UNIVERSITY OF SOUTHERN QUEENSLAND

RADIAL-BASIS-FUNCTION CALCULATIONS OF HEAT AND VISCOUS FLOWS IN MULTIPLY-CONNECTED DOMAINS

A dissertation submitted by

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Dedication

To my family.

Certification of Dissertation

I certify that the idea, experimental work, results and analyses, software and conclusions reported in this dissertation are entirely my own effort, except where otherwise acknowledged. I also certify that the work is original and has not been previously submitted for any other award.

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ENDORSEMENT

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Finally, I would like to dedicate this work to my parents. I am greatly indebted to my family for much unconditional support, understanding and love over the years and for endlessly encouraging me in academic pursuits.

Notes to Readers

To facilitate the reading of this thesis, a number of files are included on the attached CD to provide animation of some numerical results in this thesis. The contents of the CD include:

1. thesis.pdf: An electronic version of this thesis;

2. Chapter3-Circular-Circular-Annuli-velocity.wmv: An animation showing the evolution of velocity field of the buoyancy flow in a concentric circular-circular annulus using a Cartesian grid 36×36 ($Pr = 0.71, Ra = 10^4$) (Section 3.4.3, Chapter 3);

3. Chapter3-Square-Circular-Annuli-velocity.wmv: An animation showing the evolution of velocity field of the buoyancy flow in a concentric square-circular annulus using a Cartesian grid 36×36 ($Pr = 0.71, Ra = 10^5$) (Section 3.4.4, Chapter 3);

4. Chapter4-Rotating-cylinder.wmv: An animation showing the evolution of the flow between a rotating circular cylinder and a fixed square cylinder using a Cartesian grid 26×26 (Section 4.5.1, Chapter 4).

Abstract

This PhD research project is concerned with the development of accurate and efficient numerical methods, which are based on one-dimensional integrated radial basis function networks (1D-IRBFNs), point collocation and Cartesian grids, for the numerical simulation of heat and viscous flows in multiply-connected domains, and their applications to the numerical prediction of the material properties of suspensions (i.e. particulate fluids). In the proposed techniques, the employment of 1D-IRBFNs, where the RBFN approximations on each grid line are constructed through integration, provides a powerful means of representing the field variables, while the use of Cartesian grids and point collocation provides an efficient way to discretise the governing equations defined on complicated domains.

Firstly, 1D-IRBFN-based methods are developed for the simulation of heat transfer problems governed by Poisson equations in multiply-connected domains. Derivative boundary conditions are imposed in an exact manner with the help of the integration constants. Secondly, 1D-IRBFN based methods are further developed for the discretisation of the stream-function - vorticity formulation and the stream-function formulation governing the motion of a Newtonian fluid in multiply-connected domains. For the stream-function - vorticity formulation, a novel formula for obtaining a computational vorticity boundary condition on a curved boundary is proposed and successfully verified. For the stream-function formulation, double boundary conditions are implemented without the need to use external points or to reduce the number of interior nodes for collocating the governing equations. Processes of implementing cross derivatives and deriving the stream-function values on separate boundaries are presented in detail. Thirdly, for a more efficient discretisation, 1D-IRBFNs are incorporated into the domain embedding technique. The multiply-connected domain is transformed into a simply-connected domain, which is more suitable for problems with several unconnected interior moving boundaries. Finally, 1D-IRBFN-based methods are applied to predict the bulk properties of particulate suspensions under simple shear conditions.

All simulated results using Cartesian grids of relatively coarse density agree well with other numerical results available in the literature, which indicates that the proposed discretisation schemes are useful numerical techniques for the analysis of heat and viscous flows in multiply-connected domains.

Papers Resulting from the Research

Journal Papers

1. N. Mai-Duy, K. Le-Cao and T. Tran-Cong (2008) A Cartesian grid technique based on one-dimensional integrated radial basis function networks for natural convection in concentric annuli, *International Journal for Numerical Methods in Fluids*, 57, p. 1709–1730.

2. K. Le-Cao, N. Mai Duy and T. Tran-Cong (2009) An effective integrated-RBFN Cartesian-grid discretisation to the stream function-vorticity-temperature formulation in non-rectangular domains, *Numerical Heat Transfer, Part B*, 55, p. 480–502.

3. K. Le-Cao, N. Mai-Duy, C.-D. Tran and T. Tran-Cong (2010) Numerical study of stream-function formulation governing flows in multiply-connected domains by integrated RBFs and Cartesian grids, *Computer & Fluids Journal*, 44(1), p. 32–42.

4. K. Le-Cao, N. Mai-Duy, C.-D. Tran and T. Tran-Cong (2010) Towards the analysis of shear suspension flows using radial basis functions, *CMES: Computer Modeling in Engineering & Sciences*, 67(3), p. 265–294.

Conference Papers

1. K. Le-Cao, N. Mai-Duy and T. Tran-Cong (2007) Radial basis function calculations of buoyancy-driven flow in concentric and eccentric annuli. In P. Jacobs, T. McIntyre, M. Cleary, D. Buttsworth, D. Mee, R. Clements, R. Morgan and C. Lemckert (eds). *The 16th Australasian Fluid Mechanics Conference*, Gold Coast, QLD, Australia, 3-7 December. *Proceedings of The 16th Australasian Fluid Mechanics Conference* (CD), p. 659–666. The University of Queensland (ISBN 978-1-864998-94-8).

2. K. Le-Cao, C.-D. Tran, N. Mai-Duy and T. Tran-Cong (2009) Direct simulation of two-dimensional particulate shear flows using radial basis functions. In R.P. Jagadeeshan, W. Li, A. Jabbarzadeh, H. See, R. Tanner (Scientific Committee). *The 5th Australian-Korean Rheology Conference*, Sydney, NSW, Australia, 1-4/Nov/2009. Abstract Book, p. 19.

3. K. Le-Cao, N. Mai-Duy, C.-D. Tran and T. Tran-Cong (2010) A new integrated-RBF-based domain-embedding scheme for solving fluid flow problems. In N. Khalili, S. Valliappan, Q. Li and A. Russell (eds). *The 9th World Congress on Computational Mechanics* and *4th Asian Pacific Congress on Computational Mechanics (WCCM/APCOM 2010)*, Sydney, Australia, 19-23/Jul/2010. *IOP Conference Series: Materials Science and Engineering*, Vol. 10, Paper No. 012021, 10 pages. IOP Publishing (ISSN 1757-899X (Online) and ISSN 1757-8981 (Print)).

4. K. Le-Cao, N. Mai-Duy, C.-D. Tran and T. Tran-Cong (2010) Integrated-RBF calculations for direct simulation of shear suspension flows. *International Conference on Computational & Experimental Engineering and Sciences(ICCES MM'10)*, Busan, South Korea, 17-21/Aug/2010. ICCES journal. Tech Science Press (ISSN: 1933-2815 (online)) (accepted, 30/Nov/2010)

5. D. Ho-Minh, K. Le-Cao, N. Mai-Duy and T. Tran-Cong (2010) Simulation

of fluid flows at high Reynolds numbers using radial basis function networks. In G.D. Mallinson and J.E. Cater (eds). *17th Australasian Fluid Mechanics Conference*, Auckland, New Zealand, 5-9/Dec/2010. *Proceedings of 17th Australasian Fluid Mechanics Conference*, Paper No 139, 4 pages. The University of Auckland (ISBN: 978-0-86869-129-9).

Contents

Acronyms & Abbreviations

1D-IRBFN	One-Dimensional Indirect/Integrated Radial Basis Function Network
BEM	Boundary Element Method
CFD	Computational Fluid Dynamics
DNS	Direct Numerical Simulations
DRBFN	Direct/Differentiated Radial Basis Function Network
FDM	Finite Difference Method
FEM	Finite Element Method
FVM	Finite Volume Method
IRBFN	Indirect/Integrated Radial Basis Function Network
MQ	MultiQuadric
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
SVD	Singular Value Decomposition

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