Stochastic Dual Dynamic Programming for Operation of DER Aggregators under Multidimensional Uncertainty

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Abstract--The operation of aggregators of distributed energy resources (DER) is highly complex, since it entails the optimal coordination of a diverse portfolio of DER under multiple sources of uncertainty. The large number of possible stochastic realizations that arise, can lead to complex operational models that become problematic in real-time market environments. Previous stochastic programming approaches resort to two-stage uncertainty models and scenario reduction techniques to preserve the tractability of the problem. However, two-stage models cannot fully capture the evolution of uncertain processes and the a priori scenario selection can lead to suboptimal decisions. In this context, this paper develops a novel stochastic dual dynamic programming (SDDP) approach which does not require discretization of either the state space or the uncertain variables and can be efficiently applied to a multi-stage uncertainty model. Temporal dependencies of the uncertain variables as well as dependencies among different uncertain variables can be captured through the integration of any linear multidimensional stochastic model, and it is showcased for a porder vector autoregressive (VAR) model. The proposed approach is compared against a traditional scenario-tree-based approach through a Monte-Carlo validation process, and is demonstrated to achieve a better trade-off between solution efficiency and computational effort.

Index Terms-- Aggregator, distributed energy resources, multidimensional uncertainty, stochastic dual dynamic programming, vector autoregressive modeling.

I. NOMENCLATURE

A. Indexes

- *t* Index of time periods, running from 1 to *T*.
- *b* Index of energy storage (ES) units, running from 1 to *B*.
- f Index of flexible loads (FL), running from 1 to F.
- *w* Index of wind turbines (WT), running from 1 to *W*.
- *m* Index of micro-generators, running from 1 to *M*.

B. Parameters

C_t^{grid}	Energy market price at period t.
C_m^{gen}	Operating cost of micro-generator m .
C^{dem}	Cost of demand shedding.

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$E_{0,b}$	Initial energy level of ES unit <i>b</i> .
η_h	Round-trip efficiency of ES unit b.
E_{h}^{max}	Maximum energy level of ES unit <i>b</i> .
E_{h}^{min}	Minimum energy level of ES unit <i>b</i> .
P_{h}^{s}	Maximum power rating of ES unit <i>b</i> .
$D_{t,f}^{base}$	Baseline demand of FL f at period t .
S_f	Load shifting limit of FL f at period t .
$P_t^{g,sell}$	Maximum power sold to the market at period t .
$P_t^{g,buy}$	Maximum power bought from the market at period
	t.
P_m^{gen}	Maximum power rating of micro-generator <i>m</i> .
$P_{t,w}^{wind}$	Available wind power output of WT <i>w</i> at period <i>t</i> .
$P_{t-n,w}^{wind}$	Available wind power output of WT w , n periods
	before <i>t</i> .
W_w^{max}	Power capacity of WT w.
$\varphi_{n,w}$	n^{th} autoregressive coefficient associated with
	power output of WT w.
$\mathcal{E}_{t,w}$	White noise term associated with power output of
	WT w at period t .
d_t^{inf}	Inflexible demand at period <i>t</i> .
Δt	Temporal resolution of the market.
C. Varia	bles
p_t^{grid}	Power sold to (positive)/bought from (negative) the
	market at period t.
$p_{t.m}^{gen}$	Power output of micro-generator m at period t .
p_t^{dem}	Demand shed at period t .
$e_{t,b}$	Energy level of \vec{ES} unit b at period t.
$p_{t,h}^s$	Power input (positive) / output (negative) of ES unit
1 0,0	b at period t.
$d_{t,f}^{sh}$	Change of demand of FL f at period t due to load
	shifting.
$d_{t,f}^{total}$	Demand of FL f at period t after load shifting.
p_{tw}^{wind}	Dispatched wind power output of WT w at period t.

II. INTRODUCTION

A. Background

A FUNDAMENTAL feature of the emerging *Smart Grid* paradigm involves the integration of a large number of *distributed energy resources* (DER), such as flexible loads, renewable and controllable micro-generators and energy

storage units, in order to support the economic operation of the future low-carbon power system [1]-[2]. However, the large number, small individual size and inherent stochasticity characterizing these DER have complicated system scheduling and market coordination. Furthermore, driven by the wide integration of renewable generation in power systems, there is a general consensus to move energy trading as close as possible to real-time [3]-[4], which intensifies the complexity of DER coordination.

These challenges have triggered the introduction of DER aggregators, which group a substantial number of DER (not necessarily located in the same local area or connected through the same distribution network) and represent them in the market, by coordinating their operation according to market opportunities and their individual preferences and requirements [1], [5]. Nevertheless, the operation of a DER aggregator constitutes a challenging high-dimensional stochastic problem, since it entails the optimal real-time coordination of a large number of diverse DER, subject to numerous sources of uncertainty, such as the output of renewable micro-generators and consumers' demand and flexibility patterns.

Existing literature modeling the optimal operation of DER aggregators has mainly focused on stochastic programming (SP) approaches, employing two-stage uncertainty models [6]-[14]. A simplified representation of the uncertainty space is usually adopted, utilizing various scenario generation and reduction techniques. In [6], a risk-constrained two-stage stochastic formulation is presented, where stochasticity is captured through a Monte-Carlo scenario based approach and uncertain parameters are assumed to follow normal zero-mean distributions. Ignoring potential dependencies among these uncertainties, 5000 scenarios are initially sampled and are then reduced to 100 for computational tractability purposes. Authors in [7] model uncertainties surrounding load and renewable power output using univariate normal distributions, fit to historical data. In [8]-[9], a fixed number of scenarios is generated via Monte-Carlo simulation to model net wind output and day-ahead (DA) price uncertainty. In [10], a simplistic uniform distribution is assumed to capture the uncertainty associated with the demand of flexible loads. Authors in [11]-[12] develop scenarios describing electric vehicle (EV) uncertain characteristics. In both cases a very limited number of scenarios (100 and 185 respectively) are retained for a 24h scheduling horizon. A wide variety of uncertain variables, including solar irradiance, wind speed, inflexible demand and the availability of distributed generation (DG), energy storage units and the main grid are explored in [13]-[14]. In both cases, even though a large number of scenarios is initially sampled, only a small number is preserved after the scenario reduction procedure, resulting in a coarse coverage of the uncertainty space.

B. Motivation

These previous SP approaches employ two-stage uncertainty models that exhibit fundamental limitations in capturing the evolution of stochastic processes in a real-time

market environment. More specifically, the typical two-stage stochastic problem formulation consists of (i) a first-stage problem to identify the optimal day-ahead schedule of resources, carried out under uncertainty (ii) multiple secondstage problems, where each problem corresponds to one possible uncertainty realization and is tasked with identifying the optimal schedule re-adjustment given the first-stage commitment and that uncertainty realization; note that the second stage assumes perfect information. It is critical to highlight that this problem structure uses a very simplistic uncertainty description; it is assumed that following the submission of the day-ahead schedule, the uncertainty for the next 24 hours is automatically revealed and the schedule can be fully re-optimized under perfect information. However, in practice, uncertainty is revealed only gradually (i.e. only for the current hour). This necessitates the movement from a twostage scenario fan to a multi-stage scenario tree description capable of capturing inter-temporal resolution. As such, typical two-stage problem formulations severely underestimate the effect of uncertainty. Despite their shortcomings, the existing literature has largely focused on such two-stage stochastic programming approaches due to their simplicity. More sophisticated uncertainty descriptions using multi-stage scenario-trees entail a combinatorial explosion of the possible realizations, possibly leading to intractability.

For the same reasons, the existing literature also largely disregards multivariate dependencies among the uncertain variables. However, simplistic descriptions of uncertainty can lead to inefficient solutions since they disregard parts of the uncertainty structure which can be leveraged to make more informed decisions. For example, temporal dependencies may exist between the current state and future evolution of an uncertain variable. In addition, different uncertainty sources may not be independent since relationships exist between different uncertain variables, due to confounding factors such as weather conditions. In [15] for example, the authors demonstrate that ignoring the stochastic dependence characterizing the multivariate uncertainty around wind farms' output, can lead to suboptimal planning and operation decisions. In a similar vein, authors in [16] present numerous composite modeling approaches for capturing complex nonlinear dependency patterns in large power system datasets. Finally, in [17] a multivariate conditional parametric model is proposed for forecasting power output from multiple wind farms, while modeling their spatio-temporal dependence; authors show substantial improvement in forecast quality when the dependence structure is explicitly modeled.

These insights reveal fundamental limitations of the approaches adopted in [6]-[14], in dealing effectively with the high-dimensional stochastic problem faced by the DER aggregator. When such approaches consider multivariate uncertainty in a multistage context, a combinatorial explosion of the possible realizations is expected as the planning horizon expands, and the problem soon becomes intractable. Consequently, the deployment of scenario reduction techniques is unavoidable. In general, such techniques

determine the final set of considered scenarios 'a priori' (i.e. before the actual problem is solved), according to relevant distance metrics. Therefore, the impact of the stochastic process on the problem is not explicitly evaluated and potentially important scenarios may be ignored. All in all, the requirement for computational tractability imposes limits on the number of scenarios that can be considered, inadvertently leading to disregarding temporal and multivariate dependencies.

C. Contributions

In order to overcome the computational tractability problems associated with the incorporation of multi-stage uncertainty and multivariate dependence effects in the DER aggregator operation problem, this paper will be based on *stochastic dual dynamic programming (SDDP)*. This method was firstly introduced in [18]-[19] for the optimal scheduling of hydrothermal generation systems, driven by the need to model the reservoir interconnections for the future inflow sequences. SDDP has been used to model a variety of operational problems [20]. The ability of SDDP to refine solution quality around areas of the state space most likely to occur ('areas of interest') instead of searching the entire state space, facilitates the solution of high dimensional problems.

The fundamental contribution of this paper lies on developing a novel SDDP approach for the optimal operation of a DER aggregator in real-time markets, which can be efficiently applied to a multi-stage uncertainty model and can handle both time-dependent and multivariate uncertainty, through the integration of a vector autoregressive (VAR) model of order p. The traditional SDDP algorithm is extended to account for potential temporal and cross-variable correlations of the stochastic process. This is the first work that employs VAR models for the representation of multivariate uncertainty, driven by their ability to associate current realizations of each of the uncertain variables with the previous p instances of the entire set of uncertain variables. In this way, both temporal dependencies of uncertain variables and dependencies among different uncertain variables are captured. The stochastic process is integrated in the SDDP algorithm and constitutes an internal part of the solution process, explicitly considering stochastic dependencies.

The proposed SDDP approach is compared against a traditional scenario-tree-based approach with varying tree complexities, in a case study involving an aggregator with diverse generation, demand and storage resources, and facing uncertainty regarding the level of demand to be served and the available wind power output. In order to meaningfully and comprehensively compare the two approaches, Monte-Carlo validation for different demand and wind power starting points is carried out. The proposed approach is demonstrated to achieve a better trade-off between solution efficiency and computational performance, since it yields a similar aggregator's expected cost with the one achieved by the most complex scenario trees, while it exhibits a similar computational performance with the simplest scenario trees. The computational superiority of the proposed approach

becomes more significant when longer operating horizons are investigated, further demonstrating the scalability potential of the extended SDDP algorithm to large-scale problems. Finally, the proposed approach is demonstrated to yield better solutions compared to the traditional SDDP framework, which does not consider temporal and cross-variable dependencies of the stochastic process. The proposed framework is bound to increase in importance in the future as the number of controllable elements and sources of uncertainties increase, rendering operational problems even more computationally challenging.

D. Paper Structure

The rest of this paper is organized as follows: Section III describes and formulates the DER aggregator's optimal operation problem. The basic principles of the SDDP algorithm are outlined in Section IV and its novel extension for incorporating a multivariate stochastic model is detailed in Section V. The application of the developed model on the examined DER aggregator problem and illustrative results are presented in Section VI. Finally, Section VII discusses conclusions of this work.

III. PROBLEM DEFINITION

In this section, we formulate the multistage SP problem of the optimal operation of a DER aggregator participating in a real-time market and facing uncertainty regarding the level of inflexible demand to be served and the available wind power output. We assume that the aggregator portfolio consists of a number of WTs with uncertain power output, controllable micro-generators, uncertain inflexible demand, a group of flexible loads and energy storage units. Therefore, the aggregator must schedule the use of the available wind power output, the amount of energy that will be bought/sold from/to the market, the output of the micro-generators, the flexible loads' consumption levels and the storage units' charging/discharging schedule at each time period t, in order to minimize the overall cost. Wind output and demand can be curtailed, if required. Fundamentally, this is a multistage stochastic problem with recourse, where the stochastic outputs are gradually revealed and decisions are made considering both the already observed and the anticipated outputs.

It is assumed that all the stochastic variables are defined in a common and complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$. Then, ω_t corresponds to the random variable at period t and $\omega_t = (d_t^{inf}, P_{t,w}^{wind}) \in \Omega_t$ represents the realization of the stochastic process ω at time t, where d_t^{inf} and $P_{t,w}^{wind}$ correspond to the inflexible demand and wind realizations at t, respectively.

We first present the mathematical formulation pertaining to a single stage t. Equation (1) is the objective function at period t and comprises of the cost of the energy transactions (buying or selling) with the market, the cost of using the micro-generator and the demand shedding cost. In (1), x_t corresponds to the decision variables vector and is defined as $x_t \triangleq [p_t^{grid}, p_{t,m}^{gen}, p_t^{dem}, p_{t,b}^s, d_{t,f}^{sh}, p_{t,w}^{wind}]$ where $x_t \triangleq x_t(\omega_t)$, but ω_t has been dropped for notational convenience. Decisions are taken at the beginning of each period t, where the uncertainty for the current period has been resolved. This is a fundamental difference compared to scenario tree approaches, since decisions are made sequentially, as the stochastic process has been realized.

$$c_t^{\mathrm{T}} \cdot x_t = -C_t^{grid} \cdot p_t^{grid} + \sum_m C_m^{gen} \cdot p_{t,m}^{gen} + C^{dem} \cdot p_t^{dem} \quad \forall t \quad (1)$$

The operational characteristics of energy storage unit b are expressed by (2)-(5) [21], [22]. Constraint (2) expresses the energy balance in the storage unit. Constraint (3) corresponds to its maximum depth of discharge and state of charge ratings. Constraint (4) represents its power limits. In order to maintain energy neutrality, the storage energy content at the start and the end of the operating horizon are assumed equal (5).

$$e_{t,b} = \begin{cases} E_{0,b} \cdot \eta_b + p_{t,b}^s \cdot \Delta t, & \text{if } t = 1\\ e_{t-1,b} \cdot \eta_b + p_{t,b}^s \cdot \Delta t, & \text{if } 1 < t \le T \end{cases} \quad \forall b \qquad (2)$$

$$E_{b}^{min} \le e_{t,b} \le E_{b}^{max} \qquad \forall t,b \quad (3)$$

$$-P_{b}^{s} \le p_{t\,b}^{s} \le P_{b}^{s} \qquad \forall t, b \quad (4)$$

$$e_{T,b} = E_{0,b} \qquad \qquad \forall b \qquad (5)$$

The flexibility of the demand side is primarily associated with the ability to shift the operation of some loads from periods of higher prices to periods of lower prices [23]. In other words, load reduction during certain periods is accompanied by a load recovery effect during preceding or succeeding periods. This implies that the representation of demand flexibility requires the consideration of time-coupling operational characteristics. In this paper, the generic, technology-agnostic model of [24] has been employed for the representation of this time-shifting flexibility, expressed by (6)-(8). The variable $d_{f,t}^{sh}$ represents the change of demand with respect to the baseline level $D_{t,f}^{base}$ at period t due to load shifting, taking negative/positive values when demand is moved away from/towards t. Constraint (7) expresses the limits of demand change at each period due to load shifting as a ratio S_f ($0 \le S_f \le 1$) of the baseline demand; $S_f = 0$ implies that load f does not exhibit any time-shifting flexibility, while $S_f = 1$ implies that the whole demand can be shifted in time. Finally, constraint (8) ensures that load shifting is energy neutral within the operation horizon i.e. the total size of demand reductions is equal to the total size of demand increases (load recovery), assuming without loss of generality that load shifting does not involve energy losses.

$$d_{t,f}^{total} = D_{t,f}^{base} + d_{t,f}^{sh} \qquad \forall t, f \tag{6}$$

$$-S_f \cdot D_{t,f}^{base} \le d_{t,f}^{sh} \le S_f \cdot D_{t,f}^{base} \qquad \forall t, f$$
(7)

$$\sum_{t} d_{t,f}^{sh} = 0 \qquad \qquad \forall f \qquad (8)$$

Moreover, lower and upper limits of the power associated with the wind turbines, the market and the micro-generators are expressed in (9)-(11) respectively, and the portfolio power balance is ensured by (12).

$$p_{t,w}^{wind} \le P_{t,w}^{wind} \qquad \forall t, w \qquad (9)$$

$$-P_t^{g,buy} \le p_t^{grid} \le P_t^{g,sell} \qquad \forall t \tag{10}$$

$$p_{t,m}^{gen} \le P_m^{gen} \qquad \forall t,m \tag{11}$$

$$\sum_{b} p_{t,b}^{s} - \sum_{w} p_{t,w}^{wind} + p_{t}^{grid} + \sum_{f} d_{t,f}^{total} + d_{t}^{inf} - p_{t,m}^{gen} = 0 \qquad \forall t \qquad (12)$$

Finally, the multistage SP is formulated as in (13), where C is the total operating cost, $\omega_t | \omega_{t-1}$ corresponds to the probability of ω_t conditioned on ω_{t-1} and $\mathbb{E}_{\omega}[\cdot]$ denotes the expectation of the terms in the bracket over the stochastic process ω . Equation (13) clearly demonstrates the recursive nature of the problem, where decisions are made sequentially as the stochastic process unfolds and more information becomes available to the aggregator. This structure is perfectly aligned with the concept of SDDP that follows.

$$C = \min_{x_1} \left[c_1^{\mathrm{T}} \cdot x_1 + \mathbb{E}_{\omega_2} \left[\min_{x_2} c_2^{\mathrm{T}} \cdot x_2 + \mathbb{E}_{\omega_3 \mid \omega_2} \left[\min_{x_2} c_3^{\mathrm{T}} \cdot x_3 + \mathbb{E}_{\omega_T \mid \omega_{T-1}} \left[\min_{x_T} c_T^{\mathrm{T}} \cdot x_T \right] \right] \right] \right]$$
s.t. (1) - (12)

IV. STOCHASTIC DUAL DYNAMIC PROGRAMMING

The key principle of SDDP is that the original multistage stochastic problem can be decomposed into a series of singlestage master problems and sub-problems, with appropriate dual variables employed for their coordination. At each stage t, the corresponding master problem M_t captures the immediate costs (i.e. for the current stage), along with the approximation of the costs for the remaining stages of the operating horizon, which are incurred as a consequence of the current stage decisions. When M_t is solved, the decisions pertaining to stage t are optimized. Then, the respective subproblem S_{t+1} optimizes the *future costs* incurred by the aggregator at the remaining stages of the operating horizon (i.e. stages t + 1 to T), for given stage t decisions.

Thus, the problem expressed in (13) is now reformulated as in (14), where M_1 represents the first-stage master problem and \mathcal{M}_1 corresponds to the optimal costs for the entire operating horizon, which are incurred by the immediate first stage decisions. Then, $S_2(x_1, \omega_2)$ represents the respective optimal value for the sub-problem S_2 related to M_1 , where the future cost for horizon $t' = \{2, ..., T\}$ is minimized, given the optimal decisions x_1 of M_1 for realization $\omega_2 \in \Omega_2$. The realizations ω_t are derived either from scenario tree structures, in case there is a finite number of scenarios, or by sampling an appropriate forecasting model (e.g. ARIMA models), when there is a continuous distribution [25].

$$\mathcal{M}_{1}: \quad \begin{array}{l} \mathcal{C} = \mathcal{M}_{1} = \min_{x_{1}} \left[c_{1}^{T} x_{1} + \mathbb{E}_{\omega_{2}} [\mathcal{S}_{2}(x_{1}, \omega_{2})] \right] \\ \text{s.t.} \quad (1) - (12) \end{array}$$
(14)

The same decomposition process is repeated for every stage of the operating horizon, until t master problems and t-1sub-problems (no sub-problem pertains to stage t) have emerged. The problem formulation for the subsequent stages is illustrated in (15), which constitutes a generalization of (14) and symbols are used in the same context. It is assumed that the terminal cost $S_{T+1}(x_T, \omega_{T+1}) = 0$, but it could be any convex function representing future costs after *T*.

$$\mathcal{M}_{t}(x_{t-1},\omega_{t}) = \min_{x_{1}} \left[c_{t}^{\mathrm{T}} x_{t} + \mathbb{E}_{\boldsymbol{\omega}_{t+1}|\boldsymbol{\omega}_{t}} [\mathcal{S}_{t+1}(x_{t},\omega_{t+1})] \right]$$
s.t.
(15)

In essence, the algorithm determines the optimal decision set x_t for each t by building a piece-wise linear outer approximation of $\mathbb{E}_{\omega_{t+1}|\omega_t}[S_{t+1}(x_t, \omega_{t+1})]$ and identifying an approximate future cost function $\mathfrak{S}_{t+1}(x_t, \omega_{t+1})$ of \mathcal{S}_{t+1} at each stage t [26]. Therefore, a set of linear constraints ('Benders' cuts') is gradually built and appended to each master problem M_t and the expectation term in (14) - (15) is replaced by the terms a_2 and a_{t+1} respectively, which represent the future cost for stages 2 to T and t + 1 to T, respectively. The future cost terms are constrained by the inequalities given in (16) for the aggregator problem. Each linear constraint in (16) is expressed in terms of the change in the sub-problem's optimal objective function value, with respect to the master problem's state variables, which are defined as $s_t \triangleq \{e_{t,b}, d_{t,f}^{agg}\}$. The state variables s_t constitute the coupling terms between M_t and S_t , and the impact of changes in their values on S_{t+1} is captured via the respective dual variables, $\lambda \triangleq \{\lambda_{t,b,k,i}^E, \lambda_{t,f,k,i}^D\}$ for iteration *i*. For the DER aggregator problem, the coupling elements comprise of the energy level of each energy storage unit $e_{t,b}$ and the

aggregate demand shifting $d_{t,f}^{agg}$ at stage t.

$$\alpha_{t+1} \geq S_{t+1,k}^{i} \left(e_{t,b,k,i}, d_{t,f,k,i}^{agg} \right) + \sum_{b} \lambda_{t,b,k,i}^{E} \cdot \left(e_{t,b} - e_{t,b,k,i} \right) + \sum_{f} \lambda_{t,f,k,i}^{D} \cdot \left(d_{t,f}^{agg} - d_{t,f,k,i}^{agg} \right)$$
 $\forall k, i$ (16)

The approximation of the future cost functions \mathfrak{S}_{t+1} is obtained through an iterative process which involves the successive solution of all M_t (forward pass) and then all S_{t+1} (backward pass). The basic steps of the solution process of the algorithm are illustrated in Fig. 1. The main purpose of the forward pass is to drive the solution process towards 'interesting' areas of the state space, which are highly likely to occur. Therefore, during the forward pass calculation, at each iteration i, 'areas' of the state space that have substantial impact on the objective function $s_{t,k,i}$ are identified and stored for use during the backward pass calculation when the approximation of S_{t+1} is constructed. By identifying and focusing solution search around such areas of interest, naïve discretization of the state space (which is common in dynamic programming approaches) is avoided and significant computational savings are achieved. Each run of the forward pass involves the sampling of different values of the stochastic variables $\omega_{t,k,i}$. However, multiple runs of the forward pass $k = \{1, ..., NK\}$ pertaining at the same stage t can be performed simultaneously at each iteration *i*. The parallelization of the solution process introduces further computational benefits.



Fig. 1. Illustrative flowchart of the solution process of the SDDP algorithm.

During the backward pass, the approximation of the future cost functions \mathfrak{S}_{t+1} is gradually improved for $t = \{1, ..., T-1\}$ at each iteration with Bender's cuts being constructed for the areas of the state space $s_{t,k,i}$ identified during the forward pass. Consequently, optimal decisions for M_t , corresponding to the values of the state variables $s_{t,k,i}$ obtained during the forward pass run, are applied on S_{t+1} with their impact on S_{t+1} captured through the corresponding dual variables. For each point of interest k identified at the forward pass, the backward pass is solved for all the $s = \{1, ..., NS\}$ different samples of the stochastic variables, leading to $k \cdot s$ problems for each period.

In order to alleviate the computational burden that arises due to the large number of problems, the sub-problems referring to the same period can be solved in parallel. The subproblems S_{t+1} consist of all the constraints included in master problems M_t , augmented by constraints (17) - (18), which are introduced for the dual variable calculation. The auxiliary variables $\tilde{e}_{t,b,s}$ and $\tilde{d}_{t,f,s}^{agg}$ are deployed to capture the impact of the $s_{t,k,i}$, which have been identified during the forward pass, on the optimal value of the sub-problems S_{t+1} . After solving all the sub-problems referring to stage t, a single cut for each point of interest k is built as per (16) at each iteration i. The required values for the dual variables $\lambda_{t,b,k,i}^{E}$, $\lambda_{t,f,k,i}^{D}$ and the optimal future cost $a_{t,k,i}$ per point k is calculated from (19)-(21), as the expected value over the NS runs. The derived cuts from S_{t+1} are appended to the master problem M_t pertaining to period t.

$$\tilde{e}_{t,b,s} = e_{t,b,k,i} \qquad :\lambda_{t,b,k,s,i}^{E} \quad \forall t, b, k, s, i \quad (17)$$

$$\tilde{d}_{t\,f\,s}^{agg} = d_{t\,f\,k\,i}^{agg} \quad :\lambda_{t,f,k,s,i}^{D} \quad \forall t, f, k, s, i \quad (18)$$

$$\lambda_{t,b,k,i}^{E} = \sum_{s}^{S} \lambda_{t,b,k,s,i}^{E} / NS \qquad \forall t, b, k, i \qquad (19)$$

$$\lambda_{t,f,k,i}^{D} = \sum_{s} \lambda_{t,f,k,s,i}^{D} / NS \qquad \forall t, f, k, i \qquad (20)$$

$$a_{t,k,i} = \sum_{s} \alpha_{t,k,s,i} / NS \qquad \forall t,k,i \qquad (21)$$

Iterations between forward and backward passes continue until \mathfrak{S}_{t+1} has reached an acceptable accuracy level and an optimal solution of a target quality has been achieved. More specifically, upper and lower bounds of the optimal solution are defined and calculated at each iteration, and the algorithm terminates when the distance between these bounds is within a tolerance value. The upper and lower bounds are given by (22)-(23), and convergence of the two bounds is checked at the end of each forward pass run, according to (25), where \overline{z}_{mean}^{i} , σ^{i} represent the mean value and standard deviation of the forward pass optimal objective function values at iteration i [26].

$$\frac{z^i}{z^i} = \mathcal{M}_1 \tag{22}$$

$$\overline{z}^{t} = \sum_{t,k} c_{t,k}^{\mathrm{T}} x_{t,k,i} / NK$$
(23)

$$\sigma^{i} = \sqrt{\frac{1}{K-1} \sum_{t,k} \left(c_{t,k}^{\mathrm{T}} x_{t,k,i} - \overline{z}_{mean}^{i} \right)^{2}}$$
(24)

$$\overline{z}_{mean}^{i} - \frac{1.96 \cdot \sigma^{i}}{\sqrt{NS}} \le \underline{z} \le \overline{z}_{mean}^{i} + \frac{1.96 \cdot \sigma^{i}}{\sqrt{NS}}$$
(25)

V. SDDP EXTENSION FOR CAPTURING MULTIVARIATE DEPENDENT UNCERTAINTY

The SDDP algorithm presented in the previous section entails the sampling of realizations $\{\omega_t\}_{t=1}^T$, which can be obtained either from a continuous or a discrete stochastic process $\boldsymbol{\omega}$. Sampling from a continuous distribution exponentially increases the problem complexity, when the operating horizon and the dimensionality of the stochastic process are enhanced, and may lead to intractability issues. This intractability is mainly overcome by assuming that $\boldsymbol{\omega}$ is stage-wise independent (i.e. $\boldsymbol{\omega}$ does not exhibit temporal correlation), a prevailing assumption in literature (e.g. see [18], [19], [25]-[28]). The temporal independence assumption implies that the future cost function at each *t* do not depend on the evolution of $\boldsymbol{\omega}$, which allows cut-sharing among all M_t/S_t belonging at the same time-period at the cost of ignoring timedependence of $\boldsymbol{\omega}$.

An alternative approach lies in deriving a discretized representation (e.g. scenario tree), where a trade-off between representation complexity and accuracy exists [29]. When multidimensional uncertainty is considered, in order to avoid a combinatorial explosion, this approach ends up with very simple tree structures, unable to accurately capture $\boldsymbol{\omega}$.

Both approaches presented above deal with the issue of computational tractability by resorting to simplification. However, the proposed algorithm can accommodate the incorporation of stochastic models which capture complex relationships in $\boldsymbol{\omega}$ (e.g temporal and among the stochastic variables), as long as the linearity of the stochastic problem is preserved, so that the computation of the required dual variables is not inhibited. VAR models constitute suitable candidates, since they can articulate complex stochastic relationships, while retaining the linearity of the problem. A VAR model depicts the evolution of a multivariate stochastic process $\boldsymbol{\omega}$, as a linear function of the previous p instances of the process. Consequently, a p^{th} order VAR model has been adopted to represent the y-dimensional stochastic process $\boldsymbol{\omega}$, as expressed in (26), where $\boldsymbol{\omega}_t$ corresponds to the random variables at t, A_p correspond to p time-invariant $y \times y$ matrices, ω_{t-p} represents the values of $\omega_t p$ time-periods before t and ε_t is a y×1 error term matrix. The error matrix ε_t should contain zero-mean terms (i.e. $\mathbb{E}(\varepsilon_t) = 0$), while $\mathbb{E}(\varepsilon_t \varepsilon_t') = K$ and $\mathbb{E}(\varepsilon_t \varepsilon_{t-p}') = 0, \forall p$ should stand for their covariances, denoting that there is no serial correlation in ε_t .

$$\boldsymbol{\omega}_{t} = \sum_{p} A_{p} \cdot \boldsymbol{\omega}_{t-p} + \boldsymbol{\varepsilon}_{t} \qquad \forall t \qquad (26)$$

The consideration of temporal and cross-variable correlations of $\boldsymbol{\omega}$ and the explicit integration of the stochastic model in the solution process of the algorithm, suggest that additional state variables, expressing the evolution of $\boldsymbol{\omega}$ until the current stage, should be introduced. Therefore, the new expanded state variable vector is to $S_t \triangleq$ $\{e_{t,b}, d_{t,f}^{agg}, \omega_{t-1}, \cdots, \omega_{t-p}\}$ depending on the order of the VAR model and the modified cut is expressed in (27), where $S_{t+1,k}$ has been enhanced with the expanded s_t and a new term representing the impact of the previous p instances of ω_t on S_{t+1} has been added. The dual variables associated with $\omega_{t-p,k,i}$ are obtained according to (28) - (29) when problem S_t is solved, where $\tilde{\omega}_{t-p,s}$ are slack variables introduced for the calculation of $\lambda_{t,k,s,i}^{\omega}$. We should stress out that a complex multidimensional stochastic model, collapses into naïve white error sampling, when integrated in the SDDP formulation, while only increasing the state space dimensionality. However, it is crucial that $\omega_{t-p,k,i}$ refer to deterministic and already-resolved quantities, when M_t/S_t are solved, so they comprise inputs of the model. Consequently, the state space expansion does not entail increased computational burden and complexity.

$$\begin{aligned} a_{t+1} \geq \\ \mathcal{S}_{t+1,k}^{i-1} \left(e_{t,b,k,i}, d_{t,f,k,i}^{agg}, \omega_{t-1,k,i}, \cdots, \omega_{t-p,k,i} \right) \\ &+ \sum_{b} \lambda_{t,b,k,i}^{E} \cdot \left(e_{t,b} - e_{t,b,k,i} \right) \\ &+ \sum_{b} \lambda_{t,f,k,i}^{D} \cdot \left(d_{t,f}^{agg} - d_{t,f,k,i}^{agg} \right) \\ &+ \sum_{p} \lambda_{t,k,i}^{\omega} \cdot \left(\omega_{t-p} - \omega_{t-p,k,i} \right) \\ &\tilde{\omega}_{t-p,s} = \omega_{t-p,k,i} \qquad : \lambda_{t,k,s,i}^{\omega} \qquad \forall p,k,s,i \qquad (28) \\ \lambda_{t,k,i}^{\omega} = \sum_{s} \lambda_{t,k,s,i}^{\omega} / NS \qquad \forall p,k,s,i \qquad (29) \end{aligned}$$

Another important implication is that when sampling $\boldsymbol{\omega}$ from a VAR model, output cannot be forced to conform to the underlying process bounds (e.g. maximum wind power output). Ex-post adjustment of these offending realizations to within the acceptable domain (e.g. negative values corrected to 0) undermines model convexity [28]. This issue can be mitigated by employing penalty variables for each of the terms of ω_t with upper and lower limits. For the DER aggregator model, this refers to both terms of ω_t , which physically cannot be negative and cannot exceed their maximum values. Thus, variables $y_t = [y_{t,w}^{w1}, y_{t,w}^{w2}, y_t^{d1}, y_t^{d2}]$ are introduced and penalized with factor c^{pen} , which is a vector consisting of 2(1 + w) elements; appropriate penalty terms are appended to all M_t and S_t , as in (30). Constraints (31)-(36) are included to restrict $\boldsymbol{\omega}_t$ and (9) has been omitted, while $P_{t,w}^{wind}$ and D_t^{inf} are the values sampled from (26) for the available wind power output and demand to be served.

$$\mathcal{M}_{t}(x_{t-1},\omega_{t}) = \min_{x_{1}} \left[c_{t}'x_{t} + c^{pen'}y_{t} + \mathbb{E}_{\omega_{t+1}|\omega_{t}}[\mathcal{S}_{t+1}(x_{t},\omega_{t+1})] \right]$$
(30)

s.t. (1) - (8), (10) - (12), (26)

$$p_{t,w}^{wind} \le P_{t,w}^{wind} + y_{t,w}^{w1} + y_{t,w}^{w2}$$
 (31)

$$M_t: \quad y_{tw}^{w1} \le W_w^{max} - P_{tw}^{wind} \tag{32}$$

$$y_{t,w}^{w1} \le 0 \tag{33}$$

$$d_t^{inf} = D_t^{inf} + y_t^{d1} + y_t^{d2}$$
(34)

$$y_t^{d1} < D^{max} - D_t^{inf} \tag{35}$$

$$y_t^{d1} \le 0 \tag{36}$$

We should emphasize that, when S_t is solved for an out-ofbounds value of $\boldsymbol{\omega}_t$, the specific realization of $\boldsymbol{\omega}_t$ does not constitute a valid point of the state space and the respective dual variables do not interpret the real impact of $\boldsymbol{\omega}_t$ on S_t . As Finally, even though a VAR model has been selected for the examined DER aggregator problem, any linear stochastic model could be incorporated in the traditional SDDP algorithm in a straightforward way, following the process outlined in this section.

VI. CASE STUDY

A. Description, Data and Implementation

The examined study focuses on a DER aggregator participating in a real-time market, operating a portfolio consisting of one wind turbine, one controllable microgenerator, inflexible and flexible demand and one energy storage unit, and facing uncertainty regarding the level of inflexible demand to be served and the available wind power output. The deterministic parameters of the case study are presented in Table I. In order to capture these uncertainties, historical wind power output and demand data from the Northern UK area and for a period of one month (so that seasonal effects are avoided) have been employed [30]. A market resolution of $\Delta t = 1h$ is considered and two different operating horizons of 6 and 24 hours are investigated. The required models have been implemented in MATLAB R2015a [31] and FICO Xpress [32] on a 3.33 GHz Intel Xeon computer.

TABLE I **Case Study Parameters Energy Storage Parameters** E_b^{max}/E_b^{min} 700 kWh/140 kWh P_b^s 280 kW $E_{0,b}$ 350 kWh 0.9 **Flexible Demand Parameters** $D_{t,f}^{base}$ 200 kW 0%,10%,20%,30% System Parameters $W_1^{max}/P_t^{g,sell}$ 700 kW/0 kW P. g,buy/Pgen 300 kW/300 kW Ca^{grid}, Ca^{gen} 0.15 £/kWh/1 £/kWh

B. 6h Operating Horizon Case Study

The 6h horizon case study aims at comparing the proposed SDDP approach against a traditional stochastic programming approach employing scenario trees. Regarding the latter approach, a multivariate copula model [16] has been fit to the available data, and 5,000 scenarios have been sampled from this model. In order to comprehensively investigate the performance of this approach, six scenario trees of varying complexity have been formed based on the initial set of 5,000 scenarios (Table II), by employing a standard scenario reduction process based on Kantorovich distance [33].

Regarding the proposed SDDP approach, the dependencies of the multidimensional uncertainty set are captured through a VAR(1) model, which has been fit to the same set of 5,000 scenarios. Autoregressive models require that the fitted data are stationary i.e. statistical properties are time-invariant. In order to achieve that, time-series differentiation [34] has been employed in the case of wind power data, while for inflexible demand data, mean subtraction and division by the standard deviation -to remove the diurnal component- has preceded the differentiation process.

TABLE II	
Scenario Tree Structure for the 6h Horizon	Study

Complexity	Structure	Number of	Number of
Level	(nodes per stage)	nodes	scenarios
C1	1-1-1-1-1	6	1
C2	1-2-2-2-2-2	11	2
C3	1-2-4-6-8-10	31	10
C4	1-2-4-8-12-16	43	16
C5	1-2-4-8-16-32	63	32
C6	1-3-9-27-81-243	364	243

In order to meaningfully compare the solution efficiency of the seven implemented models (six models corresponding to the six different scenario trees and one model corresponding to the extended SDDP approach, which is denoted SDDPe in the remainder), the first-stage decisions taken by each model are used for Monte Carlo validation i.e. given the obtained firststage decisions, the aggregator's expected cost over the whole set of 5,000 realizations is calculated. In this way, we can capture the impact of sub-optimal first-stage decisions on subsequent operation. In order to ensure that this comparison is not prone to the selection of the starting point, this process is repeated for 10 starting points regarding wind power output (corresponding to 5% - 95% of the wind turbine's capacity W_w^{max} with a step of 10%) and 24 starting points for inflexible demand (representing typical demand levels at different hours of the day). Thus, a problem of 1.2 million scenarios has been solved.

For benchmarking purposes, the results of two additional models are reported. Firstly, the traditional SDDP algorithm, where temporal and cross-variable independence of the stochastic process is assumed, is simulated. In this context, a lattice approach, where all scenarios are equiprobable and stochastic transitions do not consider the previous realizations of the stochastic process, is adopted. Secondly, an idealized deterministic model is also implemented, where the aggregator is assumed to have perfect information regarding the future evolution of uncertain variables. In that case, optimal first stage decisions are obtained for each scenario separately.

Fig. 2 compares the expected monthly aggregator cost obtained from each of the six scenario tree models, the proposed (SDDPe) and traditional SDDP models and the model involving perfect information. These costs are derived by taking the expectation over all the 1.2 million realizations and extrapolating the costs of the examined 6-hour horizon to a monthly horizon. The performance of the scenario tree approach is improved as the complexity of the employed tree is enhanced, given that the representation of the stochastic process becomes more accurate and thus better-informed decisions are made. The proposed SDDPe model massively outperforms the two simpler scenario tree models C1 and C2 by 10-20%, while showcasing a benefit of approximately 0.5-3% with respect to models C3-C5. The most complex tree model (C6) slightly outperforms the SDDPe approach (by 0.6%). As expected, the perfect information model yields the lowest cost, but the savings it yields compared to SDDPe and C6 are relatively small (1-1.5%). Finally, it can be observed that the proposed SDDPe approach provides better results than the traditional one, exhibiting a benefit of 4.3%. Intuitively, disregarding the temporal characteristics of the stochastic process by assuming stage-wise independence leads to poorer first-stage decisions.



Fig. 2. Expected cost overall starting points for SDDP, SDDPe, different scenario tree types and under perfect information conditions in the 6h horizon case study.

In order to compare the computational performance of the different models, Table III presents the number of decision variables, total constraints and time-coupling constraints corresponding to each of the seven models, while Fig. 3 the respective computational showcases times. The computational complexity of the scenario tree approach is exponentially aggravated as the complexity of the employed tree is enhanced, as demonstrated in Table III. Regarding the traditional and the proposed SDDP models, the respective numbers correspond to the total number of variables and constraints included in a single-stage problem (M_t/S_t) -which are solved sequentially- multiplied by the number of such single-stage problems. These numbers are lower than the numbers corresponding to all scenario trees. It should be noted that time-coupling constraints which significantly contribute to problem complexity are inherently avoided by SDDP and SDDPe.

The proposed SDDPe approach also involves less computational time than scenario trees C4-C6, as depicted in Fig. 3. The average computation time of the SDDPe approach was 1.15s and the algorithm converged after 5-7 iterations. These computational advantages are driven by the fact that SDDP involves stage-wise decomposition of the optimization problem. The computational performance of the traditional SDDP algorithm is slightly better than the proposed one due to the stage-wise independence assumption. However, the computational advantage of SDDP and SDDPe is not particularly pronounced due to the small operation horizon of the study.

All in all, the combination of the insights from Fig. 2 and Table III demonstrate that SDDPe achieves a better trade-off between solution efficiency and computational performance with respect to scenario-tree-based approaches, since it yields a similar aggregator's expected cost with the one achieved by the most complex scenario trees, while it exhibits a similar computational performance with the simplest scenario trees.

TABLE III Number of Decision Variables, Total and Coupling Constraints for SDDP, SDDPe and Different Scenario Tree

Types.				
	Decision Variables	Total Constraints	Time-coupling Constraints	
SDDP	19*11	20*11	0	
SDDPe	19*11	20*11	0	
C1	36	68	7	
C2	66	126	14	
C3	126	258	70	
C4	186	390	112	
C5	378	886	224	
C6	2184	5584	1701	



Fig. 3. Simulation time for SDDP, SDDPe and different scenario tree types in the 6h horizon case study.

Fig. 4 presents the expected cost savings achieved by the proposed SDDPe model with respect to the scenario tree models C1, C2, C3 and C5 for the different wind power output starting points, where positive (negative) values signify that SDDPe outperforms (underperforms) the scenario tree solution. First of all, SDDPe outperforms the two simpler scenario tree models C1 and C2 for all starting points. However, this benefit is significantly higher for starting points involving lower wind power and diminishes as we move to starting points involving higher wind power. This effect emerges as low wind power starting points indicate that less wind power is likely available in the operating horizon, increasing the impact of strategic decisions regarding the use of flexible DER in the aggregator's portfolio. On the other hand, high wind power starting points indicate that abundance of wind power is likely available in the operating horizon, and strategic decisions have minor impacts on the aggregator's expected cost.

Compared to the scenario tree models C3 and C5, SDDPe provides consistently better results for middle and high wind power starting points, and slightly worse results for low wind power starting points. This effect emerges since low wind power output starting points are more likely to yield scenarios with negative wind power outputs, in which case the construction of meaningful cuts within the SDDPe algorithm is inhibited and thus the solution quality deteriorates. These results justify the authors' decision to employ multiple starting points in the comparison of the different models.



Fig. 4. Difference in expected cost between SDDPe and C1, C2, C3 and C5 respectively for different wind power output starting points in the 6h horizon case study.

Fig. 5 compares the expected aggregator cost obtained from the different models in a similar logic with Fig. 2, but this comparison is now made for different cases regarding the extent of demand flexibility in the aggregator's portfolio, as expressed by the load shifting limit S_f . The particularly interesting result lies in the fact that the simpler scenario tree models C1-C3 exhibit an increase in expected cost with increasing demand flexibility. This counter-intuitive result emerges because the effect of a strategic decision based on less accurate information is intensified when the aggregator manages a more flexible portfolio. In other words, the aggregator commits more deeply on a poor decision, which has an irreversible effect on its strategy. Even though the increased demand flexibility levels persist over the entire operating horizon, the loss induced by the poor first stage strategic decisions is not recovered in the subsequent periods. On the other hand, the more complex scenario tree models C4-C6 as well as the proposed SDDPe model always exploit additional demand flexibility in a beneficial way for the aggregator, as indicated by the decreasing expected cost in Fig. 5.



Fig. 5. Expected cost for $S_f = \{0\%, 10\%, 20\%, 30\%\}$ for SDDPe, different scenario tree types and under perfect information conditions.

C. 24h Operating Horizon Case Study

The 24h horizon study aims at highlighting the computational benefits of the proposed SDDP approach in dealing with large-scale problems. In this context, the

operating horizon of the DER aggregator problem is expanded to 24h and a population of 200,000 scenarios has been sampled from the same multivariate copula model used in Section VI-B [16]. In order to examine the impact of the scenario tree size on computation time, five scenario trees (i.e. D1-D5) of varying complexity (Table IV) have been constructed by deploying the same scenario reduction process used in Section VI-B.

TABLE IV Scenario Tree Structure for the 24h Horizon Study

Complexity	Structure	Number	Number of
Level	(nodes per stage)	of nodes	scenarios
D1	1-	24	1
	1-1-1-1-1-1-1	24	1
	1-2-4-8-16-32-48-72-108-160-		
D2	245-365-455-570-710-890-1110-	10.041	2,500
	1390-1530-1680-1850-2040-	18,041	
	2255-2500		
D3	1-2-4-8-16-32-64-128-192-288-	91,942	20,000
	432-650-970-1460-2190-3280-		
	4100-5125-6400-8050-10100-		
	12600-15850-20000		
D4	1-2-4-8-16-32-64-128-256-450-		
	785-1370-2060-3090-4630-6945-	226 801	50,000
	10420-13025-16280-20350-	220,891	
	25435-31795-39745-50000		
D5	1-2-4-8-16-32-64-128-256-512-		
	1024-2048-3070-4610-6910-	207.015	90,000
	10370-15550-23300-29160-	597,915	
	36450-45650-57200-71550-90000		

Fig. 6 illustrates the simulation time for the SDDP, SDPPe and the different scenario tree models. It is evident that the computational benefits of the proposed SDDP approach are significantly more pronounced, when solving large-scale stochastic problems. Fig. 6 demonstrates that the SDDPe algorithm massively outperforms every scenario tree model, except from D1, which comprises of only one scenario. The simulation time of the proposed model is similar to the simplest scenario trees (i.e. D1-D2), while the simulation time savings reach 98%, when compared to D5.



Fig. 6 Simulation time for SDDP, SDDPe and different scenario tree types in the 24h horizon case study.

We should note that the reported times refer solely to the solution of the optimization problem, while the requirements of the scenario reduction process -which in this case study involved several hours- are not included. As discussed in the previous case study, the traditional SDDP approach exhibits slightly lower computation time than the proposed one due to the independence simplifications.

VII. CONCLUSIONS

This paper develops a novel SDDP approach for the optimal operation of a DER aggregator in real-time markets under multidimensional uncertainty. Temporal dependencies of the uncertain variables as well as dependencies among different uncertain variables are captured via a p order VAR model, which is integrated in the SDDP framework. This approach can efficiently cope with the combinatorial explosion of the problem without resorting to ill-informed scenario reduction techniques, since it does not require discretization of either the state space or the uncertain variables.

The proposed approach is compared against a traditional scenario-tree-based approach with varying tree complexities, in a case study involving an aggregator with diverse generation, demand and storage resources, and facing uncertainty regarding the level of demand to be served and the available wind power output. In order to meaningfully and comprehensively compare the two approaches, Monte-Carlo validation for different demand and wind power starting points is carried out. The proposed approach is demonstrated to achieve a better trade-off between solution efficiency and computational performance, since it yields a similar aggregator's expected cost with the one achieved by the most complex scenario trees, while it exhibits a similar computational performance with the simplest scenario trees. The computational superiority of the proposed approach becomes more significant when longer operating horizons are investigated, further demonstrating the scalability potential of the extended SDDP algorithm to large-scale problems. Finally, the proposed approach is demonstrated to yield better solutions compared to the traditional SDDP framework, which does not consider temporal and cross-variable dependencies of the stochastic process.

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IX. BIOGRAPHIES

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