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Statistical evaluation of data from tractor guidance systems

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Abstract

Statistical tools are discussed for the analysis of data collected from tractor guidance systems. The importance of both accuracy and precision is discussed, and statistical tools for analysis are considered which incorporate important features of the data. In particular, accuracy is modelled using a generalized least squares model incorporating autocorrelation, and precision using a gamma generalized linear model. The methods are applied to data collected during an experiment conducted with a *Trimble* receiver used with a *Beeline* tractor guidance system. Three different scenarios are considered, then compared: a tractor simulating ploughing a field; the tractor pulling a plough with the receivers on the tractor; the tractor pulling a plough with the *Trimble* receiver on the plough. The change in the precision and accuracy between the scenarios is discussed. Data was collected over repeated swaths for each scenario. After discussing specific statistical techniques for analysis of this type of data, the collected data are analysed; major conclusions are: the data collected from the *Trimble* receiver showed evidence of autocorrelation in the offsets; the implement recorded a variance about three times that recorded by the tractor.

Introduction

Agricultural guidance systems have attracted a growing band of supporters, both operational and in research. Wilson (2000) gave an overview of the research invested in the guidance systems, and forecasts the interest to continue. Supporters of guidance system technology claim advantages such as:

- reducing driver fatigue: guidance systems reduce the effort associated with maintaining accurate vehicle paths (Thuilot et al., 2002; Wilson, 2000); Kocher et al. (2000a) cited a study in which 80% of respondents noted fatigue as the greatest hindrance to performance using non-guided tractors; likewise, Kaminika et al. (1981) showed, in a laboratory experiment, a degradation in steering accuracy when the operator had to share attention between tasks.
- reducing costs: accuracy is increased by reducing ‘skip’ (missed sections) and ‘double-up’ (repeated application) in sections of the field (Thuilot et al., 2002);
- increasing productivity: higher operating speeds are possible (Thuilot et al., 2002);
- improved quality: the driver can focus attention elsewhere to ensure better quality (Thuilot et al., 2002);
- improved safety (Zhang, Reid and Noguchi, 1999);
- less impact on the environment (Bongiovanni and Lowenberg-Deboer, 2004; Wilson, 2000);
- enabling night work with some systems (Wilson, 2000).

Guidance systems are used for planting, hoeing, application of fertilizer, application of pesticides, tillage, etc.

In this paper, we specifically focus on data gathered from using an agricultural guidance system based on Global Positioning Systems (GPS) to maintain accuracy. For such systems, two GPS receivers are required, each requiring an antenna, a controller box containing a computer, and cables. One GPS receiver is located on the tractor, and is called a roving receiver. The second is stationary at a known position, and located near the paddock; this is called the base (or reference) station, and is used to eliminate errors occuring in the roving receiver and hence enhancing accuracy. This is called 'differential GPS', or DGPS. The roving receiver determines the location of the tractor, applying corrections received from the base receiver. In autonomous systems, the information received by the controller is used to automatically steer the tractor. In semi-autonomous systems, these corrections are displayed on an in-cabin screen; the tractor driver uses these visual cues to adjust the heading of the tractor to maintain the required path.

GPS is crucial to these guidance systems. Wilson (2000) identified two limitations with GPS in tractor guidance systems which he anticipated would be overcome eventually with technological developments: the range of field conditions for which accurate methods are needed (such as steep terrain, or interruption of satellite or differential corrections) is diverse; and the time delay for signal processing at high speeds. In a more general study of precision agriculture technology, Robert (2002) identified several barriers—categorized as socio-economic, agronomic and technological—limiting the mainstream use of guidance systems. These include: inadequate skills of farmers unfamiliar with, and possibly skeptical or afraid of, technology; inefficient use, or misinterpretation of, the information; the potential need for using agroconsultants; and incorrect calibration of the equipment.

Other practical impediments exist to the uptake of tractor guidance technology:

- Understanding the different claims of the manufacturers makes it hard to compare systems directly;
- No international standards are available for comparing accuracy (White, 2003);
- Little independent analysis of performance claimed by manufacturers is available (Kocher et al., 2000a), especially in an operational setting;

- The human-factor still exists (ease-of-use; callibration; driver fatigue is still an issue with semi-autonomous systems).

In White (2003), an attempt to provide test protocols for studying accuracy of agriculture guidance systems was developed with engineers, GPS experts and manufacturers. Kocher et al. (2000a) developed a systematic procedure for testing systems in a variety of conditions.

Others have discussed statistical analysis also. For example, Taylor et al. (2003) used a complex Voronoi step interpolation method to analyse correlated data; Kocher et al. (2000b) considered statistical methods based on linear models.

In this study, we are interested in the accuracy of the final path of travel using the GPS-based agricultural guidance technology discussed. The distance between the actual and desired path of travel is called the 'offset' (or 'lateral distance'). Using a commercially available tractor guidance system in conjunction with an additional receiver to log offsets, we compare the accuracy and precision of the specification of the manufacturers; and compare the accuracy and precision of the results in three scenarios (given below) statistically. Thus, the accuracy inherently incorporates the accuracy of the GPS used, and the physical tractor guidance itself. Others have studied guidance systems performance from a physical viewpoint (for example, Cordess et al., 2000; and Thuilot et al., 2002, who examined curved paths).

We focus, in particular, on the accuracy and precision of the systems. By accuracy, we mean the tendency to travel on the desired (target) path on average; by precision, we mean the tendency to be close to the desired path at all times. Both issues are jointly relevant: having a mean accuracy of 0 mm is meaningless if the tractor misses the target by 1000 mm half the time, and -1000 mm the other half; in this case, the accuracy is good while the precision is bad. In general, accuracy is measured using means; precision using standard deviations and variances.

The next section discusses the experiment and the three scenarios; the methods of analysis are then considered, some of which have never been used in the context of this type of data. Then the data is summarized and examined; then accuracy and precision are analyzed.

Materials and methods

The data were collected during simulated ploughing of a paddock on September 2002 in 'Irri South', an irrigation farm at Macalister, Queensland, Australia. The paddock had a previous crop of cotton sown on raised beds and harvested in March 2002. The remaining stubble was ploughed into the soil for the (southern hemisphere) winter. A 27 mm rainfall event prior to data collection caused the soil to be moist during testing. The simulated ploughing considered here ran diagonally across the raised beds.

A *Beeline* Navigator (hereafter just *Beeline*) autonomous tractor guidance system was used to automatically steer the tractor on a predefined path. The *Beeline* system determines the coordinates of the tractor and adjusts the path of the tractor accordingly. In this study, a Caterpillar 95E tractor was used; the *Beeline* electronically actuates the tractor's steering system based on positional information received from the DGPS.

To record the actual path of travel, a *Trimble* MS750 RTK (real time kinematic) receiver (hereafter just *Trimble*) was used, operating at 5 Hz. Differential corrections for post-processing were recorded by a *Trimble* 4700 MSi and logged to a separate TSC 1 controller. Both the *Trimble* and *Beeline* base stations were sited together about 500 m from the test paddock.

There were three aspects of the experiment:

- In the first scenario, the *Trimble* receiver was placed on the tractor and it travelled up and down the paddock ('parallel swathing');

- In the second scenario, the *Trimble* receiver remained on the tractor which pulled an implement (an 8.5 m chisel plough) up and down the paddock;

- In the third scenario, the *Trimble* receiver was relocated to the center-line of the implement (a chisel plough) pulled by the tractor up and down the paddock.

For the first two scenarios, the *Beeline* antenna was centred laterally on the tractor cabin (2.5m above the ground), and the *Trimble* antenna was mounted 376 mm to the left of the *Beeline* antenna. (This lateral displacement was known, and the data suitably adjusted before this analysis.) For scenario three, the *Trimble* receiver was relocated to the implement 5.7 m from the towing point of the tractor and centred laterally. The third scenario is of ultimate interest: the path of the implement. The other two scenarios allow quantitative comparisons of this path to the unencumbered tractor (scenario one) and the path of the tractor while pulling the implement (scenario two).

Each swath was between two fixed coordinates about 900 m apart. (A swath is a run up or down the paddock between turns of the tractor at either end.) The same coordinates were used for each scenario and the tractor driven at about 8 km h^{-1} for each scenario, up the paddock, then back down, and so on. The swaths were 8.5 m apart, which was the width of the implement used in scenarios two and three.

The desired path of travel was identified as follows. The tractor was located at a reference coordinate (waypoint A) at one end of the paddock, and this coordinate logged onto the *Trimble* and *Beeline* controllers. The tractor was then driven to the other end of the paddock (about 900m); the tractor was stopped and this reference coordinate (waypoint B) logged into the *Trimble* and *Beeline* controllers. The *Beeline* tractor guidance system was then used to autonomously guide the tractor between waypoints A and B, and subsequent swaths successively 8.5m apart, while the *Trimble* receiver logged offsets.

The data from the *Trimble* receiver for each scenario was processed and the offset given in millimetres. In this paper, we consider the offset in two, rather than three, dimensions (see Cordess et al., 2000).

The *Trimble* receiver logged data about every second to give some 400 observations per swath for most of the data collection, but at about every 0.7 second for swaths 1 and 2 of scenario two (despite the epoch recording rate being set to one second intervals) giving about 580 observations per swath; see Table 1. There was no obvious explanation for this change in recording frequency. This change in frequency of recording does not detract from the analysis since the analysis fits separate models within each swath. In all cases, data corresponding to turns of the tractor at the end of each swath were edited out of the collected data.

The data recorded by the instruments were analysed after the experiment, based on two broad objectives:

1. A suitable statistical model was identified for modelling the accuracy (using means) of each tractor guidance system in the three scenarios;

2. A suitable statistical model was identified for modelling the precision (using variances) of each tractor guidance system in the three scenarios.

For more information on the experiment, see Hill (2002). The data are available from `http://www.sci.usq.edu.au/staff/dunn/Datasets/applications/science/guide.html`

The manufacturers' claims for the accuracy of the *Beeline* tractor guidance system are that the offsets will be between ± 20mm 95% of the time. The claim appears to be based on the offsets having a normal distribution around the ideal mean of zero, and effectively state the standard deviations are 10 mm. (For normal distributions, 95% of observations are within two standard deviations either side of the mean, and 68% within one standard deviation.) The data collected for the paper suggest the normality assumption is quite reasonable; see the Q–Q plots (also called rankit plots, or normal probability plots; see Weisberg, 1985) in Figure 1 for scenario one; the plots are similar for the other two scenarios.

— FIGURE 1 ABOUT HERE—

The *Trimble* receiver used to record locations claims an accuracy of about $\pm$ 20 mm 95% of the time. This additional *Trimble* receiver, rather than the offsets logged by the *Beeline* receiver forming part of the *Beeline* tractor guidance system, was used to record locations for two reasons. Firstly, the offsets logged by the *Beeline* receiver automatically have potential outiers deleted (removing the time-series feature); secondly, we are ultimately interested in the path of travel of the implement placed on the tractor, which can only be determined by an additional receiver (the *Beeline* receiver must remain located on the tractor to enable automatic guidance).

Each scenario uses the *Beeline* automatic guidance system to steer the tractor and a *Trimble* receiver to record offsets. Each of these components has associated errors; since the same combination is used in each scenario, the variance of this combined error remains approximately constant. This means that any changes in the variances of the recorded offsets from one scenario to another is due to the change in the setup of the scenarios, as discussed above.

The statistical models

In their comparison of an autonomous GPS and three DGPS guidance systems, Coyne et al., (2003) identifi ed the presence of autocorrelation in the recorded offsets.

Their study considered four receivers, four tractor speeds, and three different paths (curved, straight and flattened Figure 8). They modelled the mean of each treatment group rather than the individual logged offsets; thus standard techniques (such as analysis of variance, or ANOVA) assuming independence could be used for analysis. The use of data summaries could potentially conceal vital information. Kocher et al. (2000a) also noted autocorrelations, but Kocher et al. (2000b) appeared to use general linear models for analysis, hence ignoring autocorrelation. Here, we were interested in the individual logged offsets and the modelling techniques were chosen to reflect this.

Model accuracy using a generalized least squares model

In this study, the mean offsets (accuracy) were modelled using generalized least squares (GLS) if significant autocorrelation was detected, or a standard regression model otherwise. Arguably, swath is a random effect and direction a fixed effect. However, we were particularly interested in the differences between the given swaths in terms of reproducibility; accordingly, swath and direction were both treated as fixed effects in what follows.

A generalized least squares (GLS) model takes the form

$$\mathbf{y} = \mathrm{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \tag{1}$$

where $\boldsymbol{\beta}$ is an $p \times 1$ vector of unknown parameters, X is a $n \times p$ design matrix, $\mathbf{y}$ is the $n \times 1$ response vector, and $\boldsymbol{\epsilon}$ is the $n \times 1$ error vector. In standard regression, $\boldsymbol{\epsilon} \sim N(\mathbf{0}, \sigma^2)$, so that $\mathrm{var}(\boldsymbol{\epsilon}) = \sigma^2 \mathrm{I}_n$ for $\sigma^2 > 0$ and an $n \times n$ identity matrix I_n. The GLS model allows for autocorrelation in the errors. Let $\boldsymbol{\Sigma}$ be a symmetric positive-definite matrix; then $\mathrm{var}(\boldsymbol{\epsilon}) = \sigma^2 \boldsymbol{\Sigma}$. The estimate of $\boldsymbol{\beta}$ is then

$$\widehat{\boldsymbol{\beta}} = (\mathrm{X}^T \boldsymbol{\Sigma}^{-1} \mathrm{X})^{-1} \mathrm{X}^T \boldsymbol{\Sigma}^{-1} \mathbf{y} \tag{2}$$

(for example, see Weisberg, 1985). Many different structures of the autocorrelation can be used by changing the form of the correlation matrix $\boldsymbol{\Sigma}$. A common structure of $\boldsymbol{\Sigma}$ is the autoregressive form of order 1 (written AR(1)). In this case, consider two observations recorded at discrete times i and j. The correlation matrix is

$$\mathbf{\Sigma} = \begin{bmatrix} 1 & \rho & \rho^2 & \dots & \rho^{|i-j|} \\ \rho & 1 & \rho & \dots & \rho^{|i-j|-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho^{|i-j|} & \rho^{|i-j|-1} & \rho^{|i-j|-2} & \dots & 1 \end{bmatrix}. \qquad (3)$$

In practice, an estimate of ρ, say $\widehat{\rho}$, must be used (see Pinhiero and Bates (2000) for details). Other autocorrelation structures are also possible, including an exchangeable correlation (each off-diagonal entry of $\mathbf{\Sigma}$ is ρ) and unstructured (each off-diagonal entry of $\mathbf{\Sigma}$ is unrelated to any other off-diagonal entry). If $\mathbf{\Sigma} = \mathrm{I}_n$, we have independence, equivalent to regression.

Model precision using a generalized linear model

The variances of the offsets around the modelled mean (precision) were modelled using generalized linear models (or GLMs; see McCullagh and Nelder, 1989). A GLM may be defined as follows. Independent responses $Y_1, \dots, Y_n$ (here the variances of the offsets about swath means) are observed such that

$$Y_i \sim \mathrm{ED}(\mu_i, \phi/w_i) \qquad (4)$$

where the w_i are known prior weights; and $\mathrm{ED}(\mu, \phi)$ indicates the observations are from an exponential dispersion model distribution (see Jørgensen, 1997), such as the gamma used here, with mean μ_i and dispersion parameter $\phi > 0$. In this application, these prior weights were the sample sizes from which the variances were computed; effectively, these prior weights give relatively more importance to observations based on more data.

The means μ_i are related to linear predictors through a known monotonic link function

g,

$$g(\mu_i) = \mathbf{x}_i^T \boldsymbol{\beta} \tag{5}$$

where $\mathbf{x}_i$ is a vector of covariates and $\boldsymbol{\beta}$ is a vector of unknown regression parameters. Here we use the log-link $g(\mu_i) = \log \mu_i$. Let q be the dimension of $\boldsymbol{\beta}$. To avoid unnecessary complications, we assumed the design matrix X with rows $\mathbf{x}^T$ had full column rank. The maximum likelihood estimator $\hat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$ was computed using the well-known iteratively reweighted least squares algorithm proposed by Nelder and Wedderburn (1972). This iteration uses working weights given by

$$\frac{w_i}{V(\mu_i)\dot{g}(\mu_i)^2} \tag{6}$$

where, for the gamma distribution considered here, $V(\mu_i) = \mu_i^2$. The special case of the gamma distribution used here (the χ^2 distribution) sets $\phi = 2$.

Let σ_i^2 represent the estimated variance of the offsets about the modelled mean in scenario i (for each receiver, we use as replications the estimate from each swath). A suitable model for the variances is

$$\log(\sigma_i^2) = \beta_0 + \beta_1 S, \tag{7}$$

where S is a dummy variable for the scenario number. The logarithm ensures the variances remain positive, and is the link function in the language of GLMs.

Using treatment contrasts—as we do throughout for all categorical variables—means β_0 is the level of precision in the base level (scenario one unless otherwise stated). Rewrite

$$\sigma_i^2 = \exp(\beta_0)\exp(\beta_1 S); \tag{8}$$

thus, $\exp(\beta_0)$ is the modelled variance in the first scenario, and $\exp(\beta_1 S)$ represents the loss in precision (increase in variance) in later scenarios compared to the baseline.

The variances are modelled using the form above, based on a gamma GLM. The error variances from a model based on a normal distribution (as used here) have a χ^2 distribution, a special case of a gamma distribution. The idea of modelling the variances using the gamma distribution and with a logarithm link function is seen, for example, in Smyth (1988).

All the analyses in this study were done in R (2005), using the R package `nlme` (Pinhiero and Bates, 2000) to fit the GLS models.

Results

General

In scenario one, the tractor covered five swaths; four swaths were done for the other two scenarios. The number of observations was almost equal within each swath (see Table 1), except that the frequency data logged by the *Trimble* receiver changed (inexplicably) in scenario two between swaths 2 and 3.

— TABLE 1 ABOUT HERE —

The tendency to record positive offsets possibly suggests a calibration problem with the equipment. As a typical example, the data from scenario one are plotted over time in Figure 2. The data show evidence of autocorrelation (see swaths 2 and 3 in Figure 2). A typical autocorrelation function for the offsets by swath are shown in Figure 3, where strong autocorrelation is evident. Considering the partial autocorrelation function (not shown) in conjunction with the autocorrelation function suggests an AR(1) model (see, for example, Chatfield, 1996) within swath is appropriate.

— FIGURE 2 ABOUT HERE —

— FIGURE 3 ABOUT HERE —

Boxplots of the offsets by swath are given in Figure 4 for scenario one. The offsets are symmetrically distributed about the medians within each swath (and are approximately normally distributed on closer inspection; see Figure 1).

— FIGURE 4 ABOUT HERE —

Analysis of accuracy

Swath and direction effects are usually both important features, but neither consistently the most important feature. Swath is arguably a random effect and not a fixed effect; since we are particularly interested in the variations across the given swaths, swaths will be treated as fixed effects here. The fitted models are summarized in Table 2. Consider an example of a fitted model; the fitted model for the offsets recorded by the *Trimble* receiver in scenario one is

$$\mathbf{y} = \underset{(0.54)}{10.71} + \underset{(1.22)}{2.46}\, S_2 - \underset{(1.21)}{3.99}\, S_3 + \underset{(1.21)}{1.51}\, S_4 + \underset{(1.20)}{1.39}\, S_5 \tag{9}$$

with the AR(1) term $\hat{\rho} = 0.58$, where S_i is 1 in swath i, and is zero otherwise. (The figures in parentheses under the estimated parameters are the computed standard errors.) The first estimated parameter is the estimated mean straight-line path (allowing for the autocorrelation) in swath 1; the subsequent estimated parameters are the differences between the swath 1 mean and the mean in the others swaths, after allowing for the effects of modelled autocorrelation. For example, swath 3 is about 4 mm further to the left of the mean path of swath 1 (after accounting for autocorrelation).

— TABLE 2 ABOUT HERE —

The results from the *Trimble* receiver consistently show signs of autocorrelation in the offsets. This is probably a result of the *Beeline* autonomous guidance system requiring a finite time to correct the tractor path. The autocorrelation terms for the *Trimble* models are large; ignoring this autocorrelation when modelling the offsets would lead to different (and incorrect) models.

A plot of the modelled path over the data (not shown) indicates that the models fit well, and taking account of autocorrelation in the model substantially improves the fit (as expected). For model diagnostics, normalized residuals are used; these residuals are the standardized residuals pre-multiplied by the inverse square-root factor of the estimated error correlation matrix. Q–Q plots (also called normal probability plots) show that the normalized residuals are normally distributed; as an example, Figure 5 shows the Q–Q plots and autocorrelation function of the normalized residuals from the swath 1 in scenario one. The plots show the distribution of the residuals closely resemble a normal model with little autocorrelation.

— FIGURE 5 ABOUT HERE —

Analysis of precision

In this section, we compare the variances of the errors (the inverse of precision) computed from the offsets about the swath means, a proxy for the ideal path of travel for the tractor. The summary data are given in Table 1 where the 'variance' column is of interest here.

A gamma GLM with a log-link was used to model the variances for each scenario, as previously discussed, using the sample size from which the variances were computed as a prior weight. The fitted model for the *Trimble* receiver is:

$$\log(\sigma^2) = \underset{(0.032)}{5.00} - \underset{(0.054)}{0.1058}\, S_2 + \underset{(0.048)}{1.11}\, S_3 \tag{10}$$

where the standard errors of the parameters are in parentheses; S_2 is one for scenario two and is zero otherwise; S_3 is one for scenario three and is zero otherwise. The model shows the difference in variance between scenarios one and two is slightly significant ($z = -2.33$; $P = 0.020$); however, the significance of the difference is large between the variances in scenarios one and three ($z = 23.2$, $P \approx 0$). Following the methods developed earlier, the variances have increased by a factor of about $\exp(1.11) = 3.0$ from scenario one to three.

In all the above, the dispersion parameter is set to $\phi = 2$ as required for the χ^2 distribution. The usual estimates of ϕ suggest $\phi = 2$ might be too small. However, with such small sample sizes (the model is based on just 13 variances across three scenarios), the results from the small sample sizes do not suggest changing this in light of the theory.

In summary, the *Trimble* results indicate the implement has significantly reduced precision; more specifically, the variances of the offsets of the implement compared to that of the tractor increased threefold.

Conclusions

In this paper, we take data from three scenarios as recorded by a *Trimble* receiver when the tractor is autonomously guided by a *Beeline* autonomous tractor guidance system. The accuracy and precision of each scenario is analysed using statistical techniques.

We used a generalized least squares model for modelling accuracy using an AR(1) correlation structure. The autocorrelation was very evident in the data probably because the autonomous guidance system took a finite time to correct the tractor path.

The precision (inverse of variance) was analysed using a gamma generalized linear model with $\phi = 2$ (effectively a χ^2 model). The variance of the path of implement is about three times that of the path of the tractor itself, comparing scenarios one and three for the *Trimble* receiver data.

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Footnotes

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Other footnotes

4: The given P-values for the direction or swath model are for comparing to the linear model (no autocorrelation considered) for that effect. The AR(1) autocorrelation is noted and followed by the estimate of the AR(1) term ρ (denoted $\widehat{\rho}$); and a nominal 95% confidence interval for this parameter. The P-value is for comparing this model to the same model without the AR(1) component.

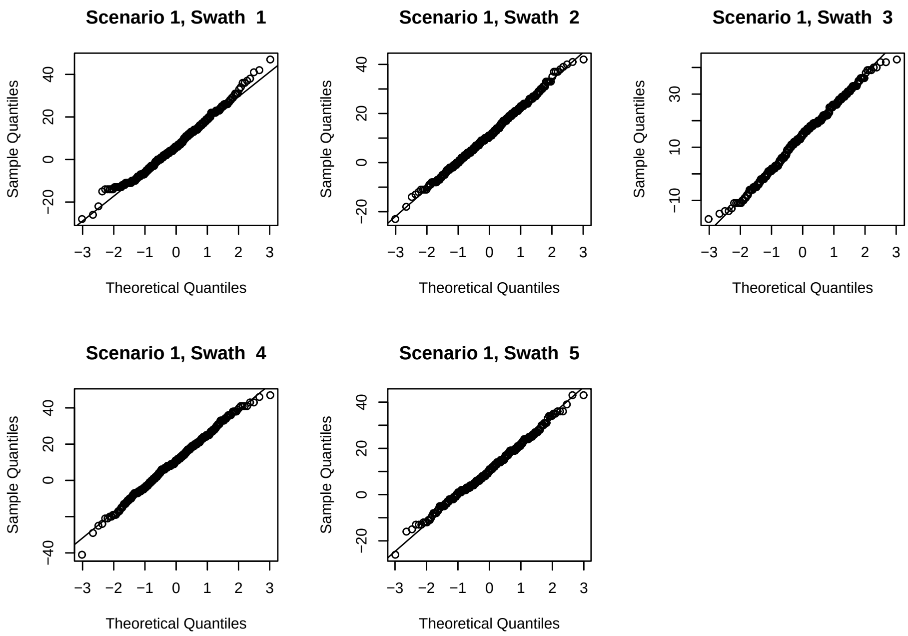

Figure 1: Q–Q plots of the data for scenario one. The solid lines are the target normal distribution. In every case, there is no evidence of non-normality in the data.

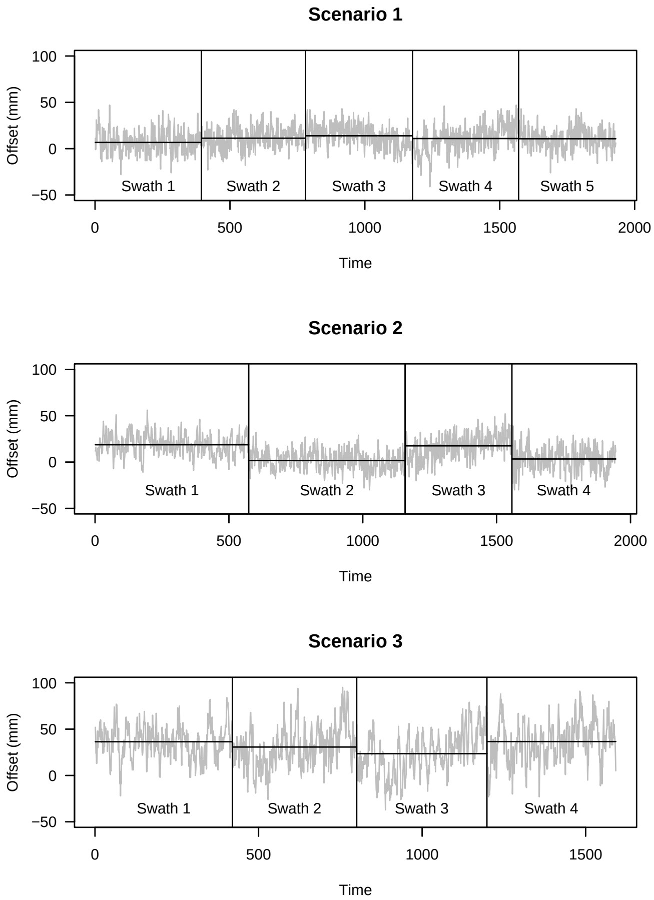

Figure 2: Plots over time of the offsets recorded by the *Trimble* receiver for each scenario. The swaths are separated by vertical lines, and the mean for each swath shown by a horizontal line. The same vertical scale is used on each plot.

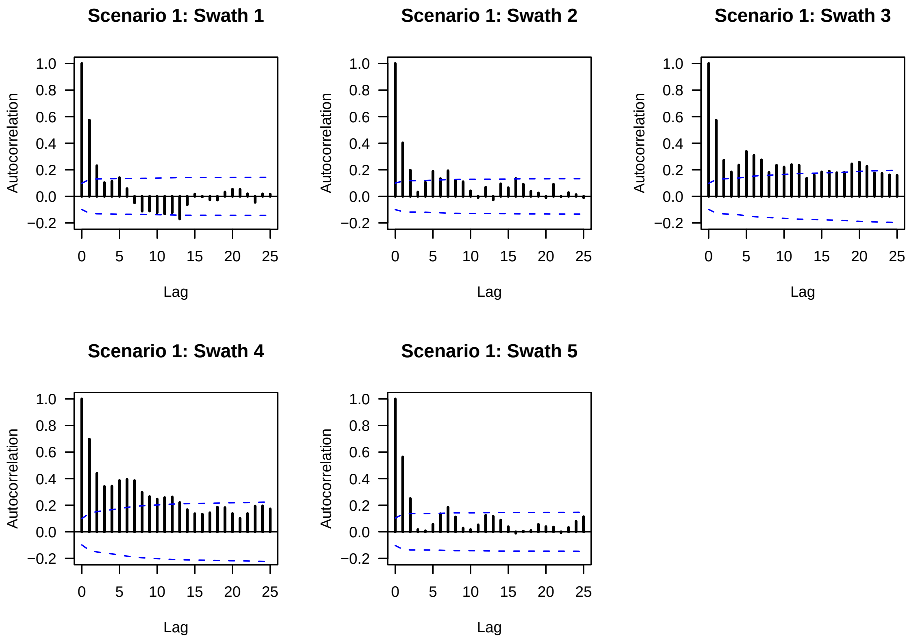

Figure 3: The autocorrelation function for offsets by swath as recorded by the *Trimble* receiver for scenario one. The dashed lines are the nominal 95% confidence intervals for detecting significant autocorrelations. There is strong evidence of low-order autocorrelation.

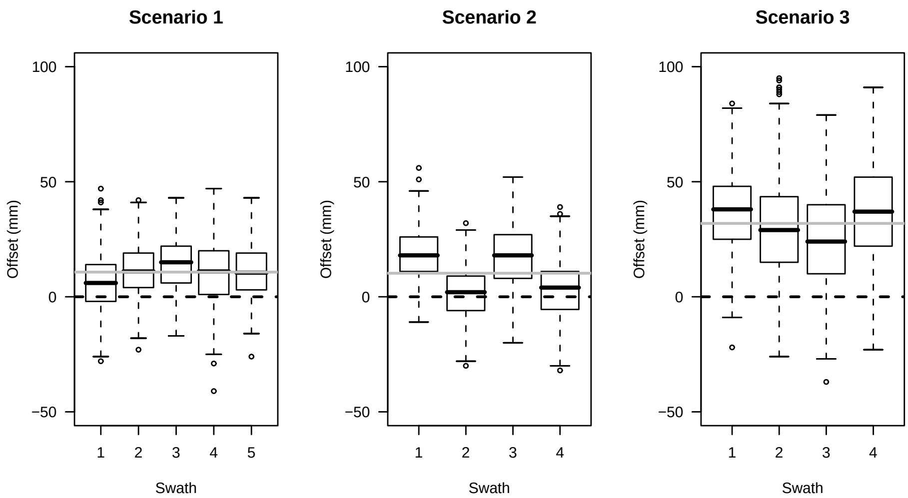

Figure 4: Boxplots of the offsets recorded by each receiver by swath for each scenario. The thick horizontal dashed lines are the target offsets of zero millimetres; the thick horizontal solid lines are the mean offsets computed as the overall mean offset for each receiver. The middle line in each box represents the *median* offset within each swath.

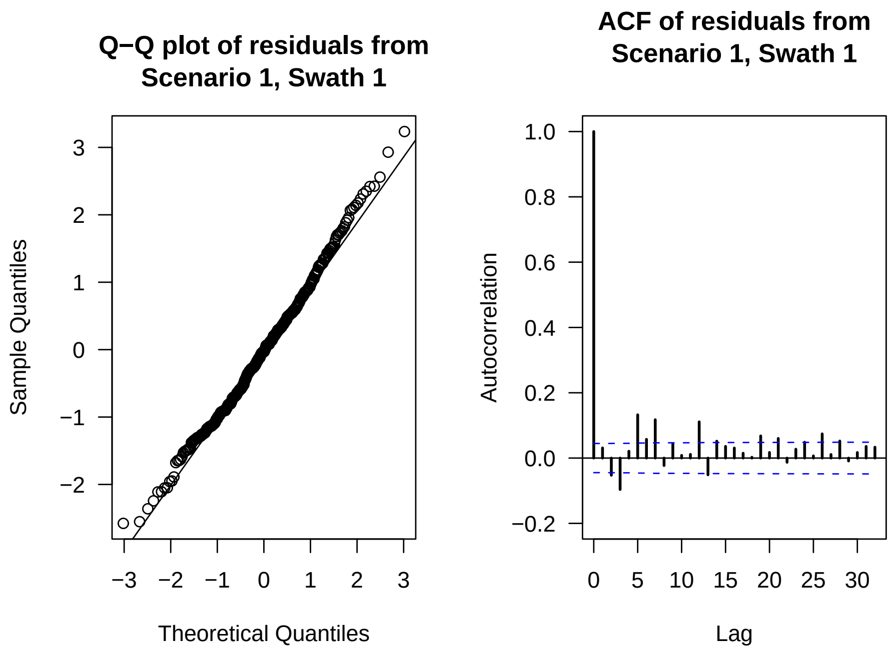

Figure 5: Two residual plots from the fitted models. As examples, swath 1 from scenario one is used. Left, a Q–Q plot of the residuals suggesting the distribution of residuals closely follows a normal distribution; right, the autocorrelation function showing the reduction in autocorrelation evident in the residuals. Both plots indicate the fitted model is adequate.

Table 1: The number of observations, the means, standard deviations (both in millimeters) and variance (in millimetres-squared) of the recorded offsets by swath for each scenario. (Note the frequency data logged by the *Trimble* receiver changes from swath 1 and 2 to swath 3 and 4 in scenario two.)

Swath	Obs.	Mean	Std Dev	Variance
		Scenario 1		
1	394	6.72	12.10	146.23
2	386	11.33	11.10	123.03
3	397	13.91	11.77	138.57
4	393	10.90	14.38	206.67
5	360	10.72	11.08	122.73
Overall	**1930**	**10.71**	**12.37**	**153.08**
		Scenario 2		
1	574	18.7	11.0	101.46
2	584	1.6	10.1	164.74
3	399	17.5	12.8	160.29
4	388	3.3	12.7	102.49
Overall	**1945**	**10.24**	**14.00**	**196.01**
		Scenario 3		
1	420	36.41	17.37	301.86
2	380	30.67	22.35	499.60
3	398	23.54	22.92	525.20
4	394	36.62	22.05	486.09
Overall	**1592**	**31.88**	**21.87**	**478.39**

Table 2: The models fitted to the offsets for each receiver in each scenario. Scenarios 2 and 3 are the same for the *Beeline* receiver[4].

Scenario	Model	P-value
1	swath effect	$P \approx 0.0011$
	AR(1) autocorrelation:	
	$\widehat{\rho} = 0.58\ (0.546, 0.620)$	$P < 0.0001$
2	direction effect	$P \approx 0$
	AR(1) autocorrelation:	
	$\widehat{\rho} = 0.68\ (0.646, 0.712)$	$P \approx 0$
3	swath effect	$P \approx 0$
	AR(1) autocorrelation:	
	$\widehat{\rho} = 0.78\ (0.749, 0.812)$	$P < 0.0001$