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PREDICTION DISTRIBUTION OF GENERALIZED GEOMETRIC SERIES DISTRIBUTION AND ITS DIFFERENT FORMS

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ABSTRACT

The prediction distribution of generalized geometric series distribution (GGSD) and of its truncated and size-biased forms is derived and studied under the non-informative and beta prior distributions. The prediction distributions for all the models are beta distribution, but the parameters of the prediction distributions depend on the choice of the prior distribution as well as the model under consideration.

KEY WORDS

Prediction distribution, prior and posterior distributions, beta distribution, size-biased and zero-truncated generalized geometric series distribution.

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1. INTRODUCTION

The predictive inference is directed towards inference involving the observables, rather than the parameters. The predictive method had been the most popular statistical tool before the diversion of interest in the inferences on parameters of the models. Predictive inference uses the realized responses from the *performed experiment* to make inference about the behavior of the unobserved future responses of the *future experiment* (cf. Aitchison and Dunsmore (1975, p.1)). The outcomes of the two experiments are connected through the same structure of the model and indexed by the common set of parameters. The prediction distribution forms the basis of all predictive inference. For details on the predictive inference methods and wide range of applications of prediction distribution interested readers may refer to Aitchison and Sculthorpe (1965), Fraser and Haq (1969), Guttman (1970), Haq and Khan (1990), Geisser (1993), and Khan (2002, 2006).

In this paper, Bayesian method is used to derive the prediction distribution of future responses from the generalized geometric series distribution (GGSD), conditional on the observed responses. The non-informative and beta prior distributions of the parameter of the GGSD are used to derive the posterior distribution of the parameters. Combining the distribution of the future responses and the posterior distribution the prediction distribution is obtained. Two special cases of the GGSD, namely size-biased and zero-truncated generalized geometric series distributions are considered. It is interesting to note that the prediction distribution follows the beta (or scaled) distribution with varying parameters for different forms of the GGSD, and under different priors. Prediction distributions for a single future response as well as that of a set of future responses are derived.

The probability function of generalized geometric series distribution (GGSD) was proposed by Mishra (1982) by using the results of the lattice path analysis as:

$$P(X=x)=\frac{1}{1+\beta x}\binom{1+\beta x}{x}\alpha^{x}(1-\alpha)^{1+\beta x-x}\ ;\ 0<\alpha<1,\ |\alpha\beta|<1,\ x=0,1,2,\ldots$$

and 0 otherwise. (1.1)

It can be seen that at β=1 , the model (1.1) reduces to simple geometric distribution and is a particular case of Jain and Consul's (1971) generalized negative binomial distribution in the same way as the geometric distribution is a particular case of the negative binomial distribution.

The first four moments of GGSD (1.1) are given as

$$\mu_1'=\frac{\alpha}{1-\alpha\beta}, \tag{1.2}$$

$$\mu_2=\frac{\alpha(1-\alpha)}{(1-\alpha\beta)^3}, \tag{1.3}$$

$$\mu_3=\frac{\alpha(1-\alpha)}{(1-\alpha\beta)^5}\left[1-2\alpha+\alpha\beta(2-\alpha)\right], \tag{1.4}$$

$$\mu_4=\frac{3\alpha^2(1-\alpha)^2}{(1-\alpha\beta)^6}+\frac{\alpha(1-\alpha)\left[1-60+6\alpha^2+2\alpha\beta(4-9\alpha+4\alpha^2)+\alpha^2\beta^2(6-6\alpha+\alpha^2)\right]}{(1-\alpha\beta)^7}. \tag{1.5}$$

Various interesting properties and estimation of the parameters of the model in (1.1) have been discussed by Mishra (1982), Singh (1989), Mishra and Singh (1982), Hassan (1995) and Hassan et al. (2002, 2007 and 2008). They found that this distribution provides much better fit to many real life data than the geometric distribution and various compound geometric distributions proposed earlier by many authors. A brief list of authors and their works can be seen in Johnson and Kotz (1969), Johnson et al. (1992) and Consul and Famoye (2006).

In this paper, the prediction distribution of generalized geometric series distribution (GGSD) and of its truncated and size-biased form is defined and studied under non-informative and beta prior distributions.

1.1 Truncated Generalized Geometric Series Distribution (TGGSD)

A discrete random variable is said to have truncated generalized geometric distribution of its probability mass function is given by

$$P_1(X=x)=\frac{1}{1+\beta x}\binom{1+\beta x}{x}\alpha^{x-1}(1-\alpha)^{1+\beta x-x};0<\alpha<1,\ |\alpha\beta|<1,\ \mathrm{x}=1,2... \qquad (1.6)$$

The moments of zero truncated GGSD in (1.6) are obtained as

$$\mu_1' = \frac{1}{(1-\alpha\beta)}, \qquad (1.7)$$

$$\mu_2 = \frac{\alpha\beta(1-\alpha)}{(1-\alpha\beta)^3}, \qquad (1.8)$$

$$\mu_3 = \frac{\alpha\beta(1-\alpha)}{(1-\alpha\beta)^5}\left[1-2\alpha+\alpha\beta(2-\alpha)\right]. \qquad (1.9)$$

1.2 Size-Biased Generalized Geometric Series Distribution (SBGGSD)

A size biased GGSD is obtained by taking the weight of the GGSD as X. So from (1.1) and (1.2), the probability function of the size biased GGSD is found to be

$$P_2(X=x)=(1-\alpha\beta)\binom{\beta x}{x-1}\alpha^{x-1}(1-\alpha)^{1+\beta x-x};\ 0<\alpha<1,\ |\alpha\beta|\prec 1,\ x=1,2,\cdots \qquad (1.10)$$

When β = 1, the probability function of the SBGGSD in (1.10) reduces to size-biased GSD with probability function as

$$P_3(X=x)=\alpha^{x-1}(1-\alpha)^2;\ 0<\alpha<1,\ x=1,2,\cdots \qquad (1.11)$$

The moments of the SBGGSD are obtained as

$$\mu_1' = \frac{(1-\alpha^2\beta)}{(1-\alpha\beta)^2}, \qquad (1.12)$$

$$\mu_2 = \frac{1}{(1-\alpha\beta)^4}\left[2\alpha^3\beta-4\alpha^2\beta+2\alpha\beta\right], \qquad (1.13)$$

$$\begin{aligned}\mu_3' = \frac{1}{(1-\alpha\beta)^6}\Big[&\alpha^3(1-\alpha\beta)^3+6\alpha^2(1-\alpha)(1-\alpha\beta)^2)\\ &+\alpha(1-\alpha)\Big[7-11\alpha-4\alpha\beta(2-\alpha)(1-\alpha\beta)\\ &+\left\{1-6\alpha+6\alpha^2+2\alpha\beta(4-9\alpha+4\alpha^2)+\alpha^2\beta^2(6-6\alpha+\alpha^2)\right\}\Big]\Big].\end{aligned} \qquad (1.14)$$

Other higher moments of the SBGGSD can be obtained, if need.

1.3 The Prior Distributions

Now we define the two prior distributions for the parameter α of the GGSD. Assuming that the parameter α follows a non-informative prior distribution we define the prior probability function as

$$h(\alpha) \propto \frac{1}{\alpha}. \tag{1.15}$$

Similarly, for the beta prior distribution of the parameter α, the probability function is defined as

$$h(\alpha) \propto \alpha^{u-1}(1-\alpha)^{v-1}, \tag{1.16}$$

where u and v are the two hyper-parameters of the beta prior distribution. Prediction distributions of the GGSD and its different forms are derived using the above two prior distributions.

In the next Section we provide the prior distributions and prediction distribution of the GGSD model. The prediction distribution for the truncated GGSD model is covered in Section 3. Section 4 derives the prediction distribution for the size biased GGSD model.

2. PREDICTION DISTRIBUTION OF GGSD

Let $x_1, x_2, ..., x_n$ be a random sample of size n from the GGSD defined in (1.1). The joint probability function of the realized sample responses is given by

$$p(x_1,...,x_n \mid \alpha) = \prod\left[\frac{1}{1+\beta x_i}\binom{1+\beta x_i}{x_i}\alpha^{x_i}\left(1-\alpha\right)^{1+\beta x_i - x_i}\right],$$

where the symbol $\prod$ stands for the product of all the forthcoming terms for $i = 1, 2, ..., n$.

Let x^f be a future response from the GGSD (1.1). Let the probability function of x^f be $h(x^f \mid \alpha)$). So the prediction distribution of x^f, conditional on the realised responses $x_1, x_2, ..., x_n$, is defined as

$$g\left(x^f \mid x_1, x_2, ..., x_n\right) \propto h(x^f \mid \alpha)\, p\left(\alpha \mid x_1, x_2, ..., x_n\right), \tag{2.1}$$

where $p\left(\alpha \mid x_1, x_2, ..., x_n\right)$ is the posterior probability function of α.

2.1 Prediction Distribution of GGSD Under Non-Informative Prior:

Under non-informative prior distribution, the prediction distribution of x^f becomes

$$g\left(x^f \mid \underline{x}\right) \propto \left[\alpha^{x^f}\left(1-\alpha\right)^{1+\beta x^f - x^f}\alpha^{\sum_{i=1}^{n} x_i}\left(1-\alpha\right)^{n+\beta\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i}\right]\frac{1}{\alpha}$$

$$= \alpha^{x^f + y - 1}\left(1-\alpha\right)^{1+\beta x^f - x^f + n + \beta y - y}, \tag{2.2}$$

where $y = \sum_{i=1}^{n} x_i$ and $\underline{x} = (x_1, x_2, ..., x_n)$.

Clearly

$$x^f \mid \underline{x} \sim B\Big(x^f + y,\ \ \mathrm{n} + \beta\big(\mathrm{x}^{\mathrm{f}} + y\big) - \big(x^f + y\big) + 2\ \Big), \tag{2.3}$$

that is, a beta distribution with appropriate arguments. The mean and variance of the prediction distribution can easily be obtained by using the properties of the beta distribution.

Thus

$$\text{Mean} = \frac{a}{a+b}, \tag{2.4}$$

$$\text{Variance} = \frac{ab}{(a+b)^2 (a+b+1)}, \tag{2.5}$$

where $a = x^f + y$ and $b = \mathrm{n} + \beta\big(\mathrm{x}^{\mathrm{f}} + y\big) - \big(x^f + y\big) + 2$.

Now let $z^f = \big(x_1{}^f, x_2{}^f, ..., x_m{}^f\big)$ be a set of m independent future responses from the GGSD in (1.1). Then for the non-informative prior distribution, the prediction distribution of z^f is obtained as

$$g\big(z^f \mid \underline{x}\big) \propto \alpha^{z + y - 1} (1-\alpha)^{n+m+\beta(z + y)-(z + y)}, \tag{2.6}$$

where $z = \sum_{j=1}^{m} x_j{}^f$.

So in the conventional notation, the prediction distribution can be written as

$$z^f \mid \underline{x} \sim B\big(z + y,\ \ \mathrm{n} + m + \beta(z+y) - (z+y) + 1\ \big). \tag{2.7}$$

For this distribution the moments are given as

$$\text{Mean} = \frac{a'}{a'+b'}, \tag{2.8}$$

$$\text{Variance} = \frac{a'b'}{(a'+b')^2 (a'+b'+1)}, \tag{2.9}$$

where $a' = z + y$ and $b' = \mathrm{n} + m + \beta(z+y) - (z+y) + 1$.

2.2 Prediction Distribution of GGSD Under Beta Prior

Using the posterior distribution under the beta prior B (u, v), the prediction distribution of a single future response x^f is given by

$$g\left(x^f \mid \underline{x}\right) \propto \alpha^{x^f+y+u-1}\left(1-\alpha\right)^{1+v+\beta x^f-x^f+n+\beta y-y}. \tag{2.10}$$

Therefore $x^f \mid \underline{x} \sim B\left(x^f+y+u,\ \mathrm{n}+\beta\left(x^f+y\right)-\left(x^f+y\right)+v+2\right)$. The mean and variance are

$$\text{Mean} = \frac{a''}{a''+b''}, \tag{2.11}$$

$$\text{Variance} = \frac{a''b''}{\left(a''+b''\right)^2\left(a''+b''+1\right)}, \tag{2.12}$$

where $a''=x^f+y+u$ and $b''=\mathrm{n}+\beta\left(\mathrm{x}^{\mathrm{f}}+y\right)-\left(x^f+y\right)+v+2$.

Similarly, the prediction distribution of a set of m future responses, $z^f=(x_1{}^f, x_2{}^f, \ldots, x_m{}^f)$ from the model under the beta prior distribution is obtained as

$$g\left(z^f \mid \underline{x}\right) \propto \alpha^{z+y+u-1}\left(1-\alpha\right)^{n+m+\beta(z+y)-(z+y)+v-1}, \tag{2.13}$$

where $z=\sum_{j=1}^{m} x_j{}^f$.

Thus $z^f \mid \underline{x} \sim B\left(z+y+u,\ \mathrm{n}+m+\beta\left(\mathrm{z}+y\right)-\left(z+y\right)+v\right)$. So the prediction distribution under the beta prior is a beta distribution. Unlike the prediction distribution under the non-informative prior, here the prediction distribution depends on the hyper-parameter of the prior distribution.

The mean and variance of the distribution are

$$\text{Mean} = \frac{a'''}{a'''+b'''}, \tag{2.14}$$

$$\text{Variance} = \frac{a'''b'''}{\left(a'''+b'''\right)^2\left(a'''+b'''+1\right)}, \tag{2.15}$$

where $a'''=z+y+u$ and $b'''=\mathrm{n}+m+\beta\left(\mathrm{z}+y\right)-\left(z+y\right)+v$.

It is interesting to note that due to the use of different priors, the shape of the prediction distribution changes; although in both the cases the underlying distribution is the beta distribution.

3. PREDICTION DISTRIBUTION OF TRUNCATED GGSD

Let x^f be a future response from the truncated generalized geometric series distribution (TGGSD) as defined in (1.6). Let the probability function of x^f

be $h(x^f \mid \alpha)$). So the prediction distribution of x^f , conditional on the observed responses $x_1, x_2, ..., x_n$, is defined as

$$g\left(x^f \mid x_1, x_2, ..., x_n\right) \propto h(x^f \mid \alpha) p\left(\alpha \mid x_1, x_2, ..., x_n\right), \tag{3.1}$$

where $p\left(\alpha \mid x_1, x_2, ..., x_n\right)$ is the posterior distribution of α.

3.1 Prediction Distribution of TGGSD Under Non-Informative Prior

Under non-informative prior distribution, the prediction distribution of x^f becomes

$$g\left(x^f \mid \underline{x}\right) \propto \left[\alpha^{x^f - 1}(1-\alpha)^{1+\beta x^f - x^f} \alpha^{\sum_{i=1}^{n} x_i - n} (1-\alpha)^{n+\beta\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i}\right] \frac{1}{\alpha}$$

$$= \alpha^{x^f + y - n - 2}(1-\alpha)^{1+n+\beta(x^f+y)-(x^f+y)}, \tag{3.2}$$

where $y = \sum_{i=1}^{n} x_i$ and $\underline{x} = (x_1, x_2, ..., x_n)$.

Clearly

$$x^f \mid \underline{x} \sim B\left(x^f + y - n - 1,\ \mathrm{n} + \beta\left(\mathrm{x}^{\mathrm{f}} + y\right) - \left(x^f + y\right) + 2\right). \tag{3.3}$$

Thus

$$\text{Mean} = \frac{\tilde{a}}{\tilde{a} + b}, \tag{3.4}$$

$$\text{Variance} = \frac{\tilde{a}b}{(\tilde{a}+b)^2(\tilde{a}+b+1)}, \tag{3.5}$$

where $\tilde{a} = x^f + y - n$ and $b = \mathrm{n} + \beta\left(\mathrm{x}^{\mathrm{f}} + y\right) - \left(x^f + y\right) + 2$.

Now let $z^f = (x_1, x_{2,} ..., x_m)$ be a set of m future responses from the TGGSD. Then for the non-informative prior distribution, the prediction distribution of z^f is found to be

$$g\left(z^f \mid \underline{x}\right) \propto \alpha^{z + y - (n+m) - 1}(1-\alpha)^{n+m+\beta(z+y)-(z+y)}, \tag{3.6}$$

where $z = \sum_{j=1}^{m} x_j{}^f$.

So in the conventional notation, the prediction distribution can be written as

$$z^f \mid \underline{x} \sim B\left(z + y - n - m,\ \mathrm{n} + m + \beta\left(\mathrm{z} + y\right) - \left(z + y\right) + 1\right). \tag{3.7}$$

Note the prediction distribution under non-informative prior is a beta distribution. The moments of the distribution are given as

$$\text{Mean} = \frac{\hat{a}}{\hat{a}+b'}, \tag{3.8}$$

$$\text{Variance} = \frac{\hat{a}b'}{\left(\hat{a}+b'\right)^2\left(\hat{a}+b'+1\right)}, \tag{3.9}$$

where $\hat{a} = z+y-n-m$ and $b' = \mathrm{n}+m+\beta\left(\mathrm{z}+y\right)-\left(z+y\right)+1$.

3.2 Prediction Distribution of TGGSD Under Beta Prior

Using the posterior distribution under the beta prior B (u, v), the prediction distribution of a single future response x^f is given by

$$g\left(x^f \mid \underline{x}\right) \propto \alpha^{x^f+y+u-n-2}\left(1-\alpha\right)^{n+\beta x^f-x^f+\beta y-y+v}. \tag{3.10}$$

Therefore $x^f \mid \underline{x} \sim B\left(x^f+y+u-n-1,\ \mathrm{n}+\beta\left(\mathrm{x}^{\mathrm{f}}+y\right)-\left(x^f+y\right)+v+1\right)$. The mean and variance are

$$\text{Mean} = \frac{\hat{a}}{\hat{a}+b''}, \tag{3.11}$$

$$\text{Variance} = \frac{\hat{a}b''}{\left(\hat{a}+b''\right)^2\left(\hat{a}+b''+1\right)}, \tag{3.12}$$

where $\hat{a} = x^f+y+u-n-1$ and $b'' = \mathrm{n}+\beta\left(\mathrm{x}^{\mathrm{f}}+y\right)-\left(x^f+y\right)+v+1$.

Similarly, the prediction of a set of m future responses, $z^f = (x_1{}^f, x_2{}^f, \ldots, x_m{}^f)$ from the model under the beta prior can be given as:

$$g\left(z^f \mid \underline{x}\right) \propto \alpha^{z+y-(n+m)+u-1}\left(1-\alpha\right)^{n+m+\beta(z+y)-(z+y)+v-1}, \tag{3.13}$$

where $z = \sum_{i=1}^{m} x_j{}^f$.

Thus $z^f \mid \underline{x} \sim B\left(z+y-n-m+u,\ \mathrm{n}+m+\beta\left(\mathrm{z}+y\right)-\left(z+y\right)+v\right)$. So the prediction distribution under the beta prior is a beta distribution. Unlike the prediction distribution under the non-informative prior, here the prediction distribution depends on the hyper parameter as expected. The mean and variance of the distribution are

$$\text{Mean} = \frac{\breve{a}}{\breve{a}+b'''}, \tag{3.14}$$

$$\text{Variance} = \frac{\breve{a}b'''}{\left(\breve{a}+b'''\right)^2\left(\breve{a}+b'''+1\right)}, \qquad (3.15)$$

where $\breve{a} = z+y-n-m+u$ and $b''' = \mathrm{n}+m+\beta\left(z+y\right)-\left(z+y\right)+v$.

4. PREDICTION DISTRIBUTION OF SIZE BIASED GGSD

Let x^f be a future response from the size biased generalized geometric series distribution (SBGGSD) in (1.10). So the prediction distribution of x^f , conditional on the observed responses $x_1, x_2, ..., x_n$, is given by

$$g\left(x^f \mid x_1, x_2, ..., x_n\right) \propto h(x^f \mid \alpha)\, p\left(\alpha \mid x_1, x_2, ..., x_n\right). \qquad (4.1)$$

4.1 Prediction Distribution of TGGSD Under Non-Informative Prior

Under the non-informative prior, the prediction distribution of x^f becomes

$$g\left(x^f \mid \underline{x}\right) \propto (1-\alpha\beta)^{n+1}\left[\alpha^{x^f-1}\left(1-\alpha\right)^{1+\beta x^f - x^f}\ \alpha^{\sum_{i=1}^{n} x_i - n}\ \left(1-\alpha\right)^{n+\beta\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i}\right]\frac{1}{\alpha}$$

$$= (1-\alpha\beta)^{n+1}\alpha^{x^f+y-n-2}\left(1-\alpha\right)^{1+n+\beta(x^f+y)-(x^f+y)}, \qquad (4.2)$$

where $y = \sum_{i=1}^{n} x_i$ and $\underline{x} = (x_1, x_2, ..., x_n)$.

Clearly

$$x^f \mid \underline{x} \sim (1-\alpha\beta)^{n+1} B\left(x^f + y - n - 1,\ \ \mathrm{n}+\beta\left(\mathrm{x}^{\mathrm{f}}+y\right)-\left(x^f+y\right)+2\ \right). \qquad (4.3)$$

Now let $z^f = (x_1{}^f, x_2{}^f, ..., x_m{}^f)$ be a set of m future responses from the SBGGSD in (1.10). Then for the non-informative prior distribution, the prediction distribution of z^f is obtained as

$$g\left(z^f \mid \underline{x}\right) \propto (1-\alpha\beta)^{n+1}\ \alpha^{z+y-(n+m)-1}\left(1-\alpha\right)^{n+m+\beta(z+y)-(z+y)}, \qquad (4.4)$$

where $z = \sum_{j=1}^{m} x_j{}^f$.

So in the conventional notation, the prediction distribution can be written as

$$z^f \mid \underline{x} \sim (1-\alpha\beta)^{n+1} B\left(z+y-n-m,\ \ n+m+\beta\left(z+y\right)-\left(z+y\right)+1\ \right). \qquad (4.5)$$

This is a scaled beta distribution with a scaling factor $(1-\alpha\beta)^{n+1}$.

4.2 Prediction Distribution of SBGGSD Under Beta Prior

Using the posterior distribution under the beta prior B (u, v), the prediction distribution of a single future response x^f is given by

$$g\left(x^f \mid \underline{x}\right) \propto (1-\alpha\beta)^{n+1} \alpha^{x^f+y+u-n-2} \left(1-\alpha\right)^{n+\beta x^f - x^f+\beta y-y+v} . \qquad (4.6)$$

Therefore

$$x^f \mid \underline{x} \sim (1-\alpha\beta)^{n+1} B\left(x^f + y + u - n - 1,\ \ \mathrm{n}+\beta\left(\mathrm{x}^{\mathrm{f}}+y\right)-\left(x^f+y\right)+v+1\right).$$

where

$$\hat{a} = x^f + y + u - n - 1 \text{ and } b'' = \mathrm{n}+\beta\left(\mathrm{x}^{\mathrm{f}}+y\right)-\left(x^f+y\right)+v+1 .$$

Similarly, the prediction of a set of m future responses, $z^f = \left(x_1{}^f, x_2{}^f, ..., x_m{}^f\right)$ from the model under the beta prior can be given as:

$$g\left(z^f \mid \underline{x}\right) \propto (1-\alpha\beta)^{n+m} \alpha^{z+y-(n+m)+u-1} \left(1-\alpha\right)^{n+m+\beta(z+y)-(z+y)+v-1} , \qquad (4.7)$$

where $z = \sum_{j=1}^{m} x_j{}^f$.

Thus

$$z^f \mid \underline{x} \sim (1-\alpha\beta)^{n+m} B\left(z+y-n-m+u,\ \ \mathrm{n}+m+\beta\left(\mathrm{z}+y\right)-\left(z+y\right)+v\right).$$

This is a scaled beta distribution with a scaling factor $(1-\alpha\beta)^{n+m}$ which depends on the sample sizes of the realized and future responses.

4. CONCLUDING REMARKS

The prediction distributions for the future response(s) from the GGSD have been obtained in this paper. The non-informative as well has beta prior distributions are used to derive the posterior as well as the prediction distributions. Two special cases, namely the truncated and size bias GGSDs, are also studied. In all the cases, the prediction distribution follows beta distribution. Of course, the parameters of the beta distributions are different for different models as well as for the different choice of the prior distribution. However, in all cases the parameters of the prediction distribution depend on the size of the realized sample as well as that of the unobserved future sample. The mean and variance of the prediction distributions are also provided.

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