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On The Teaching Of Analyzing The Effects Of Parameter Changes On The Graph Of Function

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Abstract

The proficiency level of some tenth and eleventh graders on analyzing the effects of parameter changes on the graph of function is observed in this project. The result shows the level is low. It seems that those students did not sufficiently work on math problems involving deep reasoning. The students used to solve math problems by plugging in formulas only. In this project, the causes are found and reported. At the end part, an alternative teaching approach is proposed.

Key Words: parameter, graph of function, alternative teaching.

I.   INTRODUCTION

It would seem that students' understanding of the effect of parameter changes on the graph of function is low. This can be seen from the common practices of the teachers and students when they discuss the graph of quadratic functions. They merely use formula for finding the vertex, axis-symmetry and plot some points to sketch the graph. They didn't make connection the graph they sketched and the basic quadratic graph.

Commonly, the students use formulas to solve problems. However, they seem do not understand the formula and the concepts behind it. This is of course is not sufficient according to the national standard documents (Peraturan Pemerintah, 2006), that require reasoning.

If this hypothesis is true then a lesson design must be formulated such that students can develop their understanding on the graph quadratic functions. In particular, the students

can develop their analytical skill on parameter changes. That's why this work starts from answering the following questions:

1) Do students understand the effect of parameter changes on the graph of functions?

2) What are the causes that make the students difficult to understand the effect of parameter changes?

3) What is an alternative teaching strategy that may help the students understand the effect of parameter changes?

The project's objective is to test the hypothesis of student difficulty, identify the causes, and provide an alternative teaching strategy. The new teaching strategy is expected to improve the quality of learning and the interest of the students towards mathematics.

Using the new teaching strategy, several students are examined. This paper reports the results.

II. DISCUSSION

If we let parameters be $a, b, c,$ and d and the parameters have geometry effect into the function through some general formula, like $y = af(b(x + c)) + d$, then each parameter characterizes some unique geometric effect. The parameter changes of course relate to some geometric transformations. They are namely *dilation*, *translation* and *reflection*. For example, this figure shows the effect of parameter changes in the graph of quadratic function $g(x) = 2(x - 3)^2 + 2$ from the basic graph $f(x) = x^2$.
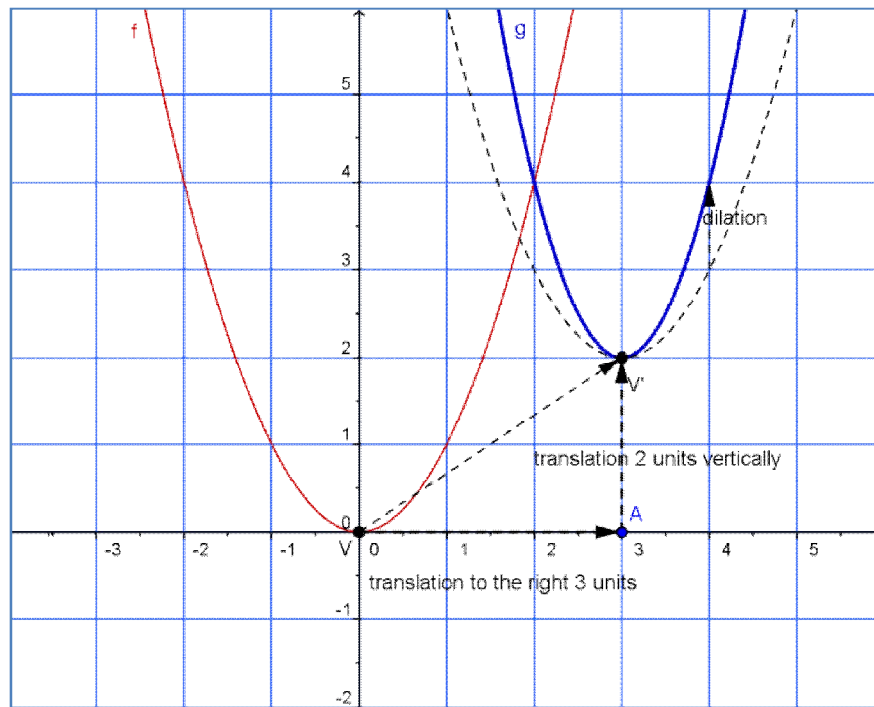
Figure 1: Transformation function $g(x) = 2(x-3)^2 + 2$ from $f(x) = x^2$

In the beginning of the project, five students were given problems. The problems assess the students' understanding on the geometric effects in parameter value changes with respect to the original function graph. From the observations on the students' answers and interviews, the students show some degree of difficulty in sketching the function graph. The students could not work with general functions that do not have explicit formula. Moreover, they used to draw the graph of a function by plotting a point by a point on the coordinate plane. Thus, they did not utilize the particular properties of each parameter in drawing the graph. In other words, they did not draw the graph qualitatively, instead they draw it quantitatively. This figure shows the answer of a student solving the given problem.

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6.   Jelaskan perubahan letak dan bentuk parabola $y = (x+1)^2$ menjadi menjadi parabola $y = -2(x-2)^2 - 1$
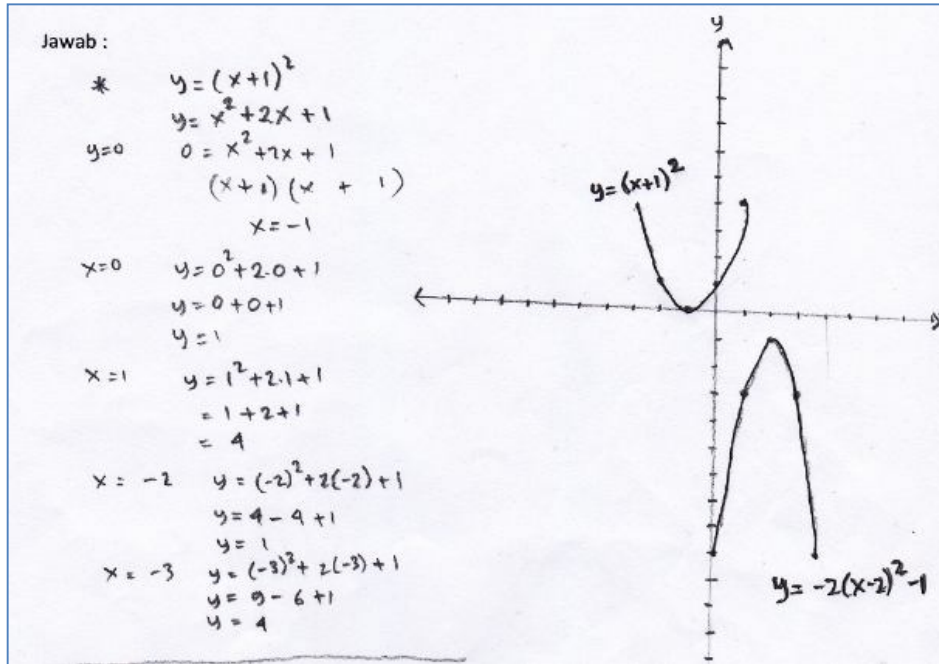

Figure 2: The answer of a student to solve a problem given

Another student answers this task using the vertex formula. The answer can be seen in this figure.
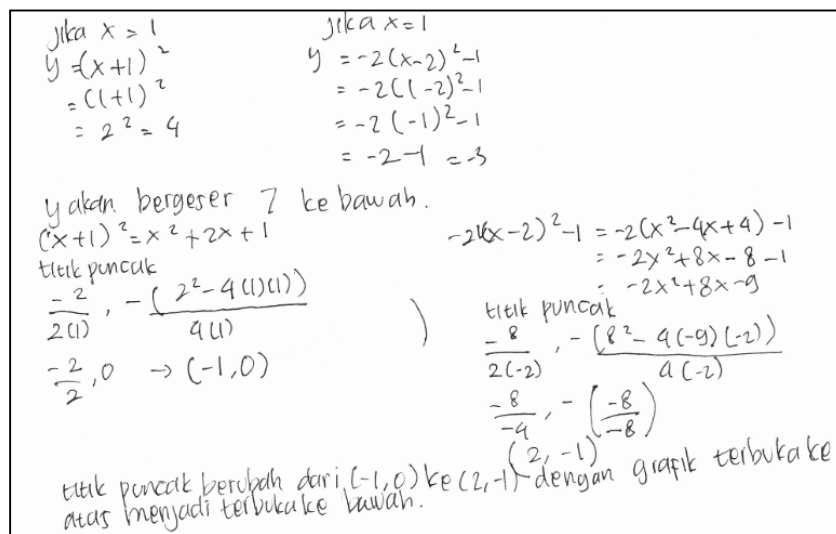

Figure 3: The answer of a student to solve a problem given

From another given task, it is observed as well that the students have difficulties in completing a perfect square. That is they are unable to transform the general quadratic form $f(x) = ax^2 + bx + c$ into the vertex-form $y = a(x + p)^2 + q$. The students did not realize that the parameter changes in the vertex-form $y =$

$a\,(x\,+\,p)^2\,+\,q$ is easy to analyze. This is another reason why completing a perfect square needs to be addressed.

We are curious if this competence is stated in the Content Standards (Peraturan Pemerintah, 2006) documents. After examining the documents, we can say that the competence "analyzing the effects of the function parameter changes" is not explicitly stated. This situation perhaps is one reason why our students could not perform the task. Because the competence is not stated, teachers do not consider it as important. Moreover, state exams never assess this important competence. In addition because there is no mathematical learning processes standard, math teachers are not aware of this competence.

Unlike the Content Standards (Peraturan Pemerintah, 2006), the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989), i.e., curriculum and assessment standards for school mathematics, states that in grade 9 to grade 12 students should be able to analyze the effect of parameter changes on the graph of functions.

As a further comparison of math contents, the International General Certificate Syllabus of Secondary Education (IGCSE), Cambridge International Mathematics (2010), states that students must have competence in describing and identifying with the use of verbal language transformation graph of $y\,=\,f\,(x)$ to graph of the function $y\,=\,f\,(x)\,+\,k$, $y\,=\,kf\,(x)$ and $y\,=\,f\,(x\,+\,k)$. This means that the national math standards documents are not complete enough.

To improve the students' mastery on the competence "analyze the effect of parameter changes on the graph of functions", technology is incorporated in the teaching strategy. In particular, we utilize the application GeoGebra. This application is chosen because it is flexible, so the users can modify it into their needs. Moreover, the application is freeware, so students can use it for free. GeoGebra is able to represent functions in the forms: formula, graph, and numerical table, simultaneously. This capability helps the students to analyze the effect of parameter changes on the graph of functions. This application also helps the students focus on the qualitative analysis, instead of complicated calculations. By using the application, the students can learn to improve

International Seminar and the Fourth National Conference on Mathematics Education 2011
Department of Mathematics Education, Yogyakarta State University
Yogyakarta, July 21-23 2011

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their communication and reasoning skills. Moreover, the students can maintain their positive attitudes in learning mathematics.
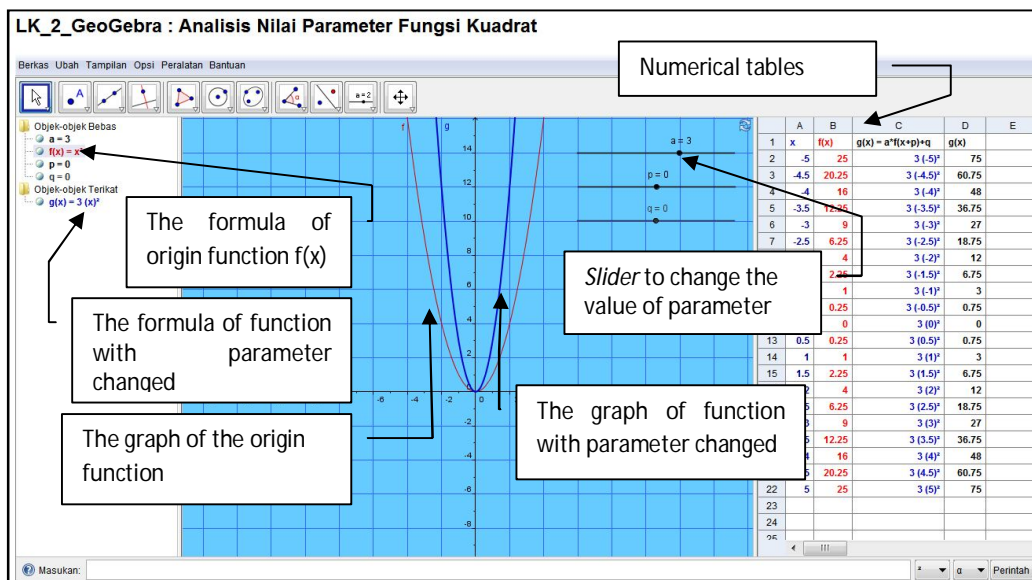
The following figure shows a worksheet in GeoGebra.

Figure 4: GeoGebra worksheet to analyze the effects of the parameter changes on the transformation of quadratic functions graph

By sliding the *slider*, students can easily observe directly the geometric effects. They can observe simultaneously the changes on the formula, graph, and table. With the utilization of GeoGebra, the students can explore and make generalizations from his observations.

According to Brodie (2010), a key role in reasoning is on the question formulization by the teachers. In the case where we utilize GeoGebra, the questions are formulated through the worksheets, prepared by the teachers. The sequence of questions in the worksheet guides the students to generalize, reason, and prove. The questions also build students' ability to communicate the reasoning, make connections, and solve problems. The experience in using GeoGebra also helps the students to differentiate between mathematical proof and drawing. The students should understand that GeoGebra or any other drawing tools cannot be considered as a math proof. Instead, math proof must always rely on reasoning.

In designing the alternative instruction, the "backward design" approach (Wiggin, 2005) is used. This design process starts with identifying the expected outcomes, determining the assessment that can be accepted, and designing the learning and teaching process.

To determine the achievement of the students to the learning outcomes, interviews with some students were conducted. Some findings are as follows:

    a. The subjects can determine the changes the characteristic of parabola by analyzing the effects of parameter changes. This figure shows how the subject analyzes the effects of parameter changes.
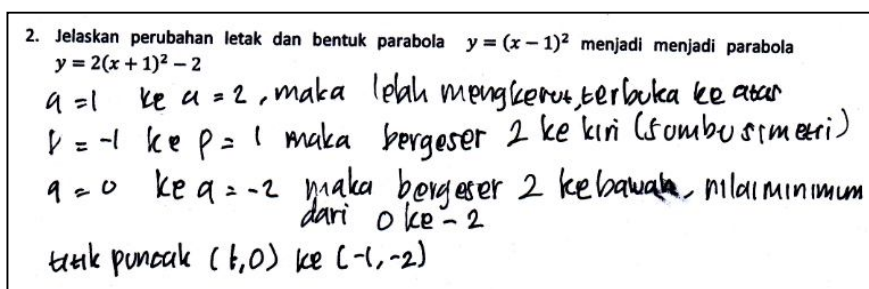

Figure 5: The answer of a student to analyze the effect of parameter changes

    b. The subjects can generalize the geometric effects of parameter changes on general functions. This figure shows the subject can analyze the effect of parameter 3 on general functions, $f(x + 3)$.
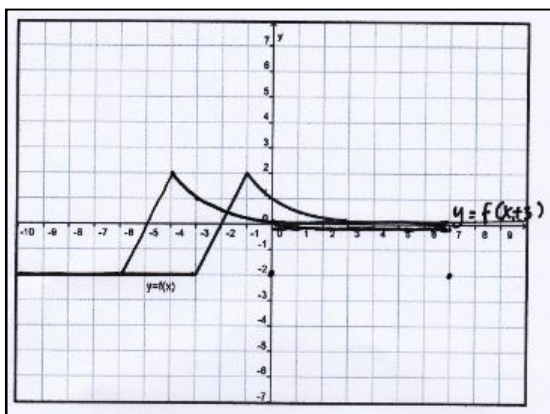

Figure 5: The answer of a student to analyze the effect of parameter changes on general function.

    c. By the interviews, the subjects show some improvement on their positive attitude towards math. Using GeoGeobra makes student easily learn the effect of parameter changes on the graph of functions.

III. CONCLUSIONS AND SUGESTIONS

This work shows that students' understanding on the competence "analyze the effect of parameter changes on the graph of functions" in general is low. This may be caused by the lack of explicitness and detailed formulation of the National Standards documents. The Standar Isi 2006 (BSNP) document does not sufficiently and explicitly the competence. This is of course in contrast with the NCTM curriculum and evaluation standards for school mathematics and the Cambridge Syllabus, where the competence is stated clearly and explicitly.

The alternative learning and teaching strategy incorporating GeoGebra shows some positive signs. The strategy shows some signs on the improvement on the quality of students' reasoning. It also improves the students' interest towards mathematics. To further strengthen the conclusions of this project, a further research with more subjects is recommended.

IV. BIBLIOGRAPHY

Brodie, K.. (2010). *Teaching Mathematical Reasoning in Secondary School Classrooms*, New York : Springer.

Peraturan Pemerintah (2006), *Peraturan Pemerintah, Nomor 22, Tahun 2006, tentang Standar Isi Mata Pelajaran Matematika SD/MI, SMP/MTs, SMA/MA*.

National Council of Teacher of Mathematics.(1989). *Curriculum and Evaluation Standards for School Mathematics.*

Syllabus International General Certificate of Secondary Education. (2008). *Mathematics 9709 for Examination June and November 2010*, University of Cambridge.

Wiggins, G. & Tighe, J.M.. (2005). *Understanding by Design*, Association for Supervision and Curriculum Development, Alexandria Virginia, USA.