

# Statistical modelling of electron devices based on an equivalent-voltage approach

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**Abstract** — Active device modelling and statistical description of the device behaviour are two key aspects in the design of high-yield integrated circuits. A new empirical approach is here proposed which is capable of describing the effects of process parameter variations on the electron device electrical response by means of only a few statistical parameters. The model can be easily identified on the basis of conventional electrical measurements. Preliminary validation results from experimental data, simulations using the Trew analytical model and simulations of a modified Curtice model are provided in the paper.

## I. INTRODUCTION

The availability of a good statistical description [1,3,5] for the dispersions in the parameters of active device models is a key point in integrated circuit design. The goal is to provide the circuit designer with suitable tools for yield optimization and design centering [1]. The most common way of considering foundry process variations in the design of an IC is the Montecarlo analysis. The statistical parameters that can be used in MC analysis are partially provided in the foundry design kits and are obtained by empirical observation of the component spreading by means of automated measurements over a large number of samples. However, information available from the foundries is usually limited to the spread of some key parameters without including their mutual correlations. This is especially true when considering the parameters of the active device models and may lead to conservative yield estimations.

On the basis of the above considerations, an empirical approach is here adopted for the development of a new, compact, nonlinear statistical model, which can be easily identified from conventional measured data. The proposed model, based on an empirical approach [4] previously used for the modelling of low-frequency dispersion effects in III-V FETs due to traps and thermal phenomena, is capable of describing the variations in the electrical performance due to process parameter dispersion using a small number of statistical variables.

## II. THE PROPOSED STATISTICAL MODEL

Let us consider an intrinsic field-effect transistor which can be sufficiently described<sup>1</sup> by adopting the well-

known charge controlled quasi-static common-gate formulation:

$$\mathbf{i}(t) = \Phi\{\mathbf{q}(t)\} + \frac{d\mathbf{q}(t)}{dt} \quad (1)$$

$$\mathbf{q}(t) = \Psi\{\mathbf{v}(t)\} \quad (2)$$

where  $\mathbf{i} = [i_s \ i_d]^T$ ,  $\mathbf{q} = [q_{gs} \ q_{gd}]^T$ ,  $\mathbf{v} = [v_{gs} \ v_{gd}]^T$  represent, respectively, the vectors of the source and drain currents, the gate source and gate drain equivalent charges, which are dealt with as state-variables, and the intrinsic port voltages. Moreover,  $\Phi$  and  $\Psi$  are suitable purely algebraic nonlinear functions. Substituting (2) in (1), the equivalent voltage-controlled formulation is obtained:

$$\mathbf{i}(t) = \mathbf{F}\{\mathbf{v}(t)\} + \mathbf{C}\{\mathbf{v}(t)\} \frac{d\mathbf{v}(t)}{dt} \quad (3)$$

where  $\mathbf{F}$  and  $\mathbf{C}$  are purely algebraic functions defined as:

$$\mathbf{F}\{\mathbf{v}\} = \Phi\{\Psi\{\mathbf{v}\}\}, \quad \mathbf{C}\{\mathbf{v}\} = \frac{d\Psi\{\mathbf{v}\}}{dt}$$

In the following, the device described by (1) and (2) will be referred to as the “Reference Device” (**RD**).

Let us now consider a generic device  $k$  affected by a parameter variation  $\Delta\mathbf{P}^{(k)}$ , where  $\Delta\mathbf{P}^{(k)} = [\Delta P^{(k)}_1 \dots \Delta P^{(k)}_n]^T$  accounts for all the spreads in the foundry process that lead to dispersion in the characteristics of a population of devices. In such a case, both the currents and charges have to be  $\Delta\mathbf{P}$ -dependent quantities, and (1) and (2) become:

$$\mathbf{i}^{(k)}(t) = \Phi_p\{\mathbf{q}^{(k)}(t), \Delta\mathbf{P}^{(k)}\} + \frac{d\mathbf{q}^{(k)}(t)}{dt} \quad (4)$$

$$\mathbf{q}^{(k)}(t) = \Psi_p\{\mathbf{v}(t), \Delta\mathbf{P}^{(k)}\} \quad (5)$$

where  $\Phi_p\{\mathbf{q}^{(k)}(t), \mathbf{0}\} = \Phi\{\mathbf{q}^{(k)}(t)\}$ ,  $\Psi_p\{\mathbf{v}(t), \mathbf{0}\} = \Psi\{\mathbf{v}(t)\}$ .

The voltage-controlled formulation is now:

$$\mathbf{i}^{(k)}(t) = \mathbf{F}_p\{\mathbf{v}(t), \Delta\mathbf{P}^{(k)}\} + \mathbf{C}_p\{\mathbf{v}(t), \Delta\mathbf{P}^{(k)}\} \frac{d\mathbf{v}(t)}{dt} \quad (6)$$

where  $\mathbf{F}_p\{\mathbf{v}(t), \mathbf{0}\} = \mathbf{F}\{\mathbf{v}(t)\}$ ,  $\mathbf{C}_p\{\mathbf{v}(t), \mathbf{0}\} = \mathbf{C}\{\mathbf{v}(t)\}$ .

The currents and charges in (4) and (5) can be related to the corresponding quantities in the reference device in the following way:

$$\mathbf{q}^{(k)}(t) = \Psi_p\{\mathbf{v}(t), \Delta\mathbf{P}^{(k)}\} = \Psi\{\mathbf{v}(t)\} + \Delta\mathbf{q}^{(k)} \quad (7)$$

$$\begin{aligned} \mathbf{i}^{(k)}(t) &= \Phi_p\{\mathbf{q}^{(k)}(t), \Delta\mathbf{P}^{(k)}\} + \frac{d\mathbf{q}^{(k)}(t)}{dt} = \\ &= \Phi\{\mathbf{q}^{(k)}(t)\} + \Delta\mathbf{i}^{(k)} + \frac{d\{\mathbf{q}^{(k)}(t)\}}{dt} \end{aligned} \quad (8)$$

<sup>1</sup> It is not strictly necessary that the device itself be accurately described by a charge-controlled formulation. It is just necessary that the charge-

controlled approximation be accurate in dealing with statistical dispersions, which are a second order effect in the device behavior.

Eqs. (7) and (8) can also be expressed in an alternative form, where  $\Delta\mathbf{q}^{(k)}$  and  $\Delta\mathbf{i}^{(k)}$  are replaced by equivalent voltage perturbations  $\Delta\mathbf{v}_q^{(k)}$  and  $\Delta\mathbf{v}_i^{(k)}$ , which satisfy the equivalence relations:

$$\mathbf{q}^{(k)}(t) = \Psi_p \{ \mathbf{v}(t), \Delta\mathbf{P}^{(k)} \} = \Psi \{ \mathbf{v}(t) + \Delta\mathbf{v}_q^{(k)} \} \quad (9)$$

$$\begin{aligned} \mathbf{i}^{(k)}(t) &= \Phi_p \left\{ \mathbf{q}^{(k)}(t), \Delta\mathbf{P}^{(k)} \right\} + \frac{d\mathbf{q}^{(k)}(t)}{dt} = \\ &= \mathbf{F} \{ \mathbf{v}(t) + \Delta\mathbf{v}_q^{(k)} + \Delta\mathbf{v}_i^{(k)} \} + \mathbf{C} \{ \mathbf{v}(t) + \Delta\mathbf{v}_q^{(k)} \} \cdot \frac{d \{ \mathbf{v}(t) + \Delta\mathbf{v}_q^{(k)} \}}{dt} = \\ &= \mathbf{F} \{ \mathbf{v}(t) + \Delta\mathbf{v}_q^{(k)} + \Delta\mathbf{v}_i^{(k)} \} + \tilde{\mathbf{C}} \{ \mathbf{v}(t) \} \cdot \frac{d\mathbf{v}(t)}{dt} \end{aligned} \quad (10)$$

where:

$$\Delta\mathbf{v}_q^{(k)} = \mathbf{H}_q(\mathbf{v}, \Delta\mathbf{P}^{(k)}) \quad (11)$$

$$\Delta\mathbf{v}_i^{(k)} = \mathbf{H}_i(\mathbf{v}, \Delta\mathbf{P}^{(k)}) \quad (12)$$

$$\tilde{\mathbf{C}} \{ \mathbf{v}(t) \} = \mathbf{C} \{ \mathbf{v}(t) + \Delta\mathbf{v}_q^{(k)} \} \cdot \left( \mathbf{1} + \frac{d\Delta\mathbf{v}_q^{(k)}}{d\mathbf{v}} \right) \quad (13)$$

the only requirement being the invertibility of the  $\Psi$  and  $\mathbf{F}$  functions with respect to charges and voltages, respectively.

Eqs. (9) and (10) show that for a population of devices, any device  $k$  excited by port voltages  $\mathbf{v}$ , can be described in terms of the reference device, excited by equivalent port voltages  $\mathbf{v} + \Delta\mathbf{v}$ .

Fig. 1 depicts the more convenient common-source formulation of the model, obtained after the application of suitable linear transformations to the common-gate formulation<sup>2</sup>.

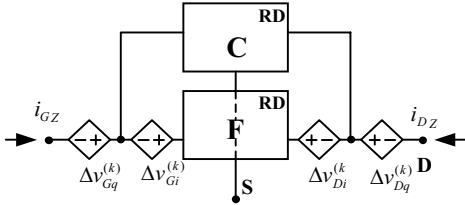


Fig1 Scheme of the proposed statistical model where the intrinsic RD is perturbed by suitable input and output generators to model the static and dynamic dispersions affecting a certain device  $k$

In this scheme we have three generators to describe the effects of process variations. The external ones,  $\Delta\mathbf{v}_{Gq}^{(k)}$  and  $\Delta\mathbf{v}_{Dq}^{(k)}$  originate from the charge function  $\Psi$ , and have effect both on the reactive and on the resistive parts of the drain current, whereas the internal generators  $\Delta\mathbf{v}_{Gi}^{(k)}$  and  $\Delta\mathbf{v}_{Di}^{(k)}$  originate from the current function  $\mathbf{F}$ , and affect only the resistive part of the drain current. In order for the static output current to vanish for any  $v_G$  whenever  $v_D$  equals zero, it will be required that the sum  $\Delta\mathbf{v}_{Di}^{(k)} + \Delta\mathbf{v}_{Dq}^{(k)}$  also vanish for  $v_D=0$ .

A further simplification of the model is possible if we linearize  $\Delta\mathbf{v}$  with respect to the vector of parameter variations  $\Delta\mathbf{P}$ . Thus:

$$\Delta\mathbf{v}_q^{(k)} = \mathbf{H}_q(\mathbf{v}, \Delta\mathbf{P}^{(k)}) \cong \sum_{j=1}^n \mathbf{h}_{qj}(\mathbf{v}) \Delta P_j^{(k)} \quad (14)$$

where:

$$\mathbf{h}_{qj}(\mathbf{v}) = \left. \frac{\partial \mathbf{H}_q}{\partial \Delta P_j} \right|_{\Delta P_j=0} \quad (15)$$

<sup>2</sup>Although so far the formulation of the statistical model has been in terms of equivalent voltages to model an intrinsic field-effect transistor, an analog formulation with shunt current generators, suitable for modelling devices with a non-negligible input current (i.e. HBTs), is straightforward.

is the sensitivity of  $\Delta\mathbf{v}_q$  to process parameter variation  $\Delta P_j$ . The sum in (14) can be decomposed into  $m$  sums:

$$\sum_{j=1}^n \mathbf{h}_{qj}(\mathbf{v}) \Delta P_j^{(k)} = \left( \sum_j \mathbf{h}_{qj}(\mathbf{v}) \Delta P_j^{(k)} \right)_1 + \dots + \left( \sum_j \mathbf{h}_{qj}(\mathbf{v}) \Delta P_j^{(k)} \right)_m \quad (16)$$

where each term groups the  $\mathbf{h}_{qj}(\mathbf{v})$  which show a similar behavior in  $\mathbf{v}$ . Presumably, many of the  $\mathbf{h}_{qj}(\mathbf{v})$  will be similar and so  $m$  in (16) will be much smaller than the original number of parameters  $n$ . Due to the similarity of the  $\mathbf{h}_j$  functions, we can write for the 1<sup>th</sup> sum in (16):

$$\left( \sum_j \mathbf{h}_{qj}(\mathbf{v}) \Delta P_j^{(k)} \right)_1 \cong \mathbf{f}_{q1}(\mathbf{v}) \cdot \sum \mathbf{b}_{qj} \Delta P_j^{(k)} \quad (17)$$

where we have assumed that all the  $\mathbf{h}_j$  functions in the 1<sup>th</sup> sum can be sufficiently approximated by a function  $\mathbf{f}_q$ , which depends only on the input voltage  $\mathbf{v}$ , and a suitable scaling factor  $\mathbf{b}_{qj}$  for each parameter  $P_j$ <sup>3</sup>. As a result, a higher compactness is achieved in the number of parameters used to describe the statistic dispersions. Eq. (14) then becomes:

$$\Delta\mathbf{v}_q^{(k)} = \begin{bmatrix} \Delta\mathbf{v}_{Gq}^{(k)} \\ \Delta\mathbf{v}_{Dq}^{(k)} \end{bmatrix} \cong \begin{bmatrix} f_{Gq1}(\mathbf{v}) \cdot \gamma_{Gq1}^{(k)} + \dots + f_{Gqm}(\mathbf{v}) \cdot \gamma_{Gqm}^{(k)} \\ f_{Dq1}(\mathbf{v}) \cdot \gamma_{Dq1}^{(k)} + \dots + f_{Dqm}(\mathbf{v}) \cdot \gamma_{Dqm}^{(k)} \end{bmatrix} \quad (18)$$

where

$$\gamma_q^{(k)} = \begin{bmatrix} \gamma_{Gq}^{(k)} \\ \gamma_{Dq}^{(k)} \end{bmatrix} = \begin{bmatrix} \left( \sum_j b_{Gqj} \Delta P_j^{(k)} \right)_1, \dots, \left( \sum_j b_{Gqj} \Delta P_j^{(k)} \right)_m \\ \left( \sum_j b_{Dqj} \Delta P_j^{(k)} \right)_1, \dots, \left( \sum_j b_{Dqj} \Delta P_j^{(k)} \right)_m \end{bmatrix} \quad (19)$$

analogously, (12) becomes:

$$\Delta\mathbf{v}_i^{(k)} = \begin{bmatrix} \Delta\mathbf{v}_{Gi}^{(k)} \\ \Delta\mathbf{v}_{Di}^{(k)} \end{bmatrix} \cong \begin{bmatrix} f_{Gi1}(\mathbf{v}) \cdot \gamma_{Gi1}^{(k)} + \dots + f_{Gim}(\mathbf{v}) \cdot \gamma_{Gim}^{(k)} \\ f_{Di1}(\mathbf{v}) \cdot \gamma_{Di1}^{(k)} + \dots + f_{Dim}(\mathbf{v}) \cdot \gamma_{Dim}^{(k)} \end{bmatrix} \quad (20)$$

In Section II, we will show some preliminary results obtained with  $\gamma_q^{(k)}$  vector of dimension  $2 \times 1$ , which means using the two generators  $\Delta\mathbf{v}_{Gq}^{(k)}$  and  $\Delta\mathbf{v}_{Dq}^{(k)}$ , each of them affected by a single parameter to describe the dispersion in the displacement current of each device. Instead, a  $\gamma_i^{(k)}$  of dimension  $1 \times 2$  (i.e. a single generator with two parameters) will be used to model dispersions in the conduction current.

We will also show that the  $\mathbf{f}$  functions can be adequately approximated by a constant term and linear dependence on  $v_G$  and  $v_D$ . After the last considerations, the proposed model for the perturbation voltages becomes:

$$\Delta\mathbf{v}_{Gq}^{(k)} = f_{Gq1}(\mathbf{v}) \cdot \gamma_{Gq1}^{(k)} + \dots + f_{Gqm}(\mathbf{v}) \cdot \gamma_{Gqm}^{(k)} = \gamma_{Gq}^{(k)} (1 + a_{Gq1} v_G + a_{Gq2} v_D) \quad (21)$$

$$\Delta\mathbf{v}_{Dq}^{(k)} = f_{Dq1}(\mathbf{v}) \cdot \gamma_{Dq1}^{(k)} + \dots + f_{Dqm}(\mathbf{v}) \cdot \gamma_{Dqm}^{(k)} = \gamma_{Dq}^{(k)} (1 + a_{Dq1} v_G + a_{Dq2} v_D) \quad (22)$$

$$\Delta\mathbf{v}_{Gi}^{(k)} = f_{Gi1}(\mathbf{v}) \cdot \gamma_{Gi1}^{(k)} + \dots + f_{Gim}(\mathbf{v}) \cdot \gamma_{Gim}^{(k)} = \gamma_{Gi}^{(k)} (a_{Gi1} v_G + a_{Gi2} v_D) \quad (23)$$

$$\Delta\mathbf{v}_{Di}^{(k)} = -\Delta\mathbf{v}_{Dq}^{(k)} \quad (24)$$

where we have adopted a constant term equal to 1 for  $\Delta\mathbf{v}_{Gq}^{(k)}$  and  $\Delta\mathbf{v}_{Dq}^{(k)}$ , without losing generality. Thus, the whole population of devices affected by random foundry

<sup>3</sup> The fact that the dispersions caused by the different parameters can be grouped into few voltage-dependent functions scaled by a much larger number of voltage independent factors is not unreasonable. In fact, it is no surprise that as the device turns off, the variance in the different characteristics tends to disappear (i.e. all devices, although coming from a manufacturing process with statistical dispersions, tend to look the same)

process variations can be generated by perturbing the reference device with four voltage sources at the input and output ports. Each of this voltage sources is defined as the product of a device-independent linear function of the input voltages, and a device-dependent factor.

### III. MODEL EXTRACTION

Model extraction involves the calculation of the six  $a$  coefficients for the whole population of devices, and the four  $\gamma^{(k)}$  factors for each device (Eqs (21)..(23)). From Fig. 1 it can be seen that the identification procedure consists of two uncoupled problems: since  $\Delta v^{(k)}_{Gq}$  and  $\Delta v^{(k)}_{Dq}$  affect both the reactive and the resistive parts of the device, they are calculated in the first place, using the measured capacitance matrixes. After that,  $\Delta v^{(k)}_{Gi}$ , which influences only the resistive part of the drain current, is calculated in order to fit the measured trans and output conductances, and the magnitude of the measured static drain current.

For each device  $k$ ,  $\Delta v^{(k)}_{Gq}$  and  $\Delta v^{(k)}_{Dq}$  are obtained by minimizing the relative error between the measured and the model-generated capacitance matrixes over a certain bias grid in  $v_G, v_D$ . In mathematical terms, the following  $V^{(k)}$  function is minimized for each device:

$$V^{(k)}(\mathbf{U}_{Gq}^{(k)}, \mathbf{U}_{Dq}^{(k)}) = \sum_{v_G} \sum_{v_D} \left( \sum_{i=1}^2 \sum_{j=1}^2 \frac{|C_{ij}^{(k)}(v_G, v_D) - \tilde{C}_{ij}^{(k)}(v_G, v_D, \mathbf{U}_{Gq}^{(k)}, \mathbf{U}_{Dq}^{(k)})|}{|C_{ij}^{(k)}(v_G, v_D)|} \right)^2 \quad (25)$$

where  $\mathbf{U}_{Gq}^{(k)} = [\gamma^{(k)}_{Gq}, \gamma^{(k)}_{Gq} a_{Gq1}, \gamma^{(k)}_{Gq} a_{Gq2}]$  and  $\mathbf{U}_{Dq}^{(k)} = [\gamma^{(k)}_{Dq}, \gamma^{(k)}_{Dq} a_{Dq1}, \gamma^{(k)}_{Dq} a_{Dq2}]$  each include the three parameters that define  $\Delta v^{(k)}_{Gq}$  and  $\Delta v^{(k)}_{Dq}$ . And

$$C(v_G, v_D) = \text{Im} \left( \frac{\mathbf{Y}_{meas}(v_G, v_D, \varpi)}{\varpi} \right)_{\varpi = 2\pi \cdot 1\text{MHz}}, \quad \tilde{C}^{(k)}(v_G, v_D, \mathbf{U}_{Gq}^{(k)}, \mathbf{U}_{Dq}^{(k)})$$

are the measured low-frequency (typically 1MHz) capacitance matrix and the model-generated capacitance matrix defined in (13), respectively.

The values of  $a_{Gq1}$  and  $a_{Gq2}$  are then extracted for the whole population of devices. If the model were exact, they could be extracted by simply averaging the ratios  $U^{(k)}_{Gq2}/U^{(k)}_{Gq1}$  and  $U^{(k)}_{Gq3}/U^{(k)}_{Gq1}$  over the entire device population. In practice, the  $U^{(k)}_{Gq}$  are not perfectly aligned in the  $[U^{(k)}_{Gq1}, U^{(k)}_{Gq2}, U^{(k)}_{Gq3}]$  space but rather form a constellation of points around a line that passes through the origin. Thus, the line which, passing through the origin, best fits in the least-squares sense the constellation of  $U^{(k)}_{Gq}$  is calculated;  $a_{Gq1}$  and  $a_{Gq2}$  are then the slopes of this line. Once  $a_{Gq1}$  and  $a_{Gq2}$  are known,  $\gamma^{(k)}_{Gq}$  can be calculated for each device by minimizing an error function analogous to  $V^{(k)}$  in (25) but that depends only on  $\gamma^{(k)}_{Gq}$ . An analogous procedure is followed for the calculation of  $a_{Dq1}$ ,  $a_{Dq2}$  and  $\gamma^{(k)}_{Dq}$  from the  $\mathbf{U}_{Dq}^{(k)}$ .

The second phase of the extraction involves the calculation of  $\Delta v^{(k)}_{Gi}$ , through the minimization of the relative error between the measured and the model-generated static drain current, output and trans conductances over a certain bias grid in  $v_G, v_D$ . The identification procedure is analogous to that of  $\Delta v^{(k)}_{Gq}$ .

### IV. PRELIMINARY MODEL VALIDATION

Due to the considerable effort that would be required to obtain experimental data for a large (and accordingly statistically representative) population of devices, the proposed model has been preliminary validated using three different tests that include data from real measured devices, devices generated using an analytical model, and devices generated after perturbing some parameters of an empirical model.

As a first validation test, the proposed model was used to describe the dispersions observed in the static characteristic of a population of seven 10x60um PHEMT devices. Fig 2 shows the statistical dispersions encountered in the DC characteristics of the seven devices. As can be seen from the figures, the variations are not negligible. Similar behavior can be observed for the static output and transconductance.

The model has been identified for this population of devices according to the procedure described in Section III. Device number 5 was chosen as the reference device from which the rest of the population was generated. Since only static measurements were available, just  $\Delta v^{(k)}_{Gi}$  was extracted in order to fit the measured static drain current and output conductances. A  $\gamma_i^{(k)}$  of dimension 1x2 was used. Table 1 shows the extracted parameters that define the generator  $\Delta v^{(k)}_{Gi}$  in (23). Figs. 3 and 4 show the measured and the model-generated output current and transconductance for two devices of this population. The results for the rest of the devices were also very good.

The second test that we used to preliminary validate the model involved the use of the well-known Trew analytical model [2], which was used to generate a population of 30 GaAs MESFETs. DC I/V characteristics and related differential conductances were generated as a function of process parameter variations. The Trew model includes the effects of arbitrary non-uniform doping profiles and accepts as input data both the device geometries and material parameters. This model represents a good trade-off between accuracy and computational effort when compared with more complex numerical models, and has been adopted by many authors working in the field of yield optimization.

After choosing a Reference Device, the proposed statistical model was extracted to fit the output current and static transconductance for the 30 devices generated with the Trew model. In this case, a  $\gamma_i^{(k)}$  of dimension 1x1 gave good results. Fig. 5 shows the histograms of the drain current and static transconductance at one bias point for the original and the reconstructed population of devices. As can be seen from the figures, the proposed model is able to reconstruct the statistical properties of the DC characteristics of a population of devices by means of just one statistical parameter.

The last validation set that was used to test the model was a population of 200 2x50um intrinsic devices generated with a scalable 3-port PHEMT Curtice-type empirical model. Process dispersions were artificially accounted for by randomly perturbing the device width and temperature. The reference device was chosen as the

one having  $W=50\mu\text{m}$  and  $T=293\text{K}$ . In this case, both the static and dynamic part of the model were extracted following the procedure in Section III. A  $\gamma_q^{(k)}$  of dimension  $2 \times 1$  and a monodimensional  $\gamma_i^{(k)}$  were enough to model the dispersions found in the population of 200 devices. Fig. 6 shows the histogram of the elements of the original and the reconstructed capacitance matrix at one bias point, together with their mean values and standard deviations. Also in this case, the fitting is very good; similar accuracy was observed for the static part of the model (drain current, output and trans conductances) along a typical class-A load line

## V. CONCLUSIONS AND FUTURE WORK

A new nonlinear technology-independent statistical electron device model which is based on only a few statistical parameters has been presented. The simplicity of the approach could enable a foundry to easily provide information on electron device dispersion which can be directly used for Montecarlo analysis and yield optimization in IC design. Measurement and simulation results, which preliminary confirm the validity of the proposed approach, have been provided by considering the DC characteristics of GaAs MESFETs and PHEMTs. The results obtained so far are encouraging and justify further experimental tests on the model.

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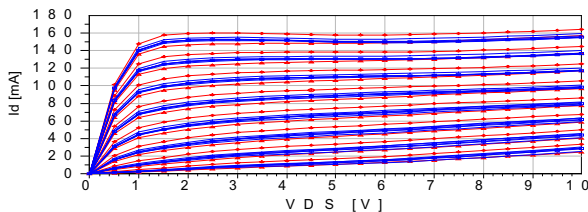


Fig. 2 DC characteristics of a population of 7  $10 \times 60 \mu\text{m}$  PHEMT devices. In (o) and ( $\Delta$ ), the devices with the maximum and minimum current at each  $V_{GS}$ . In ( $\square$ ), the device chosen as the RD.

$a_{G11}, a_{G12} = (1e-3, -1.026e-3)$					
device	$\gamma_{G11}^{(k)}$	$\gamma_{G12}^{(k)}$	device	$\gamma_{G11}^{(k)}$	$\gamma_{G12}^{(k)}$
1	-0,0178	1	5	Reference Device	
2	0,0391	0,9997	6	-0,0121	-2,95
3	-0,005	1,019	7	0,006	1
4	0,0165	0,9999			

Table 1. Parameters of the  $\Delta v^{(k)}_{Gi}$  generators extracted for a population of 7 PHEMT devices.

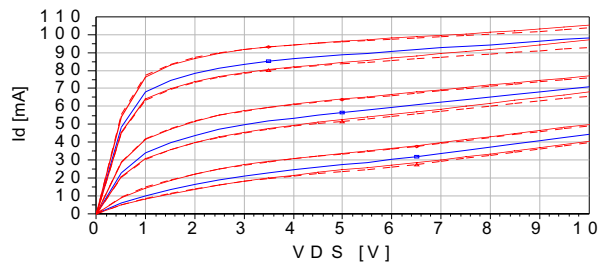


Fig. 3 DC characteristics for  $V_{GS} = [-0.7, -0.55, -0.4]$  V. In ( $\square$ ), the RD. Original (continuous line) and modelled (dotted line) devices. The last ones where obtained by perturbing the RD with suitable  $\Delta v^{(k)}_{Gi}$ .

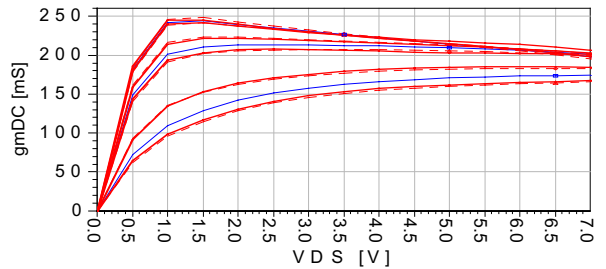


Fig. 4 DC transconductance for  $V_{GS} = [-0.7, -0.55, -0.4]$  V. In ( $\square$ ), the RD. Original (continuous line) and modelled (dotted line) devices. The last ones where obtained by perturbing the RD with suitable  $\Delta v^{(k)}_{Gi}$ .

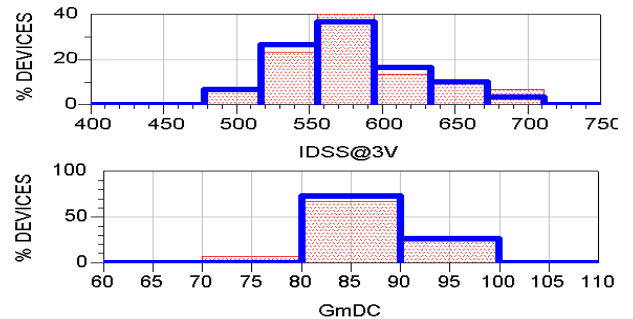


Fig. 5 Histograms of the original (thick line) and the model-generated (shaded area) drain current (top) and static transconductance (bottom) for a population of 30 MESFETs generated with the Trew model.

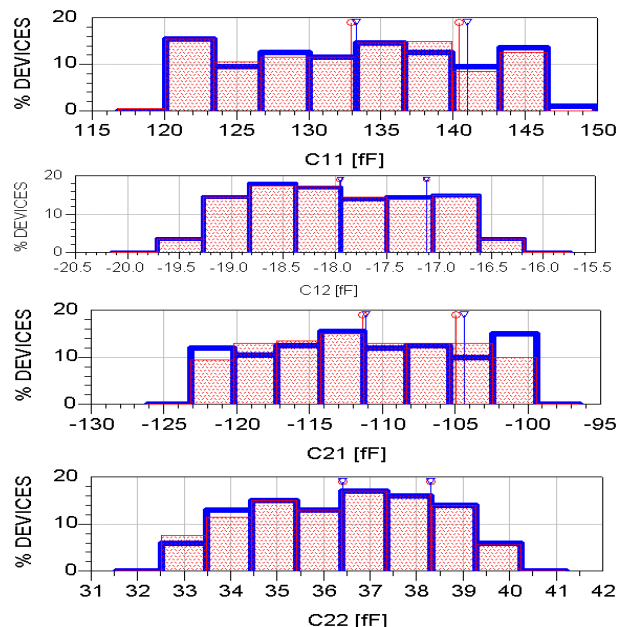


Fig 6 Histograms of the original (thick line) and the model-generated (shaded area) capacitance matrix for a population of 200 devices generated with a Curtice-type model. The thin bars indicate the mean and the mean plus the standard deviation for the original ( $\nabla$ ) and the model-generated population (o).