A FAST AND ACCURATE MICROSTRIP ARRAY MODEL FOR THE ANALYSIS OF INTEGRATED PASSIVE COMPONENTS OF COMPLEX TOPOLOGY

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ABSTRACT

The paper describes a fast and easy to implement approximate model for coupled microstrip arrays of arbitrary cross sections. The model is based on the extension of quasi-static computational concepts to the dynamic case, via the introduction of frequency-dependent capacitance models. The results obtained are in good agreement with those provided by more complex computational methods such as the spectral-domain approach.

I. INTRODUCTION

The analysis of a planar array of coupled microstrips with unequal gaps and widths can be performed in general by the Spectral-Domain Approach (SDA) [1]. This technique is accurate and physically sound, but its efficient implementation requires a very sophisticated software environment, usually based on the preparation of look-up tables and their numerical interpolation [2]. It is thus considerably difficult to handle from a programming viewpoint.

In this paper we present a simpler solution of the same problem, based on available formulae for single and symmetric coupled microstrips and on a formal extension of static concepts to the dynamic case. The main features of the present method are as follows. From the theoretical viewpoint, it is purely heuristic, and does not share the character of field-theoretical analysis tool which is typical of the SDA. From the practical viewpoint, it is computationally fast and easy to implement, and can analyze a microstrip array, including the effects of dispersion and losses, with virtually the same accuracy as the SDA. Thus the method is believed to represent an interesting proposal in view of all those microwave CAD situations requiring the availability of a handy multiple-microstrip model.

II. THE ANALYSIS ALGORITHM FOR LOSSLESS ARRAYS

The broadband behavior of a lossless transmission line with inhomogeneous dielectric is fully characterized once its effective permittivity and characteristic impedance are known as functions of frequency. In quasi-static conditions, these quantities are related to the transmission-line capacitances per unit length, C, C^0 (in dielectric and in air, respectively) by the equations

$$\varepsilon_{eff} = \frac{C}{C^0} \qquad Z_c = \frac{1}{v_0 \sqrt{C\, C^0}} \tag{1}$$

where v_0 is the velocity of light *in vacuo*. Equations (1) can be inverted to yield

$$C = \frac{\sqrt{\varepsilon_{eff}}}{v_0\, Z_c} \qquad C^0 = \frac{1}{v_0\, Z_c \sqrt{\varepsilon_{eff}}} \tag{2}$$

From a physical viewpoint, C and C^0 are frequency-independent static capacitances, and (1), (2) are only valid in the zero-frequency limit. However, it is quite obvious that once the dispersion laws for ε_{eff} and Z_c have become known in some way (e.g., as results of SDA-based calculations), equations (2) can be used to formally define two frequency-dependent capacitances which are related to the true values of effective permittivity and characteristic impedance by the

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static equations at any frequency. This leads us to the basic idea underlying our modeling approach: the dispersive properties of a microstrip array are found by applying the standard quasi-TEM (i.e., static) computational procedure to a frequency-dependent capacitance model. In this paper we adopt the simple capacitance model obtained from fig. 1 when the conductances are set to zero, which has been found to provide excellent results in the static case [3].

For this approach to be practically useful, we need a fast procedure for computing the frequency-dependent capacitance models of a given array. To do so, we relate the array capacitances of fig. 1 to the capacitances of a set of isolated microstrips, and to the even- and odd-mode capacitances of a set of symmetric coupled microstrips, whose geometries are derived from the original array geometry in the way detailed later on. The basic relationships used are exactly the same as for the static case [3, 4]. The frequency-dependent capacitances of the single and symmetric-coupled microstrips are derived by means of (2) from explicit formulae available in the technical literature [5 - 7].

Let us consider the array cross section depicted in fig. 2. The total number of coupled microstrips is denoted by m (≥ 2); the width of the j-th microstrip is indicated by W_j ($1 \leq j \leq m$), and the gap between the j-th and the (j + 1)-th microstrips by S_j ($1 \leq j \leq m-1$).

We now introduce the following sets of elementary microstrip systems associated with the given array:

A) m uncoupled microstrips having the same substrate as the array and widths W_j ($1 \leq j \leq m$).

B) m-1 symmetric coupled-microstrip pairs having the same substrate as the array and widths and gaps given by W_j, S_j, respectively ($1 \leq j \leq m-1$).

C) m-1 symmetric coupled-microstrip pairs having the same substrate as the array and widths and gaps given by W_j, S_{j-1}, respectively ($2 \leq j \leq m$).

The capacitances introduced at points B), C) above can be directly obtained by (2) from the explicit expressions for the even- and odd-mode effective dielectric constants and characteristic impedances reported in [7].

At this stage the air and dielectric array capacitance matrices C^0, C can be directly found at any given frequency, following [4]. The scattering matrix of an array of length ℓ can now be computed according to the same reference.

Note that, since the capacitance matrices C^0, C are frequency-dependent, so are the phase velocities and the voltage and current distributions of the quasi-TEM modes of the microstrip array. The above procedure thus provides an approximate model for the dispersive behavior of such modes.

III. DIELECTRIC LOSSES

Dielectric losses are computed by a first-order perturbational approach based on the assumption that the circuit properties of the array may be described by the conductance-capacitance model shown in fig. 1. Since the conductances play the role of small perturbations, the capacitances are practically the same as in the lossless case.

For an isolated microstrip, an approximate evaluation of the transverse conductance per unit length due to a finite dielectric conductivity σ_d is [8]

$$G = \frac{\sigma_d}{\varepsilon_0} C^0 \frac{\varepsilon_{eff} - 1}{\varepsilon_r - 1} \tag{3}$$

where ε_r is the relative dielectric constant of the substrate. Of course, (3) may be used to compute the even- and odd-mode conductances for a symmetric pair of coupled microstrips.

At this stage, for each of the elementary microstrip systems associated with the array as described at points A, B, C of section II, we can compute a conductance per unit length or an even- and an odd-mode conductance per unit length. Such conductances may then be combined in a similar way as it is done for the capacitances to derive the conductance matrix **G** of the microstrip array. The attenuation constant of the i-th quasi-TEM mode due to dielectric losses is then given by the first-order perturbational formula

$$\alpha_i^d \cong \frac{P_i^d}{2\, P_i} = \frac{\mathbf{x}_i^T\, \mathbf{G}\, \mathbf{x}_i}{2\, v_i\, \mathbf{x}_i^T\, \mathbf{C}\, \mathbf{x}_i} \tag{4}$$

where P_i^d, P_i are the power dissipated in the dielectric and the power carried by the i-th mode, respectively (^{T} denotes transposition), v_i its phase velocity and $\mathbf{x}_i$ the associated eigenvector [4].

IV. CONDUCTOR LOSSES

Conductor losses are also computed by a perturbational approach based on an approximate evaluation of the power dissipated in the microstrip array when any of the quasi-TEM modes is individually excited. In turn, the dissipation is related by simple empirical equations to the loss properties of the elementary microstrip systems associated with the array (points A, B, C of section II). The latter will be discussed first.

For a single microstrip and for the even or odd mode of a symmetric coupled-microstrip pair, we use the following expression of the attenuation constant due to conductor losses [9]:

$$\alpha^c = \sqrt{\frac{\pi \mu_0 \rho_c f \varepsilon_{eff}}{\varepsilon_{eff0}}} \frac{K F}{W Z_0}$$

where:

ρ_c = ohmic resistivity of the metal conductors
ε_{eff0} = zero-frequency effective dielectric constant
K = current distribution factor [9]
F = empirical correction factor
W = strip width
Z_0 = zero-frequency characteristic impedance.

The impedance Z_0 is computed by the explicit formula given in [9] for the single microstrip, and by the expressions of ref. [10] for the even and odd mode of a coupled-microstrip pair. The current distribution factor is given by [9]

$$K = \exp\left[-1.2 \left(\frac{Z_0 \sqrt{\varepsilon_{eff0}}}{\eta_0} \right)^{0.7} \right] \quad (6)$$

Finally, the correction factor F was empirically determined in order to provide the best fit between (5) and a set of experimental results obtained by the resonance method [10] for both single and coupled microstrips.

For an isolated microstrip the correction factor is given by

$$F = F_\infty = [1 + 0.64 \exp(-0.65\, u)] \quad (7)$$

where u = W/H. For the even and odd modes of a coupled pair with strip spacing equal to S, we have

$$F_E = F_\infty + 0.34\, [1 - 3.59 \exp(-5.49\, u)] \exp(-1.02\, g)$$
$$F_O = F_\infty + 0.5\, [1 - 3.71 \exp(-1.93\, u)] \exp(-0.635\, g) \quad (8)$$

where g = S/H.

Now let the i-th quasi-TEM mode of the array be individually excited with unit amplitude. The perturbational expression for the attenuation constant of this mode due to conductor losses is

$$\alpha_i^c \cong \frac{P_i^c}{2 P_i} = \frac{1}{v_i\, \mathbf{x}_i^T\, \mathbf{C}\, \mathbf{x}_i} \sum_{j=1}^{m} p_{ij} \quad (9)$$

where P_i^c is the power dissipated (per unit length) in the entire array, and p_{ij} is the power dissipated in the j-th strip in the same conditions. Note that the j-th strip carries the current $I_{ij} \exp[-j(\omega/v_i)z]$ where I_{ij} is the j-th element of the i-th mode current vector $\mathbf{I}_i$.

The computation of p_{ij} proceeds as follows. The dissipated power depends on both the total current carried by the strip and the current distribution on the strip surface, which is modified by the presence of the adjacent strips of the array (proximity effect). In order to approximately evaluate this effect, we make the assumption that the perturbations of the current distribution due to left-hand and right-hand adjacent strips are independent, and can thus be evaluated separately. The j-th microstrip is thus considered as consisting of two parts of equal widths $W_j/2$, and the ohmic losses in each half are independently evaluated in the way described below.

To find the power dissipated in the right-hand half of the j-th strip, namely p_{ijR}, we consider the symmetric coupled-microstrip pairs defined at point B) in section II, and denote by α_{jRE}, α_{jRO} and Z_{jRE}, Z_{jRO} the even- and odd-mode attenuation constants and characteristic impedances, respectively ($1 \leq j \leq m-1$). In the practical implementation of our model, the attenuation constants are given by (5), (6) and (8), and the characteristic impedances are computed according to ref. [7]. Assuming that the j-th coupled microstrips carry the currents $\pm I_{ij}$, the powers dissipated by the even and odd modes are

$$p_{ijRE} = \alpha_{jRE} Z_{jRE} I_{ij}^2$$

$$(1 \le j \le m - 1) \quad (10)$$

$$p_{ijRO} = \alpha_{jRO} Z_{jRO} I_{ij}^2$$

In the presence of the i-th quasi-TEM mode, the power lost in the right-hand half of the j-th strip lies some way between the even- and odd-mode values (10), because the voltage x_{ij+1} on the (j+1)-th strip is different from $\pm x_{ij}$. We thus take a linear combination of (10), in such a way that the even- and odd-mode values be reobtained for $x_{ij+1} = x_{ij}$ and $x_{ij+1} = - x_{ij}$, respectively. The final expression is

$$p_{ijR} = \frac{|x_{ij} + x_{ij+1}|\, p_{ijRE} + |x_{ij} - x_{ij+1}|\, p_{ijRO}}{|x_{ij} + x_{ij+1}| + |x_{ij} - x_{ij+1}|} \qquad (1 \le j \le m - 1) \quad (11)$$

The computation of the left-hand side proximity effect is carried out in a similar way, making use of the symmetric coupled-microstrip pairs defined at point C) in section II. The formulas are similar to (10), (11), with the subscript R replaced by L (for "left").

For the left-hand half of the first strip and for the right-hand half of the m-th strip, we simply use 1/2 of the dissipated power in an isolated microstrip of width W_1, W_m, respectively. The latter are given by

$$p_{i1\infty} = \alpha_{1\infty} Z_{1\infty} I_{i1}^2$$

$$(12)$$

$$p_{im\infty} = \alpha_{m\infty} Z_{m\infty} I_{im}^2$$

where by α_∞, Z_∞ we denote the isolated microstrip attenuation constant (due to conductor losses) and characteristic impedance, respectively. α_∞ is given by (5), (6), (7), while Z_∞ is computed according to ref. [6]. The final expressions for the conductor losses in the strips to be used in (9) are then

$$p_{i1} = \frac{1}{2} p_{i1\infty} + \frac{1}{2} p_{i1R}$$

$$p_{ij} = \frac{1}{2}(p_{ijL} + p_{ijR}) \qquad (1 \le j \le m - 1) \quad (13)$$

$$p_{im} = \frac{1}{2} p_{imL} + \frac{1}{2} p_{im\infty}$$

Finally, the surface roughness can be taken into account by the approach proposed in [9].

V. RESULTS

As an example of application, fig. 3 shows the topology of an interdigitated capacitor built on GaAs substrate by microstrip techniques. To analyze this device we used the decomposition method outlined in [14], whereby the component is subdivided into a number of elementary building blocks including bends, T-junctions, gaps, single-microstrip sections, and one coupled-microstrip array representing the main body of the device structure, as represented in fig. 4. The measured [15] and computed scattering parameters of this capacitor are compared in fig. 5. The calculations were carried out directly (i. e. requiring no look-up tables or similar devices) and took 3.2 CPU seconds overall on a VAX 8800. These results show that the array model developed in this paper represents a powerful and accurate, yet relatively simple tool for the broad-band simulation of MMIC components.

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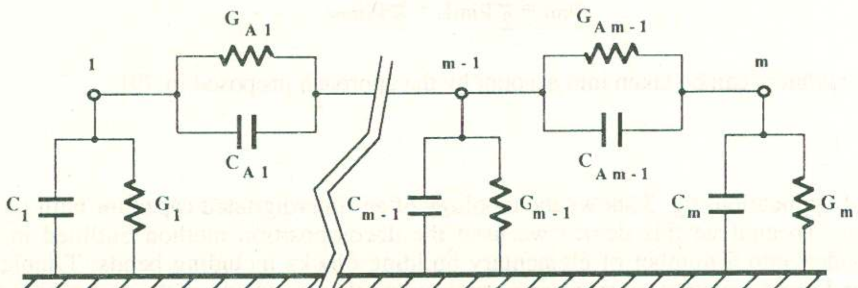
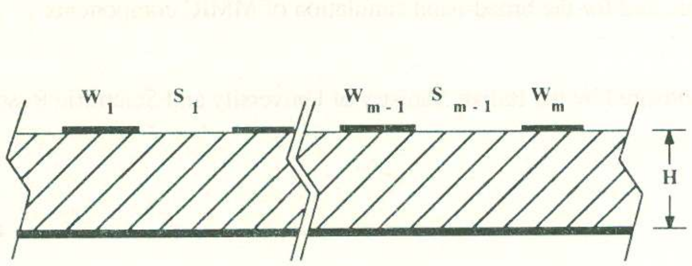


Fig. 1. Conductance-capacitance model of a microstrip array



. Fig. 2. Microstrip array cross section

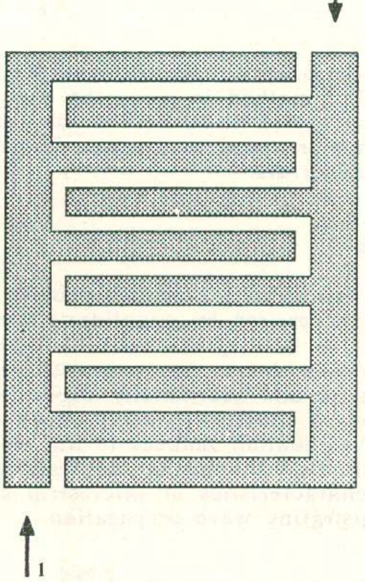


Fig. 3. Topology of an interdigitated capacitor

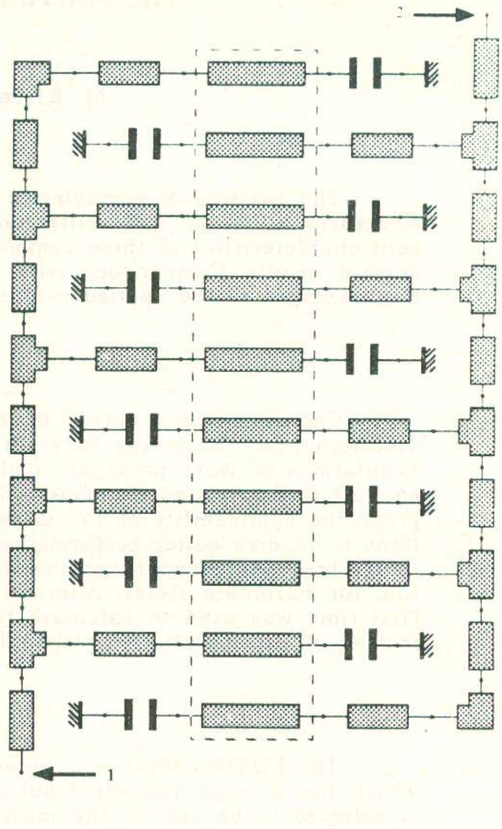


Fig. 4. Segmentation adopted in the analysis of the interdigitated capacitor of fig. 3

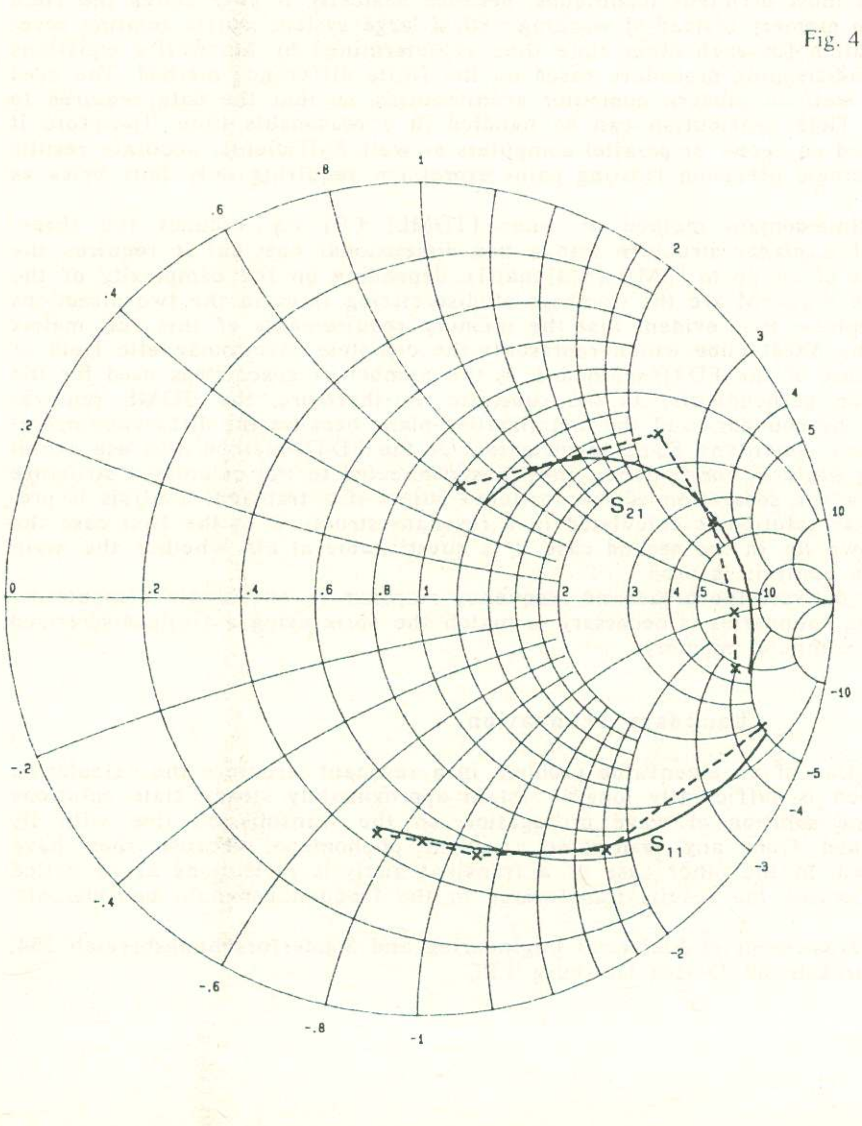


Fig. 5. Comparison between the measured and computed S-parameters of the interdigitated capacitor of fig. 3