Determination of FET noise parameters from 50 Ω noise figure measurements using a distributed noise model

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***Abstract* – A method for measuring FET noise parameters is presented. It is based on the determination of a distributed noise model from 50 Ω noise figure measurements without needing a tuner, and taking into account the propagation effects along device electrodes. Experimental results up to 40 GHz are given.**

INTRODUCTION

Distributed circuit models for FETs have been proposed to include propagation effects along the device electrodes when the transistor dimensions, in particular its gate width, become of the same order of magnitude than the wavelength [1]. In [2] a method is proposed for extracting the elemental distributed noise sources of the device from measurements of its four noise parameters through the simplifying assumption of no correlation between noise sources (Pospieszalski's model [3]). In this paper, we propose the use of the smooth frequency dependence of the FET intrinsic noise sources in distributed-element circuit models for the determination of the FET noise parameters. This method is a generalization of the method proposed in [4], where a lumped-element circuit model was used. The method determines the noise sources of an elemental section of a general correlated intrinsic noise-model, which includes a correlation coefficient, using only the device noise figure for a matched (50 Ω) source (F_{50}) measured at a number of frequency points. Therefore, a tuner is not required. The distributed model is based in the so-called slice model [2],[5]. Experimental results of measured PHEMT's noise parameters up to 40 GHz are presented.

DISTRIBUTED NOISE-MODEL

To model the distributed effects in FETs, a semi-distributed model composed by a number N of equal device elemental sections (slices) is considered here [2],[5]. The circuit elements, associated to an elemental section, are shown in Figure 1. It is assumed that gate, drain and source electrodes behave as transmission lines propagating waves through the section. The section width W_u' is defined as $W_u'=W_u/N$, where W_u is the gate-width corresponding to a single gate finger. The FET total gate-width is $W = W_u \cdot N_{bd}$, where N_{bd} is the number of gate fingers. Each elemental section is modelled as a noisy six-port (Figure 2). The hybrid

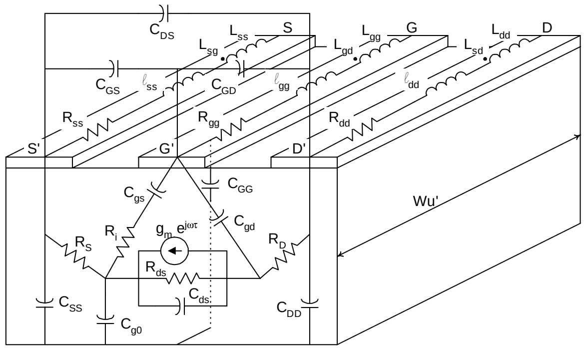

Figure 1: A section (slice) of the small-signal distributed FET model

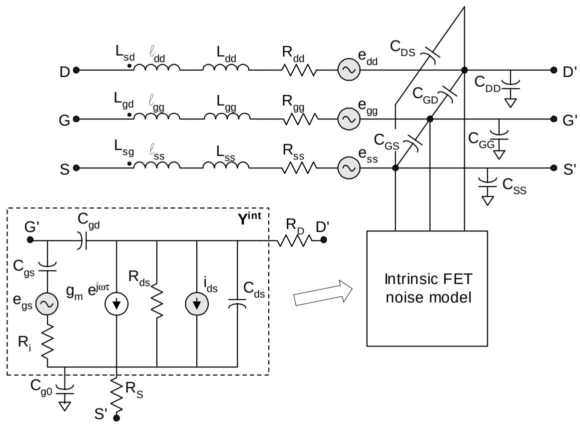

Figure 2: Schematic of an elementary FET section including the associated noise sources

configuration (e_{gs} - i_{ds}) [6] is selected for the intrinsic FET noise sources because its noise matrix ($\mathbf{C^{int}}$) is basically frequency independent. Also, the thermal noise contribution of the passive elements (R_D, R_S, R_{gg}, R_{dd}, R_{ss}) is considered.

The FET total noise-matrix in its ABCD configuration, $\mathbf{C_{AT}}$, is written in terms of the elemental section noise correlation matrix, $\mathbf{C^{int}}$:

$$\mathbf{C_{AT}} = \mathbf{C_{ext}} + \sum_{n=1}^{N} \mathbf{M}_n \cdot \mathbf{C_Y^{GD}} \cdot (\mathbf{M}_n)^+ + \sum_{n=1}^{N} \mathbf{M}_n \cdot \mathbf{P_M} \cdot \mathbf{C^{int}} \cdot (\mathbf{M}_n \cdot \mathbf{P_M})^+ \quad (1)$$

where $\mathbf{C_{ext}}$ is the correlation matrix for the contributions of electrodes and pads, $\mathbf{C_Y^{GD}}$ is the admittance noise-matrix of the gate-to-drain passive port (element C_{gd} in Figure 2), $\mathbf{P_M}$ is the transformation matrix from the selected intrinsic noise source configuration (in this paper, hybrid) to the admittance configuration, $\mathbf{M_n}$ is a transformation matrix for the slice *n,* which is a function of the equivalent circuit elements only, and the last summation term in (1) is the contribution of all the individual slices ($\mathbf{C^{int}}$) to the total noise. $\mathbf{C_{ext}}$ and $\mathbf{M_n}$ are matrices computed from the analysis developed to embed FET total noise-matrix in ABCD configuration ($\mathbf{C_{AT}}$) from the elemental intrinsic correlation matrix ($\mathbf{C^{int}}$) to the pad contributions [7]. Note that $\mathbf{C_{ext}}$, $\mathbf{C_Y^{GD}}$, $\mathbf{P_M}$ and $\mathbf{M_n}$ are functions of the FET S-parameters and the room temperature only. From $\mathbf{C_{AT}}$ in (1), the FET noise parameters are readily computed using well-known formulas [7].

The FET noise figure $F(Z_S^i)$ ($=F_{50}$) [7], which is a function of the source impedance Z_S^i, is measured at N_f frequency points (where $i = 1, ..., N_f$, is the frequency index) and expressed as a function of $\mathbf{C_{AT}}$. Then, using (1) the following equation is obtained at each frequency:

$$\Delta^i = \sum_{n=1}^{N} \mathbf{Z} \cdot \mathbf{M}_n \cdot \mathbf{P_M} \cdot \mathbf{C^{int}} \cdot (\mathbf{Z} \cdot \mathbf{M}_n \cdot \mathbf{P_M})^+ \quad (2)$$

where the left-hand term Δ^i is:

$$\Delta^i = 4kT_0 \operatorname{Re}\{Z_S^i\}\left[F(Z_S^i) - 1\right] - \mathbf{Z} \cdot \mathbf{C_{ext}} \mathbf{Z}^+ - \sum_{n=1}^{N} \mathbf{Z} \cdot \mathbf{M}_n \mathbf{C_Y^{GD}} (\mathbf{Z} \cdot \mathbf{M}_n)^+ \quad (3)$$

where k is the Boltzmann constant, T_0=290K, $\mathbf{Z}=[1 \; (Z_S^i)^*]$ and the matrices in (2-3) are evaluated at each frequency. Equation system (2) can also be written as:

$$\left[\Delta^i\right] = \sum_{n=1}^{N} \left[X_1 \quad X_2 \quad X_3 \quad X_4\right]_n^i \cdot \begin{bmatrix} C_{11}^{\text{int}} \\ C_{22}^{\text{int}} \\ \operatorname{Re}\{C_{12}^{\text{int}}\} \\ \operatorname{Im}\{C_{12}^{\text{int}}\} \end{bmatrix} \quad (4)$$

where $\mathbf{X_m}$ (with $\mathbf{m}$ =1, 2, 3, 4) is a column vector, with X_m^i elements (i=1...N_f), whose are functions of Z_S^i. Therefore, if a frequency redundancy is considered, the system (4) is an over-determined linear equation system where the unknowns are C_{11}^{int}, C_{22}^{int}, Re(C_{12}^{int}), Im(C_{11}^{int}), which are the elements of the intrinsic noise correlation matrix $\mathbf{C^{int}}$ corresponding to an elemental section.

Assuming a smooth frequency dependence for the unknowns C_{11}^{int}, C_{22}^{int}, Re(C_{12}^{int}), Im(C_{11}^{int}), they are interpolated using an L-order polynomial [4]:

$$C_{ij}^{\text{int}} = \sum_{l=0}^{L} f^l a_l^{ij}, \qquad C_{ij}^{\text{int}} = C_{11}^{\text{int}},\ C_{22}^{\text{int}},\ \operatorname{Re}\{C_{12}^{\text{int}}\} \text{ and } \operatorname{Im}\{C_{12}^{\text{int}}\} \quad (5)$$

In this work, a linear approximation is considered (L=1) [4] and (4) is solved by least squares using pseudo-inverse calculation. The computed coefficients are used as initial values in an optimisation algorithm that estimates a_0^{ij} and a_1^{ij} for the best fit of the computed $F(Z_S^i)$ in (4) to the measured noise figure in (3), using a robust Huber error function.

EXPERIMENTAL RESULTS

The FET intrinsic elemental noise sources in a distributed noise model of a 0.2 μm gate-length, 4x15 μm gate-width ED02AH PHEMT from PML, biased with V_{DS}=1.5V, I_{DS}=17.4mA, were determined using the procedure proposed in this paper. The determination of the transistor equivalent circuit elements, which are needed to compute the matrices required in (2)-(3), is made by applying scaling rules to a lumped-element model (with the same topology as the intrinsic zone of an elemental section, Figure 2 (Y^{int})). The electrode capacitances and inductances are calculated from odd and even mode decomposition using the analytical method proposed in [8]. The transistor noise figure was measured with the experimental set-up described in [9].

Assuming the Pospieszalski's model where $C_{11}^{int} \approx 4 \cdot k \cdot T_a \cdot R_i$, (where T_a is the room temperature), $C_{12}^{int}=C_{21}^{int}=0$, and then, at each frequency point there is only one unknown, C_{22}^{int}, that can be obtained from (4):

$$C_{22}^i = 4k\frac{T_d}{R_{ds}} = \frac{\Delta^i - X_1^i 4kT_aR_i}{X_2^i} \quad (6)$$

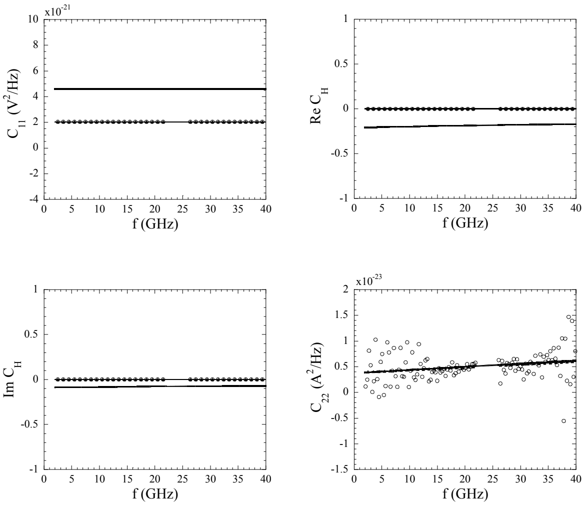

Figure 3: Individual slice PHEMT noise matrix $\boldsymbol{C}^{int}$*, from direct extraction (equation (6), Pospieszalski's model) (o), interpolating the results of the direct extraction (▲), and using the linear polynomial frequency approximation (equation (5)) (-)*

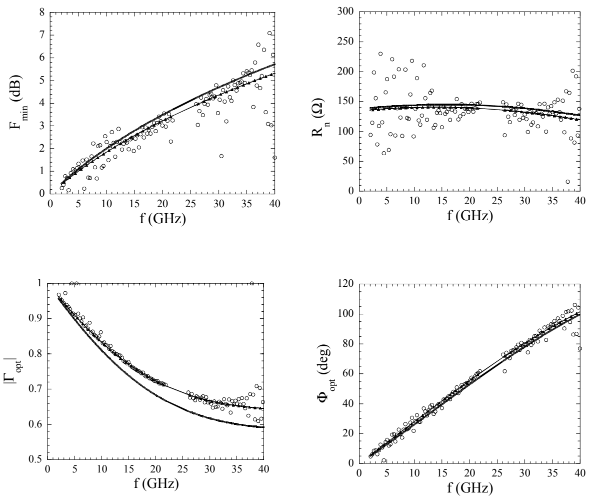

Figure 4: PHEMT noise-parameters from direct extraction of the individual noise matrix $\boldsymbol{C}^{int}$ *(equation (6), Pospieszalski's model) (o), interpolating the results of the direct extraction (▲), and using the lineal polynomial frequency approximation (equation (5)) (-)*

Figure 3 compares the extracted intrinsic correlation matrix for an elemental section of the PHEMT through direct extraction of Pospieszalski's model using (6), and those obtained after optimization using the procedure described previously (equations (4)-(5)). The direct extraction exhibits a large ripple produced by measurement errors. In fact, the elements $X_1^{\,i}$, $X_2^{\,i}$, $X_3^{\,i}$, and $X_4^{\,i}$ in (4), which depend on the FET S-parameters and the source impedance, Z_S^i, are weight coefficients for the unknowns C_{ij}^{int} which one fitted to Δ^i, computed from FET S-parameters, Z_S^i and the transistor noise figure, $F_{TRT}^{\,i}$. Thus the S-parameters and noise-power measurements uncertainties produce errors in the pseudo-inverse calculation (equations (4) and (6)); in particular at the lower frequency band, f < 10 GHz, where the transistor noise figure is very small compared to the receiver noise figure. In contrast, when the frequency redundancy, in combination with an optimization algorithm and the robust Hubert error function is used, the large measurement errors are neglected. Figure 3 also shows that the intrinsic correlation matrix elements associated to the hybrid configuration used in this paper (gate voltage and drain current sources) are nearly frequency independent and can be interpolated using a linear polynomial.

In Figures 3(b-c) the real and imaginary parts of the correlation factor, $\mathbf{C_H}$, respectively, are presented, where $\mathbf{C_H}$ is defined as:

$$C_H = \frac{C_{12}^{\text{int}}}{\sqrt{C_{11}^{\text{int}} \cdot C_{22}^{\text{int}}}} \qquad (7)$$

It can be observed that $\mathbf{C_H}$ is different from zero in contrast with the Pospieszalski's model, where it assumes $C_{12}^{int}=C_{21}^{int}=0$. This difference affects mainly to $|\Gamma_{opt}|$, in agreement with the literature [4] and, as shown in Figure 4, where the PHEMT noise parameters computed from estimated $\mathbf{C}^{\mathbf{int}}$ are presented. Also, in Figure 4 can be observed that F_{min}, R_n and Φ_{opt} obtained from the optimized intrinsic noise matrix are close to the interpolation of the results obtained from direct extraction of the Pospieszalski's model using (6). The differences between R_n and F_{min} are due to the differences in C_{11}^{int}, mainly, and C_{12}^{int}. Therefore, the method proposed here reduces the measurement errors in the determination of noise parameters.

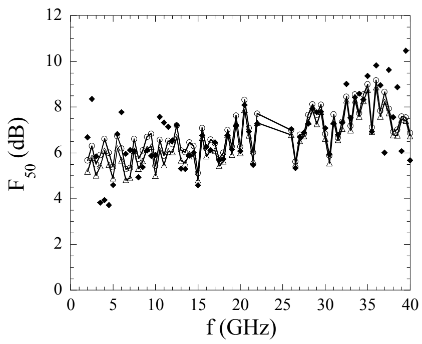

Figure 5: PHEMT noise figure (F_{50}) measurement (♦), and computed from the extracted intrinsic correlation matrix, $\boldsymbol{C}^{int}$*, using a distributed noise model (o), and a lumped noise model (Δ)*

In Figure 5 the measured noise figure (F_{50}) and the noise figure calculated from the extracted intrinsic correlation matrix using the method proposed here (which uses a distributed model), and the method described in [4] (which uses a lumped model) are compared. It can be observed that both methods give a similar fit over the entire frequency band. The measured noise figure shows a large ripple in the low frequency band because the transistor noise figure is small compared to the receiver noise figure, and thus the measurement uncertainty is larger. This effect is translated to the extracted intrinsic matrix and noise

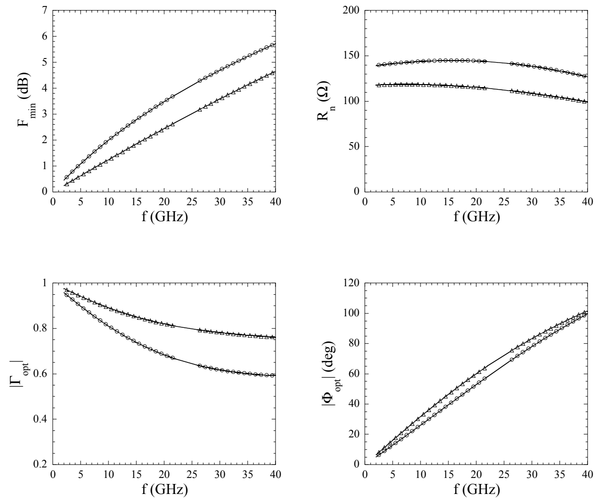

Figure 6: PHEMT noise- computed from the extracted intrinsic correlation matrix, $\boldsymbol{C}^{int}$, using a distributed noise model (o), and a lumped noise model (Δ)

parameters, as shown in Figures 3 and 4, where at the lower frequency band large deviations are observed. Figure 6 compares the four noise parameters of the PHEMT calculated using both models (lumped and distributed) showing some discrepancies which are a consequence of the different circuit model topology.

CONCLUSIONS

A method for the measurement of the four noise parameters of a FET based on the determination of its distributed noise-model from noise figure (F_{50}) measurements only, without needing a tuner, has been presented. Arbitrary intrinsic FET noise models (which include correlation) with a smooth frequency dependence can be extracted. In this paper, the hybrid configuration for the intrinsic noise matrix of an elemental section, C^{int}, has been used. The method has been applied to a PHEMT. Measured and fitted F_{50} results exhibit good agreement up to 40 GHz, very similar to that obtained with a lumped model. The transistor noise parameters computed using both models present some discrepancies, due to the difference between model topologies. Experimental results up to 40 GHz show that the elements of $\mathbf{C}^{int}$ (hybrid) have a linear frequency dependence and the proposed method reduce the uncertainty in the determination of the inner noise sources.

ACKNOWLEDGEMENT

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