Generalized Kruithof Approach for Traffic Matrix Estimation

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***Abstract*— In this paper, the traffic matrix estimation problem is formulated as an nonlinear optimization problem based on the generalized Kruithof approach which uses the Kullback distance to measure the probabilistic distance between two traffic matrices. In addition, an algorithm using the affine scaling method is provided to solve the constraint optimization problem.**

***Index Terms*— Traffic Matrix, Kruithof Method, Affine Scaling Method, Kullback distance**

I. Introduction

A traffic matrix is defined as a matrix whose elements represent the amount of traffic demand between a given origin to destination node pair in a network. It plays an important role in a variety of network applications such as network dimensioning, planning, optimization, and traffic engineering. However, due to financial and technical difficulties in measuring and determining the traffic matrix directly, an inference approach is the subject of great interest. The methods infer the traffic matrix with the given observed link loads that can be obtained from routers using the SNMP protocol.

Such inference methods rely upon solving systems of equations that are highly under-constrained. The number of unknown variables, which is the number of origin and destination pairs in a network, increases in proportion to the square of the number of nodes while the number of constraints, which is the number of links in a network, increases linearly. Therefore, as the size of the traffic matrix increases, the problem becomes increasingly under-constrained. When a problem is under-constrained, infinite numbers of solutions satisfies the problem. The traffic matrix problem is to find one solution among the infinite numbers of solutions.

Kruithof method [1] has been widely used in telephony network to balance a given fraction matrix with the expected row and column totals. However, the method lacks the ability to accommodate extra information because it was originally introduced to cooperate with the row and column totals only. To address this problem, the Kruithof problem has been generalized as an nonlinear optimization problem [2] using the Kullback distance.

In this paper, the traffic matrix estimation problem is formulated as an nonlinear optimization problem based on the generalized Kruithof approach which uses the Kullback distance to measure the probabilistic distance between two traffic matrices. The idea of the approach is to select one solution among infinite numbers of solutions by minimizing the Kullback distance from the prior solution. The proposed method is compared with the previously known methods, which are the LP methods [6][7], the least squared [4], and the Information Theory approach [5].

The nonlinear optimization problem is solved using the affine scaling method which is one of the interior point methods which cut across the interior of the feasible area to reach an optimum solution. The affine scaling method is the simplest implementation of all interior point methods, as well as it has the only interior point strategy which approaches a solution by monotonically decreasing the original objective function [3]. We do not provide the detail implementation of the method but a strategy to find a starting point for the affine scaling method by the geometric analysis.

The rest of this paper is organized as follows: In Section II, we explain the under-constrained problem of traffic matrix estimation. Then, in Section III, we explain the theory of our problem formulation. Section IV describes a strategy to accelerate the convergency of the formulated problem by the geometric analysis of the problem. Section V provides the comparison result among deterministic methods. In Section VI, a simulation test-bed is proposed and an experiment is described. Lastly, results and discussions are presented in Section VII.

II. Under Constraints Problem For Traffic Matrix Estimation

Estimating a traffic matrix can be described by the vector equation (1).

$$Y = AX \tag{1}$$

where Y is the vector of measured link loads, A is a routing matrix, and X is the vector of traffic demands. In an IP network, the routes can be obtained by noting that most intra-domain routing protocols (eg OSPF and IS-IS) are based on a shortest path algorithm such as the well-known Dijkstra or Bellman-Ford algorithms; also, link volumes in an IP network are typically available from SNMP data. The traffic demands X are unknown, and need to be estimated from the given Y and A. However, it turns out that there may be an infinite set of traffic demands satisfying the given information because linear equation (1) is an under-constrained system. This can be illustrated by the following example.

In Fig. 1, the three node network has two links with three flows. These three flows need to be estimated from measurements of the two link loads which are 12 and 16

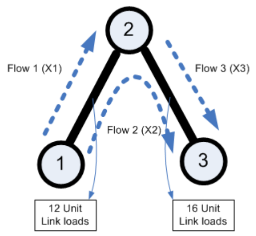

Fig. 1. 3 node network example.

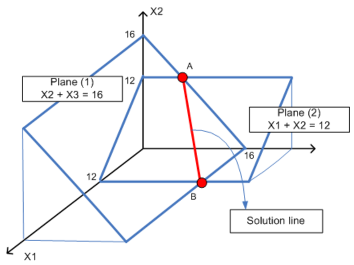

Fig. 2. Solution space

respectively. The sum of flows 1 and 2 are equal to the measured link load which is 12 and the sum of flow 2 and 3 is the same as the measured link loads 16. The two constraints are illustrated in Fig. 2 based on equations $X_1 + X_2 = 12$ and $X_2 + X_3 = 16$ respectively. This is an under-constrained problem because the number of unknown variables is more than the number of constraints. Therefore, the problem defines a *solution plane* rather than giving an unique point for the solution. In Fig. 2, the line AB represents the solutions which satisfy both constraints. Whatever technique is used for traffic matrix estimation, a solution from the method should lie on the line AB to satisfy the inter-link measurement constraints.

III. Problem Formulation

As mentioned previously, there are infinite numbers of solutions to satisfy the vector equation (1). The traffic matrix estimation problem is to select one of the infinite solutions. The solution is chosen by calculating the closeness from the given prior solution because it is assumed that the prior solution represents some characteristics of the real traffic matrix. The closeness can be represented as the Euclidean distance [4], or the Kullback distance [5]. In other words, when a prior solution is given, a solution is chosen from the feasible area by minimizing the Euclidean or Kullback distance.

We formulated the nonlinear optimization problem using the Kullback distance. It minimizes the Kullback distance between the prior solution and the feasible area, which satisfies the constraint (5). The approach is different from the previously suggested method [5] using the Kullback distance. The difference will be discussed in Section V.

Suppose that there are two traffic matrices M and X. The former is a prior traffic matrix, and the latter is the unknown traffic matrix. The both matrices are $n \times n$ matrices, and elements of the traffic matrices are represented as lowercase m_{ij} and x_{ij}. Note that $p_{ij} = x_{ij}/T_X$ and $q_{ij} = m_{ij}/T_M$ where T_X and T_M are total demands denoted by $\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}$ and $\sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}$. Therefore, the total sums of p_{ij} and q_{ij} are equal to one as follow.

$$\sum_{i=1}^{n}\sum_{j=1}^{n} p_{ij} = 1 \quad \text{and} \quad \sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij} = 1 \tag{2}$$

The probabilistic distance between p_{ij} and q_{ij}, called Kullback distance $K(p_{ij}, q_{ij})$, is defined below.

$$K(p_{ij}, q_{ij}) = \sum_{i=1}^{n}\sum_{j=1}^{n} p_{ij} \log \frac{p_{ij}}{q_{ij}} \tag{3}$$

Then, x_{ij}/T_X and m_{ij}/T_M are replacing p_{ij} and q_{ij} respectively. After all, the optimization problem is formulated by minimizing the Kullback distance from the given prior solution to the feasible area, which satisfies the constraint (5).

$$\text{Minimize} \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{x_{ij}}{T_X}\{\log \frac{x_{ij}}{m_{ij}} - \log \frac{T_X}{T_M}\} \tag{4}$$

Subject to

$$\sum_{k=1}^{K} A_{\ell k} X^{(k)} = Y_\ell \quad (\ell = 1, .., L) \quad \forall X^{(k)} \geq 0 \tag{5}$$

where Y is the vector of measured link loads, A is a routing matrix, and X is the vector of traffic demands. $(X = x^1_{(1,1)}, x^2_{(1,2)}, ..., x^{K-1}_{(n-1,n)}, x^K_{(n,n)})$ and, K and L are the number of flows and the number of links respectively.

IV. A Choice of the Starting Point for Affine Scaling Method

Any existing interior point method requires a feasible starting point, and the choice of the feasible starting point effects on the convergency speed of the interior point algorithm. In our implementation, the starting point is chosen as follows.

A point x^0 is selected by minimizing the Euclidean norm from a zero coordinate O using the pseudo-inverse method $x^0 = A^T(AA^T)^{-1}Y$. However, the point is far from the optimum solution and does not have a physical meaning except it is on the feasible region. The point x^0 needs to be moved to the other feasible point which produces a smaller objective value. From the geometric analysis in Fig.3, the moving direction can be decided.

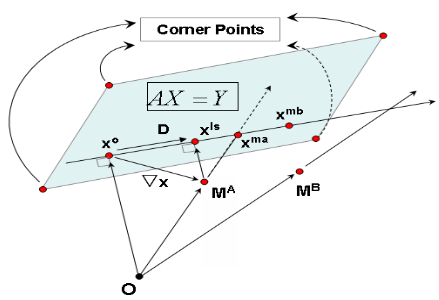

Fig. 3. Geometric Analysis

Assume that there is a prior solution M^A. The optimization process is to find a "probabilistically" closest point on the feasible region from the prior solution M^A in terms of calculating the Kullback distance. The Kullback distance between the prior solution M^A and any point on the line extended from line $\overline{OM}^A$, is zero.

For instance, if the coordinate of the prior solution M_A is m_i (i=1,...,n), any point on the extended line $\overline{OM}^A$ can be represented as k times m_i ($k \times m_i$). Therefore, the Kullback distance between the prior point m_i and any point ($k \times m_i$) on the extended line $\overline{OM}^A$ becomes zero ($\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{km_{ij}}{kT_M}\{\log\frac{km_{ij}}{m_{ij}} - \log\frac{kT_M}{T_M}\} = 0$). If the extended line $\overline{OM}^A$ goes through the feasible region, the cross point x^{ma} between the extended line $\overline{OM}^A$ and the feasible region becomes the optimum solution. Therefore, the searching direction D to obtain the optimum solution x^{ma} from the pseudo point x^0, can be represented as follow.

$$D = P\nabla x = [I - A^T(AA^T)^{-1}A]\nabla x \tag{6}$$

where $P = I - A^T(AA^T)^{-1}A$ is the projection matrix into the null space of A and $\nabla x = M^A - x^0$ (M^A is the prior point and x^0 is the pseudo-inverse result). The direction ∇x is projected onto the hyperplane which is the feasible region formed by $AX = Y$ so that any movement along the direction D can keep the feasibility. Then, the optimum solution x^{ma} is represented as follow.

$$x^{ma} = x^0 + \alpha D \tag{7}$$

α is decided by a line search which is one-dimensional minimization technique between $0 < \alpha < \alpha_{max}$.

$$\alpha_{max} = \min\{\frac{-(x_i^0)}{D_i}\} \quad (D_i \in (D_i < 0)) \tag{8}$$

From the basic calculation, α minimizes the objective function $f(x^0 + \alpha D)$ when the directional derivative $\frac{d}{d\alpha}f(x^0 + \alpha D)$ is equal to zero. In this case, the minimum of the objective function $f(x^0 + \alpha D)$ is zero so that the point x^{ma} becomes the optimum solution (Only one iteration is required to find the optimum solution).

An another case exists to make the problem more complex. Let's consider another prior solution M^B in Fig.3. In this time, the extended line $\overline{OM}^B$ does not go through the feasible area. The searching direction D and the line search are applied same as the previous case. Suppose that the point x^{mb} is the minimum point from the line search, then the point x^{mb} can be a starting point for the affine scaling method if all elements of the point x^{mb} are positive. However, the point x^{mb} may contain some negative elements which have no physical meaning. To overcome the problem, Iterative Proportional Fitting (IPF) is applied as suggested in [4].

V. Difference From the Other Deterministic Approaches

The technique suggested by the authors belongs to the deterministic techniques, which mean that the link load measurements are regarded as solid constraints rather than as statistical data. Table 1 shows the problem formulation of these deterministic approaches with their objective functions and the constraints.

	Objective Functions	Constaint
Linear Program	$(Max)\sum_{i=1}^{n}\sum_{j=1}^{n}\omega_{ij}x_{ij}$	$Ax \leq Y$,
Least Square	$(Min)\sum_{i=1}^{n}\sum_{j=1}^{n}(x_{ij} - m_{ij})^2$	$Ax = Y$
Information Theory	$(Min)\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{x_{ij}}{T_X}\{\log\frac{x_{ij}}{m_{ij}}\}$	
Generalized Kruithof	$(Min)\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{x_{ij}}{T_X}\{\log\frac{x_{ij}}{m_{ij}} - \log\frac{T_X}{T_M}\}$	

Table 1. Different deterministic approaches for the traffic matrix estimation

The objective function of the LP approaches [6][7] uses w_{ij} as a weight for OD pair x_{ij}. When the w_{ij} represents the hop counts of each OD pair x_{ij}, the objective function is parallel to the hyperplane which satisfies the $AX = Y$. The hop counts of each OD pair is equivalent to the sum of each column of A. For instance, each column and each row of A represent each flow and each link respectively. Therefore, a column sum of the matrix A means how many links the flow, involved the column, goes through.

When the objective function is parallel to the feasible region formed by $AX = Y$, infinite range of the same optimum solutions are possible. To select one of them as an solution, two main algorithms are available in the LP problem. One is the Simplex Method which chooses an optimum solution from the corner points of the feasible space. The other is the Interior Point Method (IPM) which cuts across the interior of the feasible area to reach an optimum solution.

In Fig. 3, while the Simplex Method selects a solution among the corner points, the iteration of the interior point method starts from any point satisfying with $Ax \leq Y$, then improves the objective value following a direction which is constant in a linear program (equal to $\nabla_x f(x) = w_{ij}$), since the objective value decreases most rapidly along this direction. The interior point method chooses a solution which is a cross point between a line extended from the prior solution to the search direction and the feasible region.

For the least square approach [4], after the prior solution M^A is obtained, a line is drawn perpendicular to the feasible region from the prior solution M^A. The point x^{ls}, which is "geographically" closest to the prior solution M^A, is the solution of the least square method.

While the least square approach selects a geographically closest point in the feasible region from the prior solution M_A, the information theory [5] and the generalized Kruithof approach chooses a "probabilistically" closest point defined as the Kullback distance.

The difference between the information theory and the generalized Kruithof approach is that the nonlinear objective function of the Kruithof approach has zero Kullback distance between the prior solution M^A and any point on the line extended from line $\overline{OM^A}$.

Therefore, if the extended line $\overline{OM^A}$ goes through the feasible region, the Kruithof approach selects the point x^{ma} as an optimum solution. On the other hands, in the information theory approach case, the point x^{ma} does not give zero objective value, and also is not necessary to be the smallest objective value in the feasible area. In other words, the x^{ma} may not be chosen as the optimum solution. It was also mentioned in [5] that the result of the information theory approach was similar that of the least square method with the square root weight.

Let's see the below Fig. 4 and Fig. 5, which demonstrate the variation of objective value of the Kruithof approach and the information theory respectively with the given prior solution $m_1 = 1$ and $m_2 = 1$.

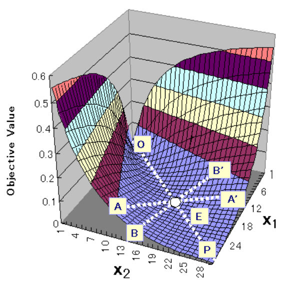

Fig. 4. Generalized Kruithof - the objective value by varying two elements x1 and x2 with the given prior solution (m1:1,m2:1)

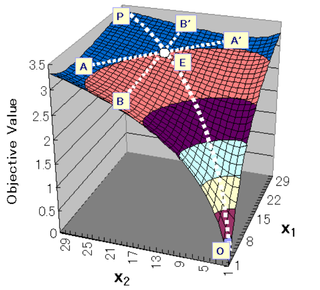

Fig. 5. Infomation theory - the objective value by varying two elements x1 and x2 with the given prior solution (m1:1,m2:1)

In the Kruithof case in Fig. 4, the objective values are zero along the line $\overline{OP}$ because the probabilistic distributions of each point along the line $\overline{OP}$ ((1,1),(2,2),...,(n,n)) is the same as that of the prior solution (1,1). If the figure is cut vertically following the line $\overline{AA'}$ or $\overline{BB'}$ which represents a possible feasible region, the parabola contour defined by the intersection is observed in each case and the parabola contour has the minimum point E which is one of points on the line $\overline{OP}$.

However, in the information theory case in Fig. 5, while the minimum point of the parabola contour by cutting the line $\overline{AA'}$ is a point E on the line $\overline{OP}$, the minimum point of the parabola contour by cutting the line $\overline{BB'}$ is not a point E but a point somewhere between B and E. It means that the information theory does not choose the point E as an optimum solution even though the point E has the minimum probabilistic distance to the prior solution. In addition, the objective value of the information theory seems to increase in proportion to the Euclidian distance from the prior point(1,1), then it indicates the objective function of the information theory behaves similar to that of the least square method.

VI. Simulation Tools & Network Topology

To judge the performance of the proposed method, the linear programming approach, the least squared method, and the information theory approach have been implemented. Table 2. outlines the implementations. A simulation program, using the C^{++} language, was built to simulate a network and to generate traffic distributions for the network.

	Implementations
Linear Program	Simplex, and Interior Point Method (GLPK)
Least Square	SVD and IPF
Information Theory	SVD and IPF with the square root weights
Generalized Kruithof	Affine Scaling Method

Table 2. Implementation of the deterministic methods

GLPK [8] (GNU Linear Programming Kit), which is an open source libraries, offers C and C^{++} libraries to solve linear programming and related problems.

The decomposition technique, Single Value Decomposition (SVD), is used to obtain the inverse of the matrix AA^T, because the normal inversion of AA^T can be very numerically inaccurate. The inverse of the matrix AA^T is required to obtain the least square result x^{ls} and the square root weighted least square x^{wls}. In [5], the information theory approach can be approximated using the square root weighted least square method.

$$\begin{aligned} x^{ls} &= M + A^T(AA^T)^{-1}(Y - AM) \qquad (9) \\ x^{wls} &= M + w(Aw)^T((Aw)(Bw)^T)^{-1}(Y - AM) \end{aligned}$$

where

$$\begin{aligned} w_{ij} &= \sqrt{M_i} \qquad (i = j) \\ w_{ij} &= 0 \qquad (i \neq j) \end{aligned}$$

M is the given point, and the point is projected on the hyperplane formed by $AX = Y$. As mentioned previously, the point x^{ls} and x^{wls} may contain some negative elements. In that case, IPF is applied to overcome the problem.

To validate those methods, the RMSE (Root Mean Squared Error) and RMSRE (Root Mean Square Relative Error) were used, which provide an overall metric for the errors in the estimates. RMSRE was calculated on the largest 75% of the flows as suggested in [4]. The reason is to protect the RMSRE from being dominated by small flows.

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}(\hat{x_{ij}} - x_{ij})^2} \qquad (10)$$

$$\text{RMSRE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\{\frac{\hat{x_{ij}} - x_{ij}}{x_{ij}}\}^2} \qquad (11)$$

One network topology (containing 16 nodes) was created as shown in Fig. 6. The sixteen node network had the same topology as that used in [9]. The topology represents the Sprint PoP-level network consisting 70 links.

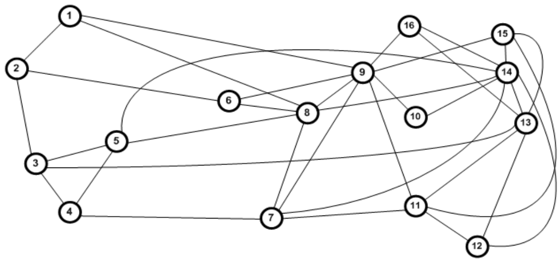

Fig. 6. Network Topologies

gMatVec [10] was used to manipulate matrices in the implementation of the affine scaling method. gMatVec is a small C^{++} matrix/vector template library provided by GNU Free Software Foundation.

VII. Results And Discussion

A. *Accuracies of the methods with the variation of the prior solution*

The accuracy of each method is dependent on how the prior solution represents the characteristic of the real traffic matrix. For instance, if an old traffic matrix is used as the prior solution, each element of the current traffic matrix is likely to increase proportionally. However, if an estimating technique such as choice model [11] is used to produce a prior solution, the solution becomes close enough to the real solution, so that the solution requires to be refined to satisfy the inter-link measurement constraint.

The following figures 7 and 8 show the variations of RMSE and RMSRE of the three different approaches namely the least square, the information theory approach and the generalized Kruithof approach, as the prior solution moves on the extended line $\overline{OM^A}$ in Fig.3.

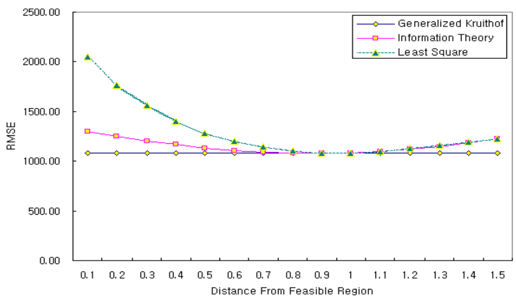

Fig. 7. The variation of RMSE as the prior point approaches to the feasible region.

The X-axis of the figures 7 and 8 represent the ratio between the lengthes of $\overline{OM^A}$ and $\overline{OX^{ma}}$ in Fig.3. Therefore, when the prior solution is on the feasible region, $\overline{OM^A}$ and $\overline{OX^{ma}}$ are matched (The ratio becomes one). The result of the generalized Kruithof does not change as the prior

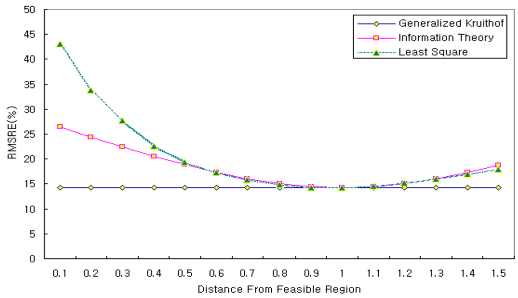

Fig. 8. The variation of RMSE as the prior point approaches to the feasible region.

solution follows the extended line $\overline{OM^A}$, while the RMSE and RMSRE of the least square and the information theory decreases, and then increases again after the prior solution passes through the feasible region. The accuracies of the three methods becomes trivial as the prior solution gets closer to the feasible region. It can be seen from the Fig.3, the prior solution M^A, the least square solution x^{ls} and the generalized Kruithof solution x^{ku} form a triangle. The triangle becomes smaller as the prior solution M^A is getting closer to the feasible region. That is why the difference of those methods becomes trivial as the prior solution is close to the feasible region.

B. *Comparison among the deterministic approaches*

The figures 9,10,11,12, and 13 plot the estimated traffic matrix elements against the synthetic traffic matrix, which is generated artificially according to [7]. The results are obtained when the prior solution has 0.3 distance from the feasible region in Fig 7 and 8. The solid diagonal line shows where the synthetic traffic matrix is estimated exactly and the dotted lines shows $\pm 20\%$ of the RMSRE (Root Mean Squared Error) of the estimated flows.

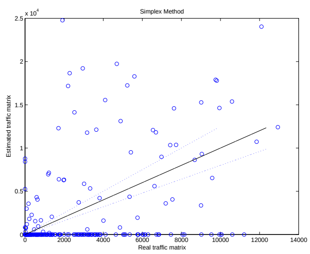

Fig. 9. Simplex Method

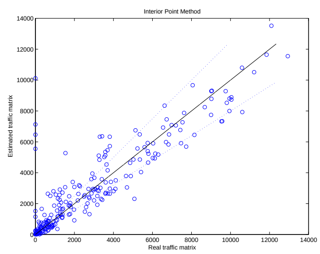

Fig. 10. Interior Point Method

Figures 9 and 10 show that the two LP methods produce very different results in terms of estimating the distribution of the synthetic traffic matrix – although the optimal values of both methods are the same. The Simplex Method estimates many elements as zeros. In Fig. 9, many estimates lie on the X-axis, however, the rest of the elements are over-estimated to compensate for the zero estimates. This result explains why Medina et al reported in [11] that the errors with the LP method are so high that it could not be used in practical

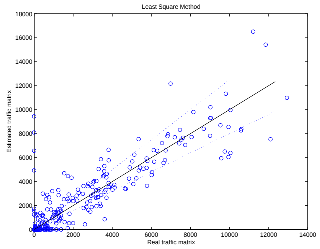

Fig. 11. Least Squared Method

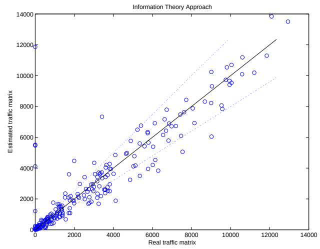

Fig. 12. Information Theory Approach

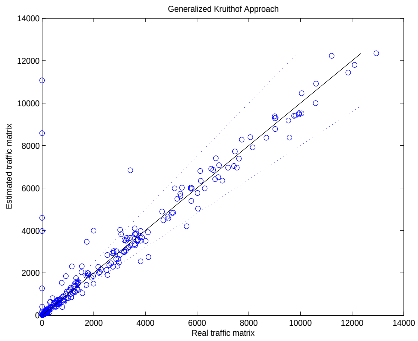

Fig. 13. Generalized Kruithof Approach

networks. However, the estimates from the interior point method are scattered along the solid roughly without zero estimation. The interior point method was implemented using the GLPK [8], and it uses a priori solution obtained by [12].

Figures 11, 12, and 13 represent the results of the least square, the information theory approach, and the generalized Kruithof approach respectively. The information theory and the generalized Kruithof approaches estimate the large elements more accurately than the least square method. Specially, in the generalized Kruithof case, large estimated elements are well scattered within the dashed lines.

	RMSE	RMSRE
Simplex Method	4710.6	107.3%
Interior Point Method	1333.9	26.4%
Least Square Method	1556.2	27.5%
Information Theory	1203.7	22.3%
Generalized Kruithof	1083.9	14.2%

Table 3. RMSE and RMSRE of deterministic approaches

Table 3 shows the RMSE (Root Mean Square Error) and the RMSRE (Root Mean Square Relative Error) of the deterministic approaches shown in Fig. 9, 10, 11, 12, and 13. While the simplex method produces over 100 % of RMSRE, the interior point method reduces the RMSRE by more than 70 %. Generalized Kruithof method produced less RMSRE and RMSE around by 8% and by 120 respectively compared to the information theory approach.

VIII. Conclusions

A new non-linear optimization problem was formulated based on the generalized Kruithof method, which uses the Kullback distance as the measurement of closeness between the given prior solution and a point in the feasible region. The non-linear optimization problem was solved using the affine scaling method which is the simplest implementation of all interior point methods, as well as it has the only interior point strategy which approaches a solution by monotonically decreasing the original objective function.

A strategy to accelerate the convergency of the affine scaling method was developed by the geometric analysis of the problem. The strategy finds a starting point and a searching direction since the choice of these effect the convergency speed of the interior point algorithm.

Four deterministic approaches, which are the simplex method, the interior point method, the least square method, and the information theory approach, have been implemented to compare with the proposed approach. The first two LP approaches show very different results each other (Simplex had an average error of 107.3% while interior point method had 26.4% only) although both use the same problem formulation. The next three methods do not have much different when the given prior solution is close to the feasible region, however the difference becomes noticeable as the prior solution moves far away from the feasible region.

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