

Ponder this

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The focus of this set of problems is Euclidean geometry. Since the ancient times, many magnificent results and theorems have been discovered and proved in this area of mathematics. Nevertheless, Euclidean geometry continues to serve as a source of wonderful properties and interesting problems. The geometry problems here were all posed by Igor Sharygin, whose talent in geometry problem posing is well known to mathematicians all over the world.

Problem set 3

- 1. Let *M* be a point in the interior of quadrilateral *ABCD* such that *AMB* and *CMD* are isosceles triangles with the angle at vertex *M* equal to 120° . Prove that there exists a point *N* such that triangles *BNC* and *DNA* are equilateral.
- 2. Let *ABCD* be a cyclic quadrilateral such that CD = AD + BC. Prove that angle bisectors at vertices *A* and *B* meet at the side *CD*.
- 3. Let *ABCD* be a cyclic quadrilateral with perpendicular diagonals. Prove that mid-points of all sides and the perpendiculars dropped from the diagonal intersection point to the sides lie on a circle.
- 4. Let *AC* be a diameter of a circle. Point *E* lies on *AC*. Draw chord *BD* through the point *E* in a way such that the area of quadrilateral *ABCD* will have the greatest value.
- 5. An angle bisector at vertex A of the parallelogram ABCD meets the straight lines BC and CD at the points K and L respectively. Prove that the centre of a circle which contains points C, K and L lies on a circle which contains points B, C and D.

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