EXACT SOLUTION OF LOSSY ASYMMETRICAL COUPLED DIELECTRIC SLAB WAVEGUIDES

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ABSTRACT

This paper gives an exact characteristic equation for asymmetrical coupled dielectric slab waveguides with losses in both the guiding and surrounding regions. For the lossless case the solution of a single transcendental equation is all that is required for the evaluation of the propagation constant.

INTRODUCTION

There has been renewed interest in dielectric and non-radiative dielectric waveguides recently [1, 2, 3]. These structures provide very low-loss transmission and are likely to be the preferred transmission line medium for future 3D MMICs at upper mm-wave frequencies. Many dielectric waveguide components such as directional couplers [4] and filters [5] rely on composite structures for there operation. For each of these components, accurate values for the modal propagation constants are required. In general, solutions of coupled structures are obtained using coupled mode theories [6, 7]. The coupled mode is approximated by a weighted sum of the modes that exist on the isolated structures. However, these methods are only valid for large separations or well-confined modes [8]. Furthermore, coupled mode theory does not work well for asymmetrical guides [8]. Marcuse [9] presented a technique for the solution of compound slab waveguides with width and permittivity asymmetry. Unfortunately, his method is cumbersome, requiring numerical techniques to find the eigenvalue of an 8×8 determinant. In this paper, we derive an exact transcendental characteristic equation for the general asymmetrical coupled slab waveguide, the roots of which can easily be found. Furthermore, the technique can be used for the calculation of losses in coupled dielectric waveguides, which is an important consideration at mm-wave frequencies.

THEORY

Consider two parallel slab waveguides, A and B, separated by a distance $2D$ and with thicknesses $2a$ and $2b$, respectively (Fig. 1). Guide A occupies region 2 and has a relative permittivity ε_a. Guide B occupies region 4 and has a relative permittivity ε_b. Regions 1, 3 and 5 all have a relative permittivity ε_2. In general, the permittivity in each of the regions will be complex, $\varepsilon_n = \varepsilon'_n(1\text{-}j\tan\delta_n)$. All five regions have permeability μ_0. We make the usual slab assumption that all the field components are independent of y and that the z dependence is $\exp(-\gamma_z z)$, where $\gamma_z = \alpha_z + j\beta_z$. α_z is the longitudinal attenuation constant and β_z is the longitudinal phase constant. The wave equation then reduces to a one dimensional Helmholtz equation:

$$\frac{d^2}{dx^2}\Phi_y(x) + k_x^2\Phi_y(x) = 0 \tag{1}$$

where $\Phi_y = E_y$ for TE modes and $\Phi_y = H_y$ for TM modes. We choose the following fields over the five regions

1: $\Phi_y(x) = A_1 e^{\alpha_{x2}[x+(D+2a)]}$

2: $\Phi_y(x) = A_2\cos\{k_{xa}[x+(D+2a)] - \phi_a\}$

3: $\Phi_y(x) = A_3\cosh[\alpha_{x2}x] + A_4\sinh[\alpha_{x2}x]$

4: $\Phi_y(x) = A_5\cos\{k_{xb}[x-(D+2b)] + \phi_b\}$

5: $\Phi_y(x) = A_6 e^{-\alpha_{x2}[x-(2b+D)]}$

where $A_1 \ldots A_6$ are amplitude constants and ϕ_a and ϕ_b are constant phase terms. The transverse propagation constants are given by $\alpha_{x2}^2 = \varepsilon_2 k_0^2 - \gamma_z^2$, $k_{xa}^2 = \gamma_z^2 - \varepsilon_a k_0^2$ and $k_{xb}^2 = \gamma_z^2 - \varepsilon_b k_0^2$ where k_0 is the free-space wave number. For TE modes, E_y and H_z must be continuous at the boundaries between the different regions. For TM modes H_y and E_z must be continuous. Therefore, using Maxwell's equations to calculate H_z (E_z), and equating E_y (H_y) and H_z (E_z) at the four boundaries gives eight equations for the boundary conditions of the TE (TM) modes. These equations can be combined to yield

$$2D = \frac{1}{2\alpha_{x2}} \ln\left[\frac{\sin(2k_{xa}a)\sin(2k_{xb}b)}{\sin(2\phi_a - 2k_{xa}a)\sin(2\phi_b - 2k_{xb}b)}\right]. \qquad (2)$$

where $\phi_a = \tan^{-1}(\rho_a \alpha_{x2}/k_{xa})$ and $\phi_b = \tan^{-1}(\rho_b \alpha_{x2}/k_{xb})$ with $\rho_a = \rho_b = 1$ for TE modes and $\rho_a = \varepsilon_a/\varepsilon_2$ and $\rho_b = \varepsilon_b/\varepsilon_2$ for TM modes. We see that α_{x2}, k_{xa} and k_{xb} are all functions of γ_z. Thus, the right hand side of eqn. (2) is a function of a single variable - the longitudinal propagation constant γ_z. Therefore if a, b, D, ε_a, and ε_b are specified, we can solve eqn. (2) for all possible solutions of γ_z.

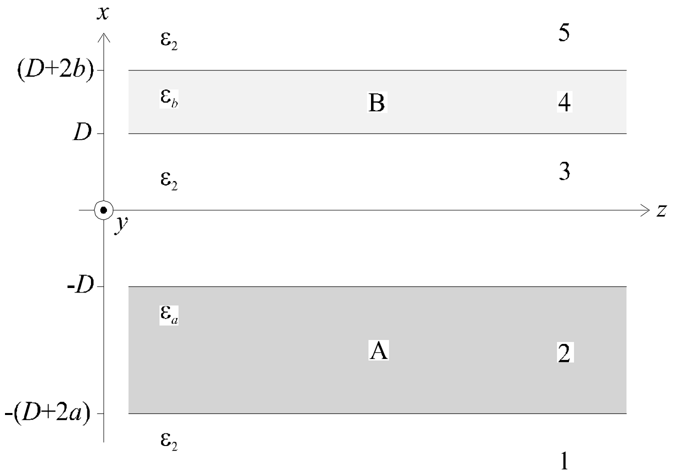

Fig. 1. Geometry of two parallel dielectric slab waveguides.

RESULTS

Fig. 2 shows the normalised separation $2D/\lambda_0$ plotted against the normalised propagation constant β_z/k_0 for different $2a/\lambda_0$ ratios. Both guides are identical, *i.e.* a=b and ε_a=ε_b=2.07. From Fig. 2 we see that, for the given $2a/\lambda_0$ ratios, there are two solutions to eqn. (2). The solution with the largest value of β_z corresponds to the lowest order (even) mode. The solution with the smallest value of β_z corresponds to the next higher order (odd) mode. This can be readily seen be substituting β_z into the field equations. It should be noted that for further increases in frequency, higher order modes would propagate. However, in general, coupled structures are limited to the two-mode case. We clarify Fig. 2 with an example. For a structure with a=b=0.5mm, $2D$=6.0mm and λ_0=10.0mm, Fig. 2 shows that two modes exist: an odd mode with β_z/k_0=1.012 and an even mode with β_z/k_0=1.068. Solutions with values of β_z/k_0 approaching unity correspond to modes near to low frequency cut-off. It is seen that in the low frequency case, $2a/\lambda_0$=0.1, the odd mode is cut-off until the guide separation $2D$ is larger than $0.4\lambda_0$. As expected the even and odd mode propagation constants tend to the value of the propagation constant of the isolated guides for increasing separation.

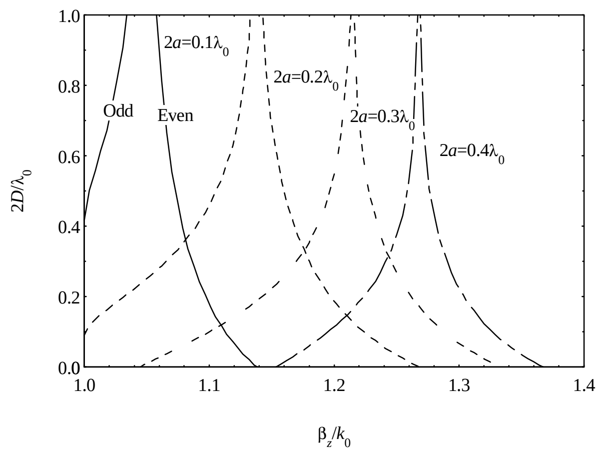

Fig. 2. Normalised separation $2D/\lambda_0$ against β_z/k_0 for several different $2a/\lambda_0$ ratios. $2a$=$2b$ and ε_a=ε_b=2.07.

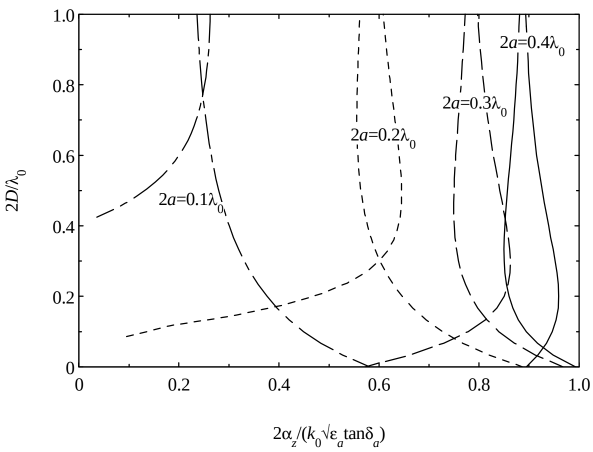

Fig. 3. Normalised attenuation constant for TE mode symmetrical coupled slab waveguide. $2a$=$2b$, ε_2=1, ε'_a= ε'_b =2.07 and $\tan\delta_a$= $\tan\delta_b$ =3.0×10^{-4}

For completeness Fig. 3 shows the attenuation constant α_z for varying separation with $2a$=$2b$, ε_2=1, ε'_a= ε'_b =2.07 and $\tan\delta_a$= $\tan\delta_b$ =3.0×10^{-4}. Notice that the odd mode solutions increase with $2D$, reach a maximum and then decrease slightly as they converge to the isolated value. For the even modes, α_z decreases as $2D$ increases, reaches a minium and then increases with $2D$, finally converging to the isolated value. This means that for some values of $2D$ the even mode of the coupled structure has a lower attenuation constant than that of the isolated dielectric waveguide. This may have some implications for low-loss propagation.

Once the propagation constant is determined from eqn. (2) the values of α_{x2}, k_{xa}, k_{xb}, ϕ_a and ϕ_b can be calculated. Thus, by applying the appropriate boundary conditions the field amplitude constants $A_1 \ldots A_6$ can be evaluated. Fig. 4 shows field plots for the even and odd modes of a TE guide for $2a$=$\lambda_0/2$ and ε_a=ε_b=2.0. Fig. 4 (a), (b) and (c) show the symmetrical case with $2D$=$\lambda_0/4$, $2D$=$\lambda_0/2$ and $2D$=λ_0, respectively. Fig. 4 (d), (e) and (f) are for the same guide separations as above but with a width asymmetry of a/b=2. Similarly, Fig. 4 (g), (h) and (I) are for a/b=4 and Fig. 4 (j), (k) and (l) are for a/b=8.

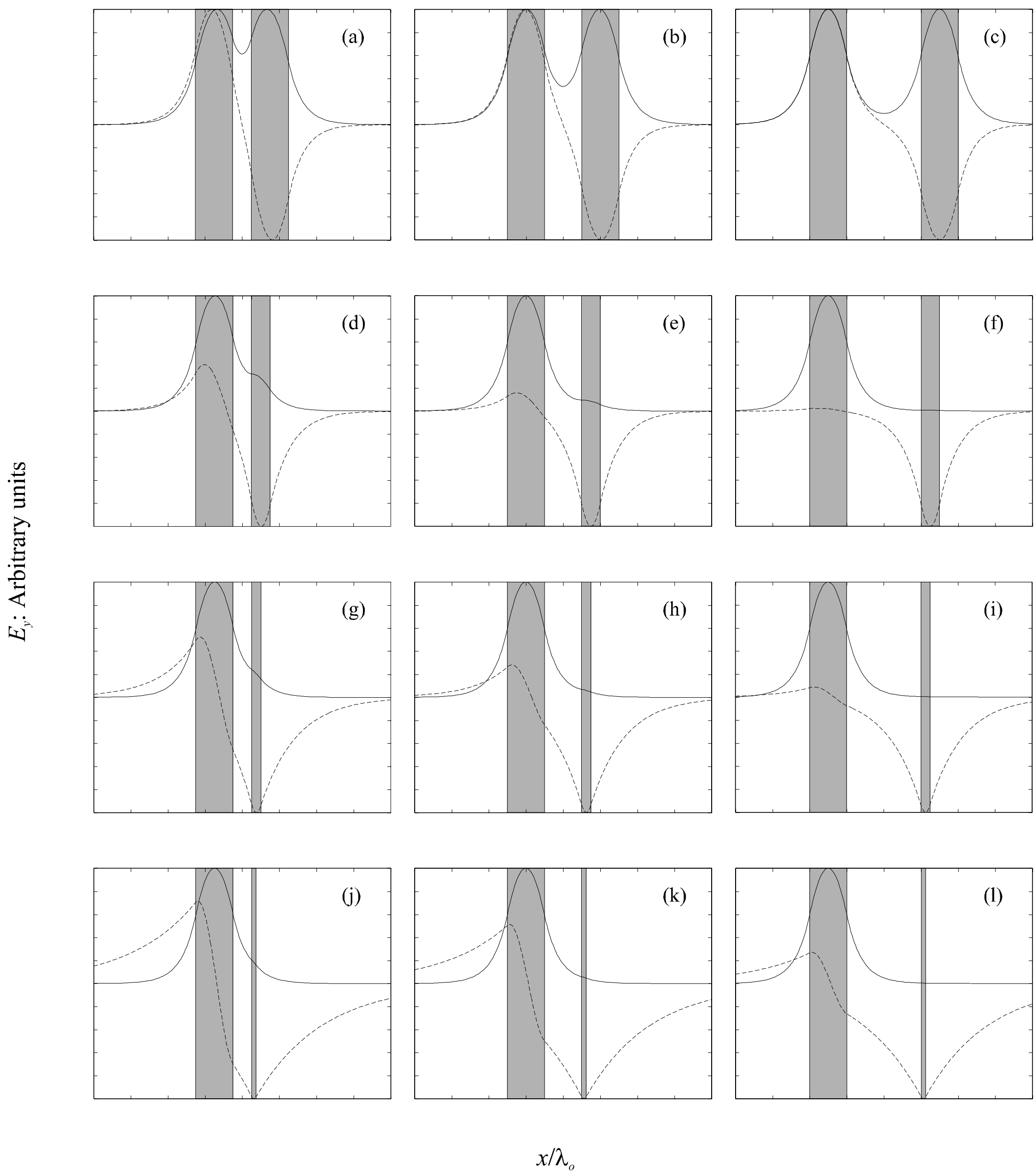

Fig. 4. Effects of separation and width asymmetry on the modal electric field profiles for TE coupled slab waveguide. Solid lines show the even modes, dashed lines show the odd.

CONCLUSIONS

In summary, an exact transcendental characteristic equation has been presented for both the TE and TM solutions of asymmetrical coupled slab waveguides with width and permittivity asymmetry. The solution of this new equation is simpler than previous methods and, unlike coupled mode theories, is exact. The technique is therefore valid for all guide separations and asymmetries. Furthermore, the technique can be used to calculate the attenuation constant when dielectric losses are present, which is often the case at millimetre-wave frequencies.

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