Change of dynamic response of pultruded composite components used in advanced composite structures due to fatigue and fracture

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ABSTRACT: The change of dynamic properties of a pultruded SHS beam sections due to fatigue loading and a crack was investigated. A significant change in the fundamental frequencies and flexural stiffness of the pultruded beam due to fatigue loading was observed. There were no significant changes in the fundamental frequencies of the beam which has a purposely created large crack on the surface. However, some noticeable discrepancies were observed in the frequency spectrum which was measured close to the crack.

1 INTRODUCTION

Advanced fibre composites which were primarily developed for defence and aerospace industries have made inroads into the fast growing and high volume civil infrastructure industry nearly a decade ago. Pultruded sections are a popular type of composites used in advanced hybrid composite constructions as pultrusions have comparatively higher fibre volume fraction and higher strength. The pultruded sections are produced by a pultrusion process which is a continuous manufacturing process to produce structural sections that has high fibre volume. This is a high volume production process that has many advantages over traditional composite production processes. The pultrusions are widely being used in hybrid composite constructions such as bridge girders and railway sleepers where harsh environmental conditions and severe fatigue loading are present. It has been reported that at the presence of heavy dynamic loading the properties of composite material such as strength and stiffness are gradually degrading (Epaarachchi & Clausen 2003). Due to this reason, unexpected failures of hybrid composite structures are inevitable during long-term operations. As such the in-depth knowledge of the long-term structural behaviour is an essential part of the designing of pultruded composite structures.

A change of the structural response is obviously a measure of a health condition of a structure. There are many indicators associated with structural response which can be monitored for long-term health or the integrity of a composite structure. The dynamic properties of structure such as the fundamental frequency of vibration is an excellent indicator of structural health of a composite structure and have been used in structural integrity monitoring for a few decades (Cawley & Adams 1978). Unfortunately, this promising health monitoring technique has not been developed for the advanced composite materials such as pultrusions which are widely used in civil infrastructure construction. This paper details an investigation performed on the change of dynamic response of pultruded composite components due to harsh operational conditions such as fatigue loading regimes and damages occurred to the component.

2 DYNAMIC RESPONSE OF PULTRUDED BEAM

2.1 *Dynamic analysis using Euler – Bernoulli beam theory*

The analysis here is performed using Euler – Bernoulli and Timeshenko Beam theory (Reddy 2004, Rao 2004)

Consider the simply supported beam shown in Figure 1.

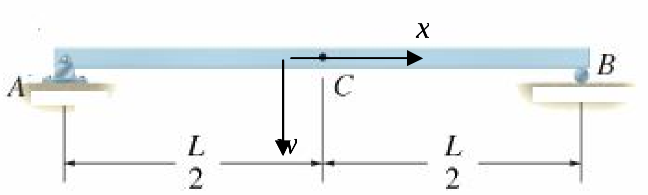

Figure 1 schematic diagram of a beam

For free lateral vibration of the beam shown the-equation of motion can be written as ;

$$EI\frac{\partial^4 w}{\partial x^4}(x,t) + \rho A(x)\frac{\partial^2 w}{\partial t^2}(x,t) = 0 \quad (1)$$

and can be solved for the fundamental frequencies of the beam as;

$$\omega_n = (n\pi)^2 \sqrt{\frac{EI}{\rho A L^4}} \quad rad/s \quad (2)$$

Where ρ is the density of the material at the section, *EI* is the flexural rigidity of the beam and *A* is cross-sectional area.

Euler – Bernoulli theory is considered as thin beam analysis as it does not includes rotary inertia and shear deformation of the beams cross-section. Using thick beam theory or Timoshenko beam theory these effects can be incorporated into the dynamic response analysis of the structure.

The lateral vibration of a Timoshenko beam AB in Figure 1 can be written as;

$$\alpha^2 \frac{\partial w^4}{\partial x^4} + \frac{\partial^2 w}{\partial t^2} - r^2\left(1 + \frac{E}{kG}\right)\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\rho r^2}{kG}\frac{\partial^4 w}{\partial t^4} = 0 \quad (3)$$

where $\alpha^2 = \frac{EI}{\rho A}$, $r^2 = \frac{I}{A}$,and *k* is Timoshenko's shear coefficient for this rectangular section *k* is taken as 5/6 and *G* is shear modulus.

Considering only the rotary inertia term of equation (3), the equation of motion can be reduced to;

$$EI\frac{\partial w^4}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} - \rho I\frac{\partial^4 w}{\partial x^2 \partial t^2} = 0 \quad (4)$$

and can be solved for the fundamental frequency

$$\text{as;}\ \omega = \sqrt{\frac{\alpha^2 n^4 \pi^4}{L^4\left(1 + \frac{n^2 \pi^2 r^2}{L^2}\right)}} \quad rad/s \quad (5)$$

Equations (2) and (5) show how the magnitude of the fundamental frequency changes with the stiffness of the beam. Therefore the change in fundamental frequency of the structure would be an indication of the change of stiffness of the beam.

Some interesting research work has been done on the vibration of cracked beams. Chondros *et al.* (1998), have developed a theory for the lateral vibration of cracked Euler-Bernoulli beams that have single edge or double edge open crack. The crack was modeled as a continuous flexibility using the displacement field in the vicinity of the crack found with fracture mechanics methods. Recently, Mei *et al.*(2006) and Mei (2005), have shown that a discontinuity in a beam such as a crack can be represented by a lumped stiffness at the location of the crack. As shown in Figure 2, a crack can be modeled using a bending, torsional and translational springs at the location of the crack. Mei *et al.* (2006), have used wave transmission and reflection approach at the discontinuity to incorporate local stiffness changes to vibration of the beam. This would make the continuous beam acts as two separate beams which are connected by the set of three type springs.

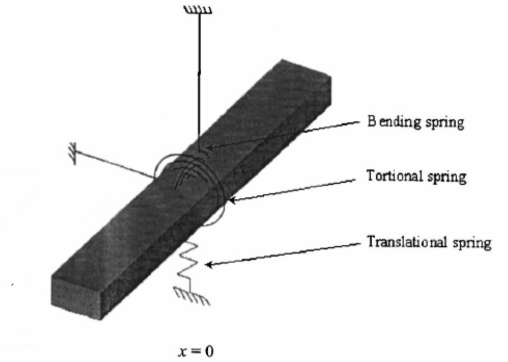

Figure 2**.** Modeling of a discontinuity in a beam using 3 springs (Mei 2005)

The local flexibility of the beam is changed due to the crack and the local change of flexibility can be represented by a "flexibility coefficient" ***C*** where is given by:

$$C = \frac{6\pi(1-\nu^2)h}{EI} f(\mu) \quad (6)$$

Here the function $f(\mu)$ is a polynomial function of the crack size μ, (Mei *et al.* 2006) and *EI* is the flexural rigidity of the beam, ν is the Poisson's ratio and *h* is the depth of the beam cross-section. Using the boundary conditions at the discontinuity, the modes of vibration of two segments of beam can be incorporated into a single characteristic equation (Mei 2005). It has been shown that fundamental frequencies of vibration were changed due to the change in stiffness at the discontinuity of a cracked beam. The models proposed by Chondros *et al* (1998) and Mei (2005) have indicated a possible change in local vibration response closer to a discontinuity such as a crack.

3 EXPERIMENTAL PROCEDURE

The mechanical properties of pultruded material was established by testing a few coupons cut from a pultruded 50 mm X 50 mm X 5mm hollow section. Two pultruded 50 mm X 50 mm X 5 mm square hollow sections by 1.3 m long were prepared for vibration testing. The locations of vibration measuring points are shown in the Figure 4. Vibration measurements were acquired by using a LMS VB8 Front end which has data acquisition rates up to 25 kHz. A PCB086C04 impulse force hammer was used to excite the system. A MEM 3–axis accelerometer MMA7260Q (1.5g @ 800mV/g) was used to measure the acceleration.

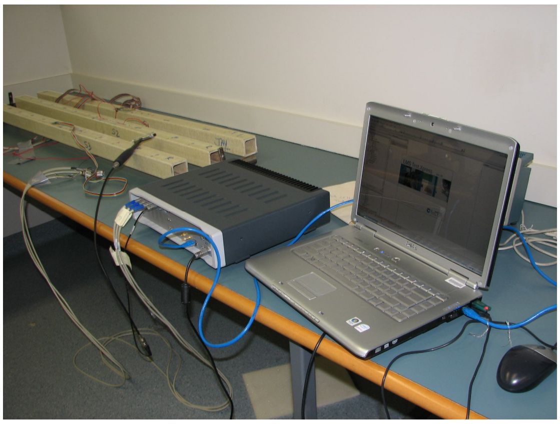

Figure 3 Experimental setup.

One of the prepared 50 mm X 50 mm X 5 mm square hollow beam was cycled in 3 point flexural fatigue rig (Fig 5) on MTS 100kN material testing machine. The fatigue loading regimes are shown in Table 1.

Table 1. Fatigue loading regimes

Regime	Load *P* (Max)	R	Cycles
1	2.6 kN	0.1	100000
2	3.5 kN	0.1	100000
3	2.2 kN	0.1	100000
3	3.5 kN	0.1	100000

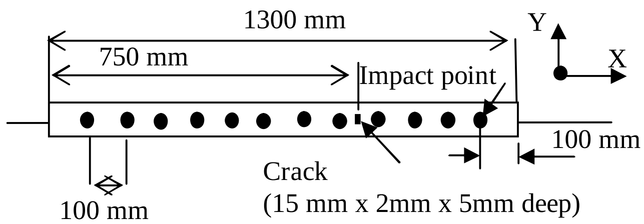

Figure 4 Locations of vibration measurement and the crack on the beam

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After each fatigue loading regime, vibration measurements were taken at each point on the beam shown in Figure 4. The beam was simply supported at both ends during the vibration measurements.

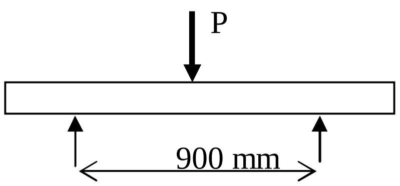

Figure 5 Schematic of the 3-point bending flexural fatigue test rig

On one pultruded 50 mm X 50 mm X 5 mm square hollow sections by 1.3 m long beam, a cut of 15mm long and 1.5 mm wide through the top flange thickness was created parallel to the transverse direction on the beam at the location shown in Figure 4. Vibration measurements were taken at each of the designated points shown in Figure 4.

All the vibration data were post-processed using LMS Text Express software. Time domain data were converted to frequency domain data using Fast Fourier Transformation (FFT). Frequency response Functions (FRF) were also obtained to extract the fundamental frequencies. The frequency results were verified by coherence analysis of the impact and acceleration signals.

A detailed finite element meshes (FEM) were created on STRAND7 FEA software for the beams which has no crack and the beam has a purposely created crack. The mesh consists of 12010 QUAD4 plate elements to simulate pultruded beam. The dynamic analysis was performed on FEM to obtain mode shapes and fundamental frequencies.

4 RESULTS AND DISCUSSION

Table 2 shows the properties of the pultruded section material used this study.

Table 2 Properties of pultrusion

Property	Value
E_1	3.15×10^{10} Pa
E_2	7.2×10^{9} Pa
G	2.9×10^{9} Pa
Poisson's ratio ν_{xy}	0.35
Poisson's ratio ν_{yx}	0.1
Ultimate tensile	471 MPa
Ultimate flexural	509 MPa
Volume fraction	0.69

Table 3 shows the calculated and experimental values of first four natural frequencies of the lateral vibration of the simply supported beam. Experimental values for fundamental frequencies show a good agreement with the calculated values. A significant differences were found in experimental natural frequency of higher modes and the predictions by the finite element analysis of the beam. This may be caused by the coupling between the flexural and torsional modes of vibration in the finite element model.

Table 3. Experimental and calculated frequencies of virgin beam

Mode	Natural Frequency (Hz)			
	Euler-Bernoli	Timeshenko	FEA	Experi-mental
1	72.5	72.4	72.1	72
2	290.0	288.8	268.5	272
3	652.4	646.5	543.7	668
4	1159.8	1141.5	1159.6	1264

Figure 6 shows a change in the flexural stiffness of the beam with the fatigue loading regimes. It can be seen that a significant change of flexural stiffness due to the applied loading regimes. Figure 7 shows a decrease in the fundamental frequency of the beam after each fatigue loading regime, as anticipated.

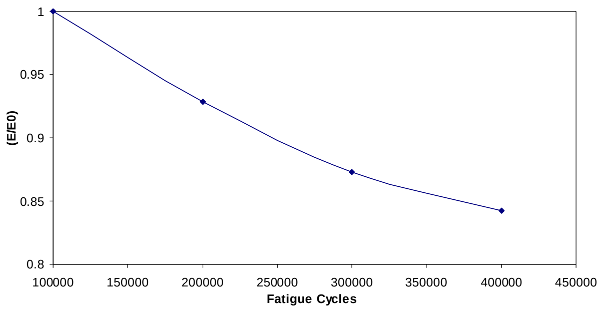

Figure 6 Change of flexural stiffness un-cracked beam undergone fatigue cycles

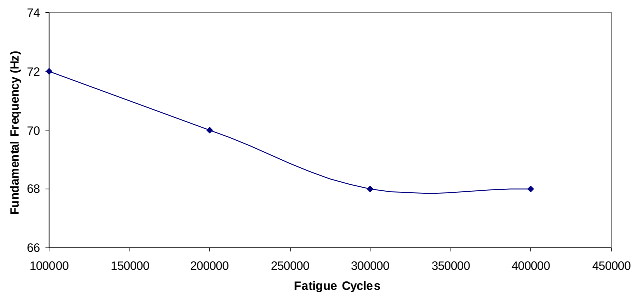

Figure 7 Change of first fundamental frequency of the un-cracked beam undergone fatigue cycles

Figure 8 shows a part of a typical frequency plot obtained for the cracked beam. The portion of the frequency plot in the Figure 8 shows the twin peak at the first fundamental frequency of 72 Hz. This trend was predominant in all vibration measurements taken along the cracked beam.

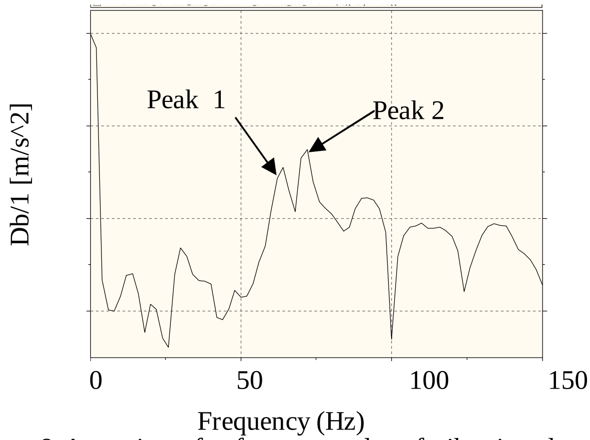

Figure 8 A portion of a frequency plot of vibration data that obtained on cracked beam, showing split peak closer to the fundamental frequency 72 Hz

The value of Peak 1 changed considerably in the vicinity of the crack as shown in Figure 9. The two peaks may be a result of a mixed mode of vibration caused by the discontinuity due to the crack. However, no significant changes of the fundamental frequency were shown by the FEA analysis when performed on the cracked beam. The first fundamental frequency obtained from FEA was 71.8 Hz for the cracked beam which is very close to the first fundamental frequency of the un-cracked beam.

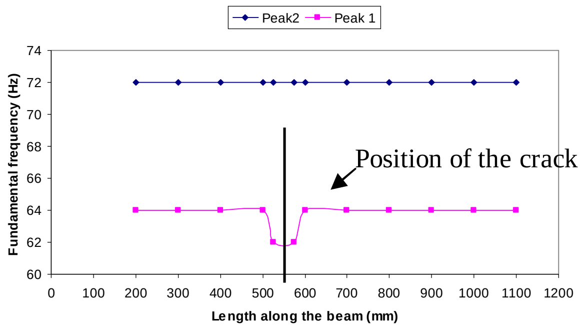

Figure 9 Fundamental frequency obtained experimentally along the cracked beam

Figures 10 and 11 shows the FEM for cracked beam at first natural frequency and the zoomed view of the crack on the beam respectively. Table 4 shows calculated and experimental fundamental frequencies of cracked beam measured at the point 400 mm from the right end of the beam (Fig 4).

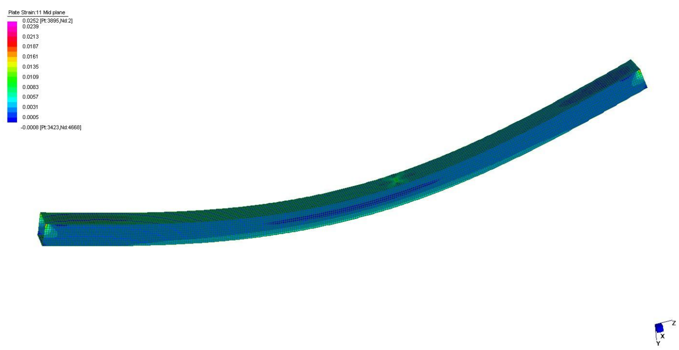

Figure 10. FEM for cracked beam @71.8 Hz

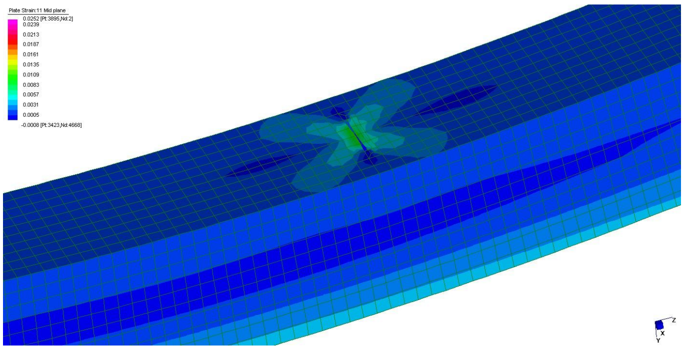

Figure 11. FEM for cracked beam @ 71.8 Hz showing the crack purposely crated.

Table 4. Experimental and calculated frequencies of cracked beam (obtained @ mid point of the beam)

Mode	FEA Hz	Experimental (Hz)
1	71.8	72 & 66
2	268.3	268
3	542.5	644

5 CONCLUSION

Long-term dynamic behavior of a pultruded 50mm x 50mm x 5mm beams, which is widely used in the construction of hybrid composite structural components, have been investigated for its dynamic response after various fatigue loading regimes and the presence of a crack. It has been found that the fundamental frequencies of the beam have changed significantly due to fatigue cycling. It was also observed a significant change of flexural stiffness due to fatigue cycling as expected.

A beam with a 15 mm long by 1.5mm wide transverse crack did not show any significant changes in the fundamental frequency of vibration due to the crack. However, a noticeable two peak split in the frequency spectrum at the first fundamental frequency was observed in all the frequency plots of vibration data of the cracked beam. The shape of the spectrum of first fundamental frequency has changed considerably in the vicinity of the crack. This observation does not suggest any significant dynamic property changes of the beam. More experimental investigations on the cracked beam and extensive mathematical modeling are required to investigate the variation of dynamic behavior at the vicinity of a crack.

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