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The Potency Of Metacognitive Learning To Foster Mathematical Logical Thinking

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Abstract

The ability of thinking logically needs to be developed due to the fact that it is an essential basic skill. Logical thinking affects that giving reason must be true, and that a sequence of assumptions is based on the high truth value. Mathematics is a subject that functions to train students to think logically. The understanding of logic will help students to arrange the proof that support through process to finally arrive at a conclusion. Currently, metacognition is viewed as an essential element of learning. It refers to someone knowledge of processes and the result itself or of that connected to the process. Metacognition is needed when student solves the task that needs argumentation and logical understanding. In order to help student to skillful think logically, mathematics learning must be designed as such so that the condition will raise the skill of metacognitive acts.

Key words: metacognitive learning, mathematical logical thinking

I. INTRODUCTION

Mathematics is one of subjects needed to learn at schooling in order to fulfill skillful students. It is important for them to pursue higher education, reaching their dreams, using it into daily life, and facing multichallenges life both simple and complex one. Skills which are improved by mathematical practice like pattern analysis, logical thinking, problem solving, and the skills to see those relationship can helps students to fulfill those mentioned needs.

Logical thinking is a key to draw a conclusion and solving complex problems. Man applies logical thinking in daily life. Although everybody has the ability to think, but it is not always the case that he can argue by good reason. It is assumed that man has universal nature to think logic and to follow logical inferences, traditionally it has defined as higher cognitive skills (Minderovic, 2006).

Anybody who is skillful in problem solving is those who can observe, clasify, measure, communicate, predict, interpret, analyze, synthesize, and draw a conclusion. They can see, organize, and give a meaning to every information given in problems

through reflective abstraction (Leongson & Limjap, 2002). They have the process ability needed to sistematically involve in mathematical tasks.

Students have to learn mathematics in order to become a better thinker and able to solve problem. Thinking logically is not a 'magic' process or genetical, but it's a mental process that is being learned (Albrecth, 1984). In the line, Lawson (1974) stressed that schooling system is not to mean to teach facts and concepts of knowledge domain only, but it's more important is to help students in gaining thinking skills. Mathematical learning should be designed to grow cognitive skill and to improve abilitiy and develop students' inner potencies. Renner and Philips (Trifone, 1987) believed that students should be given opportunities to develop their thinking ability as a basis for developing their intellectuality.

Many literatures recommend that teachers should develop students' thinking skill through teaching, to support students to reflect their learning process. Interaction between individuals can support students to communicate and explain their thinking. Learning has to be designed so that it can serve various activities that provoke students' thinking and students can control their own intellectual activities.

Costa (2001) stated that teaching of thinking at least contains four components: (1) *brain functioning*, (2) *metacognition*, (3) *great thinkers*, and (4) *epistemic cognition*. It is already founded that people solve problems by using metacognition – planing steps to work for before solving problems, monitors himself during working, following the plan, and self evaluation after finishing the work. In a classroom metacognition is being noted that there is a discussion between students about what is in their mind while thinking; a comparison between students approach when they are solving problems and drawing conclusion; identifying what has already been known, what is needed to understand, and how to obtain that knowledge; or students speak out their thinking while working.

II. DISCUSSION

2.1 Logical thinking

According to Albrecht (1984), logical thinking is a process when someone uses reasoning consistently to draw a conclusion. Problem or situation that is involving logical thinking needs a structure, relation between facts, and a sequence of reasoning that makes sense. The basis of all process of logical thinking is thinking continuously.

This process includes taking important ideas, facts, and conclusion from problems and making connection in order to become meaningful.It is also being said that thinking logically is in a steps of thinking. Whole key concepts roles as an aim: organising information avalilable to useful form and reduces problem by using known conclusion. In brief, logical thinking means to organise and to manipulate information.

The terminology of logical thinking has been declared by Suriasumantri (1996). According to him, thinking is an activity to find true knowledge. Logical thinking means as a thinking activity according to a certain pattern; logical pattern. Logic can be defined widely as an investigation to think valid. Logical thinking is thinking based on reasoning, not by feelings. Reasoning is a thinking process to draw a conclusion and it has a certain characteristic.Thinking activity is not based on reasoning such as intuition, but based on a certain pattern. Strydom (2000) defined logical thinking as thinking in cause-effect pattern, which means sequential thinking. Logical thinking follows a sequence of thought or ideas, considering what is happening and predicting something in the future: if this one happened, then the other will do also. However, this is not related to predicting. This is based on interpretation of general condition and then predicts what is going to happen at the other similar condition.

Minderovic (2006) stated that logical thinking is thinking that follows rule of logical inferences, and it is high order cognitive thinking skills.While Ioveureyes (2008) stated that logical thinking is an activity which someone is considering, reasoning, and dynamic thinking form in the process of understanding to claim objective reality of rational konwledge. Logic and reasoning is known as theoritical concepts to understand logically, and it is affirmed as structure of thinking and the analysis of emerging and developing something. Only by thinking logically, someone reaches the goal, understanding the nature of understanding the objective of the world. It is said that thinking logically is abstract thinking, a high modes of thinking, that is free of emosional aspects.Way of thinking logically includes: induction, deduction, analysis, and synthesis thinking.

Sponias (2011) defined thinking as a process based on thought to understanding the reality, and finding solution to problems. Thinking logically means thinking associated to logical principles. Logic is a set of value of reality thinking. In other word, logic is a reasoning method. In his opinion, if we want to develop logical thinking, we

have to be able to make decision following the rule of logic so that it can help us to analys the value of reality.

Logic and reasoning is known as theorical concepts to understand something logically. Lipman (2003) defined reasoning is a process of ordering and coordinating what has been found out through the inquiry. This includes finding valid ways of extending and organizing what has been discovered or invented while retaining truth. Knowledge is a result of investigating orgensied logically. Knowledge is based upon our experiences of the world; one way of extending knowledge without recourse to additional experience is through reasoning. Given what we know, by reasoning we might obtain new things. Reasoning skills according to Lipman includes: offers appropriate analogies, seeks to clarify ill-defined concept, makes relevant distinctions and connections, supports opinions with convincing reasons, provides examples and counterexamples, seeks to uncover underlying assumptions, draws suitable inferences, and makes balanced evaluative judgments.

Geisler and Brooks (1990) explain the logic as elaborating true reasoning or valid conclusion and emerging fallacies, both formal and informal. Study of true reasoning is how to think right, and how to find the truth. Here, logic is being identified as a way to draw conclusion. Valid conclusion means implication; a part of logic is when A follows B and when it is not. In brief Geisler and Brooks define that: logic is a way of thinking so that we can reach the right conclusion by understanding implication and error that is being done by others in thinking.

McInerny (2004) mentioned that whether it is being concerned as science, or arts, or skills, and also the compilation of the three – logic is a bases of our ability to think, analyze, argue, and communicate it. Logic is a core of what is said to be man intellegence. Mastery of logic starts with understanding reasoning correctly, relation between logical thinking and logical expression, knowledge of arguments, and the knowledge of illogical thinking. According to Martin (2005), logic is a science studying inference structure and is one of a base for drawing conclusion. Deciding means doing responsive toward reason and evidence. In order to reach a conclusion, someone is collecting evidences, considering, interpretating, and giving value, and this is based on logical principles. In Webster Dictionary (2009) logic is defined as the science or art of exact reasoning, or of the laws according to which the processes of pure thinking should

be conducted; the science of generalization, judgment, classification, reasoning, and systematic arrangement; the science of correct reasoning.

Reasoning has its root from phylosophy and logic. The term of reasoning comes from Shurter and Pierce (Utari Sumarmo, 1987) that is defined as a process of reaching logical conclusion based on the facts and relevan sources. There are two kind of reasoning: deductive and inductive reasoning. Similar definition is given by Jujun S. Suriasumantri (1996). (Beyer, 2001) said that reasoning is drawing conclusion systematically from onformation that follows logical rules in order to show validity of a proposition. Reasoning is a process drawing conclusion from observation or declaring hypothesis and believe.

Quellmalz (1987: 89) compared componets of reasoning extracted from the literatures of phylosophy and psychology.

Table 1 Comparison of Reasoning Skills Proposed in Psychology and Philosophy

Critcal Thinking Skills (Philosopy)	Problem-solving Strategics (Psychology)	Probable Dominant Cognitive Process (Psychology)
Clarification Identify or formulate a question Analyze mayor components Defining important terms	Identify the problem Identify essential elements and terms	Analogical Analysis and comparison
Judge credibility of support, the source, and observations	Identify appropriate information, content, and procedural schemata	Analogical Analysis and comparison; component evaluation
Inference Deduction Induction Value judgments Fallacies	Connect and use information to solve the problem	Infer/interpret relationship among components
Using criteria to judge adequacy of solution	Evaluate success of the solution	Evaluate effectiveness of specific and general strategies

Quellmalz also listed kinds of higher order reasoning skills as follows:

1. Analyze: identify the components of a process or the features of objects.
2. Compare: compare the properties of objects or events.

3. Infer/interpret: draw conclusions, make predictions, pose hypotheses, tests, and explanations.
4. Evaluate: evaluate the soundness and significance of findings.

In Saskatchewan Learning (2007) logical thinking ability in mathematics is formulated as develop and be able to apply mathematical reasoning process, skills, and strategies to new situation and problems. These processes and strategies include:

- inductive and deductive thinking
- abstracting and generalizing
- exploring, identifying, and describing patterns
- verifying and proofing
- exploring, identifying, and describing relationships
- modeling and representing (concretely, visually, physically, and simbolically)
- hypothesizing and asking "what if" (mathematical play)

The concept of thinking logical is given by Jean Piaget (1896-1980) which his study is viewed as the bases of logical thinking. Piaget pictured cognitive attitude in the degree of logical structures. Piaget identified four level of cognitive development. In the level of sensory-motor, kids are experienced with physical world and reach understanding of basic symbol. At the pre-operational, symbol are used, but kids' thinking is still pre-operational, that is kids still do not understand that operation is logical, or mathematical operation can be inverted. At the level of concrete operation kids are being supported to think logically, kids for example, understand cause and effect principles. In the level of formal operation, kids are introduced to an abstract thinking.

At the level of concrete operation, kids will apply logical thinking. Kids will start developing responses about concrete objects. They show logical thinking in relation with phyisical objects. They can relate with realistic problem through nature.They are able to understand what emerges around them and they can understand possibilities from reality, but abstract concept understanding is still limited. They can remember figure of objects physically that is not around and they can not handle the ideas. In this stage they start to develop their mathemtical thinking ability.

In the level of formal operation, kids are able to use various thinking abilities used in the level three, but with additional information that they are able to think

abstract, formulating hypothesis, determining complex abstract combination, and considering various proposition to solve problem by using systematical reasoning. Deductive reasoning is main element of formal operation thinking. This is an ability to draw a valid conclusion of an argument, whether valid or invalid argument (Inhelder and Piaget, 1958).

According to Piaget the period of development of cognitive at the level of sensory motor is at 0 − 2 years old, level of pre-operasional 2 – 7 years old, level of concrete operation 7 – 11 yeras old, and level of formal operation 11 – 16 years old up to adult age. Nevertheless, it is good to remember that not all people can reach this formal operation, abstract reasoning is not universal (Trifone, 1987; Sumarmo, 1987; Yenilmez *et al*., 2005, Fah, 2009).

2.2 Metacognition

The term metacognition is first mentioned by John Flavel in 1976 (Schoenfeld, 1992) and is defined as follows:

> *Metacognition refers to one's knowledge concerning one's own cognitive processes and products or anything related to them, e.g. the learning-relevant properties of information or data. For example, I am engaging in metacognition if I notice that I am having more trouble learning A than B; if it strikes me that I should double check C before accepting it as fact; if it occurs to me that I should scrutinize each and every alternative in a multiple-choice task before deciding which is the best one…. Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of those processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete (problem solving) goal or objective.*

Following from the definition above, metacognition refers to one's knowledge about process and the result related to those processes. Flavell views the need of a method in formal educational setting which will help students to understand better, getting better result in academic, and making decision of aware and honets live. It means that the process reflects our thinking and monitors whether our thinking path approaches the goals or it goes farther. Flavell stated that metacognition as thinking about thinking (*cognition of cognition*) has two functions, namely monitor and control process of thinking.

Reys, et. al. (1998) showed metacognition as what is known about himself by others or what they believe on us and how to control and anticipate his attitude. Students

need to aware of their strength, weaknesses, and characteristic attitude and procedures and strategy to use in learning and doing matehamtics.

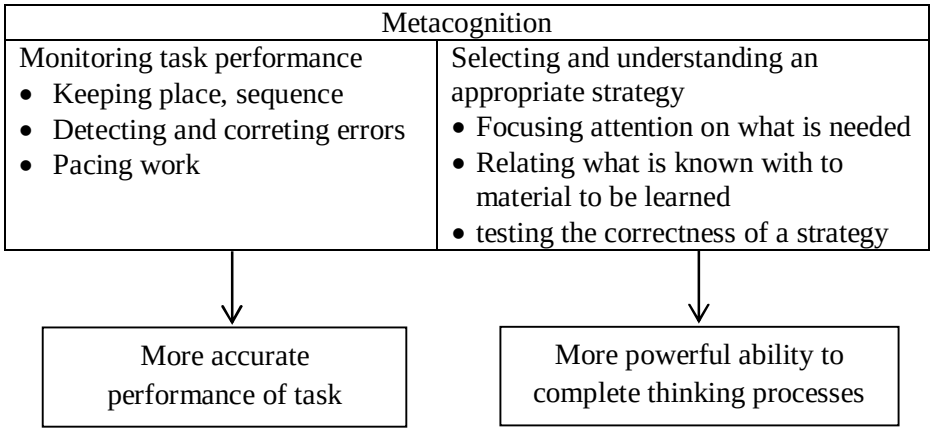
Markovits and Barroillet (2004) believe that metacognition is needed when students doing task that needs argumentation and logical understanding. Metacognition is an ability of thinking about thinking, and controling explisitely organisation process of reasoning. Metacognition as process control is often used to explain some different performance, for example in the case of someone is able to solve problem in one context, but he can not do in other problem. Besides, it is used to characterize for ability to concept abstract object and to reflect of our empirical knowledge. By using modus ponen principle "If P then Q, P is true, consequently Q must be true", students can get a concluison on the simple context. When they work on the complex concept, students is often failed in drawing a conclusion. As a consequence, it is needed the form of understanding in more complex mode and control as an addition to this complex information.

Desoete (2009) is also defined metacognition as a statement of one's knowledge and his control toward system of his cognitive. Metacognition can help in solving challenged tasks in mathematics, and has been claimed that metacognition affects the mathematical problem solving ability. Students who is clever is assumed to have a appropriate strategy and attempting to act to manipulate tasks or problems, by using knowledge has been mastered before and select appropriate behavioral learning. Metacognition is included in all aspect of mathematical problem solving, starting from the beginning of developing representation of problems up to the interpretating and checking the result obatined. Metacognition makes students to use available knowledge by his own strategy. Skill of metacognitive is needed mainly to check what has been known and what has not been known, to monitor the leraning, and to plan to learn new skills.

There are many kinds of metacognition models. Pressley, et.al in 1987 (Carr, 2010), devides metacognition into two models: (a) knowledge and metacognitive skill, (b) controling and reflection of metacognitive. Knowledge and metacognitive skills includes about when, why, and how to use strategy as working for different tasks. While control and metacognitive reflection are the process of students to monitor, control organize and learning.

Presseissen (2001) pictured out the model of metacognitive thinking skill in Figure 1. Metacognitive thinking has two main dimensions; (1) task-oriented and relates to monitoring the actual performance of a skill, and (2) strategic; it involves selecting a strategy appropriate to the circumstances and seeking feedback to affirm or alter that choice.

Figure 1.
A Model of Metacognitive Thinking skill



Source: Presseissen (2001)

The important of mathematical learning using metacognitive was suggested by Schoenfeld (1987). Metacognition is believed to have the potency of improving students meaningful learning, and creating "mathematics cultures" in the classroom is the good tool to increase metacognition, this will encourage students to think mathematically as an integral part of their daily life, increase students to make relation between various mathematical concept, and develop the feeling students communities in solving mathematical problems.

III. CONCLUSION AND SUGGESTION

Logical thinking is a key to making sound decisions and solving problems. It is a mental process in which one uses reasoning consistently to come to a conclusion. Mathematical logical thinking is an ability to use mathematical reasoning process, skills, and strategies include: inductive and deductive thinking; exploring, identifying, and describing patterns; exploring, identifying, and describing relationships; verifying and proofing; modeling and representing; hypothesizing; offering appropriate analogies,

seeking to clarify ill-defined concept; supporting opinions with convincing reasons, seeking to uncover underlying assumptions, and drawing suitable inferences.

Many studies alike shows that it is needed an attempt to increase the ability of students logical thinking. Besides, metacognition play an essensial role in order to obtain good result in learning, and it can give a suggestion that mathematical learning can be more effective, in other word, metacoginitve learning is important to students. Thus, it is important to know how to make use of aspects in metacognition, especially when and in what condition, effective aspect cognitive can be maximal develop the ability to improve students logical thinking. Teachers can do any ways to help students in developing metacognitive awareness. Metacognitive learning is a learning process that is designed by integrating writen or oral questions. These questions are to monitor and to develop students ability in making use of strategy when doing mathematical activities.

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