

# THEORETICAL AND EXPERIMENTAL ASSESSMENT OF THE NON-LINEAR SCATTERING FUNCTIONS FOR THE CAD OF NON-LINEAR MICROWAVE CIRCUITS

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## ABSTRACT

*The Non-Linear Scattering Functions have been theoretically defined and experimentally measured for the linear-equivalent design of non-linear circuits in arbitrary large signal conditions. Non-linear measures and simulations have been compared, with good agreement. Linear CAD concepts can therefore be extended to non-linear circuits in a rigorous way.*

## I-INTRODUCTION

In previous independent works an Italian group of researchers has theoretically computed [1]-[5], and a Belgian group has experimentally measured [6]-[7], the Non-Linear Scattering Functions for non-linear circuits under arbitrary conditions. The two groups are now assessing the validity of this approach, and establishing a practical linear-equivalent design methodology for arbitrary non-linear circuits.

A Large-Signal Conversion Matrix has been first introduced within the frame of Harmonic-Balance-based analysis and design of power amplifiers [1]-[3]. By extension of linear concepts, a large-signal linear-equivalent matrix was computed as a Norton equivalent of the non-linear active device; it was used for the large-signal conjugate match of a power amplifier under non-linear operations. Obviously, the large-signal Norton equivalent depends on the amplitude of the applied signal and on the external loads; it must therefore be recomputed if these conditions change. However, its computation comes for free within a Harmonic Balance frame, because it is easily derived from the Jacobian of the algorithm. In subsequent works this approach has allowed the extension of linear concepts to non-linear circuits for the definition of a Maximum Stable Conversion Gain for active Frequency Multipliers [4] and Mixers [5] in the case of potential instability of these circuits.

An advanced measurement set-up has been independently developed for the non-linear characterisation of microwave active devices and sub-circuits [6]-[8]. The set-up allows the accurate measurement of amplitude and phase of all spectral components (currently up to 20 GHz) of incident and reflected waves at the ports of a non-linear device. The set-up is based on the Nonlinear Network Measurement System (NNMS) [7], originally developed by Hewlett-Packard (now Agilent Technologies) as an improved version of the Microwave Transition Analyser. By means of this set-up, the definition and extraction of the Non-Linear Scattering Functions has been made possible for non-linear devices under arbitrary large-signal operations. These scattering functions have been used within a commercial simulator, and their capability to model the non-linear behaviour of active devices has been demonstrated [8].

Goal of the present work is to demonstrate that the Large-Signal Conversion Matrix and the Nonlinear Scattering Functions are actually equivalent representations of an arbitrary non-linear device, and that they can be used as a general and powerful tool for the CAD of non-linear microwave circuits. This demonstration will be carried out by comparison of actual measurements and computer simulations of a medium-power MESFET from the Alenia Marconi foundry.

## II-THE MEASUREMENT SET-UP

The measurement set-up is schematically shown in fig. 1 [7]. The NNMS accurately measures the amplitude and phase of all the spectral components present in the experiment up to 20 GHz. The active device is biased by means of an HP4142B semiconductor parameter analyser. The system includes two synthesised sources: the first one generates a large input signal that drives the device well into non-linear operations; the second one generates a small perturbing signal at all frequencies of interest (one at a time), i.e. the incident spectral components that cause the insurgence of reflected signals coming out of the non-linear device at all harmonically-related frequencies. This perturbing signal is made small enough to allow linearisation of the system.

As can be seen in fig. 1, a tuner allows proper large-signal matching at output of the active device at fundamental frequency. The embedding impedances seen by the device at all other harmonics are  $Z_0$  in our case; however, additional diplexers and tuners would allow in principle any harmonic loading, the only limitation being the complexity of the set-up and cost of the equipment. The perturbing signal is applied to both ports of the device, one at a time and frequency by frequency, by means of switch and directional couplers. All reflected waves at both ports and at all harmonic frequencies are detected by means of the same couplers, and are simultaneously and accurately measured by the NNMS. The incident and reflected waves are used to compute the Non-Linear Scattering Functions, that are defined as the (complex) coefficients of the linearised dependence of the reflected (outcoming) waves on the incident (incoming) waves [7]-[8]:

$$b_{k,j} = b_{k,j}^{(0)} + \sum_{j=1,2} \sum_{l=1,N} (G_{k,j,l,j} \cdot \text{Re}[a_{l,j}] + H_{k,j,l,j} \cdot \text{Im}[a_{l,j}]) \quad (1)$$

$$k = 1, \dots, N \quad i = 1, 2 \quad l = 1, \dots, N \quad j = 1, 2$$

where  $k$  and  $l$  are the indices related to the harmonic frequency, and  $i$  and  $j$  are the indices of the ports of incident and reflected waves respectively. The  $G$ 's and  $H$ 's, i.e. the (complex) Non-Linear Scattering Functions, obviously depend on the bias point, as the standard Scattering Parameters, but also on the large-signal operation state of the device, i.e. on the incident power and on the embedding loads. If these change substantially, then the scattering functions must be measured again.

The dependence of the reflected waves is modelled through two complex sets of numbers ( $G_{k,j,l,j}$  and  $H_{k,i,l,j}$ ) instead of a single set as in the case of the standard Scattering Parameters ( $S_{ij}$ ), because of the non-linear operation state of the device. In the limit of small-signal fundamental-frequency driving of the device, the two sets can be reduced to one, and in particular:

$$\begin{aligned} \text{Re}[G_{n,i,n,j}] &= \text{Im}[G_{n,i,n,j}] = \text{Re}[S_{i,j}(n\omega)] \\ -\text{Re}[H_{n,i,n,j}] &= \text{Im}[H_{n,i,n,j}] = \text{Im}[S_{i,j}(n\omega)] \end{aligned} \quad (2)$$

The determination of each scattering function requires in principle a complex measurement; two different perturbing signals must therefore be applied to determine both  $G$  and  $H$ , at least in principle. The ideal case would be to apply a real and an imaginary perturbing signals one after the other; however practical considerations suggest that many different perturbing signals (incident waves) at many random phases be applied, and that the scattering functions be found by means of a least-square fit to the redundant measured data.

In this experiment the perturbing signal is applied at fundamental frequency and at all its harmonics; in the case that the perturbing signal has a different frequency, and that the signal is small enough, the standard conversion matrix results from the measurement. The main difference with the conversion matrix lies in the fact that the perturbation is superimposed on an already applied large signal, either at fundamental or at another arbitrary frequency, thus allowing the conversion-matrix characterisation also of power amplifiers, frequency multipliers, and mixers with arbitrary input amplitude.

The scattering functions are now compared to their computer-simulated counterparts, as described in the following paragraph.

### III - THE COMPUTER SIMULATIONS

The active device, a 0.5  $\mu\text{m}$  medium power MESFET with 1mm total gate periphery from the Alenia Marconi foundry, is represented by its non-linear model, extracted from DC and standard S-parameter measurements at all bias points of interest, and including current and energy conservation constraints [9]. The model is implemented in the HP-ADS simulator as a Symbolic Defined Device.

The input signal at fundamental frequency with proper amplitude and the embedding impedances at all frequencies as in the experiment are reproduced in the computer simulation. A Harmonic-Balance analysis of the circuit yields the large-signal operation state of the device.

A perturbation of this large-signal operation state at all harmonic frequencies (including the fundamental) must now be introduced, as in the experiment. In fact, this perturbation is usually introduced during the Harmonic-Balance analysis, when the Jacobian matrix of the system is numerically computed; the Jacobian matrix is then used to numerically solve the Harmonic-Balance system by means of a Newton-Raphson iteration scheme. In the case of the Jacobian, the voltages are usually perturbed, and the currents are detected as the output of the system: Non-Linear Admittance Functions are therefore resulting for the non-linear sub-circuit, but the derivation of the corresponding Scattering Functions is straightforward.

The requirement of two complex numbers instead of only one for the modelling the linearised dependence of the outgoing signal on the incoming signal finds its counterpart in the well-know non-analytical nature of the Jacobian matrix. In fact the latter is usually computed from independent real- and imaginary-part perturbations of the port voltages, instead of a single complex perturbation, as would be the case for standard linear Scattering Parameters.

The Jacobian matrix is normally not available to users of commercial non-linear CAD programs. In previous works [1]-[5] an in-house, special-purpose program (PAOMAC) had been developed and extensively used, where the Jacobian-based Large-Signal Conversion Matrix was available; in this work the commercial HP-ADS simulator has been used, in order to assess a generally available procedure. An automatic procedure for the computation of the Jacobian matrix of the non-linear device has been developed and implemented, that yields the Non-Linear Admittance Functions; the Non-Linear Scattering Functions are easily deduced.

#### IV - COMPARISONS

A comparison has been performed between the measured and simulated data for a medium-power MESFET, biased at  $V_{ds}=5V$ ,  $V_{gs}=-1.5V$  and  $I_{ds}=60mA$  for class-A amplification. The transistor is driven from small-signal into gain compression by a 2 GHz fundamental-frequency signal at input. Both measurements and simulations obviously yield the small-signal standard S-parameters, measured by means of an HP8510C Vector Network Analyser, when driven by a small excitation signal (-10 dBm input power). As the input power increases toward compression, both simulations and measurements show a splitting of the  $G$  and  $H$  matrices, illustrating the non-analytical nature of the Jacobian. Also, at high compression the frequency-conversion terms arise, that are zero for small-signal excitation; these terms are also splitted into the two  $G$  and  $H$  terms.

In fig.2 the  $G_{2,2,2,1}$  and  $H_{2,2,2,1}$  terms (the linearised equivalents of the small-signal  $S_{2,1}$  at second harmonic frequency) are plotted on a polar plot for several input powers from small-signal to 2-dB gain compression, i.e. from -20 dBm to 12.5 dBm input power. The square represents the small-signal  $S_{2,1}$  parameter at  $2\omega$  also plotted for comparison; x-marks and circles are the measured  $G_{2,2,2,1}$  and  $H_{2,2,2,1}$  parameters, while stars and diamonds represent the simulated  $G_{2,2,2,1}$  and  $H_{2,2,2,1}$  parameters. In spite of the fact that the model is a general-purpose one, not optimised for this purpose, the agreement is remarkable. The gain compression at second harmonic frequency is clear as the transistor is driven into the non-linear region by the fundamental-frequency input signal.

In fig.3 the  $G_{4,2,2,2}$  and  $H_{4,2,2,2}$  terms, converting the 2-GHz incident wave at output into the 4-GHz reflected wave at output, are plotted on a polar plot, again for several input powers from small-signal to 2dB gain compression, i.e. from -20 dBm to 12.5 dBm input power, with the same symbol conventions. In this case the small-signal parameter is obviously zero. It is clear from the plot that the 2-to-4 GHz conversion at the output of the transistor rises from zero at small-signal to a non-negligible value at 2-dB gain compression, with a remarkable agreement between simulated and measured data. The splitting of the  $G$ 's and the  $H$ 's in the non-linear regime is also clearly shown.

In fig.4 the measured drain (output) voltage and current waveforms are plotted at 2dB gain compression, when a fifth-harmonic small perturbing signal is injected at the output port. As illustrated before, it is from this type of small perturbations that the linearised response is computed.

#### V - CONCLUSIONS

A comparison between simulated and measured data has shown that the Non-Linear Scattering Functions are a reliable and flexible representation, available to the general non-linear CAD user for the design of non-linear circuits. The Functions can be either measured by means of advanced hardware, or simulated by means of standard CAD software and advanced non-linear models. This approach allows the extension of linear concepts to non-linear circuits in a rigorous and straightforward way.

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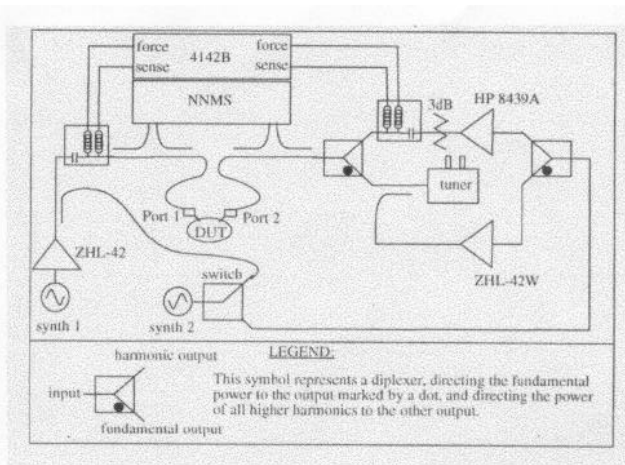


Fig. 1. The measurement set-up

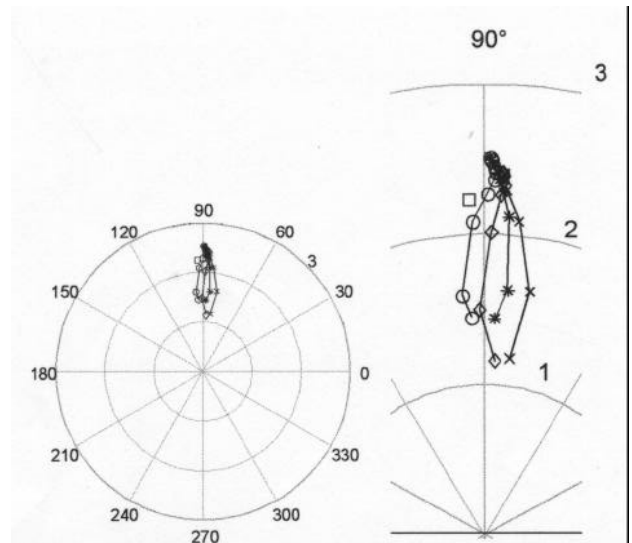


Fig. 2. Measured  $G_{2,2,2,1}$  and  $H_{2,2,2,1}$  parameters (x-marks and circles respectively) and simulated  $G_{2,2,2,1}$  and  $H_{2,2,2,1}$  parameters (stars and diamonds respectively) for several input powers from small signal to 2-dB gain compression (-20 to 13.5 dBm input power) at 4 GHz. The measured small-signal  $S_{2,1}$  at 4 GHz is also shown (square). To the right, the expanded detail.

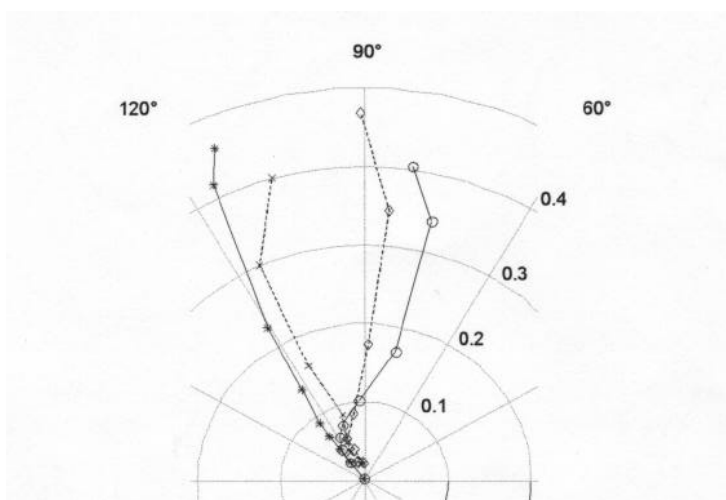


Fig. 3. Measured  $G_{4,2,2,2}$  and  $H_{4,2,2,2}$  parameters (x-marks and circles respectively) and simulated  $G_{4,2,2,2}$  and  $H_{4,2,2,2}$  parameters (stars signs and diamonds respectively) converting the 2-GHz incident wave at output into the 4-GHz reflected wave at output for several input powers from small signal to 2-dB gain compression (-20 to 13.5 dBm input power) at 2 GHz.

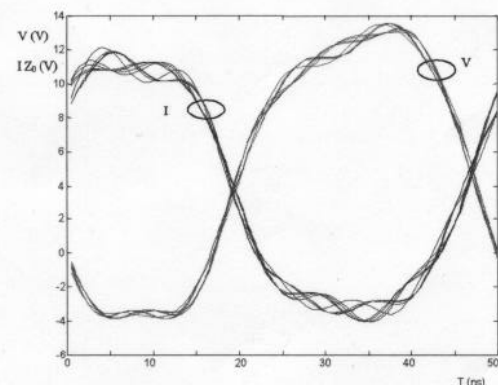


Fig. 4. Measured drain voltage and current waveforms at 2-dB gain compression; a fifth-harmonic small perturbing signal is superimposed to the large-signal waveform relative to the large input signal at fundamental frequency.