Multi-product Firms and Business Cycle Dynamics

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Abstract

Recent empirical evidence provided by Bernard et al. (2010) and Broda and Weinstein (2010) shows that a significant share of product creation and destruction in U.S. industries occurs within existing firms and accounts for a significant share of aggregate output. In the present paper, and consistent with this evidence, we relax the standard assumption of mono-product firms that is typically made in dynamic general equilibrium (DSGE) models. We develop a DSGE model with multi-product firms and endogenous markups to assess the implications of the *intra-firm* extensive margin on business cycle fluctuations. In this environment, the procyclicality of product creation emerges as a consequence of firms' strategic interactions. Due to the *proliferation* effect induced by firm-level adjustments in product scope, we show that our model embodies a quantitatively important magnification mechanism of aggregate shocks.

Keywords: Multi-product Firms, Business Cycles, Firm Dynamics

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1 Introduction

Multi-product enterprises dominate production activity in the U.S. economy. According to Bernard et al. (2010), 39 percent of firms produce more than one product and generate, on average, 87 percent of U.S. manufacturing output. In a related paper, Broda and Weinstein (2010), by using data on purchases of products by a representative sample of consumers, show that 92 percent of product creation and 97 percent of product destruction in U.S. industries occur within the boundaries of the firm, which again indicates the multi-product nature of firms. Despite this empirical evidence, multi-product firms have thus far received relatively little attention in macroeconomics. Only recently has a growing body of international trade literature incorporated multi-product firms into models of general equilibrium to address issues related to trade liberalization (see Nocke and Yeaple 2006; Eckel and Neary 2010; Mayer et al. 2010; Bernard et al. 2011).[1] These contributions show that intra-firm adjustments within multi-product firms are not limited to changes in the scale of production; in fact, the choice of product range adds a new "extensive" margin of firm adjustment that plays an important role in economy-wide shocks such as trade liberalization.

The available empirical evidence has also convincingly documented that the contribution of new products (produced at both existing and new firms) to the overall production is important enough to be a major source of aggregate output fluctuations. In this respect, Bernard et al. (2010) document the importance of adding and dropping products within firms. They show that 54 percent of firms change their mix of products within a 5-year census period −a time horizon that is consistent with the length of the business cycle− with 25 percent of firms both adding and dropping at least one product. Most importantly, the authors find that over the same period product creation accounts for a significant share of overall production (33 percent) with the gross contribution of product switching that is as large as the gross contribution of firm entry and exit. The importance of product creation and destruction is also confirmed by the work of Broda and Weinstein (2010), who show that across product groups, almost one third of the growth rate of consumption expenditures is reflected in the growth rate of expenditure shares in new product varieties.[2] The authors find that net product creation is strongly procyclical at a quarterly frequency, with the procyclicality driven mainly by creation rather than destruction. The same result is confirmed by Axarloglou (2003) at a monthly frequency for U.S. manufacturing industries and by Lee and Mukoyama (2008) for plant-level U.S. Census data. Given this evidence, two questions naturally arise: (1) what are the main driving forces behind the observed procyclical behavior of product creation?, and (2) what are the aggregate implications of firm-level adjustments along the product scope?

The present paper addresses these questions with a quantitative approach. We embed multi-product firms in a dynamic stochastic general equilibrium model (DSGE) with imperfect competition and endogenous entry. On the one hand, we are interested in building a model that is able to account for the observed cyclical properties of product creation. In particular, we identify a firm-based force, originating from strategic behavior, that explains the strong procyclicality of product creation. On the other hand, we aim to assess the role of firm-level adjustments along the product scope in shaping business cycle fluctuations. In this respect, we seek to understand the contribution of the intra-firm extensive margin to the response of the economy to productivity and demand-type shocks. Our results show that firm-level adjustments along the product range generate endogenous variations in total factor productivity that magnify the amplitudes of aggregate fluctuations.

In this paper, we link the endogenous behavior of firms with the business-cycle properties of the economy by taking firms' strategic interactions into account.[3] We thus depart from the standard Dixit-Stiglitz structure by

[1] In recent decades, the field of industrial organization has devoted considerable attention to the study of multi-product firms, mainly using partial equilibrium analysis. Important contributions on this topic include Brander and Eaton (1984), Shaked and Sutton (1990), Anderson and Palma (1992) and, more recently, Ottaviano and Thisse (1999) and Allanson and Montagna (2005).

[2] Broda and Weinstein (2010) measure products at the finest possible level of product disaggregation, the product barcode.

[3] Building on the initial contribution by Bresnahan and Reiss (1987), recent papers by Manuszak (2002), Campbell and Hopenhayn (2005), Manuszak and Moul (2008) and others have provided important evidence in support of theories of imperfect competition with strategic interaction. In contrast with the traditional Dixit and Stiglitz (1977) approach, these empirical works show that firms' pricing decisions are affected by the number of competitors they face, with markups being negatively related to the number of firms.

assuming that, in choosing their product ranges and pricing strategies, companies behave as oligopolists and not as monopolistic competitors. As a result, each firm co-ordinates its pricing decisions by internalizing demand linkages between the varieties it produces, taking into account that a price reduction for one of its products reduces the sales of all other goods in its product line. This is the so-called *cannibalization* effect, which is a distinguishing feature of multi-product firms. Moreover, because companies are large and produce a non-negligible set of varieties, they also take into account the effects of their pricing decisions on the industry's price index. In this environment, firms attempt to increase their market shares through product proliferation. We assume that a variety-level fixed production cost bounds the company's product range. Firms enter instantaneously in each period until all profit opportunities are exhausted and they pay a fixed period-by-period production cost and earn zero profits in every period. Given that firms are multi-product, market structure is then endogenously determined by the entry and exit decisions of individual producers, as well as by their optimal product scope choices.

In this environment, exogenous shocks not only affect firm entry and exit, but also generate product switching within companies that add and drop products over the business cycle. In this respect, we find that firms' product scopes increase during a macroeconomic expansion. This property stems from the assumption that product markets are oligopolistically competitive. In our model, in fact, the procyclicality of the product range results from firms using the product scope as tool to relax price competition, and this property only holds true in a strategic framework. The intuition is as follows. When product markets are oligopolistically competitive, a company chooses its optimal product scope by accounting for the effects of this choice on both its own and all other firms' pricing decisions. In this respect, we show that increasing the number of varieties produced raises each firm's own price and, as a reaction, induces rivals to cut their products' prices. Consequently, in our environment companies try to mitigate price competition by under-expanding their product scopes with respect to a situation of monopolistic competition in which firms behave in a non-strategic manner. However, the strategic effect of the product scope becomes less important as the number of companies increases. In fact, with a very large number of incumbents, this effect vanishes, and the market structure converges towards monopolistic competition. As a result, the incentive of each company to create new product varieties increases with the number of competitors in the market. Consequently, any shock that induces new firms to enter the market simultaneously prompts incumbents to increase the number of varieties produced. We call this mechanism the *proliferation* effect. As an implication of such an effect, in our model net product creation is predicted to be procyclical with product turnover due to the joint contribution of product switching within existing companies and firm entry. By contrast, in the canonical Dixit-Stiglitz framework with monopolistic competition and multi-product firms, the strategic effect of the product scope is not operative, and we show that firms' product ranges are not affected by transitory aggregate shocks. As a result, this model has the counterfactual implication that, in the short-run, new products are only introduced by new companies.

The *proliferation* effect has important implications for aggregate dynamics. We show that product range adjustments at the firm level give rise to endogenous variations in total factor productivity that magnify the amplitudes of economic fluctuations. In our model, product diversity induces increasing returns at the aggregate level. Thus, for a given amount of productive factors, output is greater the larger the number of varieties produced in equilibrium. In the canonical models of monopolistic competition, either because of the "love-for-variety" property (Chatterjee and Cooper 1993 and Bilbiie et al. 2011) or because of returns to specialization (Devereux et al. 1996), this channel affects aggregate fluctuations through the process of firm entry and exit. In addition, when the economy is populated by multi-product firms, product diversity has implications for aggregate fluctuations through intra-firm adjustments, resulting in variations in product scope. In this case, the product space depends not only on the number of active firms but also on the product ranges chosen by each of them. Consequently, any shock that fosters firm entry or expands firms' product ranges (or both) also affects aggregate output by increasing the number of available goods in the overall economy. We show that the combined effect of changes in product scope with firm entry and exit and countercyclical markup variations provides a quantitatively important, endogenous amplification mechanism. By calibrating our model to the U.S. economy, we find that output volatility increases by 94 percent relative to the canonical real business cycle (RBC) model with perfect competition, and by 19 percent relative to the mono-product firm model. These results indicate that, in our model, the amount of aggregate uncertainty required to account for the same fluctuations in actual data is necessarily lower than the amount required in the two benchmark models. Some comments are in order here. First, introducing multi-product firms in an environment with endogenous entry and oligopolistic competition substantially strengthens the weak internal propagation mechanism of the standard RBC model.[4] Second, our quantitative analysis shows that the *proliferation* effect is the main channel of shock magnification in our model. We find, in fact, that, relative to the mono-product firm model, adjustments along the product scope dampen the response of the average markup to exogenous shocks.

This paper is related to the extensive body of literature in macroeconomics that analyzes business cycle fluctuations with imperfect competition. Among the early contributions on this subject, Rotemberg and Woodford (1999) find that collusion can generate countercyclical markups, while Galí (1994) reaches a similar conclusion with a model in which variations in the composition of demand lead to variations in markups. Both Chatterjee and Cooper (1993) and Devereux et al. (1996) analyze the effects of entry and exit on business cycle dynamics and show that net business formation is procyclical. The presence of strategic interaction (with the implications stressed above) distinguishes our model from these papers, where the market structure is characterized by monopolistic competition. More recent contributions in the field include Comin and Gertler (2006), Bilbiie et al. (2011), Jaimovich and Floetotto (2008) and Etro and Colciago (2010). The first paper attempts to explain medium-term business cycle fluctuations within a model with endogenous productivity dynamics and countercyclical markups. Differently from Comin and Gertler (2006), we consider exogenous shocks and focus on a standard definition of the business cycle. Moreover, due to the presence of strategic interaction, our model generates countercyclical markups, whereas Comin and Gertler postulate a function for markups that is decreasing in the number of firms. Bilbiie et al. (2011) study the role of product creation in propagating business cycle fluctuations in a model with monopolistic competition and sunk entry costs; the authors analyze the contributions of intensive and extensive margins (i.e., changes in the production of existing goods and the range of available goods) to the economy's responses to changes in aggregate productivity. We differ from this paper in two main respects. First, in Bilbiie et al. (2011), each product unit is interpreted as a product line within a multi-product firm; due to the assumption of a continuum of goods, firm boundaries are left unspecified, without concern for strategic interactions. In our paper, we explicitly model multi-product firms by taking strategic interactions within and across firms into account. Second, the source of cyclical movements in markups is different in the two models. In Bilbiie et al. (2011), countercyclical markups are due to demand-side pricing complementarities; in our paper, countercyclical markups are related to variations in the number of competitors and occur due to supply-side considerations.[5] Both Jaimovich and Floetotto (2008) and Etro and Colciago (2010) develop an RBC model with mono-product firms and endogenous entry by taking firms' strategic interactions into account. Jaimovich and Floetotto (2008) show that oligopolistic behavior generates an important channel of shock magnification that operates through endogenous markup variations. Etro and Colciago (2010) analyze the roles of market structure and different forms of competition (Bertrand versus Cournot) in the propagation of technology shocks. We differ from these two papers by proposing a DSGE model with multi-product firms. In the context of this market structure, we show that oligopolistic behavior is responsible for the procyclicality of the product range. The latter represents an important channel of shock amplification that strengthens the endogenous magnification mechanism embodied in models with mono-product firms. Moreover, our paper provides additional insights into the role of imperfect competition in shaping business cycle fluctuations, showing that

[4] This is a well-known deficiency of the basic RBC model; see Cogley and Nason (1995) for further details.

[5] Another important difference between the two models is that the number of firms is a state variable in Bilbiie et al. (2011) because entry is subject to a sunk entry cost and a time-to-build lag. In our baseline model, there is no sunk entry cost; we assume a period-by-period, zero-profit condition so that firms enter instantaneously in each period until all profit opportunities are exploited. In Section 4.3.1, however, we study the richer dynamic problem with sunk entry costs. This alternative assumption does not alter our results; in particular, the magnification mechanism remains quantitatively significant.

cyclical variations in total factor productivity are driven by any shock that boosts aggregate demand, regardless of whether it directly affects technology.

The rest of the paper is organized as follows. Section 2 presents the model, while Section 3 solves it. Section 4 describes the calibration and the main dynamic implications of the model. Finally, Section 5 provides conclusions. The calculation details are provided in the Appendix.

2 The model

We consider an economy that consists of a finite number of imperfectly competitive multi-product firms, each producing several differentiated goods. In each period t, a zero-profit condition determines entry, exit and the equilibrium number of firms. Goods are sold by firms to a continuum (of measure one) of identical households for consumption and investment purposes, and to the government, which collects lump-sum taxes from households to finance public expenditures. All of the interactions among firms, households and the government occur in a stochastic environment where the short-run dynamics of the economy are driven by productivity, preferences and government shocks.

2.1 Households

The representative household has preferences over consumption and leisure with utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[log(C_t) - \xi_t \frac{H_t^{1+\chi}}{1+\chi} \right], \tag{1}$$

where C_t and H_t denote, respectively, consumption and hours worked by the household at time t, $\beta \in (0,1)$ is the subjective time discount factor, $\chi \geq 0$ is the inverse of Frisch labor supply elasticity, E_t denotes the mathematical expectations operator conditional on information available at time t, and ξ_t is a preference shock that follows a univariate autoregressive process of the form $log(\xi_t/\xi) = \varrho_\xi log(\xi_{t-1}/\xi) + \epsilon_t$, with $\varrho_\xi \in [0,1)$, $\xi > 0$, and ϵ_t is an i.i.d. innovation with mean 0 and standard deviation σ_ξ.

At time t, there are M_t multi-product firms that produce differentiated goods. We assume that C_t is a consumption aggregator that combines all of the product varieties from multi-product firms:

$$C_t = \left[\sum_{i=1}^{M_t} x_t^C(i)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{2}$$

where $\theta > 1$ denotes the elasticity of substitution between any two firms and $x_t^C(i)$ is a composite consumption good that aggregates all of the products from company i. Thus:

$$x_t^C(i) = \left[\sum_{j=1}^{N_t(i)} x_t^C(i,j)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}. \tag{3}$$

In Eq. (3), $x_t^C(i,j)$ denotes the purchase of product j manufactured by firm i, $N_t(i)$ is the number of varieties produced by firm i, and $\gamma > 1$ is the elasticity of substitution among these differentiated goods. Product varieties are grouped into nests, with goods within a nest being produced by the same firm. In what follows, we assume that $\gamma > \theta$, which means that the degree of substitutability between varieties within nests is larger than the degree of substitutability between nests; this assumption is consistent with the empirical evidence provided by Broda and Weinstein (2010).[6]

[6] This is usually referred to as the *market segmentation* case (see Section 4.1 for a discussion). This industry configuration arises

Households' preferences (2) and (3) exhibit the love-of-variety property. That is households are biased towards a dispersed consumption of varieties. As one can observe, the variety effect presents both an inter- and an intra-firm component, with C_t and $x_t^C(i)$ being equivalent to the standard Dixit-Stiglitz consumption aggregates. It is worth noting that, while the inter-firm component of the variety effect can always be eliminated by choosing an appropriate specification of (2), the intra-firm component of the variety effect cannot instead be removed because it turns out to be a necessary requirement for incorporating multi-product firms into the model. As we will see later, the reason is that in choosing their product ranges, firms trade scope economies (due to the presence of a firm-level fixed cost) and market power (due to product differentiation and love of variety) against the cost associated with launching a new variety. Because of this property, without the intra-firm component of the variety effect, firms would not find it profitable to produce more than one variety, and hence, in equilibrium, the model would collapse to one with mono-product firms.

The representative household holds one asset, the capital stock K_t, which is assumed to evolve over time according to the following law of motion:

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{4}$$

where $\delta \in (0,1)$ is the rate of depreciation of the capital stock, and I_t denotes the investment aggregator that combines all of the investment goods produced by the multi-product firms:

$$I_t = \left[\sum_{i=1}^{M_t} x_t^I(i)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}. \tag{5}$$

As before, $x_t^I(i)$ is a composite investment good that aggregates all of the products from company i:

$$x_t^I(i) = \left[\sum_{j=1}^{N_t(i)} x_t^I(i,j)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}, \tag{6}$$

where $x_t^I(i,j)$ is the purchase of the investment good j manufactured by firm i.

The representative household supplies labor services per unit of time and rents capital to firms. The labor and capital markets are perfectly competitive, so households take the wage rate w_t paid per unit of labor services and the rental rate r_t paid per unit of capital as given. In addition, the representative household is entitled to the receipt of pure profits from the ownership of firms, Π_t, and a net transfer, T_t, from the government. The flow budget constraint is then given by the following equation:

$$\sum_{i=1}^{M_t}\sum_{j=1}^{N_t(i)} p_t(i,j)\left[x_t^C(i,j) + x_t^I(i,j)\right] \leq w_t H_t + r_t K_t + \Pi_t + T_t, \tag{7}$$

where $p_t(i,j)$ is the price of variety j produced by firm i.

To solve for the consumption optimization problem, we use a three-stage utility maximization procedure. In the first stage, households choose consumption and investment goods within a nest. In the second stage, expenditures are instead allocated across nests. Finally, in the third stage, households choose consumption and leisure over time.

Let us analyze consumption decisions first. In the first stage, the representative household maximizes $x_t^c(i)$ subject to the expenditure constraint on firm i's products, $\sum_{j=1}^{N_t(i)} p_t(i,j)x_t^C(i,j) \leq e_t^C(i)$. Thus, we obtain the

when nest $i \in [1, M]$ corresponds to firm i, which produces N_i close substitute varieties of the good. The theoretical literature (see Brander and Eaton 1984) has also proposed an alternative case, which is denoted *market interlacing*. Under this industry configuration, each nest $i \in [1, M]$ is occupied by N_i firms and consists of varieties produced by different firms; in this case, each manufacturer produces less closely related products.

demand function for consumption good j in nest i:

$$x_t^C(i,j) = \frac{e_t^C(i)}{p_t(i,j)^{\gamma} q_t(i)^{1-\gamma}}, \tag{8}$$

where $q_t(i) = \left[\sum_{j=1}^{N_t(i)} p_t(i,j)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$ is the price index corresponding to firm i. In the second stage, the representative household maximizes C_t subject to the budget constraint on composite goods, $\sum_{i=1}^{M_t} q_t(i) x_t^C(i) \leq e_t^C$; this yields:

$$x_t^C(i) = \frac{e_t^C}{q_t(i)^{\theta} q_t^{1-\theta}}, \tag{9}$$

where $q_t = \left[\sum_{i=1}^{M_t} q_t(i)^{1-\theta}\right]^{\frac{1}{1-\theta}}$ is the aggregate consumption price index. Due to the "love-for-variety" assumption, the price indexes $q_t(i)$ and q_t are decreasing in $N_t(i)$ and M_t, respectively. Because $e_t^C(i) = q_t(i) x_t^C(i)$, with Eqs. (8) and (9), we can write the consumption demand for variety j produced by firm i as:

$$x_t^C(i,j) = \frac{e_t^C}{q_t^{1-\theta} p_t(i,j)^{\gamma} q_t(i)^{\theta-\gamma}}. \tag{10}$$

Following a similar procedure, the aggregate demand for the investment good j produced by firm i reads as:

$$x_t^I(i,j) = \frac{e_t^I}{q_t^{1-\theta} p_t(i,j)^{\gamma} q_t(i)^{\theta-\gamma}}, \tag{11}$$

where e_t^I is the total expenditure on investment goods.

In the third stage, the representative household chooses sequences of consumption C_t and hours worked H_t to maximize the inter-temporal utility function (1) under the law of motion for capital (4) and the lifetime budget constraint (7). Note that by virtue of Eqs. (10) and (11), the latter can be rewritten as follows:

$$e_t^h = q_t \left(C_t + I_t\right) \leq w_t H_t + r_t K_t + \Pi_t + T_t.$$

where $e_t^h \equiv e_t^C + e_t^I$ denotes households' total expenditures on consumption and investment goods at date t. The first-order conditions for an interior maximum are then given by the following two equations:

$$\xi_t H_t^{\chi} = \frac{w_t}{q_t C_t}, \tag{12}$$

$$\frac{1}{C_t} = \beta E_t \left\{ \frac{1}{C_{t+1}} \left[(1-\delta) + \frac{r_{t+1}}{q_{t+1}} \right] \right\}. \tag{13}$$

Under the assumption of a perfectly competitive labor market, Eq. (12) describes the supply of hours for working activities, while Eq. (13) is the well-known Euler equation that provides the inter-temporal optimality condition for consumption.

2.2 The Government

In each period $t > 0$, the government collects lump-sum taxes from households to finance nominal government spending, $q_t G_t$. We assume that real government expenditures, G_t, are exogenous and stochastic, and evolve over time according to a univariate autoregressive process of the form $log(G_t/G) = \varrho_g log(G_{t-1}/G) + \epsilon_t^g$, where $\varrho_g \in [0,1)$, ϵ_t^g is an i.i.d. innovation with mean 0 and standard deviation σ_g and G is the steady-state value of government spending.

As in Ravn et al. (2006), we assume that the government allocates spending over intermediate goods $x_t^G(i,j)$ to maximize the quantity of a composite good produced according to the following technology:

$$G_t = \left[\sum_{i=1}^{M_t} x_t^G(i)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{14}$$

where $x_t^G(i)$ is given by:

$$x_t^G(i) = \left[\sum_{j=1}^{N_t(i)} x_t^G(i,j)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}. \tag{15}$$

The resulting demand for each intermediate good $x_t^G(i,j)$ by the public sector is:

$$x_t^G(i,j) = \frac{e_t^G}{q_t^{1-\theta} p_t(i,j)^{\gamma} q_t(i)^{\theta-\gamma}}, \tag{16}$$

where e_t^G denotes nominal government expenditures.

Given Eqs. (10), (11), and (16), the aggregate demand for variety j produced by firm i, $x_t(i,j)$ can then be expressed as the following:

$$x_t(i,j) = \frac{e_t}{q_t^{1-\theta} p_t(i,j)^{\gamma} q_t(i)^{\theta-\gamma}}, \tag{17}$$

where $e_t \equiv e_t^C + e_t^I + e_t^G$ denotes total expenditure for consumption, investment and government-provided goods at date t. As can be observed, the demand for each individual variety depends negatively on its price and positively on both the nest-level and industry-level price indexes.

2.3 Firms

In this economy, firms decide their pricing policies, production plans, and product scopes. In every period t, entries and exits of multi-product firms into and out of the existing market are determined by a zero-profit condition. Incumbent firms face the same production technology with capital and labor as inputs. In addition to a fixed production cost per variety, we assume that there exists a firm-level fixed cost that must be paid regardless of the size of the firm's product range. Thus, the overall production of a typical firm i that produces $N_t(i)$ varieties at time t can be written as:

$$\sum_{j=1}^{N_t(i)} x_t(i,j) = \sum_{j=1}^{N_t(i)} \left[z_t k_t(i,j)^{\alpha} h_t(i,j)^{1-\alpha} - \phi_v\right] - \phi_f, \tag{18}$$

where $k_t(i,j)$ and $h_t(i,j)$ denote, respectively, the amount of capital and labor employed by firm i in producing variety j, ϕ_v is the fixed cost per variety, ϕ_f is the firm-level fixed cost and $\alpha \in [0,1]$. We assume that z_t is an economy-wide productivity shock at time t, the log of which follows a stationary AR(1) process with persistence parameter $\varrho_z \in (0,1]$ and a normally distributed innovation, ε_t, with a mean of zero and a standard deviation of σ_z.

We model firms' decisions as a two-stage game. In the first stage, firms choose the number of varieties to produce and determine the sizes of their product ranges. In the second stage, they act as Bertrand-Nash competitors in the product market. The model is solved by backward induction using the subgame perfect-Nash equilibrium concept.

2.4 Pricing

When setting the price of each product, a multi-product firm does not take the prices of the other varieties that it produces as given. Rather, it internalizes demand linkages between the varieties within its nest. A firm takes it into account that a price reduction for one of its products negatively affects the sales of all of the other varieties that it produces. This effect is called the *cannibalization* effect. Moreover, because companies are not small relative to the size of the market (they produce a non-negligible set of varieties), they take the aggregate price index into consideration when making a pricing decision.[7]

A typical firm i chooses a pricing rule for each variety within its nest to maximize profits:

$$\pi_t(i) = \sum_{j=1}^{N_t(i)} \left[p_t(i,j)x_t(i,j) - w_t h_t(i,j) - r_t k_t(i,j) \right]. \tag{19}$$

As shown in Appendix B, a Nash equilibrium in prices emerges when each firm i charges the same price for all of the product varieties within its nest:

$$p_t(i,j) = p_t(i) = mc_t \frac{[\theta - (\theta-1)\epsilon_t(i)]}{(\theta-1)\,[1-\epsilon_t(i)]}, \quad \forall j \in [1, N_t(i)]. \tag{20}$$

In this equation, $mc_t \equiv w_t^{1-\alpha} r_t^{\alpha} / \left[z_t (1-\alpha)^{1-\alpha} \alpha^{\alpha} \right]$ is the marginal cost of producing one more variety and $\epsilon_t(i)$ is firm i's market share:

$$\epsilon_t(i) \equiv \frac{q_t(i)x_t(i)}{e_t} = \left[\frac{q_t(i)}{q_t} \right]^{1-\theta} = \frac{N_t(i)^{\frac{1-\theta}{1-\gamma}} p_t(i)^{1-\theta}}{\sum_{i=1}^{M_t} N_t(i)^{\frac{1-\theta}{1-\gamma}} p_t(i)^{1-\theta}}, \tag{21}$$

where $x_t(i) \equiv x_t^C(i) + x_t^I(i) + x_t^G(i)$ denotes the total demand for consumption, investment and government goods produced by firm i. As can be seen, the market share of a typical firm is increasing in the number of its own varieties and decreasing in the number of varieties produced by rivals. This feature of the model, driven by the intra-firm component of the variety effect (the price index $q_t(i)$ is decreasing in $N_t(i)$), explains the reason why firms may find profitable to offer multiple products to their consumers.

2.5 Product scope

In the first stage of the game, firms anticipate the subsequent price competition and determine their product ranges by playing a Nash game. Thus, when choosing a product scope, a firm takes the number of competitors and their product ranges as given. As shown in Appendix C, the gross profits of firm i can be expressed as:

$$\pi_t(i) = e_t \epsilon_t(i) L_t(i) - mc_t \left[N_t(i)\phi_v + \phi_f \right], \tag{22}$$

where $L_t(i) \equiv [p_t(i) - mc_t] / p_t(i) = 1/ [\theta - \epsilon_t(i)\,(\theta - 1)]$ is the Lerner index of market power. Firm i maximizes profits with respect to $N_t(i)$ by accounting for the strategic effect of the product range decision, that is, the effect of its product range choice on both its own and all other firms' pricing decisions. Now, the first-order condition for an interior maximum can be written as the following:

$$\frac{e_t \epsilon_t(i) L_t(i)}{N_t(i)} \eta_t(i) = mc_t \phi_v, \tag{23}$$

where $\eta_t(i) = \theta L_t(i) \cdot [\partial \epsilon_t(i) / \partial N_t(i)] \cdot [N_t(i)/\epsilon_t(i)]$ is the elasticity of the variable profits of firm i with respect to the size of its product range, $N_t(i)$. The left-hand side of Eq. (23) provides the benefit of expanding the product range by one unit: opening a new product line increases the firm's market share and leads to higher profits. However, a greater number of products involves higher proliferation costs; the right-hand side of Eq. (23) represents the cost of adding one more variety to the nest. As it is evident from Eq. (21), in the absence of the intra-firm component

[7] To account for all of these effects, in Appendix A we first provide the derivation of the elasticity of the price index with respect to the price of a product variety. Then, we determine the demand elasticities of a good in response to variations in its own price and other goods' prices within the same firm.

of the variety effect ($q_t(i) = p_t(i)$), the market share of a typical company i, $\epsilon_t(i)$, would not depend on its product range, $N_t(i)$; in such a case, according to Eq. (22), the profit function would be decreasing in $N_t(i)$ and thus each firm would optimally produce only one variety.

3 Symmetric rational expectations equilibrium

From now on, we restrict our attention to symmetric equilibria. In solving the model, we first examine product- and firm-level variables and then turn to aggregate variables.

3.1 Product- and firm-level variables

Under the assumption of symmetry, each company produces the same number of varieties and presents the same market share, that is, $N_t(i) = N_t$, and $\epsilon_t(i) = 1/M_t$ for every $i \in [1, M_t]$. This implies that the price of each product variety reduces to:

$$p_t = mc_t \mu(M_t) = mc_t \frac{[(M_t - 1)\theta + 1]}{(\theta - 1)(M_t - 1)}, \tag{24}$$

where $\mu(M_t)$ denotes the mark-up ratio that is a decreasing function of the number of firms.

Using the fact that $q_t(i) = \left[\sum_{j=1}^{N_t(i)} p_t(i,j)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$, and substituting $x_t^C(i)$, $x_t^I(i)$ and $x_t^g(i)$, respectively from (3), (6) and (15) into (21), output per product (*intensive margin*) is written as:

$$x_t = \frac{e_t}{p_t N_t M_t}, \tag{25}$$

and, consequently, output per firm can be easily expressed as $x_t N_t = e_t/(p_t M_t)$.

Each firm chooses its optimal product scope by accounting for the effect of its product range choice on both its own and all other firms' pricing decisions. As shown in Appendix C, in our strategic set-up, firms use the product range as a tool to relax price competition. In fact, an increase in the number of a firm's varieties raises the firm's own price and reduces other firms' prices. Consequently, to mitigate price competition in the second stage of the game, firms contract their product range in the first stage, reducing the number of product varieties offered. The elasticity of variable profits with respect to the size of each firm's product scope measures the extent to which firms under-expand their product scopes with respect to a situation of monopolistic competition in which firms' strategic interactions are neglected:

$$\eta_t = \frac{(\theta - 1)}{(\gamma - 1)} \left[\frac{\theta(M_t - 1)^2}{M_t \theta (M_t - 1) + \theta - 1}\right].$$

As can be easily ascertained, the term in the square brackets (which is smaller than one) is an increasing function of the number of firms, M_t. Intuitively, the strategic effect of the product scope becomes less important as M_t increases so that the incentive to create new varieties increases with the number of firms. When the latter becomes very large, the term in the square brackets tends toward one, and the elasticity η_t tends toward $(\theta - 1) / (\gamma - 1)$, as in monopolistic competition. Now, replacing η_t from the above equation into Eq. (23) and then imposing symmetry, the number of varieties per firm (*intra-firm extensive margin*) can be written as:

$$N_t = \frac{e_t \theta (M_t - 1)}{\phi_v (\gamma - 1) p_t [M_t \theta (M_t - 1) + \theta - 1]}. \tag{26}$$

Finally, using (24) and (25) in (22) and requiring that firms make zero profits, we obtain:

$$x_t N_t [\mu(M_t) - 1] = \phi_v N_t + \phi_f. \tag{27}$$

This condition, which relates the number of firms (*inter-firm extensive margin*) to the output per product and the product scope, must hold in every period.

3.2 Aggregate variables

In a symmetric equilibrium, $h_t(i,j) = h_t$ and $k_t(i,j) = k_t$ for every $i \in [1, M_t]$ and $j \in [1, N_t(i)]$. Then, aggregate hours and aggregate capital are equal to $H_t = M_t N_t h_t$ and $K_t = M_t N_t k_t$, respectively. The total number of firms M_t can be found by combining technology (18) with the zero-profit condition (27):

$$M_t = z_t K_t^{\alpha} H_t^{1-\alpha} \frac{\mu(M_t) - 1}{\mu(M_t)\,(\phi_v N_t + \phi_f)}. \tag{28}$$

The clearing of the goods market requires that in equilibrium $Y_t \equiv C_t + I_t + G_t$, where Y_t denotes the aggregate level of production. The latter can be easily found by plugging (3) into (2), (6) into (5) and (15) into (14) after using Eqs. (27) and (28) to simplify:

$$Y_t = z_t K_t^{\alpha} H_t^{1-\alpha} \frac{M_t^{1/(\theta-1)} N_t^{1/(\gamma-1)}}{\mu(M_t)}. \tag{29}$$

In what follows, we use the price p_t as the *numéraire* and set it to one; thus, we can rearrange Eq. (24) and write the marginal cost of production mc_t as $1/\mu(M_t)$. With this normalization, the equilibrium wage rate, the rental rate and the price index, respectively, read as follows:

$$w_t = \frac{(1-\alpha) z_t}{\mu(M_t)} K_t^{\alpha} H_t^{-\alpha}, \tag{30}$$

$$r_t = \frac{\alpha z_t}{\mu(M_t)} K_t^{\alpha-1} H_t^{1-\alpha}, \tag{31}$$

$$q_t = M_t^{\frac{1}{1-\theta}} N_t^{\frac{1}{1-\gamma}}. \tag{32}$$

4 Results

In this section, we first calibrate the model to the U.S. economy, and we then quantitatively characterize the response of our model to three types of exogenous shocks: technology shocks, z_t, preference shocks, ξ_t, and government-spending shocks, G_t. We study the model's dynamics at both the firm and aggregate levels.

4.1 Calibration

The model has a total of 17 parameters, $\omega = \{\beta,\, \chi,\, \xi,\, \theta,\, \gamma,\, z,\, \delta,\, \alpha,\, \phi_v,\, \phi_f,\, G,\, \varrho_z,\, \sigma_z,\, \varrho_g,\, \sigma_g,\, \varrho_\xi,\, \sigma_\xi\}$. Because many of these are standard in the business cycle literature, we use common values. Specifically, we calibrate the model to the U.S. economy assuming that each period corresponds to a quarter. We thus set the subjective discount factor, β, equal to 0.99, which implies a yearly nominal interest rate of 4 percent. Following Ravn et al. (2006), we set the labor income share (wH/qY) to 0.75, the investment share (I/Y) to 0.18, the government consumption share (G/Y) to 0.12 and the Frisch elasticity of labor supply to 1.3. These restrictions imply that the capital elasticity of output in production, α, is 0.25, the capital depreciation rate, δ, is 0.025, the inverse of the Frisch elasticity of labor supply, χ, is roughly 0.77, and the steady-state level of government purchases is 0.0510. As in Prescott (1986), the preference parameter ξ is chosen to ensure that, in the steady state, the representative consumer devotes 1/4 of his time to labor activities. Concerning stochastic shocks, we normalize the long-run rate of technology, z, to 1, and we set the autocorrelation coefficients, ϱ_z, ϱ_g, and ϱ_ξ to 0.97. Following King and Rebelo (1999) and

Table 1: **Calibration**

Parameter	Value	Description
β	.9902	Subjective discount factor
α	0.25	Capital Elasticity of output
χ	0.77	Inverse of Frish elasticity
δ	0.025	Capital depreciation rate
ξ	12.44	Preference parameter
z	1	Long-Run rate of technology
γ	11.5	Within-brand elasticity of substitution
θ	10	Across-firm elasticity of substitution
ϕ_f	0.057	Firm-level fixed cost
ϕ_v	0.003	Variety-level fixed cost
G	0.0510	Steady state level of government purchases
ϱ_z, ϱ_g, ϱ_ξ	0.9702	Persistence parameter of exogenous shocks
σ_ε	0.0072	Standard error of innovations in the rate of technology
σ_g	0.0089	Standard error of government-spending shocks
σ_ξ	0.0232	Standard error of preference shocks

Collard and Dellas (2006), we set the standard deviations of the technology shock, σ_z, and government-spending shock, σ_g, to 0.0072 and 0.0089, respectively. The standard deviation of the preference shocks σ_ξ is chosen such that, conditional on all of the other parameters, the volatility of hours worked in the model matches its empirical counterpart of 1.79%.[8] This calibration restriction implies that the standard deviation of preference shocks is set to 0.0232, a value that is consistent with the available empirical evidence.[9]

Turning to the firm-related parameters $\{\gamma,\, \theta,\, \phi_v,\, \phi_f\}$, we set the across-brand elasticities of substitution, θ, to 10. This value is consistent with the empirical evidence provided by Cogley and Sbordone (2008) for the U.S. economy and is intermediate among the values typically used in the RBC literature. The within-brand elasticity of substitution, γ, is set equal to 11.5 in accordance with the empirical evidence provided by Broda and Weinstein (2010) for multi-product firms in the U.S. economy. More specifically, the value assigned to γ corresponds to the median of the distribution of the estimated elasticities obtained by Broda and Weinstein (2010), using data for 122 product groups in the U.S. economy.[10] To assign values to the fixed cost parameters $\{\phi_f,\, \phi_v\}$, we introduce two calibration restrictions. First, as in Jaimovich and Floetotto (2008), we assume that the long-run aggregate markup, μ, takes a value of 1.3.[11] Then, using the expression for the mark-up ratio, $\mu = [(M-1)\,\theta + 1] \,/\, [(\theta - 1)\,(M-1)]$, we can find the number of firms, M, that (given the value assigned to θ) causes the equilibrium mark-up rate, μ, to be equal to 1.3:

$$M = 1 + \frac{1}{\mu(\theta - 1) - \theta}. \tag{33}$$

As a second restriction, we assign a value of 3.5 to the long-run product scope, N. According to the evidence reported by Bernard et al. (2010), this number corresponds to the average number of products produced by manufacturing firms in the U.S. economy over the period 1987-1997.[12] The introduction of this second restriction is

[8] This number has been taken from King and Rebelo (1999).

[9] For example, Galí and Rabanal (2004) in estimating a DSGE model for the U.S. economy find that the standard deviation of preference shocks of the type described in the present paper is 0.025.

[10] See Table 8 (page 717) in Broda and Weinstein (2010).

[11] This value is intermediate among those typically used in RBC models with monopolistically competitive product markets. For instance, Rotemberg and Woodford (1999) assume an average markup of 20%, while Bilbiie et al. (2011) assume a markup of 36%.

[12] See Table 1 (page 79) in Bernard et al. (2010).

necessary because in our framework, unlike in models of mono-product firms, knowing the equilibrium number of firms does not allow us to select values for the fixed-cost parameters. In fact, in a model with multi-product firms, the zero-profit condition (28) involves the two fixed-cost parameters, ϕ_f and ϕ_v, and the equilibrium product scope, N. Thus, although the latter can be determined with (26), we still do not have sufficient information to simultaneously determine ϕ_f and ϕ_v. This requires the introduction of another calibration restriction in addition to (33). With the latter and the restriction $N = 3.5$ in combination with equations (26) and (28), we set the fixed cost parameters ϕ_v and ϕ_f to 0.0031 and 0.0568, respectively. This implies that, in the steady state, sunk fixed costs absorb approximately 20% of the gross production of each firm. Table 1 summarizes the set of calibrated parameters.

4.2 Firm-level Dynamics

We now examine the dynamics at the firm level and compare our model's implications to those of two different scenarios. In the first, firms are mono-product and behave as oligopolists (Figure 1). For this economy, we retain the parameterization used in the multi-product firm model, but we adjust the value of fixed-cost parameters to ensure that the long-run number of active firms, M, is equal in the two economies.[13] By doing this, we remove any long-run effects associated with the number of active firms; thus, any difference in firm dynamics across the two economies can be directly imputed to short-term adjustments in the intra-firm extensive margin (the number of products per firm). This allows us to more clearly disentangle the impact of firms' product scopes on entry/exit decisions over the course of the business cycle. In the second scenario, firms are still multi-product but behave as monopolistic competitors (Figure 3). Unlike the first case, computations based on this economy are obtained by retaining the same parameterization as that summarized in Table 1. Thus, comparing our model with this alternative economy allows us to assess the overall impact of introducing firms' strategic interactions on firm-level dynamics. The impulse-response functions summarize the dynamic responses of a selected variable to a 1% shock. Row 1 of Figures 1 and 3 refers to technology shocks, row 2 to preference-type shocks and, finally, row 3 to government-spending shocks. From left to right, we depict the dynamic responses of the number of firms (inter-firm extensive margin), product scope (intra-firm extensive margin), output per variety (intensive margin), and output per firm. The continuous lines refer to the firm-level dynamics in our model, while the dashed lines denote those of the two alternative scenarios.

4.2.1 Impulse-response functions

In Figure 1, we plot the firm-level dynamics of our model and compare them with those obtained in a set-up in which firms are mono-product and behave as oligopolists. In this alternative scenario, each company takes the effects of its own price decisions on the aggregate price index into account. As shown in Figure 1, the dynamic responses of firm-level variables to the three types of shocks are qualitatively identical. Therefore, in what follows we will focus on the upper panels of this figure, that is when the short-run dynamics of the economy are driven by technology shocks.

The first panel of Figure 1 shows that an unexpected boost in productivity has a positive impact on firm entry, both when companies are multi-product and when they are single-product. Intuitively, for given factor prices in these two economies, a positive technology shock decreases on impact real marginal costs. This effect creates profit opportunities that encourage the entry of new firms, which takes place until the economy reaches a zero-profit equilibrium.[14] This finding has two important implications. First, a positive technology shock triggers

[13] More precisely, when firms are mono-product, there is only the firm-level fixed cost, ϕ_f; the latter is adjusted to ensure that the long-run number of active firms, M, is equal in the two economies with and without multi-product firms.

[14] More generally, Figure 1 shows that any shock to the agents' environment that generates new profit opportunities induces net business formation. Intuitively, a positive preference-type shock raises the marginal utility of consumption and induces an expansion in labor supply by reducing the value of leisure with respect to consumption. The increase in labor supply introduces downward

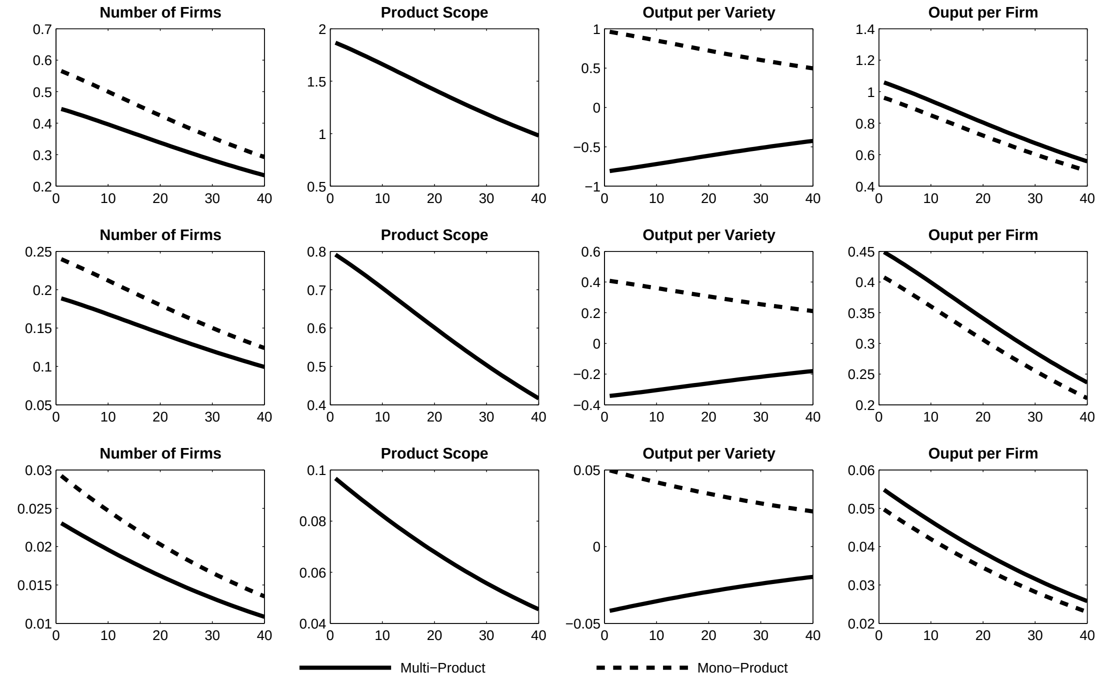

Figure 1: **Firm-level Dynamics: Multi-product vs Single-product Firms**. The picture shows the responses of a subset of firm-level variables to a 1% variation in exogenous temporary shocks. The first row of panels refers to a technology shock, the second row to a preference shock and the third row to a government-spending shock. Impulse responses are measured in per cent deviations from steady state. Horizontal axes display the number of quarters after the shock.

countercyclical markup variations (see Figure 5). This feature is driven by the competitive effect associated with the entry of new firms: when the market structure is endogenous, an increase in the number of active firms makes competition tougher and pushes incumbents to cut their markup ratios.[15] As a result, any shock that fosters the entry of new firms on impact also causes a decline in the average markup. Second, as shown in the second panel of Figure 1, an exogenous boost in productivity causes an expansion in firms' product ranges. As explained more extensively in Appendix C, this model's feature is a direct consequence of the strategic effect of the product range. In particular, because the latter becomes less important as the number of firms increases, the entry of new companies raises the incentives of each producer to introduce new product lines. Therefore, a positive technology shock, by inducing the entry of new firms, also triggers an expansion in the size of each firm's product range.

The positive responses of both business formation, M_t, and firms' product ranges, N_t, imply that net product creation, $M_tN_t - M_{t-1}N_{t-1}$, increases on impact of an expansionary shock. This property implies that in our model net product creation and final output are strongly positively correlated. We find in fact that the model-based coefficient of contemporaneous correlation between these two variables is equal to 0.70.[16] In other words, consistent with the evidence reported in Broda and Weinstein (2010), Axarloglou (2003) and Lee and Mukoyama (2008), our model predicts that net product creation moves procyclically over the course of the business cycle. Three main features of this result are worth emphasizing. First, by comparing the first two panels of Figure 1, we note that the response of the product scope to a technology shock is stronger than that of the number of firms,

pressure on wages and raises firms' profitabilities. With free entry, enhanced profit opportunities lead to firm entry. An expansionary government-spending shock is similar to a positive preference shock in that it increases labor supply at given factor prices. If the number of firms is held constant, the real wage would decline; by increasing profits on impact, this shock stimulates entry.

[15] Because of this property, in the symmetric equilibrium the average markup is a decreasing function of the number of active firms, M_t, as is apparent from Eq. (24).

[16] This number refers to HP-filtered variables ($\lambda = 1600$) and is computed using the procedure explained in Uhlig (1995).

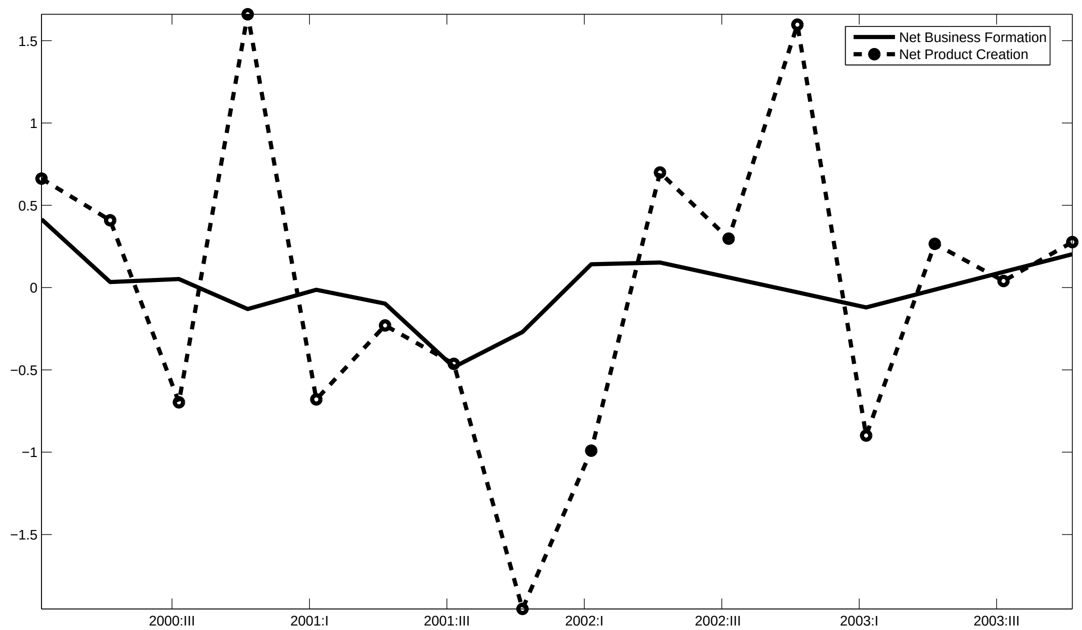

Figure 2: **Net Product Creation versus Net Business Formation**. The picture depicts quarterly series of net product creation (dashed line) and net business formation (continuous line) over the period 2000-2003. Figures for net product creation are taken from Broda and Weinstein (2010) (http://www.aeaweb.org/articles.php?doi=10.1257/aer.100.3.691). Net business formation is defined as the difference between openings and closings of establishments over the total number of establishments. These figures are taken from the Bureau of Labor Statistics (http://data.bls.gov/cgi-bin/dsrv?bd). Both net product creation and net business formation have been detrended with the HP-filter ($\lambda = 1600$).

being about 4 times larger on impact. A similar pattern can also be observed with respect to preference shocks and government-spending shocks.[17] In our model, therefore, the contribution of the intra-firm extensive margin to the overall fluctuations of product creation is substantially more important than the contribution of the entry of new firms. For example, in the case of a positive technology shock, we find that new varieties produced at existing firms account on impact for 71 percent of total product creation. This prediction of the model is in line with the empirical evidence provided by Broda and Weinstein (2010) who show that most product turnover in the U.S. economy occurs within the boundaries of the firm.[18] Second, our model predicts that demand-type shocks are the main driving forces of fluctuations in net product creation. The results from the variance decomposition show that approximately 2/3 of the overall volatility of net product creation is explained by preference and government-spending shocks.[19] Finally, in our model net product creation is predicted to be more volatile than net business formation ($M_t - M_{t-1}$). This model's feature can be tested against data by comparing the quarterly figures of net product creation provided by Broda and Weinstein (2010) with a measure of net business formation. This is done in Figure 2 where we graph the HP-detrended series of net product creation along with the difference between the openings and closings of establishments− a row measure of net business formation− over the period 2000-2003. [20] Although this evidence must be interpreted with some caution given the extremely short period of time considered, Figure 2 seems to confirm our model's predictions: net product creation and net business formation are, in fact,

[17]As a result, we find that the product scope is approximately 4 times more volatile than business formation.

[18]In this respect, the authors find that most product creation occurs within the firm because the market share of new products is four times the market share of new firms.

[19]This number refers to the decomposition of the asymptotic variance implied by the model.

[20]The choice of the time horizon in this comparison is constrained by the availability of data of product creation.

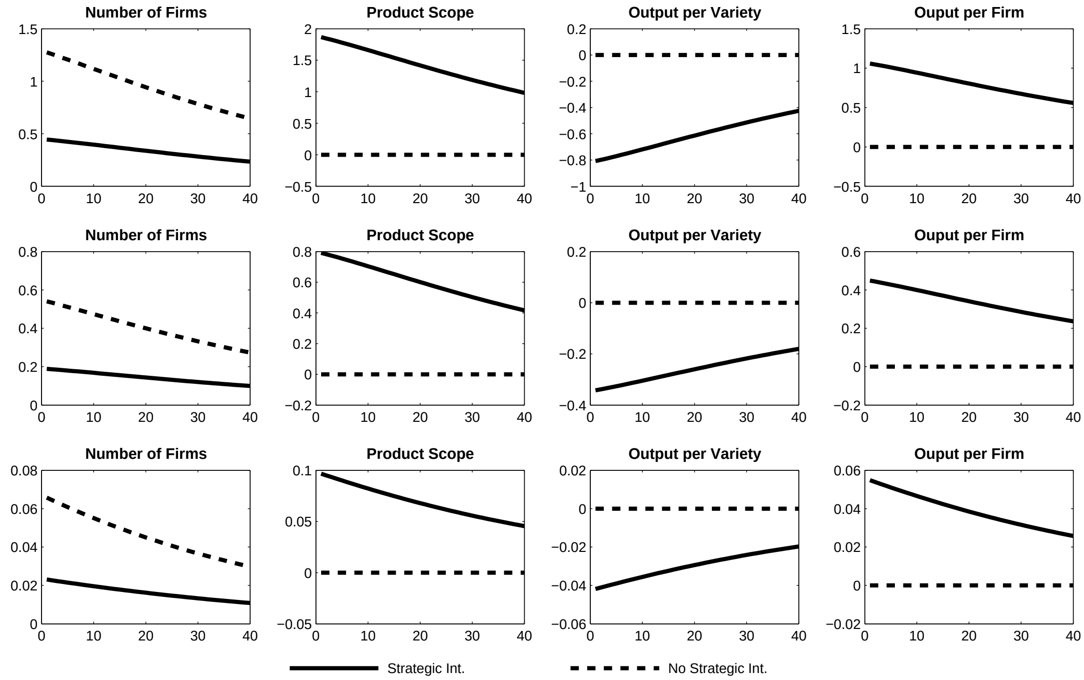

Figure 3: **Firm-level Dynamics: the role of Strategic Interaction**. The picture shows the responses of a subset of firm-level variables to a 1% variation in exogenous temporary shocks. The first row of panels refers to a technology shock, the second row to a preference shock and the third row to a government-spending shock. Impulse responses are measured in per cent deviations from steady state. Horizontal axes display the number of quarters after the shock.

positively correlated in the data (0.32), but with the former variable which is much more volatile than the latter (about 4.65 times more volatile).

It is also interesting to note that in our model adjustments along the intra-firm extensive margin act as a sort of barrier to entry over the business cycle. As shown in the first column of Figure 1, the dynamic response of the inter-firm extensive margin in the model with mono-product firms is stronger than its counterpart in the model with multi-product firms. Because the two models are calibrated to ensure that the long-run number of active firms is equal in the two scenarios, the mono-product firm model is characterized by a larger number of competitors throughout the transition to the steady state. Intuitively, a larger number of products per firm increases the burden of proliferation costs that incumbents must incur. Consequently, in the model with multi-product companies, the maximum number of firms that the market can accommodate with non-negative profits is smaller.

The third panel of Figure 1 shows that output per variety responds positively to a technology shock when firms are mono-product and negatively when firms are multi-product. This is because output per variety coincides with output per firm in a mono-product firm model. In our framework, when companies expand the sizes of their product ranges, the quantity of each variety produced decreases. Due to the *cannibalization* effect, multi-product firms reduce the production of their individual varieties when expanding their product scopes; the opening of a new product line cannibalizes the sales of existing products.

Finally, the fourth panel shows that output per firm responds positively to a technology shock in both models. Interestingly, the impact is more pronounced when firms are multi-product. This finding indicates that the increase in the intra-firm extensive margin is so large that it not only dominates the decrease in the intensive margin, but it also makes output per firm expand more in multi-product firms than it does in single-product ones.

In Figure 3, we compare our model with a different set-up in which firms are multi-product but behave as monopolistic competitors. As before, we restrict our attention to technology shocks (first row). The first panel of this figure shows that an expansionary technology shock has a positive impact on the number of firms in both models. The response is larger under monopolistic competition; in the latter case, the adjustment to a shock occurs only along the inter-firm extensive margin. As shown by the second and third columns, respectively, of Figure 3, the product scope and output per variety do not respond to a technology shock when firms behave as monopolistic competitors. Now, the intra-firm extensive margin is not working because the strategic effect of the product scope, which is responsible for the procyclicality of the product range, is missing in a model where firms' strategic interactions are neglected.[21] Moreover, there is no adjustment along the intensive margin because an expansionary technology shock fosters the entry of new companies without modifying the size of each firm's product range or the price of each product variety. As shown in Appendix D, output per variety, $x_t = e_t/(M_t N_t p_t)$, remains constant because the number of firms, M_t, varies proportionally with the aggregate demand for consumption, investment and government goods, e_t. Finally, the fourth column of Figure 3 shows that the response of output per firm to a technology shock is nil under monopolistic competition; this follows from the constancy of both the product scope and output per variety.

A clear picture emerges from the above analysis: in and of itself, the presence of multi-product firms is of limited interest for the analysis of firm-level dynamics. Instead, introducing firms' strategic interactions is fundamental to addressing both the countercyclicality of markups and the procyclicality of product creation.

4.3 Aggregate Dynamics

Next, we study the implications of multi-product firms and strategic interaction for aggregate dynamics. Our aim in this section is to assess the contribution of the intra-firm extensive margin to the economy's response to changes in the technology shock, z_t, preference shock, ξ_t, and government-spending shocks, G_t. As in the previous section, we address this issue by performing several numerical experiments that compare the properties of our model with those of other alternative setups, including the canonical RBC model with perfect competition. The results are summarized in Figures 4 and 5, where we display the impulse response functions of selected endogenous variables, and in Table 2, where we quantify the aggregate implications of product creation. Model-based second-order moments involved in the computation of the statistics reported in Table 2 are the theoretical moments of HP-filtered data ($\lambda = 1600$), computed as in Uhlig (1995). Additionally, in Table 2, to evaluate how sensitive our results are to alternative calibrations, we report statistics for the baseline case ($\mu = 1.3$) and for alternative values of the long-run average markup, μ.

4.3.1 Impulse-Responses

The first row of Figure 4 displays the impulse responses of output, consumption, investment and real wage (w_t/q_t) to a 1% temporary increase in the rate of exogenous technology, z_t. The response of the model with multi-product firms is shown with a solid line. For comparison, the picture also depicts the responses of (i) the RBC economy with perfect competition (line with the + marker) and (ii) the economy with mono-product firms (dashed line). Comparing our model with the perfectly competitive economy is useful for assessing the joint aggregate implications of the multi-product nature of firms, oligopolistic competition and firm entry, whereas comparing our model with an identical economy with mono-product firms allows us to directly assess the specific contribution of the process of product creation.

Starting with the first comparison, we see that in our environment, as in the standard RBC model, output and its components all increase in response to an unexpected boost in productivity. However, as is evident from the impulse response functions, variables in our model are substantially more responsive than those in the canonical

[21] As a result, this model predicts that product creation is pro-cyclical but with the counterfactual implication that product turnover depends only on the contribution due to the entry of new firms.

RBC model. In other words, the impact of a temporary technology shock on the main aggregates is magnified in the presence of endogenous market structure and multi-product firms. To identify the channels through which the amplification mechanism operates, it is useful to define an appropriate measure of total factor productivity (TFP). Formally, given Eq. (29), the *effective* rate of TFP can be simply defined as follows:[22]

$$TFP_t = \frac{Y_t}{H_t^{1-\alpha} K_t^{\alpha}} \equiv z_t \left(\frac{M_t^{\frac{1}{\theta-1}} N_t^{\frac{1}{\gamma-1}}}{\mu(M_t)} \right). \tag{34}$$

In addition to the purely exogenous rate of technology z_t, in our model movements in TFP are also driven by an endogenous component, which is the term in brackets in Eq. (34). Accordingly, there are two sources of endogenous amplification: oligopolistic competition and product diversity. The first factor affects the effective TFP by inducing countercyclical markup variations through the process of firm entry and exit (see Figure 5).[23] As discussed in the previous section, a positive technology shock creates new profit opportunities that induce firm entry. The increase in M_t, in turn, leads to a decline in the aggregate average markup, $\mu(M_t)$. With lower markups, producers now need to sell more goods to recover fixed costs.[24] Because TFP is measured only in terms of actual sales, the decline in the average markup is accompanied by an increase in the effective TFP.

The second source of endogenous amplification is related to the property that product diversity at the firm level induces increasing returns at the aggregate level.[25] This feature is a direct consequence of the assumption that households' preferences exhibit the "love-for-variety" property. In a model with mono-product firms, this property amplifies the effects of technology shocks through the process of firm entry and exit (see Chatterjee and Cooper 1993 and Bilbiie et al. 2011).[26] The amplification mechanism can be explained as follows. If households' preferences display the "love-for-variety" property, this implies that a larger range of available products increases utility. Thus, when an entrant introduces a new differentiated good into the market, the bundle of goods expands and households derive higher utility from a given nominal spending amount. Consequently, the price index that tracks the composition of the variety basket declines. Given that the latter is the relevant price for households, the entry of new firms also shifts the schedules of both labor and saving supply to the right.[27] Therefore, with an unexpected boost in the rate of technology, the resulting positive net-entry response encourages households to work harder and accumulate more capital, thus amplifying the effect of the original shock.[28] In our model with multi-product firms, the variety effect also presents an intra-firm component that depends on the product range, N_t. As the latter moves in response to temporary shocks, in this economy, product diversity affects final output not only through the entry of new firms, but also through adjustments along the firms' product scopes. In particular, because the strategic effect of the product scope becomes weaker as the number of firms increases, a positive technology shock fosters firm entry and expands firms' product ranges. The resulting larger set of available goods in the overall economy boosts the aggregate level of output, thus amplifying the effect of the shock. This feature of the model is captured by the two terms $M_t^{\frac{1}{\theta-1}}$ and $N_t^{\frac{1}{\gamma-1}}$ in Eq. (34), which provide the contribution of the inter-firm and intra-firm extensive margins to the effective TFP, respectively.

[22] We use the word *effective* to distinguish total factor productivity from the purely exogenous technology shock, z_t.

[23] This source of amplification is, however, not related to the multi-product nature of the firm because it works similarly in a model of oligopolistic competition with mono-product firms (see for instance Jaimovich and Floetotto 2008).

[24] This follows from the zero-profit condition (27), which implies that the ratio of fixed costs to sales, $(\phi_f + \phi_v N_t)/(N_t x_t)$, being equal to $\mu(M_t) - 1$, increases in the average markup, $\mu(M_t)$.

[25] That is, for a given amount of productive factors, final output is greater the larger the number of varieties produced.

[26] Devereux et al. (1996) obtain the same result in an alternative set-up where households derive utility from a homogeneous consumption good produced by a competitive sector that aggregates intermediate goods using a CES production function. In such an environment, the mechanism that relates the process of entry and exit to economic fluctuations is driven by increasing returns to specialization at the aggregate level instead of the "love-for-variety" property.

[27] To ascertain this, notice that with mono-product firms, the labor supply is still defined as in Eq. (12) but with the price index q_t equal to $M_t^{1/(1-\theta)}$. *Ceteris paribus*, a larger number of active firms shifts the labor supply schedule to the right, and thus households are more willing to work. A similar conclusion holds true for saving supply.

[28] For further details on this mechanism, see Chatterjee and Cooper (1993).

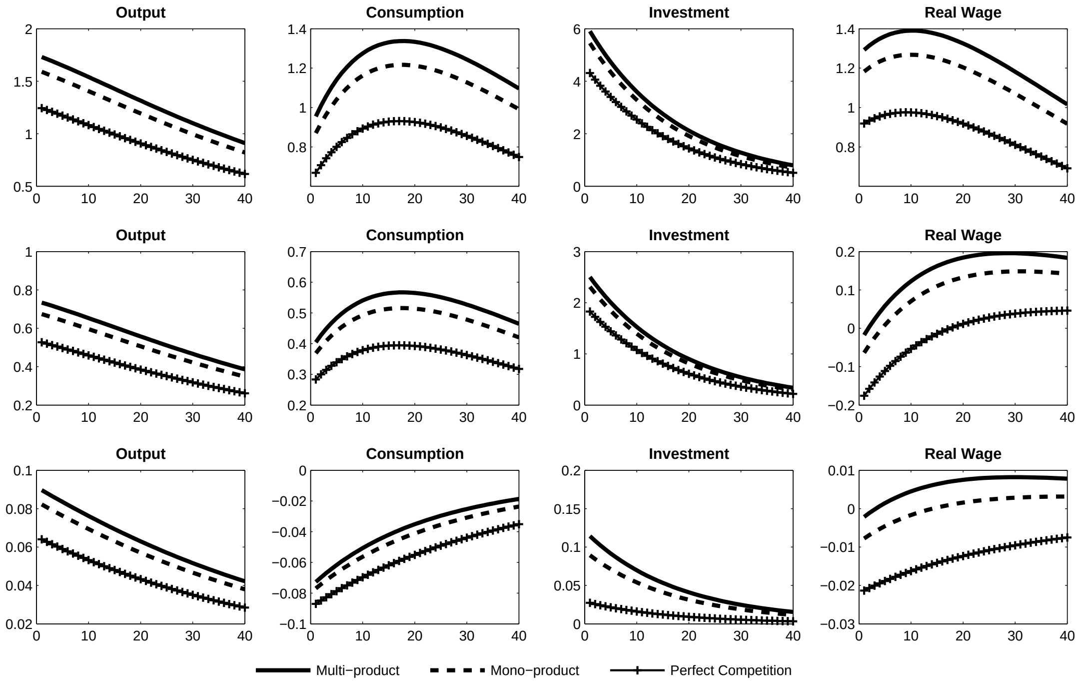

Figure 4: **Aggregate Dynamics**. The picture shows the responses of selected aggregate variables to a 1% variation in exogenous temporary shocks. The first row of panels refers to a technology shock, the second row to a preference shock and the third row to a government-spending shock. Impulse responses are measured in per cent deviations from steady state. Horizontal axes display the number of quarters after the shock.

The endogenous amplification that is induced by the combined effect from both product diversity and oligopolistic competition is evident in the impulse response functions. Focusing on the first panel of Figure 5, we see that the response of the effective TFP substantially deviates from technology shocks (line with the + marker), being larger both on impact (about 45% larger) and during the transition back to the steady state. Exactly as in the RBC economy with a purely exogenous rate of technology z_t, in our model, changes in the effective rate of TFP translate into aggregate demand and supply fluctuations by directly affecting the relevant prices for both households and firms, w_t/q_t and r_t/q_t. It follows, therefore, that the boosting effect on TFP is transmitted to the rest of the economy through the usual channels.[29] This explains the reason why we find that output, consumption and investment are all more responsive in the multi-product firm model than in the canonical RBC model.

To evaluate the contribution of the multi-product nature of firms to the above amplification effect, we first incorporate preferences displaying the "love-for-variety" property into an oligopolistically competitive set-up with single-product firms. Then, we compare our framework with this alternative economy. In such a model, the endogenous amplification of technology shocks is driven by the combined effect of both product diversity (inter-firm component of the variety effect) and countercyclical markup variations, thereby allowing for a direct comparison with the multi-product economy. The first rows of figures 4 and 5 perform such a comparison in terms of impulse response functions, showing that TFP, output and its components are all more responsive when firms are multi-product rather than mono-product (compare the continuous and dashed lines). Thus, the presence of multi-product firms strengthens the endogenous amplification mechanism embodied in a model with firm entry and oligopolistic competition. To analyze this result in further detail, Figure 5 also depicts the two endogenous components of TFP (in deviation from the steady state), namely the average markup (second column) and the variety effect, $\hat{V}_t^e$ (third

[29] Dividing both (30) and (31) by q_t yields (i) $w_t/q_t = (1-\alpha)TFP_t(K_t/H_t)^{\alpha}$ and (ii) $r_t/q_t = \alpha TFP_t(H_t/K_t)^{(1-\alpha)}$. These equations are identical to those that would be obtained in the canonical RBC model but with TFP_t replaced by z_t.

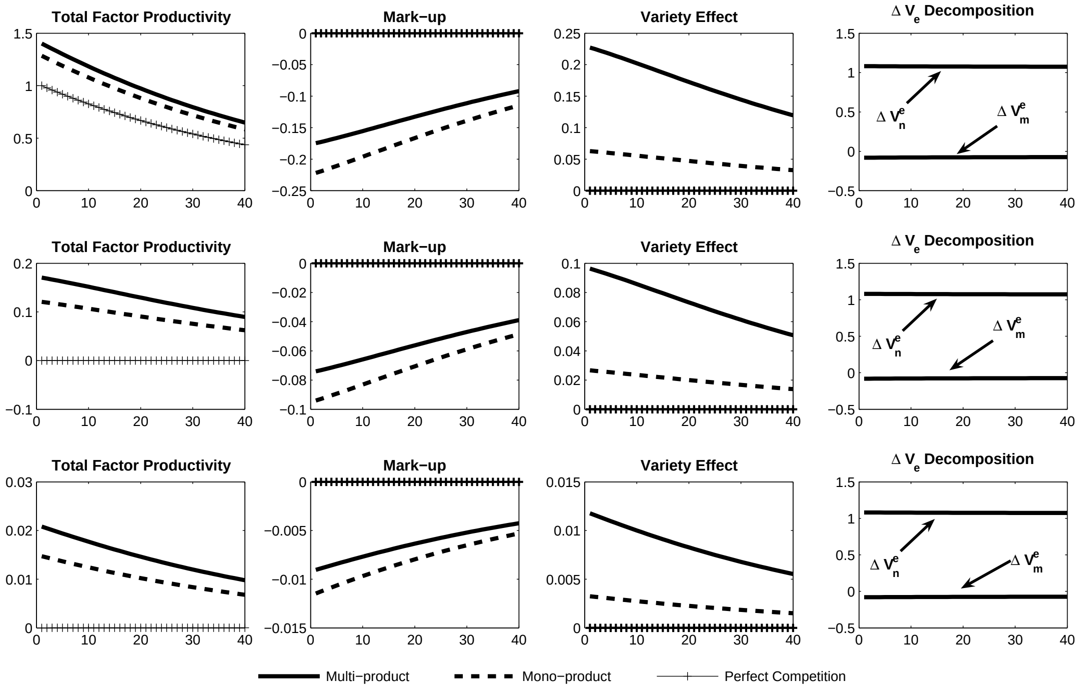

Figure 5: **The Endogenous Amplification Mechanism**. The picture shows the responses of selected aggregate variables to a 1% variation in exogenous temporary shocks. The first row of panels refers to a technology shock, the second row to a preference shock and the third row to a government-spending shock. Impulse responses are measured in per cent deviations from steady state. Horizontal axes display the number of quarters after the shock.

column). The latter, according to Eq. (34), is simply defined as:

$$\hat{V}_t^e = \frac{\hat{m}_t}{\theta - 1} + \frac{\hat{n}_t}{\gamma - 1}. \tag{35}$$

The picture clearly shows that the larger response of TFP in the model with multi-product firms is entirely driven by the variety effect. In fact, relative to the economy with mono-product firms, adjustments along the intra-firm extensive margin dampen the response of average markup, but magnify the response of the variety effect. The fourth column of Figure 5, in turn, clarifies that the stronger response of the variety effect is due to its intra-firm component, that is, the *proliferation* effect.[30] In fact, this picture decomposes the change in the response of the variety effect between the two models ($\Delta V_t^e = \hat{V}_t^{e,multi}$-$\hat{V}_t^{e,mono}$) into the contribution due to changes in the number of firms ($\Delta V_{m,t}^e = \Delta\hat{m}_t/[\Delta V_t^e(\theta-1)]$), and the contribution due to changes in the number of varieties per firm ($\Delta V_{n,t}^e = \hat{n}_t/[\Delta V_t^e(\gamma-1)]$), showing that the variability of $\hat{V}_t^e$ is essentially due to firms' adjustments along the product scope. In our model, therefore, the response of the intra-firm extensive margin is so important that it not only compensates for the effect of both the total number of firms and the markup but also strengthens the response of the effective TFP. It is worth emphasizing, however, that this result depends crucially on the presence of strategic interactions among firms. As we will see shortly, shutting off such a channel (for instance, by assuming monopolistically competitive producers) implies no differences between the response of the economy with and without multi-product firms.

The second and third rows of figures 4 and 5 display the response of the three economies under analysis to

[30] As explained in Section 2, the intra-firm component of the variety effect is an essential ingredient of the model with multi-product firms. Its inter-firm component, instead, can be eliminated from the model. This change to consumers' preferences, however, would not affect our results appreciably; computations are available from the authors upon request.

a 1% temporary decrease in the preference shock, ξ_t, (positive demand shock), and 1% temporary increase in the exogenous real government-spending shock, respectively. These two figures confirm our previous results: the presence of multi-product firms in an environment with endogenous market structure strengthens the response of output (first column of Figure 4), induces countercyclical markup variations (second column of Figure 5), and generates endogenous movements in TFP (first column of Figure 5). Moreover, the specific contribution of multi-product firms to the endogenous amplification is again driven by the *proliferation* effect that, relative to the economy with mono-product firms, is strong enough to magnify movements in TFP (see the last column of Figure 5).

With demand shocks of the type described in our model, the amplification effect operates through a labor demand channel. The mechanism can be explained as follows. In our environment, as in the model of perfect competition, government and preference shocks are similar in that they affect the economy by increasing aggregate demand and labor supply. However, in the economy with multi-product firms and an endogenous market structure, unexpected boosts in aggregate demand and labor supply on impact create profit opportunities that attract the entry of new firms. As explained above, this effect makes competition tougher and leads incumbents to cut their markups and expand their product scopes. The resulting endogenous boost in TFP in turn shifts the labor demand schedule (30) to the right. In the perfectly competitive economy, this channel is missing because the labor demand schedule is unaffected by both preference and government-spending shocks. In this economy, the expansion of labor supply in combination with unchanged labor demand implies that the real wage unequivocally falls. By contrast, in the economy with multi-product firms, the increase in labor demand compensates the expansion in labor supply, and thus provides upward pressure on the equilibrium real wage. As a result, in this economy, the real wage declines on impact, but by a smaller amount than its counterpart in the perfectly competitive economy (see Figure 4). The resulting higher equilibrium real wage raises current labor income and thus stimulates consumption and investment by households. This explains the reason why the response of output to demand-type shocks is magnified in the presence of multi-product firms and endogenous market structure.

Beyond the aggregate effects of product creation, the above analysis presents two related implications that are of particular interest. First, our results provide additional insights into the role of imperfect competition in shaping business-cycle fluctuations. As already mentioned, recent contributions in this topic have shown that the interaction between firm entry and imperfect competition leads to endogenous variations in TFP that magnify the impact of technology shocks.[31] Our model generalizes this result, showing that, with an endogenous market structure, cyclical variations in TFP are driven by any shock that boosts aggregate demand, regardless of whether it directly affects technology. This result is akin to the contribution of Devereux et al. (1996) that shows that, in an economy with endogenous entry and increasing returns to specialization, government-spending shocks of the type described in our model trigger procyclical movements in productivity. Second, a large body of literature has convincingly documented the failure of standard Solow residuals to accurately measure purely exogenous technology shocks. For instance, Hall (1991), Evans (1992), and Finn (1995) document that measured Solow residuals co-vary with exogenous government-spending instruments, such as military expenditures. In this respect, Hall (1991) suggests that the endogeneity of Solow residuals might be interpreted as evidence in support of the existence of market power and increasing returns in the U.S. economy. He also argues that, in such a circumstance, Solow residuals may change endogenously in response to aggregate shocks that do not directly affect technology. The results provided in the present paper support such a conjecture. In our model, endogenous variations in productivity are a consequence of any aggregate shock, and this effect is precisely triggered by the interaction between market power (imperfect competition) and increasing returns (product diversity).

Turning to the main focus of the paper, we now quantify the aggregate implications of product creation. This is done in Table 2 where, for key aggregates, we compare second-order moments implied by the model with multi-product firms with those implied by the perfectly competitive economy (first block) and by the model with mono-

[31] See, for instance, Jaimovich and Floetotto (2008) and Etro and Colciago (2010).

Table 2: **The Aggregate Implications of Product Creation.**

Ratios	Panel 1 Oligopolistic Competition $\mu = 1.2$	Panel 1 Oligopolistic Competition $\mu = 1.3$	Panel 1 Oligopolistic Competition $\mu = 1.4$	Panel 2 Monopolistic Competition $\mu = 1.3$	Panel 3 Forward-Looking Entry $\mu = 1.2$	Panel 3 Forward-Looking Entry $\mu = 1.3$	Panel 3 Forward-Looking Entry $\mu = 1.4$
$\sigma_z^2/\sigma_{TFP}^2$	0.61	0.44	0.32	0.75	0.61	0.44	0.33
$\sigma_y^2/\sigma_{y,RBC}^2$	1.53	1.94	2.42	1.30	1.43	1.82	2.30
$\sigma_c^2/\sigma_{c,RBC}^2$	1.57	2.03	2.60	1.32	1.42	1.85	2.38
$\sigma_i^2/\sigma_{i,RBC}^2$	1.50	1.87	2.30	1.28	1.40	1.78	2.22
$\sigma_h^2/\sigma_{h,RBC}^2$	1.09	1.16	1.22	1.06	0.97	1.04	1.11
$\sigma_{TFP}^2/\sigma_{TFP,MONO}^2$	1.11	1.26	1.38	1	1.12	1.26	1.38
$\sigma_y^2/\sigma_{y,MONO}^2$	1.09	1.19	1.26	1	1.02	1.12	1.20
$\sigma_\mu^2/\sigma_{\mu,MONO}^2$	0.45	0.62	0.75	-	0.42	0.59	0.71
$\sigma_m^2/\sigma_{m,MONO}^2$	0.45	0.62	0.75	1	0.42	0.59	0.71
$\sigma_{Ve}^2/\sigma_{Ve,MONO}^2$	4.02	13.1	28.4	1	3.80	12.4	27.1

Note: Variances of TFP, output (y), consumption (c), investment (i), hours worked (h), net business formation (m), markup (μ), and the variety effect (V^e) are exact theoretical moments for HP-filtered data ($\lambda = 1600$). The subscripts MONO and RBC respectively refer to computations performed by using the model with mono-product firms, and with the RBC model with perfect competition. Monopolistic competition refers to computation based on a model with endogenous entry and monopolistically competitive products market. Forward-Looking Entry refers to computations based on a model in which entry is a forward-looking decision by firms, as in Bilbiie et al. (2011)

product firms (second block). The first row of Table 2 compares the volatilities of technology shocks, σ_z^2, and effective TFP, σ_{TFP}^2. The ratio $\sigma_z^2/\sigma_{TFP}^2$ measures the marginal contribution of the exogenous factor z_t to the overall TFP volatility, and thus provides information on how strongly the response of TFP deviates from technology shocks.[32] We thus take this statistic as a measure of the strength of the endogenous amplification mechanism. In this respect, results reported in the table make it clear that the model with multi-product firms embodies a very powerful endogenous amplification mechanism: the estimates of $\sigma_z^2/\sigma_{TFP}^2$ range from a minimum of 0.32 to a maximum of 0.61, meaning that the effective rate of TFP is between 63% and 212% more volatile than the technology shock (depending on the value chosen for the markup ratio, μ). Alternatively, this result shows that, in our framework, between 39% and 68% of the overall TFP volatility is explainable by endogenous components.[33]

The effects of such endogenous movements in TFP on the main economic aggregates are illustrated from the second to the fifth row of Table 2. The reported statistics show that the magnitude of macroeconomic fluctuations are greatly amplified in the presence of multi-product firms and endogenous market structure. For example, relative to the RBC economy, in our framework output volatility increases by 53% when $\mu = 1.2$, by 94% in the baseline calibration $\mu = 1.3$, and by 142% when $\mu = 1.4$ (see the first row of Table 2).[34] A similar pattern emerges with respect to volatilities of consumption and investment, while the effect on hours worked is relatively milder.

The last five rows of Table 2 evaluate the specific contribution to the amplification effect of adjustments along the intra-firm extensive margin. As we did above, this analysis is performed by comparing the second order moments implied by our model with those implied by the mono-product framework. The results reported in the table make it

[32] In detail, indicating the endogenous component in (34) with ω_t, the log-deviation of TFP with respect to the steady state can be written as $T\hat{F}P_t = \hat{z}_t + \hat{\omega}_t$. This implies that $\sigma_{TFP}^2 = \sigma_z^2 + \sigma_\omega^2 + 2Cov(\hat{\omega}_t, \hat{z}_t)$, where $Cov(\cdot)$ denotes the covariance operator. Hence, σ_z^2 captures the marginal contribution of the exogenous technology shock, while $\sigma_\omega^2 + 2Cov(\hat{\omega}_t, \hat{z}_t)$ measures the contribution of endogenous components to TFP volatility.

[33] Denoting the endogenous component in the effective TFP with $\hat{\omega}_t$, it is possible to prove that $Cov(\hat{\omega}_t, \hat{z}_t) > 0$. Thus, it follows from the previous footnote that the fraction of TFP volatility explained by endogenous components can be simply computed as 1-$\sigma_z^2/\sigma_{TFP}^2$.

[34] Among other things, these results indicate that in our model, the amount of volatility of technology shocks required to account for the same fluctuations in actual data is necessarily lower than that required by the standard RBC model.

clear that allowing for product switching as an additional source of resource reallocation substantially strengthens the amplification mechanism embodied in a model with an endogenous market structure and mono-product firms. For example, the table shows that when μ is set to 1.3, both TFP and final output are substantially more responsive to shocks in the presence of multi-product firms, with volatilities that increase by 26% and 19%, respectively. Furthermore, the reported statistics clearly show that the *proliferation* effect is crucial for the endogenous amplification. On the one hand, we see that relative to the economy with mono-product firms, if product markets are monopolistically competitive, volatilities of key aggregates are unaffected by the presence of multi-product companies (see the second block of panel 2). In such an environment, firms do not adjust their product ranges in response to temporary shocks, and thus the *proliferation* effect is not operative.[35] On the other hand, we see that the dampening effect on the entry of new firms implies that both net business formation and average markup are substantially less volatile in the multi-product firm model. For example, relative to the framework with mono-product firms, in our model markup volatility decreases by 65% when $\mu = 1.2$, by 48% in the baseline calibration $\mu = 1.3$, and by 25% when $\mu = 1.4$ (see the eighth row of Table 2). The same magnitudes characterize the decline in the volatility of business formation. These numbers confirm our previous conclusions based on impulse-response functions: the proliferation effect is quantitatively so strong that it more than compensates for the dampening effect on average markup and net business formation. This is also confirmed by noting that, because of firm-level adjustments in product scope, the variety effect in our model is dramatically more volatile that its counterpart in the mono-product firm model (see the last row of panel 1).

We conclude our analysis by checking the robustness of our results. In particular, we relax the assumption that the process of entry/exit is governed by a static zero-profit condition by assuming that the entry decision is forward-looking. In the third panel of Table 2, we report computations based on an alternative formulation in which the entry decision is forward-looking (as in the model of Bilbiie et al. 2011). In this environment, each prospective entrant decides whether to enter the market based on the post-entry present values of the firms, which depend on all future profit flows and on the exogenous exit probability.[36] Relative to our baseline specification in which firms' entry is governed by a period-by-period zero-profit condition, such an alternative framework captures a well-known property of the U.S. data: the coexistence of countercyclical markup movements and procyclical aggregate profits. However, relying on this alternative structure rather than on our baseline set-up does not in any respect modify the main conclusions of our paper. As can be seen by comparing the first and the third panels of Table 2, the results obtained in the two formulations are very similar. In particular, the magnification mechanism remains quantitatively important when entry is a forward-looking decision.

5 Conclusions

Recent empirical work by Bernard et al. (2010) and Broda and Weinstein (2010) indicate that a significant share of product creation and destruction in U.S. industries occurs within existing firms and accounts for an important share of aggregate output. Consistent with this empirical evidence, in this paper, we relax the standard assumption of mono-product firms that is typically made in dynamic general equilibrium models. We develop a DSGE model with multi-product firms and endogenous strategic interactions. Oligopolistic behavior gives rise to countercyclical markups and procyclical product creation: in our framework, any shock that boosts aggregate demand fosters firm entry and leads to an expansion in firms' product ranges. Due to the proliferation effect induced by firm-level adjustments in product scope, we show that in this environment the presence of multi-product firms endogenously

[35] As a result and given that the average markup is constant, in this economy the endogenous amplification (see block 1 of panel 2) is triggered by the inter-firm component of the variety effect, which is independent on the multi-product nature of firms. The overall effect is, however, quantitatively modest when compared to the model where firms adjust their product ranges (compare block 1 of panels 1 and 2).

[36] See Appendix F for a detailed presentation of this specification of our model.

magnifies the amplitude of economic fluctuations.

The model has a number of potentially interesting extensions. First, the framework could be extended to an open-economy context to address optimal coordination of monetary and fiscal policy and international business cycle issues. Given the omnipresence and empirical importance of multi-product firms in the global economy, such an extension would be of great interest in the analysis of the relationship between international trade and aggregate macroeconomic fluctuations across countries.

Another important extension would be to introduce nominal rigidities in the form of sticky prices and to study the implications of multi-product firms for the analysis of inflation dynamics. Because pricing and product scope decisions by firms are interrelated, adjustments along the intra-firm extensive margin might affect the magnitude of price changes and thus the properties of inflation dynamics. Moreover, when prices are rigid, firms may react to market changes with their product ranges, thereby potentially affecting the aggregate implications of nominal rigidities on both resource allocation and inflation dynamics. These issues are the focus of our future research.

A Price and demand elasticities

In this Appendix, we first derive the elasticity of the price index with respect to the price of a product variety. Then, we determine the demand elasticities of a good in response to variations of its own price and other goods' prices within the same nest.

Transforming the demand function $x_t(i,j)$ (17) into logarithms yields the following:

$$\ln x_t(i,j) = (\theta - 1)\ln q_t + \ln e_t - \gamma \ln p_t(i,j) - (\theta - \gamma)\ln q_t(i).$$

By using the fact that $q_t(i) = \left[\sum_{j=1}^{N_t(i)} p_t(i,j)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$, it is easy to show that $\partial \ln q_t(i)/\partial \ln p_t(i,j) = [p_t(i,j)/q_t(i)]^{1-\gamma}$. Thus, the effect of $p_t(i,j)$ on the market price index q_t can be expressed as:

$$\frac{\partial \ln q_t}{\partial \ln p_t(i,j)} = \frac{\partial \ln q_t}{\partial \ln q_t(i)} \frac{\partial \ln q_t(i)}{\partial \ln p_t(i,j)} = \left[\frac{q_t(i)}{q_t}\right]^{1-\theta} \left[\frac{p_t(i,j)}{q_t(i)}\right]^{1-\gamma}.$$

Using this result, we can write the two demand elasticities as follows:

$$\frac{\partial \ln x_t(i,j)}{\partial \ln p_t(i,j)} = -\gamma - (\theta-\gamma)\left[\frac{p_t(i,j)}{q_t(i)}\right]^{1-\gamma} + (\theta - 1)\left[\frac{q_t(i)}{q_t}\right]^{1-\theta}\left[\frac{p_t(i,j)}{q_t(i)}\right]^{1-\gamma}, \tag{A.1}$$

$$\frac{\partial \ln x_t(i,k)}{\partial \ln p_t(i,j)} = -\,(\theta-\gamma)\left[\frac{p_t(i,j)}{q_t(i)}\right]^{1-\gamma} + (\theta - 1)\left[\frac{q_t(i)}{q_t}\right]^{1-\theta}\left[\frac{p_t(i,j)}{q_t(i)}\right]^{1-\gamma}, \text{ for } k \neq j. \tag{A.2}$$

B Pricing

In this Appendix, we determine the optimal pricing strategy; this corresponds to solving the second stage of the game. Firm i maximizes profits (19) under the constraint (18). The Lagrangian for this maximization problem is as follows:

$$\mathcal{L} = \sum_{j=1}^{N_t(i)} [p_t(i,j)x_t(i,j) - w_t h_t(i,j) - r_t k_t(i,j)] + \lambda \left\{ \sum_{j=1}^{N_t(i)} \left[z_t k_t(i,j)^{\alpha} h_t(i,j)^{1-\alpha} - \phi_v\right] - \phi_f - \sum_{j=1}^{N_t(i)} x_t(i,j) \right\}.$$

The optimality conditions are:

$$\frac{\partial \mathcal{L}}{\partial p_t(i,j)} = 0 \Rightarrow x_t(i,j) + \sum_{j=1}^{N_t(i)} [p_t(i,j) - \lambda] \frac{\partial x_t(i,j)}{\partial p_t(i,j)} = 0, \tag{B.1}$$

$$\frac{\partial \mathcal{L}}{\partial h_t(i,j)} = 0 \Rightarrow -w_t + (1-\alpha)\lambda z_t k_t(i,j)^{\alpha} h_t(i,j)^{-\alpha} = 0, \tag{B.2}$$

$$\frac{\partial \mathcal{L}}{\partial k_t(i,j)} = 0 \Rightarrow -r_t + \alpha \lambda z_t k_t(i,j)^{\alpha-1} h_t(i,j)^{1-\alpha} = 0. \tag{B.3}$$

Combining Eqs. (B.2) with (B.3), we obtain the Lagrange multiplier λ amounting to:

$$\lambda = \frac{w_t^{1-\alpha} r_t^{\alpha}}{z_t (1-\alpha)^{1-\alpha}\alpha^{\alpha}},$$

which corresponds to the marginal cost of producing one more variety, mc_t. Using (B.2) and (B.3) and replacing λ

with mc_t, the total cost of production by firm i boils down to:

$$\sum_{j=1}^{N_t(i)} [w_t h_t(i,j) + r_t k_t(i,j)] = \sum_{j=1}^{N_t(i)} mc_t z_t k_t(i,j)^{\alpha} h_t(i,j)^{1-\alpha} = mc_t \left\{ \sum_{j=1}^{N_t(i)} [x_t(i,j) + \phi_v] + \phi_f \right\}.$$

Consequently, firm i's profits are written as:

$$\sum_{j=1}^{N_t(i)} [p_t(i,j)x_t(i,j) - mc_t x_t(i,j)] - mc_t [N_t(i)\phi_v + \phi_f]. \tag{B.4}$$

Using Eqs. (A.1) and (A.2) into Eq. (B.1) and replacing λ with mc_t, we obtain the following:

$$x_t(i,j) - \tfrac{\gamma x_t(i,j)}{p_t(i,j)} [p_t(i,j) - mc_t] - \sum_{k=1}^{N_t(i)} \tfrac{x_t(i,k)}{p_t(i,j)} [p_t(i,k) - mc_t] \left\{ \theta - \gamma - (\theta - 1) \left[\tfrac{q_t(i)}{q_t}\right]^{1-\theta} \right\} \left[\tfrac{p_t(i,j)}{q_t(i)}\right]^{1-\gamma} = 0.$$

Substituting for $x_t(i,j)$ using (17) in the above equation yields:

$$e_t \left[\frac{q_t(i)}{q_t}\right]^{1-\theta} - \gamma e_t \left[\frac{q_t(i)}{q_t}\right]^{1-\theta} \tfrac{[p_t(i,j)-mc_t]}{p_t(i,j)} = \sum_{k=1}^{N_t(i)} x_t(i,k) [p_t(i,k) - mc_t] \left\{ \theta - \gamma - (\theta - 1) \left[\tfrac{q_t(i)}{q_t}\right]^{1-\theta} \right\}. \tag{B.5}$$

The right-hand side of this equation is the same for all $j \in [1, N_t(i)]$. We conclude that firm i charges the same price for all of the product varieties that it produces, that is, $p_t(i,j) = p_t(i)$ for all $j \in [1, N_t(i)]$.

In what follows, we solve Eq. (B.5) for $p_t(i)$. Before proceeding, we must first observe that the term $[q_t(i)/q_t]^{1-\theta}$ in Eq. (B.5) amounts to firm i's market share $\epsilon_t(i) \equiv q_t(i)x_t(i)/e_t$, where $x_t(i) \equiv x_t^C(i) + x_t^I(i) + x_t^G(i)$ denotes total demand of consumption, investment and government-provided goods produced by firm i. This can be easily verified by using Eqs. (3), (6), (15) and (17). Now, because $q_t(i) = \left[\sum_{j=1}^{N_t(i)} p_t(i,j)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$ and $q_t = \left[\sum_{i=1}^{M_t} q_t(i)^{1-\theta}\right]^{\frac{1}{1-\theta}}$, firm i's market share $\epsilon_t(i)$ can be written as:

$$\epsilon_t(i) \equiv \left[\frac{q_t(i)}{q_t}\right]^{1-\theta} = \frac{N_t(i)^{\frac{1-\theta}{1-\gamma}} p_t(i)^{1-\theta}}{\sum_{i=1}^{M_t} N_t(i)^{\frac{1-\theta}{1-\gamma}} p_t(i)^{1-\theta}}. \tag{B.6}$$

Then, using Eqs. (17) and (B.6) into Eq. (B.5) and rearranging terms yields:

$$p_t(i) = mc_t \frac{[\theta - (\theta - 1)\epsilon_t(i)]}{(\theta - 1)[1 - \epsilon_t(i)]}. \tag{B.7}$$

As can be seen, Eq. (B.7) coincides with Eq. (20) in the text.

C Product scope

In this Appendix, we determine firms' optimal product scope. To proceed, we rearrange Eq. (B.4) to write firm i's gross profits as a function only of the market share $\epsilon_t(i)$. Substituting $x_t(i,j)$ from (17) into (B.4) and then using $q_t(i) = \left[\sum_{j=1}^{N_t(i)} p_t(i,j)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$ and (B.6) to simplify yields:

$$\pi_t(i) = e_t \frac{\epsilon_t(i)[p_t(i) - mc_t]}{p_t(i)} - mc_t [N_t(i)\phi_v + \phi_f].$$

By using the definition of the Lerner index of market power, $L_t(i) \equiv [p_t(i) - mc_t] / p_t(i) = 1/ [\theta - \epsilon_t(i)(\theta - 1)]$, in the above equation, we can write the gross profits of firm i as:

$$\pi_t(i) = e_t \epsilon_t(i) L_t(i) - mc_t [N_t(i)\phi_v + \phi_f] .$$

Differentiating $\pi_t(i)$ with respect to $N_t(i)$ yields the first order condition:

$$\frac{\partial \pi_t(i)}{\partial N_t(i)} = \frac{e_t \epsilon_t(i) L_t(i)}{N_t(i)} \eta_t(i) - mc_t \phi_v = 0, \tag{C.1}$$

where $\eta_t(i) = \theta L_t(i) \cdot [\partial \epsilon_t(i)/\partial N_t(i)] \cdot [N_t(i)/\epsilon_t(i)]$ is the elasticity of variable profits of firm i with respect to the size of its product range, $N_t(i)$. In what follows, we first calculate $\eta_t(i)$; then, we use this result in (C.1) and determine the optimal firms' product scope. Now, differentiating $\epsilon_t(i)$ from Eq. (B.6) with respect to $N_t(i)$, we obtain:

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = \frac{(\theta - 1)}{(\gamma - 1)} \frac{\epsilon_t(i)}{N_t(i)} - (\theta - 1) \epsilon_t(i) \left[\frac{1}{p_t(i)} \frac{\partial p_t(i)}{\partial N_t(i)} - \frac{1}{q_t} \frac{\partial q_t}{\partial N_t(i)} \right]. \tag{C.2}$$

The first term on the right hand side of Eq. (C.2) gives the increase in the market share resulting from the opening of a new product line when firms' strategic interactions are neglected (monopolistic competition). In such a case, the Lerner index of market power $L_t(i)$ is equal to $1/\theta$ so that the elasticity of variable profits with respect to the level of product proliferation amounts simply to $(\theta - 1) / (\gamma - 1)$. Given the assumption that $\theta < \gamma$, this elasticity is lower than one. This feature of the model reflects the *cannibalization* effect: the opening of a new product line reduces the sales of the firm's existing varieties so that the introduction of a new variety leads to a less than proportionate increase in variable profits. The second term on the right side of Eq. (C.2) captures the *strategic* effect of the product scope, that is, the effect of the firm's product range choice on both its own and all other firms' pricing decisions. As we will see shortly, the partial derivatives $\partial p_t(i)/\partial N_t(i)$ and $\partial q_t/\partial N_t(i)$ in the square brackets of Eq. (C.2) are respectively positive and negative; thus, the *strategic* effect of the product range decision always leads to a contraction in firms' product scope with respect to monopolistic competition.

To proceed with the calculation of $\partial \epsilon_t(i)/\partial N_t(i)$, we first compute the partial derivative $\partial q_t/\partial N_t(i)$. Because $q_t = \left[\sum_{k=1}^{M_t} N_t(k)^{\frac{1-\theta}{1-\gamma}} p_t(k)^{1-\theta} \right]^{\frac{1}{1-\theta}}$, after some rearrangement, we can express $\partial q_t/\partial N_t(i)$ as:

$$\frac{\partial q_t}{\partial N_t(i)} = q_t^{\theta} \left[\sum_{k=1}^{M_t} N_t(k)^{\frac{1-\theta}{1-\gamma}} p_t(k)^{-\theta} \frac{\partial p_t(k)}{\partial N_t(i)} + \frac{1}{1-\gamma} N_t(i)^{\frac{1-\theta}{1-\gamma}-1} p_t(i)^{1-\theta} \right]. \tag{C.3}$$

Now, we show that the first term in the square brackets in the above equation is equal to zero. In fact, by using Eq. (B.7), we write the inverse of the markup as:

$$\frac{p_t(i)}{p_t(i) - mc_t} = \theta - \epsilon_t(i)(\theta - 1).$$

Summing the above equation over i yields:

$$\sum_{i=1}^{M_t} \frac{p_t(i)}{p_t(i) - mc_t} = 1 + (M_t - 1)\theta,$$

which indicates that the sum of inverse markups over all firms is independent of firms' product scopes. Differentiating the above equation with respect to $N_t(i)$, we obtain:

$$\sum_{k=1}^{M_t} \frac{mc_t}{\left[p_t(k) - mc_t\right]^2} \frac{\partial p_t(k)}{\partial N_t(i)} = 0.$$

Under symmetry, this equation boils down to $(M_t - 1)\partial p_t(k)/\partial N_t(i) + \partial p_t(i)/\partial N_t(i) = 0$. Using this result in (C.3) and imposing $p_t(k) = p_t$ and $N_t(k) = N_t$ for every $k \in [1, M_t]$, the first term in the square brackets of Eq. (C.3) cancels out, and we can write $\partial q_t/\partial N_t(i)$ as:

$$\frac{\partial q_t}{\partial N_t(i)} = -q_t^{\theta} \frac{q_t(i)^{1-\theta}}{(\gamma - 1)\, N_t(i)}.$$

As anticipated, the partial derivative $\partial q_t/\partial N_t(i) < 0$ so that an increase in the level of product proliferation $N_t(i)$ reduces the aggregate price index q_t. Plugging $\partial q_t/\partial N_t(i)$ back into (C.2) yields:

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = \frac{(\theta - 1)}{(\gamma - 1)} \frac{\epsilon_t(i)}{N_t(i)} \left[1 - \epsilon_t(i)\right] - (\theta - 1) \frac{\epsilon_t(i)}{p_t(i)} \frac{\partial p_t(i)}{\partial N_t(i)}. \tag{C.4}$$

Now, we need to compute the partial derivative $\partial p_t(i)/\partial N_t(i)$. Thus, we differentiate $p_t(i)$ from Eq. (B.7) with respect to $N_t(i)$ to obtain:

$$\frac{\partial p_t(i)}{\partial N_t(i)} = \frac{mc_t}{(\theta - 1)\left[1 - \epsilon_t(i)\right]^2} \frac{\partial \epsilon_t(i)}{\partial N_t(i)}.$$

Replacing $\partial p_t(i)/\partial N_t(i)$ into Eq. (C.4) and then using Eq. (B.7) to simplify yields:

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = \frac{(\theta - 1)}{(\gamma - 1)} \frac{\epsilon_t(i)}{N_t(i)} \frac{\left[1 - \epsilon_t(i)\right]^2 \left[\theta - \epsilon_t(i)\,(\theta - 1)\right]}{\left\{\theta\left[1 - \epsilon_t(i)\right] + (\theta - 1)\,\epsilon_t(i)^2\right\}}. \tag{C.5}$$

Because $\partial \epsilon_t(i)/\partial N_t(i) > 0$, we obtain that $\partial p_t(i)/\partial N_t(i) > 0$, which indicates that increasing the number of a firm's varieties raises the firm's own price. In light of these results, one can easily comprehend the reason why the *strategic* effect of the product range decision leads to a contraction of the product scope. When firms' strategic interactions are neglected, an increase in the level of proliferation $N_t(i)$ raises the market share, $\epsilon_t(i)$, without affecting the price, $p_t(i)$, which remains constant. In our strategic set-up, firms proliferate less because they use the product range as a tool to relax price competition. According to Eq. (B.7), price $p_t(i)$ is positively related to the market share $\epsilon_t(i)$. Thus, an increase in the level of proliferation $N_t(i)$ raises the market share $\epsilon_t(i)$, which then elevates price $p_t(i)$. Because $(M_t - 1)\partial p_t(k)/\partial N_t(i) + \partial p_t(i)/\partial N_t(i) = 0$, the increase in $p_t(i)$ is accompanied by a decrease in other firms' prices. Consequently, firm i tries to mitigate price competition in the second stage of the game by under-expanding its product scope in the first stage.

To conclude, we replace $\epsilon_t(i) = 1/M_t$ into Eq. (C.5), and then we use this equation to derive the elasticity of variable profits of a typical firm i with respect to the size of its product range:

$$\eta_t = \frac{(\theta - 1)}{(\gamma - 1)} \left[\frac{\theta(M_t - 1)^2}{M_t\theta\,(M_t - 1) + \theta - 1}\right].$$

Finally, plugging η_t into the first-order condition (C.1) and then using (B.7) to simplify, we write the optimal product scope as:

$$N_t = \frac{e_t\theta\,(M_t - 1)}{\phi_v\,(\gamma - 1)\, p_t\left[M_t\theta(M_t - 1) + \theta - 1\right]}.$$

As can be easily ascertained, the elasticity of variable profits with respect to the level of product proliferation is an increasing function of the number of firms, M_t. When the latter becomes very large, this elasticity tends to $(\theta - 1)\,/\,(\gamma - 1)$, as in the case of monopolistic competition. Intuitively, the larger the number of firms, the smaller the impact that a change in the market share $\epsilon_t(i)$ has on the price $p_t(i)$ (see Eq. (B.7)). Because the *strategic* effect of the product scope becomes less and less important as M_t increases, the incentive to create new varieties increases with the number of firms. This feature of the model is responsible for the procyclicality of the product scope. Intuitively, a positive technology shock $z_t > 0$ is equivalent to an expansion of the market because it increases the aggregate demand of consumption and investment goods, e_t. This, in turn, fosters the entry of new firms into the product market. Because the elasticity of variable profits with respect to the level of product proliferation is an increasing function of M_t, the increase in the equilibrium number of firms is accompanied by an expansion of the product scope.

D Monopolistic competition

In this Appendix, we solve the model of monopolistic competition when firms are multi-product. Following a similar procedure to that used in Appendix C, we write the gross profits of firm i as:

$$\pi_t(i) = e_t\epsilon_t(i)L_t(i) - mc_t\left[N_t(i)\phi_v + \phi_f\right].$$

Because firms' strategic interactions are neglected under monopolistic competition, the optimal price is a constant mark-up over marginal cost:

$$p_t(i) = mc_t\frac{\theta}{(\theta-1)}. \tag{D.1}$$

The Lerner index of market power $L_t(i)$ is equal to $1/\theta$; consequently, firm i's profits can be written as:

$$\pi_t(i) \quad = \quad e_t\frac{\epsilon_t(i)}{\theta} - mc_t\left[N_t(i)\phi_v + \phi_f\right].$$

We now differentiate the above equation with respect to $N_t(i)$. Because the *strategic* effect of the product scope is absent, the elasticity of variable profits with respect to the level of product proliferation is equal to $(\theta-1)/(\gamma-1)$. Solving the first-order condition under symmetry and then using (D.1) to simplify yields:

$$N_t = \frac{e_t}{\phi_v\left(\gamma-1\right)p_tM_t}. \tag{D.2}$$

Substituting this result back into firm i's profits and replacing mc_t with $p_t\left(\theta-1\right)/\theta$, we obtain the following:

$$\pi_t(i) = \frac{e_t\left(\gamma-\theta\right)}{\left(\gamma-1\right)M_t\theta} - p_t\frac{(\theta-1)}{\theta}\phi_f.$$

Under free entry, firms enter until their profits fall to zero. Setting $\pi_t(i) = 0$, and solving for M_t yields:

$$M_t = \frac{e_t\left(\gamma-\theta\right)}{p_t\left(\gamma-1\right)\left(\theta-1\right)\phi_f}. \tag{D.3}$$

The equilibrium number of varieties produced by firm i can be found by substituting (D.3) into (D.2):

$$N_t = \frac{\phi_f}{\phi_v}\frac{(\theta-1)}{(\gamma-\theta)}.$$

Finally, by replacing N_t and M_t into $x_t \equiv e_t/\left(M_tN_tp_t\right)$, we easily obtain output per firm:

$$x_t = \phi_v\left(\gamma-1\right).$$

As can be seen, N_t and x_t are determined only by exogenous parameters. A positive technology shock $z_t > 0$ increases the aggregate demand of consumption and investment goods, e_t, and fosters the entry of new firms into the product market, M_t. Now, as Eq. (D.3) shows, M_t varies proportionally to e_t. Consequently, under monopolistic competition, the product range and output per variety are not affected by the technology shock.

E Forward-looking entry decision

We have so far assumed that the process of entry/exit is governed by the static zero-profit condition. In this Appendix, we modify this hypothesis by assuming that entry is a forward-looking decision, as in Bilbiie et al. (2011). The rest of the model is left unchanged.

In every period, there are M_t incumbents producing multiple products and an unbounded mass of prospective entrants. Entering firms are forward looking and thus correctly anticipate their future profits, $\pi_t(i)$, and the probability, δ_m, of exiting the market. Therefore, entrants in period t compute their expected post-entry value, $v_t(i)$, as:

$$v_t(i) = E_t \sum_{s=t}^{\infty} (1-\delta_m)^{s-t} r_{t,s} \pi_s(i),$$

where $r_{t,s}$ is the stochastic discount factor. There is a sunk entry cost, ψ, expressed in terms of units of output. The equilibrium equates the present discounted value of future profits (expressed in real terms) to the upfront entry cost:

$$v_t(i)/q_t = v_t/q_t = \psi, \, \forall i \in [1, M_t].$$

The assumption made on the timing of entry and exit implies that the number of incumbents at time t is given by:

$$M_t = (1-\delta_m) M_{t-1} + M_t^e,$$

where M_t^e is the number of firms entering the market at time t. Incumbents produce goods for both investment and consumption purposes via a Cobb-Douglas technology. We assume that the same production function is used for the creation of new firms. Therefore, the technology of setting up M_t^e new firms is given by:

$$\psi M_t^e = z_t \left(H_t^e\right)^{1-\alpha} \left(K_t^e\right)^{\alpha},$$

where H_t^e and K_t^e are the amounts of labor and capital, respectively, devoted to the creation of new firms.

In this environment, households may invest both in capital and in new firms and receive pure profits from the firms they own. Therefore, the flow budget constraint of a representative household is:

$$\sum_{i=1}^{M_t} \sum_{j=1}^{N_t(i)} p_t(i,j) \left[x_t^C(i,j) + x_t^I(i,j)\right] + v_t M_t^e \leq w_t H_t + r_t K_t + M_t \pi_t.$$

The first-order conditions for the household's optimal plan are described by Eqs. (8)-(13) and augmented by the first-order condition with respect to M_t^e:

$$\frac{v_t - \pi_t}{q_t} = \beta \left(1-\delta_m\right) E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-1} \frac{v_{t+1}}{q_{t+1}} \right\}.$$

The market clearing condition in the goods sector is $Y_t^g \equiv C_t + I_t + G_t$, where Y_t^g denotes total production in this sector. Because $e_t = q_t(C_t + I_t + G_t)$, we easily obtain the following:

$$Y_t^g \equiv C_t + I_t + G_t = \frac{e_t}{q_t} = \frac{M_t N_t x_t p_t}{q_t}.$$

Normalizing the price p_t to 1 and, then, substituting for $N_t x_t$ using (18), the above equation can be rewritten as:

$$Y_t^g = \frac{z_t (K_t^g)^\alpha (H_t^g)^{1-\alpha} - (\phi_v N_t + \phi_f) M_t}{q_t},$$

where K_t^g and H_t^g denote, respectively, the total amounts of capital and labor employed in the goods sector. Making similar substitutions into Eq. (19) and then plugging in w_t and r_t respectively from Eqs. (30) and (31), individual profits expressed in real terms, π_t/q_t, can be written as:

$$\begin{aligned}
\frac{\pi_t}{q_t} &= \frac{1}{q_t M_t}\left[z_t (K_t^g)^\alpha (H_t^g)^{1-\alpha} - (\phi_v N_t + \phi_f) M_t - w_t H_t^g - r_t K_t^g\right] \\
&= \frac{1}{M_t}\left\{\left[1 - \frac{1}{\mu_t(M_t)}\right] z_t \frac{(K_t^g)^\alpha (H_t^g)^{1-\alpha}}{q_t} - \frac{(\phi_v N_t + \phi_f) M_t}{q_t}\right\} \\
&= \left[1 - \frac{1}{\mu_t(M_t)}\right]\frac{Y_t^g}{M_t} - \frac{\phi_v N_t + \phi_f}{\mu_t(M_t) q_t}.
\end{aligned}$$

Aggregate output, capital and labor in the overall economy are given, respectively, by:

$$\begin{aligned}
Y_t &= Y_t^g + \frac{v_t}{q_t} M_t^e, \\
K_t &= K_t^g + K_t^e, \\
H_t &= H_t^g + H_t^e.
\end{aligned}$$

Because capital and labor receive the same return in both sectors, the following condition must hold in equilibrium:

$$\frac{K_t^e}{H_t^e} = \frac{K_t^g}{H_t^g}.$$

Numerical experiments based on this alternative set-up are performed by calibrating the model with the same restrictions of the baseline multi-product firm model (see Section 4.1). Additionally, as in Bilbiie et al. (2011), we set the exit probability, δ_m, to 0.025, while the sunk entry-cost, ψ, is pinned down from the steady-state equilibrium as a result of our calibration restrictions.

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