toward an anticipatory view of design', *Design Studies*, 28(4): 411-436.

Book Chapters

Zamenopoulos, T. and Alexiou, K. (2003) 'Structuring the plan design process as a coordination problem: the paradigm of distributed learning control coordination'. In *Advanced Spatial Analysis: the CASA Book of GIS*, P. Longley and M. Batty (eds), ESRI Press: US, pp 407-426. ISBN 1589480732.

Papers in Proceedings

Zamenopoulos, T. and Alexiou, K. (2005) 'Linking Design and Complexity: a review', *In ECCS'05 Satellite Workshop Proceedings: Embracing Complexity in Design*, Open University: UK, pp 91-102.

Zamenopoulos, T. and Alexiou, K. (2005) 'The Problem of Design in Complexity Research', *In ECCS'05 Conference Proceedings*, Bourgine, P., Képès, F., Schoenauer, M. (eds), pp 137-138.

Zamenopoulos, T. and Alexiou, K. (2003) 'Computer-aided creativity and learning in distributed cooperative human-machine networks'. In *Digital Design: research and practice, Proceedings of the 10th International Conference on CAAD Futures 2003*, Chiu, M-L., Tsou, J-Y., Kvan, T., Morozumi, M. and T. Jeng (eds), Kluwer Academic Publishers: Dordrecht, The Netherlands, pp 191-201.

Alexiou, K. and Zamenopoulos, T. (2002) 'A Control Based Approach to Artificial Design and Plan Generation'. In *Proceedings of the Conference Common Ground, Design Research Society International Conference*, D. Durling and J. Shackleton (eds), Staffordshide University Press: UK, pp 13.

Zamenopoulos, T. and Alexiou, K. (2002) 'Learning to be creative and the creative memory - a discussion motivated by a control-based coordination model'. In *Learning and Creativity Workshop notes, Seventh International Conference on Artificial Intelligence in Design (AID'02),* Cambridge: UK. [Available from URL: http://www.cad.strath.ac.uk/AID02_workshop/Workshop_webpage.html]

Zamenopoulos, T. and Alexiou, K. (2002) 'Artificial Design and Plan Generation by Learning. A model of control-based coordination'. *In AID '02 Accepted Posters*. Available in CD-ROM.

Alexiou, K. and Zamenopoulos, T. (2002) 'Artificial Design and Planning Support: Interactive Plan Generation and Coordination in Distributed Decision-Making'. In *Design and Decision Support Systems in Urban Planning, Proc. of the 6th International Conference*, Eindhoven University of Technology: The Netherlands, pp 1-11.

Alexiou, K. and Zamenopoulos, T. (2001) 'A Connectionist Paradigm in the Co-ordination and Control of Multiple Self-Interested Agents' in the *Proceedings of the 2nd National Conference Input 2001: Democracy and Technologies*, Tremiti Islands, Italy, 27-29 June 2001.

Working Papers

Zamenopoulos, T. and Alexiou, K. (2004) 'Design and Anticipation: towards an organisational view of design systems'. CASA Working Paper 76, University College London, Centre for Advanced Spatial Analysis. [Available from URL: http://www.casa.ucl.ac.uk/working_papers/paper72.pdf]

Alexiou, K. and Zamenopoulos, T. (2002) 'Designing Plans: A Control Based Coordination Model'. CASA Working Paper 48, University College London, Centre for Advanced Spatial Analysis. [Available from URL: http://www.casa.ucl.ac.uk/working_papers/paper48.pdf]

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