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Developing Students Ability To Write Mathematical Proof By Polya Method

Kodirun
Mathematics Department of
Faculty of Mathematics and Natural Sciences of University of Haluoleo Kendari
Email: kodirun_zuhry@yahoo.co.id

Abstract

Both writing and reading a proof is equally not easy. Some mathematicians attested that students found difficulties in mathematical proving. Mathematics and mathematics education experts like Jones (1997, 2001), Weber (2001), and Smith (2006) found that difficulty in proof writing is due to: lack of theorem and concept understanding, lack of proving ability, and there is a teaching-learning process that unites with the subject. So, it is truly needed that class of writing proof in order to help to generate students' ability to do mathematical proving. Polya method is going to be that purpose.

Keywords: mathematics proof, Polya method.

I. INTRODUCTION

Both reading and writing mathematical proof are equally difficult. Some mathematicians and mathematics educators confirmed that mathematical proof is a difficult matter that was done students, even teachers and experts. Smith (2006) stated that many undergraduate students who finished their study did not make to develop the work of proving at the end of their study. Weber (2001) stated that students experienced difficulties in proving caused by lacking of concepts supporting in proof writing, lack of understanding theorems or concepts and as a consequence becoming misused them in proof construction, and lack of strategy to construct proof. Jones (1997, 2001) stated that students' difficulty in writing proof is because of teaching-learning process of proving. This teaching-learning process is conducted by the fact that the proof model is in traditional proof of geometry, i.e. two column model of proof. As a consequence, mathematical proof is an activity that is too important to ignore, then in proving teachers and lectures are very essential to help students to understand the process of proving that is understanding oriented. A mathematical proof is being accepted or refuted by mathematics community, so students need help in producing a work of proof.

The aims of mathematics learning (Depdiknas, 2006) are at least that students of middle schooling have to able to make use of reasoning of patterns and properties, to do mathematical manipulation in drawing general conclusion, constructing proofs, or

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giving explanation to the truth of mathematics notion and of mathematics statement. Those abilities must already been mastered by university students, both freshmen and sophomore even all undergraduate mathematics students, since the ability of proving has already been given when they were still at the middle school. The way of thinking and reasoning should be delivered to students so that when they enrol into a university class those abilities have been no more such a trouble. This is very important to face mathematics subject involving many kinds of proving. Subjects of mathematics major that assimilate proving are those learning mathematical structures. They are: Abstract Algebra, Linear Algebra, Probability Theory, Real Analysis, and Calculus. Other subject that includes reasoning is Introduction to Mathematics for sophomore.

Mathematical proof is not a special topic that is taught solely through a certain kind of subject. It is integrated into subjects including topics in which involves proving like subjects mentioned before. Especially, proving method is mentioned but too little and too narrow when students were to proof mathematical problems. As a consequence, it is a must to find models in order to develop students’ ability as they write a mathematical proof. This article is to bridge in helping students to develop proving ability by making use of Polya method.

II. MATHEMATICAL PROOF AND MATHEMATICAL REASONING

Mathematical language is a base of mathematics reasoning, so that it must be carefully learned in order students can make use of it correctly. Concept of variable is essential and it is important to learn from the beginning. Standard connectives (and, or, not, if ... then ..., if and only if) and quantifiers (for every, for all, there exists, none) are used in little bit different mode in mathematical language rather than in daily language, so big attention of them are necessarily to be boosted.

Students are hoped to be familiar with mathematical language and sentences, when they learn mathematical concepts and how they express them all. They are also hoped being able to explain their works carefully both written and oral explanation; they become better in writing and explaining the validity of an informal argument; and they are hoped being able to write more proper mathematical model in solving mathematical problems.

Mathematical proof is an argument started with postulates or assumptions and followed by drawing conclusion that holds with an argumentation method (Mitchell & Johnson, 2008). Why do we have to prove? The important answer is that we are doing proof because deliver questions. Two essential questions are (1) is this really true? Do we really know that it is indeed true, or do we believe it to be true because other people who is an expert telling you that this has already been investigated, or is it because the thing will always true and hold for every example done? (2) If it is true, what are the main reasons to check its truth? What is now happening here which helps us to understand and remember this unusual fact? Is there anyone who is going to help us in different way, for similar problem? It might be possible that the problem to solve is just similar to previous one without our alertness.

There are two kinds of mathematical reasoning, i.e., inductive reasoning and deductive reasoning. Inductive reasoning consists of recognizing pattern and reasoning by analogy, from a smaller part to whole part. By considering some examples or testing evidence, students must abstract or generalize in order to make a conjecture (mathematical sentence that is not known yet its truth). They may test their conjecture by using other examples. However, by using a set of data to test it does not mean that we have already doing a substitute of a proof, and we hope that these differences make it to all students clear.

In deductive reasoning, a new statement is obtained from assumption that is stated clearly and from a statement which has already been shown its truthfulness by using some rules of logic. So, proof is an explanation about why a mathematical sentence is true. Deductive reasoning gives a method of deriving new truth from any known information and a way in order to differentiate between understanding and believing.

Some mathematicians differ in giving their opinions of mathematical proof. Nevertheless, those differences are because of their views only. Krantz (2007) stated that a proof is a rhetorical tool to ensure others that mathematics proposition is true or valid. While Stylianides (2007) suggests a concept of proof in more comprehensive way namely proof is a mathematical argument which is having a form of a sequence of statements; the statement must be in certain characteristics and its truth is being accepted. These characteristics consist of (1) using a statement which its truth is accepted by mathematics community and without any advanced explanation, (2) using a form of valid reasoning and being known by mathematics community, and (3) it is

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communicated in a proper form and it is well-known by mathematics community.

III. ROLE OF PROVING IN MATHEMATICS AND MATHEMATICS TEACHING-LEARNING PROCESS

Mathematics proof is an essential component in mathematics and plays urgent role which differentiates mathematics from other subjects (Tall, 2002). However, the process of proving and writing proof seems to be a ritual work that is not recognized well by many people (Smith, 2006; Weber, 2001). Proving is often simplified as a standard deductive representation, for example it is just like old geometry Euclid proof in the two-column table of proof in proving geometry theories and conjectures.

The aims of proving in mathematics learning is little bit different from the role of proving in mathematical research, its main role is to demonstrate the validity of propositions or conjectures. The main role of proving in mathematical learning is to explain why a proposition is true (Hersh, 1993).

In order students to get something that is useful form proof understanding, students must demonstrate that they understand the important nature of mathematical proving through their answers toward questions follows. Answers to the questions that provide them to firstly students are to complete steps of proving process given by way of developing statement related to reason or giving reason toward statements given. Secondly, students must develop a relation between steps in the proving process by way of identifying steps – that is, a step is preceded by other steps and in order to deduct proposition generated in one step each. Thirdly, students have to find erroneous in proving process given. Fourth, students must able to evaluate validity a written proof given to them. Finally, it is a must that students able to differentiate and able to justify given problems by empirical explanation, proving by examples, and also proving by axiomatic system.

Blue print for teaching-learning process that can be used to assess students' proof construction ability consists of giving help by, first, students have to be able to outline a written and proof process. Secondly, students have to be able to identify mathematical knowledge needed in order to prove of proving a problem or a conjecture. Thirdly, students must be able to complete steps that are left as blanks in a proving process. Fourth, students must be able to provide clues to others to construct a proof. Fifth,

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students must be able to adapt a proving process into new situation where some of elements has been left blank or has been changed or some of assumptions have been modified. Finally, students must be able to find alternative proofs from a given proof.

IV. POLYA METHOD TO DEVELOP THE ABILTIY TO WRITE MATHEMATICAL PROOF

A well known book for mathematics community and mathematics education community is *How to Solve It* - a work of George Polya (1973). In this book, Polya provided direction to solve mathematical problems. This direction has been used by generations of mathematicians, and is hoped that we are also going to make use of it now. This is the direction and will be make it appropriate in order to fit the goals of paper in your hand.

First. "Understanding the problem." It is easy to say but difficult to do, of course. What is going to do? Make sure that all words in the problem is recognized and comprehended. The usage of words in the book at your hand or other books needs to be anticipated. Consider the statement available in order to give picture in a careful manner of what is known and what is going to do. When any figure is needed, then draw it. Should we need some examples to make the problem clearer? Check all required. Do we need to show the truth of the statement? Is it true or is it wrong? When what is going to do has been recognized, then next steps will follow smoothly.

Second. "Devising a plan." How do you work your problem? Here, understanding of what is going to do next needs to be increased because previous steps have been done before. Do we have any experiences of seeing or doing like this before? If you do not have any kind of these experiences before, or you have never read some kinds of such a problem from other books, or you have not been doing such a problematic, then what you are going to do is that you have to do all mentioned above. Notice all texts and recall your problem in your mind, read carefully all notes related to the problems, notice exercises or theorems you already know that looks like to the problem in hand. It might be possible that some ideas in proofs or in theorems needs to be applied, or some previous problems can be used as examples and examples how to do the problem. Mathematics is being developed by its own as well as problems in the book can be very

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sure they are built from their own. If you are in working for problem solving is truly stuck, try to answer simple problem first, by questioning similar question. When a method is already been suggested to be used, then try it.

Third. "Carrying out the plan." Solve the problem. Notice the answer you write. Is every sentence correct? Sometimes, it is difficult to observe the errors of the answer you find out. Left it for a while and then recheck it later. Is every sentence still looked alright?

Fourth. "Looking Back." Polya suggested to check the result and to check its argument. If you are allowed to do so, ask your teacher to check your answer, it is a good way to give your teacher to check the truth of your answer. The answer can also be delivered to your friend to check. If he cannot understand what you have written, then ask him to read aloud so that you can understand and recognize the errors. When your written answer is believed to be true, then make it perfect by writing in tidy way.

V. AN EXAMPLE OF POLYA METHOD.

Prove the following theorem by using Polya Method.

Theorem:

Let N be a normal subgroup of a group G. If $a \equiv b \pmod N$, *and* $c \equiv d \pmod N$, *then* $ac \equiv bd \pmod N$.

First. "*Understanding the problem.*" All right, before we begin, let us identify the hypothesis and the conclusion. What are they? The hypothesis is $a \equiv b \pmod N$, and $c \equiv d \pmod N$, these are what we are going to start with. What does it means by $a \equiv b \pmod N$, and $c \equiv d \pmod N$? Ok, we do not know yet, so let us thinking of it. We start with $a \equiv b \pmod N$, and so that we will understand and recognize $c \equiv d \pmod N$; the first thing is that $a \equiv b \pmod N$ means that for a, and b elements of group G, then a is congruent to b provided that ab^{-1} is an element of N. Also, note $c \equiv d \pmod N$, this has similar meaning to those already explained before, that is that for c, and d elements of group G, then c is congruent to d provided that cd^{-1} is an element of N. We have been defining everything, we have understood the problem and we feel sure – ready to start solving the problem. Are there any things else needed that is not being well understood? What is N? It is a normal subgroup of a group G. Then N must have this property

$aN = Na$ for every a an element of G. What is the conclusion that we wanted to? The conclusion is that $ac \equiv bd \pmod{N}$, and we know the meaning of $ac \equiv bd \pmod{N}$, because we have already know the meaning of $a \equiv b \pmod{N}$.

Second. "*Devising a plan.*" We have already known it, notation that we are using is, a, b, c, d, all are elements of G, however the result of the operation of them are elements of N.

We want to proof that if ab^{-1} is an element of N, and cd^{-1} is another element of N, then $(ac)(bd)^{-1}$ is an elements of N. By observing of what we already know and what we are going to show then this is about time to do the plan.

Proof

Third. *"Carrying out the plan."* Given that N is normal subgroup of group G. Let m, and n are elements of N, such that $ab^{-1} = m$ and $cd^{-1} = n$. Since N is a subgroup and we want to prove that $ac \equiv bd \pmod{N}$, and we know the meaning that $ac \equiv bd \pmod{N}$ is meant to be $(ac)(bd)^{-1}$ an element of N. Next, $(ac)(bd)^{-1} = acd^{-1}b^{-1} = anb^{-1}$. An element an is a member of left cosset of aN. Since N is a normal subgroup then $aN = Na$. So, $an = n_1a$, for some n_1 element of N. As a consequent, $(ac)(bd)^{-1} = acd^{-1}b^{-1} = anb^{-1} = n_1ab^{-1} = n_1m$ is an element of N; and it seems that $(ac)(bd)^{-1}$ is an element of N. This shows that ac is congruent to bd modulo N; and we write it by $ac \equiv bd \pmod{N}$.

Fourth. "*Looking back.*" Let us admire the work proving above. This is such a beautiful work. The sentences are written in a good manner and complete, points, and every symbol is written in clear definition and very careful. We were saying where wanted to start, i.e., what are the assumptions, and we ended with a conclusion that we are going to find. The end of this work of proving is usually closed by box or it is written by Q.E.D. *(quod erat demonstrandum*) means that it has been demonstrated.

VI. CLOSING REMARK

Polya method is hopefully can help in improving students' ability to write proof. The sequence of the process of proving is really easy to follow so the possibility of increment of writing proof ability is going to improve. In order to study the development of students proving ability, then writing journal of mathematical proof will also be useful to observe and follow the process of improvement of students' ability.

VII. **REFERENCES**

Depdiknas. 2006. Peraturan Menteri Pendidikan Nomor 22, 23, 24 Tahun 2006 tentang Standar Isi dan Standar Kompetensi Lulusan Pendidikan Dasar dan Menengah. Jakarta: Depdiknas.

Hungerford, T. W. (...). *Abstract Algebra*. Saunders College Publising. Philadelphia.

Jones, K. 1997. Student teachers' conceptions of mathematical proof. *Mathematical Education Review,* Vol 9, pp. 21-32. Diambil pada tanggal 30 September 2010 dari http://eprint.soton.ac.uk/41245/1/Jones_student_teachers_conceptions_proof_MER_1997.pdf

Jones, K. 2001. Providing a Foundation for Deductive Reasoning: Students' Interpretations when Using Dinamic Geometry Software and Their Evolving Mathematical Explanations. *Educational Studies in Mathematics* 44: 55-85. Kluwer Academic Publishers. Printed in the Netherlands.

Krantz, S. G. (2007). *The History and Concept of Mathematical Proof*. Diambil pada tanggal 11 Januari 2011 dari www.math.wustl.edu/~sk/eolss.pdf

Polya, G. 1973. *How to Solve It. A New Aspect of Mathematical Method*. Penerbit Princeton University Press, Princeton, New Jersey.

Smith, J. C. 2006. A sense-making approach to proof: strategies of students in traditional and problem-based number theory courses. *Journal of Mathematical Behavior* 25 2006 73-90.

Stylianides, A. J. (2007). Proof and proving in schools mathematics. *Journal for Research in Mathematics Education*. Vol. 38 (3), 289-321.

Weber, K. (2001). Student difficulty in constructing proofs: The need for strategic knowledge. *Educational Studies in Mathematics*, 48, 101 – 119. Diambil pada tanggal 2 Februari 2011 dari http://www.springerlink.com/content/jq2cdlg17au1lr90/