A performance-based optimization method for topology design of continuum structures with mean compliance constraints

Qing Quan Liang[a,*] and Grant P. Steven[b]

[a] *Liang Computing Pty Ltd, 4/52 Orpington Street, Ashfield, NSW 2131, Australia*
[b] *School of Engineering, University of Durham, Durham, DH1 3LE, UK*

Abstract

A performance-based optimization (PBO) method for optimal topology design of linear elastic continuum structures with mean compliance constraints is presented in this paper. The performance-based design concept is incorporated in continuum topology optimization, which is treated as the problem of improving the performance of a continuum design domain in terms of the efficiency of material usage and overall stiffness. A simple scheme is employed in the proposed method to suppress the formation of checkerboard patterns. Two energy-based performance indices are derived for quantifying the topology performance of plane stress structures and plates in bending. Performance-based optimality criteria incorporating performance indices are proposed, and can be used in any continuum topology optimization methods for compliance minimization problems to obtain the optimum. Numerical examples are provided to demonstrate the effectiveness and validity of the PBO method in producing optimal topologies of continuum structures.

Keywords: Topology optimization; Performance index; Performance-based design; Compliance; Finite elements

* Corresponding author. E-mail: qqliang@hotmail.com

1. Introduction

The performance-based design is currently a popular design concept in the filed of structural engineering. This design concept describes the required and possessed performance of a structure being concerned. The performance is classified into serviceability, restorability, safety and cost. The performance of a structure is quantified by performance indices. Structural responses, such as stresses and displacements, are used as performance indices to evaluate the performance of structures. Many design codes of practice have been changing from the limit state design to the performance-based design. The intent is to provide owners and designers with alternative performance objectives for different structures and needs. Performance objectives are qualitatively expressed by non-engineering terms that can be easily understood by the owners and community.

The cost performance has increasingly become an important consideration in structural design because of the limited material resources, technological competition and environmental impacts. Structural optimization methods are efficient tools for achieving the cost-performance objective, and can be categorized into sizing, shape and topology optimization. The importance of topology optimization is justified by the fact that it can significantly improve the cost performance of a structure when comparing with sizing and shape optimization. In an integrated design process, topology optimization can be used as a preprocessing tool for sizing and shape optimization.

The shape optimization problem of continuum structures has been solved by Zienkiewicz and Campblell [1] using a sequential programming technique. A survey on continuum shape optimization has been conducted by Haftka and Grandhi [2]. Topology optimization of continuum structures as a generalized shape optimization problem has received considerable attentions in recent years, as demonstrated by the work of Rozvany et al. [3]. In 1988, Bendsøe and Kikuchi [4] introduced the homogenization-based optimization (HBO) method, which is shown to be effective in generating optimal topologies of continuum structures. In the HBO method, topology optimization is transformed to the problem of material redistribution within a design domain constructed by composite material with microstructures. The effective material properties of the composite material are calculated using the theory of homogenization. The homogenization-based design concept has been used to solve the problems of maximum stiffness topology design by Suzuki and Kikuchi [5], Díaz and Bendsøe [6], Tenek and Hagiwara [7], Bendsøe et al. [8], and Krog and Olhoff [9], frequency optimization by Díaz and Kikuchi [10] and Ma et al. [11], and optimization against buckling by Min and Kikuchi [12].

Simple approaches to topology optimization of continuum structures have been reported in the literature. The density function approach proposed by Mlejnek and Schirrmacher [13] and Yang and Chuang [14] uses the material density of each finite element as the design variable to solve the topology optimization problem. The effective material properties can be evaluated using the relationships between the material density and Young's modulus, as suggested by Gea [15]. Alternatively, the optimal topology of a continuum structure can be generated by removing underutilized elements from the structure in an evolutionary manner. This kind of optimization techniques is known as the hard kill optimization (HKO) method, which has been reported by Rodriguez and Seireg [16], Atrek [17], Rozvany et al. [18], Xie and Steven [19,20], and Hinton and Sienz [21]. Without element elimination, Mattheck and Burkhardt [22] and Baumgartner et al. [23] suggested a soft kill optimization (SKO) method. In the SKO, by setting the Young's modulus equal to the effective stress of elements in an optimization process, the optimal topology that represents an efficient load-carrying mechanism in the design domain can be characterized by the variation in its modulus.

For the maximum stiffness topology design, the mean compliance of a structure is commonly used in most of the existing optimization methods as the objective function, and the constraint is imposed on a somewhat arbitrarily specified material volume. In stress-based optimization methods, either the number of iterations or the material volume is used as the termination condition for determining the optimum. Obviously, appropriate criteria for identifying the optimum are still lacking in these continuum topology optimization methods that can lead to many locally optimal solutions. To overcome this problem, performance-based optimality criteria in terms of performance indices have been developed and incorporated in the performance-based optimization (PBO) method by Liang et al. [24-26] to obtain the optimum. In practice, the PBO method for continuum structures with displacement constraints has been proposed by Liang et al. [27,28] as a rational and efficient tool for automatically generating optimal strut-and-tie models for the design and detailing of structural concrete, which includes reinforced and prestressed concrete structures.

In this paper, an extension of the PBO method for topology design of linear elastic continuum structures with mean compliance constraints is presented. The performance-based design concept is incorporated in continuum topology optimization. The performance objective is to minimize the weight of a structure for a required stiffness performance. Element removal criteria based on the strain energy density of elements are described together with a simple scheme for suppressing the formation of checkerboard patterns. Performance-based optimality criteria are formulated for determining optimal topologies from the optimization process. A performance-based optimization procedure is presented. Numerical examples are examined to show the efficiency of the PBO method as a practical design tool.

2. Performance objective

The performance-based optimal design is to design a structure or structural component that can perform physical functions in a specified manner throughout its design service life at minimum cost or weight. The performance of an optimized design for stiffness can be expressed by its weight and associated strain energy. For practical purposes, the minimum material quantity should be sought by using optimization methods rather than specified by the designer. It is realistic to treat the weight of a structure as the objective function and structural responses such as the mean compliance or displacements as constraints since limitations on structural responses are specified in design codes. The performance objective of topology optimization is to seek a minimum-weight design with an acceptable mean compliance. This can be expressed in mathematical forms as follows:

$$\text{minimize} \quad W = \sum_{e=1}^{n} w_e(t) \tag{1}$$

$$\text{subject to} \quad C \le C^* \tag{2}$$

$$t^L \le t \le t^U \tag{3}$$

in which W is the total weight of a structure, w_e is the weight of the eth element, t is the thickness of elements, C is the absolute value of the mean compliance of the structure, C^* is the prescribed limit of C, and n is the total number of elements in the structure, t^L is the lower limit of element thickness and t^U is the upper limit of element thickness. Since the thickness of a continuum structure has a significant effect on the weight of the final design, it is treated as one of the design variables in order to obtain the best design. To simplify the optimization problem, the uniform sizing of element thickness is considered in the proposed method.

3. Element removal criteria

By performing a finite element analysis on a continuum structure under applied loads, it can be found that some regions of the structure are not as effective in carrying loads as other regions. Removing these underutilized regions from a structure can result in a higher performance design. The criteria for element removal can be formulated on the basis of the design sensitivity analysis of constraints with respect to design variables. In the PBO method, the design sensitivity analysis is to investigate the effect of element removal on the changes of the mean compliance of a continuum structure. Approximate concepts are employed in the design sensitivity analysis.

Consider a linearly elastic continuum structure under applied loads and boundary conditions. By modeling the structure with finite elements, the equilibrium equation for the structure can be written by

$$[K]\{u\} = \{P\} \tag{4}$$

where $[K]$ is the stiffness matrix of the structure, $\{u\}$ is the displacement vector and $\{P\}$ is the nodal load vector. When the eth element that is less effective in carrying loads is removed from a design domain, the stiffness matrix and displacement vector of the structure are changed accordingly. Eq. (4) can be rewritten as

$$([K]+[\Delta K])(\{u\}+\{\Delta u\})=\{P\} \tag{5}$$

in which $[\Delta K]$ is the changes of the stiffness matrix and $\{\Delta u\}$ is the change of nodal displacements vector. The loads applied to the structure are assumed to be unchanged in an optimization process. Since only the *e*th element is removed from the structure, the change of the stiffness matrix can be derived as follows

$$[\Delta K]=[K_r]-[K]=-[k_e] \tag{6}$$

where $[K_r]$ is the stiffness matrix of the resulting structure and $[k_e]$ is the stiffness matrix of the *e*th element. The change of displacement vector due to element elimination can approximately be obtained from Eqs. (4) and (5) by neglecting higher order terms as

$$\{\Delta u\}=-[K]^{-1}[\Delta K]\{u\} \tag{7}$$

The strain energy or mean compliance of a structure is represented by

$$C=\frac{1}{2}\{P\}^{\mathrm{T}}\{u\} \tag{8}$$

The change of the strain energy of a structure due to the removal of the *e*th element can approximately be derived as follows

$$\Delta C = \frac{1}{2}\{P\}^{\mathrm{T}}\{\Delta u\} = -\frac{1}{2}\{P\}^{\mathrm{T}}[K]^{-1}[\Delta K]\{u\} = -\frac{1}{2}\{u\}^{\mathrm{T}}[\Delta K]\{u\} = \frac{1}{2}\{u_e\}^{\mathrm{T}}[k_e]\{u_e\} \tag{9}$$

in which $\{u_e\}$ is the displacement vector of the *e*th element. Eq. (9) indicates that the change of the strain energy of a structure due to the removal of the *e*th element is equal to the strain energy of the *e*th element. This means that the element strain energy is a measure of element contribution to the overall stiffness performance of a structure, and is denoted as

$$c_e = \frac{1}{2}\{u_e\}^{\mathrm{T}}[k_e]\{u_e\} \tag{10}$$

To achieve the performance objective, it is obvious that a small number of elements with the lowest strain energy should be systematically removed from a design domain. For continuum structures modeled with different size finite elements, the element strain energy per unit weight, which is defined as the strain energy density of the element, should be calculated for element elimination. The strain energy density of the *e*th element is

$$\gamma_e = \frac{|c_e|}{w_e} \tag{11}$$

Checkerboard patterns often present in optimal topologies generated by continuum topology optimization methods when elements whose numerical stability is not guaranteed are used in the finite element analysis. The presence of checkerboard patterns leads to difficulty in interpreting and manufacturing optimal structures. As a result of this, it is desirable to suppress the formation of checkerboard patterns in continuum topology optimization. An investigation conducted by Jog et al. [29] shows that using higher-order elements whose numerical stability is guaranteed by the Babuska-brezzi condition can prevent checkerboard patterns from occurring. However, the use of higher-order elements significantly increases the computational cost. Youn and Park [30] suggested a density redistribution method, which is shown to be effective in suppressing the formation of checkerboard patterns. A similar scheme to the density redistribution method is incorporated in performance-based optimization algorithms. In this scheme, the nodal strain energy densities of an element are calculated by averaging the strain energy densities of neighboring elements as follows

$$\zeta_{nd} = \frac{1}{M}\sum_{e=1}^{M}\gamma_e \tag{12}$$

where ζ_{nd} is the nodal strain energy density and M is the number of elements that connect to that node. The strain energy density of each element can be recalculated from the nodal strain energy densities at the nodes of that element by

$$\zeta_e = \frac{1}{Q}\sum_{nd=1}^{Q}\zeta_{nd} \tag{13}$$

in which ζ_e is the recalculated strain energy density of the eth element and Q is the number of nodes in the element. This simple scheme can effectively suppress the formation of checkerboard patterns as demonstrated by examples presented in this paper.

For structures subject to multiple loading conditions, a logical AND scheme can be used in optimization algorithms to take account of the effects of different loading conditions on optimal designs, as suggested by Liang et al. [25]. In the logical AND scheme, an element is eliminated from the design domain only if its strain energy density (ζ_e) is the lowest for all loading conditions. A loop is used to count elements with the lowest strain energy density until they made up the specified amount that is the element removal ratio times the total number of elements in the initial design domain. The element removal ratio (R) for each iteration is defined as the ratio of the number of elements to be removed to the total number of elements in the initial design domain. The element removal ratio is not changed in the whole optimization process.

4. Performance-based optimality criteria (PBOC)

4.1 General

By gradually eliminating elements with the lowest strain energy densities from a design domain, the distribution of strain energy densities will consequently become more and more uniform. The uniform strain energy density has been used as an optimality condition in truss topology optimization approaches, and can be derived by using the Kuhn-Tucker condition [31]. However, the uniform condition of element strain energy densities in a continuum structure under applied loads is seldom achieved by numerical topology optimization methods even if the mean compliance constraint has been violated. This means that a minimum-weight design with an acceptable stiffness performance is not necessarily a structure in which the distribution of element strain energy densities is absolutely uniform. Therefore, the uniformity of element strain energy densities cannot be incorporated in continuum topology optimization methods as a termination condition for determining the optimum. Performance-based optimality criteria are proposed here for obtaining optimal topologies.

4.2 PBOC for plane stress structures

In design problems with element thickness or cross-sectional design variables, an infeasible design in an optimization process can be converted into a feasible one by the scaling procedure. Due to its simplicity and efficiency, this scaling procedure has been used in truss topology optimization by Kirsch [32]. The scaling design concept has been utilized to develop performance indices for evaluating the performance of structural topologies and shapes subject to stress and displacement constraints by Liang et al. [24,26].

For plane stress continuum structures, the stiffness matrix of a structure is a linear function of element thickness. Therefore, for structures with the mean compliance constraint, the element thickness can be uniformly scaled to keep the mean compliance constraint active at each iteration in the optimization process. By scaling the initial design with a factor of C_0 / C^*, the scaled weight of the initial design can be expressed by

$$W_0^{\mathrm{s}} = \left(\frac{C_0}{C^*} \right) W_0 \tag{14}$$

in which W_0 is the actual weight of the initial design domain and C_0 is the strain energy of the initial design under applied loads. Similarly, by scaling the current design with respect to the mean compliance limit, the scaled weight of the current design at the *i*th iteration can be determined by

$$W_i^{\mathrm{s}} = \left(\frac{C_i}{C^*} \right) W_i \tag{15}$$

where C_i is the strain energy of the current structure under applied loads at the ith iteration and W_i is the actual weight of the current structure at the ith iteration.

The performance of the resulting structure at the ith iteration can be evaluated by the performance index, which is proposed as

$$PI_{ES} = \frac{W_0^s}{W_i^s} = \frac{(C_0 / C^*)W_0}{(C_i / C^*)W_i} = \frac{C_0 W_0}{C_i W_i} \quad (16)$$

It is seen that the performance index is composed of the strain energy and the weight of the structure. The performance index composed of strain energy is called the energy-based performance index in order to distinguish it from the stress-based performance index [24] and the displacement-based performance index [26]. By systematically eliminating elements with the lowest strain energy densities from a continuum structure, the performance of the structure can gradually be improved. The higher value of the performance index means the better performance of a structural topology. The performance characteristics of structural topologies in an iterative optimization process can fully be demonstrated by the weight-compliance curve as shown in Fig. 1. Performance characteristics indicate whether a proposed design for specified performance is feasible. In addition, they provide a measure of the success of the design configurations.

To obtain the optimal topology, the performance-based optimality criterion (PBOC) for plane stress structures with the mean compliance constraint is proposed as

$$maximize\ PI_{ES} = \frac{C_0 W_0}{C_i W_i} \tag{17}$$

This PBOC means that the optimal topology of a continuum structure under applied loads is found when its associated strain energy and material consumption are a minimum. The optimal topology obtained represents an efficient load-carrying mechanism within the design domain. It can be observed from Eq. (17) that the optimal topology does not depend on the mean compliance limit. Since the performance index is a dimensionless number, the uniform scaling of element thickness does not affect its values. Therefore, the element thickness of an initial design domain can be assumed and needs not to be changed in the finite element analysis and optimization process. The performance index can be employed to monitor the optimization history so that the optimum can be identified from the performance index history. Scaling the thickness of an optimal topology obtained can satisfy the actual mean compliance constraint.

For a structure subject to multiple loading cases, the performance index of a structure at each iteration can be calculated by using the strain energy of the structure under the most critical loading case in the optimization process.

4.3 PBOC for plates in bending

When the thickness of a bending plate is uniformly scaled, the displacement vector of the scaled plate can be represented by

$$\{u^s\} = \frac{1}{\varphi^3}\{u\} \tag{18}$$

where $\{u^s\}$ is the scaled displacement vector, and φ is the scaling factor. It can be seen from Eq. (8) that the mean compliance of a bending plate is proportional to the displacement vector. If the thickness of a bending plate is reduced by a factor φ, the mean compliance will increase with a factor of $1/\varphi^3$. In order to satisfy the mean compliance constraint, the plate needs to be scaled by a factor

$$\varphi = \left(\frac{C}{C^*} \right)^{1/3} \tag{19}$$

By following the scaling procedure, the performance index of a bending plate at the *i*th iteration can be derived as follows

$$PI_{EP} = \left(\frac{C_0}{C_i} \right)^{1/3} \frac{W_0}{W_i} \tag{20}$$

It is seen that the performance index is a measure of structural responses and the weight of a plate in the optimization process, and thus quantifies the performance of a bending plate. Therefore, the performance-based optimality criterion for plates in bending with a mean compliance constraint can be proposed as

$$maximize\ PI_{EP} = \left(\frac{C_0}{C_i} \right)^{1/3} \frac{W_0}{W_i} \tag{21}$$

It should be noted that for a bending plate under multiple load cases, the strain energy of the plate under the most critical load case should be used in the calculation of the performance index.

5. Performance optimization procedure

The finite element method is used in numerical topology optimization techniques as a modeling and analytical tool. Based on the information obtained from the results of the finite element analysis (FEA), underutilized elements can be identified, as discussed previously. The performance of a structural topology can then gradually be improved by systematically eliminating these underutilized elements from the structure. The process of FEA and performance improvement is repeated until the termination criterion is satisfied. The main steps of the performance-based optimization procedure are given as follows:

(1) Model the initial design domain with fine finite elements. Applied loads, material properties and support conditions are specified. Non-design regions that are not removed in an optimization process are defined by specifying the number of their material properties to a different number from that of design regions.

(2) Perform a linear elastic finite element analysis on the structure.

(3) Evaluate the performance of the resulting topology using Eq. (16) for plane stress structures and Eq. (20) for plates in bending.

(4) Calculate the strain energy densities of elements (ζ_e) under each loading case.

(5) Remove *R* (%) elements with the lowest strain energy densities (ζ_e) from the design domain.

(6) Check continuity of the resulting structure. The continuity constraint affects resulting topologies in the optimization process. It is assumed that two elements are connected

together if they have at least one common edge. Any element that is not connected with other elements is considered as a singular element, which is removed from the model.

(7) Check the symmetry of the resulting structure. A scheme for checking the symmetry of resulting structures is employed in the optimization algorithm. Extra elements are removed from the structure to maintain the symmetry of resulting structures under an initially symmetrical condition.

(8) Save data for the current structure. The data for structures generated in the optimization process is saved to files so that the optimization history can be kept track.

(9) Repeat step (2) to (8) until the performance index is less than unity or kept constant in later iterations.

(10) Plot the performance index history and select the optimum. The optimal topology that corresponds to the maximum performance index can be identified from the performance index history.

6. Numerical examples

6.1 Verification of the PBO method

The Michell truss shown in Fig. 2 is known as an optimal solution, which was obtained by using the analytical method by Michell [33]. This example is to show that whether the PBO method proposed for topology design problems with mean compliance constraints can reproduce the Michell truss. A continuum structure shown in Fig. 3 is used as the initial structure for deriving the Michell truss, as adopted by Bendsøe et al. [34]. The initial structure is discretized into 110×80 four-node plane stress elements. The circular non-design domain constructed approximately by rectangular elements is treated as the fixed support where no deformations are allowed. A tip load is applied to the centre of the free end as illustrated in Fig. 3. The Young's modulus of material E = 200 GPa, Poisson's ratio ν = 0.3, and the thickness of elements t = 5 mm are assumed in the analysis. The element removal ratio R = 2% is employed in the optimization.

The performance characteristics of the Michell structure in the optimization process are demonstrated by the dimensionless weight-compliance curve shown in Fig. 4. It is seen that by gradually removing elements from the design domain, the mean compliance of the Michell structure increases with the reduction in its weight. In addition, this curve indicates the variation rate of the weight and mean compliance in the optimization process. The performance index history of the Michell structure is presented in Fig. 5. The performance index in the optimization process increases when elements with the lowest strain energy density are gradually removed from the design domain. The maximum performance index is 1.33, which occurs at iteration 14. The optimal topology corresponding to the maximum performance index is shown in Fig. 6(a). It is observed that the optimal topology obtained is a continuum-like structure with a few small holes inside rather than a discrete Michell truss. This demonstrates that continuum topology optimization may or may not result in truss-like optimal structures so that is a more general approach than the truss topology optimization method.

In order to generate a truss-like structure, the optimization process is continued. The resulting topologies at iterations 17 and 23 are shown in Fig. 6(b) and (c), respectively. It can be seen from these figures that when more and more elements are removed from the design domain, the resulting topology is gradually evolved towards a truss-like structure. If the resulting structure is to be designed as a truss, the topology shown in Fig. 6(c) agrees extremely well with the Michell truss and solutions produced using the HBO method by Suzuki and Kikuchi [5] and Bendsøe et al. [34].

It is observed from Fig. 5 that the performance of resulting topologies at iterations from 14 to 23 decreases only slightly. This indicates that the material volume that is needed to construct these structures is almost the same while satisfying the same mean compliance limit. In other words, the structure can be designed by selecting one of these topologies shown in Fig. 6. The performance index is a useful tool, which assists the selection of the best topology in structural design when the structural performance, aesthetic and construction constraints are taken into consideration.

6.2 Layout design of bridge structures

In this example, the PBO method is used to find the best layout of a bridge structure under uniformly distributed traffic loading in the conceptual design stage. The design domain and support conditions of a bridge structure are illustrated in Fig. 7, where the bottom supports are fixed. The continuum design domain is modeled with 90 $\times$ 30 four-node, plane stress elements. The two rows of elements below the loading level are treated as the non-design domain, which represents the bridge deck. The uniformly distributed loading is modeled by applying a 500 kN point load per node. The Young's modulus of material E = 200 GPa, Poisson's ratio ν = 0.3 and the thickness of elements t = 300 mm are used in the analysis. The R = 1% is adopted in the optimization process.

Fig. 8 shows the performance characteristic of the bridge structure in the optimization process. In the early stages of the optimization process, element elimination only has a minor effect on the overall stiffness performance of the bridge structure. However, after iteration 65, further element removal leads to a large increase in the mean compliance, as indicated by the plateau of the curve. This can also be seen from the performance index history of the bridge structure presented in Fig. 9, which shows that the performance of the structure drops sharply after iteration 65. This is because the load-carrying mechanism is destroyed by further element elimination. The maximum performance index is 1.40, which occurs at iteration 56.

The topology optimization history of the bridge structure is presented in Fig. 10. It is observed that the part below the bridge deck is systematically removed since they have no contributions to the structural efficiency. The optimal topology obtained is shown in Fig. 10(c). This optimum design indicates a well-known tie-arch bridge structural system that has commonly been used in bridge engineering. In the design of bridge structures, the designer usually needs to consider various important aspects, such as structural performance, economy, aesthetic and constructability. The aesthetic issue may weight over the economical aspects if the beauty of a bridge is of importance to the surrounding environment. To select a bridge form that not only has a good looking but also has a high structural performance, the performance index plays an important role. It is seen from Fig. 10 that the performance of the topology obtained at iteration 64 is almost the same as that of the optimum. However, the form shown in Fig. 5.12 (d) looks better than the optimum. Therefore, it is suggested that the topology shown in Fig. 10(d) shall be used as the final design proposal for the bridge layout. The arch of the bridge can be constructed by using either concrete or steel trusses, which are the structural form used in the Sydney Harbor Bridge.

6.3 Plate in bending

A clamed square plate (400 × 400) under a concentrated load of 500 N applied at its centre is optimized using the PBO method. The design domain of the plate in bending is divided into 50 × 50 four-node plate elements. The Young's modulus E = 200 MPa, Poisson's ratio ν = 0.3 and the thickness of the plate t = 5 mm are assumed in the analysis. Four elements around the loaded point are frozen so this region is not removed during the optimization process. The element removal ratio R = 1% is adopted.

The performance characteristics of the plate in bending are fully captured using the PBO method and are demonstrated in Fig. 11. The performance index history of the plate is shown in Fig. 12, where the maximum performance index is 2.13. After reaching its highest performance, the plate fails due to further element removal, as indicated by the sharp drop of the performance index shown in Fig. 12. The topology optimization history of the plate in bending is presented in Fig. 13. It is seen that the corners of the plate have been removed since these regions have the less contribution to the overall stiffness performance.

6.4 Effects of finite element meshes

This example is to investigate the effects of finite element meshes on the optimal topologies of continuum structures optimized by the PBO method while other conditions are fixed. The Michell type structure with a simply supported condition is used as the test example. Fig. 14 shows the design domain for the simply supported Michell structure under a concentrated load of P = 100 kN. The design domain is divided into three different meshes, such as 70 × 35, 100 × 50 and 120 × 60, using four-node plane stress elements. The Young's modulus E = 200 GPa, Poison's ratio ν = 0.3 and the thickness of all elements t = 10 mm are specified. Plane stress conditions are assumed in the finite element modeling. To eliminate the effects of element removal ratio on final solutions, the element removal ratio R = 1% is used for all cases. The structure is analyzed and optimized by using the PBO method for three different finite element discretizations.

Fig. 15 shows the performance index histories for the Michell structure modeled with different finite element meshes. It is observed from Fig. 15 that the discrepancies of the performance index value between different meshes increase with the increases in the iteration numbers. It is shown that the finer the mesh used to model the structure, the higher the performance of the optimal topology obtained. The maximum performance indices of optimized structures for meshes 70 × 35, 100 × 50 and 120 × 60 are 1.53, 1.60 and 1.67, respectively. It can be observed that performance indices reach the peak values at different iterations for the structure optimized using different meshes for the same element removal ratio. After reaching the peak, performance indices decrease and finally drop very sharply. It is also observed that more iterations are usually needed for a structure modeled using a finer mesh to obtain the optimum.

Optimal topologies obtained using three different meshes are presented in Fig. 16. The optimal topology generated using 70 × 35 finite elements indicates a truss-like structure as shown in Fig. 16(a). By inspection, it is seen that the in-plane member size of the truss is approximately proportional to the axial force carried by that member. In other words, the distribution of element strain energy density within the optimal topology is approximately uniform since elements with the lowest strain energy density are systematically eliminated from the design domain. It is also seen from Fig. 16(a) and (b) that these optimal topologies are almost identical. However, the optimal topology with a finer mesh discretization exhibits a truss-like structure in which the in-plane member size is smaller than that with a coarse one.

The optimal solution obtained by Michell [33] using the analytical method indicates a pined-joint truss with finite members. The strain filed in the Michell truss was assumed to be linearlized. In addition, compression and tensile members of the Michell truss are subjected to the same allowable stress. It should be noted that the Michell truss is theoretical optima, which is not necessarily a practical design. It is noted that a more accurate solution can be achieved by using a finer mesh in the finite element analysis. To see whether the optimal structure can approach the Michell truss, a 120 $\times$ 60 mesh is used to divide the design domain. The optimal topology obtained using 120 $\times$ 60 elements is presented in Fig. 16(c). It is seen that a more Michell truss-like structure is obtained by using the finer elements in continuum topology optimization. If elements were refined infinitely, the optimal structure would be towards the Michell truss.

This investigation shows that the optimal structure converges to the theoretical optima as sufficiently fine elements are used. Even coarse mesh can produce a rough idea of the optimal structure. In the conceptual design stage, if the PBO method is used to find the primary layout of an optimal structure, a coarse mesh can be used to solve the optimization problem. After obtaining the optimal topology, shape optimization techniques can be employed to further improve its performance.

7. Conclusions

This paper has presented a PBO method incorporating the uniform sizing of element thickness for topology design of linear elastic continuum structures with mean compliance constraints. The optimal topology of a continuum structure is generated by gradually deleting elements with the lowest strain energy densities from a continuum design domain. Two energy-based performance indices have been developed for evaluating the topology performance of plane stress structures and of bending plates subject to mean compliance constraints. Performance-based optimality criteria have been proposed to allow for the optimal topology to be identified from the optimization history. Performance characteristics of structural topologies in an optimization process are expressed by the weight-compliance curves.

The PBO method can produce optimal topologies that might be obtained by analytical methods and other continuum topology optimization approaches. It is shown that simply recalculating the strain energy densities of elements can suppress the formation of checkerboard patterns in the proposed method. Continuum topology optimization methods may or may not result in truss-like optimal structures. This study also indicates that the finite element mesh has considerable effects on the optimal topologies and their performance. The finer the mesh used to model a structure, the higher the performance of the optimal topology obtained.

Performance indices formulated in this paper are very useful tools, which assist the structural designer in the selection of the best topology in structural design when considering the required performance, aesthetic and construction constraints. They can also be used to rank the efficiency of structural topologies and shapes generated by different structural optimization methods. Performance-based optimality criteria proposed herein can be incorporated in any continuum topology optimization methods for compliance optimization problems to determine optimal designs. The mathematical formulation of the PBO method is simple to understand by practicing engineers. Moreover, the performance-based optimization concept incorporated in the proposed method is consistent with the performance-based design concept being adopted in current building codes of practice. Therefore, the PBO method is a valuable design tool for practicing engineers in the performance-based layout design of continuum structures.

References

[1] O.C. Zienkiewicz and J.S. Campbell, Shape optimization and sequential linear programming, In: Optimal Structural Design, R.H. Gallagher and O.C. Zienkiewicz (eds), 1973 (Wiley, New York 1973) 109-126.

[2] R.T. Haftka and R.V. Grandhi, Structural shape optimization—A survey, Comp. Methods Appl. Mech. Engrg. 57(1) (1986) 91-106.

[3] G.I.N. Rozvany, M.P. Bendsøe and U. Kirsch, Layout optimization of structures, Appl. Mech. Rev. 48 (1995) 41-119.

[4] M.P. Bendsøe and N. Kikuchi, Generating optimal topologies in structural design using a homogenization method, Comp. Methods Appl. Mech. Engrg. 71 (1988) 197-224.

[5] K. Suzuki and N. Kikuchi, A homogenization method for shape and topology optimization, Comp. Methods Appl. Mech. Engrg. 93 (1991) 291-318.

[6] A.R. Díaz and M.P. Bendsøe, Shape optimization of structures for multiple loading condition using a homogenization method, Struct. Optim. 4 (1992) 17-22.

[7] L.H. Tenek and I. Hagiwara, Static and vibrational shape and topology optimization using homogenization and mathematical programming, Comp. Methods Appl. Mech. Engrg. 109 (1993) 143-154.

[8] M.P. Bendsøe, A.R. Díaz, R. Lipton and J.E. Taylor, Optimal design of material properties and material distribution for multiple loading conditions, Int. J. Numer. Methods Engrg. 35 (1995) 1449-1170.

[9] L.A. Krog and N. Olhoff, Optimum topology and reinforcement design of disk and plate structures with multiple stiffness and eigenfrequency objectives Comp. & Struct. 72 (1999) 535-563.

[10] A.R. Díaz and N. Kikuchi, Solution to shape and topology eigenvalue optimization problems using a homogenization method, Int. J. Numer. Methods Engrg. 35 (1992) 1487-1502.

[11] Z.D. Ma, N. Kikuchi and H.C. Cheng, Topological design for vibrating structures, Comp. Methods Appl. Mech. Engrg. 121 (1995) 259-280.

[12] S. Min and N. Kikuchi, Optimal reinforcement design of structures under the buckling load using the homogenization design method, Struct. Engrg. Mech. 5(5) (1997) 565-576.

[13] H.P. Mlejnek and R. Schirrmacher, An engineer's approach to optimal material distribution and shape finding, Comp. Methods Appl. Mech. Engrg. 106 (1993), 1-26.

[14] R.J. Yang and C.H. Chuang, Optimal topology design using linear programming, Comp. & Struct. 52(2) (1994) 265-275.

[15] H.C. Gea, Topology optimization: a new microstructure-based design domain method, Comp. & Struct. 61(5) (1996) 781-788.

[16] J. Rodriguez and A. Seireg, Optimizing the shapes of structures via a rule-based computer program, Computing in Mech. Engrg. ASME, 40(1) (1985) 20-29.

[17] E. Atrek, (1989). "Shape: a program for shape optimization of continuum structures." In: Computer Aided Optimization Design of Structures: Applications, C.A. Brebbia and S. Hernandez (eds), 1989 (Computational Mechanics Publications, Southampton, 1989) 135-144.

[18] G.I.N. Rozvany, M. Zhou and T. Birker, Generalized shape optimization without homogenization, Struct. Optim. 4 (1992) 250-252.

[19] Y.M. Xie and G.P. Steven, A simple evolutionary procedure for structural optimization, Comp. & Struct. 49(5) (1993) 885-896.

[20] Y.M. Xie and G.P. Steven, Evolutionary structural optimization for dynamic problems, Comp. & Struct. 58 (1996) 1067-1073.

[21] E. Hinton and J. Sienz, Fully stressed topological design of structures using an evolutionary procedure, Engrg. Computations 12 (1995) 229-244.

[22] C. Mattheck and S. Burkhardt, A new method of structural shape optimization based on biological growth, Int. J. of Fatigue, May (1990) 185-190.

[23] A. Baumgartner, L. Harzheim and C. Mattheck, SKO (soft kill option): the biological way to find an optimum structure topology, Int. J. of Fatigue, 14(6) (1992) 387-393.

[24] Q.Q. Liang, Y.M. Xie and G.P. Steven, Optimal selection of topologies for the minimum-weight design of continuum structures with stress constraints, Proc. Instn Mech. Engrs. J. Mech. Engrg. Sci. Part C, 213(C8) (1999) 755-762.

[25] Q.Q. Liang, Y.M. Xie and G.P. Steven, Optimal topology design of bracing systems for multistory steel frames, J. Struct. Engrg. ASCE, 126(7) (2000) 823-829.

[26] Q.Q. Liang, Y.M. Xie and G.P. Steven, Optimal topology selection of continuum structures with displacement constraints, Comp. & Struct. 77(6) (2000) 635-644.

[27] Q.Q. Liang, Y.M. Xie and G.P. Steven, Topology optimization of strut-and-tie models in reinforced concrete structures using an evolutionary procedure, ACI Structural Journal, 97(2) (2000) 322-330.

[28] Q.Q. Liang, Y.M. Xie and G.P. Steven, Generating optimal strut-and-tie models in prestressed concrete beams by performance-based optimization, ACI Structural Journal, 98(2) (2001) 226-232.

[29] C.S. Jog, R.B. Haber and M.P. Bendsøe, Topology design with optimized, self-adaptive materials. Int. J. Numer. Meth. Engrg. 37 (1994) 1323-1350.

[30] S.-K. Youn and S.-H. Park, A study on the shape extraction process in the structural topology optimization using homogenized material, Comp. & Struct. 62(3) (1997) 527-538.

[31] G.I.N. Rozvany, Structural Design via Optimality Criteria, 1989 (Kluwer, Dordrecht, 1989).

[32] U. Kirsch, Optimal design based on approximate scaling, J. Struct. Engrg. ASCE, 108(ST4) (1982) 888-909.

[33] A.G.M. Michell, The limits of economy of material in frame-structures, Phil. Mag. 8 (1904) 589-597.

[34] M.P. Bendsøe, A.R. Díaz and N. Kikuchi, Topology and generalization layout optimization of elastic structures, In: Topology Design of Structures, M.P. Bendsøe and C.A. Mota Soares (eds), 1993 (Kluwer Academic Publishers, Dordrecht, 1993) 159-206.

Figures

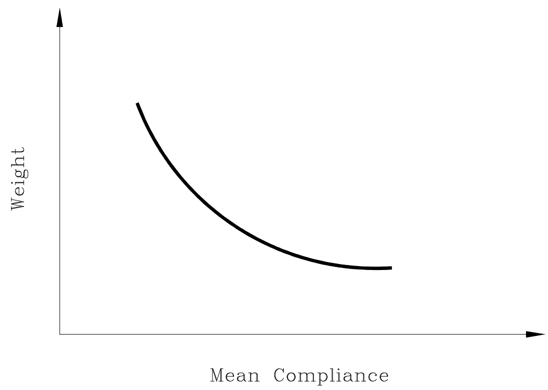


Fig. 1. Typical performance characteristics of structural topology

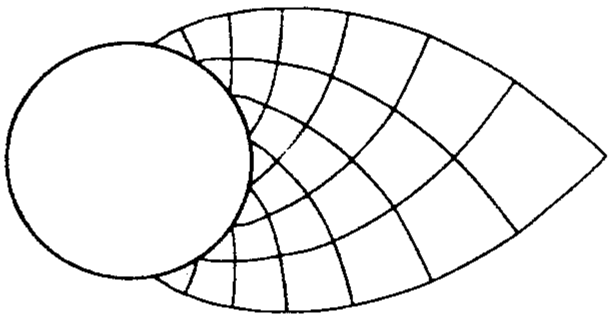


Fig. 2. Michell structure

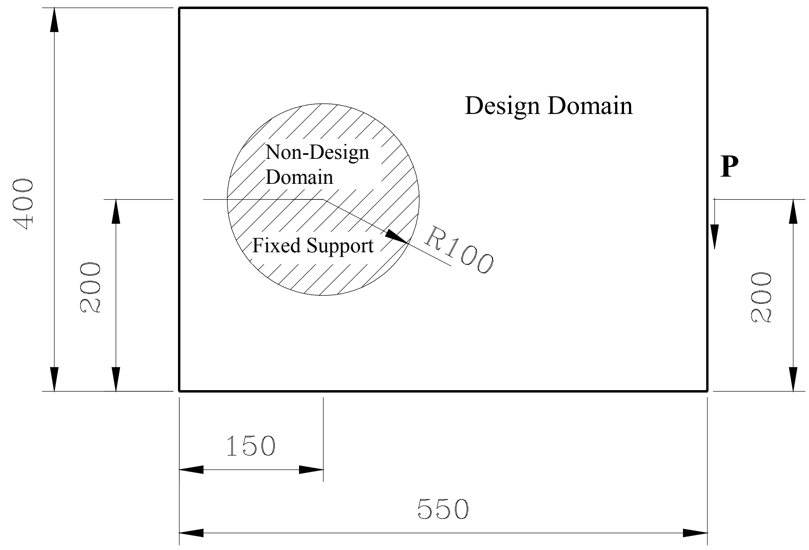


Fig. 3. Design domain of Michell structure

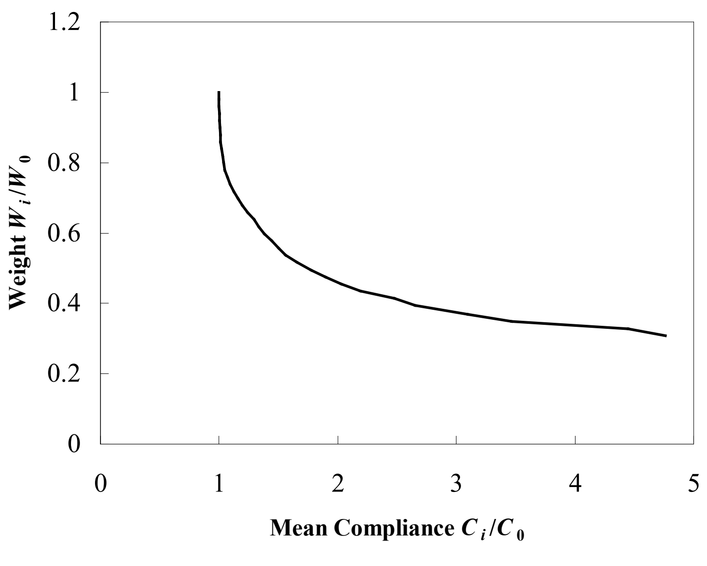
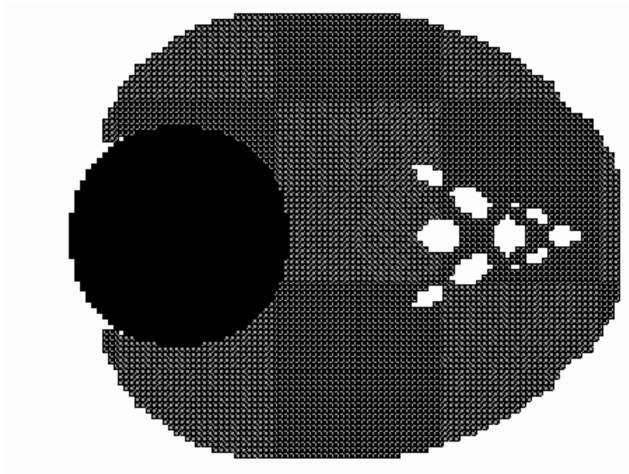
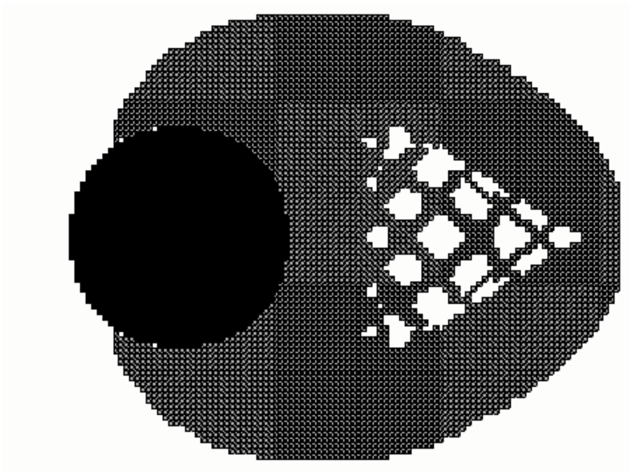


Fig. 4. Performance characteristics of Michell structure

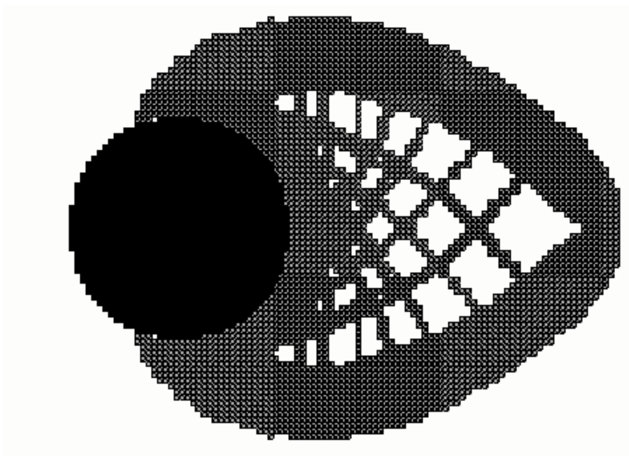
Fig. 5. Performance index history of Michell structure



(a) Optimal Topology at iteration 14, $PI_{ES} = 1.23$



(b) Topology at iteration 17, $PI_{ES} = 1.22$



(c) Topology at iteration 23, $PI_{ES} = 1.19$

Fig. 6. Topology optimization history of Michell structure

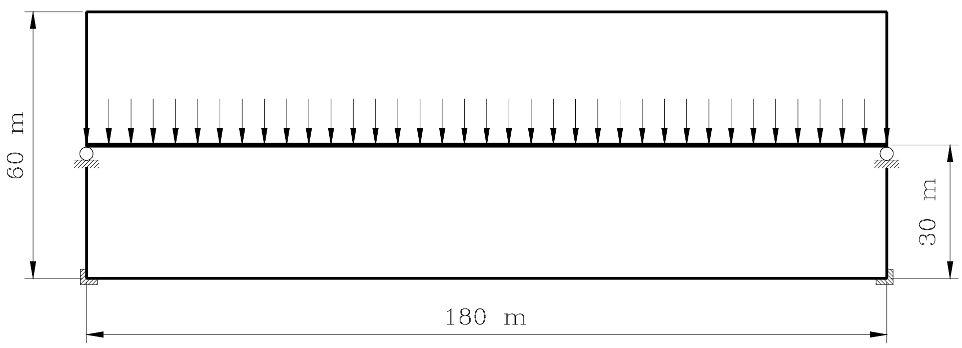


Fig. 7. Design domain of a bridge structure

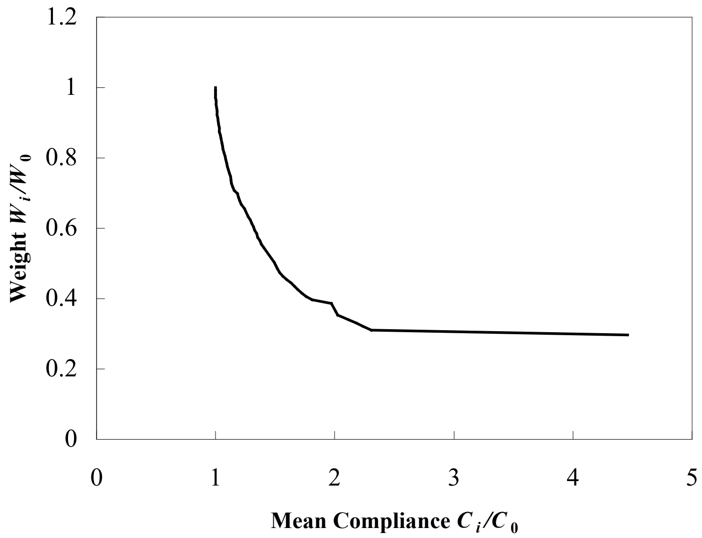


Fig. 8. Performance characteristics of the bridge structure

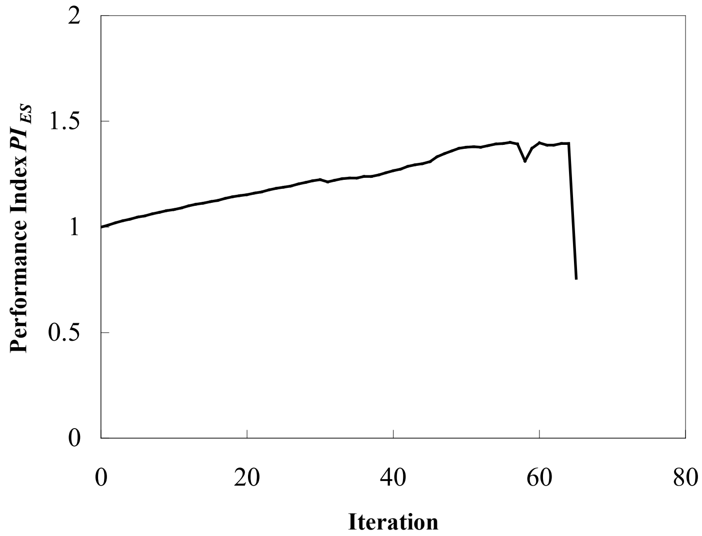
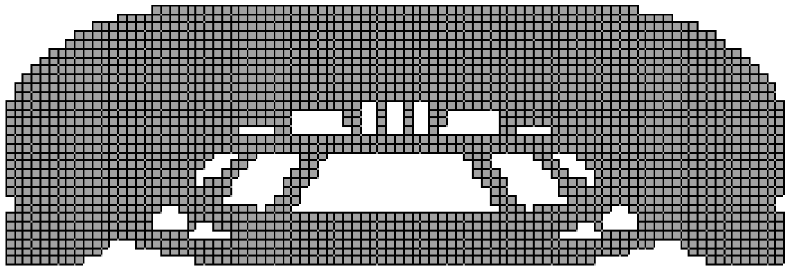
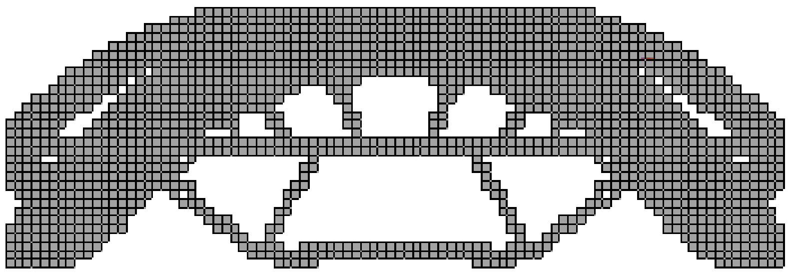


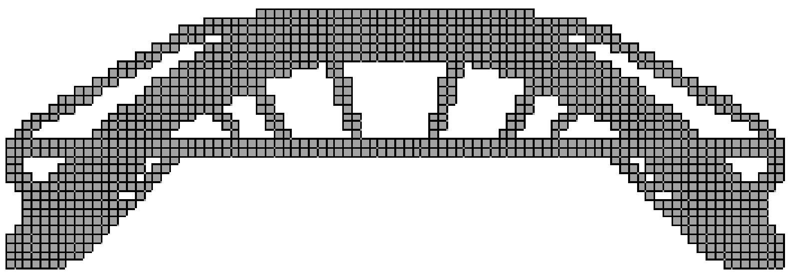
Fig. 9. Performance index history of the bridge structure



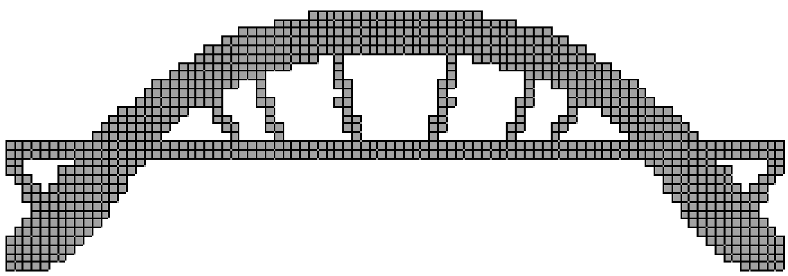
(a) Topology at iteration 20, PI_{ES} = 1.15



(b) Topology at iteration 40, PI_{ES} = 1.26



(c) Optimal topology at iteration 56, PI_{ES} = 1.40



(d) Topology at iteration 64, PI_{ES} = 1.39

Fig. 10. Topology optimization history of the bridge structure

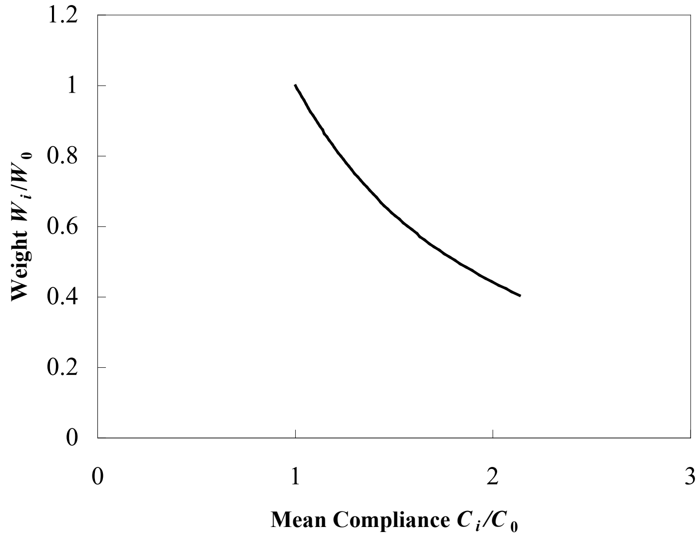


Fig. 11. Performance characteristics of the plate in bending

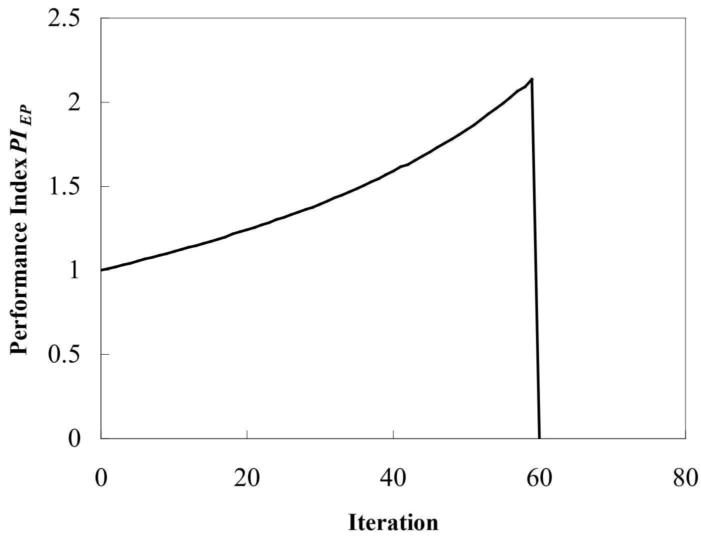
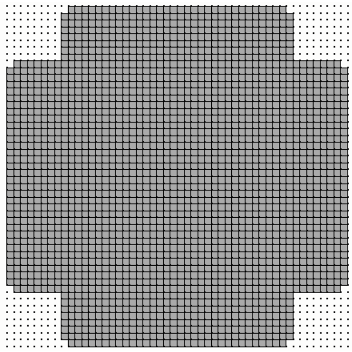
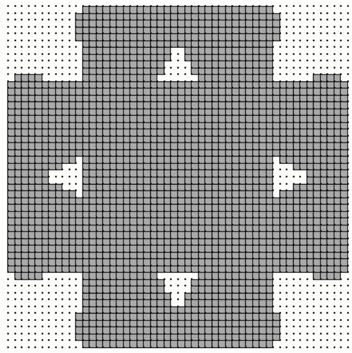
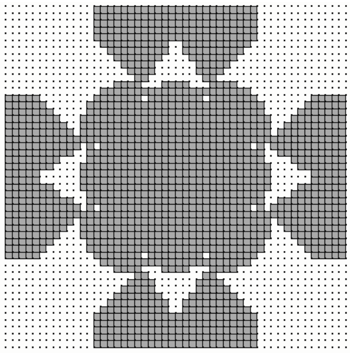
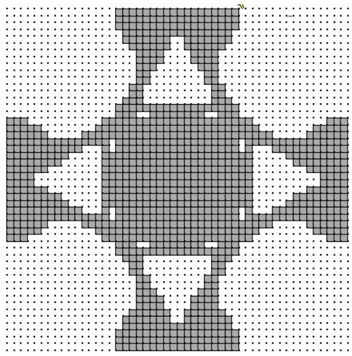


Fig. 12. Performance index history of the plate in bending

(a) Topology at iteration 10, $PI_{EP} = 1.11$ (b) Topology at iteration 20, $PI_{EP} = 1.24$

(c) Topology at iteration 40, $PI_{EP} = 1.59$ (d) Optimum at iteration 59, $PI_{EP} = 2.13$

Fig. 13. Topology optimization history of the plate in bending

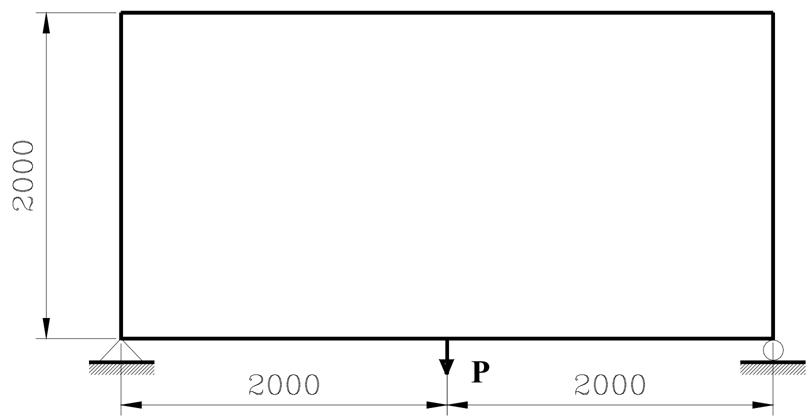


Fig. 14. Design domain of simply supported Michell Structure

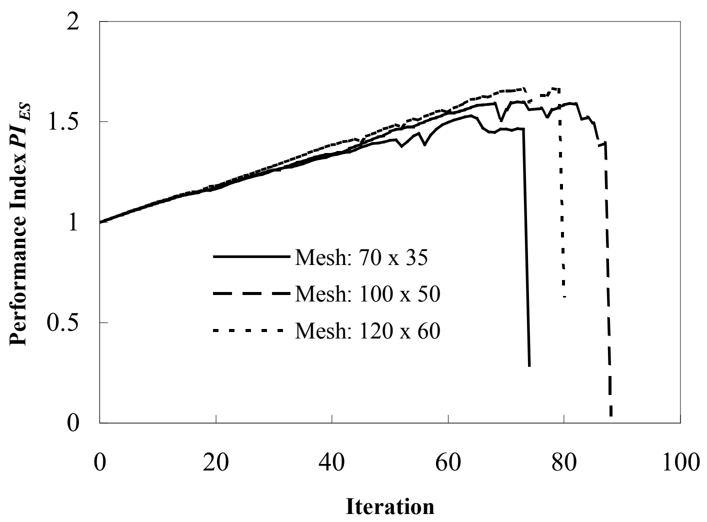
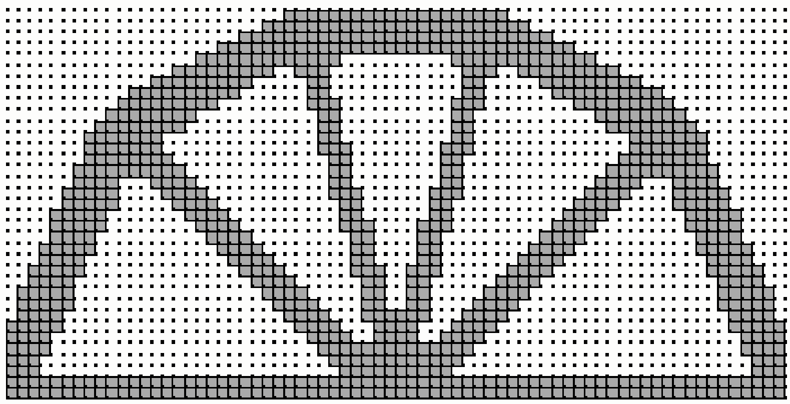
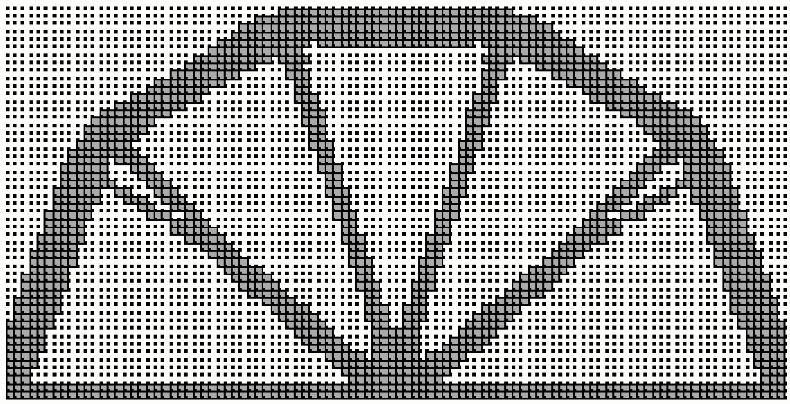


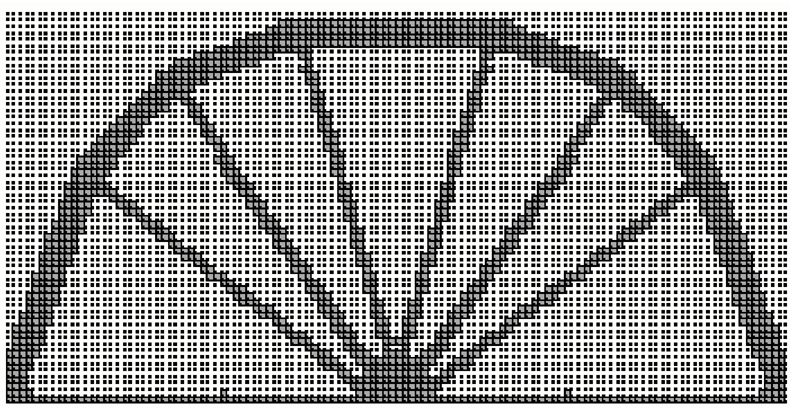
Fig. 15. Performance index histories of simply supported Michell structure



(a) Optimum obtained using a 70 × 35 mesh, PI_{ES} = 1.53



(b) Optimum obtained uisng a 100 × 50 mesh, PI_{ES} = 1.60



(c) Optimum obtained using a 120 × 60 mesh, PI_{ES} = 1.67

Fig. 16. Mesh effects on optimal topologies of Michell structure