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# Empirical Asset Pricing in High-Frequency Markets

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## Abstract

The first chapter provides a brief overview of the theoretical grounding of the so-called realized-measures and summarizes their statistical properties. Motivated by the superior statistical accuracy of high-frequency measures I test if they add economic value in a low-frequency investment context. I compare the profitability of the same investment strategy against two implementations of its trading signals: one that conventionally uses daily returns (LF) and the other that takes advantage of high-frequency (HF) returns. I use different lengths of the formation period to verify if the HF implementation can leverage the superior amount of observations. Although economic differences favour the HF implementation, the evidence is not statistically significant. Nonetheless, the HF implementation is more robust to the choice of parameters and provides, for the most illiquid stocks, strong economic benefits that are inversely increasing in the length of the formation period.

The second chapter moves the focus onto the intraday level and seeks to establish the existence of an intraday momentum effect. Motivated by limited evidence of intraday predictability both in the cross-section of US stock returns (see Heston, R. A. Korajczyk, and Sadka, 2010) and in the time-series of the aggregate stock market (see Gao et al., 2015), I reconsider the time-series dimension using all common US stocks from 1993 until 2010 and, building on this, I present the cross-sectional dimension with new and complimentary evidence. I find that statistical time-series predictability does not imply economic profitability, whereas cross-sectional sorts on past performance see stocks, which lost or won the most in the morning, earn (positive returns) above the rest of the cross-section in the afternoon, and especially during the last half-hour of trading. The effect is robust to stock characteristics, the day-of-week effect, variations in the formation and holding periods, but exhibits some dependence on the sample period, suggesting that specific market mechanisms or frictions play a relevant role on intraday price formation.

The third chapter looks at some claims about price acceleration constituting an informative trading signal. We build several empirical measures of acceleration and compare them to other traditional equity signals from the academic literature. We find that buying stocks whose returns are decelerating and shorting stocks whose returns are accelerating, produces a wide spread in returns. Moreover, while these profits are not explained by the state-of-the-art equity factor models, the cross-sectional variation of the average returns of portfolios sorted on the acceleration signals, are reconciled once we add our *la5* factor – a simple reversal strategy with a lookback of one week. Taken together, our results cast doubt on acceleration being a separate phenomenon and suggest that the lookback period in trending strategies has been shrinking over time.

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# Declaration of originality

I herewith certify that this thesis constitutes my own work and that all material, which is not my own work, has been properly acknowledged.

*Oleg Komarov*



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# Chapter 1

## Low-Frequency Investment with High-Frequency Measures: Is it Profitable?

### 1.1 Introduction

The past fifteen years have witnessed significant advances in financial econometrics relative to the use of high-frequency returns in the estimation of the so-called measures of realized variation.<sup>1</sup> The success of these measures lies in the appeal of their statistical properties, discussed below, and in the simplicity of their implementation, made possible by the ever growing availability of high frequency data. However, while applications of realized variation offer significant statistical gains in fields that range from risk-management to derivative pricing, there is no evidence that these measures provide tangible economic gains. This paper addresses this gap in the literature and proposes a methodology to measure

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<sup>1</sup>Among the most influential theoretical results are [Andersen, Bollerslev, Diebold, and Labys \(2001, 2003\)](#) and [Barndorff-Nielsen and Shephard \(2002a,b\)](#), especially for univariate measures like the realized volatility, and [Barndorff-Nielsen and Shephard \(2004\)](#) for multivariate measures like the realized covariance (RC) or the realized regression.

the economic impact of statistical measures based on high frequency returns, as opposed to the same measures calculated with daily returns.

Before going into further details, I give a brief overview on the implementation of these measures and on their statistical benefits. In their simplest form, these measures are sums of squared intraday returns sampled over a fixed horizon, usually a day, and at sufficiently high frequency, e.g. at intervals of 5 minutes. The mechanics are trivial in comparison to competing parametric models like (G)ARCH and Stochastic Volatility (SV) and virtually requires no calibration. Moreover, the theoretical link with the Quadratic Variation allows to treat volatility (the price-path variation) as an ex-post observable quantity and, therefore, to derive estimators which are approximately unbiased and consistent (see again Andersen, Bollerslev, Diebold, and Labys (2001)).

In other words, high-frequency measures provide estimates and forecasts of volatility which are more accurate than those derived from daily returns or from competing models. For example, in the context of derivatives pricing, returns standardized by Realized Volatility (RV) are nearly Gaussian. In contrast, returns standardized by (G)ARCH or SV exhibit excess kurtosis (see Andersen, Bollerslev, Diebold, and Labys (2000) among others). In terms of forecasting accuracy, Andersen, Bollerslev, Diebold, and Labys (2003) show that RV and Realized Covariance (RC) are preferable over GARCH and related approaches. In yet another example, P. R. Hansen and Lunde (2006a) use the realized measure as the optimal benchmark, due to the consistency of RV, and rank competing volatility models against it.

While it is beyond the scope of this paper to summarize the whole progress of this field of literature, it is worth noting that the underlying theoretical framework is flexible to the inclusion of jumps and that the estimation mechanics can be adapted to deal with real world frictions like microstructure noise.<sup>2</sup> In fact, a great deal of research has explored

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<sup>2</sup>A good overview on the advances and challenges of high-frequency financial econometrics is given by Barndorff-Nielsen and Shephard (2007), Andersen, Bollerslev, and Diebold (2010) and McAleer and Medeiros (2008).

the two strands of financial time-series mentioned above in search of an optimal solution. However, the focus has remained statistical in nature and no study has tried to assess the potential economic value of this statistical accuracy.

Hence, this work tries to quantify the economic gains that generate from the increased statistical accuracy of realized measures. Since the task is naturally broad, I suggest a methodology that quantifies the economic contribution in the context of a tradable strategy in US equities. More specifically, I implement a trading strategy from a risk factor building signals based on low-frequency returns. Then, I reproduce the same strategy using signals based on high-frequency returns and compare the economic performance of the two implementations. Apart from the frequency of returns, all other implementation details are kept identical, i.e. I use the same: investable universe, formation period, rebalancing scheme and trading costs. With this approach, any difference in economic performance between the two versions of the same strategy should exclusively come from differences in the statistical accuracy of high-frequency measures.

This work is relevant from both an academic and a practitioner's perspective. In the former case, the discrepancy between the high- and low-frequency versions of an investment strategy can be interpreted as a test of the weak-form of market efficiency postulated by [Fama \(1970, 1991\)](#). That is, if the market already incorporates past public information, then the price at the close-of-day should reflect all intraday fluctuations, and therefore, it should not be possible to obtain significant economic gains by using high-frequency data. However, from a practical perspective, as the number of observations diminishes, e.g. by shortening the formation period, the estimation of the signal from daily observations becomes noisier than its equivalent based on high-frequency data. Hence, establishing a clear trade-off between the length of the estimation horizon and the added value of high-frequency measures is of direct benefit to e.g. asset management applications.

For the choice of the strategy, I use the Betting-Against-Beta (BAB) factor by [Frazzini and Pedersen \(2014\)](#) since it is relatively easy to implement and yet it involves the

estimation of covariances. It is important to note that the implications should not be interpreted as conclusions about the BAB factor itself and that other eligible factors could have been used in its place. However, this analysis cannot be applied to momentum-like factors, because holding-period returns do not depend on the cumulation frequency. Another arguably eligible example is the strategy based on realized skewness by [Amaya et al. \(2015\)](#). As the authors point out, there is a fundamental theoretical difference between historical and realized skewness which impedes a clear direct comparison. Lastly, although the estimation methodology is central to realized measures, the main focus of the paper is not directed towards finding the optimal combination of parameters that maximises the performance of the high-frequency strategy.

The remainder of the paper is organized as follows: this section concludes with an overview of the theoretical framework underlying the estimation of the realized betas for the high-frequency version of the BAB factor. [Section 1.2](#) describes the data adopted in the analysis with an emphasis on the challenges posed by high-frequency trades. Specifically, the section details the procedure used to link the CRSP and TAQ datasets, summarizes the cleaning steps for high-frequency trades and proposes solutions to the big-data challenges inherent to such tasks. The general methodology on the construction of the portfolios and preliminary statistics are presented in [section 1.3](#), and a comparison of the performance between the high- and low-frequency versions of the BAB portfolio, with several robustness checks, are given in [section 1.4](#). Finally, [section 1.5](#) concludes.

### 1.1.1 The theoretical framework of high-frequency measures

I assume that asset returns can be described by a discrete-time one-factor model

$$r_i = \alpha_i + \beta_i r_0 + \epsilon_i \tag{1.1}$$

where the  $r_0$  is the return on a systematic risk factor which is assumed uncorrelated with the asset-specific risk  $\epsilon_i$ . The  $\beta$  measures the contribution of the systematic return to that of the  $i$ -th asset and is generally not observed, but can be estimated with a regression of its returns on the returns of some market portfolio. In a simple linear regression with e.g. daily returns, the beta is equivalent to

$$\beta_i = \frac{\text{cov}(r_i, r_0)}{\text{var}(r_0)}. \quad (1.2)$$

However, to extend the same approach to a high-frequency setting, additional structure is required. I will leverage previous works on realized betas by Bollerslev and B. Y. Zhang (2003), Barndorff-Nielsen and Shephard (2004) and Andersen, Bollerslev, Diebold, and Wu (2005), Andersen, Bollerslev, Diebold, and Wu (2006) and provide a brief overview of the relevant results.

To this end, let log-prices  $p_i$  evolve over a fixed-time interval  $[0, T]$  according to a continuous-time diffusive process defined on some filtered probability space  $(\Omega, \mathcal{F}, (\mathcal{F})_{t \geq 0}, \mathbb{P})$ . Assume the process is a semi-martingale of the form

$$dp_{i,t} = \mu_{i,t} dt + \beta_i \sigma_{0,t} dW_{0,t} + \sigma_{i,t} dW_{i,t}, \quad (1.3)$$

where  $\mu_{i,t}$  is a predictable drift of finite variation,  $\sigma_{0,t}$  and  $\sigma_{i,t}$  are continuous càdlàg processes respectively capturing the volatility of the market portfolio and of the  $i$ -th asset, and  $W_{i,t}$  and  $W_{0,t}$  are standard Brownian processes independent of each other. Moreover, I assume that the market portfolio is only exposed to systematic risk, i.e.

$$dp_{0,t} = \mu_{0,t} dt + \sigma_{0,t} dW_{0,t}. \quad (1.4)$$

This continuous-time setting is consistent with [equation \(1.1\)](#) since we can rewrite [equa-](#)



tion (1.4) in terms of its diffusive part and substitute back into equation (1.3) to get

$$dp_{i,t} = (\mu_{i,t} - \mu_{0,t}) dt + \beta_i dp_{0,t} + \sigma_{i,t} dW_{i,t},$$

where the return of the asset, expressed in differential terms, depends on a firm-specific drift term, the return of the market portfolio and the firm-specific diffusive component.

From known arguments, the quadratic covariation between the processes in equations (1.3) and (1.4), and the quadratic variation of the market portfolio, over the period  $(0, t]$ , are

$$\begin{aligned} [p_i, p_0]_t &= \beta_i \int_0^t \sigma_{0,s}^2 ds, \\ [p_0, p_0]_t &= \int_0^t \sigma_{0,s}^2 ds, \end{aligned}$$

which we can use to express the realized beta over the period  $(0, t]$  as the ratio of these two terms, i.e.

$$\beta_i = \frac{[p_i, p_0]_t}{[p_0, p_0]_t}. \quad (1.5)$$

To estimate the quantities in equation (1.5), we can refer to the general definition of quadratic variation and covariation for two stochastic processes  $X_t$  and  $Y_t$

$$\begin{aligned} [X, X]_t &= \lim_{\|\Pi\| \rightarrow 0} \sum_{k=1}^n (X_{\tau_k} - X_{\tau_{k-1}})^2 \\ [X, Y]_t &= \lim_{\|\Pi\| \rightarrow 0} \sum_{k=1}^n (X_{\tau_k} - X_{\tau_{k-1}})(Y_{\tau_k} - Y_{\tau_{k-1}}) \end{aligned}$$

where the limit is taken over random partitions of the interval  $[0, t]$ , which are informally defined as  $\Pi = \{0 = \tau_0 \leq \tau_1 \leq \dots \leq \tau_n = t\}$ , such that the mesh of the partition, given by the longest sub-interval, goes to zero, i.e.  $\|\Pi\| = \max_k(\tau_k - \tau_{k-1}) \rightarrow 0$ . By this definition, it is possible to approximate the quadratic variation and covariation with the sum of discrete-time increments of the price, by sampling the processes as frequently as

possible.

Concretely, if log-prices are observed at  $n$  equally-spaced intervals over the trading day  $[t, t + 1)$ , I denote the log-return of an asset over the  $k$ -th intraday period  $[(k - 1)/n, k/n]$  by  $r_{t:k} \equiv p_{t+k/n} - p_{t+(k-1)/n}$ . To keep notation simple, the estimation horizon is normalized to a single day. By this convention, the daily realized covariance (RC) with the market portfolio and the realized variance (RV) are defined

$$\begin{aligned} \text{RC}_{t,i} &= \sum_{k=1}^n r_{i,t:k} r_{0,t:k} + r_{i,t}^{\text{ON}} r_{0,t}^{\text{ON}} \\ \text{RV}_{t,i} &= \sum_{k=1}^n r_{i,t:k}^2 + \left(r_{i,t}^{\text{ON}}\right)^2 \end{aligned} \tag{1.6}$$

where  $r_t^{\text{ON}}$  is the overnight return of an asset or the market portfolio, and is defined as the difference between the open price of the current day minus the previous-day price at close, i.e.  $r_t^{\text{ON}} = p_t^{\text{OPEN}} - p_{t-1}^{\text{CLOSE}}$ . The expression for  $\text{RV}_{t,0}$  follows immediately from  $\text{RV}_{t,i}$ .

The inclusion of the overnight component in the estimation of the realized measures follows in spirit [P. R. Hansen and Lunde \(2005\)](#), which suggest to weight its influence in order to reduce excessive noise. In order to keep the specification as simple as possible I include the full overnight return but test the robustness of the estimates in [section 1.4](#) by comparing it with results that exclude  $r^{\text{ON}}$  altogether. Moreover, since TAQ prices are not adjusted for corporate events, e.g. splits or distributions, the overnight component is backed out from CRSP daily total returns and the details of the procedure are outlined in [section 1.A.1](#).

The measures in [equation \(1.6\)](#) are linear and increasing in time and can be extended to a multi-day horizon of length  $h$ , by simply summing the daily estimates

$$\begin{aligned} \text{RC}_{t,i}^{(h)} &= \sum_{s=0}^{h-1} \text{RC}_{t-s,i} \\ \text{RV}_{t,i}^{(h)} &= \sum_{s=0}^{h-1} \text{RV}_{t-s,i} \end{aligned} \tag{1.7}$$

As the number of sampling points  $n$  increases, the RC and RV respectively converge to the quadratic covariation and variation in [equation \(1.5\)](#). Formally,

$$\begin{aligned} \text{RC}_{t,i}^{(h)} &\xrightarrow{p} [p_i, p_0]_{(t-h+1,t)} \\ \text{RV}_{t,i}^{(h)} &\xrightarrow{p} [p_i, p_i]_{(t-h+1,t)}, \end{aligned} \tag{1.8}$$

as  $n \rightarrow \infty$ , and in the same manner, the estimated high-frequency beta converges to the integrated beta, that is

$$\hat{\beta}_i^{\text{HF}} \xrightarrow{p} \beta_i.$$

To summarize, the framework presented in this section provides the necessary grounding to estimate betas both with daily returns, through a simple linear regression, and with high-frequency returns by exploiting the theory of quadratic variations which underpins the estimation of the realized variance and covariance.

## 1.2 Data

The sample includes all US equities that belong to both the Center for Research in Security Prices (CRSP) and the Trades And Quotes (TAQ) databases, with a coverage that extends to the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX or currently NYSE MKT), the NYSE Arca exchange and NASDAQ's National Market System (NMS, OTC trades only due to TAQ limitations).

I only keep common stocks, identified as such by a CRSP Share Type Code of 10 or 11, and in order to mitigate the spurious effects induced by microstructure issues, I exclude observations that belong to microcaps. That is, holding period returns do not include observations from stocks which either had, on the previous day, a price below \$5 or a market capitalization in the lowest New York Stock Exchange (NYSE) decile. Moreover, to alleviate the impact of stale prices, I require stocks to be sufficiently liquid. Specifically,

I only keep the stock-date pairs that had at least 79 observations on the previous day, which is equivalent to a series having on average a price every 5 minutes during a 9:30 to 16:00 trading session. Finally, all returns from CRSP are adjusted for corporate events, i.e. stock splits and distributions, and for delistings as in [Beaver, McNichols, and Price \(2007\)](#).

Consequently, the sample has a total of 8924 equities with an average of more than 1800 stocks per day, and covers the period from January 1993, first date of availability of the TAQ, until May 2010.<sup>3</sup>

**Description of the TAQ dataset.** High-frequency data are collected by NYSE through the Consolidated Tape System and distributed as the Trade and Quote (TAQ) database directly by NYSE Market Data or by secondary vendors, e.g. Tick Data, Thomson Reuters or Wharton Research Data Services (WRDS). The database is organized into the Master Records table, which holds historical meta-information about securities like the symbol or the company name, the Trades table, which collects time-series with traded prices, the Quotes table, which has time-series with quoted prices and a National Best Bid Offer (NBBO) table which derives the NBBO series from the Quotes table. The data are available either as a monthly or a daily subscription, the latter giving exclusive access to the NBBO table and having time-series of prices with a resolution of the timestamp up to the millisecond instead of a second. Since the primary identifier for TAQ securities is a modified version of the trading symbol, which appends a suffix to differentiate among types of issue, it is not straightforward to match those series in other databases like CRSP. A solution to this problem is discussed in the following section.

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<sup>3</sup>Days with partial trading times, either due to recurring festivities like Christmas' Eve, or due to major disruptions like the power outage of 2003/08/15, are excluded from the analysis.

### 1.2.1 Linking TAQ to CRSP

This section introduces the identification problem of securities across data sources and gives a detailed account of the matching procedure.

**Series identification problem.** Working with separate databases usually poses the problem of how to uniquely identify the same stock across the different sources. A similar situation applies to the interaction between CRSP and TAQ, since the former database uses the PERMNO for most of its data as its unique identifier, while the latter relies on a special version of the trading symbol. To overcome this segregation, I link stocks through the historical CUSIP and adopt the PERMNO as the fundamental unit of analysis also for TAQ data. While any other unit qualifier, e.g. the CUSIP, can also solve the identification problem, the PERMNO tracks a stock through its entire life, i.e. across firm events and changes in the name, symbol and CUSIP. Overall, I match 98% on average and always at least 91% of the daily TAQ observations.

[Table 1.1](#) show the relevant fields from the TAQ master records and CRSP's *msenames* table which are used to establish the link. Both tables report the trading symbol (called TSYMBOL in CRSP), the historical CUSIP, i.e. a database independent code that identifies North American securities, and one or two dates, depending on whether the end-range of the record is implicitly subsumed by the start of the next period (as in TAQ). Additionally, the *msenames* table has the PERMNO which is inherited by the TAQ master records after matching.

Even though both tables contain the trading symbol, which TAQ uses to index its data, it is preferable for a few reasons to match securities through the historical CUSIP. First, the TSYMBOL often does not coincide for the same security with the TAQ's SYMBOL. In fact, the latter usually includes a suffix to identify the type of the issue, as with e.g. AAPR which appears in the last row of [table 1.1](#) where PR stands for *private*.<sup>4</sup> Second, in contrast with

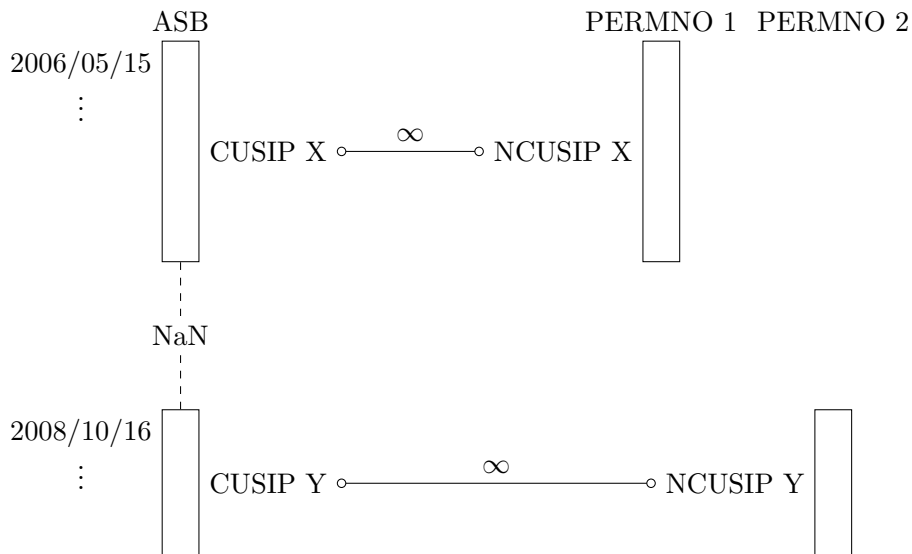
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<sup>4</sup>The complete list of suffixes can be found in the appendices to the TAQ manuals: [Daily TAQ](#) and [Monthly TAQ](#).

**Table 1.1:** an extract of stock-specific information used to match securities between TAQ and CRSP databases. The top panel contains records from CRSP’s *mnames* table, and the lower panel reports TAQ master records. The upper panel lists the Permno, CRSP’s proprietary unique stock identifier, the trading symbol (Tsymbol), the name of the company (Comnam), the historical 8-character CUSIP (Ncusip), and the date range on which the meta-information applies (Namedt, Nameendt). The lower panel reports TAQ’s Symbol, the Name of the issue, the 12-character TAQ Cusip, and the first date of validity of the meta-information record. TAQ records report information for each issue, i.e. at a lower granularity than CRSP. The panels are horizontally aligned to highlight the closest match between the columns.

<b>CRSP: mnames</b>					
Permno	Tsymbol	Comnam	Ncusip	Namedt	Nameendt
10000	OMFGA	Optimum Manufacturing	68391610	19860107	19861203
10000	OMFAC	Optimum Manufacturing	68391610	19861204	19870309
10000	OMFGA	Optimum Manufacturing	68391610	19870310	19870611
10001	GFGC	Great Falls Gas	39040610	19860109	19931121
10001	EWST	Energy West	29274A10	19931122	20040609
10001	EWST	Energy West	29274A10	20040610	20041018
10001	EWSTE	Energy West	29274A10	20041019	20041226
10001	EWST	Energy West	29274A10	20041227	20080204
10001	EWSTD	Energy West	29274A20	20080205	2008030
<b>TAQ: master records</b>					
	Symbol	Name	Cusip	Date	
	A	Agilent Technologies	00846U101000	20060118	
	AA	Alcoa	013817101000	20031013	
	AAC	Ableauctions.com	00371F206001	20041109	
	AACC	Asset Accep Cap	04543P100002	20040205	
	AACE	Ace Cash Express	004403101002	20040102	
	AAI	Airtran Holdings	00949P108000	20060118	
	AAME	Atlantic American	048209100002	20040102	
	AAON	Aaon	000360206002	20040102	
	AAP	Advance Auto Parts	00751Y106000	20060118	
	AAPL	Apple Computer	037833100001	20060316	
	AAPL	Apple Computer	037833100002	20060214	
	AAPR	Alcoa 3.75 Cum Pfd	013817200001	20060301	

the symbol, the CUSIP cannot be reused by different issues. This means that matches built on the CUSIP do not require meeting conditions on date ranges and hence avoid altogether potential discrepancies in the dates across the databases. [Figure 1.1](#) presents a real-life example of a case of symbol re-use, on the left-side of the diagram, and the correspondence to a different CUSIP/PERMNO on the right-side.

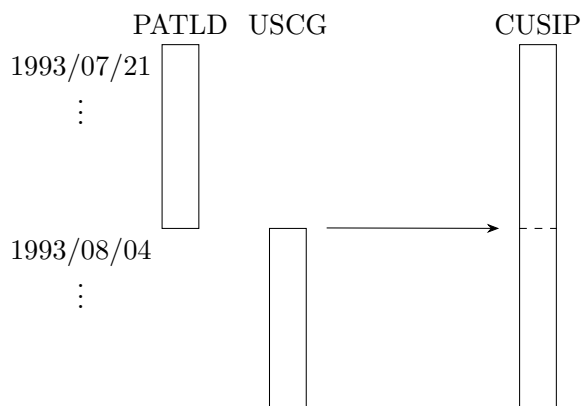


**Figure 1.1:** the same symbol can be used at different points in time by two or more companies. On the left side, the two bars are interrupted by a sequence of NaNs (Not-a-Number) but represent the same time-series which is identified by one symbol. In fact, the two portions of the time-series can be mapped to two different TAQ CUSIPs which in turn will link ( $\infty$ ) to separate CRSP NCUSIPs. The right side shows the correct separation of the two time-series and their final correspondence to CRSP PERMNOs. The example is a real case extracted from TAQ.

Finally, the CUSIP avoids fragmentation of a time-series by tracking the issue across symbol changes, as [figure 1.2](#) illustrates. To sum it up, the CUSIP offers a simpler and more reliable way of linking the two databases.

**The linking procedure.** The actual procedure consists of some initial data handling followed by two main steps: matching series through the CUSIP and dropping duplicate series.

The initial manipulations concern the format of the CUSIP. TAQ reports an extended



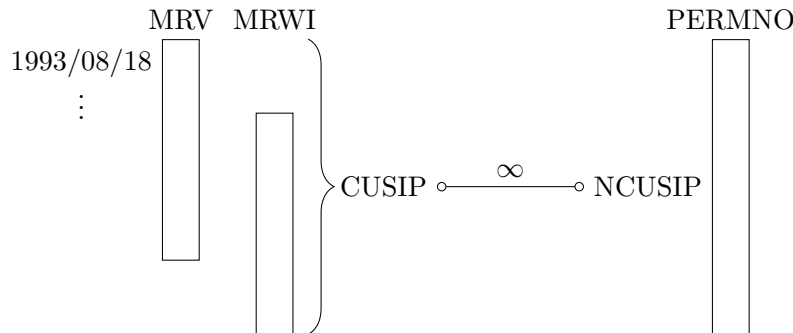
**Figure 1.2:** the same stock issue can change its symbol. On the left side, the two bars represent two separate time-series identified by a different symbols, while in fact, they should be treated as a single series. The right side maps the two symbols to the same CUSIP and correctly recombines the fragments into one series. The example is a real case extracted from TAQ.

12-character CUSIP (see [table 1.1](#)) while CRSP uses the 8-character format, which does not include the check digit from the standard 9-character format. Hence, I drop the last 4 characters from the TAQ's format, which represent the check-digit and a 3-digit code that distinguishes between NYSE, AMEX, and NASDAQ issues. Furthermore, records with all-null or empty identifiers are excluded altogether on both sides. After the CUSIPs are readied, the match is established through them, and the PERMNO from [table 1.1](#) is implicitly ported to TAQ data respecting the pairings between symbol and date-ranges.

The second step handles duplicates which arise from a security sharing the same CUSIP for its main issue and special ones. [Figure 1.3](#) outlines this problem for MRV, which labels the main issue, and MRWI which denotes a *when issued* series. Special issues usually have the symbol as the main one but extended with a suffix. Therefore, most of the conflicts are solved by sorting the symbols in alphabetic order and dropping the series with the longest symbol. In a few cases, it is not possible to determine the main issue from the symbols alone, which results in a negligible loss of 130 series.

A more sophisticated matching procedure, that extends the one just outlined, is described in [section 1.A.2](#). Although the results are very similar for the current sample, the





**Figure 1.3:** a stock can have secondary or special issues. On the left side, the two bars correspond to separate time-series with the main issue mapped to the symbol MRV and a secondary *when-issued* issue corresponding to symbol MRWI. Both symbols map to the same CUSIP and therefore to the same PERMNO. When such conflicts arise, all but the main issue are dropped. The example is a real case extracted from TAQ.

extended procedure could prove its worth in analyses focusing on short-lived securities and special issues.

## 1.2.2 TAQ cleaning and sampling

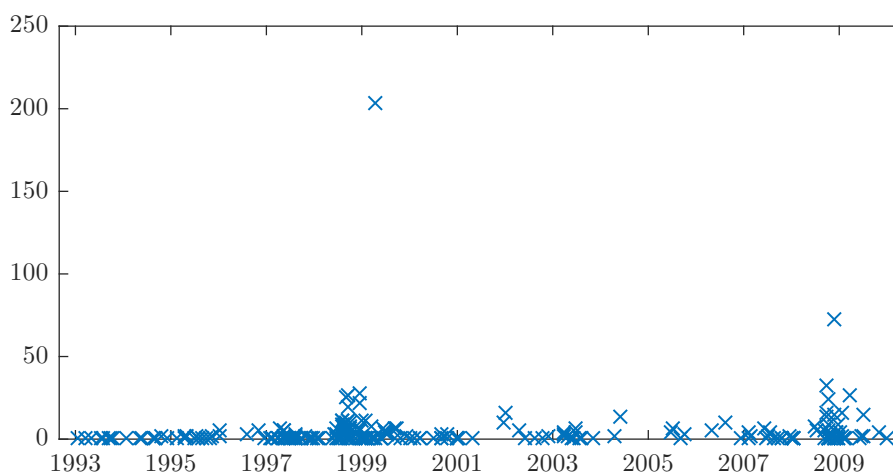
This section describes the cleaning rules applied to TAQ raw trade-data and how the cleaned data is subsequently sampled at a lower frequency.

**Cleaning high-frequency trades.** High frequency data is known to be affected by irregular or misreported trades and [Falkenberry \(2002\)](#) gives a good overview of the problem. For this reason, I follow in spirit [Barndorff-Nielsen, P. R. Hansen, et al. \(2009\)](#) and [Bollerslev, Li, and Todorov \(2016\)](#), and only keep valid prices of the trading session that have not been flagged as irregular by the central tape and which have passed a simple filter for abnormal observations. Specifically, I keep records if:

- the timestamp falls within the NYSE trading hours window which extends from 9:30 to 16:00;
- prices are not corrected, i.e. the CORR field has a value of 0;

- the trade does not qualify as ‘G’, Rule 127 or stopped stock trade, i.e. if the G127 field is either 0 or 40 (can be a Display Book-reported trade);
- sales conditions are normal, i.e. when the COND field contains only the letter codes ‘E’ (Automatic Execution), ‘F’ (Intermarket Sweep Order), ‘ ’ (No Sale Condition) or ‘@’ (Regular Trade);
- the price is positive;
- the traded amount (SIZ) is positive;
- the price is proportionally within an order of magnitude from the daily median of prices.

The filter for the outliers from the last bullet point captures only grossly misreported prices and is thus very conservative. [Figure 1.4](#) backs the claim by showing the daily count of outliers, which most of the time stays below 5-10 bad observations and is valued only on 288 days out of 17 years of data.



**Figure 1.4:** total number of (price) outliers per day in the TAQ trades. For a given stock, an intraday price is defined to be an outlier if it is an order of magnitude bigger or smaller than its median intraday price. The total count sums the outliers of all stocks in the sample. Outliers are detected on only 288 days out of the whole period of analysis that goes from 1993 until 2010 (more than 4000 days).

**Table 1.2:** example of price outlier. The table reports a slice of TAQ records together with CRSP’s PERMNO, the symbol, the name of the company, the date and time of trade, the recorded price, the size of the traded lot (Size), whether it respected G and 127 rules, if there were corrections (Corr) and the condition of the trade (Cond). Null or empty values for G127, Corr and Cond, qualify as a valid trade. The (first) traded price on 2003-10-31 is highlighted in yellow and marks the outlier.

Permno	Symbol	Name	Date	Time	Price	Size	G127	Corr	Cond
77008	XLTC	EXCEL TECHNOLOGY INC	2003-10-30	15:59:03	28.30	100	0	0	
77009	XLTC	EXCEL TECHNOLOGY INC	2003-10-30	15:59:33	28.31	100	0	0	
77010	XLTC	EXCEL TECHNOLOGY INC	2003-10-31	09:30:00	0.01	100	0	0	
77010	XLTC	EXCEL TECHNOLOGY INC	2003-10-31	09:30:01	28.13	300	0	0	
77010	XLTC	EXCEL TECHNOLOGY INC	2003-10-31	09:31:46	28.25	100	0	0	

To give a sense of what qualifies as a grossly misreported price, [table 1.2](#) provides a real case detected by the filter. The table reports a few intraday trades in Excel Technology (XLTC) at the end of October 2003. In the example, none of the records contain invalid prices or are flagged as irregular by the central tape, and in fact the G127, CORR and COND fields all contain acceptable values. However, the first price on October 31st is only \$0.01 as opposed to the prices immediately before and after which have a value of about \$28. [Falkenberry \(2002\)](#) suggests that the record erroneously reports only the decimal component of the price.

On one hand, the impact of the outlier from [table 1.2](#) can be substantial, as e.g. the average open-to-close return for the whole cross-section on the 31st of October 2003 would record a biased value of 77.23% instead of a more realistic -0.54% which is obtained by excluding XLTC. On the other hand, while the economic impact of such outliers might average out over two decades of data, [Anderson, Bianchi, and Goldberg \(2015\)](#) warn that they might induce statistical significance in results that have none. Hence, it is best to remove these observations altogether.

After high-frequency data are cleaned of its irregularities, I consolidate observations that share the same timestamp. Hence, I follow [Bollerslev, Li, and Todorov \(2016\)](#) and take the volume-weighted average of the prices and accumulate the volumes. Another alternative approach, which produces very similar results, is used by [Barndorff-Nielsen,](#)

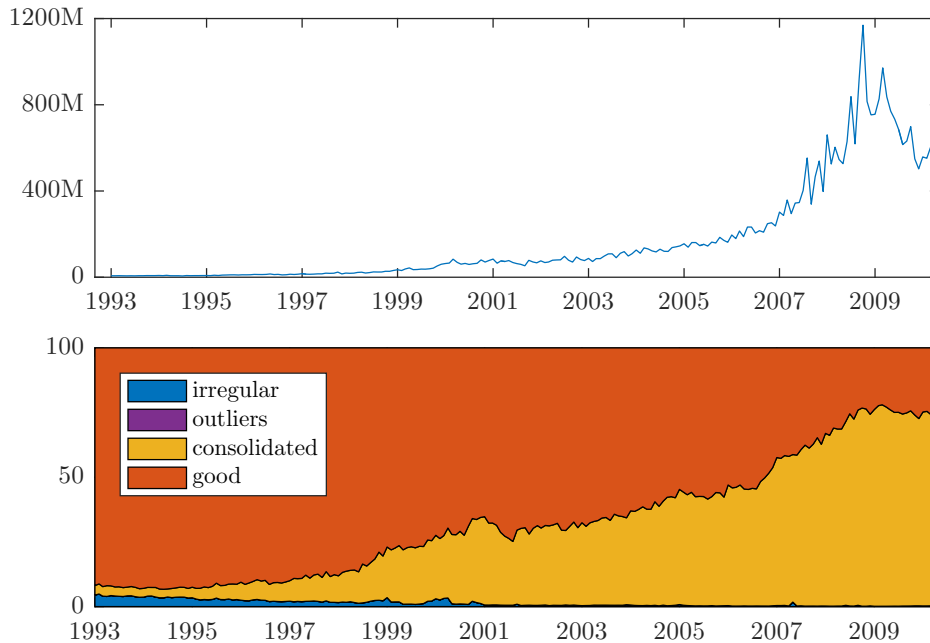
P. R. Hansen, et al. (2009) who take the median of the prices instead of the volume-weighted average.

The impact of the data handling, in terms of loss of observations, is given in [figure 1.5](#) which presents the monthly count of trades over time (top subfigure) and the proportional split among outliers, irregular observations, consolidated and good observations (bottom subfigure). The top figure shows that trading activity increased over time, steadily until 2007 and almost exponentially afterwards, with a peak in 2008. The bottom figure, instead, highlights the relative importance of the preparatory steps that precede data sampling. Most notably, we can observe that the loss of observations due to same-timestamp consolidation is dominant in the most recent years, with a clear upward trend. This result is mechanically induced by two effects, the fixed length of the trading day combined with the increased number of trades and the limited resolution of the timestamp.<sup>5</sup> Irregular trades, while increasing in absolute terms, never take more than 5% of the monthly observations, and outliers at most represent 0.02% (not visible in the figure).

**Sampling.** Prices are sampled at a fixed and relatively low frequency of 5 minutes, for a total of 79 observations per day, ready to be combined into lower frequencies as the need arises. The 5-minute interval has been one of the most widely adopted in literature since the early work by [Andersen and Bollerslev \(1997\)](#) and has been a difficult benchmark to beat in terms of optimal sampling frequency [L. Y. Liu, Patton, and Sheppard \(2015\)](#), at least for realized volatility estimates. Nonetheless, the choice of the sampling interval has received a lot of attention, since the frequency determines the trade-off between the theoretical precision in estimation, which improves as the interval shrinks, and the effects of microstructure noise, which diminishes as the interval widens. The area of research is quite vast and some authoritative references, in the univariate context, are [Lan Zhang,](#)

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<sup>5</sup>I currently use the monthly feed whose timestamp has resolution up to a second, but even with the daily feed, which also records milliseconds, we would still observe a smaller but increasing trend in observations lost to consolidation.

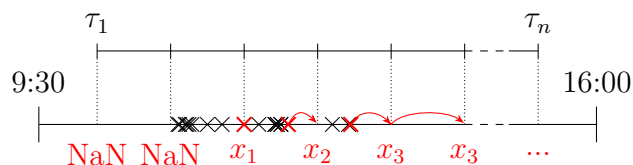


**Figure 1.5:** monthly count of total TAQ trades (top) and percentage split between types of trades (bottom). Trades classify as irregular if the traded amount is not positive, the price is null, the record has been corrected, or qualifies under the G127 rule or corresponds to an abnormal sale condition. A price of a given stock is defined to be an outlier if it is an order of magnitude bigger or smaller than the median intraday price for the same stock. Time-consolidated records corresponds to prices that share the same timestamp minus one observations which is retained. The residual trades are considered good and eligible for sampling. The proportion of outliers is very small and hence not visibly perceptible in the bottom plot.

Mykland, and Aït-Sahalia (2005), Oomen (2006), P. R. Hansen and Lunde (2006b) and Barndorff-Nielsen, P. R. Hansen, et al. (2008).

Figure 1.6 illustrates the mechanics of the sampling procedure. The figure consists of two timelines, with the bottom one representing the trading hours, and the top one being a sampling grid which marks the sampling times on the lower line. Trades are marked with a cross on the bottom line. The sampling uses previous-point interpolation, also known as forward-filling, to pick the most recent trade, at each sampling time, and marks it in red. As with the 5-minute interval, the previous-point method is yet another stronghold in high-frequency sampling and it has been shown by P. R. Hansen and Lunde (2006b) to be the better alternative to linear interpolation, used in earlier studies. The effects of the interpolation are illustrated by the red arrows. At the end of the sampling, since the grid

is fixed, all stocks will have  $n$  observations on every day.



**Figure 1.6:** example of previous-price sampling on a regular 9:00 to 16:00 trading day. Trades are executed throughout the day at times marked by the crosses. The upper grid casts  $n$  sampling points (dotted vertical lines) on the timeline and the most recent trade to each grid-point becomes marked in red to denote that it has been sampled. The arrows show the previous-price interpolation and the carry-over effects, i.e. the last trade within an interval is sampled at the next closest grid-point and if no trades are executed during the next interval, the same price is carried-over to the following point and so forth until the end of the day. Sampling points which are never preceded by any trade are filled with a NaN (Not-a-Number).

The figure encompasses three specific cases of sampling and outlines the role of interpolation. In the trivial case, represented by  $x_1$ , no interpolation occurs since the trade falls exactly on a sampling point. However, the most common case, denoted by  $x_2$ , is when the trade occurs sometime between two consecutive sampling times. As [figure 1.6](#) shows with the red arrow, the trade is brought forward in time to the next sampling point. The last example shows what happens during a period of prolonged inactivity, i.e. when prices are stale. The trade, labelled with  $x_3$ , is first interpolated to the next sampling point and also carried over to the following grid-points until a new trade happens or the day ends.

If a stock is only traded a while after the opening time, the initial sampling points will be filled with NaNs (not a number). I do not fill the series backwards with the first price of the day nor interpolate with the closing price of the previous day. Therefore, I keep the sampling method consistent with the resolution of information over time and free of simplifying assumptions about the opening auction.

To summarize, consistently with prior literature, data are cleaned of irregular trades flagged by the Central Tape. Then, grossly misreported prices are removed by a very parsimonious filter and same-timestamp records are consolidated using a volume-weighted average for prices and summing volumes. Finally, prices are sampled at the 5-minute frequency.

### 1.2.3 A big data problem

The amount of data involved in high-frequency research poses several challenges whose solutions are usually left out of the analysis. This section gives a concise overview of these challenges and their solutions.

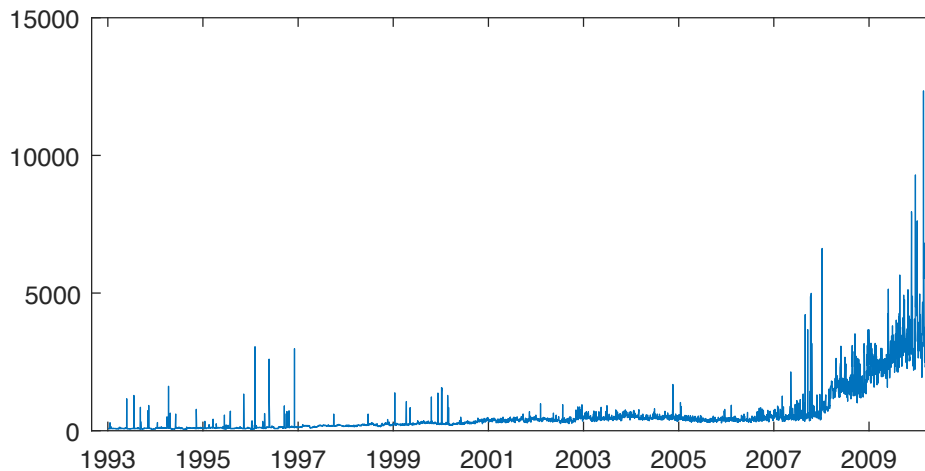
**Size matters.** With technological advances, electronic trading has become ever more pervasive and since 2007 the number of trades has soared exponentially (see [figure 1.5](#)). For example, at the end of the period of analysis, the month of May registered about 800 million trades, which is equivalent to about 15 GB of data.<sup>6</sup> On the other hand, if we were to consider the whole sample, the storage requirement would go up to almost 700 GB. Furthermore, this number represents just about 5% of the whole TAQ database and quotes constitute the remaining part. Thus, high-frequency financial data demands significant amounts of disk-space.

In addition to the storage requirement, [Figure 1.7](#) gives context to the real-time flow of data that e.g. intraday analysis or trading systems have to cope with. The figure plots the maximum number of trades per second in a day and shows that the count peaks at more than 12,000. While this statistic might not be very impressive in terms of disk-space, it stresses latency and resiliency prerequisites in real-time systems. In other words, each stream of data has to be processed and analysed by the second and these operations must also be reliably executed under unexpected increases in loads. Although these requirements might not be as stringent in research, the total time of execution becomes central when processing huge amounts of information.

**Fundamental changes in the workflow.** The examples given above, all emphasize the technical aspects of a problem that ultimately lies with adopting the correct workflow. In essence, it is not possible to approach the analysis in the usual way, i.e. the limitations

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<sup>6</sup>The estimate assumes a trade is stored in the TAQ binary format which uses 19 bytes per record and I ignore index files in the size count. Alternative formats that repeat symbol and date, and use e.g. csv files to store trades, will easily take up twice the space needed by the binary format.



**Figure 1.7:** daily series of the maximum number of TAQ trades executed in a second on a single stock. The surge after 2007 corresponds to an increase in automated High-Frequency Trading.

imposed by the data invalidate the one-action-per-task workflow, and alternative solutions must be taken.

One way to tackle Big Data problems is to scale up the infrastructure and, for example, use a clustered file system. However, using large-scale systems is just one side of the coin and handling huge amounts of data also asks for specific programming paradigms. The MapReduce, an industry standard by [Dean and Ghemawat \(2008\)](#), is one such programming model which processes data by blocks and in parallel on a distributed environment.

A concrete example on high-frequency data will clarify. Suppose a researcher is interested in the average return of a company, e.g. Apple Inc. (AAPL), over its lifetime. In the usual one-action-per-task approach, we would just load into RAM the whole series of returns and apply some function to get the desired statistic. Nonetheless, the task is slightly more complicated if the series does not fit into memory as in the case with AAPL. Therefore, we need to partition the data into  $m$  blocks and perform several actions before getting the average return. This limitation forces to decompose the mean, or any other function of interest, into a linearly additive form (or parallelize, in computing jargon). For instance, each partition of returns is summed and counted on separate instances, and the intermediate results, respectively  $s_i$  and  $N_i$ , are later consolidated together as in



$$\bar{r} = \frac{1}{N} \sum_t^N r_t = \frac{s_1 + s_2 + \cdots + s_m}{N_1 + N_2 + \cdots + N_m},$$

where  $s_i = \sum_t^{N_i} r_t$  and  $N = \sum_i^m N_i$ . On the two sides of the rightmost equality is the sample mean (LHS) and the partitioned mean (RHS), and the difference between the two lies only in the implementation. The former does not scale indefinitely while the latter does so by reducing a block of data to a mere sum and count, before processing the next partition.

Another concept that becomes necessary when working with partitioned data is caching of intermediate results. It arises when a statistic, e.g. the variance, needs some partial result, i.e. the average, that has to be calculated on data which spans several blocks. A solution is to split the data such that the partial result can be calculated from a single block. However, this is often impossible to achieve with multidimensional datasets like a cross-section of stocks. If the horizon is sufficiently long, we cannot partition the cross-section by stock, because even a single series, as with AAPL, will not fit into memory. Hence, to calculate stock variances, previously computed means need to be grouped and replicated as necessary to match the partitioning.

The actual infrastructure is based on a local implementation of the MapReduce, which executes jobs in parallel on up to 8 cores, and stores data as a collection of compressed binary files (.mat files). The parallel execution achieves a speedup of execution almost equivalent to the number of cores while compression allows to keep all data on a local hard drive, achieving a compression factor of a magnitude. In this setting, even though compression trades disk-space for CPU cycles (time), a job runs on average in an hour (recall the example of the mean).<sup>7</sup>

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<sup>7</sup>The implementation runs on a PC with an Intel Core i7-2700 CPU, 500 GB of disk space and 16 GB of RAM. Although, hardware specifications can be lowered to a CPU with 4 cores, 150 GB of disk space and 8 GB of RAM.

## 1.3 Methodology and preliminary results

This section outlines two implementations of the Betting Against Beta (BAB) factor by [Frazzini and Pedersen \(2014\)](#). It starts by presenting the general testing framework and follows with the details of the factor construction. The description concludes with some preliminary results.

The BAB factor is short high-beta securities and buys leveraged low-beta securities; I implement it in two versions, using daily and high frequency returns.<sup>8</sup> The two portfolios share the same investable universe, formation and holding periods, and are rebalanced at the beginning of each month. The choice of a common lookback ensures that the only difference in the information set comes from the sampling frequency of the returns. Moreover, the trading schedule is identical, implying the same cost-profile for the two versions. In other words, since the focus of the analysis is to uncover eventual differences in performance and because costs bear the same impact on the two strategies, they will be disregarded altogether.

### 1.3.1 Estimating betas

Consistently with [Frazzini and Pedersen \(2014\)](#), henceforward FP, I estimate low-frequency betas with a simple linear regression of stock returns on market returns (plus a constant). I use daily observations and log-returns in excess of the one month T-bill rate. The market portfolio and the risk-free rate are from professor's [Kenneth French Data Library](#).<sup>9</sup> The estimated low-frequency (LF) market beta for stock  $i$  is given by

$$\hat{\beta}_i^{\text{LF}} = \frac{\widehat{\text{cov}}(r_i, r_0)}{\widehat{\text{var}}(r_0)}, \quad (1.9)$$

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<sup>8</sup>Henceforward, depending on the context, I will interchangeably refer to the implemented BAB factors by calling them portfolios, strategies or signals.

<sup>9</sup>The market portfolio is defined as the value-weighted index of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ that have a CRSP share code of 10 or 11.

where, consistently with [equation \(1.2\)](#),  $\widehat{\text{cov}}(r_i, r_0)$  and  $\widehat{\text{var}}(r_0)$  are respectively the estimated covariance between the stock and the market returns, and the variance of the market factor.

In estimating high-frequency betas, I use the realized variants of the covariance and variance. Akin to [Bollerslev, Li, and Todorov \(2016\)](#), prices are sampled every 75 minutes starting at 9.45 am until 4.00 pm for a total of five returns per day plus the overnight. The sampling grid might seem coarse relative to the commonly adopted 5-minute grid, which produces 78 returns. However, while for univariate measures the 5-minute step strikes an optimal balance between microstructure noise and precision of estimation, and according to [L. Y. Liu, Patton, and Sheppard \(2015\)](#) there is hardly any worth in alternative specifications for realized variance, on the other hand, for multivariate measures there are additional complications. For instance, [Epps \(1979\)](#) documents that asynchronous trading biases the realized covariance towards zero as the sampling is performed at higher frequencies.

Research on optimal sampling offers several competing alternatives to circumvent the undesirable impact on covariance estimates of the Epps effect and the microstructure noise in general. Among the solutions [Barndorff-Nielsen, P. R. Hansen, et al. \(2011\)](#) have suggested using the refresh-time sampling scheme along with multivariate realized kernels, [Sheppard \(2006\)](#) adopts a scrambling procedure, [Christensen, Oomen, and Podolskij \(2014\)](#) formulate a theory of pre-averaged measures and [Lan Zhang \(2011\)](#) extends the two-scale estimation to multi-variate measures. However, along these more sophisticated solutions, it is still possible to simply lower the sampling frequency and trade-off accuracy to obtain a relative stable estimate of the realized covariance. Due to its simplicity, most of the literature follows this approach and so do I by picking an intermediate sampling interval of 75 minutes.

The estimated - realized - high-frequency (HF) betas for stock  $i$  are given by

$$\hat{\beta}_i^{\text{HF}} = \frac{\text{RC}_i}{\text{RV}_0}, \quad (1.10)$$

where the  $\text{RC}_i$  and  $\text{RV}_0$  are defined by equations (1.6) and (1.7) to be respectively the estimated realized covariance between stock and market returns, and the realized variance of the market. I adopt the S&P 500 ETF (SPY), readily available in TAQ, as a proxy for the high-frequency market returns. However, since LF betas use a different proxy, I verify that results are not affected by this choice in section 1.4 by using close-to-close returns of the SPY series.<sup>10</sup>

Betas are estimated with daily and high-frequency observations over formation periods of a year and six, three and one months. Consistent with FP, I require at least 200, 100, 50 and all trading days of non-missing data for the periods listed above. All estimation windows are rolledover each month and thus overlap, with the exception of the single-month period.

Finally, to reduce the impact of outliers, I apply the same shrinkage used by FP with their set of parameters. At each date, time-series betas are shrunk towards their cross-sectional mean. Formally, if I denote with  $\hat{\beta}_i^{\text{TS}}$  the time-series beta for the  $i$ -th stock, regardless of their low- or high-frequency nature, the shrunk beta is given by

$$\hat{\beta}_i = w_i \hat{\beta}_i^{\text{TS}} + (1 - w_i) \hat{\beta}^{\text{XS}}, \quad (1.11)$$

where, for all dates and stocks, the shrinkage factor  $w_i$  is kept constant at 0.6 and the cross-sectional average  $\hat{\beta}^{\text{XS}}$  at 1.<sup>11</sup> The impact of different values of the shrinkage parameter is assessed in section 1.4.

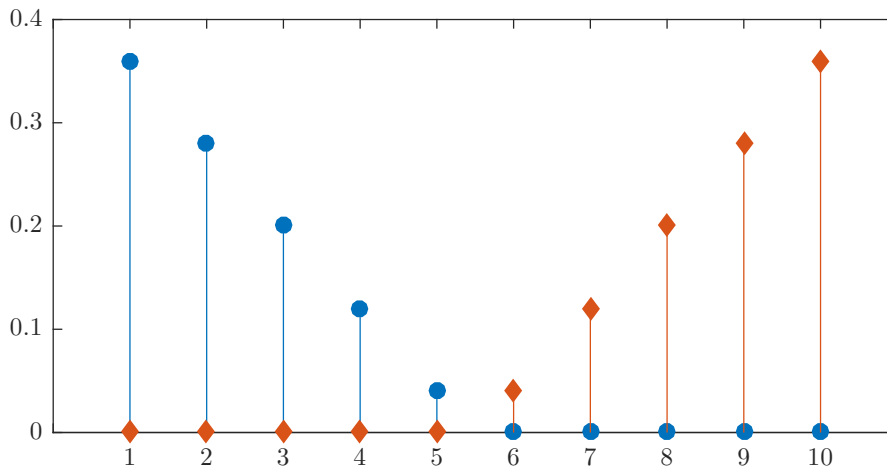
<sup>10</sup>Results are unaffected by the choice of the proxy and are available upon request.

<sup>11</sup>The same shrinkage factor as in FP is used for all formation periods. The only exception goes for the monthly window which requires a factor of 0.4 since it is more sensitive to outliers because of its shorter length.

### 1.3.2 Constructing the BAB factor

The construction of the BAB factor is exactly the same as in FP and it is here reported for clarity.

Stocks are ranked in ascending order by their estimated betas and the first half of the securities are assigned to the low-beta portfolio while the other half to the high-beta portfolio. The weights, within each portfolio, are determined by the distance of each rank from the average overall rank. Hence, contributions are linearly increasing as we approach both extremes of the ordering. [Figure 1.8](#) presents an example of weights for ten pre-ordered securities, i.e. the first stock has the lowest beta and the last one the highest beta. The weighting scheme is symmetric around the average overall rank of 5.5, with the (positive) proportions for the low- and high-beta portfolios respectively on the left and right side of the figure. Both sets of weights sum up to one by construction and details follow below.



**Figure 1.8:** example of weights for the low-beta portfolio (circles) and the high-beta portfolio (diamonds) which are used to construct the BAB factor. The sample is assumed to have a cross-section of ten stocks, already sorted in ascending order on the value of their market beta, i.e. the stocks with lowest and highest betas are respectively in position one and ten. Weights sum up to one by construction and are linearly increasing in the distance from the average rank of the beta. Stocks with a beta ranking above (below) the average have null weight and thus are not included in the low(high)-beta portfolio.

Formally, given a set of  $n$  securities, their beta ranks  $z_i = \text{rank}(\beta_{i,t})$  and the average

rank  $\bar{z} = (n + 1)/2$ , the weights for the low- and high-beta portfolios are generated by

$$\begin{aligned} w_i^L &= -\frac{(z_i - \bar{z})^-}{k} \\ w_i^H &= \frac{(z_i - \bar{z})^+}{k} \end{aligned} \tag{1.12}$$

where  $k$  normalizes the weights to one and is equal to  $k = \frac{1}{2} \sum |z_i - \bar{z}|$ , with  $x^+$  and  $x^-$  respectively equal to  $\max(x, 0)$  and  $\min(x, 0)$ . Then, both portfolios are rescaled to have a beta of one and the BAB is short high-beta securities and buys low-beta securities, with overall factor return of

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^L} (r_{t+1}^L - r^f) - \frac{1}{\beta_t^H} (r_{t+1}^H - r^f), \tag{1.13}$$

where  $\beta_t^* = \sum w_i^* \beta_{i,t}$  and  $r_{t+t}^* = \sum w_i^* r_{i,t+1}$ . The BAB is a self-financing strategy financed by the risk-free and a beta-neutral portfolio by construction, i.e. its beta is equal to zero.

### 1.3.3 Descriptive statistics

This section presents descriptive statistics and preliminary results for all estimation windows of the BAB factor. To distinguish between the high- and low-frequency versions of the strategy, I label with *bab* the portfolio whose signals are estimated with daily returns, and with *rbab* (realized BAB) its high-frequency counterpart. The labels contain a subscript to indicate the formation period of the signals which can be of a year (*y*), semester (*s*), quarter (*q*) and of one month (*m*).

**Analysis of correlations.** Table 1.3 reports the time-series average of cross-sectional correlations among the estimated betas, which are the (sorting) signals for the BAB portfolios. The table has two types of correlations coefficients, with Spearman's rank correlations appearing above the main diagonal and the conventional Pearson's coefficients below the main diagonal. For convenience, the matrix is arranged in blocks: the upper-left portion lists

**Table 1.3:** time-series average of cross-sectional correlations between market betas estimated over formation periods of a month, a quarter, a semester and a year. Spearman’s rank correlations appear above the main diagonal and Pearson’s coefficients are below the main diagonal. The matrix is arranged in blocks: the upper-left portion lists only the interactions among the low-frequency betas, i.e.  $bab_m$ ,  $bab_q$ ,  $bab_s$  and  $bab_y$ , and the lower-right portion has the correlations between the high-frequency betas only, namely  $rbab_m$ ,  $rbab_q$ ,  $rbab_s$  and  $rbab_y$ . The lower-left and upper-right blocks address the cross-interaction between the low- and high-frequency variants. Low-frequency betas are calculated from simple linear regressions of stock excess returns on market excess returns. High-frequency betas are calculated as the ratio of realized covariance between stock and market intraday returns over the realized variance of the market. Intraday returns are sampled at 75 minutes. The average correlations are calculated over the 1993-2010 period. The sample covers common US equities and daily returns are from CRSP and intraday returns are from TAQ.

	$bab_m$	$bab_q$	$bab_s$	$bab_y$	$rbab_m$	$rbab_q$	$rbab_s$	$rbab_y$
$bab_m$		0.61	0.64	0.53	0.60	0.51	0.53	0.47
$bab_q$	0.60		0.62	0.75	0.63	0.72	0.61	0.66
$bab_s$	0.64	0.61		0.76	0.87	0.73	0.77	0.68
$bab_y$	0.51	0.75	0.76		0.77	0.90	0.76	0.83
$rbab_m$	0.58	0.61	0.88	0.77		0.79	0.85	0.74
$rbab_q$	0.49	0.71	0.73	0.91	0.80		0.79	0.88
$rbab_s$	0.52	0.60	0.77	0.76	0.85	0.79		0.83
$rbab_y$	0.45	0.66	0.68	0.84	0.74	0.88	0.84	

only the interactions among the LF strategies, i.e.  $bab_m$ ,  $bab_q$ ,  $bab_s$  and  $bab_y$ , and the lower-right portion the correlations between the HF strategies only, namely  $rbab_m$ ,  $rbab_q$ ,  $rbab_s$  and  $rbab_y$ . The lower-left and upper-right blocks, instead, address the cross-interaction between the LF and HF variants.

First, all coefficients fall between 0.45 and 0.91, indicating an expected high degree of similarity between the signals. Second, there are no major differences between Spearman’s and Pearson’s correlations, suggesting the absence of outliers among the estimated betas.<sup>12</sup> Nonetheless, a few differences emerge when comparing HF and LF signals. For instance, looking at the lower-right block in [table 1.3](#), the average correlation among HF signals is about 82%, sensibly higher than the average coefficient of 64% calculated on the LF signals, in the upper-left block of the table. Especially influential in the latter average is

<sup>12</sup>Spearman’s correlation is robust to outliers since all values are first converted into ranks. Hence, no matter how extreme is an influential observation, its value is shrunk to at most the minimum or maximum rank.

the monthly signal, i.e.  $bab_m$ , which exhibits low affinity with all the other signals of the same low-frequency nature. Moreover, when paired with the HF variants, the  $bab_m$  scores the lowest correlation of 0.45 with the  $rbab_y$  and only 0.58 with  $rbab_m$ , i.e. the HF version with the same formation period. On the other hand, the converse is not true: that is, the correlation between  $rbab_m$  and the other HF signals, or the LF variants, are not necessarily the lowest coefficients among all pairings. Additionally, looking at the main diagonals of the lower-left and upper-right blocks in the [table 1.3](#), the degree of affinity between HF and LF signals with the same formation period improves as the lookback increases. For example, the correlation between  $bab_m$  and  $rbab_m$  is 0.58, and it increases to 0.84 for the yearly pair, i.e. between  $bab_y$  and  $rbab_y$ . Hence, the formation window seems to play a role in differentiating the signals, as shorter periods correspond to lower correlations. Moreover, this effect is strongest with  $bab_m$  and the other HF strategies suggesting that intraday data might also be relevant.

To summarize, although all signals are very similar to each other, there are perceivable differences driven by changes in the estimation window and some indication that HF signals bear some systematic dissimilarities from the LF alternatives.

**Descriptive statistics.** [Table 1.4](#) reports descriptive statistics of monthly returns earned by the low- and high-beta portfolios and the BAB factor. The table is split in four panels, one per estimation window of the betas, each showing side-by-side the results for the LF and HF versions of the signals. Among others statistics, I report the average return ( $avgret$ ) and standard deviation ( $std$ ), their annualized equivalents ( $annret$  and  $annstd$ ), the annualized Sharpe Ratio ( $SR$ ), the [Newey and West \(1987\)](#) standard error of the average return computed with fixed lags ( $se$ ) and its p-value ( $pval$ ), the skewness and kurtosis ( $skew$  and  $kurt$ ), and the average beta for each portfolio ( $avgbeta$ ), see [equation \(1.13\)](#), where BAB by construction has 0. Returns, and related measures, are expressed in percentage.

The results detailed for the  $bab_y$ , bottom-right panel on the left-side, are partially in



**Table 1.4:** descriptive statistics of monthly returns earned by the low- and high-beta portfolios and the Betting-Against-Beta (BAB) factor. The table lists in this order: the average return (*avgret*), standard deviation (*std*), the default-bandwidth Newey-West standard error of the average (*se*) and its pvalue (*pval*), the annualized return, standard deviation and Sharpe Ratio (*annret*, *annstd* and *SR*), the annualized downside deviation with return threshold at 0 (*downstd*), the minimum, median and maximum returns (*minret*, *medret* and *maxret*), skewness and kurtosis (*skew* and *kurt*), the maximum drawdown return, its length in months and the number of months that the price series took to recover from it (*mdd*, *mddlen* and *reclen*), the sortino ratio (*sortino*), and the time-series average of the cross-sectional averages of betas within a portfolio (BAB is beta-neutral by construction). All return are in percentage. The table is organized in panels by estimation window of the betas (month, quarter, semester, year), and each panel has statistics for portfolios based on low-frequency (Daily) and high-frequency (Intraday, shaded sections) betas. Specifically, low-frequency betas are calculated from simple linear regressions of stock excess returns on market excess returns. High-frequency betas are calculated as the ratio of realized covariance between stock and market intraday returns over the realized variance of the market. Intraday returns are sampled at 75 minutes. Portfolio formation is described in section 1.3.2. The sample covers common US equities and daily returns are from CRSP and intraday returns are from TAQ. Statistics are calculated over the entire period of analysis from 1993 until 2010.

	Daily			Intraday			Daily			Intraday		
	Low	High	BAB	Low	High	BAB	Low	High	BAB	Low	High	BAB
	<b>month</b>						<b>quarter</b>					
<i>Avgret</i>	0.68	0.21	0.47	0.66	0.36	0.30	0.74	0.26	0.48	0.84	0.34	0.50
<i>Std</i>	6.60	5.27	4.32	5.01	6.58	4.75	5.49	4.81	3.59	5.02	6.06	4.68
<i>Se</i>	0.48	0.38	0.29	0.38	0.46	0.35	0.41	0.35	0.27	0.38	0.44	0.37
<i>Pval</i>	0.16	0.58	0.10	0.08	0.44	0.39	0.07	0.46	0.08	0.03	0.44	0.18
<i>Annret</i>	8.11	2.51	5.61	7.94	4.35	3.58	8.89	3.15	5.74	10.06	4.04	6.02
<i>Annstd</i>	22.86	18.27	14.97	17.36	22.79	16.46	19.02	16.65	12.45	17.39	20.98	16.20
<i>SR</i>	0.35	0.14	0.37	0.46	0.19	0.22	0.47	0.19	0.46	0.58	0.19	0.37
<i>Downstd</i>	16.62	12.79	10.80	12.32	15.69	11.81	13.08	11.64	8.29	11.75	14.56	10.65
<i>Minret</i>	-29.49	-19.41	-21.21	-23.14	-21.51	-24.76	-21.36	-16.57	-15.69	-21.65	-20.85	-20.11
<i>Medret</i>	1.40	0.36	0.69	1.19	0.54	0.34	1.23	0.45	0.54	1.11	0.47	0.32
<i>Maxret</i>	16.79	22.46	12.81	12.16	26.80	17.61	15.68	17.51	13.96	15.87	22.16	20.99
<i>Skew</i>	-0.96	-0.07	-1.08	-0.92	-0.07	-0.78	-0.59	-0.21	-0.27	-0.70	-0.16	-0.05
<i>Kurt</i>	5.99	5.79	8.68	5.62	4.76	10.19	5.19	4.72	6.96	5.87	4.49	8.52
<i>Mdd</i>	57.97	63.94	54.54	48.95	73.92	62.99	47.25	60.18	38.61	43.66	70.11	62.60
<i>Mddlen</i>	31	108	29	21	108	23	21	108	27	21	108	23
<i>Reclen</i>	-92	-15	-115	-15	-15	50	-15	-15	11	-15	-15	25
<i>Sortino</i>	0.49	0.20	0.52	0.64	0.28	0.30	0.68	0.27	0.69	0.86	0.28	0.57
<i>Avgbeta</i>	0.66	1.57	0	0.75	1.30	0	0.66	1.62	0	0.66	1.33	0
	<b>semester</b>						<b>year</b>					
<i>Avgret</i>	0.63	0.22	0.41	0.81	0.29	0.52	0.69	0.18	0.52	0.78	0.23	0.55
<i>Std</i>	4.99	5.13	4.03	5.12	6.25	5.20	4.59	5.21	4.39	5.08	6.26	5.42
<i>Se</i>	0.37	0.37	0.32	0.39	0.45	0.40	0.35	0.39	0.35	0.39	0.47	0.44
<i>Pval</i>	0.09	0.55	0.20	0.04	0.52	0.19	0.05	0.65	0.14	0.05	0.62	0.21
<i>Annret</i>	7.54	2.65	4.89	9.67	3.45	6.23	8.33	2.13	6.20	9.42	2.77	6.65
<i>Annstd</i>	17.28	17.77	13.95	17.75	21.67	18.00	15.91	18.05	15.19	17.60	21.69	18.79
<i>SR</i>	0.44	0.15	0.35	0.54	0.16	0.35	0.52	0.12	0.41	0.54	0.13	0.35
<i>Downstd</i>	12.15	12.57	9.47	11.98	15.23	11.77	10.70	12.96	9.88	11.95	15.47	12.30
<i>Minret</i>	-19.37	-18.12	-19.34	-21.25	-22.28	-22.10	-16.34	-18.68	-19.87	-20.09	-22.19	-26.10
<i>Medret</i>	1.20	0.46	0.39	1.20	0.48	0.31	1.27	0.30	0.28	1.31	0.33	0.14
<i>Maxret</i>	15.27	16.43	15.79	16.05	20.26	21.87	13.69	16.03	16.73	16.60	19.45	21.23
<i>Skew</i>	-0.72	-0.27	-0.31	-0.59	-0.24	-0.05	-0.49	-0.38	-0.12	-0.61	-0.33	-0.15
<i>Kurt</i>	5.15	4.55	7.88	5.64	4.21	8.16	4.76	4.21	8.24	5.64	3.97	8.30
<i>Mdd</i>	46.40	62.10	49.66	44.03	70.74	62.30	42.68	63.79	54.67	40.66	71.96	66.54
<i>Mddlen</i>	21	108	23	21	108	30	21	108	23	21	108	23
<i>Reclen</i>	-15	-15	48	-15	-15	24	-15	-15	18	-15	-15	26
<i>Sortino</i>	0.62	0.21	0.52	0.81	0.23	0.53	0.78	0.16	0.63	0.79	0.18	0.54
<i>Avgsignal</i>	0.72	1.58	0	0.69	1.32	0	0.77	1.53	0	0.71	1.29	0

line with those reported by FP. For instance, the strategy scores a monthly average return of 52 basis points (bp), which is 30% smaller than the 70 bp listed by FP for US equities, and has a p-value of only 0.14 (*se* of 35 bp). The Sharpe ratio is half that of FP with a value of 0.41 versus 0.78 which reflects the relatively high standard error. Even though the Sharpe ratio does not line up with FP, the average betas of the securities in the low- and high-portfolios, with values of 0.77 and 1.53, are very close to those listed for the decile portfolios in Frazzini and Pedersen (2014).

The outcome for the other estimation periods is very similar to those described for the  $bab_y$ , although in principle not directly comparable to FP. Average returns, Sharpe ratios and, in particular, average betas do not change much across estimation windows. However, all  $rbab$  strategies exhibit slightly worse performance and statistical significance than their corresponding  $bab$  versions, an observation that already comes up in figure 1.9. Noticeable differences across LF and HF versions appear in the average beta of the high-beta portfolio, the leg that is shorted in the BAB factor. The average LF beta has a value of about 1.6 across all estimation windows while its HF counterpart settles at around 1.3. Finally, the distribution of BAB returns is not normal with the kurtosis usually above seven although the skewness is often close to zero, especially so for  $rbab_q$ ,  $rbab_s$  and  $rbab_y$ .

For a visual inspection of all strategies, figure 1.9 plots the monthly cumulated returns, rebased at one, of the low- and high-beta portfolios, defined by the first and second term of equation (1.13), and the BAB portfolio, resulting from the combination of the two components. There are a total of four pairs of side-by-side plots, vertically ordered by increasing formation period, with the  $bab$  strategies on the left and the  $rbab$  strategies on the right side. In all plots, a vertical line marks April 2001, the date when the decimalization was extended to the NASDAQ market, briefly after its introduction to the NYSE in January of the same year.<sup>13</sup>

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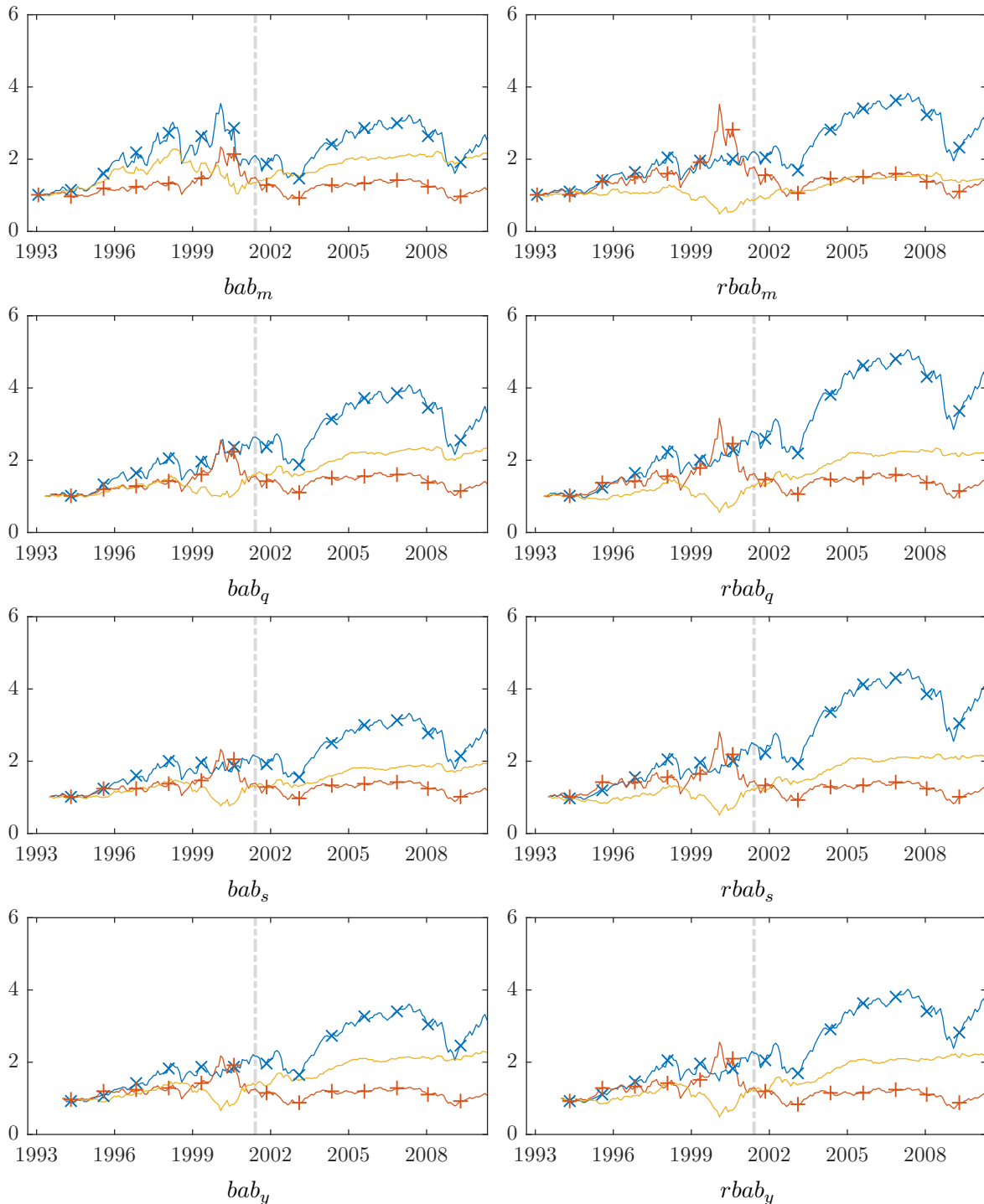
<sup>13</sup>The decimalization introduces finer increments in stock prices. Before its introduction, prices were allowed to change by pre-set fractional amounts, with 1/16 being the smallest allowed change.

The figure reflects the results anticipated by the high correlations between signals in [table 1.3](#), i.e. the cumulated returns all trace very similar paths to each other. Nonetheless, upon closer inspection of the solid lines without markers, all *rbab* strategies (on the right) underperform their *bab* counterparts (on the left) over the entire horizon, with the worst case captured by the monthly estimation window (top row of plots). Aside any differences between the HF and LF versions, it seems that the BAB factor falls short of expectations. I contextualize this graphical evidence with extensive descriptive statistics in [table 1.4](#). Preliminary results on the whole period of analysis suggest that estimating the betas, the signals for our BAB strategy, with intraday returns in place of daily observations can only worsen performance as measured by Sharpe ratios. However, for reasons outlined in the following section, these implications are not final and should be re-considered over the more recent period that starts from the decimalization of price quotes in April 2001.

### 1.3.4 Dot-com bubble and price decimalization

This section provides the motivation for focusing on the sub-period that comes after April 2001, and an update, with new evidence, to the results from the previous section.

**Excluding the pre-decimalization period.** I limit the analysis to a more recent sub-period in order to avoid the dot-com bubble and the excess noise in the estimation of realized betas. First, it is well known that internet stocks have experienced a sharp rise in prices and an even more pronounced fall during the period from January 1998 until February 2000. [Ofek and Richardson \(2003\)](#) explain this extraordinary event with the excessive optimism of some agents, the inaction of pessimistic agents limited by short-sale constraints and lockup expiration agreements. Moreover, [Estrada \(2004\)](#) finds the market beta of such companies, calculated against the S&P 500 index, to average at an impressive 2.5. Hence, it is not surprising, that the prediction of FP's model is inconsistent and the BAB factor is bound to lose money by virtually shorting internet companies. In fact, this



**Figure 1.9:** monthly cumulated returns of the low-beta (cross-marked), high-beta (plus-marked), and the Betting-Against-Beta (BAB) portfolios (no markers). On the left (right) side, portfolios are formed on low(high)-frequency betas. Betas are estimated, from top to bottom, over the past month, quarter, semester and a year. Specifically, at month-end, stocks are sorted by the rank of their market beta (the signals) into low- and high-beta portfolios, with weights linearly increasing in the distance of each rank from the average rank (see [equation \(1.12\)](#) and [figure 1.8](#)). The BAB factor is the difference in excess returns of the levered versions of the low- minus high-beta portfolios (see [equation \(1.13\)](#)). Stocks are held for a full month. Low-frequency betas are calculated from simple linear regressions of stock excess returns on market excess returns. High-frequency betas are calculated as the ratio of realized covariance between stock and market intraday returns over the realized variance of the market. Intraday returns are sampled at 75 minutes. All portfolios are re-based at one and the vertical line marks April 2001, the date of the final decimalization of price quotes. The sample covers common US equities and daily returns are from CRSP and intraday returns are from TAQ.

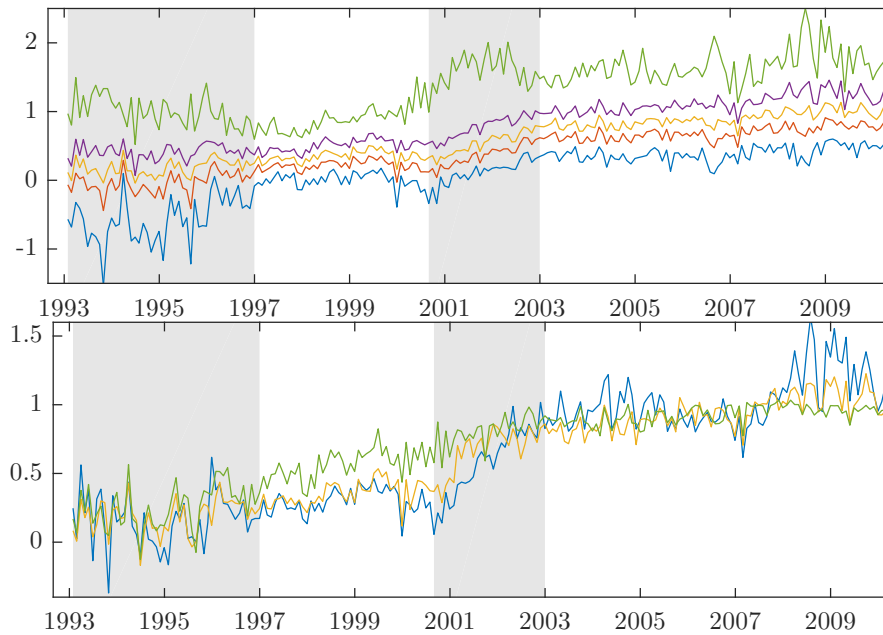
is exactly what happens in [figure 1.9](#): the high-beta portfolio, the solid line with the *plus* markers (+), outperforms the low-beta portfolio, the solid line with the *cross* markers (×). However, we should not be including these stocks in the sample at all, owing to their rigid constraints on short-sales. It is also worth noting that FP cover the period from 1926 until 2012 while I am restricted to 1993 by TAQ’s availability, and therefore the impact of the bubble is much stronger in my results.

The second reason to exclude the initial period is to avoid the noise, due to lack of liquidity, and the bias, coming from market microstructure limitations, in the estimates of realized measures. These complications are easier to show for the sub-sample of the S&P 500 constituents with betas calculated at the 5-minute frequency, which allows to emphasize the impact of the Epps effect. [Figure 1.10](#) reports the time-series of average betas for this sub-sample. The top-plot draws the time-series of the average betas for beta-sorted quintile portfolios, while the bottom plot has equivalent averages but for capitalization-sorted quintile portfolios (only bottom, mid and top portfolio averages are shown for clarity). The shaded areas mark the periods affected by the estimation issues.

The first shaded area covers the initial period that runs from 1993 through 1997 and is characterized by intense intraday illiquidity.<sup>14</sup> In fact, the price series of many US stocks and the S&P 500 ETF are very stale, i.e. sampled intraday returns are often null. This has a two-fold impact on the distribution of realized betas. First, the covariance shrinks, as a direct consequence of the Epps effect, and betas distribute almost symmetrically around zero (the time-series average of the 3rd portfolio averages at 0.2), far from the sub-sample’s (value-weighted) average beta of one. Second, illiquid stocks will have noisier beta estimates and the overall distribution will be more dispersed. In fact, the few non-null returns, which survive the cross-product of the realized covariance from [equation \(1.6\)](#), have a high chance of being substantially bigger in magnitude than those of the market

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<sup>14</sup>I define intraday activity as the daily proportion of non-null returns computed at a given frequency, e.g. 5-minute sampling, over the number of sampling points. Average results for the S&P 500 constituents and its ETF are available on request.



**Figure 1.10:** average of high-frequency betas of the cross-section of S&P 500 constituents, calculated with intraday returns sampled at 5 minutes (and excluding the overnight). The top plot shows the average betas for beta-sorted quintile portfolios and the bottom plot shows the average betas for quintile portfolios sorted on market capitalization. In the top plot, the first portfolio has the lowest values and the fifth the highest ones. The bottom plot shows only the first, third and fifth portfolios and their order follows the top plot under the 2001 mark, i.e. lowest values correspond to first portfolio and highest to fifth. High-frequency betas are calculated as the ratio of realized covariance between stock and market intraday returns over the realized variance of the market. The shaded bands cover two periods affected by estimation bias in the betas. In the first period, the whole cross-section of S&P 500 constituents is affected by the [Epps \(1979\)](#) effect. During the second period, the trend in the betas (top) is driven by the gradual improvement of liquidity in small-caps (bottom). Intraday returns are from TAQ.

proxy, since decimalization has not occurred yet. This means that the covariance will be big relative to the market variance and will compose into imprecise and extreme estimates of the realized beta. This outcome, which can be observed in the conic shape traced by the average betas of the 1st and 5th quintile portfolios, slowly vanishes over time as the liquidity of the market proxy improves.

The second shaded area covers the introduction of the decimalization, piloted in late 2000, and extends until 2003. The betas of the quintile-sorted portfolios, in the bottom plot of [figure 1.10](#), delineate an increasing trend where the median slowly converges to the expected value of one. Stocks are traded more often thanks to the finer increments in quotes and liquidity improves across the whole sub-sample. However, we observe a trend

instead of a shift in the beta-sorted quintile portfolios (top plot), because low-capitalization stocks (bottom line in the bottom plot) have a slower response to the introduction of finer quotes. While the average beta of large-caps has been gradually converging towards one since 1997, small-caps had betas consistently biased towards zero until late 2000, and only afterwards their estimates benefit from the increased liquidity.

To sum up, the period from 1993 until 2001 is not appropriate for my comparative study in view of the inconsistency of the FP model with the dot-com bubble and because of the systematic noise and bias in the estimates of realized betas. Therefore, I condition correlations, descriptive statistics and the plots with the cumulated returns on the period starting April 2001. The tables and the figure can be found in the appendix, respectively under [table A1](#), [table 1.5](#) and [figure A1](#).

**Descriptive statistics after decimalization.** After April 2001, correlations among signals are slightly higher and fall between 0.49 and 0.95. The same considerations made for [table 1.3](#) apply to [table A1](#) and hence are omitted for clarity.

More relevant is [table 1.5](#), which reports overall better performance metrics across all formation windows. Specifically, the results for the  $bab_y$  strategy are now in line with FP. In fact, the Sharpe ratio has a value of 0.72, much closer to the 0.78 reported by FP than the previous value of 0.40, and the average monthly return of 50 bp, while unchanged, is now significant at the 5% level. Most importantly, the ranking between LF and HF portfolios this time is reversed, with the  $rbab$  strategies systematically scoring higher average returns and p-values below the 5% level.

Moreover, the Sharpe ratios of the HF implementation remain stable at 0.75 when estimating betas over the past year, semester and quarter and a slightly lower value of 0.70 for the monthly window. In contrast, the LF implementation performs worse producing a Sharpe ratio of 0.58 when using semiannual or quarterly betas but records values of 0.72 and 0.71 respectively for the yearly and monthly estimation windows. Even though redu-

### 1.3. Methodology and preliminary results

**Table 1.5:** descriptive statistics of monthly returns earned by the low- and high-beta portfolios and the Betting-Against-Beta (BAB) factor after the decimalization in 2001. The table lists in this order: the average return (*avgret*), standard deviation (*std*), the default-bandwidth Newey-West standard error (*se*) of the average and its pvalue (*pval*), the annualized return, standard deviation and Sharpe Ratio (*annret*, *annstd* and *SR*), the annualized downside deviation with return threshold at 0 (*downstd*), the minimum, median and maximum returns (*minret*, *medret* and *maxret*), skewness and kurtosis (*skew* and *kurt*), the maximum drawdown return, its length in months and the number of months that the price series took to recover from it (*mdd*, *mddlen* and *reclen*), the sortino ratio (*sortino*), and the time-series average of the cross-sectional averages of betas within a portfolio (BAB is beta-neutral by construction). All return are in percentage. The table is organized in panels by estimation window of the betas (month, quarter, semester, year), and each panel has statistics for portfolios based on low-frequency (Daily) and high-frequency (Intraday, shaded sections) betas. Specifically, low-frequency betas are calculated from simple linear regressions of stock excess returns on market excess returns. High-frequency betas are calculated as the ratio of realized covariance between stock and market intraday returns over the realized variance of the market. Portfolio formation is described in [figure 1.9](#). The sample covers common US equities and daily returns are from CRSP and intraday returns are from TAQ.

	Daily			Intraday			Daily			Intraday		
	Low	High	BAB	Low	High	BAB	Low	High	BAB	Low	High	BAB
	<b>month</b>						<b>quarter</b>					
<i>Avgret</i>	0.36	-0.13	0.49	0.44	-0.11	0.54	0.37	-0.04	0.41	0.54	-0.03	0.57
<i>Std</i>	5.27	4.32	2.39	4.73	5.12	2.68	4.81	3.93	2.44	4.71	4.72	2.66
<i>Se</i>	0.60	0.48	0.23	0.58	0.54	0.26	0.57	0.42	0.25	0.56	0.50	0.25
<i>Pval</i>	0.55	0.79	0.03	0.45	0.84	0.04	0.52	0.93	0.10	0.34	0.95	0.03
<i>Annret</i>	4.33	-1.57	5.89	5.25	-1.28	6.53	4.46	-0.44	4.90	6.52	-0.38	6.89
<i>Annstd</i>	18.25	14.96	8.28	16.38	17.74	9.30	16.66	13.62	8.44	16.33	16.35	9.20
<i>SR</i>	0.24	-0.10	0.71	0.32	-0.07	0.70	0.27	-0.03	0.58	0.40	-0.02	0.75
<i>Downstd</i>	13.72	11.53	5.52	12.22	13.37	5.86	12.70	10.12	6.09	11.91	12.02	5.59
<i>Minret</i>	-18.38	-14.12	-9.40	-16.48	-15.96	-9.46	-19.62	-11.48	-10.88	-19.43	-13.24	-9.18
<i>Medret</i>	1.15	0.32	0.74	1.13	0.18	0.57	0.96	0.12	0.57	1.11	0.13	0.27
<i>Maxret</i>	12.94	9.85	6.10	11.44	11.78	9.22	11.52	10.08	6.89	11.74	11.54	8.98
<i>Skew</i>	-0.96	-0.56	-0.83	-1.03	-0.40	-0.23	-1.21	-0.32	-1.44	-1.05	-0.23	-0.11
<i>Kurt</i>	4.70	4.05	6.16	5.07	3.62	5.26	5.99	3.48	9.12	5.97	3.26	5.44
<i>Mdd</i>	50.19	43.64	16.26	48.95	48.71	16.89	47.25	38.49	16.14	43.66	42.52	14.13
<i>Mddlen</i>	21	92	8	21	92	10	21	34	8	21	34	5
<i>Reclen</i>	-15	-15	-15	-15	-15	-13	-15	-15	-15	-15	-15	12
<i>Sortino</i>	0.32	-0.14	1.07	0.43	-0.10	1.11	0.35	-0.04	0.81	0.55	-0.03	1.23
<i>Avgbeta</i>	0.71	1.51	0	0.78	1.31	0	0.69	1.57	0	0.69	1.36	0
	<b>semester</b>						<b>year</b>					
<i>Avgret</i>	0.37	-0.04	0.40	0.56	-0.04	0.60	0.47	-0.04	0.50	0.57	-0.04	0.61
<i>Std</i>	4.64	4.09	2.42	4.81	4.86	2.75	4.38	4.16	2.43	4.76	4.89	2.81
<i>Se</i>	0.56	0.44	0.23	0.57	0.51	0.27	0.53	0.45	0.24	0.56	0.53	0.28
<i>Pval</i>	0.51	0.93	0.08	0.33	0.94	0.03	0.38	0.94	0.04	0.32	0.94	0.03
<i>Annret</i>	4.39	-0.47	4.86	6.71	-0.46	7.17	5.60	-0.43	6.02	6.84	-0.50	7.34
<i>Annstd</i>	16.09	14.17	8.37	16.65	16.82	9.54	15.16	14.41	8.40	16.50	16.94	9.75
<i>SR</i>	0.27	-0.03	0.58	0.40	-0.03	0.75	0.37	-0.03	0.72	0.41	-0.03	0.75
<i>Downstd</i>	12.23	10.45	5.71	12.16	12.29	5.59	11.24	10.60	5.01	12.19	12.43	5.50
<i>Minret</i>	-18.25	-11.66	-9.64	-18.25	-13.01	-8.32	-15.87	-11.77	-6.93	-19.34	-13.45	-8.62
<i>Medret</i>	1.13	0.16	0.37	1.15	0.07	0.32	1.24	0.13	0.28	1.28	0.18	0.30
<i>Maxret</i>	10.82	10.24	6.80	11.60	11.95	9.20	9.44	10.80	8.56	10.37	12.06	10.79
<i>Skew</i>	-1.16	-0.25	-0.75	-1.02	-0.17	0.13	-1.05	-0.22	0.07	-1.17	-0.19	0.35
<i>Kurt</i>	5.71	3.28	5.68	5.63	3.20	4.83	5.02	3.21	4.74	5.95	3.13	5.48
<i>Mdd</i>	46.40	39.68	14.59	44.03	43.70	13.98	42.68	39.61	13.79	40.66	44.68	15.98
<i>Mddlen</i>	21	34	6	21	34	5	21	21	6	21	92	6
<i>Reclen</i>	-15	-15	13	-15	-15	37	-15	-15	15	-15	-15	15
<i>Sortino</i>	0.36	-0.04	0.85	0.55	-0.04	1.28	0.50	-0.04	1.20	0.56	-0.04	1.33
<i>Avgsignal</i>	0.75	1.55	0	0.73	1.35	0	0.78	1.51	0	0.74	1.33	0



cing the estimation period from a year to a semester or a quarter plays in favour of a HF implementation, as the difference in Sharpe ratios suggests, this pattern is not monotonic. For instance, the difference in performance for the monthly window is negligible. Hence, although the month-long period has many less daily observations than the intraday sampling scheme, the realized measure does not manage to leverage this statistical advantage and create a monotonic gain in terms of Sharpe ratios. Interestingly, the performance leap is achieved with essentially unchanged average betas of the low- and high-beta portfolios that form the BAB factor.

To recapitulate, after conditioning on the sub-period following April 2001, the *rbab* strategies have an economic advantage over the low-frequency *bab* analogues. This gain is picked by the difference in the Sharpe ratios and should potentially reflect the superior statistical accuracy of intraday realized measures. However, the length of the formation period does not seem to play the role it was expected to have in differentiating HF signals from LF ones. In fact, it is not self-evident whether realized measure are more accurate than low-frequency estimates. The difference in Sharpe ratios, between HF and LF, improves as the formation period shortens, but not monotonically, and reverts for the month-long estimation period, signalling the inability of realized betas to leverage the superior amount of observations. Alternatively, the results might indicate that the cross-sectional ranks of the betas are relatively stable over time. Regardless of the answer, I first need to establish, in the next section, if the observed economic advantage is statistically significant.

## 1.4 Performance evaluation

This section tests if the difference in performance between *bab* and *rbab* portfolios is statistically significant, that is, if there is an economic advantage in using the HF version of conventional LF signals. Results are reported for the base case and for specifications that gauge the robustness of the construction of the signals, of the sampling methodology

and whether the overnight component bears a significant impact for portfolios built on HF data.

**Methodology.** I adopt the [Jobson and Korkie \(1981\)](#) statistic with the correction by [Mommel \(2003\)](#) to test if the difference in Sharpe ratios is equal to zero. Specifically, given the return series of two portfolios  $A$  and  $B$ , the null hypothesis of no difference in Sharpe ratios, i.e.

$$H_0 : SR_A - SR_B = \mu_A/\sigma_A - \mu_B/\sigma_B = 0, \quad (1.14)$$

where  $\mu$  and  $\sigma$  are the estimated means and standard deviations of their respective portfolios, can be tested with the following statistic

$$z_{JK} = \frac{\sigma_B \mu_A - \sigma_A \mu_B}{\sqrt{\theta}} \sim N(0, 1), \quad (1.15)$$

with

$$\theta = \frac{1}{T - M} \left( 2\sigma_A^2 \sigma_B^2 - 2\sigma_A \sigma_B \sigma_{A,B} + \frac{1}{2} \mu_A^2 \sigma_B^2 + \frac{1}{2} \mu_B^2 \sigma_A^2 - \frac{\mu_A \mu_B}{\sigma_A \sigma_B} \sigma_{A,B}^2 \right),$$

and where  $\sigma_{A,B}$  is the estimated covariance between the two series of returns. The statistic in [equation \(1.15\)](#) is derived under IID and normally distributed returns, and because these conditions are often violated in time-series data, [Ledoit and Wolf \(2008\)](#) suggest alternative approaches. For instance, they test the null using heteroskedasticity-autocorrelation consistent (HAC) and bootstrapped Sharpe ratios. Since [table 1.5](#) reports skewness values of more than 4.77 for all series of BAB returns, I also employ both methods suggested above to guard against spurious effects induced by non-normality. The additional tests are in line with the inference drawn with the  $z_{JK}$  statistic and, in all cases, exhibit a more pronounced failure of rejecting the null. Therefore, to avoid redundancy, only the bootstrapped test is reported, and the results with the HAC Sharpe ratios remain available on request.

**Results.** [Table 1.6](#) collects, in this order, the annualized Sharpe ratios of the LF and

HF strategies, their difference ( $\Delta \text{SR}$ ), and the Jobson-Korkie (JK) and the bootstrapped Ledoit-Wolf (LW) p-values on the null in [equation \(1.14\)](#), i.e. that  $H_0 : \Delta \text{SR} = \text{SR}_{\text{HF}} - \text{SR}_{\text{LF}} = 0$ . Results are reported for all four formation periods (month, quarter, semester and year), with the base-case represented by the period going from April 2001 until May 2010, with the realized betas estimated over returns sampled at 75 minutes, including the overnight return and having the same shrinkage factor of 0.6 as in FP (see [equation \(1.11\)](#)). For completeness, I also report in successive panels the tests on the whole horizon, i.e. from 1993 until 2010, and a series of robustness checks that gauge the sensitivity of the framework.

The base case, i.e. the first four rows of [table 1.6](#), confirms the results anticipated in [section 1.3.4](#). The difference in Sharpe ratios increases as the formation period shortens from yearly to semi-annual or quarterly, but the pattern is not monotonic (top panel). In fact, the LF Sharpe ratios for the year- and month-long periods settle at about 0.71, while the portfolios formed over the two intermediate lookbacks record a performance of about 0.58. In contrast, the HF portfolios are steadier and exhibit higher Sharpe ratios of about 0.75 with the exception of 0.70 for the monthly formation period. The highest  $\Delta \text{SR}$  is of 0.17 for signals estimated over the past semester but the difference is not statistically significant with JK and LW p-values of respectively 0.21 and 0.26. Contrary to the belief that the estimation of LF betas might suffer from the lack of observations, the  $bab_m$  and  $bab_y$  achieve the same  $\text{SR}_{\text{LF}}$  of about 0.71 with a slight advantage for the LF version.

Noticing how the p-values are inevitably large, a remark is necessary. Since the focus of the paper is on testing if the HF implementation adds any economic value, we are mostly interested in checking that  $\text{SR}_{\text{HF}} > \text{SR}_{\text{LF}}$ . However, concluding the opposite is also of interest, even more so if the difference is statistically significant. Hence, I use the two-tailed test instead of a one-sided alternative and retain the more general p-values. The reader can mechanically half those values to see that in some cases, the HF implementation comes close in adding value with respect to the LF variant. As an example, the JK p-values

**Table 1.6:** comparison of annualized Sharpe ratios (SR) and tests of statistical significance of the difference in performance, i.e.  $H_0 : \Delta \text{SR} = 0$ . The table reports Sharpe ratios of the Betting-Against-Betas (BAB) portfolios based on low-frequency betas ( $\text{SR}_{\text{LF}}$ ) and high-frequency betas ( $\text{SR}_{\text{HF}}$ ), the high-minus-low difference ( $\Delta \text{SR} = \text{SR}_{\text{HF}} - \text{SR}_{\text{LF}}$ ), the Jobson and Korkie (1981) p-value with correction by Memmel (2003), and the Ledoit and Wolf (2008) bootstrapped p-value with their default specification. BAB portfolios are constructed from low- and high-frequency betas estimated over the past month, quarter, semester and a year, and results are reported for each of these cases. For a detailed description of the methodology see section 1.3. The first four rows list the performance for the base case, which is estimated over the period from 2001 until 2010 with intraday returns sampled at 75 minutes, including the overnight component, and using the same shrinkage factor as Frazzini and Pedersen (2014) of 0.6, refer to equation (1.11). Sensitivity to changes in estimation parameters is tested on the shrinkage parameter, the smoothing approach, the sampling frequency (affects HF only), the sampling method (refresh times with minimum step at 75 minutes and with pre-averaged returns), the absence of the overnight component (affects HF only), and the base case is also estimated over the full horizon for completeness, although inconsistent with BAB as described in section 1.3.4.

	$\text{SR}_{\text{LF}}$	$\text{SR}_{\text{HF}}$	$\Delta \text{SR}$	JK <i>pval</i>	LW <i>pval</i>
<b>2001-2010</b>					
month	0.712	0.702	-0.009	0.968	0.969
quarter	0.581	0.749	0.168	0.320	0.434
semester	0.580	0.752	0.172	0.214	0.256
year	0.717	0.753	0.036	0.746	0.761
<b>1993-2010</b>					
month	0.359	0.179	-0.180	0.487	0.543
quarter	0.454	0.392	-0.062	0.574	0.647
semester	0.360	0.315	-0.045	0.964	0.965
year	0.401	0.323	-0.078	0.321	0.361
<b>No shrinkage</b>					
month	0.361	0.404	0.043	0.934	0.927
quarter	0.202	0.576	<b>0.374</b>	0.007	0.030
semester	0.299	0.621	<b>0.322</b>	0.020	0.064
year	0.521	0.532	0.011	0.923	0.940
<b>Shrinkage 0.8</b>					
month	0.265	0.565	0.300	0.194	0.293
quarter	0.403	0.707	<b>0.303</b>	0.048	0.066
semester	0.457	0.726	<b>0.269</b>	0.052	0.108
year	0.650	0.697	0.046	0.680	0.762
<b>ARMA(1,1)</b>					
month	0.516	0.483	-0.033	0.732	0.765
quarter	0.487	0.621	0.134	0.402	0.560
semester	0.568	0.669	0.101	0.461	0.567
year	0.696	0.762	0.066	0.571	0.628

continues...

	$SR_{LF}$	$SR_{HF}$	$\Delta SR$	JK <i>pval</i>	LW <i>pval</i>
...continues from <a href="#">table 1.6</a> .					
<b>5min sampling</b>					
month	0.712	0.569	-0.143	0.642	0.676
quarter	0.581	0.629	0.048	0.850	0.892
semester	0.580	0.584	0.004	0.985	0.989
year	0.717	0.639	-0.078	0.740	0.782
<b>30min sampling</b>					
month	0.712	0.695	-0.017	0.949	0.947
quarter	0.581	0.696	0.115	0.570	0.685
semester	0.580	0.635	0.055	0.747	0.798
year	0.717	0.698	-0.019	0.899	0.907
<b>Refresh 75min</b>					
month	0.712	0.719	0.007	0.974	0.971
quarter	0.581	0.730	0.149	0.364	0.451
semester	0.580	0.726	0.146	0.306	0.423
year	0.717	0.700	-0.017	0.891	0.918
<b>Refresh pre-avg</b>					
month	0.712	0.682	-0.030	0.888	0.883
quarter	0.581	0.688	0.107	0.590	0.614
semester	0.580	0.731	0.151	0.481	0.516
year	0.717	0.785	0.068	0.754	0.817
<b>No overnight</b>					
month	0.712	0.543	-0.169	0.517	0.554
quarter	0.581	0.738	0.157	0.435	0.549
semester	0.580	0.742	0.162	0.314	0.377
year	0.717	0.775	0.058	0.607	0.614

for the base case, under the one-sided hypothesis, would respectively be about 0.37, 0.16, 0.11 and 0.48 for signals estimated over a year, semester, quarter and a month.

Next, in the second block of four rows, I list the robustness check performed using data on the whole horizon of analysis, i.e. since 1993. Results, are largely discussed in [section 1.3.3](#) and the conclusions should not be taken as indicative of overall average performance but are repeated here for completeness. The period from 1993 until 2001 is inconsistent with the model underlying the BAB factor. Moreover, realized measures are severely affected by the Epps effect, and the bias diminishes only after the decimalization of price quotes in April 2001. For this reason, the  $\Delta SR$  is always negative across all estimation windows. Therefore, these results should be interpreted with caution because of the uneven playing field.

Additionally, I assess the sensitivity of the signals to the shrinkage parameter used to reduce outliers in the betas (see [equation \(1.11\)](#)). The first robustness test is run without shrinking the time-series betas from [equations \(1.9\)](#) and [\(1.10\)](#) to their cross-sectional average. The second run uses an intermediate value of  $w = 0.8$ , instead of 0.6 employed by the base case. A final test is conducted by regularizing each time series of estimated betas with an ARMA(1,1) filter, rather than a cross-sectional shrinkage towards the average value.

For the robustness test with no shrinkage and for the intermediate value of  $w = 0.8$ , the absolute performance of the low- and high-frequency portfolios worsens. The impact is stronger for shorter formation periods and particularly for the LF strategies. In fact, the  $SR_{LF}$  are reduced to the point that the difference with the  $SR_{HF}$  is positive and statistically significant. For example, with  $w = 0.8$ , the Sharpe ratio of  $rbab_q$  has an advantage of 0.30 points over  $bab_q$ , and this discrepancy is significant at the 5% level. With no shrinkage at all, i.e. with  $w = 1$ , the performance advantage for the quarter-long estimation window widens to 0.37 points and has a higher statistical significance with a JK pvalue of 0.007. While this evidence cannot establish an unequivocal advantage of the HF implementation

over the LF variant, it does expose the excessive sensitivity to outliers of the LF portfolios. The robustness with the ARMA(1,1) filter does not provide additional insight other than marginally improving the performance over the base case for the HF portfolio estimated over the past year, i.e. the  $SR_{HF}$  goes from 0.75 to 0.76.

A different type of robustness is carried out on the choice of the sampling scheme for HF betas (does not affect the *bab* portfolios). As already mentioned in [section 1.2](#), optimal sampling is central to the estimation of realized-measures, but is beyond the scope of the current analysis. Hence, I experiment with sampling intervals of 5 and 30 minutes. I pick the former frequency because it is a canonical choice for the estimation of realized-variance, and since [L. Y. Liu, Patton, and Sheppard \(2015\)](#) is also arguably the best choice. The latter frequency, is an intermediate calibration step between the base case (75 minutes) and the 5-minute interval. In terms of results, the performance of the *rbab* portfolios progressively worsens as the sampling step shortens (see [table 1.6](#)), giving clear indication of the well-known estimation problems that affect multivariate measures, like asynchronous trading and microstructure noise.

To tackle potential issues deriving from asynchronous trading, instead of sampling prices at a fixed interval, I borrow the Refresh Time sampling from [Barndorff-Nielsen, P. R. Hansen, et al. \(2011\)](#). The sampling scheme, whose mechanics are illustrated by the authors in their figure 1, avoids biasing multivariate measures, like the covariance, towards zero (see [Epps \(1979\)](#)). When one of two series is rarely traded, a sufficiently high-frequency fixed-sampling scheme will create many null returns in that series. Those null returns will void the variability in any other series which is more frequently traded. Hence, to avoid this effect, the sampling is performed only when both series have traded at least once or, in other words, when both series have refreshed their prices. Additionally, to avoid microstructure noise that is stronger at higher frequencies, the sampling is refreshed only after 75 minutes have elapsed from the previous price. An alternative approach to the minimum step, in order to avoid additive microstructure noise, is to pre-average the

series as in Christensen, Kinnebrock, and Podolskij (2010).

The performance of HF portfolios, under the alternative refresh-times scheme and the minimum step or the pre-averaged series, is preserved. There are two minor improvements in the absolute values of  $SR_{HF}$  with respect to the base case: the value for the month-long estimation window raises from 0.70 to 0.72 under refresh time with the minimum step, and the value for the year-long estimation window raises from 0.75 to 79 under the refresh time with pre-averaging. All other values are marginally worse than in the base case.

Finally, Lou, Polk, and Skouras (2016) split the close-to-close return into intraday and overnight components and highlight the dominant role of the latter one. In this check, I estimate realized betas using only same-day prices in order to assess the contribution of the overnight return to the profitability of HF strategies. Changes in Sharpe ratios with respect to the base case are negligible except for the *rbab* estimated over a month which exhibits some performance regression. This effect is consistent with the results of Lou, Polk, and Skouras (2016): the co-movement between stock and market proxy is stronger overnight and is more relevant during shorter horizons, when the number of observations is relatively low.

To summarize, the results reported in [table 1.6](#) suggest that the theoretical statistical precision of high-frequency measures does not translate directly into superior and statistically significant economic gains. Moreover, although in absolute terms there is some improvement, it is not monotonic in the formation period as suggested in [section 1.1](#). In other words, there is no evident advantage in high-frequency data when using a formation period of one month. However, a series of robustness checks suggest that realized betas are less sensitive to the estimation methodology than their low-frequency counterparts.








### 1.4.1 Sorts by liquidity

The previous section has established that although the HF implementation is more robust than the LF one, the difference in performance is usually not statistically significant. Moreover, the  $\Delta SR$  does not improve monotonically as the lookback period becomes shorter, suggesting that the accuracy of high-frequency measures might not play as big of a role as supposed in [section 1.1](#).

Nonetheless, this section shows that the HF implementation does provide a strong benefit among the most illiquid stocks. In other words, while one month of daily observations is generally long enough to deliver a meaningful signal for the BAB strategy, this principle does not extend uniformly to the whole cross-section. For instance, less liquid stocks might exhibit some variability only during the trading day, while remaining stale at the daily level. To assert this claim, stocks are first sorted into five quintiles according to the [Amihud \(2002\)](#) illiquidity measure. For each group, BAB portfolios are formed from HF and LF betas estimated over the past month, quarter, semester and a year, for a total of 20 portfolios, and their performance is compared.

**Table 1.7:** difference in annualized Sharpe ratios of the Betting-Against-Betas (BAB) portfolios based on high-frequency betas minus the low-frequency version, i.e.  $\Delta SR = SR_{HF} - SR_{LF}$ . Equities are one-way sorted into quintiles by the illiquidity measure of [Amihud \(2002\)](#) and the BAB portfolios for each quintile are constructed from betas estimated over several formation periods. For a detailed description of the methodology see [section 1.3](#). The rows of the table group portfolios by liquidity, with the most to least liquid portfolios respectively in the first and last rows. The columns, instead, group portfolios by formation period of the betas, with the month, quarter, semester and year windows respectively in the first through the fourth column. The last column plots the values by row. In bold are the coefficients that have a [Jobson and Korkie \(1981\)](#) p-value with correction by [Mommel \(2003\)](#) below 5%.

	month	quarter	semester	year	
liquid	0.03	0.06	0.06	0.06	
2	-0.06	0.09	0.08	0.03	
3	-0.12	0.05	0.12	-0.02	
4	-0.16	0.26	0.16	0.05	
illiquid	<b>0.56</b>	<b>0.43</b>	<b>0.35</b>	0.21	

[Table 1.7](#) lists the difference in annualized Sharpe ratios between the HF and LF im-

plementation of the BAB portfolios, i.e.  $\Delta \text{SR} = \text{SR}_{\text{HF}} - \text{SR}_{\text{LF}}$ . Results are organized in a matrix with the most liquid portfolios in the first row and the least liquid in the last row. Columns group portfolios by lookback period with the monthly window in first position and the yearly window in last. Results in bold represent  $\Delta \text{SR}$  which have a JK p-value of 5% or lower. The coefficients are also plotted by row in the last column of the table. For completeness, the Sharpe ratios and the JK and LW p-values are reported in [table A2](#).

It is evident how the least liquid stocks largely benefit from the high-frequency implementation of the signals, and that the effect is monotonically increasing as the lookback window shortens. The  $\Delta \text{SR}$  for the month-, quarter- and semester-long estimation window are respectively 0.56, 0.43 and 0.35, and are all statistically significant, while the coefficient for the year-long period is 0.21. Also, it is noteworthy the absence of any performance differential for the most liquid group of stocks, in the first row of the table. In fact, all coefficients are about 0.06 and are insensitive to the length of the lookback.

To conclude, there is some evidence that the high-frequency implementation of the BAB portfolio is more robust than the canonical low-frequency implementation and that it provides statistically significant economic gains among the most illiquid stocks.

## 1.5 Conclusion

This paper proposes a framework to test if high-frequency realized measures add economic value in the context of asset pricing factors which are traditionally traded at a lower frequency. Although the performance of the factor improves in absolute terms with signals estimated on intraday data, the evidence is only statistically significant for the most illiquid stocks.

The empirical exercise implements the Betting Against Beta factor by [Frazzini and Pedersen \(2014\)](#), over the horizon from 2001 until 2010, using daily and intraday data sampled at 75 minutes. All other conditions are held constant, i.e. both variants of

the factors share the same investable universe (US equities), formation period, monthly rebalancing and have equal trading costs. The performance of the two versions of the BAB factor is assessed with a test on the difference in Sharpe ratios. Additionally, to verify the intuition that signals built with intraday data can leverage the higher number of observations, as opposed to a daily scale, the factors are calculated with lookbacks of a year, semester, quarter and a month.

While it does seem that the difference in performance increases as the estimation window becomes shorter, from yearly to semi-annual or quarterly, the difference for the monthly period takes a step back. The regression in the performance differential suggests that realized betas, estimated with five times the amount of daily observations, are noisy on short lookbacks. To further gauge the sensitivity of the estimated betas to estimation parameters, general results are complemented with robustness checks on the shrinkage parameter, the type and the frequency of the sampling (only affects high-frequency factors) and the exclusion of the overnight return from the realized beta.

As already noted in the preliminary analysis, which uncovers strong bias in the estimates preceding the decimalization of stock prices in 2001, high-frequency betas are severely affected by the Epps effect at lower sampling intervals. Despite that, the high-frequency measure is more robust than the conventional factor to single outliers in the cross-section of the estimated betas and the performance is more stable across estimation windows. In fact, reducing or removing the shrinkage parameter which mitigates the impact of outliers, produces a statistically significant difference in performance for the intraday version of the factor. However, the absolute performance worsens in comparison with the base case, thus only indicating a marginal benefit.

Perhaps more interesting is the effect of the overnight return. Intraday betas calculated over the past month and without the close-to-open component, exhibit much worse performance than the corresponding daily analogue. This result is consistent with [Lou, Polk, and Skouras \(2016\)](#), who argue that close-to-close performance is mainly driven by the

overnight return. Hence, the estimation of realized betas over short windows is strongly dependent on the overnight component and thus similar to the daily measures.

Finally, when the performance of the two versions of the BAB factor is compared for stocks sorted into quintiles by the liquidity measure of Amihud (2002), the HF implementation shows statistically significant economic gains for the most illiquid group. Moreover, the effect is monotonically inversely increasing in the length of the formation period. That is, the accuracy of high-frequency measures does pay off especially on shorter periods for the most illiquid stocks.

Although the findings suggest that a real-world implementation of high-frequency measures might not add any economic value to strategies traded at low frequencies, additional insight is warranted due to the specific choices of the setting. First, while the BAB factor is well known it does not entail a straightforward to understand strategy. An easier to interpret alternative could be a version of the reversal strategy, where signals are standardized by volatility estimated with daily and high frequency returns. Moreover, in addition to the test on the difference in Sharpe Ratios, a certainty equivalent measure might capture the added economic value without the shortcoming of e.g. unequal leverage of the long and short leg of the BAB factor. Finally, a preliminary step that ascertains the accuracy of the high-frequency measures in an out-of-sample exercise, would relate the statistical contribution to the economic one.

# Appendix

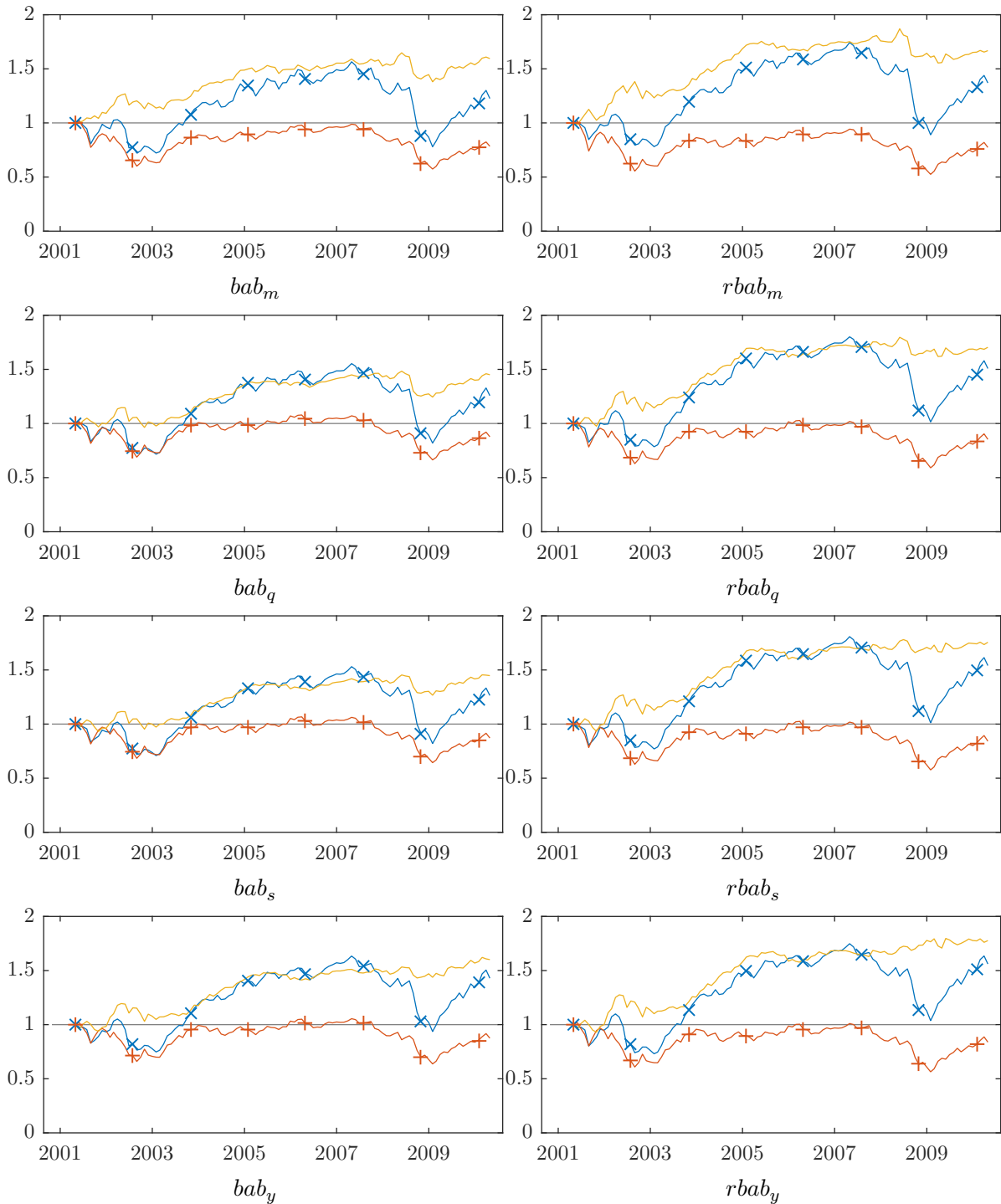
## Appendix 1.A Additional tables and figures

**Table A1:** time-series average of cross-sectional correlations between market betas estimated over formation periods of a month, a quarter, a semester and a year. Spearman's rank correlations appear above the main diagonal and Pearson's coefficients are below the main diagonal. The matrix is arranged in blocks: the upper-left portion lists only the interactions among the low-frequency betas, i.e.  $bab_m$ ,  $bab_q$ ,  $bab_s$  and  $bab_y$ , and the lower-right portion has the correlations between the high-frequency betas only, namely  $rbab_m$ ,  $rbab_q$ ,  $rbab_s$  and  $rbab_y$ . The lower-left and upper-right blocks address the cross-interaction between the low- and high-frequency variants. Low-frequency betas are calculated from simple linear regressions of stock excess returns on market excess returns. High-frequency betas are calculated as the ratio of realized covariance between stock and market intraday returns over the realized variance of the market. The average correlations are calculated over the 2001-2010 period, i.e. after the decimalization of stock quotes. The sample covers common US equities and daily returns are from CRSP and intraday returns are from TAQ.

	$bab_m$	$bab_q$	$bab_s$	$bab_y$	$rbab_m$	$rbab_q$	$rbab_s$	$rbab_y$
$bab_m$		0.68	0.70	0.60	0.65	0.57	0.58	0.53
$bab_q$	0.66		0.69	0.80	0.68	0.76	0.66	0.70
$bab_s$	0.69	0.67		0.83	0.93	0.80	0.82	0.73
$bab_y$	0.56	0.79	0.82		0.83	0.96	0.81	0.87
$rbab_m$	0.63	0.66	0.93	0.82		0.86	0.89	0.80
$rbab_q$	0.54	0.75	0.79	0.95	0.85		0.85	0.92
$rbab_s$	0.56	0.64	0.81	0.80	0.89	0.84		0.90
$rbab_y$	0.49	0.68	0.72	0.87	0.79	0.92	0.90	

**Table A2:** full complement to [table 1.7](#). Comparison of annualized Sharpe ratios (SR) and tests of statistical significance of the difference in performance, i.e.  $H_0 : \Delta \text{SR} = 0$ . The table reports Sharpe ratios of the Betting-Against-Betas (BAB) portfolios based on low-frequency betas ( $\text{SR}_{\text{LF}}$ ) and high-frequency betas ( $\text{SR}_{\text{HF}}$ ), the high-minus-low difference ( $\Delta \text{SR} = \text{SR}_{\text{HF}} - \text{SR}_{\text{LF}}$ ), the [Jobson and Korkie \(1981\)](#) p-value with correction by [Memmle \(2003\)](#), and the [Ledoit and Wolf \(2008\)](#) bootstrapped p-value with their default specification. Stocks are first sorted into quintiles by the illiquidity measure of [Amihud \(2002\)](#) and BAB portfolios are constructed from low- and high-frequency betas estimated over the past month, quarter, semester and a year, for a total of 20 combinations. For a detailed description of the methodology see [section 1.3](#).

	$\text{SR}_{\text{LF}}$	$\text{SR}_{\text{HF}}$	$\Delta \text{SR}$	JK <i>pval</i>	LW <i>pval</i>
<b>month</b>					
liquid	0.218	0.252	0.034	0.801	0.808
2	0.615	0.554	-0.061	0.740	0.744
3	0.588	0.464	-0.124	0.565	0.597
4	0.795	0.630	-0.165	0.594	0.618
illiquid	-0.210	0.349	<b>0.559</b>	0.017	0.029
<b>quarter</b>					
liquid	0.197	0.261	0.063	0.482	0.529
2	0.469	0.560	0.091	0.417	0.480
3	0.500	0.547	0.047	0.702	0.727
4	0.318	0.574	0.256	0.058	0.064
illiquid	-0.063	0.371	<b>0.434</b>	0.015	0.039
<b>semester</b>					
liquid	0.243	0.305	0.062	0.399	0.391
2	0.491	0.574	0.083	0.405	0.397
3	0.389	0.505	0.116	0.228	0.270
4	0.539	0.697	0.159	0.280	0.329
illiquid	0.033	0.379	<b>0.346</b>	0.024	0.109
<b>year</b>					
liquid	0.256	0.313	0.056	0.395	0.396
2	0.577	0.608	0.031	0.695	0.699
3	0.554	0.537	-0.017	0.799	0.780
4	0.722	0.767	0.045	0.664	0.680
illiquid	0.215	0.424	0.209	0.110	0.158



**Figure A1:** monthly cumulated returns, after decimalization of stock quotes in April 2001, of the low-beta (cross-marked), high-beta (plus-marked), and the Betting-Against-Beta (BAB) portfolios (no markers). On the left (right) side, portfolios are formed on low(high)-frequency betas. Betas are estimated, from top to bottom, over the past month, quarter, semester and a year. Specifically, at month-end, stocks are sorted by the rank of their market beta into low- and high-beta portfolios, with weights linearly increasing in the distance of each rank from the average rank (see equation (1.12) and figure 1.8). The BAB factor is the difference in excess returns of the levered versions of the low- minus high-beta portfolios (see equation (1.13)). Stocks are held for a full month. Low-frequency betas are calculated from simple linear regressions of stock excess returns on market excess returns. High-frequency betas are calculated as the ratio of realized covariance between stock and market intraday returns over the realized variance of the market. All portfolios are re-based to one. The sample covers common US equities and daily returns are from CRSP and intraday returns are from TAQ.

### 1.A.1 TAQ overnight returns

The TAQ database does not adjust price series for corporate events like splits or distribution of dividends. However, in order to retrieve the overnight return we need to take these events into account. A simple and intuitive way to calculate this overnight component is to define it as the residual return from CRSP's daily total return minus TAQ's open-to-close return. This method, while algebraically equivalent to manually adjusting the price series, has two advantages: it needs only daily and high-frequency returns, and it avoids date synchronization issues coming from the CRSP's time series for the price adjustment factor and corporate distributions.

CRSP defines the adjusted daily total return,  $R_t$ , as

$$R_t = \frac{P_t^c F_t + D_t}{P_{t-1}^c} - 1 \quad (1.16)$$

where the current close price,  $P_t^c$ , is adjusted for stock events by the factor  $F_t$  and is gross of current distributions  $D_t$ , and the previous close price,  $P_{t-1}^c$ , usually belongs to the previous day but could go as far back as ten days due to missing or invalid observations.<sup>15</sup>

Let TAQ's open-to-close return be  $R_t^{oc}$ , then the simple overnight return  $R_t^{ON}$  can be backed out from the following identity

$$\begin{aligned} 1 + R_t &= (1 + R_t^{ON})(1 + R_t^{oc}) \\ 1 + R_t^{ON} &= \frac{1 + R_t}{1 + R_t^{oc}}. \end{aligned} \quad (1.17)$$

The log-return for the overnight component is then

$$r_t^{ON} \equiv \begin{cases} \ln(1 + R_t^{ON}) & \text{for } t = 1, \dots, T \\ 0 & \text{for } t = 0 \end{cases}. \quad (1.18)$$

<sup>15</sup>In the case of a longer period of missing prices or in other special situations, CRSP fills in specific return codes which are listed in [section 1.A.3](#).



## 1.A.2 Extending the link between TAQ and CRSP

This section describes an extended procedure that matches securities across TAQ and CRSP. As mentioned in [section 1.2](#), an average of 98% TAQ daily observations are matched just by the first step, i.e. using the CUSIP. Nonetheless, additional steps might be used to increase the number of matched series if analysis aims to include short-lived securities and special issues.

Most of the following steps are in line with the WRDS linking procedure, from which I borrow the scoring mechanism.<sup>16</sup> The additional matching steps are applied in order of their presumed reliability. The rule that produces the least amount of false positive matches is assigned a lower score. The next rule is applied to the set of unmatched observations and produces a higher (worse) score. The procedure is applied until there are no rules or unmatched observations. The sample of interest can then be selected by the desired score.

### 1. CUSIP

- (a) Match TAQ's CUSIP, truncated to 8 characters, with the CRSP's NCUISP;
- (b) propagate the match to records having the same name;
- (c) if the propagated match to same-name records discovers a new CUSIP, propagate the match through it.

2. **Exact symbol and date:** exact match between symbols and TAQ's date within CRSP's start and end dates.

### 3. Partial symbol, name and monthly dates

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<sup>16</sup>The WRDS `tc1ink` sas macro by Rabih Moussawi is available on [https://wrds-web.wharton.upenn.edu/wrds/research/macros/sas\\_macros/tc1ink.cfm](https://wrds-web.wharton.upenn.edu/wrds/research/macros/sas_macros/tc1ink.cfm).

- (a) match partial TAQ symbol in CRSP and name with lowest Levenshtein distance while TAQ's month is within CRSP's start and end months;
- (b) expand match through CUSIP.

#### 4. Partial name and monthly dates

- (a) Match names with lowest Levenshtein distance with month condition;
- (b) expand match through CUSIP.

Rules are ranked from lowest to highest rate of false positives which is logically inferred from the presumed precision of their informative content. For example, the CUSIP uniquely identifies an issue of a specific firm, no matter the period of time in consideration. Hence, the date field is irrelevant since a CUSIP cannot be recycled by other issues/firms. Therefore, even if a match is misaligned in time, i.e. the CUSIP matches on both ends of the link but the date does not, the identifier is still reliably referring to the same security, unless it has been misplaced in one or both datasets. However, even in the case of erroneous attribution of the identifier, matching CUSIPs is preferred to a joint match of the symbol and date. In a similar fashion, the symbol itself can be misplaced and even if it is correct, a misalignment in the dates can actually identify different companies (because the symbol is recyclable). In simple words, we have less degrees of freedom in the latter match.

The propagations of the match are carried out through subsequent links on the meta-characteristics of the matched records. For example, step 1b applies to e.g. records that both have the same name but only one reports the CUSIP. The record with the missing CUSIP inherits the PERMNO from the record with the matched CUSIP. I assume this propagation to be more reliable than the joint match on symbol and date because names are longer and incorporate higher literal variation than symbols. Thus, the Levenshtein distance, which is defined as the minimum number of single-character edits to achieve equality of two strings (i.e. insertions, deletions or substitutions), performs better on longer texts.

In step 3a I allow partial matches on the symbol and coarser matches on the date and complement the loss in precision by adding a partial match on the name. While the symbol should match a leading substring from either end of the link, the name should satisfy a hard threshold on the Levenshtein metric. The design of the rule reflects the structure of the TAQ symbols and their suffixes. For instance, we might have a single CRSP symbol which corresponds to the first common issue of a company. TAQ on the other hand might track other issues too and will list the same CRSP symbol with a trailing letter to differentiate among issues. On the other hand, the name might contain some typos in any position of the string or can have repeated spaces on one side of the link.

The last step is mainly included to address unmatched records which do not have a symbol or a CUSIP and is the least reliable.

### 1.A.3 CRSP special return codes

Special return codes for missing price observations filled in by CRSP and corresponding SAS codes used by WRDS in their datasets. I handle those codes by replacing them with a NaN.

CRSP code	SAS code	Description
-44	.E	No valid comparison for an excess return
-55	.D	No listing information
-66	.C	No valid previous price or >10 periods before
-77	.B	Not trading on the current exchange
-88	.A	No data available to calculate returns
-99	.	No valid price (usually suspended or trading on unknown exchange)

# Chapter 2

## Intraday momentum

### 2.1 Introduction

A recent study by [Gao et al. \(2015\)](#) suggests the existence of time-series intraday predictability in the S&P500 index. They show that the return of the first half-hour of trading predicts the return of the last half-hour. I re-examine this evidence using all common US stocks from 1993 until 2010 in order to assess if and how the predictability patterns extend to the whole cross-section. I find similar statistical predictability in the time-series, which however does not translate into economic profitability. In fact, I conclude that the observed pattern is of cross-sectional nature and does not come from the time-series. In other words, cross-sectional sorts on past performance see stocks, which lost or won the most in the morning, earn positive returns and above the rest of the cross-section in the afternoon, and especially during the last half-hour of trading. Moreover, the documented effect shows some dependence on the interval of the holding period and the horizon of analysis, suggesting that specific market mechanisms or frictions play a relevant role in price formation.

While the literature has examined intraday returns at the unconditional level, e.g. providing a simple breakdown of averages by half-hours or hours, studies about intraday

predictability in returns are limited to two contributions.<sup>1</sup> On one hand, [Heston, R. A. Korajczyk, and Sadka \(2010\)](#) (henceforth HKS) apply the well-known “cross-sectional” momentum by [Jegadeesh and Titman \(1993\)](#) to a high-frequency setting. The authors analyse return periodicity in the cross-section and find patterns of continuation at half-hour intervals that are multiple of one day and last up to 40 trading days. In other words, any half-hour today is e.g. positively predicted by the same half-hour yesterday. On the other hand, [Gao et al. \(2015\)](#) investigate the intraday time-series dimension of momentum. The effect, first documented at the monthly frequency and across several asset classes by [Moskowitz, Ooi, and Pedersen \(2012\)](#), concerns individual securities and anticipates continuation of past performance. [Gao et al. \(2015\)](#) concentrate on the time-series predictability of the S&P 500 ETF and find that the first half-hour return predicts the return of the last half-hour.

My work differs from the previous two in several aspects: while [Gao et al. \(2015\)](#) use an aggregate index, I look at the whole cross-section and thus I am able to identify additional patterns of time-series predictability. Moreover, I disentangle the overnight return from the first half-hour and analyse its impact separately. With respect to the work by [Heston, R. A. Korajczyk, and Sadka \(2010\)](#), the main difference is in the formation period: they look at the impact of half-hours lagged by more than a day while I concentrate on same-day intervals. HKS dismiss the same-day focus on the ground of mechanical negative autocorrelation induced by microstructure. However, I show that this effect is only present across two contiguous half-hours and skipping one interval gets rid of the bias. Moreover, their study covers the period from 2001 until 2005, much shorter than the 1993 until 2010 horizon used here.

Hence, the current work contributes to the literature in three ways: first, it clarifies that the time-series pattern documented by [Gao et al. \(2015\)](#) is of cross-sectional nature.

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<sup>1</sup>A partial list documenting intraday average returns includes [Wood, McNish, and Ord \(1985\)](#), [Harris \(1986\)](#) and [Jain and Joh \(1988\)](#).

Second, it relates this effect to the work by Heston, R. A. Korajczyk, and Sadka (2010), complimenting it with an intraday analysis contained within the same day and finally it uncovers a structural break in 2001, which drives the results of the statistical predictability in the time-series approach.

To establish the first contribution, consistent with prior literature, I divide the trading session in 13 half-hours and focus entirely on patterns in returns within the day. The impact of the overnight return is considered separately. I look at which half-hours are statistically predicted by past half-hours of the same day. I use the methodology of pooled regressions with clustered standard errors by Moskowitz, Ooi, and Pedersen (2012). In line with Gao et al. (2015), I find that the last interval is predicted by the first and, in a more flexible specification, by the larger interval from 9:30 until 12:00. I name this pattern *last*.<sup>2</sup> Additionally, I observe that the second half of the trading day, namely from 13:30 until 15:30 is negatively related to the morning period from 9:30 to 13:00. I name this pattern *afternoon*.

Then, to gauge the periodicity shown by the time-series regressions, I sort stocks in two groups, those with positive and those with negative returns in the morning, and form equal-weighted portfolios. The return of these portfolios is calculated in the afternoon according to the two specifications summarized above. While *last* anticipates continuation and *afternoon* reversal, the performance of the two portfolios is inconsistent with both specifications. The evolution of the strategies over-time clearly shows that stocks, independent of their past performance, appreciate in the second half of the trading day. Moreover, morning losers exhibit higher returns than their winning counterparts, hence suggesting the existence of systematic differences in the cross-section. Additionally, a winners-minus-losers strategy exhibits a clear structural break around the beginning of 2001. While the date coincides with decimalization of stock quotes, I do not investigate a causal link within

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<sup>2</sup>I will interchangeably refer to the set of formation and holding periods with pattern, configuration or specification.

the scope of this research.

The second contribution is motivated by the inconsistencies between economic performance and time-series predictability. Hence, I explore differences in returns from a cross-sectional perspective for both the *last* and *afternoon* configurations. I sort stocks into ten groups based on their performance during the intraday formation period and form ten equal-weighted portfolios. The return to those portfolios is calculated over the afternoon and last half-hour. I find a U-shaped pattern in average returns where stocks that lost or gained the most in the morning, earn higher and positive returns in the second half of the trading day, and especially during the last half-hour.

The U-shaped pattern in average returns to portfolios sorted on past performance is robust to stock characteristics (I consider size, traded volume, illiquidity, tick size, volatility and skewness), the day-of-week effect, variations to formation and holding periods, but exhibits some dependence on the sample period (which implies the last contribution). Namely, under *afternoon*, returns to cross-sectional portfolios before the decimalization in 2001 are systematically different from the returns after that date. On the other hand, the U-shaped pattern in returns under the *last* configuration appears more resilient. Nonetheless, the general observation is that morning winners are more profitable than morning losers before 2001 but these circumstances are reversed afterwards, and the change is radical under *afternoon*.

I also assess the impact of including the overnight component into the formation period and I find that returns to the extreme winners and losers are amplified in a way which makes reversal a profitable strategy. That is, stocks that lost the most in the morning, inclusive of the overnight return, earn even higher returns in the afternoon, while stocks that won the most, now gain less. The outcome is independent of the holding period, i.e. persists under *last* and *afternoon*, and raises the economic opportunity to go long past losers and to short past winners.

The remainder of the study is organized as follows: [section 2.2](#) describes the data and presents preliminary results on intraday unconditional average returns. [Section 2.3](#) examines time-series predictability and implements market timing strategies. [Section 2.4](#) moves the focus from the time-series dimension to the cross-sectional one, documents the U-shaped pattern in average returns of portfolios formed on morning performance and tests the robustness of this pattern. Finally, [Section 2.5](#) concludes and suggests directions for further research.

## 2.2 Data and preliminary results

This section describes the sample and presents preliminary results on the unconditional average returns by half-hour.

The sample includes all US equities that belong to both the Center for Research in Security Prices (CRSP) and the Trades And Quotes (TAQ) databases, with a coverage that extends to the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX or currently NYSE MKT), the NYSE Arca exchange and NASDAQ's National Market System (NMS, limited to OTC trades by TAQ).

I only keep common stocks, i.e. series with CRSP Share Type Code of 10 or 11, and exclude observations that belong to microcaps in order to mitigate the spurious effects induced by microstructure issues. That is, holding period returns do not include observations from stocks which, on the previous day, either had a price below \$5 or a market capitalization in the lowest New York Stock Exchange (NYSE) decile. Moreover, to alleviate the impact of stale prices, I require stocks to be sufficiently liquid. Specifically, I only keep the stock-date pairs that had at least 79 observations on the preceding day, which is equivalent to a security being traded on average every 5 minutes during the 9:30 to 16:00 session. The resulting sample has a total of 8924 equities with an average of more than 1800 stocks per day, and covers the period from January 1993, first date of availability of



the TAQ, until May 2010.<sup>3</sup>

High-frequency data from TAQ are cleaned of irregular and misreported trades following the usual rules from Barndorff-Nielsen, P. R. Hansen, et al. (2009) and Bollerslev, Li, and Todorov (2016), and are assigned PERMNOs from CRSP. The link between the two datasets is established through the historical CUSIP and matches on average 98% of the daily TAQ observations.<sup>4</sup> Intraday prices are used to calculate half-hour returns by splitting the 9:30 to 16:00 trading day into 13 non-overlapping intervals.<sup>5</sup> I keep the overnight return separate from the first interval and I avoid backfilling the first price of the day to the opening of the session. In addition to returns, I also calculate the realized volatility over each half-hour of interest using prices sampled at 5-minute intervals.

## 2.2.1 Unconditional moments of intraday returns

This section explores patterns in the unconditional average and standard deviation of half-hour returns and compares the evidence to existing literature. For instance, Wood, McInish, and Ord (1985) and Jain and Joh (1988) find that returns are higher at the beginning and end of the trading day, with a similar pattern applying to volatility. However, Harris (1986) shows that the initial positive returns are driven by the previous close-to-open appreciation, and that generally, returns are negative right after the open.<sup>6</sup> Moreover, while the author does not address differences in standard deviation, I provide such decomposition.

Figure 2.1 shows in the top plot the cross-sectional dispersion in intraday returns for the 13 half hours and the overnight interval. For each stock and sub-interval, I calculate the average return across days. Then, for each half-hour, the 25th, 50th and 75th percentiles

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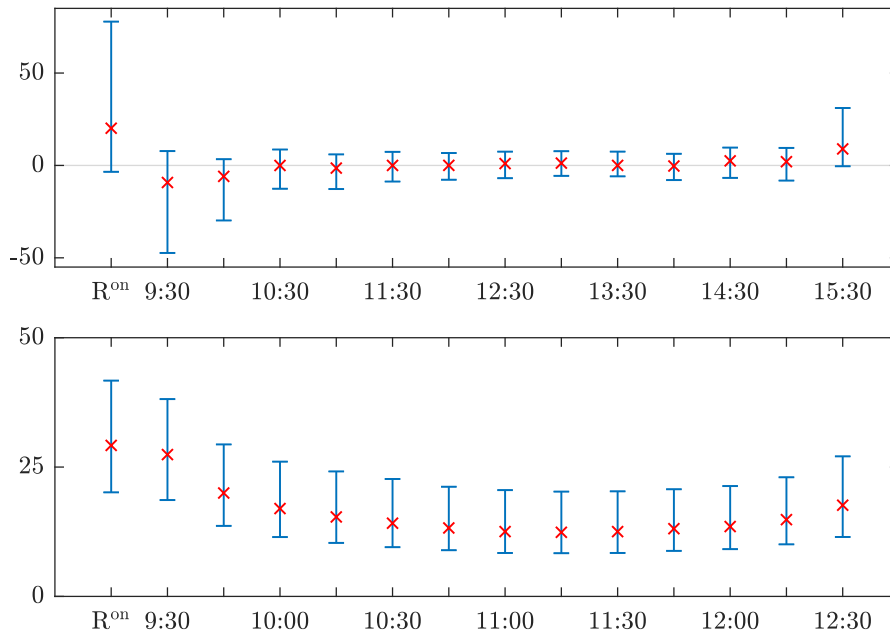
<sup>3</sup>Days with partial trading times, either due to recurring festivities like Christmas' Eve, or due to major disruptions like the power outage of 2003/08/15, are excluded from the analysis.

<sup>4</sup>For a detailed and general description of the linking procedure between TAQ and CRSP, the cleaning and sampling of high-frequency data, refer to Komarov (2016).

<sup>5</sup>All intervals are of the  $lb \leq x < ub$  type except for the last that half-hour which also includes the upper bound.

<sup>6</sup>Henceforth, I will refer interchangeably to previous close-to-open return as overnight.

of the cross-sectional distribution are respectively denoted in the plot by the lower bar, the marker and the upper bar. The bottom graph plots the cross-sectional dispersion in the standard deviation, calculated across days, for the half-hour and overnight returns. All values are annualized.



**Figure 2.1:** cross-sectional dispersion of annualized averages (top) and standard deviations (bottom) of half-hour and overnight returns. The trading day is partitioned in 13 half-hours and for every stock the time-series average and standard deviation of returns are calculated for each sub-period and the overnight time. Then, the lower bar represent the 25th percentile of the cross-sectional dispersion for a given half-hour, while the marker (x) and the upper bar, respectively delimit the median and the 75th percentile. The time on the x-axis marks the start of a half-hour. Values are expressed in percentage. The sample consists of all common NYSE stocks excluding microcaps, i.e. stocks with a price below 5\$ or market capitalization in the bottom decile, and covers the period from January 1993 to May 2010.

The pattern in unconditional returns is consistent with the results from [Harris \(1986\)](#). The first two half-hours record a median annualized return of -9.2% and -5.9%, with the lower bars stretching much further than the upper ones, indicating a general trend of negative returns at the beginning of the trading day. The last half-hour exhibits the opposite behaviour, with a value of 8.9% and a much longer upper bar than the negative one, showing overall very high returns near close-of-day. The intermediate half-hours, i.e. from 10:30 until 15:30, show absence of strong trends and exhibit contained and symmetric

dispersion with median values that range from -1.4% to 2.5% circa. The overnight return exhibits the highest annualized value, at 20.1%, and the biggest dispersion, with the upper bar stretching four times as high as the median and the lower bar going negative but close to zero.

In related works, Kelly and Clark (2011) document a risk premium for the overnight return at about 23% whereas Lou, Polk, and Skouras (2016) find that return to momentum strategies are mostly realized overnight, which they estimate at about 11%. Meanwhile, the intraday return is either strongly negative or relatively negative in both studies, thus confirming the magnitude and shape of the results presented by figure 2.1. Additionally, the reported time-series averages are not driven by a specific period of time. Figure A1 plots cross-sectional averages over time and shows that returns at beginning of the day are usually lower than those at the end, and that the overnight return is the highest.

The lower plot in figure 2.1, instead of averages, shows the cross-sectional dispersion in standard deviations. The intraday volatility draws a u-shaped pattern, with annualized median standard deviations of e.g. 27.4%, 12.5% and 17.6% for the first, middle and last half hour. The close-to-open period exhibits similar median and dispersion to the first half-hour. Overall, the differences among each sub-interval are less drastic if compared to the distribution of the averages. Nonetheless, the cross-sectional variability is still significant, with the difference between the 90th and 10th percentiles exceeding 40% volatility.

To summarize, the sample includes US common stocks matched in both the TAQ and CRSP databases and excludes microcaps, i.e. those stocks with a price below \$5 or a market capitalization in the bottom NYSE decile. Results on unconditional average returns by half-hour see negative returns in the first two intervals, positive and extremely positive returns in the last and overnight intervals respectively. The evidence is consistent with prior literature.

## 2.3 Time series momentum

In this section I build on the pattern in unconditional intraday returns from [section 2.2](#) by examining intraday time-series predictability of equity returns. After identifying two main patterns, I explore their economic significance by implementing intraday timing strategies.

### 2.3.1 Intraday predictability

To examine intraday predictability I use pooled panel regressions with standard errors clustered by time as in [Moskowitz, Ooi, and Pedersen \(2012\)](#).

**Methodology.** I split the trading day, which starts at 9:30 and ends at 16:00, in 13 disjoint half-hours and denote the intervals with  $h = 1, \dots, 13$ . I then regress the return  $r_{h,t}^*$  of half-hour  $h$  in day  $t$  on same-day returns of all previous half-hours except the contiguous interval  $h - 1$ .<sup>7</sup>

$$r_{h,t}^* = a + b_1 r_{1,t}^* + \dots + b_{h-2} r_{h-2,t}^* + e_{h,t} = a + \sum_{k=1}^{h-2} b_k r_{k,t}^* + e_{h,t}. \quad (2.1)$$

Returns are scaled by ex-ante volatility, i.e.  $r_{h,t}^* = r_{h,t} / \sigma_{h,t-1}$ , where the ex-ante variance  $\sigma_{h,t-1}^2$  is calculated as the exponential average of daily realized variances  $\text{RV}_{h,t}$  for the half-hour of interest:<sup>8</sup>

$$\sigma_{h,t-1}^2 = \sum_{i=0}^{\infty} \alpha (1 - \alpha)^i \text{RV}_{h,t-1-i}$$

and  $\alpha$  is chosen such that 83% of the weight is given to the first 60 days, i.e.  $\alpha = 2/(60+1)$ .

The use of standardized returns is justified by the evident cross-sectional differences in

<sup>7</sup>The inclusion of the contiguous interval  $h - 1$  among the explanatory variables does not affect the main results, but introduces the spurious effect of the bid-ask bounce described by [Roll \(1984\)](#). In other words, large and negative t-statistics are limited to the contiguous interval, while everything else remains unchanged. Results are reported in [table A1](#).

<sup>8</sup>An overview of the literature on realized measures is given in [Komarov \(2016\)](#). For this application, daily realized variances are calculated for each half-hour as the sum of squared returns sampled at the 5-minute frequency.

volatility plotted in [figure 2.1](#). Nonetheless, the specification in [equation \(2.1\)](#) is robust to the use of unscaled returns  $r_{h,t}$ , as sustained by results in [table A1](#).

**Results.** [Table 2.1](#) reports t-statistics and scaled coefficients (by 100) of time-series predictability regressions. Each row represents a pooled panel regression described by [equation \(2.1\)](#), where the explained half-hour is indicated by the row header and the predicting intervals are specified in the columns. For example, the first row regresses the return realized over the 15:30 – 16:00 interval onto the half-hour returns that go from 9:30 until 15:00. The second row regresses the 15:00 – 15:30 interval onto half-hours from 9:30 until 14:30, and so forth. The dependent variable from the last row is the return over the 10:30 – 11:00 interval.

Results are laid out to allow the column-wise inspection of the impact of a single half-hour on several future intervals. At the same time, the multivariate regression favours the interpretation of a predictive interval composed by several half-hours. Nevertheless, estimates are robust to the univariate equivalent of [equation \(2.1\)](#) and are reported in [table A1](#). Similarly, rows also can be pooled together into an interval that spans several half-hours, granted an appropriate cutoff is selected for the predictive period. In this fashion, I identify two main patterns from the t-statistics in [table 2.1](#):

- from the first row, the 15:30 – 16:00 return is positively predicted by the interval from 9:30 to 12:00. I will commonly refer to this combination of predictive and predicted interval as the *last* specification;
- the return from 13:30 to 15:30 is negatively predicted by the interval from 9:30 until 13:00. I will refer to this combination as the *afternoon* specification.

In particular, the predictive regression for the *last* half-hour lists t-statistics of 2.24 and 2.44 for the (explanatory) half-hours starting respectively at 9:30 and 11:30, and exhibits statistical significance at the 10% level in the 10:00 and 10:30 periods. However, the highest

**Table 2.1:** t-statistics and coefficients from time-series predictability regressions. Each row is a pooled panel regression of the return from the half-hour denoted by the row-header onto a constant and all, but the contiguous, half-hour returns of the same day that precede it. Returns are scaled by ex-ante volatility, which is measured by the exponential average of the 5-minute realized volatilities calculated over the respective half-hour. The centre of mass of the exponential average is fixed at 60 days. The t-statistics are computed from standard errors clustered by time at the daily level. Darker colours highlight statistical significance at the 1% level and lighter colours that at the 5%. The sample consists of all common NYSE stocks excluding microcaps, i.e. stocks with a price below 5\$ or market capitalization in the bottom decile, and covers the period from January 1993 to May 2010.

	t-statistics											
	c	09:30	10:00	10:30	11:00	11:30	12:00	12:30	13:00	13:30	14:00	14:30
15:30	3.46	2.24	1.92	1.77	0.78	2.44	1.12	-1.32	-0.10	-0.01	0.89	1.47
15:00	2.00	-2.04	-1.13	-2.70	-0.87	-0.32	-2.05	-2.05	0.33	-0.58	0.91	
14:30	2.58	-0.04	-0.92	1.39	0.97	0.64	0.67	0.69	2.48	0.81		
14:00	-0.87	-2.03	-0.76	-0.72	-0.49	-0.71	-0.33	-0.28	0.83			
13:30	0.11	-0.11	0.29	-1.39	-2.12	-1.31	-0.72	-1.00				
13:00	1.76	1.55	0.09	0.36	0.21	-1.67	-0.78					
12:30	2.47	0.68	0.22	0.69	-1.02	-0.69						
12:00	0.97	0.86	1.16	1.32	1.35							
11:30	0.10	0.75	-1.32	1.82								
11:00	-0.96	1.02	-1.76									
10:30	-0.10	-2.08										
	coefficients * 100											
	c	09:30	10:00	10:30	11:00	11:30	12:00	12:30	13:00	13:30	14:00	14:30
15:30	2.43	0.74	0.72	0.86	0.38	1.24	0.57	-0.75	-0.07	0.00	0.64	1.00
15:00	1.45	-0.74	-0.44	-1.46	-0.48	-0.18	-0.95	-1.28	0.24	-0.35	0.71	
14:30	1.75	-0.01	-0.31	0.59	0.44	0.30	0.27	0.31	1.21	0.36		
14:00	-0.60	-0.61	-0.29	-0.32	-0.21	-0.36	-0.14	-0.16	0.50			
13:30	0.07	-0.03	0.09	-0.56	-0.89	-0.64	-0.32	-0.56				
13:00	1.07	0.40	0.03	0.16	0.09	-0.75	-0.37					
12:30	1.39	0.18	0.07	0.27	-0.42	-0.32						
12:00	0.55	0.23	0.34	0.53	0.57							
11:30	0.06	0.21	-0.47	0.75								
11:00	-0.57	0.29	-0.61									
10:30	-0.06	-0.58										

impact is captured by the intercept with a scaled coefficient of 2.43 basis points (bp) per day and a t-statistic of 3.46. The *afternoon* combines predicted sub-intervals from 13:30 to 15:30 and loads throughout the morning more sparsely than *last*, but exhibits stronger statistical significance (all t-statistics are below -2). For instance, the half-hours starting at 9:30, 10:30, 12:00 and 12:30 negatively predict the 15:00 – 15:30 interval. The 9:30 period is also relevant for the prediction of the 14:00 – 14:30 interval, while the half-hour starting at 11:00 exhibits a t-statistic of -2.12 for the 13:30 – 14:00 period. As with *last*, the intercept is significant and has a positive sign for two of the predicted sub-intervals.

While the choice of the predictive and predicted intervals is to some extent discretionary, the focus is on identifying a general and preferably robust regularity in returns rather than on finding the optimal timing strategy.<sup>9</sup> In this spirit, the *last* and *afternoon* configuration aim to capture the tendency suggested by the t-statistics of morning returns predicting reversal in afternoon returns, but with a strong exception for the last half-hour, characterized instead by continuation. Nevertheless, [section 2.3.2](#) covers alternative specifications and compares results with [Gao et al. \(2015\)](#).

Finally, for completeness,  $R^2$  from the regressions of [table 2.1](#) are included in the first column of [table A2](#). Values range from 0.063% for the prediction of the half-hour starting at 15:00, to 0.003% for the half-hour at 10:30. In comparison, e.g. [Campbell \(1991\)](#) documents a  $R^2$  of 2.5% for monthly regressions from 1926 in equities. Considering that to longer holding periods correspond larger  $R^2$  (see, e.g. [Fama and French \(1988\)](#) and [Cochrane \(2008\)](#)), the values of my analysis remain reasonable.

### 2.3.2 Timing strategies

Next, I investigate the profitability of trading strategies based on the timing patterns defined by *last* and *afternoon* in the previous section. For each stock and day, I calculate

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<sup>9</sup>In the context of a timing strategy, the predictive interval will constitute the formation period while the predicted interval will be the holding period.

the return of the intraday formation period. For *last*, it is the return through half-hours 1 to 5, i.e. from 9:30 until 12:00, and for *afternoon* it is the return through half-hours 1 to 7, i.e. from 9:30 until 13:00. Then, I sort stocks into those which had positive and negative returns in the morning and form two equal-weighted portfolios. For the *afternoon* strategy I hold positions through half-hours 9 to 12, i.e. from 13:30 until 15:30, while for the *last* strategy, stocks are kept for the last half-hour only, i.e. from 15:30 until close at 16:00.

Mathematically this can be summarized as:

$$\begin{aligned}
 \textit{afternoon} : \quad & \begin{cases} r_t^{\text{win}} = \frac{1}{N^{\text{win}}} \sum_{i=1}^{N^{\text{win}}} r_{9:12,i} & \text{if } r_{1:7,i} > 0 \\ r_t^{\text{lose}} = \frac{1}{N^{\text{lose}}} \sum_{i=1}^{N^{\text{lose}}} r_{9:12,i} & \text{if } r_{1:7,i} < 0 \end{cases} \\
 \textit{last} : \quad & \begin{cases} r_t^{\text{win}} = \frac{1}{N^{\text{win}}} \sum_{i=1}^{N^{\text{win}}} r_{13,i} & \text{if } r_{1:5,i} > 0 \\ r_t^{\text{lose}} = \frac{1}{N^{\text{lose}}} \sum_{i=1}^{N^{\text{lose}}} r_{13,i} & \text{if } r_{1:5,i} < 0 \end{cases}
 \end{aligned} \tag{2.2}$$

where  $N^*$  is the number of stocks with positive or negative returns in the formation period and  $r_{\text{start:end}}$  is short-hand for intraday return of day  $t$  through half-hours  $h = \text{start}, \dots, \text{end}$ . The winners-minus-losers and long-only portfolios are simply the difference and the sum of the win and lose portfolios:

$$\begin{aligned}
 r_t^{\text{WML}} &= r_t^{\text{win}} - r_t^{\text{lose}} \\
 r_t^{\text{long}} &= r_t^{\text{win}} + r_t^{\text{lose}}.
 \end{aligned} \tag{2.3}$$

Table 2.2 reports descriptive statistics of daily returns earned by the equal-weighted portfolios of morning losers (lose) and winners (win), the winners-minus-losers (WML) timing strategy and the long-only portfolio (win plus lose) from equation (2.3). Among other statistics, I list the average return (*avgret*) and standard deviation (*std*), the Newey and West (1987) standard error of the average with default-bandwidth (*se*) and its pvalue (*pval*), the annualized return, standard deviation and Sharpe Ratio (*annret*, *annstd* and



**Table 2.2:** descriptive statistics of daily returns earned by the equal-weighted portfolio of morning losers (lose), winners (win), the winners-minus-losers (WML) timing strategy and the long-only variant (win plus lose). Specifically, stocks are sorted every day into winners and losers based on the performance over the intraday formation period, and the return of equal-weighted portfolios is calculated for the holding period. The table is organized in two panels. On the left is the *last* timing specification, with formation period from 9:30 until 12:00 and holding period from 15:30 to 16:00, and on the right is the *afternoon* timing specification, with formation period from 9:30 until 13:00 and holding period from 13:30 to 15:30. The table lists in this order: the average return (*avgret*), standard deviation (*std*), the default-bandwidth Newey-West standard error of the average (*se*) and its pvalue (*pval*), the annualized return, standard deviation and Sharpe Ratio (*annret*, *annstd* and *SR*), the annualized downside deviation with return threshold at 0 (*downstd*), the minimum, median and maximum returns (*minret*, *medret* and *maxret*), skewness and kurtosis (*skew* and *kurt*), the maximum drawdown return, its length in months and the number of months that the price series took to recover from it (*mdd*, *mddlen* and *reclen*), the sortino ratio (*sortino*), and the time-series average of the cross-sectional averages of returns during the formation period (*avgsignal*). Asterisks in the *reclen* value indicate truncation from end-of-sample period. The average signal for WML and long-only is omitted. Returns and related measures are expressed in percentage. The sample consists of all common NYSE stocks excluding microcaps, i.e. stocks with a price below 5\$ or market capitalization in the bottom decile, and covers the period from January 1993 to May 2010.

	<i>last</i>				<i>afternoon</i>			
	win	lose	WML	long	win	lose	WML	long
<i>avgret</i>	0.037	0.038	-0.001	0.075	0.025	0.007	0.018	0.032
<i>std</i>	0.354	0.365	0.106	0.712	0.569	0.611	0.227	1.159
<i>se</i>	0.005	0.005	0.002	0.010	0.007	0.008	0.004	0.014
<i>pval</i>	0.000	0.000	0.680	0.000	0.001	0.326	0.000	0.022
<i>annret</i>	9.318	9.527	-0.209	18.846	6.304	1.861	4.443	8.164
<i>annstd</i>	5.621	5.798	1.675	11.297	9.036	9.706	3.600	18.406
<i>SR</i>	1.658	1.643	-0.125	1.668	0.698	0.192	1.234	0.444
<i>downstd</i>	3.756	3.802	1.252	7.498	6.147	6.423	2.808	12.413
<i>minret</i>	-3.260	-3.269	-1.077	-6.358	-3.671	-3.957	-7.457	-7.628
<i>medret</i>	0.040	0.035	-0.001	0.080	0.050	0.013	0.014	0.065
<i>maxret</i>	3.346	3.511	1.259	6.857	7.003	8.380	1.937	13.348
<i>skew</i>	-0.106	0.044	-0.784	-0.078	0.615	1.392	-8.757	0.836
<i>kurt</i>	20.019	19.580	19.776	19.973	15.357	22.737	282.823	16.194
<i>mdd</i>	14.529	9.253	33.958	23.132	13.620	53.671	54.608	30.398
<i>mddlen</i>	637	380	3620	637	1107	2828	3506	1121
<i>reclen</i>	477	28	9*	46	728	3073	4*	714
<i>sortino</i>	2.481	2.506	-0.167	2.514	1.025	0.290	1.582	0.658
<i>avgsignal</i>	0.016	-0.016	-	-	0.017	-0.017	-	-

$SR$ ), skewness and kurtosis ( $skew$  and  $kurt$ ), and the time-series average of the cross-sectional averages of returns during the formation period ( $avgsignal$ ).<sup>10</sup> Returns and related measures are expressed in percentage. The table summarizes results for the *last* and *afternoon* specifications respectively on the left and right sides.

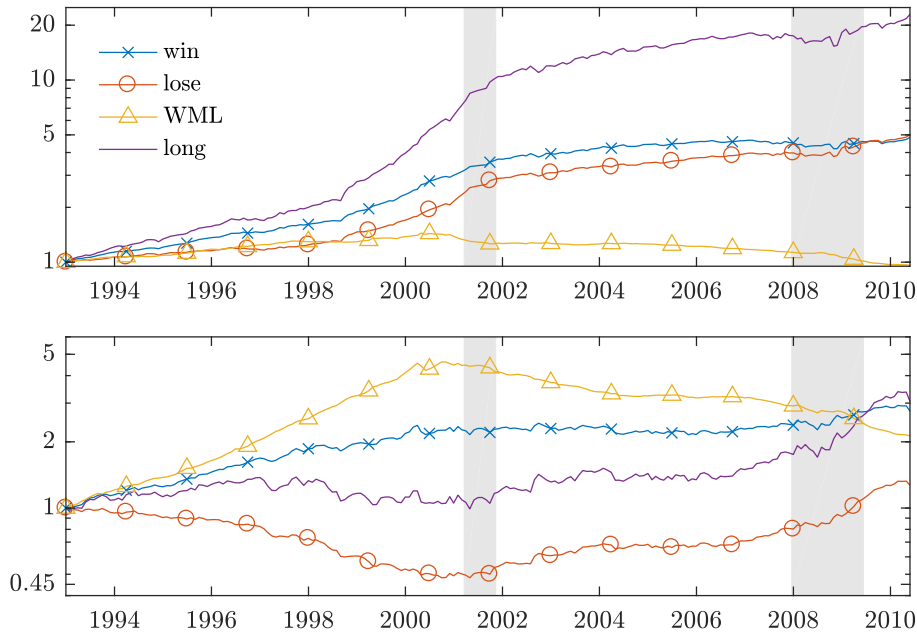
First, the average daily return of the lose portfolio under the *last* specification is inconsistent with the predictability results from [table 2.1](#). Instead of continuing the negative performance, morning losers earn an average of 3.8 bp per day. Second, the reversal predicted with the *afternoon* pattern is also not supported by the profitability of the morning winners and losers. The former group, instead of the anticipated negative performance, exhibits a positive return of 2.5 bp per day, and the latter group records a statistically insignificant daily value of 0.7 bp. In terms of WML returns, the *last* and *afternoon* respectively record an average of -0.1 and 1.8 bp, hence, denying continuation in the former case and reversal in the latter. Finally, the long-only variant suggests that, no matter the performance recorded in the morning, stocks appreciate in the second half of the day, and especially so in the last half-hour. The evidence strongly suggests absence of predictability and confirm the unconditional pattern in intraday returns shown in [figure 2.1](#).

The summary statistics of equal-weighted portfolios do not offer any insight on the evolution of the strategies over-time. For this purpose, [figure 2.1](#) plots the monthly cumulated returns earned by equal-weighted portfolios of morning winners and losers, of the winners-minus-losers (WML) trading strategy and the long-only variant.

The portfolio with the morning losers from the *last* strategy constantly appreciates over-time, strengthening the conclusions drawn from [table 2.2](#) and again rejecting any pattern of performance continuation. For the *afternoon* specification, the win and lose portfolios display momentum until 2001 but reversal afterwards, the latter being consistent with the predictability results. However, rather than a convergence to the expected behaviour,

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<sup>10</sup>Daily returns are annualized by a factor of 252 and daily standard deviations by the square root of 252.



**Figure 2.1:** monthly cumulated returns earned by equal-weighted portfolios of morning winners (cross-marked) and morning losers (circle-marked), the winners-minus-losers (WML) trading strategy (triangle-marked) and the long-only variant (winners plus losers, no markers). Shaded areas denote NBER recessions. Specifically, stocks are sorted every day into winners and losers based on the performance over the intraday formation period, and the return of equal-weighted portfolios is calculated for the holding period. In the top plot is the *last* timing specification, with formation period from 9:30 until 12:00 and holding period from 15:30 to 16:00, and in the bottom plot is the *afternoon* timing specification, with formation period from 9:30 until 13:00 and holding period from 13:30 to 15:30. The sample consists of all common NYSE stocks excluding microcaps, i.e. stocks with a price below 5\$ or market capitalization in the bottom decile, and covers the period from January 1993 to May 2010.

the *afternoon* strategy is now mimicking *last*, where morning losers appreciate faster than the winners. The absence of predictability is also confirmed by the low average rate of correct predictions under *afternoon*, which is equal to 45.1% for the whole sample and to a slightly higher 48.0% after 2001. Similar estimates come from *last*, with averages of 43.7% and 49.1% for the whole sample and for the period after 2001. Predictability aside, [figure 2.1](#) suggests the presence of a structural break in January 2001, which coincides with the introduction of decimalization in stock prices.<sup>11</sup> While decimalization had an undeniably strong economic impact on financial markets, it is outside the scope of this

<sup>11</sup>The decimalization was launched as a pilot in September 2000 following orders by SEC (2000b) and SEC (2000a), and was fully implemented by NYSE in January 2001. NASDAQ and other regional markets followed at the end of March 2001.

analysis to discuss the microeconomic foundations that might support (or not) intraday predictability.<sup>12</sup> Notwithstanding, the patterns in the t-statistics from [table 2.1](#) only really appear after 2001, as the sub-period analysis of predictability suggests in [table A1](#).

**Alternative specifications.** In related work, [Gao et al. \(2015\)](#) find that the return of the first half-hour predicts the return of the last half-hour. Their work differs in a few aspects: first, their analysis is based on the SP500 ETF only, second, they include the overnight return in the definition of the first half-hour return and finally, they cover the sample period from 1993 until 2013.<sup>13</sup> Even though I am able to closely match their predictability results, using the first half-hour as the sole predictive interval still produces inconsistent results in terms of win and lose portfolios. Moreover, the inclusion of the overnight return, while marginal in terms of predictability (see [table A1](#)), produces an unexpected reversal effect with average returns for the lose, win and WML portfolios of 1.6 (3.7), 5.7 (3.8) and -4.2 (-0.1) bp per day (values in parenthesis are from [table 2.2](#)). The impact of the overnight return is analysed in more detail in [section 2.4.4](#).

Additionally, I implement other strategies and report the correlations with the *last* and *afternoon* in [table 2.6](#). For instance, I exclude the first half-hour from the formation period of the *last* specification, setting it to the interval from 10:30 until 12:00. Next, since the second-last half-hour, i.e. the one starting at 15:00, has the most of the significant t-statistics among the whole *afternoon*, I set it aside in a separate specification (the formation period remains from 9:30 to 13:00). I also check the relevance of the first half-hour in predicting the second-last half-hour by excluding it from the formation period. The

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<sup>12</sup>For an analysis of the impact of the decimalization on market quality, see [Bessembinder \(2003\)](#), while for a more general overview see [SEC \(2012\)](#). In general, tick-size reduction plays a fundamental role in liquidity provision and its effects have been analysed by [Goldstein and A. Kavajecz \(2000\)](#) after prices, in 1997, went from being quoted in eights to sixteenths. From another perspective, [Angel \(1997\)](#) and [Schultz \(2000\)](#) give evidence that (wider) relative tick-sizes act as incentive for the market dealer and that strategic splits can promote trading in the stock. Hence the connection between optimal tick sizes and liquidity. In this respect, [C. A. Ball and Chordia \(2001\)](#) show that the optimal spread for large stocks was bound by the tick-size prior to decimalization.

<sup>13</sup>They also exclude days with less than 500 trades which effectively sets the beginning of their sample period to 1997.

evidence from the correlations in [table 2.6](#) is examined in more detail in the next section, but confirms that alternative specifications are only marginally different from *last* and *afternoon*.

To summarize, I investigate the existence of intraday predictability in returns and implement timing strategies based on the *last* and *afternoon* patterns. The former predicts continuation of morning performance in the last half-hour, while the latter predicts reversal of morning performance in the afternoon (excluding the last interval). However, the economic performance of the trading strategies is inconsistent with the results from the predictability regressions. In fact, I observe a general tendency of stocks to earn positive returns in the afternoon, especially in the last half-hour, and morning losers to appreciate faster than morning winners. The evidence is consistent with the pattern of unconditional returns from [figure 2.1](#) and hints at a cross-sectional phenomenon rather than a time-series one. Additionally, the evolution of the timing strategies uncover a structural break at the beginning of 2001 which coincides with the price decimalization of US stocks.

## 2.4 Cross-sectional patterns

[Section 2.3](#) does not find economic evidence supporting intraday predictability. However, the systematic appreciation that stocks exhibit in the second half of the day, with past winners earning higher returns than past losers, raises the question whether there are significant and systematic cross-sectional differences. I address this question in this section and sort stocks on their performance and on several stock characteristics. I find that stocks which either performed very poorly or extremely well in the morning, keep earning in the afternoon more than others. I also investigate how the uncovered pattern behaves before and after the decimalization in 2001, whether it is exclusive to a particular day of the week and how it is affected by the inclusion of the overnight return in the formation signal.

**Methodology.** I use the *last* and *afternoon* specifications from [section 2.3](#) to be consistent with the time-series analysis, and explore variations to those in [section 2.4.4](#). As in [section 2.3.2](#), for each stock and day I calculate the return of the intraday formation period. For *last*, it is the return from 9:30 until 12:00, and for *afternoon* it is the return from 9:30 until 13:00. Then, I sort stocks into deciles based on their performance and form ten equal-weighted portfolios. Finally, the performance of the ten portfolios is measured from 13:30 until 15:30 according to the *afternoon* timing, and over the last half-hour of the trading day as defined by the holding period in *last*.

**Results.** [Table 2.1](#) reports descriptive statistics of daily returns (akin to [table 2.2](#)) earned by decile portfolios formed on past performance and by an equal-weighted portfolio of all stocks. Portfolios are ordered from lose to win, i.e. in ascending order of the formation signal. In particular, the table lists the average return (*avgret*) and its Newey and West (1987) standard error with default-bandwidth (*se*), the annualized return, standard deviation and Sharpe Ratio (*annret*, *annstd* and *SR*), skewness and kurtosis (*skew* and *kurt*), and the time-series average of the cross-sectional averages of returns during the formation period (*avgsignal*). Returns and related measures are expressed in percentage. The table summarizes results for the *last* and *afternoon* specifications respectively in the top and bottom panels.

First, the annualized average return of 9.4% earned by the portfolio of all stocks under the *last* definition is in line with the median value of 8.9% provided in [section 2.2.1](#) by the unconditional average return of the last half-hour. Second and most importantly, the average returns of the ten portfolios form a U-shaped pattern, i.e. stocks that lost or gained the most in the morning, tend to earn higher and positive returns in the second half of the trading day, and especially during the last half-hour. For instance, under *last* the lose and win portfolios respectively earned an impressive 6.2 and 7.7 bp per day, which translate to 15.6 and 19.4% in annualized terms. On the other hand, most of the intermediate

portfolios, with stocks belonging to deciles 2 through 9, recorded average returns below 3 bp and never higher than 4 bp per day. It is also worth pointing out that standard errors create a similar U-shape, and while the lose and win portfolios exhibit larger dispersion in average returns than the intermediate deciles, the second moments are not big enough to offset the superior performance. In fact, not only the win and lose portfolios have the highest Sharpe Ratios, they also have positive skewness, unlike portfolios 3 through 7, and the lowest values of kurtosis. In other words, stocks belonging to the two extreme deciles, earn positive returns during the last half-hour more consistently than those from the other deciles.

In regard to *afternoon*, the U-shaped pattern in average returns uncovered by *last* is slightly weaker. The lose and win portfolios still have the highest values at 6.2 and 2.2 bp but the latter return is much lower than the 7.7 bp from the *last* equivalent. This indicates that morning losers keep earning superior returns in the afternoon, i.e. from 13:30 until 15:30, but past winners are closer to stocks from intermediate deciles. Most indicative of this difference in the cross-sectional behaviour are skewness and kurtosis: portfolios exhibit monotonically decreasing values in both moments. For example, skewness goes from 3.88 to -0.05 from lose-to-win of the cross-sectional deciles. Similarly, kurtosis starts at 78.43 on the first decile and ends at 10.86 on the last decile, with the first (last) five portfolios having higher (lower) estimates than those shown under *last*. In short, the upside potential of morning winners is somewhat more limited during the interval from 13:30 to 15:30 than in the last half-hour of trading.

**Table 2.1:** descriptive statistics of daily returns earned by equal-weighted decile portfolios formed on past performance. Every day, stocks are sorted into ten groups based on their performance during the intraday formation period. Then, ten equal-weighted portfolios are formed and their return is calculated over the holding period. Additionally, the return for an overall equal-weighted portfolio is reported in the last column. The table is organized in two panels. In the top panel is the *last* timing specification, with formation period from 9:30 until 12:00 and holding period from 15:30 to 16:00, and in the bottom panel is the *afternoon* timing specification, with formation period from 9:30 until 13:00 and holding period from 13:30 to 15:30. The table lists in this order: the average return (*avgret*) and the default-bandwidth Newey-West standard error of the average (*se*), the annualized return, standard deviation and Sharpe Ratio (*annret*, *annstd* and *SR*), the minimum, median and maximum returns (*minret*, *medret* and *maxret*), skewness and kurtosis (*skew* and *kurt*), and the time-series average of the cross-sectional averages of returns during the formation period (*avgsignal*). Returns and related measures are expressed in percentage. The sample consists of all common NYSE stocks excluding microcaps, i.e. stocks with a price below 5\$ or market capitalization in the bottom decile, and covers the period from January 1993 to May 2010.

	<i>last</i>											
	lose	2	3	4	5	6	7	8	9	win		all
<i>Avgret</i>	0.062	0.040	0.030	0.028	0.026	0.025	0.024	0.026	0.033	0.077		0.037
<i>Se</i>	0.007	0.006	0.005	0.005	0.005	0.005	0.004	0.005	0.005	0.007		0.005
<i>Annret</i>	15.625	10.085	7.553	6.984	6.496	6.245	5.949	6.628	8.234	19.413		9.423
<i>Annstd</i>	7.473	6.454	5.819	5.562	5.402	5.352	5.334	5.441	5.830	7.068		5.691
<i>SR</i>	2.091	1.563	1.298	1.256	1.203	1.167	1.115	1.218	1.412	2.747		1.656
<i>Minret</i>	-3.500	-3.441	-3.359	-3.290	-3.431	-3.092	-3.007	-3.074	-3.047	-3.744		-3.203
<i>Medret</i>	0.050	0.038	0.033	0.028	0.032	0.029	0.026	0.028	0.033	0.074		0.039
<i>Maxret</i>	4.349	4.349	3.603	3.245	3.174	3.424	3.444	3.388	3.771	6.036		3.468
<i>Skew</i>	0.648	0.513	-0.153	-0.185	-0.212	-0.238	-0.136	0.068	0.045	0.455		-0.045
<i>Kurt</i>	15.579	20.457	21.167	21.487	20.928	21.451	20.989	20.739	18.335	19.623		20.039
<i>Avgsignal</i>	-0.038	-0.018	-0.011	-0.007	-0.003	0.001	0.005	0.009	0.016	0.038		0.000
	<i>afternoon</i>											
	lose	2	3	4	5	6	7	8	9	win		all
<i>Avgret</i>	0.062	0.009	0.000	0.000	0.003	0.009	0.012	0.020	0.021	0.022		0.016
<i>Se</i>	0.011	0.009	0.008	0.007	0.007	0.007	0.007	0.007	0.008	0.010		0.007
<i>Annret</i>	15.620	2.146	0.106	0.027	0.828	2.328	2.972	4.992	5.182	5.666		4.046
<i>Annstd</i>	14.333	11.328	10.199	9.063	8.604	8.320	8.346	8.618	9.509	11.689		9.454
<i>SR</i>	1.090	0.189	0.010	0.003	0.096	0.280	0.356	0.579	0.545	0.485		0.428
<i>Minret</i>	-5.361	-5.238	-4.050	-3.921	-3.810	-3.891	-3.587	-3.679	-4.062	-5.382		-3.901
<i>Medret</i>	0.048	0.016	0.005	0.013	0.003	0.023	0.020	0.034	0.044	0.072		0.034
<i>Maxret</i>	20.770	16.698	13.892	9.293	6.764	6.250	6.988	6.928	7.036	7.524		7.558
<i>Skew</i>	3.884	3.667	3.445	1.986	1.371	1.049	1.085	0.795	0.519	-0.046		1.251
<i>Kurt</i>	78.426	82.276	67.662	33.455	22.733	18.962	20.011	17.176	14.073	10.862		21.230
<i>Avgsignal</i>	-0.041	-0.019	-0.012	-0.007	-0.003	0.001	0.005	0.010	0.018	0.042		0.000



### 2.4.1 Stock characteristics

The previous section uncovered the U-shaped pattern in the cross-section of average returns, with morning winners and losers earning more in the afternoon, and especially during the last half-hour, than stocks from intermediate deciles. In this section, I explore if this pattern is driven by cross-sectional differences in stock characteristic like e.g. size.

**Selecting stock features.** Among the numerous stock characteristics I pick a classical one like size (*size*), others which are strongly related to intraday activity like illiquidity (*illiq*), volume (*vol*) and tick-size (*tick*), a feature emerging from recent results, i.e. the realized skewness (*skew*), and for a purely descriptive perspective the realized volatility (*std*).<sup>14</sup> I use the logarithm of market capitalization from the previous month to proxy for *size*, the logarithm of the previous-month illiquidity measure of Amihud (2002) for *illiq*, the logarithm of the average daily volume for *vol*, and the average of the relative tick-size for *tick*.<sup>15</sup> For *skew*, I take the average of the daily realized skewness calculated as in Amaya et al. (2015) and *std* is defined as the average of daily realized volatilities. Averages are taken over the past quarter and rolled on a daily basis. Finally, realized measures are calculated with intraday returns sampled at the 5-minute frequency.

Since many of the proposed characteristics are related to each other, only relevant stock attributes are considered for the double sorts in order to avoid redundancy in the results.

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<sup>14</sup>Banz (1981) finds that small stocks outperform bigger stocks. Amihud (2002) suggests that an increase in expected illiquidity should be compensated with higher returns. Similarly, Amihud and Mendelson (1986) show that stock returns increase in the bid-ask spread, and hence in the relative tick-size which sets the lower boundary for the spread. Admati and Pfleiderer (1988) propose a theoretical framework where liquidity and informed traders will trade during periods of high volume. Amaya et al. (2015) find stock returns with lower realized skewness outperform those with higher skewness.

<sup>15</sup>The Amihud illiquidity measure is defined as:

$$illiq_t = \frac{1}{252} \sum_{n=0}^{252} \frac{|r_{t-n}|}{vol_{t-n} p_{t-n}}.$$

The absolute tick size was fixed at 1/8 of a dollar until June 1997 when it was reduced to a 1/16 and finally to a 1/100 in April 2001. The *tick* is then the size of the tick divided by the price from the previous close.

For this purpose, [table 2.2](#) displays the time-series average of cross-sectional correlations between stock characteristics. The table has two types of correlations, with Spearman’s rank coefficients appearing above the main diagonal and the conventional Pearson’s coefficients below the main diagonal. Specifically, at each point in time, I calculate the cross-sectional correlation between the values of two characteristics. This produces a time-series of coefficients for each pairing which I then average and summarize in the table.

**Table 2.2:** time-series average of cross-sectional correlations between stock characteristics. Spearman’s rank correlations appear above the main diagonal and Pearson’s coefficients are below the main diagonal. Definition of characteristics: *size* is the logarithm of the previous-month market capitalization; *vol* is the logarithm of the average daily volume; *skew* is the average of the daily realized skewness from [Amaya et al. \(2015\)](#); *illiq* is the logarithm of the previous-month illiquidity measure from [Amihud \(2002\)](#); *tick* is the average of the relative tick-size; *std* is the average of the daily realized volatility. Average values are computed over the past quarter and rolling on a daily basis. Realized measure are calculated with intraday returns sampled at the 5-minute frequency. Colours highlight coefficients above 0.5 (green) and below -0.5 (orange). The sample consists of all common NYSE stocks excluding microcaps, i.e. stocks with a price below 5\$ or market capitalization in the bottom decile, and covers the period from January 1993 to May 2010.

	<i>size</i>	<i>vol</i>	<i>skew</i>	<i>illiq</i>	<i>tick</i>	<i>std</i>
<i>size</i>		0.55	0.16	-0.87	-0.60	-0.67
<i>vol</i>	0.59		0.04	-0.65	-0.09	-0.12
<i>skew</i>	0.14	0.05		-0.11	-0.09	-0.12
<i>illiq</i>	-0.88	-0.62	-0.12		0.63	0.63
<i>tick</i>	-0.45	-0.14	-0.11	0.50		0.62
<i>std</i>	-0.58	-0.11	-0.14	0.57	0.59	

From the first row, *size* is positively correlated to *vol*, with a coefficient of 0.55, and is negatively correlated to *illiq*, *tick* and *std*, with values of -0.87, -0.60 and -0.67, correspondingly. This type of strong affinity is expected since small stocks tend to be illiquid, hence sporting less trading volume, to be more volatile and have wider relative tick-sizes in order to attract market dealer’s interest (see [Angel \(1997\)](#) and [Schultz \(2000\)](#)). The implications just outlined between market capitalization and the other features naturally translates into the negative dependence between *illiq* and *vol*, with a Spearman correlation of -0.65, and positive pairwise associations between *illiq*, *tick* and *std*, with values that

fare in the low 0.60s.<sup>16</sup> The *skew* remains orthogonal to all other characteristics since it preserves the sign, in contrast with *std*, and because it describes past returns rather than company qualities.

From the analysis of correlations, I choose to report results from the double sorts for *size*, since it is easier to interpret than *illiq* and has longer-standing contributions in the literature, for *skew*, because of its orthogonality to the other features, and for *std*, which diversifies on the evidence. On the other hand, I do not report results from the double sorts on attributes related to trading activity since they do not add qualitative information on top of that provided by the sort on market capitalization.

**Double sorts.** Every day, stocks are sorted into five groups based on their performance during the intraday formation period. Stocks are also allocated independently to three groups sorted on either *size*, *std* or *skew*. The intersection of the two sorts produces 15 groups which will form equal-weighted portfolios. The return of the portfolios is calculated over the holding period. As explained at the beginning of this section, formation and holding periods are defined by the *last* and *afternoon* configurations.

**Table 2.3** collects average annualized returns earned by equal-weighted portfolios formed on past performance and a stock attribute, and reports t-statistics in parentheses. Each combination yields 15 values arranged into three-row by five-column matrices. Columns are sorted from lose to win, i.e. in ascending order of the return realized in the formation period, and rows are sorted in ascending order of the stock characteristic, e.g. for the *size* characteristic into small, medium and large stocks. The table summarizes results for the *last* specification on the left and for *afternoon* on the right side.

A general inspection of the table shows that the U-shaped pattern from the cross-sectional sort on performance (see **table 2.1**), also persists after controlling for stock attributes. For example, let us consider the *last* specification: the shape of average returns

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<sup>16</sup>The negative dependence between volume and illiquidity arises mechanically since the former appears in the denominator of the Amihud measure.

across the small, medium and large *size* groups is clearly parabolic, i.e. extreme morning winners and losers earn more than the intermediate portfolios. More specifically, small stocks from the first (lose), third and fifth (win) groups, have an annualized return of 19.1, 12.6 and 21.5%, on average. As a side note, the same findings extend to *vol*, *illiq* and *tick*, but are omitted for conciseness.<sup>17</sup>

A minor exception to the usual U-pattern, which I will explore in more detail in the next section, arises for the low-volatility group. Returns decrease monotonically as morning performance improves. In particular, while morning losers yield 7.9% in the afternoon, morning winners average at 1.1% only. Otherwise, the medium- and high-volatility groups conform with the previous results. This description suggests that the less volatile stocks can recover from a drawdown with a sudden surge, but are not likely to jump up if they were already earning a profit.

The U-pattern aside, *size*, *std* and the unreported features are still responsible for cross-sectional variation in returns. Considering market capitalization again, e.g. the lose group exhibits decreasing returns of 19.1, 9.6 and 7.4%, as we look at small, medium and then large stocks. However, the influence of *skew* on the cross-section is less pronounced, but at the same time the U-shape is clearly outlined. For instance, the maximum difference in returns between the low- and high-skewness groups is of only 2.1% (win column). In contrast, *size* sets a distance of 16.3% between small and large stocks (win column), and *std* gets an even bigger 18.8% between the low and high portfolios (win column).

Most of the conclusions drawn for *last* also apply to the *afternoon* specification, with two caveats. First, returns are generally lower, consistently with the unconditional results by half-hour (section 2.2), and, as observed in table 2.1, the parabolic pattern is weaker. For example, low-skewness stocks from the first (lose), third and fifth (win) performance groups, exhibit estimates of 7.0, -0.2 and 3.2% respectively, sign that the pattern is not as well defined as under *last*, where the profit of the win group is on par with that of the losers.

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<sup>17</sup>Results for double sorts on the excluded characteristics are available upon request.

Second, the impact of features on average returns is not always coherent across performance groups, which is probably also due to lower statistical significance as t-statistics are often smaller than 2 in absolute terms. Let us consider *size*: smaller stocks are more lucrative than bigger ones in the first and last portfolios but this scenario is reversed for the second and third group. Similarly, average returns are inversely proportional in the *std* for all but the win portfolio which behaves just the opposite. On the other hand, the effect of *skew* is largely consistent across groups formed on morning returns but it is proportionally increasing in the values of skewness, which is in opposition with the influence shown under *last*.

**Table 2.3:** average annualized returns earned by equal-weighted portfolios formed on past performance and stock characteristics. Every day, stocks are sorted into five groups based on their performance during the intraday formation period. Stocks are allocated independently to three groups sorted on either *size*, *std* or *skew*. The intersection of the two sorts produces 15 equal-weighted portfolios and their return is calculated over the holding period. Definition of characteristics: *size* is the logarithm of the previous-month market capitalization, *std* is the average of the daily realized volatility and *skew* is the average of the daily realized skewness from Amaya et al. (2015). Average values are computed over the past quarter and rolling on a daily basis. Realized measure are calculated with intraday returns sampled at the 5-minute frequency. In parentheses are t-statistics calculated from the default-bandwidth Newey-West standard errors of the average. The table is organized in two panels. In the left panel is the *last* timing specification, with formation period from 9:30 until 12:00 and holding period from 15:30 to 16:00, and in the right panel is the *afternoon* timing specification, with formation period from 9:30 until 13:00 and holding period from 13:30 to 15:30. Returns are expressed in percentage. The sample consists of all common NYSE stocks excluding microcaps, i.e. stocks with a price below 5\$ or market capitalization in the bottom decile, and covers the period from January 1993 to May 2010.

	<i>last</i>						<i>afternoon</i>					
	lose	2	3	4	win		lose	2	3	4	win	
<i>size</i>												
small	19.1 [10.7]	13.1 [8.3]	12.6 [8.4]	13.3 [8.9]	21.5 [12.0]		10.5 [4.0]	-3.4 [-1.6]	1.2 [0.6]	3.9 [1.9]	6.8 [2.8]	
medium	9.6 [6.4]	6.3 [5.2]	5.5 [4.9]	5.6 [5.0]	10.8 [7.5]		8.8 [3.5]	1.0 [0.5]	0.9 [0.5]	3.2 [1.9]	3.0 [1.3]	
large	7.4 [4.7]	4.5 [4.1]	2.7 [2.7]	1.9 [1.8]	5.3 [3.9]		7.5 [2.8]	2.8 [1.5]	2.3 [1.5]	3.9 [2.4]	4.3 [2.0]	
<i>std</i>												
low	7.9 [6.0]	3.8 [3.8]	2.0 [2.1]	0.7 [0.8]	1.1 [1.0]		17.2 [8.1]	5.2 [3.4]	3.0 [2.2]	3.1 [2.2]	-0.1 [-0.1]	
medium	9.4 [6.5]	6.3 [4.9]	5.4 [4.5]	5.2 [4.4]	8.1 [5.8]		11.1 [4.5]	0.9 [0.5]	1.9 [1.1]	3.0 [1.7]	4.2 [2.0]	
high	16.4 [8.9]	13.1 [7.7]	13.4 [8.2]	13.8 [8.6]	19.9 [10.7]		4.2 [1.5]	-5.7 [-2.3]	-3.8 [-1.6]	1.9 [0.8]	4.9 [1.8]	
<i>skew</i>												
low	12.8 [7.8]	7.8 [6.1]	6.5 [5.6]	6.4 [5.4]	13.6 [9.0]		7.0 [2.8]	-0.4 [-0.2]	-0.2 [-0.1]	3.9 [2.2]	3.2 [1.4]	
medium	11.5 [7.4]	6.7 [5.4]	6.0 [5.3]	6.4 [5.6]	12.2 [8.1]		8.4 [3.1]	0.4 [0.2]	1.3 [0.8]	2.0 [1.2]	5.0 [2.2]	
high	11.1 [7.1]	6.3 [5.2]	4.9 [4.5]	4.8 [4.3]	10.1 [7.1]		9.4 [3.7]	0.8 [0.4]	2.1 [1.3]	3.9 [2.3]	5.0 [2.3]	

## 2.4.2 Sub-period analysis

In section [section 2.3.2](#) I pinpoint a potential structural break at the beginning of 2001. Both [figure 2.1](#) and the robustness by sub-period on the predictability analysis covered in [table A1](#), highlight a change in the behaviour of morning winners and losers after the suggested date. Moreover, the timing coincides with the introduction of the decimalization of US quotes, which bore deep microstructural implications on day-trading (see e.g. [Bessembinder \(2003\)](#)). In this section, I split the horizon at the end of April 2001 in two sub-periods, and form double-sorts on each interval. Implications for the simple cross-sectional sort on past performance are easily deduced.<sup>18</sup>

[Table 2.4](#) reports the sub-period analysis of average annualized returns earned by equal-weighted portfolios independently formed on past performance and a stock attribute. Methodology and organization of results mimic [table 2.3](#). Additionally, the upper panel presents estimates for the period up to April 2001 and the lower panel presents estimates for the subsequent interval. The t-statistics of the averages are listed in [table A4](#).

Two main points arise from a general overview of the estimates. First, the parabolic pattern in average profits is the strongest before April 2001 and overall under *last*. While this is generally true across all stock attributes, small stocks generate the highest observed returns with e.g. the first (lose), third and fifth (win) groups listing annualized values of 20.7, 17.4 and 34.4% respectively. Instead, *afternoon*'s profile takes the shape of a right-smirk with the first three groups usually yielding negative profits. As an illustration, low-volatility stocks from the first group generate one of the worst performances with an annualized value of -10.7%. In summary, unconditional on the timing specification, the first sub-period favours morning winners.

Second, the period following April 2001 sees morning losers come ahead. Under *last*,

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<sup>18</sup>[Figure A2](#) plots monthly cumulated returns earned by equal-weighted portfolios formed on past performance. The structural break is particularly visible for the *afternoon* specification on the win and lose portfolios.

while returns are generally lower than during the first sub-period, the U-shaped pattern morphs into a left-smirk. However, this transition comes from morning losers losing upside potential as e.g. high-volatility stocks settle onto a new low of 9.2% from a previous value of 31.9% whereas the annualized profit of morning losers, from the same stock category, is essentially unchanged at 16.3%. The transition is even more pronounced under the *afternoon* specification. If earlier, the shape of average annualized profits was that of a right-smirk, returns are now monotonically decreasing with the win portfolios yielding between -3.5 and 1.5% (the sole positive estimate).

To conclude, while the U-shaped pattern is strong under *last* and before April 2001, it becomes a left-smirk when sorting stocks by *std* in the second sub-period. Additional investigations on the reason of this behaviour are warranted. Moreover, the (weaker) parabolic profile under *afternoon* in [table 2.3](#) is the combination (average) of the right-smirk and decreasing monotonic patterns from the two estimation intervals.



**Table 2.4:** sub-period analysis of average annualized returns earned by equal-weighted portfolios independently formed on past performance and stock characteristics. The sorts are formed as in table 2.3. The sample period is split at the end of April 2001. Definition of characteristics: *size* is the logarithm of the previous-month market capitalization, *std* is the average of the daily realized volatility and *skew* is the average of the daily realized skewness from Amaya et al. (2015). Average values are computed over the past quarter and rolling on a daily basis. Realized measure are calculated with intraday returns sampled at the 5-minute frequency. The table is organized in two panels. In the left panel is the *last* timing specification, with formation period from 9:30 until 12:00 and holding period from 15:30 to 16:00, and in the right panel is the *afternoon* timing specification, with formation period from 9:30 until 13:00 and holding period from 13:30 to 15:30. Returns are expressed in percentage. The sample consists of all common NYSE stocks excluding microcaps, i.e. stocks with a price below 5\$ or market capitalization in the bottom decile, and covers the period from January 1993 to May 2010.

	<i>last</i>						<i>afternoon</i>					
	lose	2	3	4	win		lose	2	3	4	win	
<b>Pre April 2001</b>												
<i>size</i>												
small	20.7	17.1	17.4	18.6	34.4		-1.7	-15.9	-4.8	1.0	12.6	
medium	12.5	8.8	7.9	8.5	18.3		-3.7	-9.0	-5.2	2.4	10.0	
large	9.2	5.8	3.2	2.6	8.5		-0.1	-3.1	-1.9	3.5	9.1	
<i>std</i>												
low	12.4	6.0	3.0	1.0	1.5		12.5	-0.8	-1.1	2.7	1.8	
medium	11.4	8.4	7.8	8.1	14.7		-2.6	-9.9	-3.0	2.3	11.8	
high	16.5	15.6	17.2	19.1	31.9		-10.7	-17.8	-14.3	-2.6	11.0	
<i>skew</i>												
low	15.1	10.6	8.8	8.7	22.5		-6.3	-10.8	-8.2	2.4	8.0	
medium	13.8	9.3	8.2	9.5	19.8		-3.4	-8.6	-3.9	0.0	12.0	
high	12.3	7.9	6.2	6.4	15.2		-3.3	-7.9	-2.1	3.1	11.6	
<b>Post April 2001</b>												
<i>size</i>												
small	17.6	9.5	8.4	8.4	9.6		21.7	8.1	6.6	6.5	1.5	
medium	6.9	3.9	3.2	2.9	4.0		20.3	10.2	6.4	3.9	-3.5	
large	5.7	3.4	2.3	1.2	2.3		14.5	8.1	6.2	4.3	-0.1	
<i>std</i>												
low	3.8	1.9	1.1	0.5	0.7		21.4	10.5	6.7	3.4	-1.9	
medium	7.7	4.3	3.4	2.6	2.3		23.3	10.6	6.3	3.7	-2.6	
high	16.3	10.8	10.1	9.0	9.2		17.6	5.1	5.5	6.0	-0.5	
<i>skew</i>												
low	10.8	5.2	4.5	4.4	5.6		18.8	9.0	7.1	5.3	-1.0	
medium	9.4	4.4	4.0	3.6	5.4		18.9	8.5	6.0	3.9	-1.2	
high	10.0	4.8	3.7	3.4	5.6		20.7	8.6	5.8	4.7	-0.9	

### 2.4.3 Day of the week































French (1980) finds that returns are lower on Monday than on other days of the week. While I have investigated the impact of stock characteristics and the period of analysis on stocks sorted by morning performance, the substantial difference documented across days of the week might constitute a relevant driver of the patterns observed so far. In this section, I investigate the difference in returns by day of the week and by sub-period and find that the U-shaped pattern is not caused by a specific day.

Table 2.5 reports average annualized returns earned by equal-weighted portfolios formed on past performance and grouped by day of the week. Every day, stocks are sorted into five groups based on their performance during the intraday formation period (see *last* and *afternoon* specifications). Then, five equal-weighted portfolios are formed and their return is calculated over the holding period and reported for each day of the week. The analysis is carried out on the whole horizon and on the two sub-periods created by splitting the sample at the end of April 2001. The t-statistics are listed in table A5.

The estimates on the whole sample are consistent with the weekend effect. Under *last*, Monday returns are economically lower with e.g. the lose portfolio recording a mere 5.5% while the same group on Friday scores 19.2%, in annualized terms. For the *afternoon* specification, the phenomenon is even stronger since the equal-weighted portfolios yield negative estimates between -1.5 and -9.1%, while on the other days, performance is always positive. The pattern in average profits is parabolic on all days during the last half-hour and for *afternoon* from Wednesday onwards, Monday being the biggest exception with negative but increasing returns. This is in line with the findings from the previous sections.

The estimates for the days of the week combined with the sub-period analysis are consistent with the observations from section 2.4.2. For example, average returns for the *afternoon* configuration change in shape from a right-smirk to a monotonically decreasing pattern. These results from the previous section simply combine with the Monday effect.

**Table 2.5:** average annualized returns earned by equal-weighted portfolios formed on past performance and grouped by day of the week. Every day, stocks are sorted into five groups based on their performance during the intraday formation period. Then, five equal-weighted portfolios are formed and their return is calculated over the holding period and reported for each day of the week. The analysis is carried out on the whole horizon and on the two sub-periods created by splitting the sample at the end of April 2001. The table is organized in two panels. In the left panel is the *last* timing specification, with formation period from 9:30 until 12:00 and holding period from 15:30 to 16:00, and in the right panel is the *afternoon* timing specification, with formation period from 9:30 until 13:00 and holding period from 13:30 to 15:30. Returns are expressed in percentage. The sample consists of all common NYSE stocks excluding microcaps, i.e. stocks with a price below 5\$ or market capitalization in the bottom decile, and covers the period from January 1993 to May 2010.

	<i>last</i>						<i>afternoon</i>					
	lose	2	3	4	win		lose	2	3	4	win	
<b>Whole sample</b>												
Mon	5.5	3.6	3.2	3.7	10.6		-7.8	-9.1	-4.5	-1.5	-2.5	
Tue	10.2	6.2	5.8	6.4	13.6		9.8	0.8	1.8	4.2	3.5	
Wed	14.4	7.6	5.7	6.2	12.7		12.4	1.5	3.3	7.6	12.4	
Thu	14.8	8.5	7.9	7.8	15.7		9.4	2.3	1.8	4.6	7.1	
Fry	19.2	10.6	8.4	8.0	16.4		19.7	4.5	3.9	4.4	5.9	
<b>Pre April 2001</b>												
Mon	1.4	3.6	3.3	5.2	17.5		-18.3	-15.3	-6.2	1.4	8.1	
Tue	11.4	7.3	6.9	8.3	19.4		0.4	-9.9	-6.3	-1.1	3.8	
Wed	20.0	12.7	9.7	10.3	22.6		-0.9	-8.9	-2.6	7.5	20.6	
Thu	17.6	10.3	9.4	10.2	24.3		0.5	-3.9	-2.6	5.0	15.1	
Fry	24.8	14.8	12.2	11.3	27.2		4.8	-8.3	-2.8	2.0	11.4	
<b>Post April 2001</b>												
Mon	9.2	3.7	3.1	2.4	4.3		1.9	-3.3	-3.1	-4.2	-12.2	
Tue	9.0	5.1	4.7	4.7	8.1		18.5	10.8	9.4	9.2	3.2	
Wed	9.3	2.9	2.1	2.3	3.6		24.7	11.2	8.8	7.6	4.8	
Thu	12.1	6.9	6.4	5.7	7.8		17.6	8.0	5.8	4.2	-0.3	
Fry	14.1	6.8	5.0	4.9	6.4		33.3	16.3	10.1	6.5	0.9	

As for the whole period, Mondays report lower returns across all performance groups under *afternoon*, and for the win group during the first sub-period and for the lose group after April 2001 under *last*.

The evidence does not suggest that the U-shaped pattern concentrates on a particular day of the week.

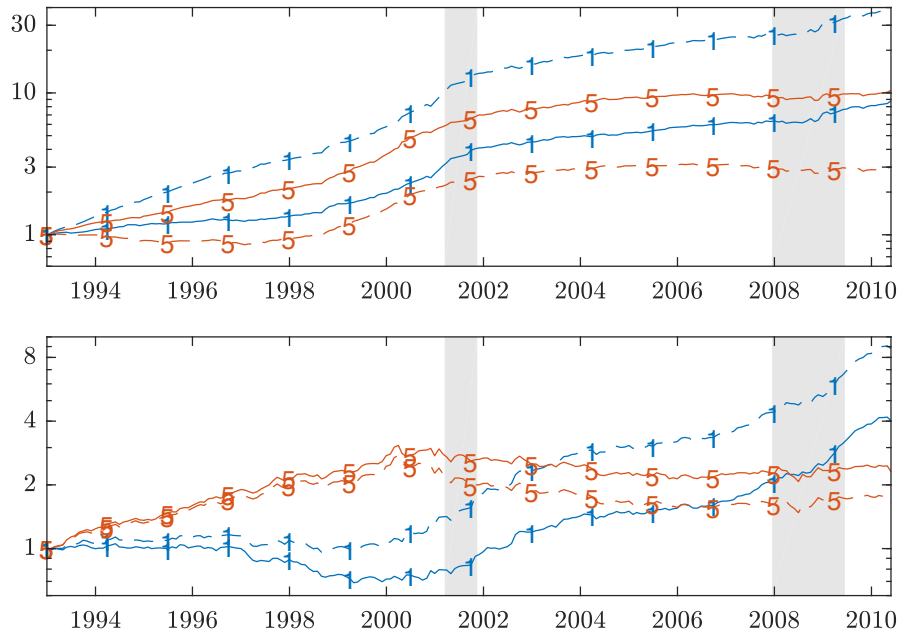
#### 2.4.4 Overnight return and alternative specifications

In this section, I consider alternatives to the *last* and *afternoon* specifications introduced in [section 2.3.2](#). First, I assess how the inclusion in the signal of the overnight return affects the performance of the cross-sectional equal-weighted portfolios. Second, I introduce variations in the formation and holding periods by e.g. excluding the first half-hour from the signal, or by investing only during the second-last half-hour.

**Overnight return.** Rogalski (1984) and Harris (1986) give an early account about the relevance of the overnight return in the context of the weekend effect. More recently, Kelly and Clark (2011) estimate the magnitude of investing during trading hours against the non-trading periods and find the latter case to be overwhelmingly more profitable than the former, and Lou, Polk, and Skouras (2016) find similar evidence in an application to momentum strategies. Moreover, the unconditional breakdown of the day into half-hours and the overnight component in [figure 2.1](#), confirms these findings. Hence, it is natural to ask how the cross-sectional sorts are affected when *last* and *afternoon* compound the overnight return into their definition of morning signals.

[Figure 2.1](#) plots monthly cumulated returns earned by equal-weighted quintile portfolios formed on past performance with (dashed line) or without the overnight return (solid line). For clarity, I only show the bottom and top quintiles. The *last* and *afternoon* configurations are respectively displayed in the top and bottom axes.

The impact of the overnight return on the cross-sectional sorts is unequivocal and



**Figure 2.1:** monthly cumulated returns earned by equal-weighted quintile portfolios formed on past performance including the overnight return (dashed lines) and without it (solid lines). Only the bottom and top quintiles are displayed. Shaded areas denote NBER recessions. Specifically, stocks are sorted every day into five equally populated groups based on the performance over the intraday formation period plus the eventual overnight component. Then, the return of equal-weighted portfolios is calculated for the holding period. In the top plot is the *last* timing specification, with formation period from 9:30 until 12:00 and holding period from 15:30 to 16:00, and in the bottom plot is the *afternoon* timing specification, with formation period from 9:30 until 13:00 and holding period from 13:30 to 15:30. The sample consists of all common NYSE stocks excluding microcaps, i.e. stocks with a price below 5\$ or market capitalization in the bottom decile, and covers the period from January 1993 to May 2010.

independent of the specification: the returns of the equal-weighted portfolios are amplified in a way which makes trading on reversal profitable. The lose portfolios (first quintiles) average at 5.1 and 3.5 bp per day on the *last* and *afternoon* specifications without the overnight component (solid lines) but improve by about 50% to 8.6 and 5.3 bp when the close-to-open return is included in the signals (dashed lines). The win portfolios (fifth quintiles), on the other hand, average at 5.4 and 2.1 bp per day on the two intraday specifications and worsen on average by 45% to 2.6 and 1.3 bp after the inclusion of the overnight return. Hence, the inclusion of the overnight return in the signal creates the opportunity for a cross-sectional reversal where a losers-minus-winners strategy would yield between 6-7 bp per day, whether positions are held from 1:30 to 15:30 or only during

the last half-hour of trading.

**Alternative formation and holding periods.** This section builds on the predictability patterns from [section 2.3](#) and defines additional intraday specifications as variations to the commonly used *last* and *afternoon*. To recall, the former defines the formation period from 9:30 until 12:00 and the holding period from 15:30 to 16:00. The latter defines the formation period from 9:30 until 13:00 and the holding period from 13:30 to 15:30.

In modifications to *last*, the formation period is adjusted to the interval from 10:30 to 13:00, or to only include the first half-hour as in [Gao et al. \(2015\)](#), while the holding period is kept unchanged in both variants. As an alternative to *afternoon*, its holding period is shortened to the second-last half-hour, and in one case the formation period remains unchanged while in the other I use the 10:30 to 13:00 interval (excludes first half-hour).

[Table 2.6](#) reports correlations between timing strategies of winners-minus-losers (WML) and cross-sectional quintile portfolios based on alternative intraday specifications. WML portfolios are formed for each specification as in [table 2.2](#), and pairwise correlations between different strategies are reported under the time-series panel. Likewise, five cross-sectional portfolios are formed for each specification as in [table 2.1](#), and five correlation matrices are calculated between alternative specifications of the same quintile. Then, the average correlation across quintiles is displayed under the cross-sectional panel.

Conclusions are similar for both time-series and cross-sectional portfolios. As expected, high coefficients cluster around the *last* and *afternoon* specifications showing that alternative intraday strategies are only marginally different from the base cases. This is especially true for the cross-sectional sorts, where all variations of the *last* configuration correlate above 0.90 among each other, while estimates from alternatives to *afternoon* all score above 0.60. On the other hand, (cross) combinations of *last* and *afternoon* variants record values in the low 0.20 or below.

## 2.5 Conclusion and further research

Literature on intraday predictability in returns is limited to a cross-sectional contribution by Heston, R. A. Korajczyk, and Sadka (2010) and a time-series investigation by Gao et al. (2015). Motivated by this scarcity of evidence I provide new and complimentary findings by using US common stocks data from 1993 until 2010. I divide the trading day in half-hours and study how returns in the afternoon are predicted by those in the morning. I find similar patterns of statistical predictability to those in Gao et al. (2015), i.e. morning returns anticipate continuation in the last half-hour, and I extend them with additional evidence of reversal in the afternoon. However, the economic profitability of timing strategies based on such periodicities turns out to be inconsistent with the statistical estimates. Moreover, the performance of these strategies over time suggests the existence of systematic differences in the cross-section.

By looking at the average return of ten cross-sectional portfolios formed on morning performance, I uncover a U-shaped pattern in average returns where stocks that lost or gained the most in the morning, earn higher and positive returns in the second half of the trading day, and especially during the last half-hour. For example, stocks that ranked in the bottom and top decile, respectively earn in the last half-hour 6.2 and 7.7 bp per day or about 15.6 and 19.4% in annualized terms. The finding is fundamentally different from the evidence documented by Heston, R. A. Korajczyk, and Sadka (2010), and is robust to: stock characteristics (I consider size, traded volume, illiquidity, tick size, volatility and skewness), the day-of-week effect, and variations to formation and holding periods. However, the pattern shows some dependence on the period of analysis. Specifically, morning winners are more profitable than morning losers before the decimalization in 2001, but these circumstances are reversed afterwards.

What is causing the cross-sectional U-shaped pattern in average returns during the last half-hour of trading remains an open question. I conjecture two interrelated effects. First,

institutional trading by e.g. indexed funds might induce significant positive price pressure, and second, trading mechanisms, such as the closing auction, and/or limits to speculative short sales and excessive depreciations establish a floor on the losses and leave the upside potential untouched. This latter effect would then be responsible for the differences observed in the average returns of the cross-section. There is established literature for most of the elements in the conjecture. For example, the last minutes of trading account for most of the daily return (see Cushing and Madhavan, 2000) and this phenomenon is consistent with institutional investors strategically timing their activity during periods of high volume (see Admati and Pfleiderer, 1988), i.e. at the open and close of the session. Moreover, Edelen and Warner (2001) document that institutional order imbalances bear impact on contemporaneous returns. Additionally, Amihud and Mendelson (1987) document a significant effect of trading mechanisms, like the opening auction, on the stock price. More research is needed to establish whether there is a positive price impact by institutional investors during the last half-hour of trading and whether market frictions and/or the closing auction play a role in the superior appreciation of morning losers. The decimalization of stock quotes in 2001 might prove to be relevant to the latter. Understanding the impact of decimalization on the U-shaped cross-sectional pattern might also improve statistical predictability by allowing the inclusion of regime switches, as Pesaran and Timmermann (1995) suggest.

Studies have investigated the impact of institutional trading on prices (CL1993; LVS1992), finding some evidence. However, the authors used quarterly data or it covered a short period of time. To test whether institutional trading is causing a positive price pressure at the end of the day, intraday signed volume and daily fund flow is needed for at least the S&P 500 ETF. By relating these two quantities, it should be possible to verify the asserted positive price pressure. In particular, since ETFs rebalance daily, price pressure should follow directly from net fund flow. Alternatively, since such data is proprietary, a weaker test could be implemented by excluding end-of-month days. As Etula et al. (2016)



document, most institutional traders rebalance at the end of the month. In the presence of strong positive fund flow and excluding those days, the price pressure should weaken significantly.

**Table 2.6:** correlations between timing strategies of winners-minus-losers (WML) and cross-sectional quintile portfolios based on alternative intraday specifications. Alternative holding and formation periods are defined in the table, where *last* and *afternoon* correspond to point A) and E). The vwap variations define buy and sell prices as 5-minute volume-weighted averages. WML portfolios are formed for each specification as in table 2.2, and pairwise correlations between different strategies are reported under the time-series panel. Likewise, five cross-sectional portfolios are calculated under alternative specifications of the same quintile. Then, the average correlation is displayed under the cross-sectional panel. Coefficients above 0.45 are highlighted in green. The sample consists of all common NYSE stocks excluding microcaps, i.e. stocks with a price below 5\$ or market capitalization in the bottom decile, and covers the period from January 1993 to May 2010.

	signal		hpr	
A)	9:30 - 12:00		15:30 - 16:00	
B)	9:30 - 12:00		15:30 - 16:00 vwap	
C)	10:30 - 13:00		15:30 - 16:00	
D)	9:30 - 10:00		15:30 - 16:00	
E)	9:30 - 13:00		13:30 - 15:30	
F)	9:30 - 13:00		13:30 - 15:30 vwap	
G)	9:30 - 13:00		15:00 - 15:30	
H)	10:30 - 13:00		15:00 - 15:30	

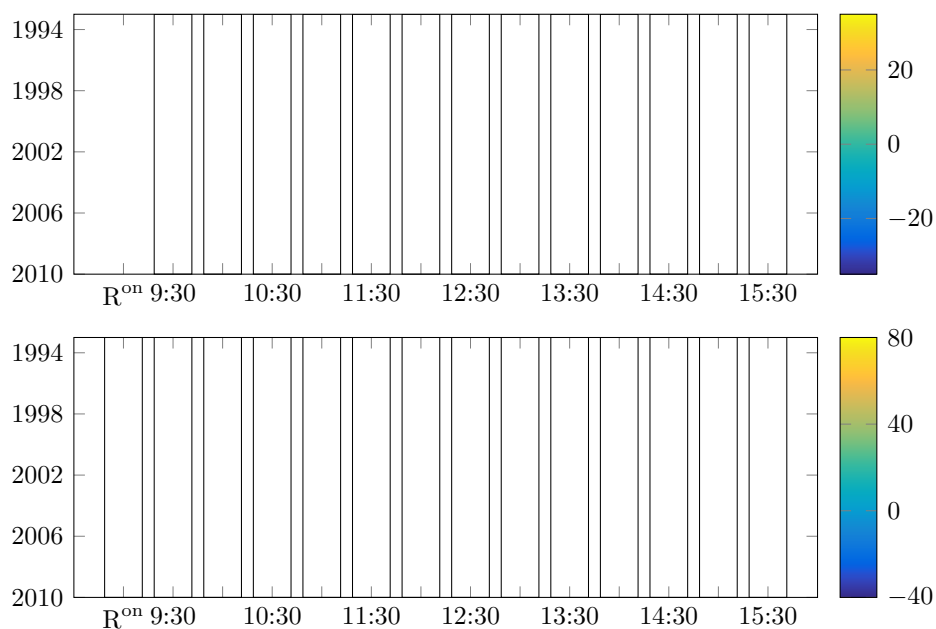
time-series								
	A)	B)	C)	D)	E)	F)	G)	H)
A)	1.00							
B)	0.87	1.00						
C)	0.53	0.46	1.00					
D)	0.66	0.57	0.20	1.00				
E)	0.18	0.18	0.13	0.13	1.00			
F)	0.21	0.20	0.16	0.14	0.94	1.00		
G)	0.15	0.17	0.10	0.11	0.65	0.64	1.00	
H)	0.10	0.11	0.06	0.05	0.46	0.46	0.63	1.00

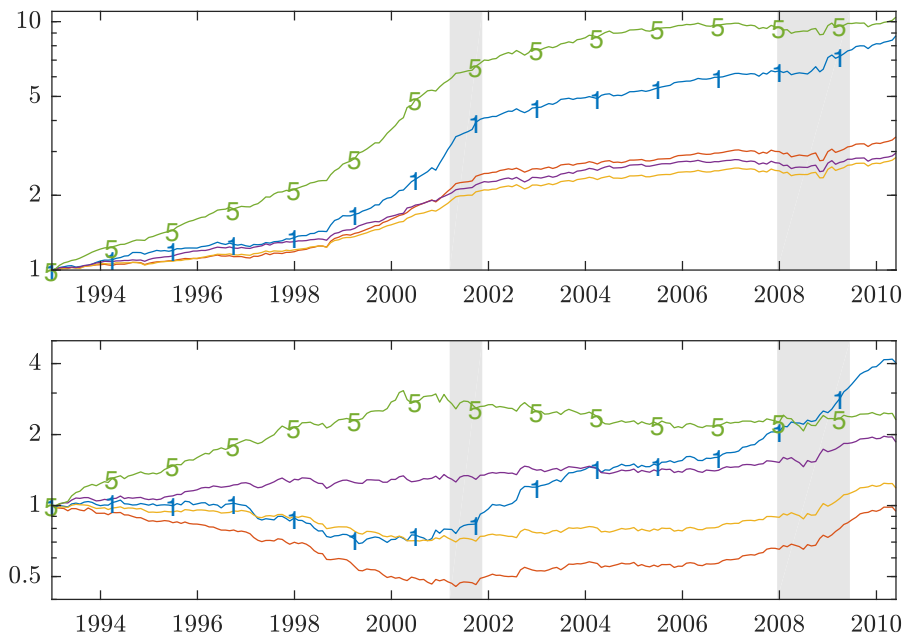
cross-sectional								
	A)	B)	C)	D)	E)	F)	G)	H)
A)	1.00							
B)	0.96	1.00						
C)	0.96	0.93	1.00					
D)	0.98	0.94	0.96	1.00				
E)	0.21	0.17	0.21	0.21	1.00			
F)	0.20	0.17	0.20	0.20	0.98	1.00		
G)	0.23	0.19	0.23	0.23	0.63	0.61	1.00	
H)	0.23	0.18	0.22	0.23	0.62	0.60	0.98	1.00

# Appendix

## Appendix 2.A Additional tables and figures



**Figure A1:** annualized cross-sectional average of half-hour returns over time. The trading day is partitioned in 13 half-hours and, at each point in time, the cross-sectional average of returns is calculated for each sub-period and the overnight time. The top plot focuses on intraday averages by dropping the overnight return estimate. The time on the x-axis marks the start of a half-hour. Values are expressed in percentage and darker colours correspond to lower returns. The sample consists of all common NYSE stocks excluding microcaps, i.e. stocks with a price below 5\$ or market capitalization in the bottom decile, and covers the period from January 1993 to May 2010.



**Figure A2:** monthly cumulated returns earned by equal-weighted quintile portfolios formed on past performance. Shaded areas denote NBER recessions. Specifically, stocks are sorted every day into five equally populated groups based on the performance over the intraday formation period. Then, the return of equal-weighted portfolios is calculated for the holding period. For clarity, only the first (lose) and the fifth (win) quintiles are marked. In the top plot is the *last* timing specification, with formation period from 9:30 until 12:00 and holding period from 15:30 to 16:00, and in the bottom plot is the *afternoon* timing specification, with formation period from 9:30 until 13:00 and holding period from 13:30 to 15:30. The sample consists of all common NYSE stocks excluding microcaps, i.e. stocks with a price below 5\$ or market capitalization in the bottom decile, and covers the period from January 1993 to May 2010.

**Table A1:** t-statistics from time-series predictability regressions. Each panel assesses the robustness of the base case (see table 2.1) with respect to single changes in methodology or in the period of analysis. In this order: A) univariate equivalent of the equation (2.1); B) regressions with non-standardized returns; C) include all previous half-hour in the RHS of the regression; D) include the overnight return as a separate regressor; E) excludes period of the financial crisis from December 2007 until May 2009; F) uses sample until April 2001 (pre-decimalization); G) uses sample after April 2001 (post-decimalization); H) uses periods from the beginning until end of 2000, then from May 2002 until June 2007 and from start of 2009 until May 2010 (bull markets); I) uses periods from 2000 until May 2002 and from June 2007 until end of 2009 (bear markets).

A) univariate												
c	09:30	10:00	10:30	11:00	11:30	12:00	12:30	13:00	13:30	14:00	14:30	
15:30	2.17	1.76	1.71	0.75	2.42	1.17	-1.36	-0.03	-0.06	0.83	1.46	
15:00	-2.04	-0.97	-2.66	-0.82	-0.29	-1.95	-1.97	0.47	-0.63	0.96		
14:30	0.00	-1.00	1.42	0.94	0.61	0.62	0.51	2.43	0.64			
14:00	-2.01	-0.70	-0.68	-0.47	-0.71	-0.32	-0.31	0.84				
13:30	-0.12	0.39	-1.37	-2.10	-1.30	-0.66	-0.96					
13:00	1.53	0.06	0.33	0.21	-1.66	-0.74						
12:30	0.66	0.19	0.69	-1.03	-0.68							
12:00	0.82	1.07	1.26	1.32								
11:30	0.76	-1.43	1.88									
11:00	1.05	-1.77										
10:30	-2.08											
B) no standardization												
c	09:30	10:00	10:30	11:00	11:30	12:00	12:30	13:00	13:30	14:00	14:30	
15:30	6.10	1.47	1.80	2.28	1.93	3.52	1.08	-1.42	0.99	1.04	2.00	2.02
15:00	1.55	-0.09	0.93	-1.75	1.37	0.17	-0.48	-1.34	0.67	0.45	1.26	
14:30	2.54	1.87	-0.69	1.10	-0.15	-0.63	-0.19	1.09	0.94	-1.75		
14:00	-0.57	-0.67	0.32	-1.15	0.01	-1.38	-1.84	-1.47	-1.03			
13:30	0.59	-0.90	-0.32	-1.91	-2.26	-1.29	-1.24	-2.06				
13:00	2.07	1.88	1.20	-0.02	-0.13	-1.48	-1.66					
12:30	1.76	1.95	0.08	0.78	-1.15	-0.12						
12:00	0.32	1.57	0.66	-0.07	-0.55							
11:30	0.20	0.77	-1.92	-0.46								
11:00	-1.74	0.41	-3.87									
10:30	0.06	-1.34										

continues...

...continues from table A1

C) no skip

	c	09:30	10:00	10:30	11:00	11:30	12:00	12:30	13:00	13:30	14:00	14:30	15:30
15:30	3.53	2.05	1.81	1.53	0.71	2.37	0.99	-1.44	-0.07	-0.05	0.92	1.03	-1.35
15:00	2.08	-2.04	-1.16	-2.64	-0.84	-0.30	-2.02	-2.03	0.40	-0.56	0.78	-3.85	
14:30	2.55	-0.09	-0.94	1.36	0.95	0.62	0.66	0.68	2.49	0.59	-3.44		
14:00	-0.86	-2.02	-0.75	-0.77	-0.56	-0.75	-0.36	-0.33	0.41	-4.23			
13:30	0.23	-0.01	0.30	-1.34	-2.10	-1.41	-0.80	-1.55	-1.89				
13:00	1.88	1.59	0.10	0.40	0.16	-1.73	-1.28	-8.96					
12:30	2.50	0.71	0.26	0.75	-0.96	-0.93	-3.08						
12:00	0.97	0.87	1.12	1.37	1.32	-5.80							
11:30	0.10	0.76	-1.33	1.79	-0.89								
11:00	-0.96	0.97	-1.99	-4.37									
10:30	-0.20	-2.29	-2.43										
10:00	-2.61	-5.07											

D) overnight

	c	R <sup>on</sup>	09:30	10:00	10:30	11:00	11:30	12:00	12:30	13:00	13:30	14:00	14:30
15:30	2.59	0.53	2.89	2.24	2.13	0.90	2.19	2.04	-0.93	-0.61	-0.18	0.89	1.66
15:00	0.96	0.41	-1.35	-0.61	-2.65	-0.12	-0.06	-1.47	-1.28	1.34	0.42	1.41	
14:30	1.88	-0.24	-0.29	-0.75	1.32	1.34	1.05	0.27	0.10	1.92	0.68		
14:00	-1.26	-0.84	-0.49	-0.54	-1.16	-0.54	0.04	0.08	-0.34	0.20			
13:30	-0.34	0.48	-0.24	-0.14	-1.23	-1.80	-1.27	-0.28	-1.80				
13:00	1.53	1.21	3.17	0.63	0.23	0.22	-2.15	-1.46					
12:30	1.16	-0.14	-0.96	-0.92	-0.29	-2.27	-0.89						
12:00	0.20	0.87	0.16	1.24	0.46	-1.04							
11:30	0.72	2.56	0.50	-1.02	0.61								
11:00	-0.92	2.10	2.25	-1.60									
10:30	0.63	-1.10	-0.60										
10:00	-1.56	-3.34											

E) exclude crisis 2007-2009

	c	09:30	10:00	10:30	11:00	11:30	12:00	12:30	13:00	13:30	14:00	14:30
15:30	4.02	3.17	1.77	1.35	0.82	2.21	-0.06	-0.16	-0.53	-0.19	1.72	0.81
15:00	1.64	-2.38	-1.85	-1.09	-1.12	0.64	-2.29	-0.58	-0.52	-0.67	2.47	
14:30	1.50	-0.98	-1.55	0.54	0.98	-0.74	-0.56	1.09	2.19	1.21		
14:00	-0.77	-2.76	-1.21	0.39	-0.63	-0.72	0.98	0.00	0.86			
13:30	0.95	-0.41	0.38	-0.97	-1.61	-0.84	-0.48	-1.05				
13:00	1.70	1.35	0.43	0.91	-0.60	-0.77	-0.64					
12:30	3.17	-0.71	0.79	-0.05	-1.57	-0.66						
12:00	1.39	0.27	1.08	1.69	1.78							
11:30	0.31	1.14	-1.07	1.82								
11:00	-0.25	0.88	-1.02									
10:30	-0.23	-3.04										

continues...

...continues from [table A1](#)

F) pre-decimalization												
	c	09:30	10:00	10:30	11:00	11:30	12:00	12:30	13:00	13:30	14:00	14:30
15:30	6.10	-1.47	0.06	-0.49	0.00	1.08	-0.73	-0.28	-1.07	-0.80	0.54	0.50
15:00	0.65	-1.96	-0.19	-0.28	2.12	-0.37	0.08	0.46	1.98	1.66	3.05	
14:30	-0.11	-1.62	0.04	-0.27	-0.32	-0.26	-0.44	2.10	2.69	-0.68		
14:00	-1.32	-0.43	1.58	-0.31	0.91	-0.74	1.21	0.05	-0.66			
13:30	0.46	-1.41	-0.82	-1.42	-1.06	-0.75	-0.94	-1.37				
13:00	1.13	1.72	1.05	1.21	-0.12	-1.42	0.81					
12:30	-0.55	0.88	2.40	1.55	1.45	2.06						
12:00	-1.20	2.44	2.00	2.44	3.60							
11:30	-1.39	2.09	-0.12	3.01								
11:00	-1.59	2.53	1.23									
10:30	0.58	2.64										
G) post-decimalization												
	c	09:30	10:00	10:30	11:00	11:30	12:00	12:30	13:00	13:30	14:00	14:30
15:30	1.66	2.86	2.05	1.86	0.80	2.25	1.38	-1.30	0.08	0.10	0.78	1.41
15:00	1.83	-1.63	-1.18	-2.73	-1.47	-0.22	-2.12	-2.28	-0.07	-1.23	0.15	
14:30	2.55	0.62	-1.03	1.61	1.10	0.69	0.82	-0.02	1.83	1.10		
14:00	-0.53	-1.93	-1.33	-0.58	-0.67	-0.40	-0.67	-0.25	1.13			
13:30	-0.09	0.31	0.44	-0.93	-1.94	-1.07	-0.56	-0.45				
13:00	1.34	1.02	-0.29	0.03	0.27	-1.35	-0.98					
12:30	2.60	0.44	-0.75	0.25	-1.44	-1.67						
12:00	1.28	0.20	0.47	0.82	0.67							
11:30	0.55	0.11	-1.41	1.03								
11:00	-0.39	0.42	-2.46									
10:30	-0.27	-3.20										

continues...

...continues from [table A1](#)

H) Bull markets												
c	09:30	10:00	10:30	11:00	11:30	12:00	12:30	13:00	13:30	14:00	14:30	
15:30	3.53	3.55	1.48	0.77	1.35	3.74	2.18	1.30	-1.72	0.21	1.21	0.98
15:00	0.98	-1.89	-0.36	0.11	-1.64	0.22	-2.40	0.15	0.10	-1.33	-0.37	
14:30	1.96	0.17	-1.51	1.38	1.08	-0.22	-0.62	0.93	1.46	1.79		
14:00	-0.16	-2.12	-1.05	0.91	-0.07	-0.34	1.20	0.93	1.11			
13:30	0.70	-0.71	-0.41	-0.55	-0.84	-0.74	0.54	1.40				
13:00	1.03	0.70	1.01	0.91	0.47	-0.57	-1.00					
12:30	3.14	-1.19	0.17	-0.14	-1.13	-1.52						
12:00	2.93	0.40	1.40	1.77	1.34							
11:30	-0.17	2.03	0.02	2.19								
11:00	0.77	0.92	-0.84									
10:30	0.91	-3.64										



















I) Bear markets												
c	09:30	10:00	10:30	11:00	11:30	12:00	12:30	13:00	13:30	14:00	14:30	
15:30	1.29	-0.19	1.06	1.22	-0.14	-0.02	0.25	-3.21	1.33	0.17	-0.32	1.62
15:00	1.47	-0.33	-0.04	-2.85	0.07	-1.19	-0.46	-2.88	0.72	-0.01	0.72	
14:30	2.06	1.05	0.17	1.13	0.14	0.87	1.35	-0.62	1.46	-0.96		
14:00	-1.27	-0.44	0.11	-2.32	-0.59	-0.82	-2.30	-0.74	-0.09			
13:30	-1.30	0.16	0.39	-1.48	-1.90	-1.44	-1.31	-1.78				
13:00	1.51	1.32	-0.59	-0.19	0.54	-1.43	0.06					
12:30	0.39	2.49	-0.12	1.28	-0.07	0.55						
12:00	-2.00	0.89	0.85	0.31	-0.07							
11:30	0.28	-0.84	-1.72	0.28								
11:00	-2.86	0.36	-2.84									
10:30	-0.80	1.25										end

**Table A2:** R-squared coefficient from predictability regressions. The first column contains coefficients from the base-case regressions of [table 2.1](#) and the remaining column represent variants to the base-case as outlined in [table A1](#), with the exception of variant A), which is not reported for compactness.

	B)	C)	D)	E)	F)	G)	H)	I)	
15:30	0.051	0.093	0.447	0.112	0.039	0.006	0.134	0.083	0.303
15:00	0.063	0.040	0.199	0.081	0.048	0.062	0.144	0.032	0.529
14:30	0.025	0.037	0.094	0.030	0.023	0.023	0.047	0.036	0.092
14:00	0.011	0.044	0.175	0.009	0.016	0.013	0.032	0.023	0.146
13:30	0.017	0.097	0.390	0.039	0.010	0.018	0.024	0.009	0.159
13:00	0.009	0.018	0.316	0.048	0.006	0.010	0.014	0.009	0.031
12:30	0.004	0.009	0.229	0.031	0.007	0.033	0.021	0.011	0.043
12:00	0.007	0.004	0.070	0.011	0.009	0.017	0.005	0.011	0.011
11:30	0.010	0.007	0.012	0.026	0.009	0.022	0.011	0.015	0.033
11:00	0.006	0.019	0.037	0.033	0.002	0.006	0.021	0.003	0.052
10:30	0.003	0.005	0.176	0.007	0.007	0.006	0.016	0.016	0.006
10:00	-	-	0.023	0.053	-	-	-	-	-



**Table A3:** average counts of constituents from portfolios formed independently on past performance and a stock characteristic. For details, refer to [table 2.3](#).

	<i>last</i>						<i>afternoon</i>					
	lose	2	3	4	win		lose	2	3	4	win	
<i>size</i>												
small	156	100	93	105	148		157	101	93	103	149	
medium	117	120	120	128	119		117	121	120	127	119	
large	89	134	142	143	96		88	134	144	142	95	
<i>std</i>												
low	68	131	149	143	79		68	131	150	142	79	
medium	114	114	109	118	116		114	115	109	117	116	
high	161	90	79	92	149		161	91	79	91	149	
<i>skew</i>												
bottom	123	110	107	113	118		123	110	107	112	118	
medium	113	112	113	119	113		113	113	114	118	113	
top	106	114	117	122	112		106	114	117	121	112	

**Table A4:** t-statistics for sub-period analysis of average annualized returns earned by equal-weighted portfolios independently formed on past performance and stock characteristics from [table 2.4](#).

	<i>last</i>					<i>afternoon</i>				
	lose	2	3	4	win	lose	2	3	4	win
<b>Pre April 2001</b>										
<i>size</i>										
small	8.6	8.5	8.8	10.0	15.6	-0.4	-5.3	-1.5	0.4	3.7
medium	6.0	6.4	6.7	7.0	9.9	-1.0	-3.6	-2.7	1.2	3.4
large	3.8	4.0	2.6	2.0	4.2	-0.0	-1.1	-1.0	1.8	3.2
<i>std</i>										
low	6.5	5.2	2.9	0.9	1.2	3.7	-0.4	-0.7	1.6	0.9
medium	5.8	5.6	5.6	5.9	8.1	-0.7	-3.9	-1.4	1.1	4.2
high	6.3	6.6	7.4	8.8	12.9	-2.6	-5.1	-4.1	-0.8	2.7
<i>skew</i>										
low	6.3	6.7	6.4	6.3	11.5	-1.7	-4.2	-3.8	1.1	2.5
medium	6.5	6.3	6.3	7.0	10.2	-0.8	-3.4	-2.0	0.0	4.0
high	5.7	5.4	5.0	5.0	7.9	-0.8	-3.1	-1.1	1.6	3.9
<b>Post April 2001</b>										
<i>size</i>										
small	7.1	4.1	3.9	3.8	3.9	6.0	2.7	2.3	2.1	0.4
medium	3.3	2.1	1.8	1.6	1.9	6.0	3.5	2.4	1.4	-1.1
large	3.0	2.0	1.5	0.8	1.3	4.4	3.1	2.4	1.6	-0.0
<i>std</i>										
low	2.2	1.2	0.7	0.3	0.4	7.9	4.7	3.0	1.5	-0.7
medium	3.7	2.2	1.8	1.4	1.1	7.2	3.6	2.3	1.3	-0.8
high	6.5	4.6	4.5	4.1	3.7	4.5	1.5	1.6	1.7	-0.1
<i>skew</i>										
low	4.9	2.7	2.5	2.3	2.7	5.5	3.2	2.6	1.9	-0.3
medium	4.3	2.3	2.2	2.1	2.6	5.5	3.0	2.2	1.4	-0.4
high	4.6	2.6	2.1	1.9	2.8	6.2	3.1	2.2	1.7	-0.3

**Table A5:** t-statistics for average annualized returns earned by equal-weighted portfolios formed on past performance and grouped by day of the week from [table 2.5](#).

	<i>last</i>					<i>afternoon</i>				
	lose	2	3	4	win	lose	2	3	4	win
<b>Whole sample</b>										
Mon	1.3	1.2	1.2	1.3	2.9	-1.2	-1.8	-1.1	-0.4	-0.5
Tue	3.0	2.2	2.1	2.3	4.0	1.3	0.2	0.4	1.0	0.6
Wed	3.0	1.8	1.5	1.6	2.8	1.8	0.3	0.7	1.6	2.0
Thu	3.9	3.0	3.0	3.0	4.7	1.7	0.5	0.5	1.2	1.5
Fry	5.9	4.5	3.8	3.6	5.8	3.9	1.1	1.0	1.1	1.3
<b>Pre April 2001</b>										
Mon	0.2	0.9	1.1	1.5	3.2	-1.8	-2.2	-1.1	0.3	1.1
Tue	2.7	2.5	2.7	3.1	5.3	0.0	-1.2	-1.2	-0.2	0.5
Wed	4.2	4.0	3.8	4.0	6.3	-0.1	-1.4	-0.5	1.4	2.3
Thu	3.3	2.8	3.0	3.1	5.3	0.1	-0.7	-0.6	1.1	2.5
Fry	5.9	5.3	5.0	4.7	8.6	0.8	-1.9	-0.7	0.5	2.0
<b>Post April 2001</b>										
Mon	1.6	0.8	0.7	0.5	0.9	0.2	-0.5	-0.5	-0.7	-1.6
Tue	1.7	1.1	1.0	1.0	1.5	2.1	1.5	1.4	1.3	0.4
Wed	1.2	0.4	0.3	0.3	0.5	2.5	1.5	1.3	1.1	0.6
Thu	2.4	1.7	1.7	1.4	1.7	2.1	1.2	0.9	0.6	0.0
Fry	2.9	1.8	1.4	1.3	1.5	4.4	2.5	1.7	1.1	0.1

# Chapter 3

## Acceleration and reversal

### 3.1 Introduction

Previous literature has established that historical stock prices may contain information about future returns. Hence, it is possible to create trading strategies based on signals extracted from past price changes (returns). In this work, motivated by [Xiong and Ibbotson \(2015\)](#), we attempt to analyse how the concept of return acceleration fits in the research picture, and if the long-short strategies based on such concept are profitable in absolute terms, and relative to simpler momentum strategies which only use cumulative past returns.

Intuitively, we are not just interested in the level that a price path has reached but also in how this new point has been achieved. From this perspective, a price path could be categorised into an upward or downward straight line (noise aside) or into an upward or downward launch ramp. Understanding if these four configurations matter, is relevant to several fields of literature. For instance, there is evidence of market overreaction, first documented by [De Bondt and R. Thaler \(1985\)](#) and [Howe \(1986\)](#), which would be consistent with a ramp-like path. This, in turn, is related to price formation and market efficiency, or in other words to how acceleration in returns comes into play and whether it is an anomaly (see related work by [R. Ball and Kothari, 1989](#); [Beaver, Lambert, and Morse,](#)

1980). Finally, since we are looking at the path in its entirety, we can see if and how acceleration is related to specific calendar regularities, like the month-end effect (seminal works are by De Bondt and R. H. Thaler, 1987; Lakonishok and Smidt, 1988), and if institutional trading is the driving force that justifies the whole picture (see Etula et al., 2016). Further motivation and literature is discussed in [section 3.1.1](#).

We build several empirical measures of acceleration and compare them to other signals from the academic literature. We show that acceleration measures are mostly independent from traditional price-based signals such as 1-month reversal and 12-2-month momentum. Then, we study the properties of equity portfolios formed by sorting stocks into deciles based on acceleration. We find that there is a large spread in average returns for these portfolios, and for the low-minus-high (LMH) strategies, which go long stocks whose returns are decelerating and short stocks whose returns are accelerating. Moreover, the LMH strategy produces large positive mean returns, even after controlling for transaction costs.

In the next step, we analyse the risk properties of the acceleration portfolios and the LMH strategy. We show that the classic and contemporary state-of-the-art equity factor models do not explain the mean returns exhibited by acceleration portfolios. In particular, even the 1-month reversal, which is fairly similar to acceleration, does not add much explanatory power. However, the reversal factor constructed on a shorter lookback period of 5 days, is able to price the acceleration-based portfolios.

The remainder of the paper is organized as follows: this section concludes with the motivation for the empirical analysis in relation to the current literature. [Section 3.2](#) describes the data and provides the details on how acceleration measures are constructed. [Section 3.3](#) shows that our acceleration-sorted portfolios produce a robust monotonic pattern in mean returns. We then present the formal analysis on the risk-return trade-off in [section 3.4](#). We conclude the study and provide suggestions for further research in [section 3.5](#).

### 3.1.1 Acceleration

Extensive academic literature explores how information from past prices predicts future returns in the cross-section.<sup>1</sup> However, the academic community was predominantly interested in understanding how price *changes* can forecast returns. Instead, in this paper we would like to explore, from a predictability point of view, how helpful are *changes in changes*. In particular, we question whether the speed of the price moves adds any information to the simple content given by the direction and size of these moves (already previously studied).

From another perspective, if we look at the cumulative return over the formation period (lookback) of a signal, for example a month, we only get the point of arrival and lose information in between. In other words, fixing the formation period, it is interesting to explore if price formation *within* the period is also useful for the formulation of a trading strategy. In this context, a study of acceleration becomes a natural candidate since it also implicitly picks how price has moved during the lookback.

Subsequently, we found that similar work has only been carried-out by [Xiong and Ibbotson \(2015\)](#), two researchers of the financial industry. They claim that twelve-month momentum and one-month reversal are two faces of the same phenomenon, which they call acceleration. In particular, [Xiong and Ibbotson \(2015\)](#) focus on a twelve-month lookback and define an empirical measure of medium-term acceleration to be the difference in the average returns over the last six months minus the first six months.<sup>2</sup> The authors find that acceleration is informative about the probability and the magnitude of a stock price reversal.

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<sup>1</sup>Classic references are [De Bondt and R. Thaler \(1985\)](#) for long run reversal, [Jegadeesh and Titman \(1993\)](#) for medium-term momentum, and [Jegadeesh \(1990\)](#) for short-run reversal.

<sup>2</sup>This is not the only work that attempts to combine the informative content of momentum and reversal in order to provide a single-factor explanation. [Han, Zhou, and Zhu \(2016\)](#) also look into combining various price signals with different lookbacks into a single factor. They call this factor a trend, and show that a strategy based on it outperforms the traditional short-term reversal, medium-term momentum and long-run reversal strategies while not being spanned by them.

Our work is different since we look at a lookback period of one month and the shorter formation window has two implications. First, if acceleration underlies a deviation of the price from fundamental values, then such deviation is likely to be transitory, which might be economically consistent and hence profitable only in the short term, rather than over a year. Second, even [Xiong and Ibbotson \(2015\)](#) argue that a contrarian position should also be profitable, i.e. going long the least accelerated stocks and shorting those that accelerated the most. If that is the case, it would be relevant to compare which of the two contrarian strategies – short-run reversal or acceleration – has the most informative signal. However, for a fair comparison, the strategies should share the same lookback, and it is preferable to level with reversal since it has been already extensively researched.

We do not exclude ex-ante the possibility that acceleration is related to the traditional reversal. For instance, [Da, Q. Liu, and Schaumburg \(2014\)](#) argue that not every stock which has a negative (positive) performance during the previous month, should reverse in the next period. In particular, signals built on past returns should also account for fundamentals – news about cashflows and discount rates – in order to lock-in a “*cleaner*” reversal. The intuition is as follows: if a stock went down due to a deterioration in expected profitability, a successive recovery can only follow from an improvement in the profitability outlook. Separately, differences in the cross-section of returns can arise from different exposures to the risk factor. That is, if a stock outperformed another one simply because it is riskier, and assuming that the risk factor on average realizes positive returns, there is no guarantee that the riskier stock will underperform in the following period. [Da, Q. Liu, and Schaumburg \(2014\)](#) conclude that returns of the “*cleaned*” reversal strategy are stronger than the traditional reversal and link the economic motivation to liquidity risk and investor-sentiment. Since it seems unlikely that prices reflect news every month in a way that leads to visible acceleration patterns, it could well be the case that our acceleration measure is capturing in a simpler way the “*cleaned*” reversal.

We also do not dispute whether any of the theories put forward for reversal could

actually lie behind acceleration. Hence, we do not take a particular stand on what might cause acceleration and, as with reversal, there could be competing theories coming both from the behavioural finance and rational expectations literature.<sup>3</sup> Our main task is to empirically establish whether acceleration is a separate phenomenon as e.g. argued by Xiong and Ibbotson (2015).

## 3.2 Data and methodology

This section defines our sample, the period of coverage and the main data used for the analysis. The methodology on signal construction is outlined next, and a description of the factor models, used to adjust and test the abnormal returns of our strategies, concludes.

Our sample consists of common stocks from the Center for Research in Security Prices (CRSP), i.e. those with a share-type code of 10 or 11, and excludes observations from the holding period which either had, at the end of the previous month, a price below \$5 or a market capitalization in the lowest New York Stock Exchange (NYSE) decile. Consequently, our panel has a total of 9569 equities with an average of 4052 stocks per month, and covers the period from January 1993 until December 2014. The CRSP database provides return data adjusted for firm events, e.g. stock splits or distributions, which we complement with delisting returns, as in Beaver, McNichols, and Price (2007), to avoid selection bias.

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<sup>3</sup>Rational expectations theories of reversal are often based on liquidity issues. Avramov, Chordia, and Goyal (2006) argue that reversal strategy returns are stronger for lower liquidity stocks. Nagel (2012) suggests average returns to reversal strategy reflect the market maker's liquidity-bearing capacity; moreover, there is significant time variation in this liquidity provision. So and S. Wang (2014) find consistent evidence for this story via inventory risk channel by studying reversal profits around scheduled information releases. An alternative rational agents story is that by Johnson (2016), who claims that delegated fund management with performance evaluation versus the benchmark can generate reversal patterns. Finally, the story in Etula et al. (2016) combines the institutional asset management friction, argued to be related to the monthly cash management cycle, with the limited liquidity-bearing capacity by market makers to explain aggregate market-level reversals. There is still an unsettled empirical debate on whether the higher share of institutional investors increases [Johnson (2016) and Etula et al. (2016)] or decreases [Cheng et al. (2016)] return reversals.



**Signal construction** All signals are built with daily observations at the end of the formation month which precedes the holding period. We consider five specifications throughout the analysis and, in particular, we focus on proxies for acceleration in returns.

Our first proxy for acceleration is denoted by  $acA$ , which stands for area-of-acceleration, and its construction is shown in [figure 3.1](#).

**FIGURE 3.1** ABOUT HERE

The figure plots a solid line of hypothetical prices, denoted by  $P_0, P_1, \dots, P_N$ , and a dashed straight line of prices, identified by  $I_1, \dots, I_{N-1}$ , which are interpolated between the two ends of the formation period. Then, the signal estimates the area formed by the two lines by adding up the deviations of actual prices from the interpolated ones (see the red vertical lines in the figure). Formally, by defining the interpolated prices with  $I_t \equiv \frac{P_N - P_0}{N}t$ , the signal becomes

$$acA \equiv \sum_{t=1}^{N-1} (I_t - P_t), \quad (3.1)$$

where  $N$  counts the number of trading days of the formation month and  $P_0$  is the last price from the preceding month.<sup>4</sup> Intuitively, the closer the price series resembles an increasing parabolic path, much alike that of a jumping ramp, the greater will be the area enclosed by the interpolated line.

Our second proxy for acceleration, which we call  $ac5$  (acceleration five), is more easily defined as the difference of the average return over the last five days -  $la5$  - minus the average return over the first five days -  $fi5$  - of the formation month. Hence, the signal is simply

$$ac5 \equiv fi5 - la5. \quad (3.2)$$

By looking again at [figure 3.1](#), we can give a geometric interpretation of [equation \(3.2\)](#) and highlight the link between  $ac5$  and  $acA$ . In fact, the average return in  $fi5$  can be

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<sup>4</sup>The sum in [equation \(3.1\)](#) is taken over days  $1, 2, \dots, N - 1$  since on the interpolation extremes, i.e. for  $t \in \{0, N\}$ , the difference between the interpolated and actual prices is null by construction

approximated by the (negative) slope of the line going through  $P_0$  and  $P_5$ . Similarly,  $la5$  can be approximated by the (positive) slope of the line going through prices  $P_{N-5}$  and  $P_N$ . Hence, keeping this geometric representation in mind, if the slope for  $fi5$  steepens, i.e. in our example becomes more negative, the area enclosed by the path of actual and interpolated prices, widens. This combined effect translates in both  $ac5$  and  $acA$  having a higher signal. Thus, the connection between the two acceleration proxies is established by noting that the variation in the signals preserves sign and magnitude. The same conclusion applies to changes in  $la5$  and across different market conditions.

Together with the accelerations proxies, we include  $fi5$  and  $la5$  as standalone signals in order to give additional insight into which part of the formation period might be contributing towards our strategies. This interpretation is possible thanks to the direct role that  $la5$  and  $fi5$  play in the construction of  $ac5$  and their implicit connection to  $acA$ . Finally, since we focus on a one-month formation period, a natural choice is to include the reversal strategy by Jegadeesh (1990). The signal, which we label with  $rev$ , is simply the prior-month return.

As general remarks on signal construction, to avoid the bias induced by the bid-ask bounce, we use mid-to-mid returns as e.g. in Da, Q. Liu, and Schaumburg (2014) and Lou, Polk, and Skouras (2016).<sup>5</sup> Moreover, we skip stocks that have missing daily observations during the formation period. Finally, each signal is de-measured by its industry-average as in Novy-Marx (2013). We follow standard literature practice, e.g. Goyal (2014), and use the Standard Industry Classification (SIC) codes from the COMPUSTAT-merged database to classify each stock into one of the 49 Fama and French industry portfolios, which are taken from professor Kenneth French's Data Library.

**Transaction costs.** We consider a cost-minimization exercise in section 3.3 to gauge the profitability of our strategies. Here, we briefly present the methodology applied for

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<sup>5</sup>Mid-to-mid returns are calculated from firm-event adjusted data and the procedure is described in section 3.B.

the robustness and how the transaction costs ( $Tcosts$ ) are estimated using bid-ask spreads, henceforth simply spreads, and portfolio turnover.<sup>6</sup> Specifically, we calculate the daily spread of a stock as the difference in bid/ask prices at close over the mid price, i.e.  $(P_t^{Ask} - P_t^{Bid})/P_t^{Mid}$ . Then, at the end of the formation period, we only keep stocks from the cheapest quintile by spread and construct value-weighted portfolios with those equities. Our approach is similar to the cost-minimizing methods examined in [Novy-Marx and Velikov \(2015\)](#). The authors perform conditional double sorts in order to decrease the bid-ask spread component: for each size decile they keep equities from the lowest (effective) bid-ask spread decile.<sup>7</sup> For each portfolio, we calculate its (value-weighted) average spread and turnover, where the latter captures the percentage change in the constituents after rebalancing.<sup>8</sup> Then, we calculate the cost by portfolio as the product between its turnover and average spread. Finally, the  $Tcost$  of the LMH is defined as the sum of the individual costs for the *High* and *Low* portfolios. In other words,  $Tcosts_{LMH} = spread_{Low} \times turnover_{Low} + spread_{High} \times turnover_{High}$ . Price data are from CRSP.

**Test specifications and risk factors.** We consider some of the classical and most recent models from the empirical equity pricing literature. For instance, we use the well known [Fama and French \(1993\)](#) 3-factor model with their market (MKT), size (small-minus-big, SMB) and book-to-market (high-minus-low, HML) factors. We also consider the [Fama and French \(2015\)](#) extended 5-factor model (FF5), which adds a robust-minus-weak (RMW) factor, defined as the difference in return between stocks with robust and weak profitability, and the conservative-minus-aggressive (CMA) factor, which captures the difference in return between stocks with low and high investment. Another specification borrows

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<sup>6</sup>We assume that the nominal size of our strategies is negligible and hence bears no market impact during trading.

<sup>7</sup>The sample preserves the statistical properties reported for the whole universe in [table A1](#).

<sup>8</sup>The average bid-ask spread by portfolio does not change significantly from one month to another. Hence, the robustness test is not affected whether ex-ante or ex-post costs are taken. Results are available upon request.

the so-called *q-factors* model by Hou, Xue, and Lu Zhang (2015), which in addition to the market and size, also includes factors based on their own measure of investments (investment-to-asset, IA) and profitability (ROE). Furthermore, we also test three specifications which extend the FF5. The first model is augmented with the liquidity factor (LIQ) from Pastor and Stambaugh (2003). The second adds the time-series momentum (TSMOM) factor from Moskowitz, Ooi, and Pedersen (2012), and the cross-sectional momentum (UMD) and short-term reversal (STREV) factors from professor Kennet French's Data Library. Finally, the last specification adds our own short-term reversal factor based on *la5* i.e. it shorts low-signal stocks and buys high-signal stocks. Models specifications, factor descriptions and the sources of the data are summarized in [table A2](#).

### 3.3 Preliminary analysis

This section first shows that acceleration is not a mere proxy for other price-based signals typically considered in the academic literature. Second, it uncovers a monotonic pattern in mean returns of portfolios sorted on acceleration, leading to high profitability of low-minus-high (LMH) strategies which are robust to transaction costs.

**Signal comparison.** [Table 3.1](#) reports time-series averages of cross-sectional correlations among signals. Pearson (linear) and Spearman (rank) coefficients fill respectively the lower and upper triangular section of the matrix. Since both versions are very similar, hereafter we will only refer to linear correlations. Numbers in bold indicate a value greater than 0.3 in absolute terms.

[TABLE 3.1](#) ABOUT HERE

The correlation matrix builds on a broader set of signals (cf. [section 3.2](#)) to provide supporting evidence for a twofold claim: acceleration does not depend on other traditional signals of the financial literature, but its proxies are correlated among each other.

To support the first part of the claim, we add *mom*, *trd*, *std<sub>0</sub>* and *std*. The first of the list is momentum (see Jegadeesh and Titman, 1993), and is defined as return over the previous twelve months skipping the most recent. The listed *trd* signal is the regression coefficient of prices on a time-trend, i.e. it is the coefficient  $b$  from the time-series regression specified by  $P_t = a + bt + \varepsilon$  with  $t = 1, 2, \dots, N$ . The last two, measure daily volatility over the formation month, where the former assumes a null mean, i.e. adds up squared returns, while the latter is the usual standard deviation.

For the second part of the claim, we include *acR* and *acH*. The former is the regression coefficient of prices on a quadratic time-trend, i.e. the  $b_2$  in  $P_t = a + b_1t + b_2t^2 + \varepsilon$ . The latter is a variation of *ac5* where instead of five days we take the last and first half of the formation month and, eventually, skip one day in between if the number of trading days ( $N$ ) is odd.<sup>9</sup>

Evidence shows that the acceleration signals are uncorrelated with reversal, momentum or volatility, with correlations coefficients ranging between -0.05 and 0.07, but they are correlated among each other, with coefficients starting at 0.66. Another evident pattern about acceleration is the positive dependence with *la5* compensated by an almost equal negative correlation with *fi5*. This behaviour is consistent with the definition of *ac5* in [equation \(3.2\)](#) which takes  $la5 - fi5$ , and generalizes to the other acceleration signals. Hence, acceleration seems to capture information which is not already included in other traditional signals of the academic literature.

**Portfolio sorts and LMH strategies.** At the beginning of the holding period, we sort stocks into deciles based on the signal and form 10 portfolios weighted by market capitalization.<sup>10</sup> Then, the low-minus-high (LMH) strategy goes long the low-signal portfolio

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<sup>9</sup>In addition to the acceleration proxies reported in the correlation matrix we extensively explored variants of the *ac5* and time-trend regressions. Results do not change qualitatively and are available upon request.

<sup>10</sup>Market capitalization is defined as the product between number of outstanding shares and stock price. Our main conclusions remain unchanged if we use equal-weighted portfolios.

and shorts the high-signal portfolio. The whole procedure is repeated for each signal, and portfolios are rebalanced monthly.

Table 3.2 collects monthly gross-of-costs descriptive statistics on decile portfolios and on the LMH strategies. Panel A reports the average returns of the 10 portfolios, a sparkline plotting those values and the average return of the LMH strategy. Panel B provides detailed statistics on the series of LMH returns, where, among others, AC(1) is the lag-one autocorrelation, SP the Sharpe Ratio, MDD the maximum drawdown, len the length of the MDD and rec the time it took the strategy to recover afterwards.<sup>11</sup> Each row from the panels refers to a specific signal.

TABLE 3.2 ABOUT HERE

According to Panel A, signals *la5*, *ac5* and *acA* show an increasing monotonic pattern in portfolio returns which translates into economically conspicuous profits for the LMH strategies, with monthly (annualized) values averaging at 1.59 (19.08), 1.23 (14.76) and 1.22% (14.64%). In contrast, *rev*'s return profile is flat and the strategy grosses a monthly negative gain of -0.11%, while *rev*, at best, exhibits a hump-shaped pattern with the corresponding LMH strategy settling on a value of 17 bps.<sup>12</sup>

Panel B focuses on LMH series and their characteristics. All strategies have relatively high standard deviation, with values ranging between 4.92 and 6.05%. However, as already anticipated in Panel A, while *la5*, *ac5* and *acA* compensate volatility with returns, *rev* and *fi5* fail to do so. Furthermore, if we look at Sharpe Ratios, the gap between *la5*, *acR* and *ac5* is only marginal with monthly (annualized) values of respectively 0.27 (0.94), 0.25 (0.87) and 0.24% (0.83%). In strong contrast, the *rev* and *fi5* strategies have negligible Sharpe Ratios, their monthly minimums are almost four times worse compared to *acA* and have they have the worst profiles in terms of maximum drawdown (MDD).

<sup>11</sup>Detailed descriptive statistics at the portfolio level are reported in the appendix under table A1.

<sup>12</sup>This result is consistent with Lou, Polk, and Skouras (2016), who look at a very similar sample horizon and find no profitability in one-month reversal strategy.

We complement descriptive statistics with a plot of cumulative LMH returns in [figure 3.2](#). For comparability, all strategies are rebased to zero at the beginning of the period of analysis and scaled by their respective standard deviation.<sup>13</sup>

The figure outlines strong resemblance between *la5*, *acA* and *ac5* on one side, and *rev* and *fi5* on the other side. The former group realises steady gains until 2000 when it increases its pace and makes the most of the *dotcom* bubble until 2002. Follows a flat period, whose end in 2009, coincides with the beginning of a new positive cycle, stretching until the end of the analysis; this last trend sees e.g. *acA* gaining more than 9% per year. Moreover, the first group shows countercyclical behaviour and remains profitable during NBER recessions, which are marked in the figure by the grey bands. Lastly, the differences in MDD, its length and recovery time, which might have favoured the acceleration signals (see Panel B in [table 3.2](#)), materialize during the flat period. It becomes clear from the graph, that those numbers should be interpreted with caution, since none of the strategies outperforms the others.

**FIGURE 3.2** ABOUT HERE

The picture is quite different for *fi5* and *rev*, with the former being still except for the last two years when it closes in negative territory. The reversal strategy, while not profitable overall, and consistently so with [Lou, Polk, and Skouras \(2016\)](#), at first trends persistently upwards until mid 2004, but then changes direction and retraces back until the end of the study. In comparison, *la5*, which is intuitively the same kind of reversal strategy but with a formation period of one week, earns money for most of the time when *rev* does not. The example seems to suggest that the choice of the lookback length is crucial and might dependent on the historical period.

Additionally, we examine closer the average holding period return. [Figure A1](#) presents the breakdown of the holding period by trading days. We notice that *acA*, and *ac5*as

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<sup>13</sup>Since strategies in the figure have 1% volatility, they are not representative of the profitability reported in the tables.

well as *la5* earn positive returns beyond the first trading week. On the contrary, *rev* is only profitable in the initial five trading days and then fluctuates around zero. This result is interesting because it suggests the traditional reversal signal with one-month lookback is only informative about the one-week ahead LMH returns, while the one-week lookback contains information about the strategy profitability in the subsequent weeks. This observation gives supporting motivation for keeping the holding period at one month for *la5*. Lastly, the mean returns for *fi5* always stay roughly flat at zero, again hinting that the signal is uninformative.

Essentially, strategies can be grouped by profitability with *la5*, *acA* and *ac5* on the good side and the *rev* and *fi5* on the other side. It might come unexpected, that the *la5* is bunched together with the acceleration strategies, since the former is a short-term variation of *rev*. Yet, *la5* (together with *fi5*) is a fundamental constituent of *ac5* (see [equation \(3.2\)](#)), and more so if we consider that *fi5* bears no contribution to the profitability of *ac5*. Hence, not only *la5* is intrinsically related to the acceleration strategies, but evidence suggests that *de facto* it might be their only driver. Under this perspective, it is only natural that the three strategies of the profitable group have similar risk-return profiles.

**Profitability net of costs.** In the previous section we identified a profitable signal group in *la5*, *acA*, and *ac5*. Gross-of-costs, only market makers can take advantage of these investment opportunities by transacting at about mid prices. Here, we show in a cost-minimization exercise, that LMH returns remain statistically significant even after transaction costs, such that, also buy-side investors can feasibly earn a profit by implementing acceleration and weekly reversal strategies in the equity market.<sup>14</sup> Thus, to keep LMH profitable in the presence of bid-ask spread (proportional transaction cost), we only keep stocks from the cheapest quintile and form value-weighted portfolios with the remaining equities. Details on the estimation of portfolio costs, *Tcosts*, and the methodology

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<sup>14</sup>For an analysis of price impact on reversal strategies see e.g. Avramov, Chordia, and Goyal (2006) and R. Korajczyk and Sadka (2004).



for the conditional sorts is outlined in [section 3.2](#).<sup>15</sup>

[Table 3.3](#) reports the results of the cost analysis in a similar fashion to [table 3.2](#). In addition to the gross average returns by portfolio, the sparklines and the gross strategy return (*LMH*), the table lists the transaction cost (*Tcost*) of each strategy and its gross standard error (*se*). Two implications are evident: first, the return profiles are preserved even after conditioning on the cheapest quintile. In fact, returns of conditional portfolios have the same monotonic pattern as that depicted by the whole cross-section and gross LMH returns are unchanged (compare Panel A of [table 3.2](#)). Second, after subtracting the estimated cost from the LMH of *la5*, *acA* and *ac5*, the obtained difference is still greater than twice its gross *se*. For instance, the three strategies are left with a difference which is significant at the 5% level, with values (in parenthesis are  $1.96 \times se$ ) respectively of about 1.40 (0.99), 0.98 (0.86) and 0.88% (0.88%).<sup>16</sup> [Table 3.3](#) under Panel B shows that most companies from the cheapest spread quintile are of large capitalization. There are roughly ten times more firms from the highest market capitalization quintile than in the lowest. Hence, stocks that have low quoted bid-ask spread tend to be big firms.

**TABLE 3.3** ABOUT HERE

Our results are different from those of Avramov, Chordia, and Goyal (2006) for the weekly reversal. In their empirical design, they consider equally-weighted portfolios and their starting sample does not exclude microcaps as we do. Hence, the difference in the conclusion about transaction costs are mostly coming from small stocks, which have a larger influence in their design.

To summarize, in opposition to the standard 1-month reversal factor, there is a strong and increasing monotonic pattern in average portfolio returns for *acA*, *ac5*, and *la5* as we move from a low-signal portfolio to a high-signal portfolio. This makes LMH strategies based

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<sup>15</sup>Fixed costs or short-selling costs are assumed to be negligible.

<sup>16</sup>Our estimates of transaction costs are conservative, as the effective bid-ask spread is typically lower than the quoted analogue (see Avramov, Chordia, and Goyal, 2006).

off those signals, which go long the former portfolio and short the latter, very profitable in gross terms. In fact, we argue that these profits survive the reasonable transaction cost estimates, provided we impose a cost-minimizing approach which restricts our tradable stock universe. Finally, we notice from our preliminary analysis that acceleration signals do not profile better than *la5*, except for certain stand-alone strategy risk characteristics such as duration of maximum drawdown. Next, we explore in more detail the risk-return properties of our return series.

### 3.4 Spanning by factor models

In this section we look at the risk properties of the LMH strategies and the decile portfolios. First, we look at risk-adjusted LMH returns by running time-series regressions on some of the classical and most recent factor models from the empirical equity pricing literature (for a description see [section 3.2](#)).<sup>17</sup> Subsequently, we formally test whether the aforementioned models can price the cross-section of our portfolios. To preview our results, we find that the risk-adjustment plays in favour of *la5*, *acA* and *ac5*, and that the models struggle to explain the returns of those strategies. However, when we include the *la5*'s LMH as a factor, the puzzle seems to be accounted for. Furthermore, this factor helps pricing the acceleration and reversal portfolios, suggesting strong dependence in the returns of our portfolios.

**Empirical setting.** In the absence of arbitrage opportunities, if test assets are constructed to be zero-cost, the Euler equation implies that the risk-adjusted return of the  $i$ -th asset should be zero, i.e.  $E_t[R_{i,t+1}M_{t+1}] = 0$ , where  $M_t$  is some existing stochastic discount factor (SDF). We also assume the SDF is affine in the factors:

$$M_t = 1 - (f_t - \mu_f)'b, \quad (3.3)$$

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<sup>17</sup>Both the dependent and independent variables in the regression are consistently gross-of-costs.

where  $f_t$  is the set of pricing factors and  $\mu_f$  is their unconditional mean, and the  $b$ 's are the factor loadings. This specification implies a beta-pricing model where the expected excess-return of the  $i$ -th portfolio, the  $R_{i,t}$ , depends on the price of the risk factors, i.e. the  $\lambda$ s, and the corresponding risk sensitivities, denoted by  $\beta_i$ :

$$E[R_{i,t}] = \lambda' \beta_i, \quad (3.4)$$

where the factor prices are obtained from the loadings  $b$  and the covariance matrix of factor returns  $V_f$ , i.e.  $\lambda \equiv V_f b$ . The  $\beta_i$  are equivalent to the regression coefficients from the excess return of the  $i$ -th portfolio on the factors plus a constant, i.e.  $\beta_i \equiv V_f^{-1} cov(R_{i,t}, f_t)$ .

**Univariate analysis.** Table 3.4 displays the results of monthly time-series regressions of LMH strategy returns on the risk factors and a constant. Model specifications vary across columns while the risk factors, corresponding to the regression coefficients, are labelled along the rows (see table A2 for details). The first row with the  $\alpha$  identifies the constant of the regression, and the last row reports the adjusted  $R^2$  coefficient, i.e. a measure of goodness of fit. Results are grouped into panels denominated with the signal underlying the LMH strategy.

TABLE 3.4 ABOUT HERE

The mean excess return of the *rev* strategy, reported in the first column, is economically small and statistically insignificant. After controlling for the factors, the risk-adjusted return captured by the  $\alpha$  tends to be negative, although insignificantly so in all but one case. The excess-return of the strategy does not load on any of the factors, except for the market return and, unsurprisingly, the reversal factors. However, when we control in the last specification for our *la5*, the risk-adjusted return worsens to -60 bp and becomes significant at the 5% level. Additionally, the adjusted  $R^2$  sensibly improves in comparison to earlier specifications but remains at a modest 16% in absolute terms. The analysis confirms

the disappointing profitability of the reversal strategy based on a 1-month lookback period.

Similar conclusions hold for *fi5*. Neither the raw nor the risk-adjusted return are large enough to be statistically different from zero. The strategy loads positively on the market and 1-month reversal, and negatively, to some extent, on the cross-sectional momentum factor (UMD). Since the signal underlying *fi5* is independent of *la5* (see [table 3.1](#)), it does not come as a surprise that *la5*'s coefficient in the last specification is null.

The panel for *la5* presents a totally different scenario. The MKT factor loading is only significantly positive in model (2). At the same time, the loading on the HML factor becomes negative and significant and so does the IA factor. Nevertheless, the risk-adjusted return for *la5* tends to be higher than the raw return in almost all cases, although the standard error increases as well, but the coefficient remains strongly significant. More relevantly, adding other price-related factors does not help explaining the puzzlingly positive *la5* returns, and even the loading on STREV is within one standard error from zero.  $R_{adj}^2$  is again very low across all models with the highest achieved value of 11%.

Finally, acceleration strategies *ac5* and *acA* seem to be largely unrelated to any of the traditional factors. In contrast to *rev* and *fi5*, they do not load positively on the market and the 1-month reversal factors, and even show some evidence of negative loadings. Also, they do not exhibit strong negative coefficients on the HML and IA factors as does *la5*. However, once the 5-day reversal LMH strategy is added as an additional factor to the model by Fama and French (2015), in place of the 1-month reversal, the monthly  $\alpha$  drops by 80 bps to a low of 40 bps, a three-fold drop. Admittedly, the coefficient remains marginally significant.  $R_{adj}^2$  also jumps - from below 10% for models (1 – 5) to about 35% scored on the last model. In factor specification (6), there is also stronger evidence of a negative loading on the market.

To sum up, the current state-of-the-art empirical equity factor models provide a poor explanation for the positive returns on the 5-day reversal and acceleration strategies. The informative content of the latter is not subsumed by standard risk factors, but a large

proportion of this information seems to be explained by *la5*. Nonetheless, acceleration strategies seem to provide some value, especially if considered in a diversification context. Next, we move from a univariate time-series analysis, to study how well the chosen factor-models jointly price multiple test assets.

**Joint tests.** We use time-series and cross-sectional joint tests. Since all our factors are tradable assets, we can apply the methodology described in Cochrane (2005, ch. 12.1). In particular, tradability allows us to interpret the intercept, from a time-series regression of test-asset returns on the returns of the factors, in terms of pricing errors. The null for these tests assumes all pricing errors are jointly equal to zero. We use the general method of moments (GMM) by Lars Peter Hansen (1982) applied to the moment conditions in equation (3.4) to estimate the  $\beta$  parameters and test the null via Hansen's  $J$ -test.<sup>18</sup>

We also use two versions of the cross-sectional asset-pricing test from (see Cochrane, 2005, ch. 12.2), where the estimation of the  $\lambda$  parameter is part of the objective. First, we compute the  $HJ$  distance measure of Lars P. Hansen and Jagannathan (1997), and report simulated  $p$ -values for the null hypothesis that the  $HJ$  distance is equal to zero as in Jagannathan and Z. Wang (1996). The long-run covariance matrix of the sample moments is estimated with Newey and West (1987) and the optimal number of lags is selected according to Andrews (1991). We estimate the parameters of the unconditional Euler equation (see footnote 18) with the SDF given in equation (3.3) by GMM. One of the reasons we also calculate the  $HJ$  statistic, is that it allows to compare results across competing models, while the  $J$ -test and its analogues do not.

In addition, we also implement the traditional Fama-MacBeth-type two-pass OLS regression. In the first step, we estimate the betas by running time-series regressions of asset excess-returns on a constant and the returns of the factors:

$$R_{i,t} = a_i + f_t' \beta_i + \varepsilon_{i,t} \tag{3.5}$$

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<sup>18</sup> We only test the unconditional moments, i.e. the instrument set contains only a constant.

In the second step, we estimate at each point in time the  $\lambda_t$ , with cross-sectional regressions of asset excess-returns on the betas from the first step:

$$R_{i,t} = \hat{\beta}'_i \lambda_t + \alpha_t. \quad (3.6)$$

We then compute the risk price of the factors as the average of these slope coefficient estimates, i.e.  $\hat{\lambda} = E_T \hat{\lambda}_t^{\text{OLS}}$ .<sup>19</sup> The null of the test assumes the cross-sectional pricing errors  $R_{i,t} - \lambda' \beta_i$  from the beta model in [equation \(3.4\)](#) are jointly equal to zero. Statistical inference is based on [Shanken \(1992\)](#) errors.

We use the same factors and model specifications as in the univariate time-series analysis of the LMH strategies ([table A2](#)). [Table 3.5](#) collects the results from the tests, with sections of four rows for each signal studied in the paper. In brackets are the p-values for the corresponding asset pricing tests. In bold are tests statistically significant at the 5% level.  $\chi^2_{TS}$  denotes the results of the time-series tests on the null of jointly zero pricing errors. The distribution of the statistic is  $\chi^2(N)$  where  $N$  corresponds to the number of test assets (which in this case is nine). Long-run variance-covariance matrix of the sample moments is estimated with [Newey and West \(1987\)](#) and optimal number of lags according to [Andrews \(1991\)](#).  $\chi^2_{XS}$  denotes the cross-sectional tests with the null of jointly zero cross-sectional pricing errors. The distribution of the test statistic is  $\chi^2(N - K)$  where  $K$  degrees of freedom are lost in the estimation of the  $K$ -dimensional parameter  $\lambda$ . *HJ* dist refers to the [Lars P. Hansen and Jagannathan \(1997\)](#) distance. The null of the corresponding test assumes the asset pricing model is correctly specified, i.e. *HJ* dist = 0. To save space, we do not report the results for *rev* and *fi5* as we previously showed how neither the raw nor the risk-adjusted return of their LMH strategies create enough spread in mean returns, i.e. they are already very close to 0. The price of risk estimates and the cross-sectional  $R^2$  coefficients are reported in [tables A3](#) and [A4](#).

<sup>19</sup>Note that in the second stage regression we do not add a constant to capture the common over- or under-pricing in the cross section of excess returns.

TABLE 3.5 ABOUT HERE

The test assets used in the empirical exercise of Panel A are the portfolios 2 – 10 from the univariate sorts on the signals described in [table 3.2](#). In particular, we fund each long position in portfolios 2 – 10 out of the corresponding short position in portfolio 1. This way, our test assets are in excess return form and lever the portfolio exposure to the factor of interest as an alternative to the risk-free funding.

For *la5* we notice from the time-series joint test that neither [Fama and French \(1993\)](#) nor the more recently proposed factors are able to price the set of nine portfolios. The null is strongly rejected with p-values scoring lower than 1%. However, the cross-sectional tests do not support this rejection. We attribute this disagreement to the loss of degrees of freedom from  $\lambda$ 's estimation. For instance, in the case of model specification (5), which has five factors, the  $\chi^2$  test statistic has only  $9 - 8 = 1$  degrees of freedom. This also explains why most prices of risk have a large standard error, and the point estimates are statistically insignificant ([table A3](#)). Finally, as we add the LMH based on *la5* as a factor to the [Fama and French \(2015\)](#) in specification (6), even the time-series test cannot reject the null of a correctly specified model. In fact  $\lambda$ 's point estimate for the LMH *la5* is the only statistically significant risk factor and has a value of 1.60 which is economically indistinguishable from the time-series average for the LMH *la5* of 1.59. Since the latter is the  $\lambda$  estimate in the time-series test, the result is quite comforting.

Test outcomes for portfolios based on *acA* and *ac5* are very similar to those of *la5*. In the time-series tests, the traditional factor models struggle at explaining the mean returns of these portfolios and the pricing errors are jointly statistically different from zero. Once the *la5*-based factor is added to the specification, no spreads in mean returns are left unexplained, with p-values for the *acA* and *ac5* respectively standing at 27% and 18%. Nonetheless, the issue with the cross-sectional tests is visible. Interestingly, the only significant estimate of risk-price is again that for the *la5* factor when we use the test-assets based on *ac5*. The point estimate of 2.34, while higher than the time-series average of 1.60,

stays within one standard error from the latter, which is reassuring.

Since the cross-sectional tests often have insufficient degrees of freedom, we augment the set of test-assets with the 32 portfolios, triple-sorted independently on size, operating profitability and investment, studied by Fama and French (2015). These portfolios are all in excess of the one-month risk-free rate. The results are presented in table 3.5 under Panel B.

For *la5*, the time-series test is again rejected by the traditional models. However, a word of caution is in order: the null is also rejected when the only test-assets are the 32 Fama-French portfolios.<sup>20</sup> Not surprisingly, even the model specification in (6) is now rejected; although, we never suggested that adding our *la5* factor to the traditional models, should produce *the* model which prices all assets. Therefore, it is somewhat of minor interest for this paper to be able to (not) price two sets of assets at the same time, a task known in advance to be difficult.

Instead, we focus our attention on the outcomes of the cross-sectional joint tests: the first four model specifications are now strongly rejected by the data with p-values all below 1%. Same conclusions are reached by looking at the *HJ* distance, which is statistically different from zero for the same group of models;<sup>21</sup> for the model by Hou, Xue, and Lu Zhang (2015), the statistical significance is marginal at the 10% level. The MKT and HML price of risk estimates are significantly positive for Fama and French (1993) model (see table A4). Nevertheless, the  $\lambda$  on HML is not different from zero once RMW and CMA are added as factors (consistent with the conclusions of Fama and French, 2015), while that on MKT remains significant.<sup>22</sup> For the model specifications (5-6), neither of the

<sup>20</sup>This is in line with Fama and French (2015) who use a similar time-series test based on Gibbons, Ross, and Jay Shanken (1989, GRS): “The GRS test easily rejects all models considered for all LHS portfolios and RHS factors. To save space, the probability, or p-value, of getting a GRS statistic larger than the one observed if the true intercepts are all zero, is not shown. [...] In short, the GRS test says all our models are incomplete descriptions of expected returns”.

<sup>21</sup>This is important, considering only Fama and French (1993) model is rejected by the cross-sectional tests when 32 Fama-French portfolios are considered, while models (2 – 6) are not rejected.

<sup>22</sup>Interestingly, the SMB price of risk in the Hou, Xue, and Lu Zhang (2015) model is marginally positive. However, since their model is rejected in many of the tests, we do not pursue its variations any further.



two cross-sectional tests rejects the null. We see that the  $HJ$  distance is 6-11 bps lower compared to models (1-4) and the p-values are comfortably above the 10% threshold. The  $\lambda$  on  $la5$  is significant at 2.27 which is within one standard error from the time-series average of 1.60.

Finally, we regard the none-rejection by the cross-sectional test of the specification in (5) as suspicious because of three reasons. First, it is the specification with the most factors, and consequently, the most parameters to estimate (less degrees of freedom). Therefore, it partly reflects the same problem described for the cross-sectional test in Panel A. Second, in unreported checks, we ran the cross-sectional tests on the [Fama and French \(2015\)](#) factors plus one factor from UMD, TSMOM and STREV at a time (instead of all three together), and all the new three specifications were rejected. Third, we noticed in the univariate time-series analysis that the LMH, based on  $la5$ , has a significant alpha with respect to this model but none of the betas, except for HML, are statistically different from zero.

For  $ac5$  and  $acA$  we consistently observe very similar test results to those of  $la5$ . We again notice that models (1-4) tend to be rejected by cross-sectional tests. Nonetheless, there is less evidence coming from the  $HJ$  statistic; admittedly, the point estimates are close to being borderline significant. Prices of risk of MKT, CMA, and RMW factors are again significantly positive. Perhaps, most importantly,  $la5$  prices well the acceleration-sorted portfolios and leaves no significant pricing errors.

To sum up, the evidence from the joint tests is consistent with the univariate time-series analysis. The traditional models cannot jointly explain the cross section of the very short-term reversal and acceleration portfolios. However,  $la5$  seems to price well the set of acceleration portfolios.

## 3.5 Conclusion

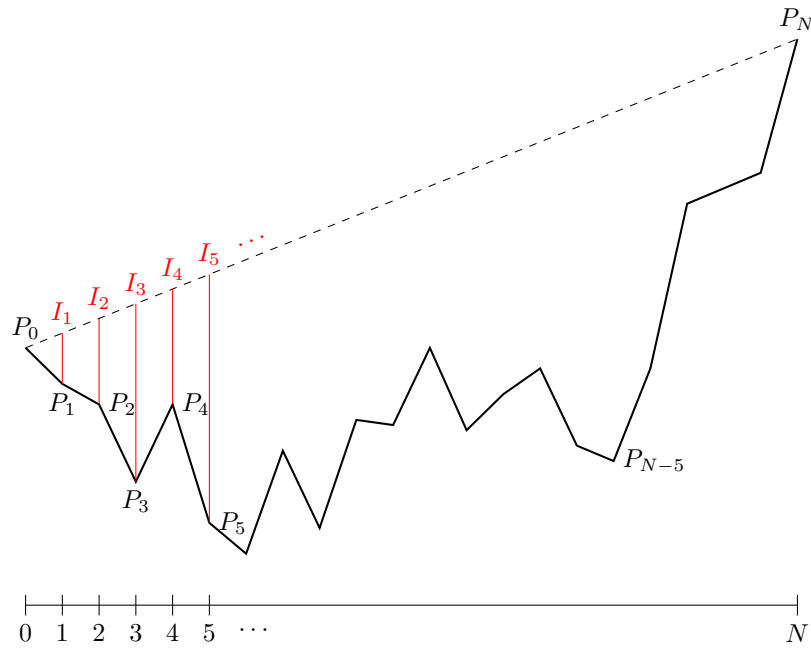
In this paper, we looked at some claims about the price acceleration providing an informative trading signal. We built several empirical measures of acceleration and compared them to the price signals from the previous academic literature. We do find that long-short (LMH) trading strategy built on acceleration delivers high profits. Moreover, those profits are not explained by the state-of-the-art equity factor models. However, the LMH profitability and the cross-sectional variation in mean returns of decile portfolios, built on acceleration, are explained once we add our *la5* factor, a simple reversal strategy with a lookback of one week. Taken together, our results cast doubt on acceleration representing a separate phenomenon.

Nonetheless, this study provides the premise for further research in reversal strategies as a function of the lookback period. In fact, our empirical analysis shows how the demand for trade immediacy (liquidity) has been met, over time, with increasing supply, thus shortening the execution of long trades. Hence, it is probable that the lookback period of a profitable reversal strategy has been shrinking. Previously, Cooper (1999) concludes for the period 1962-1993: *“a security is more likely to have greater reversals if it has incurred two, rather than just one, consecutive weeks of losses or gains”*. However, this claim is not supported by our sample over the 1993-2015 window.

Moreover, in support of our view of a shrinking optimal lookback, the mean return and the Sharpe ratio of the 1-month reversal strategy from Kenneth French’s Data Library have been steadily decreasing. Specifically, during the 1960-1970s, 1980-1990s, and 2000-2015s, the average return went respectively from 93 bps, to 37 bps and 20bps, and similarly did the Sharpe ratio, with values of 1.30, 0.53, and 0.16, correspondingly. Therefore, our finding is not a recent nor cyclical phenomenon, and most likely is part of a long-term trend. Such analysis should also partly address the debate on whether the reversal strategy returns are profitable (see Da, Q. Liu, and Schaumburg, 2014) or not (e.g. Avramov, Chordia, and

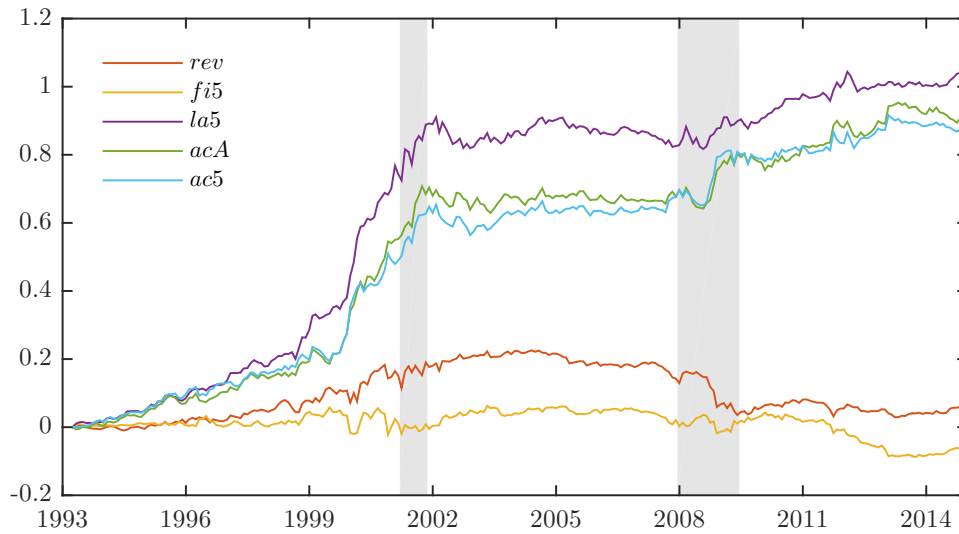
Goyal, 2006) – since the profits are bound to change over time, if the lookback window is held fixed.

Given the trend of diminishing returns in the reversal strategy, and the suggested connection with the time-varying length of its lookback, is the strategy going to disappear altogether? In this respect, it would be interesting to verify if our best performing strategy, the *la5*, is related to Etula et al. (2016). They claim that reversals are stronger following a decline in institutional, i.e. informed, trading in a given stock. Assuming this applies to our context, and continuing to rebalance the *la5* at month-end, the optimal lookback should not be much shorter than three days, which they argue corresponds to the monthly cash management cycle of institutional investors. Moreover, by adding sorts on liquidity, this exercise would relate to Avramov, Chordia, and Goyal (2006) and would potentially extend their results until 2010.



**Figure 3.1: Construction of acceleration signals**

The figure illustrates the intuition behind the construction of the accelerations signals. The figure plots a solid line of hypothetical prices, denoted by  $P_0, P_1, \dots, P_N$ , and a dashed straight line of prices, namely  $I_1, \dots, I_{N-1}$ , which are interpolated between the two ends of the lookback period, of length  $N$ . The area enclosed between the interpolated and the price-line proxies for acceleration. The vertical lines, summed together, constitute the  $acA$  signal which is a standardized Riemann sum for the estimation of the area. The  $fi5$  and  $la5$  signals are roughly the slopes of the price-line during the first and last five days in the picture.



**Figure 3.2: Cumulative returns on low-minus-high strategies**

The figure presents cumulative returns on low-minus-high (LMH) strategies. All LMH series are rebased to 1 at the beginning of the period of analysis and scaled by their respective return standard deviation. Shaded areas denote NBER recessions. For each signal stocks are sorted every month into 10 value-weighted portfolios and the LMH strategy goes long the low signal portfolio and short the high signal portfolio. Definitions of signals: *rev* is the previous month return; *fi5* is the average return over the first five trading days of the previous month; *la5* is the average return over the last five trading days of the previous month; *acA* is the sum of daily deviations of the price from the linearly interpolated line between the beginning and the end of the previous month prices; *ac5* is the difference between *la5* and *fi5*. All quantities are calculated on a monthly frequency. The equity price data are taken from the Center for Research in Security Prices (CRSP) database. The sample excludes prices below 5 US dollars and prices corresponding to market cap below the first NYSE decile at the time of portfolio formation. Signals are industry-adjusted by subtracting from each raw stock signal its respective industry average signal. The period of study is from January 1993 to December 2014.





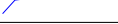
**Table 3.1: Correlations among signals**

This table presents the time series averages of the cross-sectional correlations between signals. Specifically, below the main diagonal are Spearman rank correlations; above the main diagonal are Pearson linear correlations. Definitions of signals: *rev* is the previous month return; *fi5* is the average return over the first five trading days of the previous month; *la5* is the average return over the last five trading days of the previous month; *acR* is the coefficient from the regression of prices on squared time (including constant and time) using daily observations over the previous month; *acA* is the sum of daily deviations of the price from the linearly interpolated line between the beginning and the end of the previous month prices; *ac5* is the difference between *la5* and *fi5*; *acH* is the difference between the average daily returns in the last half and the first half of the previous month; *std<sub>0</sub>* and *std* are standard deviations of daily returns over the previous month - the former assumes zero mean while the latter takes sample mean in the calculations; *mom* is the return over the previous 12 months, excluding the most recent month. All quantities are calculated on a monthly frequency. The equity price data are taken from the Center for Research in Security Prices (CRSP) database. The sample excludes prices below 5 US dollars and prices corresponding to market cap below the first NYSE decile at the time of portfolio formation. Signals are industry-adjusted by subtracting from each raw stock signal its respective industry average signal. The period of study is from January 1993 to December 2014.

	<i>rev</i>	<i>trd</i>	<i>fi5</i>	<i>la5</i>	<i>acR</i>	<i>acA</i>	<i>ac5</i>	<i>acH</i>	<i>std<sub>0</sub></i>	<i>std</i>	<i>mom</i>
<i>rev</i>		<b>0.88</b>	<b>0.46</b>	<b>0.42</b>	-0.01	-0.03	-0.04	-0.02	0.05	0.04	0.00
<i>trd</i>	<b>0.90</b>		0.19	0.27	-0.01	0.06	0.05	0.05	0.04	0.03	0.00
<i>fi5</i>	<b>0.50</b>	0.21		-0.01	<b>-0.38</b>	<b>-0.62</b>	<b>-0.69</b>	<b>-0.47</b>	0.03	0.02	0.00
<i>la5</i>	<b>0.45</b>	0.28	0.00		<b>0.49</b>	<b>0.58</b>	<b>0.65</b>	<b>0.44</b>	0.06	0.05	-0.02
<i>acR</i>	-0.02	-0.01	<b>-0.41</b>	<b>0.52</b>		<b>0.79</b>	<b>0.63</b>	<b>0.88</b>	0.02	0.02	-0.02
<i>acA</i>	-0.03	0.07	<b>-0.65</b>	<b>0.61</b>	<b>0.81</b>		<b>0.88</b>	<b>0.86</b>	0.01	0.01	-0.02
<i>ac5</i>	-0.05	0.04	<b>-0.72</b>	<b>0.68</b>	<b>0.66</b>	<b>0.89</b>		<b>0.67</b>	0.01	0.01	-0.02
<i>acH</i>	-0.03	0.05	<b>-0.51</b>	<b>0.47</b>	<b>0.89</b>	<b>0.86</b>	<b>0.70</b>		0.00	0.00	-0.02
<i>std<sub>0</sub></i>	0.12	0.10	0.08	0.07	-0.02	-0.03	-0.02	-0.02		<b>0.997</b>	-0.01
<i>std</i>	0.11	0.08	0.08	0.06	-0.02	-0.03	-0.01	-0.02	<b>0.998</b>		-0.01
<i>mom</i>	0.00	0.00	0.00	-0.01	-0.02	-0.01	-0.01	-0.01	-0.06	-0.06	







**Table 3.2: Portfolios and low-minus-high strategies - descriptive statistics**

This table presents the descriptive statistics on portfolios and low-minus-high (LMH) strategies. For each signal stocks are sorted every month into 10 value-weighted portfolios and the LMH strategy goes long the low signal portfolio and short the high signal portfolio. Definitions of signals: *rev* is the previous month return; *fi5* is the average return over the first five trading days of the previous month; *la5* is the average return over the last five trading days of the previous month; *acA* is the sum of daily deviations of the price from the linearly interpolated line between the beginning and the end of the previous month prices; *ac5* is the difference between *la5* and *fi5*. Panel A depicts the average portfolio returns. Panel B shows analysis for LMH strategies.  $AC_1$  stands for first order autocorrelation coefficient.  $SR$  is the Sharpe ratio of the strategy. Min and max denote the minimum and the maximum monthly return, respectively. The maximum drawdown (MDD) is followed by its duration in months (*len*) and the number of months it took the price series to recover after the drawdown (*rec*). Recoveries marked with an asterisk (\*) are incomplete and match the end of the period of analysis. All quantities are calculated on a monthly frequency and expressed in percent (except for *skew*, *kurt*,  $AC_1$ ,  $SR$ , *len*, and *rec*). The equity price data are taken from the Center for Research in Security Prices (CRSP) database. The sample excludes prices below 5 US dollars and prices corresponding to market cap below the first NYSE decile at the time of portfolio formation. Signals are industry-adjusted by subtracting from each raw stock signal its respective industry average signal. The period of study is from January 1993 to December 2014.

Panel A: Average returns on signal-sorted portfolios												
	<i>High</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>Low</i>		<i>LMH</i>
<i>rev</i>	0.51	0.47	0.66	0.82	0.93	0.87	0.99	1.12	0.87	0.68		0.17
<i>fi5</i>	0.82	0.84	0.87	0.58	0.79	0.93	0.87	0.93	0.82	0.72		-0.11
<i>la5</i>	-0.30	0.42	0.49	0.71	0.80	0.91	0.93	1.03	1.14	1.29		1.59
<i>acA</i>	-0.07	0.39	0.63	0.82	0.73	0.90	0.84	0.87	1.32	1.16		1.23
<i>ac5</i>	-0.06	0.64	0.72	0.79	0.88	0.79	0.76	0.88	1.11	1.16		1.22
Panel B: Descriptive statistics on LMH portfolios												
	<i>mean</i>	<i>std</i>	<i>med</i>	<i>skew</i>	<i>kurt</i>	$AC_1$	$SR$	<i>min</i>	<i>max</i>	<i>MDD</i>	<i>len</i>	<i>rec</i>
<i>rev</i>	0.17	6.05	0.05	0.28	8.17	-0.20	0.03	-25.60	29.10	69.39	106	21*
<i>fi5</i>	-0.11	5.45	-0.10	-0.99	10.49	-0.13	-0.02	-32.10	19.46	66.79	171	15*
<i>la5</i>	1.59	5.80	1.02	1.61	8.67	0.07	0.27	-13.61	30.83	29.93	78	23
<i>acA</i>	1.23	4.92	0.43	1.18	5.91	0.07	0.25	-8.90	25.61	22.26	20	65
<i>ac5</i>	1.22	5.05	0.73	1.44	8.84	0.07	0.24	-11.50	32.30	25.60	9	21

**Table 3.3: Portfolios and low-minus-high strategies - transaction cost analysis**

This table presents some descriptive statistics on individual stocks, portfolios, and low-minus-high (LMH) strategies in the equity universe restricted to the lowest bid-ask spread quintile. For each signal stocks are sorted every month into 10 value-weighted portfolios and the LMH strategy goes long the low signal portfolio and short the high signal portfolio. Definitions of signals: *rev* is the previous month return; *fi5* is the average return over the first five trading days of the previous month; *la5* is the average return over the last five trading days of the previous month; *acA* is the sum of daily deviations of the price from the linearly interpolated line between the beginning and the end of the previous month prices; *ac5* is the difference between *la5* and *fi5*. First ten columns depict the average portfolio returns. For each LMH strategy the tables also shows its mean (gross) return, standard error of the mean (*se*) computed using Newey and West (1987), average estimate of transaction costs (*Tcost*). In bold are the coefficients that remain significant net of transaction costs at the 5% level, i.e. when  $LMH - Tcost > 1.96 \times se$ . Panel B presents the analysis of the typical stock size in the lowest bid-ask spread quintile. All quantities are calculated on a monthly frequency and expressed in percent. The equity price data are taken from the Center for Research in Security Prices (CRSP) database. The sample excludes prices above first bid-ask spread quintile, below 5 US dollars, and prices corresponding to market cap below the first NYSE decile at the time of portfolio formation. Signals are industry-adjusted by subtracting from each raw stock signal its respective industry average signal. The period of study is from January 1993 to December 2014.

Panel A: descriptive statistics for decile and LMH portfolios formed from the lowest BA quintile														
	<i>High</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>Low</i>		<i>LMH</i>	<i>Tcost</i>	<i>se</i>
<i>rev</i>	0.83	0.27	0.93	0.74	0.99	1.02	0.97	1.25	1.06	0.96		0.13	0.34	0.30
<i>fi5</i>	0.90	1.03	0.70	0.77	0.62	0.99	1.02	1.09	1.09	0.50		-0.40	0.34	0.30
<i>la5</i>	-0.48	0.43	0.57	0.62	1.10	0.84	0.99	1.20	1.34	1.26		<b>1.74</b>	0.34	0.51
<i>acA</i>	-0.19	0.65	0.50	0.82	0.97	1.26	0.90	1.03	1.24	1.13		<b>1.32</b>	0.34	0.44
<i>ac5</i>	-0.11	0.64	0.79	0.68	1.03	0.74	1.00	1.13	1.06	1.10		<b>1.22</b>	0.34	0.45
Panel B: size-breakdown of stocks in the lowest BA quintile														
Size group	0-20%	20-40%	40-60%	60-80%	80-100%									
Average count	21	46	84	131	219									



**Table 3.4: Risk-adjusted low-minus-high strategies' returns**

This table presents the results of time-series regressions of low-minus-high (LMH) strategies' returns on risk factors. For each signal stocks are sorted every month into 10 value-weighted portfolios and the LMH strategy goes long the low signal portfolio and short the high signal portfolio. Definitions of signals: *rev* is the previous month return; *fi5* is the average return over the first five trading days of the previous month; *la5* is the average return over the last five trading days of the previous month; *acA* is the sum of daily deviations of the price from the linearly interpolated line between the beginning and the end of the previous month prices; *ac5* is the difference between *la5* and *fi5*. Factor models used in the analysis are described in Appendix Table [table A2](#).  $\alpha$  coefficient corresponds to the constant in the regression. [Newey and West \(1987\)](#) standard errors are shown in parentheses and statistical significance at the 10, 5 and 1% level is marked respectively with \*, \*\* and \*\*\*. All quantities are calculated on a monthly frequency. The equity price data are taken from the Center for Research in Security Prices (CRSP) database. The sample excludes prices below 5 US dollars and prices corresponding to market cap below the first NYSE decile at the time of portfolio formation. Signals are industry-adjusted by subtracting from each raw stock signal its respective industry average signal. The period of study is from January 1993 to December 2014.

	(1)	(2)	(3)	(4)	(5)	(6)
<i>rev</i>						
$\alpha$	0.17 (0.317)	-0.07 (0.313)	-0.14 (0.347)	-0.09 (0.301)	-0.16 (0.310)	N/A (0.257)
MKT		0.44*** (0.129)	0.47*** (0.137)	0.45*** (0.114)	0.43*** (0.113)	0.40*** (0.110)
SMB		-0.17 (0.227)	-0.09 (0.209)	-0.07 (0.196)	-0.09 (0.197)	-0.04 (0.196)
HML		0.01 (0.210)		0.06 (0.229)	0.08 (0.233)	0.16 (0.226)
RMW				0.21 (0.207)	0.19 (0.208)	0.28 (0.223)
CMA				-0.27 (0.294)	-0.27 (0.292)	-0.23 (0.304)
IA		-0.08 (0.303)				
ROE			0.18 (0.204)			
LIQ				0.13 (0.092)		
UMD						
TSMOM						
STREV						
<i>la5</i>						0.30*** (0.107)
$R_{adj}^2$		0.09	0.08	0.09	0.09	0.16

	(1)	(2)	(3)	(4)	(5)	(6)	
<i>fi5</i>							
$\alpha$	-0.11 (0.282)	-0.25 (0.285)	-0.25 (0.318)	-0.21 (0.291)	-0.23 (0.298)	-0.07 (0.278)	-0.29 (0.261)
MKT		0.26** (0.104)	0.25** (0.102)	0.24*** (0.086)	0.24*** (0.086)	0.04 (0.084)	0.24*** (0.082)
SMB		-0.11 (0.163)	-0.05 (0.149)	-0.05 (0.165)	-0.06 (0.167)	-0.03 (0.154)	-0.05 (0.162)
HML		0.00 (0.219)		0.08 (0.165)	0.09 (0.172)	-0.14 (0.141)	0.10 (0.164)
RMW				0.09 (0.277)	0.08 (0.282)	0.11 (0.240)	0.10 (0.282)
CMA				-0.27 (0.287)	-0.27 (0.286)	-0.03 (0.206)	-0.27 (0.288)
IA			-0.06 (0.295)				
ROE			0.05 (0.255)				
LIQ				0.03 (0.101)			
UMD						-0.21* (0.115)	
TSMOM						-0.08 (0.108)	
STREV						0.52*** (0.131)	
<i>la5</i>							0.05 (0.105)
$R_{adj}^2$		0.03	0.02	0.03	0.03	0.20	0.03
<i>la5</i>							
$\alpha$	1.60*** (0.421)	1.59*** (0.424)	1.92*** (0.471)	1.73*** (0.473)	1.69*** (0.482)	1.84*** (0.513)	N/A
MKT		0.20** (0.090)	0.08 (0.115)	0.14 (0.108)	0.14 (0.107)	0.09 (0.105)	
SMB		-0.06 (0.165)	-0.07 (0.161)	-0.12 (0.137)	-0.13 (0.137)	-0.11 (0.139)	
HML		-0.46*** (0.164)		-0.33** (0.159)	-0.32** (0.153)	-0.41** (0.166)	
RMW				-0.23 (0.284)	-0.24 (0.281)	-0.21 (0.305)	
CMA				-0.14 (0.281)	-0.14 (0.283)	-0.07 (0.298)	
IA			-0.76*** (0.233)				
ROE			-0.26 (0.161)				
LIQ					0.08 (0.076)		
UMD						-0.06 (0.126)	
TSMOM						-0.08 (0.113)	
STREV						0.13 (0.172)	
<i>la5</i>							
$R_{adj}^2$		0.09	0.11	0.09	0.09	0.09	

	(1)	(2)	(3)	(4)	(5)	(6)	
<i>acA</i>							
$\alpha$	1.23*** (0.361)	1.34*** (0.374)	1.40*** (0.410)	1.29*** (0.383)	1.33*** (0.394)	1.27*** (0.377)	0.40* (0.230)
MKT		-0.06 (0.085)	-0.07 (0.089)	-0.04 (0.079)	-0.03 (0.076)	0.07 (0.091)	-0.11* (0.065)
SMB		-0.15 (0.116)	-0.12 (0.127)	-0.14 (0.118)	-0.13 (0.119)	-0.15 (0.114)	-0.08 (0.106)
HML		-0.19 (0.188)		-0.24 (0.168)	-0.25 (0.170)	-0.13 (0.150)	-0.06 (0.143)
RMW				0.06 (0.258)	0.07 (0.259)	0.06 (0.234)	0.18 (0.162)
CMA				0.08 (0.287)	0.08 (0.283)	-0.04 (0.255)	0.15 (0.215)
IA			-0.32 (0.249)				
ROE			0.04 (0.168)				
LIQ					-0.07 (0.085)		
UMD						0.13 (0.100)	
TSMOM						-0.02 (0.109)	
STREV						-0.25** (0.124)	
<i>la5</i>							0.52*** (0.063)
$R_{adj}^2$		0.01	0.01	0.00	0.00	0.05	0.34
<i>ac5</i>							
$\alpha$	1.24*** (0.353)	1.40*** (0.358)	1.51*** (0.423)	1.37*** (0.384)	1.40*** (0.396)	1.36*** (0.368)	0.46** (0.213)
MKT		-0.12 (0.088)	-0.14 (0.100)	-0.11 (0.085)	-0.10 (0.080)	0.01 (0.085)	-0.18*** (0.067)
SMB		-0.14 (0.104)	-0.14 (0.117)	-0.15 (0.112)	-0.14 (0.115)	-0.16 (0.114)	-0.09 (0.100)
HML		-0.26 (0.193)		-0.30** (0.151)	-0.31** (0.155)	-0.18 (0.131)	-0.12 (0.122)
RMW				-0.01 (0.270)	0.00 (0.271)	-0.01 (0.249)	0.12 (0.167)
CMA				0.09 (0.286)	0.09 (0.284)	-0.03 (0.259)	0.17 (0.217)
IA			-0.35 (0.268)				
ROE			-0.07 (0.212)				
LIQ					-0.06 (0.092)		
UMD						0.14 (0.092)	
TSMOM						-0.02 (0.105)	
STREV						-0.26* (0.142)	
<i>la5</i>							0.53*** (0.075)
$R_{adj}^2$		0.02	0.02	0.02	0.02	0.07	0.35

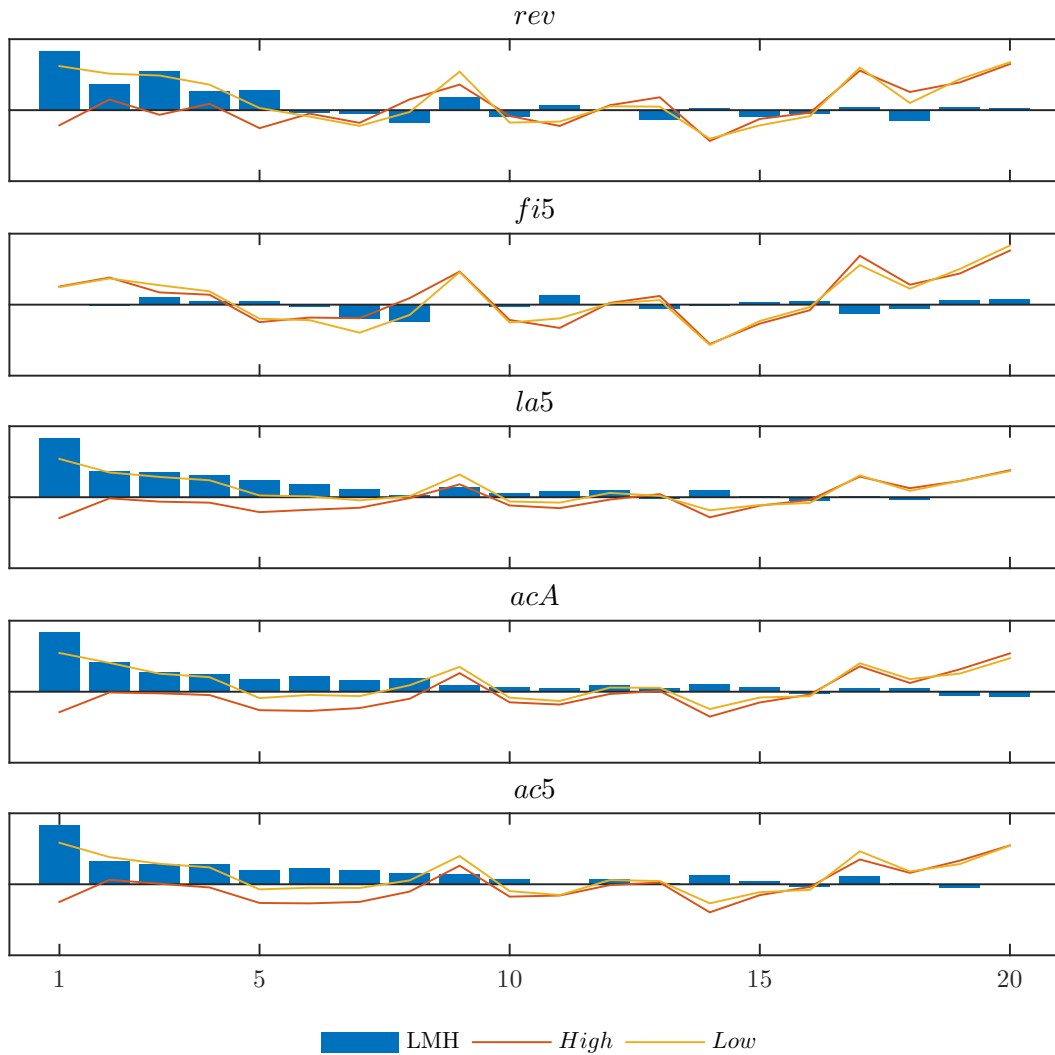
**Table 3.5: Asset pricing tests**

The table presents asset pricing results. Each section of four rows corresponds to a particular signal. For each signal stocks are sorted every month into 10 value-weighted portfolios. In Panel A portfolios 2-10 in excess of portfolio 1 serve as test assets; in Panel B Fama and French (2015) 32 portfolios - triple-sorted on size, operating profitability, and investment and taken in excess of the 1-month risk-free rate - are added to the set of test assets from Panel A. Definitions of signals: *fi5* is the average return over the first five trading days of the previous month; *la5* is the average return over the last five trading days of the previous month; *acA* is the sum of daily deviations of the price from the linearly interpolated line between the beginning and the end of the previous month prices; *ac5* is the difference between *la5* and *fi5*. For each set of test assets the columns of the table report asset pricing results for six asset pricing models described in Appendix Table A2. In brackets are the p-values for the corresponding asset pricing tests. The bolded numbers indicate p-values lower or equal to 0.05.  $\chi_{TS}^2$  denotes the time-series asset pricing test obtained via general method of moments (GMM) procedure with the null of jointly zero pricing errors from time-series regressions (see Cochrane, 2005, ch. 12.1). Long run variance-covariance matrix of the sample moments is estimated with Newey and West (1987) and optimal number of lags according to Andrews (1991).  $\chi_{XS}^2$  denotes the cross-sectional Fama-MacBeth-type asset pricing test using Shanken (1992) standard errors with the null of jointly zero cross-sectional pricing errors (see Cochrane, 2005, ch. 12.2-3). *HJ* dist refers to the Lars P. Hansen and Jagannathan (1997) distance; the null of the corresponding test is that asset pricing model is correctly specified (*HJ* dist = 0). For the corresponding price of risk estimates please consult Tables A3-A4 in Appendix. The equity price data are taken from the Center for Research in Security Prices (CRSP) database. The sample excludes prices below 5 US dollars and prices corresponding to market cap below the first NYSE decile at the time of portfolio formation. Signals are industry-adjusted by subtracting from each raw stock signal its respective industry average signal. The period of study is from January 1993 to December 2014.

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: 9 test assets						
<i>la5</i>						
$\chi_{TS}^2$ test	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.01]</b>	<b>[0.01]</b>	<b>[0.01]</b>	[0.14]
$\chi_{XS}^2$ test	[0.99]	[0.97]	[0.94]	[0.91]	[0.51]	[0.92]
<i>HJ</i> dist	0.21	0.23	0.19	0.15	0.18	0.06
<i>HJ</i> test	[0.91]	[0.87]	[0.75]	[0.77]	[0.23]	[0.93]
<i>acA</i>						
$\chi_{TS}^2$ test	<b>[0.00]</b>	<b>[0.03]</b>	<b>[0.04]</b>	<b>[0.04]</b>	<b>[0.03]</b>	[0.27]
$\chi_{XS}^2$ test	[0.55]	[0.67]	[1.00]	[0.99]	[0.91]	[0.99]
<i>HJ</i> dist	0.27	0.25	0.04	0.04	0.02	0.04
<i>HJ</i> test	[0.18]	[0.15]	[1.00]	[0.99]	[0.90]	[0.99]
<i>ac5</i>						
$\chi_{TS}^2$ test	<b>[0.00]</b>	<b>[0.02]</b>	<b>[0.04]</b>	<b>[0.05]</b>	<b>[0.02]</b>	[0.18]
$\chi_{XS}^2$ test	[0.54]	[0.45]	[0.33]	[0.95]	[0.47]	[0.44]
<i>HJ</i> dist	0.22	0.23	0.21	0.12	0.13	0.12
<i>HJ</i> test	[0.31]	[0.33]	[0.16]	[0.95]	[0.51]	[0.34]
Panel B: 9 + FF 32 test assets						
<i>la5</i>						
$\chi_{TS}^2$ test	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>
$\chi_{XS}^2$ test	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>	[0.25]	[0.23]
<i>HJ</i> dist	0.54	0.50	0.49	0.49	0.42	0.43
<i>HJ</i> test	<b>[0.01]</b>	[0.06]	<b>[0.03]</b>	<b>[0.03]</b>	[0.71]	[0.21]
<i>acA</i>						
$\chi_{TS}^2$ test	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>
$\chi_{XS}^2$ test	<b>[0.00]</b>	<b>[0.03]</b>	<b>[0.03]</b>	<b>[0.03]</b>	[0.44]	[0.64]
<i>HJ</i> dist	0.50	0.45	0.45	0.45	0.40	0.38
<i>HJ</i> test	<b>[0.05]</b>	[0.25]	[0.14]	[0.13]	[0.56]	[0.55]
<i>ac5</i>						
$\chi_{TS}^2$ test	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>
$\chi_{XS}^2$ test	<b>[0.00]</b>	<b>[0.01]</b>	<b>[0.01]</b>	<b>[0.03]</b>	[0.29]	[0.46]
<i>HJ</i> dist	0.50	0.47	0.46	0.45	0.41	0.40
<i>HJ</i> test	<b>[0.04]</b>	[0.12]	[0.10]	[0.17]	[0.49]	[0.39]

# Appendix

## Appendix 3.A Additional tables and figures



**Figure A1: Average returns on low-minus-high strategies – breakdown by day**

The figure presents average holding-period returns on low-minus-high (LMH) strategies. The monthly holding period is broken down by trading days with each bar representing the average return on a particular day. The y-axis ranges between -0.5% - 0.5% for all signals. For each signal, stocks are sorted every month into 10 value-weighted portfolios and the LMH strategy goes long the low-signal portfolio and short the high-signal portfolio. Definitions of signals: *rev* is the previous month return; *fi5* is the average return over the first five trading days of the previous month; *la5* is the average return over the last five trading days of the previous month; *acA* is the sum of daily deviations of the price from the linearly interpolated line between the beginning and the end of the previous month prices; *ac5* is the difference between *la5* and *fi5*. All quantities are calculated on a monthly frequency. The equity price data are taken from the Center for Research in Security Prices (CRSP) database. The sample excludes prices below 5 US dollars and prices corresponding to market cap below the first NYSE decile at the time of portfolio formation. Signals are industry-adjusted by subtracting from each raw stock signal its respective industry average signal. The period of study is from January 1993 to December 2014.

Table A1: Descriptive statistics on portfolios

<b>rev</b>	<i>High</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>Low</i>
<i>mean</i>	0.51	0.47	0.66	0.82	0.93	0.87	0.99	1.12	0.87	0.68
<i>std</i>	6.63	4.90	4.48	4.33	4.31	4.47	4.78	5.46	6.29	8.24
<i>med</i>	1.54	0.86	1.41	1.44	1.43	1.23	1.43	1.73	1.19	1.59
<i>skew</i>	-0.21	-0.51	-0.81	-0.69	-0.55	-0.52	-0.67	-0.96	-0.56	-0.91
<i>kurt</i>	6.61	4.39	4.31	4.02	4.75	4.71	5.20	5.77	5.67	6.57
$AC_1$	0.06	0.02	0.12	0.10	0.08	0.02	0.07	0.15	0.01	0.07
<i>turn</i>	0.88	0.90	0.90	0.89	0.89	0.89	0.89	0.91	0.90	0.86
<i>Tcost</i>	0.52	0.44	0.43	0.45	0.45	0.45	0.47	0.47	0.50	0.61
<b>fi5</b>	<i>High</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>Low</i>
<i>mean</i>	0.82	0.84	0.87	0.58	0.79	0.93	0.87	0.93	0.82	0.72
<i>std</i>	7.28	5.38	4.44	4.48	4.58	4.45	4.53	5.07	6.27	8.14
<i>med</i>	1.16	1.50	1.57	1.32	1.33	1.38	1.67	1.73	1.34	1.30
<i>skew</i>	0.05	-0.59	-0.49	-0.85	-0.82	-0.92	-0.48	-0.73	-1.09	-0.74
<i>kurt</i>	8.41	4.95	4.28	4.42	5.09	5.32	4.27	6.41	7.44	7.33
$AC_1$	0.07	0.01	0.04	0.12	0.09	0.12	0.10	0.07	0.13	0.08
<i>turn</i>	0.87	0.90	0.90	0.89	0.89	0.89	0.90	0.90	0.90	0.86
<i>Tcost</i>	0.54	0.46	0.45	0.45	0.46	0.45	0.45	0.46	0.49	0.54
<b>la5</b>	<i>High</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>Low</i>
<i>mean</i>	-0.30	0.42	0.49	0.71	0.80	0.91	0.93	1.03	1.14	1.29
<i>std</i>	6.61	5.22	4.74	4.43	4.26	4.41	4.70	4.86	5.77	8.06
<i>med</i>	0.97	1.28	1.13	1.18	1.31	1.46	1.59	1.45	1.56	1.61
<i>skew</i>	-1.06	-1.03	-0.99	-0.54	-0.65	-0.81	-0.92	-0.85	-0.83	-0.14
<i>kurt</i>	5.38	6.05	5.28	4.11	4.83	5.36	5.54	5.81	5.19	6.89
$AC_1$	0.17	0.04	0.09	0.10	0.11	0.05	0.12	0.10	0.09	0.00
<i>turn</i>	0.86	0.90	0.90	0.89	0.89	0.89	0.89	0.90	0.90	0.86
<i>Tcost</i>	0.57	0.47	0.45	0.47	0.46	0.46	0.46	0.46	0.45	0.59
<b>acA</b>	<i>High</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>Low</i>
<i>mean</i>	-0.07	0.39	0.63	0.82	0.73	0.90	0.84	0.87	1.32	1.16
<i>std</i>	7.57	5.58	4.76	4.45	4.41	4.32	4.46	4.76	5.58	7.50
<i>med</i>	0.70	1.06	1.43	1.36	1.42	1.43	1.55	1.55	1.68	1.18
<i>skew</i>	-1.04	-0.69	-1.00	-0.77	-0.73	-0.66	-0.63	-0.78	-0.60	0.16
<i>kurt</i>	5.46	5.65	5.21	4.28	4.41	4.37	3.74	4.38	5.69	7.08
$AC_1$	0.15	0.11	0.13	0.08	0.13	0.01	0.13	0.09	0.04	0.03
<i>turn</i>	0.85	0.89	0.90	0.89	0.89	0.89	0.90	0.90	0.89	0.86
<i>Tcost</i>	0.59	0.48	0.47	0.45	0.45	0.44	0.44	0.46	0.47	0.57
<b>ac5</b>	<i>High</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>Low</i>
<i>mean</i>	-0.06	0.64	0.72	0.79	0.88	0.79	0.76	0.88	1.11	1.16
<i>std</i>	7.68	5.57	4.86	4.62	4.19	4.26	4.49	4.97	5.70	7.39
<i>med</i>	0.85	1.36	1.42	1.25	1.14	1.46	1.53	1.32	1.49	1.60
<i>skew</i>	-0.98	-0.86	-0.92	-0.75	-0.59	-0.54	-0.97	-0.89	-0.69	-0.05
<i>kurt</i>	5.91	5.50	4.70	5.16	3.89	4.88	4.78	5.44	5.67	6.34
$AC_1$	0.13	0.08	0.16	0.06	0.10	0.04	0.12	0.11	0.01	0.04
<i>turn</i>	0.85	0.90	0.90	0.89	0.89	0.89	0.89	0.90	0.90	0.85
<i>Tcost</i>	0.60	0.47	0.47	0.44	0.45	0.45	0.44	0.46	0.47	0.59

**Table A2: Description of factor models**

	MKT	SMB	HML	RMW	CMA	IA	ROE	LIQ	UMD	TSMOM	STREV	<i>la5</i>
(1)	x	x	x									
(2)	x	x				x	x					
(3)	x	x	x	x	x							
(4)	x	x	x	x	x			x				
(5)	x	x	x	x	x				x	x	x	
(6)	x	x	x	x	x							x
References to related academic works												
(1)	Fama and French (1993)											
(2)	Hou, Xue, and Lu Zhang (2015)											
(3)	Fama and French (2015, FF5)											
(4)	FF5, Pastor and Stambaugh (2003)											
(5)	FF5, Jegadeesh and Titman (1993), Moskowitz, Ooi, and Pedersen (2012), Jegadeesh (1990)											
(6)	FF5, Lehmann (1990)											
	Brief description of the underlying factor exposure						Data source					
MKT	broad market minus risk-free rate						website of Kenneth French					
SMB	small minus big						website of Kenneth French					
HML	high book/market minus low book/market						website of Kenneth French					
RMW	robust minus weak operating profitability						website of Kenneth French					
CMA	conservative minus aggressive investment policy						website of Kenneth French					
IA	low minus high percentage of investments to assets						courtesy of professor Lu Zhang					
ROE	high minus low return on equity						courtesy of professor Lu Zhang					
LIQ	high minus low exposure to market liquidity						website of Lubos Pastor					
UMD	XS momentum (winners minus losers), 12-2m lookback						website of Kenneth French					
TSMOM	TS momentum (via futures market), 12m lookback						website of Applied Quantitative Research (AQR)					
STREV	XS reversal (losers minus winners), 1m lookback						website of Kenneth French					
<i>la5</i>	XS reversal (losers minus winners), 5d lookback						authors' own calculations					



**Table A3: Asset pricing tests - price of risk estimates (corresponding to Table 3.5 Panel A)**  
 In addition, the table also shows  $R^2$  from the second stage (cross-sectional) regression.

	(1)	(2)	(3)	(4)	(5)	(6)
<i>rev</i>						
MKT	0.62 (0.805)	1.06 (0.804)	0.59 (1.148)	0.67 (1.131)	0.49 (1.890)	-0.57 (1.446)
SMB	-0.03 (0.706)	0.03 (0.986)	0.28 (0.982)	0.22 (0.956)	-0.88 (1.117)	0.58 (1.024)
HML	0.80 (0.635)		-0.55 (1.150)	-0.18 (1.197)	-0.05 (1.358)	-1.19 (1.424)
RMW			0.60 (1.061)	-0.23 (1.231)	0.22 (1.592)	0.84 (1.253)
CMA			0.67 (0.630)	0.92 (0.648)	-0.18 (1.337)	0.59 (0.717)
IA		0.96 (0.634)				
ROE		-0.94 (1.942)				
LIQ				1.41 (1.335)		
UMD					3.22 (3.181)	
TSMOM					1.46 (3.723)	
STREV					0.26 (0.581)	
<i>la5</i>						1.77 (2.263)
$R^2_{adj}$	0.08	0.26	0.47	0.46	0.13	0.51

3.A. Additional tables and figures

	(1)	(2)	(3)	(4)	(5)	(6)
<i>fi5</i>						
MKT	0.03 (1.588)	0.07 (1.502)	0.26 (2.913)	0.29 (3.043)	0.27 (10.911)	0.23 (2.610)
SMB	0.65 (1.083)	0.54 (0.580)	-2.41 (2.764)	-2.40 (2.766)	-2.64 (3.602)	-2.38 (3.566)
HML	0.43 (1.271)		-2.35 (3.018)	-2.36 (3.035)	-2.58 (4.620)	-2.31 (4.514)
RMW			0.10 (1.287)	0.13 (1.548)	0.14 (2.103)	0.08 (1.534)
CMA			0.41 (0.845)	0.42 (0.870)	0.36 (1.563)	0.41 (0.815)
IA		0.58 (0.493)				
ROE		-0.80 (0.645)				
LIQ				0.08 (2.386)		
UMD					1.41 (3.516)	
TSMOM					0.46 (7.391)	
STREV					-0.44 (2.919)	
<i>la5</i>						1.11 (4.368)
$R_{adj}^2$	-0.08	0.17	0.90	0.88	0.76	0.87
<i>la5</i>						
MKT	-8.74 (7.852)	-2.21 (5.996)	-7.61 (11.434)	-7.65 (11.188)	-9.45 (19.127)	-0.27 (3.977)
SMB	-5.27 (3.558)	-4.04 (3.045)	-5.28 (3.617)	-4.08 (2.935)	-6.32 (7.688)	-1.94 (1.243)
HML	-6.28 (4.984)		-5.55 (6.440)	-4.54 (5.607)	-6.86 (12.905)	-0.97 (2.292)
RMW			0.71 (4.883)	0.01 (4.601)	1.06 (8.693)	0.04 (2.081)
CMA			-1.24 (5.149)	-1.71 (5.323)	-0.65 (10.166)	0.63 (2.078)
IA		-4.52 (3.729)				
ROE		7.61 (6.404)				
LIQ				4.71 (7.077)		
UMD					2.14 (21.396)	
TSMOM					3.54 (15.189)	
STREV					-4.52 (9.111)	
<i>la5</i>						1.60*** (0.360)
$R_{adj}^2$	0.52	0.48	0.29	0.51	-1.07	0.97

	(1)	(2)	(3)	(4)	(5)	(6)
<i>acA</i>						
MKT	-3.62 (2.230)	-4.26 (2.788)	-0.26 (3.542)	-0.38 (3.711)	-0.57 (4.520)	-0.37 (4.017)
SMB	-2.93** (1.490)	-0.21 (1.775)	-2.62 (2.702)	-2.59 (2.706)	-1.35 (5.848)	-2.78 (3.808)
HML	-3.24* (1.665)		-3.26 (2.468)	-3.33 (2.552)	-2.75 (3.347)	-3.59 (5.675)
RMW			0.89 (3.076)	0.98 (3.216)	1.66 (3.928)	1.02 (3.864)
CMA			0.53 (2.947)	0.39 (3.213)	0.14 (3.668)	0.46 (3.268)
IA		-2.43* (1.309)				
ROE		3.40* (1.745)				
LIQ				0.73 (2.982)		
UMD					2.19 (10.479)	
TSMOM					0.11 (3.133)	
STREV					-1.36 (4.300)	
<i>la5</i>						0.96 (2.147)
$R^2_{adj}$	0.36	0.21	0.98	0.98	0.99	0.98
<i>ac5</i>						
MKT	-2.10* (1.268)	-1.76 (1.890)	-1.95 (1.460)	0.57 (3.599)	1.19 (6.139)	-0.30 (0.995)
SMB	-2.65** (1.227)	-3.51 (2.370)	-2.50* (1.373)	0.14 (2.960)	0.03 (3.366)	-0.32 (0.889)
HML	-2.05** (0.929)		-2.20** (0.961)	-2.85 (2.388)	-1.67 (2.730)	0.27 (0.929)
RMW			0.96 (0.920)	2.68 (2.898)	2.65 (3.509)	0.44 (0.690)
CMA			-0.91 (0.933)	0.87 (2.565)	-2.02 (3.198)	-0.38 (0.645)
IA		-1.41 (1.061)				
ROE		0.86 (2.016)				
LIQ				-6.11 (7.541)		
UMD					5.95 (6.987)	
TSMOM					7.76 (16.022)	
STREV					-0.43 (4.638)	
<i>la5</i>						2.34*** (0.800)
$R^2_{adj}$	0.02	-0.67	-0.45	0.46	-1.29	0.54

**Table A4: Asset pricing tests - price of risk estimates (corresponding to Table 3.5 Panel B)**  
 In addition, the table also shows  $R^2$  from the second stage (cross-sectional) regression.

	(1)	(2)	(3)	(4)	(5)	(7)
<i>rev</i>						
MKT	0.84*** (0.272)	0.84*** (0.275)	0.88*** (0.272)	0.90*** (0.272)	0.90*** (0.273)	0.85*** (0.273)
SMB	0.12 (0.200)	0.37* (0.213)	0.19 (0.199)	0.18 (0.199)	0.17 (0.199)	0.18 (0.199)
HML	0.81*** (0.237)		0.09 (0.269)	0.16 (0.270)	0.33 (0.316)	0.16 (0.280)
RMW			0.37** (0.175)	0.36** (0.175)	0.33* (0.175)	0.42** (0.175)
CMA			0.34** (0.137)	0.34** (0.137)	0.30** (0.138)	0.35** (0.138)
IA		0.27* (0.142)				
ROE		0.60** (0.257)				
LIQ				-0.69 (0.662)		
UMD					2.01*** (0.637)	
TSMOM					1.40 (0.862)	
STREV					-0.18 (0.910)	
<i>la5</i>						2.28** (1.144)
$R_{adj}^2$	0.84	0.91	0.91	0.91	0.96	0.92

	(1)	(2)	(3)	(4)	(5)	(7)
<i>fi5</i>						
MKT	0.84*** (0.272)	0.84*** (0.275)	0.88*** (0.272)	0.90*** (0.272)	0.90*** (0.273)	0.85*** (0.273)
SMB	0.12 (0.200)	0.37* (0.213)	0.19 (0.199)	0.18 (0.199)	0.17 (0.199)	0.18 (0.199)
HML	0.81*** (0.237)		0.09 (0.269)	0.16 (0.270)	0.33 (0.317)	0.16 (0.280)
RMW			0.37** (0.175)	0.36** (0.175)	0.33* (0.175)	0.42** (0.175)
CMA			0.34** (0.137)	0.34** (0.137)	0.30** (0.138)	0.35** (0.138)
IA		0.27* (0.142)				
ROE		0.60** (0.257)				
LIQ				-0.69 (0.662)		
UMD					2.00*** (0.638)	
TSMOM					1.40 (0.865)	
STREV					-0.19 (0.925)	
<i>la5</i>						2.28** (1.146)
$R_{adj}^2$	0.84	0.91	0.91	0.91	0.96	0.92
<i>la5</i>						
MKT	0.84*** (0.272)	0.84*** (0.275)	0.88*** (0.272)	0.90*** (0.272)	0.90*** (0.273)	0.85*** (0.273)
SMB	0.12 (0.200)	0.37* (0.213)	0.19 (0.199)	0.18 (0.199)	0.17 (0.199)	0.18 (0.199)
HML	0.81*** (0.237)		0.09 (0.269)	0.16 (0.270)	0.33 (0.317)	0.16 (0.280)
RMW			0.37** (0.175)	0.36** (0.175)	0.33* (0.175)	0.42** (0.175)
CMA			0.34** (0.137)	0.34** (0.137)	0.30** (0.138)	0.35** (0.138)
IA		0.27* (0.142)				
ROE		0.60** (0.257)				
LIQ				-0.69 (0.662)		
UMD					2.00*** (0.638)	
TSMOM					1.40 (0.865)	
STREV					-0.18 (0.927)	
<i>la5</i>						2.27** (1.132)
$R_{adj}^2$	0.83	0.91	0.91	0.91	0.96	0.92

3.A. Additional tables and figures

	(1)	(2)	(3)	(4)	(5)	(7)
<i>acA</i>						
MKT	0.84*** (0.272)	0.84*** (0.275)	0.88*** (0.272)	0.90*** (0.272)	0.90*** (0.273)	0.85*** (0.273)
SMB	0.12 (0.200)	0.37* (0.213)	0.19 (0.199)	0.18 (0.199)	0.17 (0.199)	0.18 (0.199)
HML	0.81*** (0.237)		0.09 (0.269)	0.16 (0.270)	0.33 (0.317)	0.16 (0.280)
RMW			0.37** (0.175)	0.36** (0.175)	0.33* (0.175)	0.42** (0.175)
CMA			0.34** (0.137)	0.34** (0.137)	0.30** (0.138)	0.35** (0.138)
IA		0.27* (0.142)				
ROE		0.60** (0.257)				
LIQ				-0.69 (0.662)		
UMD					2.00*** (0.638)	
TSMOM					1.40 (0.865)	
STREV					-0.19 (0.928)	
<i>la5</i>						2.28** (1.141)
$R^2_{adj}$	0.84	0.91	0.91	0.91	0.96	0.92
<i>ac5</i>						
MKT	0.84*** (0.272)	0.84*** (0.275)	0.88*** (0.272)	0.90*** (0.272)	0.90*** (0.273)	0.85*** (0.273)
SMB	0.12 (0.200)	0.37* (0.213)	0.19 (0.199)	0.18 (0.199)	0.17 (0.199)	0.18 (0.199)
HML	0.81*** (0.237)		0.09 (0.269)	0.16 (0.270)	0.33 (0.317)	0.16 (0.280)
RMW			0.37** (0.175)	0.36** (0.175)	0.33* (0.175)	0.42** (0.175)
CMA			0.34** (0.137)	0.34** (0.137)	0.30** (0.138)	0.35** (0.138)
IA		0.27* (0.142)				
ROE		0.60** (0.257)				
LIQ				-0.69 (0.662)		
UMD					2.00*** (0.638)	
TSMOM					1.41 (0.865)	
STREV					-0.19 (0.928)	
<i>la5</i>						2.28** (1.141)
$R^2_{adj}$	0.84	0.91	0.91	0.91	0.96	0.92

## Appendix 3.B Returns from trading at mid, bid or ask

Returns are usually defined at close prices as

$$r_t := \frac{P_t}{P_{t-1}} - 1,$$

but we can similarly define them at mid prices,  $P^M$ , as

$$r_t^M := \frac{P_t^M}{P_{t-1}^M} - 1. \quad (3.7)$$

Since we are interested in working with returns rather than with prices, for reasons that will be clarified in the next section, we adjust the close-to-close return on both the current and previous days by its residual close-to-mid component which is defined as

$$r_t^{\text{CM}} := \frac{P_t^M}{P_t} - 1. \quad (3.8)$$

We can re-arrange the previous equation in terms of mid prices

$$P_t^M = (1 + r_t^{\text{CM}})P_t$$

and substitute it back into [equation \(3.7\)](#) to get an expression for the mid-to-mid return

$$\begin{aligned} r_t^M &= \frac{1 + r_t^{\text{CM}}}{1 + r_{t-1}^{\text{CM}}}(P_t/P_{t-1}) - 1, \\ &= \frac{1 + r_t^{\text{CM}}}{1 + r_{t-1}^{\text{CM}}}(1 + r_t) - 1. \end{aligned} \quad (3.9)$$

[Equation \(3.9\)](#) does not depend on prices because it corrects the close-to-close return for the distance between  $P^M$  and  $P$  which is itself a return. Since the adjustment uses same-

day quantities, it is approximately not affected by company events like stock splits or distributions.

### 3.B.1 Magnitude of the approximation

As mentioned earlier, we prefer working directly with adjusted returns because it spares us a series of manual manipulations related to company events that we would otherwise need to perform if we were to use prices.<sup>23</sup> Instead, we can use CRSP returns and apply our correction at the cost of a negligible approximation which is shown below.

Given CRSP's definition of adjusted return<sup>24</sup>

$$r_t := \frac{P_t F_t + D_t}{P_{t-1}} - 1,$$

where  $F_t$  is a factor that adjusts the price for eventual splits and  $D_t$  is the amount of the distribution, we can express it in terms of mid prices and apply the substitution with the re-arranged [equation \(3.8\)](#) to get

$$\begin{aligned} r_t^M &= \frac{P_t^M F_t + D_t}{P_{t-1}^M} - 1, \\ &= \frac{(1 + r_t^{\text{CM}})P_t F_t + D_t}{(1 + r_{t-1}^{\text{CM}})P_{t-1}} - 1, \\ &= \frac{1 + r_t^{\text{CM}}}{1 + r_{t-1}^{\text{CM}}} \left( \frac{P_t F_t + D_t \frac{1+r_{t-1}^{\text{CM}}}{1+r_t^{\text{CM}}}}{P_{t-1}} \right) - 1, \\ &\approx \frac{1 + r_t^{\text{CM}}}{1 + r_{t-1}^{\text{CM}}} (1 + r_t) - 1. \end{aligned} \tag{3.10}$$

[Equation \(3.10\)](#) shows that our correction is invariant to splits, i.e. the multiplicative factor, but bears an approximation in the additive term due to stock distributions. The

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<sup>23</sup>Although, adjusting prices for company events is not complicated per se, there are many edge cases that need to be considered in order to obtain the same results provided by CRSP.

<sup>24</sup>For additional details and the exact definition see the CRSP manuals.



last approximate equality holds strictly on all but the dividend days and on those dates only if

$$D_t \frac{1 + r_{t-1}^{\text{CM}}}{1 + r_t^{\text{CM}}} = D_t.$$

Hence the approximation is limited to a few days per year and is also small in magnitude. In fact, close-to-mid returns do not change much over time and consequently their day-to-day ratio stays close to 1. Moreover, the dividend itself is usually small relative to the daily return and bears a negligible overall impact. In light of these considerations, we can simply use [equation \(3.9\)](#).

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