
University of Southern Queensland

Faculty of Engineering & Surveying

**PHD DISSERTATION**

# WIRELESS NETWORK CODING FOR MULTI-HOP RELAY CHANNELS

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# Abstract

Future wireless communication systems are required to meet growing demands for high spectral efficiency, low energy consumption and high mobility. The advent of wireless network coding (WNC) has offered a new opportunity to improve network throughput and transmission reliability by exploiting interference in intermediate relays. Combined with network coding and self-information cancelation, WNC for two-way relay channels (TWRCs) has come to the forefront.

This dissertation focuses on exploiting WNC in multi-hop two-way relay channels (MH-TRCs). Particularly, a multi-hop wireless network coding (MH-WNC) scheme is designed for the generalized $L$-node $K$-message MH-TRC. Theoretical studies on the network throughput and performance bounds achieved by the MH-WNC scheme with different relaying strategies (i.e., amplify-and-forward (AF) and compute-and-forward (CPF)) are carried out. Furthermore, by introducing different numbers of transmission time intervals into the MH-WNC, a multiple-time-interval (Multi-TI) MH-WNC is proposed to determine an optimal MH-WNC which can achieve the best outage performance for all-scale MH-TRCs. Finally, this study extends the research on WNC one step forward from two-user networks to multi-user networks. An extended CPF joint with a dominated solution for maximizing the overall computation rate is proposed for the multi-way relay channel (mRC) in the last chapter.

The contributions of this dissertation are multifold. First, the proposed MH-WNC scheme with fixed two transmission time intervals can achieve a significantly improved network throughput compared to the non-network coding (Non-NC) scheme in the generalized $L$-node $K$-message MH-TRC. Theoretical results are derived for both multi-hop analog network coding (MH-ANC) and multi-hop compute-and-forward (MH-CPF). Moreover, both theoretical and numerical re-

sults demonstrate that the two MH-WNC schemes can be applied to different scale MH-TRCs to achieve a better outage performance compared to the conventional Non-NC scheme (i.e., MH-ANC for the non-regenerative MH-TRC with a small number of nodes, and MH-CPF for the regenerative MH-TRC with a large number of nodes.). Furthermore, a Multi-TI MH-WNC scheme is generalized with a special binary-tree model and characteristic matrix. The determined optimal MH-WNC scheme is able to provide the best outage performance and outperform the Non-NC scheme in all scale MH-TRCs. Last but not least, this dissertation provides a preliminary investigation of WNC in mRCs. The proposed dominated solution for maximizing the overall computation rate can ensure that all the nodes in the mRC successfully recover their required messages. Moreover, the extended CPF strategy is proven superior to Non-NC in the mRC with a small number of users.

# Certification of Dissertation

I certify that the ideas, designs and experimental work, results, analyses and conclusions set out in this dissertation are entirely my own effort, except where otherwise indicated and acknowledged.
I further certify that the work is original and has not been previously submitted for assessment in any other course or institution, except where specifically stated.

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# List of publications

The following publications were produced during the period of candidature:

[1] Gengkun Wang, Wei Xiang, Jinhong Yuan, and Tao Huang, "Outage performance of Analog Network Coding in Generalized Two-Way Multi-Hop Networks," in *Proc. IEEE Wireless Communications and Networking Conference*, Cancun, Mexico, Mar. 2011, pp. 759-764.

This paper was awarded the Best Paper Award by IEEE WCNC 2011, which is one of world's three most prestigious conferences in communication technologies.

[2] Gengkun Wang, Wei Xiang, Jinhong Yuan, and Tao Huang, "Outage Analysis of Non-Regenerative Analog Network Coding for Two-Way Multi-Hop Networks," *IEEE Commun. Lett.*, vol. 15, pp. 662 - 664, May 2011.

The work in the paper is presented in Chapter 4.

[3] Gengkun Wang, Wei Xiang, and Jinhong Yuan, "Multi-Hop Compute-and-Forward for Generalized Two-Way Relay Channels," accepted by *Trans. Emerging Tel. Tech. (formerly known as European Transactions on Telecommunications (ETT)).*

The work in the paper is presented in Chapter 5.

[4] Gengkun Wang, Wei Xiang, and Jinhong Yuan, "Outage Performance for Compute-and-Forward in Generalized Multi-Way Relay Channels," *IEEE Commun. Lett.*, vol. 16, pp. 2099 - 2102, Dec 2012.

The work in the paper is presented in Chapter 7.

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# Acronyms & Abbreviations

| | |
|---|---|
| AF | Amplify-and-forward |
| ANC | Analog network coding |
| AWGN | Additive white Gaussian noise |
| BER | Bit-error-rate |
| BPSK | Binary phase-shift keying |
| BS | Base station |
| BSC | Binary symmetric channel |
| CDMA | Code division multiple access |
| CDNC | Channel decoding network coding |
| CF | Compress-and-forward |
| CPF | Compute-and-forward |
| CSI | Channel state information |
| DF | Decode-and-forward |
| EF | Estimate-and-forward |
| FDMA | Frequency division multiple access |
| FFNC | Finite field network coding |
| GF | Galois field |
| LDPC | Low-density parity-check |
| LLL | Lenstra-Lenstra-Lovsz |
| LNC | Linear network coding |
| LTE | Long-term evolution |
| MAC | Multiple-access channel |
| MH-ANC | Multi-hop analog network coding |
| MH-CPF | Multi-hop compute-and-forward |

| | |
|---|---|
| MH-TRC | Multi-hop two-way relay channel |
| MH-WNC | Multi-hop wireless network coding |
| MIMO | Multi-input multi-output |
| ML | Maximum likelihood |
| mRC | Multi-way relay channel |
| MS | Mobile station |
| NC | Network coding |
| Non-NC | Non-network coding |
| P2P | Peer-to-peer |
| PNC | Physical-layer network coding |
| QPSK | Quadrature phase-shift keying |
| RA | Repeat accumulate |
| RLNC | Random linear network coding |
| SNR | Signal-noise-ratio |
| SVP | Shortest vector problem |
| TDMA | Time division multiple access |
| TI | Transmission time interval |
| TWRC | Two-way relay channel |
| WNC | Wireless network coding |
| XOR | Exclusive or |