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Investigating Mathematics in Scotland and the United States

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Abstract

This paper presents the results of an initial investigation on how educators from two different educational systems engaged in mathematics calculations. The study explored the nature of the educators' solution strategies and the extent to which these strategies adhered to standard taught algorithms or more non-traditional procedures. Our future studies hope to provide more evidence of our beliefs that teachers who only know or use traditional algorithms are not readily able to assist students with developing more sense-making strategies that not only are more efficient but also reflect flexible thinking.

Curriculum

In both the United States and Scotland, current curricula in place for mathematics teaching and learning share similar philosophies and underlying principles. In the United States the Common Core Standards (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010) reflect a balance between supporting the development of conceptual understanding and establishing computational fluency. These two aspects of students' mathematical experiences are not mutually exclusive. The Standards address Mathematical Practices that include supporting students to make sense of problems, to reason abstractly, and to construct viable arguments.

In Scotland Curriculum for Excellence (CfE) has been implemented in schools since 2010. It is founded on the principle of children as active learners who construct meaning through engaging in purposeful activity. In mathematics, this is exemplified through the specification that all children, from the early stages onwards 'should experience success in mathematics and develop the confidence to take risks, ask questions and explore alternative solutions without fear of being wrong. They will enjoy exploring and applying mathematical concepts to understand and solve problems, explaining their thinking and presenting their solutions to others in a variety of ways.' (Scottish Government, 2009, p.3).

Practice and beliefs

The relationship between teacher-held beliefs about the teaching of mathematics and mathematics learning is significant (Ernest, 1989). Students, whose teachers hold a sense-making view of mathematics teaching, are more likely to have classroom experiences that support a sense-making process. However there is evidence that many teachers do not hold this position, possibly because they themselves do not make sense of the mathematics they teach; therefore it is likely that their students may not be supported by sense-making experiences (Beswick, 2005). We believe that teachers who make sense of mathematics themselves are best able to help students to make sense of mathematics.

The study

The aim of our study was to investigate if teacher educators with various responsibilities relied on traditional strategies or non-traditional sense-making strategies.

The findings we report here are part of a larger study on how different populations of mathematics learners get answers to 15 calculations. Prior research addressed developmental levels of students as they learn to make sense and reason about mathematics (Lubinski, Cady, & Otto 2014). In this study, we explore the extent to which two populations of teacher educators used sense-making approaches to find an answer to two calculations from a calculation assessment shown to provide rich information on thinking strategies.

Methods

We administered the fifteen-item computational assessment to two groups of teachers, a United States (US) group and a Scottish group. The US educators (n=32) were all teachers with various years teaching experience. Educators were primarily high school teachers (n=7), primarily elementary teachers (n=15), K-5 special education (n=2), or other (n=8).

Scottish population of n=30 educators: primary teachers (n=6), secondary teachers (n=4), secondary mathematics students (n=12), and various other educators including support teachers and primary deputy heads and primary teacher support for learning (n=8).

The US teachers in this study had been involved to various degrees with research involving how children learn mathematics. The Scottish educators had not had extensive instruction on children's learning connected to this research.

Findings

For this paper we are focussing on all responses to two assessment items (item numbers 1 and 13) to illustrate the richness of data that were collected.
Assessment Item 1: Subtract 747 from 8000:

- Four US and seven Scottish teachers had just an answer, no work. One US teacher's answer was incorrect.
- Three US and four Scottish educators rewrote the calculation in either a vertical or horizontal format and got a correct answer but showed no work.

We chose to report on the educators whose strategies were either traditional or non-traditional. We do not comment on those teachers who only wrote an answer. All answers were correct unless otherwise indicated.

1. Traditional strategy:
    - 7 United States teachers (one incorrect answer)
    - 13 Scottish teachers (one incorrect answer)
2. Non-traditional strategy:

- 18 US teachers (one incorrect)
- 6 Scottish teachers

Both US and Scottish educators who implemented a non-traditional strategy used a count on, count down to, compensate or combination strategy. For example:

- 747 + 53 -> 800 + 200 ->1000 + 7000 = 8000. Thus, 53 + 200 + 7000 = 7253
- 8000 – 700 ->7300 – 40 ->7260 – 7 = 7253
- 8000 – 1 = 7999; 7999 – 747 = 7252; 7252 + 1 = 7253
- 8000 – 750 = 7250; 7250 + 3 = 7253

Correctness on this calculation is high in both groups with only three US teachers and one Scottish teacher getting it wrong. Of those educators whose strategies we could identify, slightly over half of them, or roughly a third of all of them, used non-traditional strategies. It is important to note, three times as many US teachers than Scottish teachers used a non-traditional strategy. These US educators have had access to research involving how children learn mathematics. We believe knowledge of this research provides teachers not only with flexibility in their own choice of solution strategies but also allows them to better understand their students' thinking and how it might be developed.

Additionally, twice as many Scottish educators used a traditional strategy than the US educators. Our future research will investigate the relationship of teachers' knowledge of strategies to teachers' ability to assist students with developing more sense-making strategies that not only are more efficient but also reflect flexible thinking.

Quiz Item 13: Place a decimal point in 0001206000 so that 04.02 X 0.30 and 0001206000 are equal. A teacher's strategy with this calculation determined answer correctness. We saw variations of rule-bound or procedural strategies and sense-making strategies (S represents Scottish educators):

- unclear, (1 S) or no answer (1 S and 4 US)
- an answer of 120.6 reflects the rule that when multiplying two numbers with digits two places to the right of the decimal point, you go over four places to the left in your answer (4 S and 6 US)
- 4.02 X 0.3, multiply and count over three (11 S and 9 US). Educators rewrote the item as 4.02 X 0.3, multiplied, and counted over from the right three places for the decimal point. They still used the multiplication in a procedural way but wit more understanding of the rule.
- 0.3 X 4. = 0.12 and 0.3 X 0.02 =0.18 so 12.06, multiply decimal numbers incorrectly, add, and get a wrong answer (1 S)
- multiply 3 X 4.02 and compensate by dividing by10 (3 S and 0 US)
- 4 X .3 = 1.2, estimate calculation and place the decimal point (0 S and 5 US, 1 wrong)
- no work, just placed the decimal point (9 S with 1 wrong and 6 US with 2 wrong). This appeared to be a strategy of using a rule, void of any sense making.

Correctness is interesting on this calculation. Nine US teachers answered incorrectly and 4 gave no answer; Nine Scottish teachers answered incorrectly and 1 gave no answer. That is, roughly a third of each group got it wrong or did not answer.

Discussion and Further Work

It was notable that three times as many US teachers as Scottish teachers used non-traditional strategies to solve problem 1. Further research would be required to explore the professional learning, as well as earlier learning experiences of the teachers, to determine the extent to which these prior experiences supported or constrained sense-making in their own mathematical processes.

The development of sense-making approaches in classrooms is challenging and an on-going struggle for teachers (Hiebert, 2013). Teachers' pedagogical practices are shaped by their beliefs as well as by their own educational experiences (Beswick, 2005). If teachers are to support children in developing relational understanding (Skemp, 1976) then, it has been argued, that they should have a profound (Ma, 1999) understanding themselves (Maclellan, 2014). There are some indications in our findings that teachers are operating from a procedural basis. This is not to suggest that the teachers lack conceptual understanding but if teachers operate at procedural level of understanding themselves we believe their conceptual understanding is limited.

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