Hadronic Weak Decays in the Heavy Quark Limit

Matthäus Bartsch

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Matthäus Bartsch

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Matthäus Bartsch
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Contents

Zusammenfassung

Den Schwerpunkt dieser Arbeit bildet die Untersuchung der Parameter $|V_{ub}|$ und γ der CKM-Matrix, die in $b \to u$-Übergängen messbar sind. Dies wird durch die Suche nach *Neuer Physik* motiviert, von der erwartet wird, dass sie die CP-Verletzung beeinflusst. In der B-Physik gibt es eine große Menge an Messdaten, die von mehreren Experimenten aufgezeichnet werden und so ein breites Fundament bilden, um die Parameter der Theorie zu bestimmen und Unsicherheiten zu kontrollieren. Eine einheitliche Betrachtung der Klasse der exklusiven Zerfälle von B_d- und B_s-Mesonen in leichte Hadronen ist geeignet einen großen Teil dieser Daten für die Theorie zu nutzen. Wir wenden eine Entwicklung in Λ_{QCD}/m_b an und drücken nicht-perturbative QCD durch Lichtkegel-Distributionsamplituden und Formfaktoren aus. Diese Vorgehensweise ist unter dem Namen QCD-Faktorisierung bekannt. Wir diskutieren zwei separate Klassen von B-Zerfällen.

Im ersten Teil der Arbeit führen wir eine phänomenologische Analyse von B-Zerfällen in 2 longitudinale Vektormesonen in nächstführender Ordnung in α_s und führender Ordnung in Λ_{QCD}/m_b durch. In einer Analyse von $|V_{ub}|$ aus $B_d \to \rho_L^+ \rho_L^-$ sind die niedrigsten Korrekturen quadratisch und ermöglichen eine sehr genaue Vorhersage. Durch diese Methode kann auch ein möglicher Anteil von Neuer Physik in der Mischungsphase von B_d-Mesonen eingeschränkt werden. Die größten Korrekturen aus der theoretischen Beschreibung von hadronischen Zerfällen stammen von Beiträgen der 1. Ordnung in Λ_{QCD}/m_b. Deswegen entwickeln wir eine Methode diese Korrekturen durch eine zusätzliche Messung eines Verzweigungsverhältnisses einzuschränken. In der Zukunft wird der Zerfall $B_s \to \phi\phi$ ein geeigneter Test für Neue Physik sein. Wir geben eine obere Schranke für die CP-Verletzung an, die auf einer experimentellen Bestimmung von hadronischen Parametern mit Unsicherheiten von unbekannter Größe beruht.

Der zweite Teil der Arbeit widmet sich der Entwicklung von notwendigen Komponenten und Werkzeugen für die Berechnung von baryonischen B-Zerfällen. Wir betrachten dazu den Zerfall $B_s \to p\bar{p}$ und diskutieren die führenden Beiträge. Aufgrund der Helizitätsunterdrückung ist es notwendig Beiträge der 1. Ordnung in Λ_{QCD}/m_b zu berechnen. Für die in diesem Zerfall auftretenden Proton-Formfaktoren geben wir explizite Formeln in nächstführender Ordnung an. Wir bestimmen dazu sowohl die fehlenden Teile der nicht-perturbativen als auch die der perturbativen QCD und geben Integralformeln für die Zerfallsamplitude an. Die auftretenden Integrale über 4 Parameter bedürfen einer Regularisierung, für die wir eine Methode zur systematischen Entflechtung von Divergenzen bei Cut-off regularisierten Integralen entwickeln.

Abstract

We mainly investigate the parameters $|V_{ub}|$ and γ of the CKM matrix that are associated with $b \to u$ transitions in electroweak theory. These investigations are motivated by the search for *New Physics*, which is expected to have an influence on CP-violation. There is a wealth of experimental data available from an active experimental community, which provides a broad foundation to determine and control parameters of the theory. In order to make use of a large amount of data we discuss exclusive charmless decays of B_d and B_s mesons to light hadrons. We apply an expansion in Λ_{QCD}/m_b and express nonperturbative QCD by light cone distribution amplitudes and form factors. This procedure is known as QCD factorization. We discuss two separate classes of B-decays.

In the first part of this thesis we perform a phenomenological analysis of B-decays to longitudinal vector mesons, $B \to V_L V_L$. We exploit the smallness of 2 parameters in the decay $B_d \to \rho_L^+ \rho_L^-$ and express CKM parameters in an expansion. We observe that for $|V_{ub}|$ such an expansion starts at second order and use this fact to provide a precise value assuming the standard model. This method also serves to constrain possible New Physics phases in the mixing of B_d mesons. A major troubling aspect of hadronic decays are the general power corrections of order 10%. Therefore we develop a strategy to constrain the power corrections with the help of an additional measurement of a branching fraction. Apart from CKM parameters, we also extract the hadronic parameter in order to check the leading power prediction. On the experimental side particularly the sector of B_s decays will be developed in the future. Among the decays into hadrons that are suitable for probes of New Physics is $B_s \to \phi\phi$. We provide an upper bound for the CP violation, based on experimental determinations of hadronic parameters that have corrections of unknown size.

The second part is devoted to the development of nesessary ingredients and tools to compute decays that involve baryons. Specifically we regard the decay $B_s \to p\bar{p}$ and discuss the leading contributions. This decay is helicity suppressed, which makes it necessary to calculate up to the first power in Λ_{QCD}/m_b. Computations at first power in baryonic systems require the input of next-to-leading soft and hard quantities. We determine the necessary power suppressed terms on the soft and the hard side and give explicit formulas for the decay amplitude. The result may be presented as an integration over 4 parameters with parameter dependent integration boundaries. We describe a systematic method to regulate these integrals. The result of the regulation is given for proton form factors. In order to systematically disentangle divergent parts for more complex integrals we develop a method for cut-off-regulated integrals.

Chapter 1

Introduction

The deepest validated formulation of physics at present is the standard model. Its theoretical formulation is the result of a series of publications [1–18], that were spurred by experimental observations in the 1950's and 1960's. The main building blocks that explain these observations, including the observed CP-violation in 1964, were published by 1973. Since then it has been firmly established by an enormous amount of experiments, which confirmed consistency predictions and measured up to now 18 of the 19 free parameters at a relative precision ranging from 0.65 for the u-quark mass to $7 \cdot 10^{-10}$ for the fine structure constant α_{em}. The precision of α_{em} translates to a similar precision for selected physical observables like the anomalous magnetic moment of the electron, which shows the good theoretical control of certain observable physical quantities. The historical development of physical theories gives an impression of the success of the standard model. For example the initial test of the theory of general relativity, the perihelion precession of Mercury, explained a deviation relative to newtonian physics of 10^{-2}. Very little though is known about the 9 additional parameters that entered after the discovery of non-zero neutrino masses [19] and the Higgs boson is currently searched for at the Large Hadron Collider.

Despite the overall success of the current framework to describe collider physics, astronomical observations indicate physics beyond the standard model. One important observation is that there is no sign of a large amount of anti-matter in the universe, which indicates a non-conservation of symmetries in nature, more precisely C- and CP-invariance [20]. Indeed the required abundance of matter before annihilation of matter and anti-matter had to be of the order of 10^{-10}, which is unattainable within the mechanism of CP-violation in the standard model [21]. It is therefore quite natural to assume that there is *New Physics* associated with CP-violation.

The indirect approach to gain deeper understanding of the independent parameters of the standard model – the search for *New Physics* – is to make precise predictions for observable quantities, which can be accurately measured. New interactions leave their traces in physics below the kinematically allowed region of their production and can have an impact on the measured values. In the past, this approach led to predictions like the top-quark mass that were verified later by direct detection. There are only few hints from these indirect measurements in colliders so far that the standard model does not suffice to

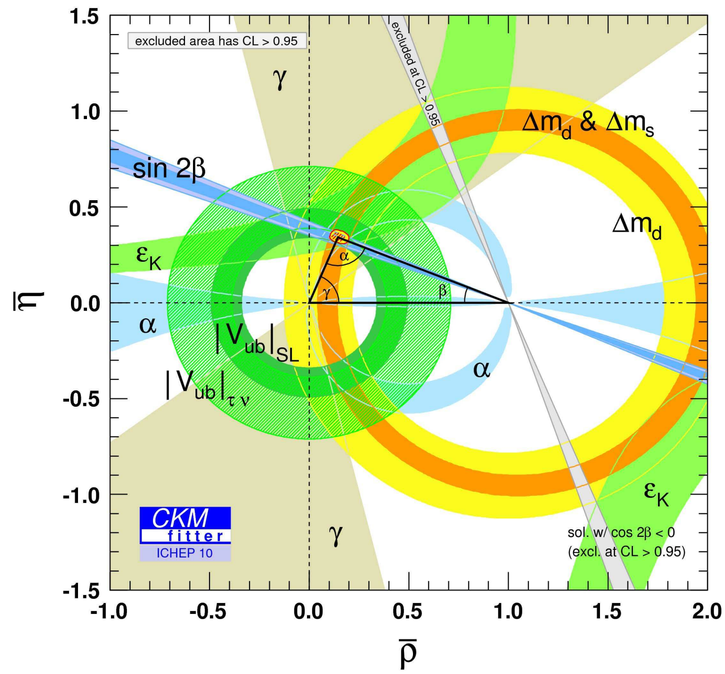

Figure 1.1: Standard model fit of CKM parameters [26].

describe the physics behind them. An example that potentially signals New Physics, is the comparison of the above mentioned anomalous magnetic moment of electron and muon. There is a difference of 3.2σ of experimental value [22,23] and theoretical prediction [24] of the muon anomalous magnetic moment. With the help of a precise measurement of other decay modes, such as $B_s \to \mu^+\mu^-$, certain classes of New Physics can be ruled out [25]. Here we see a nice interplay of several indirect tests and the importance of controlled QCD-dynamics, which is the limiting factor in understanding the anomalous magnetic moment.

The CP-violating parameters in the standard model have made significant progress only in the last decade with the start of the dedicated experiments BELLE and BABAR. They are related to the fact that a different linear combination of the 3 generations of quarks couples to W-bosons than the mass basis. In flavour changing weak decays CP-violation can be observed and the 4 independent parameters of the involved mixing matrix, the CKM-matrix, can be determined. A combined fit of several decay modes results in values that are well consistent with a unitary standard model mixing matrix [26, 27], as illustrated in Fig. 1.1. In order to extract standard model values, the CKMfitter group assumes unitarity for the fit. The independent measurements that are shown also test unitarity itself though.

There is a wealth of different decay modes, which are susceptible to different contributions of potential New Physics. The expectation of New Physics contributions in a channel is usually tied to the presence of sizable QCD corrections, which make it difficult to predict decay rates. Therefore it is important to use strategies to minimize their impact and to employ systematic techniques to calculate strong phases. For decays of B-mesons one can make use of the large mass of the b-quark compared to the scale of QCD, Λ_{QCD}, and expand decay rates in powers of Λ_{QCD}/m_b. For decays into final states composed of 2 pseudoscalar mesons this procedure systematically predicts the leading power terms, but fails to predict subleading powers [28–30]. Thus a sizable uncertainty for most decay amplitudes of the order of 10% remains, which limits the overall potential of branching ratios to extract CKM-parameters. An uncertainty of similar size is introduced by heavy-to-light form factors, which are non-perturbative objects of QCD. This limiting property is shared with other decay channels of B-mesons, e.g. semileptonic exclusive decays, which makes it advantageous to regard ratios of observables with similar QCD-behavior, where unknown leading power terms drop out [31–34]. Another well known source of QCD uncertainties is a special class of Feynman diagrams, which are known as penguin diagrams. They are calculable in the heavy quark limit [29,35] and for certain decay channels (e.g. $B \to \pi^0\pi^0$) a sizable contribution can be found.

In this thesis we make use of this power expansion to extract CKM-matrix elements from B-decays to longitudinal vector mesons: $B \to V_L V_L$ [36]. These decay channels seem particularly suited for phenomenological analysis, since vector mesons as final states have a significantly reduced impact of penguin diagrams compared to pseudoscalar mesons [37,38]. The main focus lies here on a phenomenological analysis, which tries to exploit the numerous relations of the 28 calculable channels. In particular together with an expansion of the CKM-matrix, the Wolfenstein parametrization [39, 40], hadronic uncertainties can be significantly reduced. In order to constrain the remaining uncertainty due to subleading powers in Λ_{QCD}/m_b and form factors, we suggest to use a flavour-$SU(3)$-related decay channel. In an analysis of $|V_{ub}|$ we use the fact that α, the top angle of the CKM-triangle in Fig. 1.1, is close to 90°. This ensures a well controlled determination of $|V_{ub}|$ with quadratic corrections of small parameters. The sensitivity to $|V_{ub}|$ also serves as a means to constrain an additional phase of New Physics in B–$\bar{B}$-mixing.

The systematic expansion of hadronic decay amplitudes relies on the factorization of the decay amplitude. Factorization holds, if not all particles that take part in an interaction, interact in a strong manner with all the other particles. It is then possible to regard parts of the amplitude and study these parts independently in other decay modes or determine them by nonperturbative methods. In the hypothetical case of no interaction at all, the decay amplitude factorizes completely and all particles can be regarded separately from each other. With the weak interaction switched on, the amplitude is still called factorizable, since due to the weakness of the interaction it is under good theoretical control. Possible non-factorization arises, if the strong interaction is switched on. If all particles are hadrons, they contribute to the interaction and can spoil factorization already at leading power in Λ_{QCD}/m_b and leading order in α_s, which means the whole amplitude has to be regarded as a complete object and the perturbative expansion ("Feynman diagrams") cannot be applied directly. Here we do not aim at a factorization proof to all orders in α_s and limit ourselves to finite order. Therefore we can make use of the QCD Lagrangian of the standard model and do not need to separate momentum modes at the level of the Lagrangian. A useful tool for factorization proofs to all orders in α_s is Soft Collinear Effective Theory [41, 42].

In exclusive hadronic B-decays to two mesons factorization of the amplitude holds only for the term of $\mathcal{O}(1)$ in an expansion in Λ_{QCD}/m_b. The colour structure of QCD can play an important role in the presence or absence of factorization, because diagrams may not contribute or are additionally suppressed in the number of colours. For instance it can be shown that factorization holds at first order in a combined expansion in Λ_{QCD}/m_b and $1/N_c$ [43]. The colour situation is different in decays to baryons, since they form antisymmetric invariants of $SU(3)$ instead of singlets. There are few known studies of baryonic B-decays at first power until now [44, 45], which leaves open questions about the factorization of objects with different colour structure than mesons. Necessary ingredients to perform such calculations, like the twist 4 nucleon projector, are still missing. Here we fill this gap and compute the first power of the proton form factor explicitly. The results are regularized, if necessary.

Perturbative corrections do not factorize, if gluons resolve the soft structure of the forming hadrons. This is typically indicated by diverging integrals over momentum components of single quarks. Factorization in next-to-leading order in α_s and leading power is well established for a decay of a heavy meson to 2 light mesons. Decays that involve baryons, typically have more complex perturbative corrections. For heavy baryons that decay into another heavy baryon and a meson, e.g. $\Lambda_b \to \Lambda_c \pi$, the situation is similar to the mesonic case, $B \to D\pi$ [46]. These decays can be shown to factorize at all orders in the strong coupling constant α_s. Additional types of diagrams contribute in the baryonic decay, but are suppressed. Among the decays that probe V_{ub} and therefore are of interest for this work, we can identify $\Lambda_b \to p\pi$ and $B_d \to p\bar{p}$ as promising candidates.

Here we will make the first steps to calculate the decay amplitude of $B_d \to p\bar{p}$ in the framework of QCD factorization, by focusing on an important subclass of Feynman diagrams – the annihilation diagrams. The advantage here is that the B-meson projector has only 2 parameters. The distribution amplitude of Λ_b is known since 2008 at leading twist [47] and opens the path for future studies of baryonic decays.

From the mesonic decays, $B \to MM$, it is known that these diagrams do not factorize [29]. This is signaled by unsuppressed nonperturbative contributions of soft quarks. The size of the annihilation diagrams in the mesonic case is not very well known. Estimates that rely on the degree of divergence [30] allow for large contributions, whereas light cone sum rules [48] indicate moderate contributions. For a calculation of $B_d \to p\bar{p}$ it is therefore important as a first step to calculate and regularize these diagrams. They make up an important class of tree diagrams of $B_s \to p\bar{p}$, which receives less contributions from other diagrams than $B_d \to p\bar{p}$. This makes it feasible to discuss them in the example of this decay channel.

A challenge in the calculation of $B_s \to p\bar{p}$ is the helicity suppression, which makes it necessary to make use of higher twist terms of the proton distribution amplitude. Therefore there are considerably more terms of long computation time to treat in the perturbative evaluation than in the mesonic case, which makes it necessary to develop suitable computer programs. Another challenge is posed by the regularization of divergent 4-parameter integrals, which are given by folding distribution amplitudes with Feynman diagrams. Therefore they share features of integrals over Feynman parameters, with the important difference that they are not dimensionally regulated. The very developed machinery of treating loop integrals cannot be applied here and has to be modified or new methods have to be developed. We describe a modified procedure of sector decomposition and discuss the complications that arise.

The structure of this thesis can be outlined as follows: In chapter 2 we perform a phenomenological analysis of B-decays into 2 longitudinal vector mesons, $B \to V_L V_L$. After the main ingredients for the calculation are lined out in section 2.1 we present the analytical expressions for all relevant diagrams, parameters and amplitudes in section 2.2. Here we provide expressions for the parameters that are used to describe the decay amplitudes, explicitly write down all 28 calculable decay amplitudes, give expressions for a model of the annihilation diagrams and write down the corresponding amplitudes. In section 2.3 we give an overview over the experimental situation in $B \to V_L V_L$ decays. An extensive phenomenological discussion is given in section 2.4. We start with a discussion of branching fractions and CP asymmetries and provide a detailed error analysis, including a discussion of the effects of ω-ϕ-mixing on the branching fractions for the affected decays. Subsequently we extract the parameters α, γ and $|V_{ub}|$ from the decay $B_d \to \rho^+\rho^-$ on the basis of leading power predictions. Additionally we constrain the size of a possible mixing phase from New Physics in $B - \bar{B}$-mixing. Section 2.4.5 is an update of an analysis for γ, which aims at determining the amplitude of penguin diagrams by the measurements of $B^- \to K_L^{*0}\rho_L^-$ and $B_d \to \rho^+\rho^-$. The analysis in section 2.4.6 constrains power suppressed terms by the measurements of branching fractions of the flavour-$SU(3)$-related decays $B_d \to \rho^+\rho^-$ and $B_d \to \bar{K}_L^{*0} K_L^{*0}$. The suppression of correction terms is discussed and CKM parameters and hadronic parameters are extracted and compared to results from leading power predictions. This analysis is repeated for the case of pions as final states in 2.4.7. In section 2.4.8 we derive a bound for CP violation in $B_s \to \phi\phi$. The following sections compare with the literature, deal with electromagnetic corrections and quote central values for coefficients of the analysis.

Chapter 3 deals with questions of factorization in B-decays with baryonic final states. First we lay out and develop in section 3.1 the basic ingredients in the calculation. This includes the derivation of nucleon distribution amplitudes at subleading twist, the description of a developed computer program, which is capable and fast enough in dealing with expressions of large size. The section 3.2 is devoted to nucleon form factors. They are calculated from first principles and include subleading powers. We give analytical expressions for the vector form factor and regularize it, as well as for the axial vector form factor. Section 3.3 contains an analysis of the contributing diagrams and discusses the size of their contribution. There it is also layed out how the calculation of the hard contribution is performed. The results of this calculation are given in the following, ordered by type of diagram and the complications in the regularization are discussed. Finally we conclude in chapter 4.

Chapter 2

Exclusive Mesonic B-Decays

2.1 Preliminaries

In the computation of the perturbative expansion ("Feynman diagrams") it is of great use to reduce the number of propagators to simplify calculations. In weak decays of particles with much lower mass than the W-boson mass, this can be accomplished by integrating out the W and Z boson and work with effective Hamiltonians. A basic example is given in figure 2.1. The two diagrams need to be matched at the scale M_W, i.e. the coupling

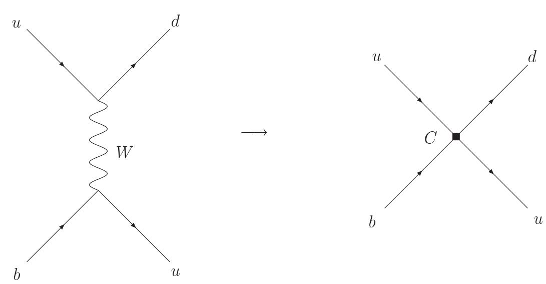

Figure 2.1: Example for the representation of Feynman diagrams by effective operators.

constant is determined. Corrections in the strong coupling constant need to be taken in account at first order for the present analysis. By solving the renormalization group equation the coupling constant C ("Wilson coefficient") can be determined at the scale m_b. This procedure sums up large logarithms $\log(M_W/\mu)$, where $\mu \sim m_b$, and is explained in more detail in [49, 50]. The full effective weak Hamiltonian for charmless hadronic B decays, without change in strangeness ($\Delta S = 0$), is given by [49]

$$\mathcal{H}_{\text{eff}}^{\Delta S=0} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left(C_1\, Q_1^p + C_2\, Q_2^p + \sum_{i=3,\ldots,10} C_i\, Q_i + C_{7\gamma}\, Q_{7\gamma} + C_{8g}\, Q_{8g} \right) + \text{h.c.} \quad (2.1)$$

where the elements of the CKM quark-mixing matrix V enter as $\lambda_p = V_{pb}V_{pd}^*$, C_i are Wilson coefficients, and the operators Q_i read

$$\begin{aligned}
Q_1^p &= (\bar{p}b)_{V-A}(\bar{d}p)_{V-A}\,, & Q_2^p &= (\bar{p}_ib_j)_{V-A}(\bar{d}_jp_i)_{V-A}\,,\\
Q_3 &= (\bar{d}b)_{V-A}\sum\nolimits_q(\bar{q}q)_{V-A}\,, & Q_4 &= (\bar{d}_ib_j)_{V-A}\sum\nolimits_q(\bar{q}_jq_i)_{V-A}\,,\\
Q_5 &= (\bar{d}b)_{V-A}\sum\nolimits_q(\bar{q}q)_{V+A}\,, & Q_6 &= (\bar{d}_ib_j)_{V-A}\sum\nolimits_q(\bar{q}_jq_i)_{V+A}\,,\\
Q_7 &= (\bar{d}b)_{V-A}\sum\nolimits_q\tfrac{3}{2}e_q(\bar{q}q)_{V+A}\,, & Q_8 &= (\bar{d}_ib_j)_{V-A}\sum\nolimits_q\tfrac{3}{2}e_q(\bar{q}_jq_i)_{V+A}\,,\\
Q_9 &= (\bar{d}b)_{V-A}\sum\nolimits_q\tfrac{3}{2}e_q(\bar{q}q)_{V-A}\,, & Q_{10} &= (\bar{d}_ib_j)_{V-A}\sum\nolimits_q\tfrac{3}{2}e_q(\bar{q}_jq_i)_{V-A}\,,\\
Q_{7\gamma} &= \frac{e}{8\pi^2}\,m_b\,\bar{d}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}b\,, & Q_{8g} &= \frac{g}{8\pi^2}\,m_b\,\bar{d}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu}b
\end{aligned} \tag{2.2}$$

Here i,j are colour indices, e_q are the quark charges, and the sums extend over $q = u,d,s,c,b$. The Wilson coefficients C_i will be taken at next-to-leading order (NLO), using the treatment of electroweak contributions described in detail in [30]. The sign conventions for the electromagnetic and strong coupling correspond to the covariant derivative $D_\mu = \partial_\mu + ieQ_fA_\mu + igT^aA_\mu^a$. With these definitions the coefficients $C_{7\gamma}$, C_{8g} are negative in the Standard Model, which is the convention usually adopted in the literature.

The effective Hamiltonian for charmless decays of B mesons with $\Delta S = 1$ can be obtained from (2.1) by interchanging d- and s-quark labels. The CKM factors governing these transitions are then $\lambda_p' = V_{pb}V_{ps}^*$.

To obtain the amplitudes for $\bar{B} \to V_{1L}V_{2L}$ decays from the Hamiltonian, the matrix elements of the operators Q_i have to be computed in QCD factorization [28–30]. To lowest order the matrix elements are expressed in terms of $\bar{B} \to V$ form factors and vector-meson decay constants. The required form factors are defined by (see e.g. [51])

$$\begin{aligned}
\langle V(p,\epsilon)|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}(p_B)\rangle \;=\; & 2m_VA_0(q^2)\frac{\epsilon\cdot q}{q^2}q^\mu + (m_B+m_V)A_1(q^2)\left[\epsilon^\mu - \frac{\epsilon\cdot q}{q^2}q^\mu\right]\\
& -A_2(q^2)\frac{\epsilon\cdot q}{m_B+m_V}\left[(p_B+p)^\mu - \frac{m_B^2-m_V^2}{q^2}q^\mu\right]
\end{aligned} \tag{2.3}$$

$$\langle V(p,\epsilon)|\bar{q}\gamma^\mu b|\bar{B}(p_B)\rangle = -2i\frac{V(q^2)}{m_B+m_V}\varepsilon^{\mu\nu\rho\sigma}p_{B\nu}p_\rho\epsilon_\sigma \tag{2.4}$$

where the momentum transfer is $q = p_B - p$ and the totally antisymmetric tensor $\varepsilon^{\mu\nu\rho\sigma}$ is normalized by $\varepsilon^{0123} = -1$.

The vector-meson decay constant f_V is given by

$$\langle V(q,\eta)|\bar{q}\gamma_\mu q'|0\rangle = -if_Vm_V\eta_\mu \tag{2.5}$$

for a vector meson with flavour content $V = \bar{q}'q$. The corresponding matrix element where $\gamma_\mu \to \gamma_\mu\gamma_5$ is zero. For energetic vector mesons $V(p,\epsilon)$ with longitudinal polarization

$$\epsilon^\mu = \frac{p^\mu}{m_V} \tag{2.6}$$

up to corrections of second order in m_V/m_B.

The factorized matrix element of a $(V-A)\otimes(V-A)$ operator then reads to lowest order (α_s^0)

$$\langle V_{1L}V_{2L}|(\bar{q}_1 b)_{V-A}(\bar{q}_2 q_2')_{V-A}|\bar{B}_q\rangle = i m_B^2 A_0^{B\to V_1}(m_{V_2}^2) f_{V_2} \tag{2.7}$$

where $V_1 = \bar{q}q_1$ and $V_2 = \bar{q}_2' q_2$ ($q_1 \neq q_2$).

The corrections at higher order in α_s are expressed in terms of calculable hard-scattering kernels and meson light-cone distribution amplitudes. The latter quantities enter through the meson projectors in momentum space. For the B meson the projector $b\bar{q}$ is given at leading power in Λ_{QCD}/m_B by [29]

$$\begin{aligned}
&\langle 0|\bar{q}^i_\alpha(z)[z,0]b^j_\beta(0)|\bar{B}(p_B)\rangle\big|_{z=z_-n_-} = \\
&= -i\frac{f_B}{4}\frac{\delta^{ji}}{N_c}\left[(\not{p}_B + m_B)\gamma_5\right]_{\beta\gamma}\int_0^1 \mathrm{d}\xi\, e^{-i\xi p_B\cdot z}\left[\phi_{B1}(\xi) + \sqrt{2}\not{n}_-\phi_{B2}(\xi)\right]_{\gamma\alpha} \\
&\equiv -\int_0^1 \mathrm{d}\xi\, e^{-i\xi p_B\cdot z} b\bar{q},
\end{aligned} \tag{2.8}$$

where the Wilson line

$$[x,y] = \mathrm{P}\exp\left[ig\int_0^1 \mathrm{d}t(x-y)_\mu t^a A^{a\mu}\left(tx+(1-t)y\right)\right] \tag{2.9}$$

is used to make the matrix element (2.8) gauge invariant. $b\bar{q}$ denotes a matrix in Dirac space displaying the flavour composition of a $\bar{B}_q$ meson in the initial state. The 4-vector $n_-^\mu = 1/\sqrt{2}\,(1,0,0,-1)$ is chosen to be in the direction of the recoiling meson V_1. The parameter ξ is the light-cone momentum fraction of the spectator quark $\bar{q}$. The distribution amplitudes are normalized as

$$\int_0^1 d\xi\,\phi_{B1}(\xi) = 1, \qquad \int_0^1 d\xi\,\phi_{B2}(\xi) = 0 \tag{2.10}$$

In the present analysis ϕ_{B2} does not enter the results and ϕ_{B1} appears only through the first inverse moment

$$\int_0^1 d\xi\,\frac{\phi_{B1}}{\xi} = \frac{m_B}{\lambda_B} \tag{2.11}$$

which defines the hadronic parameter $\lambda_B = \mathcal{O}(\Lambda_{QCD})$.

The distribution amplitudes of light vector mesons, given by matrix elements like $\langle V(p)|O(x)\Gamma O(0)|0\rangle$, can be found by expanding the composite operator $O(x)O(0)$ into local operators on the light cone. Explicitly, an operator consisting of several local operators (here 2) has the following light cone expansion, assuming a complete set of local operators $O^n_{\mu_1\ldots\mu_j}$:

$$A(x)B(0) \simeq \sum_n C_n(x^2)x^{\mu_1}\ldots x^{\mu_j}O^n_{\mu_1\ldots\mu_j}(0) \tag{2.12}$$

Here the Wilson coefficients scale as $C_n \to \sqrt{x^2}^{t-d_A-d_B} \log{(x^2m^2)^p}$, where d_A, d_B are the dimensions of the operators A, B, respectively, $t = d_n - j_n$ is the twist of operator $O^n_{\mu_1\ldots\mu_j}$, m is a mass and p an undetermined number. A way to help in diagonalizing the operator mixing matrix is to use operators with definite transformation properties under conformal transformations. In that way we gain an additional "quantum number", the conformal spin $j = 1/2(l+s)$. s is the projection of the spin on the "+"-direction. The corresponding conformal operator product expansion is given by

$$A(x)B(0) \simeq \sum_{n=0}^{\infty} C_n \left(\frac{1}{x^2}\right)^{\frac{t_A+t_B-t_n}{2}} \frac{x_-^{n+s_1+s_2-s_A-s_B}}{\mathrm{B}(j_A-j_B+j_n, j_B-j_A+j_n)} \times \int_0^1 du\, u^{j_A-j_B+j_n-1}(1-u)^{j_B-j_A+j_n-1}\mathbb{O}_n^{j_1,j_2}(ux_-)\,, \tag{2.13}$$

where j_i is the conformal spin of the operator i, s_i is the corresponding spin projection on the "+"-direction and $t_i = d_i - s_i$ is the conformal twist. $\mathbb{O}_n^{j_1,j_2}$ are conformal operators out of 2 constituent fields with conformal spins j_1, j_2. More details can be found in the review [52]. Although QCD is not a conformally invariant theory, due to mass terms and quantum corrections, the framework of conformal field theories is applicable for the determination of anomalous dimensions and their eigenfunctions. Operators with different conformal spin cannot mix under renormalization at leading order, because the counterterms are tree level to this accuracy and keep all classical symmetries. Another feature is, that exact equations of motions on the operator level can be solved order by order in the conformal expansion. The reason is, that the equations of motion don't need to be renormalized and so don't receive quantum corrections. This technique is used in [53] to determine the ρ-meson distribution amplitudes. Subtleties arise from the fact, that generators of the collinear conformal group do not commute with general spin rotations. Therefore one finds that operators with different helicity and conformal twist are connected by the Wandzura-Wilczek relations. If formulated in terms of geometric twist, this is not the case [54]. But gauge invariance is broken for distribution amplitudes of definite geometric twist and causes artificial singularities, that are cancelled exactly by total derivative operators. In the following we refer by twist to the conformal twist, which coincides with the geometric twist for the leading contribution.

For a longitudinally polarized vector meson in the final state with flavour content $\bar{q}_2 q_1$ and momentum p, the projector can be written as [37]

$$q_2\bar{q}_1 = \frac{if_V}{4}\not{p}\,\phi_{||}(x) - \frac{if_V^{\perp}}{4}m_V\frac{\not{k}_2\not{k}_1}{k_2\cdot k_1}\,\Phi_v(x) \tag{2.14}$$

Here x is the momentum fraction of the final-state quark q_1 and

$$k_1^\mu = xp^\mu + k_\perp^\mu + \frac{\mathbf{k}_\perp^2}{\sqrt{2}xm_B}n^\mu \tag{2.15}$$

$$k_2^\mu = \bar{x}p^\mu - k_\perp^\mu + \frac{\mathbf{k}_\perp^2}{\sqrt{2}\bar{x}m_B}n^\mu \tag{2.16}$$

(with $\bar{x} = 1 - x$) are the momenta of q_1 and $\bar{q}_2$, respectively. n^μ is a light-like vector with spatial direction opposite to p^μ: If $p \sim n_+$ ($p \sim n_-$) then $n = n_-$ ($n = n_+$), where $n^\mu_\pm = 1/\sqrt{2}(1, 0, 0, \pm 1)$.

The function $\phi_{||}(x)$ is the light-cone distribution amplitude of leading twist for a longitudinal vector meson. The subleading-twist amplitude $\Phi_v(x)$ has been treated in (2.14) in the Wandzura-Wilczek approximation. It gives rise to contributions suppressed by one power of Λ_{QCD}/m_b. We will nevertheless include it in order to estimate the impact of this particular source of power corrections to factorization in the heavy-quark limit.

The functions $\phi_{||}$ and Φ_v can be expanded in terms of Gegenbauer and Legendre polynomials, respectively,

$$\phi_{||}(x) = 6x\bar{x} \sum_{n=0}^{\infty} \alpha_n C_n^{3/2}(2x - 1) \tag{2.17}$$

$$\Phi_v(x) = 3 \sum_{n=0}^{\infty} \alpha_{n\perp} P_{n+1}(2x - 1) \tag{2.18}$$

where $\alpha_0 = \alpha_{0\perp} = 1$. In the Wandzura-Wilczek approximation Φ_v can be expressed in terms of the twist-2 wave function of a transversely polarized vector meson, $\phi_\perp$, as

$$\Phi_v(x) = \int_0^x du \, \frac{\phi_\perp(u)}{\bar{u}} - \int_x^1 du \, \frac{\phi_\perp(u)}{u} \tag{2.19}$$

$\phi_\perp$ has an expansion similar to (2.17) and this leads to (2.18).

Note that (2.19) implies

$$\int_0^1 dx \, \Phi_v(x) = 0 \tag{2.20}$$

even though $\Phi_v(x)$ is not necessarily antisymmetric under $x \leftrightarrow \bar{x}$ for general $\alpha_{n\perp}$. The normalization of $\phi_{||,\perp}$ is $\int_0^1 dx \, \phi_{||,\perp}(x) = 1$.

For phenomenological applications we shall truncate the expansions of $\phi_{||}$ and Φ_v and use, for a particular meson V,

$$\phi^V_{||}(x) = 6x\bar{x} \left[1 + \alpha^V_1 \, 3(2x - 1) + \alpha^V_2 \, 6(5x^2 - 5x + 1)\right] \tag{2.21}$$

$$\Phi^V_v(x) = 3(2x - 1) \tag{2.22}$$

Taking the vacuum-to-meson matrix element of a local current, the projector (2.14) reproduces (2.5), (2.6)

$$\langle V(p)|\bar{q}_1 \gamma_\mu q_2|0\rangle = -\int_0^1 dx \, \mathrm{tr} \, \gamma_\mu q_2 \bar{q}_1 = -i f_V p_\mu \tag{2.23}$$

and

$$\langle V(p)|\bar{q}_1 \Gamma q_2|0\rangle = 0 \tag{2.24}$$

for $\Gamma = 1$, γ_5, $\gamma_\mu\gamma_5$.

2.2 QCD factorization in $B \to V_L V_L$ decays

The amplitudes for the $\Delta S = 0$ decay of a $\bar{B}$ meson into a pair of light vector mesons with longitudinal polarization can be conveniently expressed as follows (the case of $\Delta S = 1$ is obtained by replacing $d \leftrightarrow s$):

$$\langle V_{1L} V_{2L} | \mathcal{H}_{\rm eff}^{\Delta S=0} | \bar{B} \rangle = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \, \langle V_{1L} V_{2L} | \mathcal{T}_p^d + \mathcal{T}_p^{{\rm ann},d} | \bar{B} \rangle \tag{2.25}$$

where

$$\begin{aligned}
\mathcal{T}_p^d &= a_1(V_1V_2)\, \delta_{pu}\, (\bar{u}b)_{V-A} \otimes (\bar{d}u)_{V-A} \\
&+ a_2(V_1V_2)\, \delta_{pu}\, (\bar{d}b)_{V-A} \otimes (\bar{u}u)_{V-A} \\
&+ a_3(V_1V_2) \sum\nolimits_q (\bar{d}b)_{V-A} \otimes (\bar{q}q)_{V-A} \\
&+ a_4^p(V_1V_2) \sum\nolimits_q (\bar{q}b)_{V-A} \otimes (\bar{d}q)_{V-A} \\
&+ a_5(V_1V_2) \sum\nolimits_q (\bar{d}b)_{V-A} \otimes (\bar{q}q)_{V+A} \\
&+ a_7(V_1V_2) \sum\nolimits_q (\bar{d}b)_{V-A} \otimes \tfrac{3}{2} e_q (\bar{q}q)_{V+A} \\
&+ a_9(V_1V_2) \sum\nolimits_q (\bar{d}b)_{V-A} \otimes \tfrac{3}{2} e_q (\bar{q}q)_{V-A} \\
&+ a_{10}^p(V_1V_2) \sum\nolimits_q (\bar{q}b)_{V-A} \otimes \tfrac{3}{2} e_q (\bar{d}q)_{V-A}
\end{aligned} \tag{2.26}$$

Here the summation is over $q = u, d, s$. The symbol $\otimes$ indicates that the matrix elements of the operators in $\mathcal{T}_p^d$ are to be evaluated in factorized form [30]. The factorization coefficients a_i include hard QCD corrections to the B-decay matrix elements at NLO, as well as electroweak effects in the systematic approximation of [30]. Note that structures with scalar and pseudoscalar currents are absent in (2.26), in contrast to the case of $B \to K\pi$ considered in [30]. Because (pseudo)scalar currents cannot create a vector meson from the vacuum, these structures can give no contribution to $B \to VV$ decays.

The term $\mathcal{T}_p^{{\rm ann},d}$ in (2.25) describes the effects of weak annihilation. These are power suppressed in the heavy-quark limit and cannot be computed in QCD factorization. We shall use model calculations to estimate this important class of power corrections to the leading, factorizable amplitudes. Weak annihilation will be discussed in Section 2.2.3.

2.2.1 Results for the parameters a_i

The factorization coefficients can be written as $a_i = a_{i,\mathrm{I}} + a_{i,\mathrm{II}}$. We find

$$
\begin{aligned}
a_{1,\mathrm{I}} &= C_1 + \frac{C_2}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi} V_V\right], & a_{1,\mathrm{II}} &= \frac{C_2}{N_c}\frac{C_F\pi\alpha_s}{N_c} H_{V_1V_2},\\
a_{2,\mathrm{I}} &= C_2 + \frac{C_1}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi} V_V\right], & a_{2,\mathrm{II}} &= \frac{C_1}{N_c}\frac{C_F\pi\alpha_s}{N_c} H_{V_1V_2},\\
a_{3,\mathrm{I}} &= C_3 + \frac{C_4}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi} V_V\right], & a_{3,\mathrm{II}} &= \frac{C_4}{N_c}\frac{C_F\pi\alpha_s}{N_c} H_{V_1V_2},\\
a^p_{4,\mathrm{I}} &= C_4 + \frac{C_3}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi} V_V\right] - \frac{C_5}{N_c}\frac{C_F\alpha_s}{4\pi} r^V_\perp V^\perp_V & &\\
&\quad + \frac{C_F\alpha_s}{4\pi N_c}(P^p_{V,2} - r^V_\perp P^p_{V,3}), & a_{4,\mathrm{II}} &= \frac{C_3}{N_c}\frac{C_F\pi\alpha_s}{N_c} H_{V_1V_2},\\
a_{5,\mathrm{I}} &= C_5 + \frac{C_6}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}(-V'_V)\right], & a_{5,\mathrm{II}} &= \frac{C_6}{N_c}\frac{C_F\pi\alpha_s}{N_c}(-H'_{V_1V_2}),\\
a^p_{7,\mathrm{I}} &= C_7 + \frac{C_8}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}(-V'_V)\right] + \frac{\alpha}{9\pi}P^{p,\mathrm{EW}}_{V,n}, & a_{7,\mathrm{II}} &= \frac{C_8}{N_c}\frac{C_F\pi\alpha_s}{N_c}(-H'_{V_1V_2}),\\
a^p_{9,\mathrm{I}} &= C_9 + \frac{C_{10}}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi} V_V\right] + \frac{\alpha}{9\pi}P^{p,\mathrm{EW}}_{V,n}, & a_{9,\mathrm{II}} &= \frac{C_{10}}{N_c}\frac{C_F\pi\alpha_s}{N_c} H_{V_1V_2},\\
a^p_{10,\mathrm{I}} &= C_{10} + \frac{C_9}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi} V_V\right] - \frac{C_7}{N_c}\frac{C_F\alpha_s}{4\pi} r^V_\perp V^\perp_V & &\\
&\quad + \frac{\alpha}{9\pi N_c}(P^{p,\mathrm{EW}}_{V,2} - r^V_\perp P^{p,\mathrm{EW}}_{V,3}), & a_{10,\mathrm{II}} &= \frac{C_9}{N_c}\frac{C_F\pi\alpha_s}{N_c} H_{V_1V_2}
\end{aligned}
\tag{2.27}
$$

where $C_i \equiv C_i(\mu)$, $\alpha_s \equiv \alpha_s(\mu)$, $C_F = (N_c^2-1)/(2N_c)$, and $N_c = 3$. The hadronic quantities $V^{(\prime)}_V$, $H^{(\prime)}_{V_1V_2}$, $P^p_{V,2}$, $P^p_{V,3}$, $P^{p,\mathrm{EW}}_{V,2}$, $P^{p,\mathrm{EW}}_{V,3}$, and $P^{p,\mathrm{EW}}_{V,n}$ are given below. All indices $_V$ in V_V, P_V, r^V are understood to refer to the emitted meson V_2.

Contributions suppressed by one power of Λ_{QCD}/m_b that arise from the twist-3 component of the vector-meson wave function have been included in the above expressions. They are related to the scalar penguin operator $(\bar{q}b)_{S-P}(\bar{s}q)_{S+P}$ and come with a factor

$$
r^V_\perp(\mu) = \frac{2m_V f^\perp_V(\mu)}{m_b(\mu) f_V} = \frac{2m_V f^\perp_V(1\mathrm{GeV})}{m_b(m_b) f_V}\left[\frac{\alpha_s(\mu)}{\alpha_s(m_b)}\right]^{-3C_F/\beta_0}\left[\frac{\alpha_s(\mu)}{\alpha_s(1\mathrm{GeV})}\right]^{C_F/\beta_0}
\tag{2.28}
$$

Here $m_b(\mu)$ is the $\overline{MS}$-mass of the b quark at scale μ, and $\beta_0 = 23/3$ for $f = 5$ flavours of quarks.

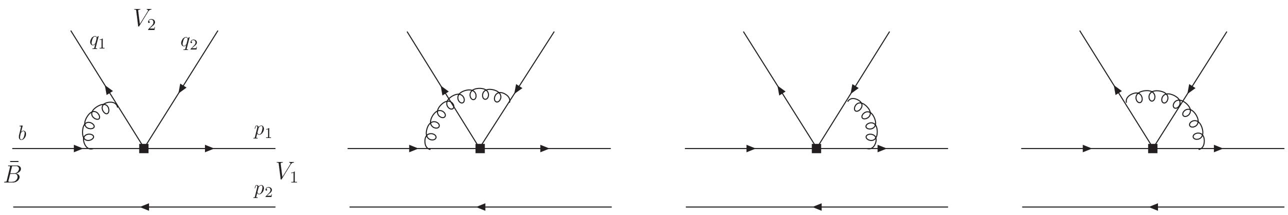

Figure 2.2: Vertex diagrams.

Vertex and penguin contributions

The vertex corrections (Fig. 2.2) are given by

$$\begin{aligned} V_V &= 12 \ln \frac{m_b}{\mu} - 18 + \int_0^1 dx\, g(x)\, \phi_{||}^V(x)\,, \\ V_V' &= 12 \ln \frac{m_b}{\mu} - 6 + \int_0^1 dx\, g(1-x)\, \phi_{||}^V(x)\,, \\ g(x) &= 3 \left(\frac{1-2x}{1-x} \ln x - i\pi \right) \\ &\quad + \left[2\, L_2(x) - \ln^2 x + \frac{2 \ln x}{1-x} - (3 + 2i\pi) \ln x - (x \leftrightarrow 1-x) \right] \end{aligned} \tag{2.29}$$

$$V_V^{\perp} = \int_0^1 dx\, \left[2\, L_2(x) - \ln^2 x - (1 + 2i\pi) \ln x - (x \leftrightarrow 1-x) \right]\, \Phi_v^V(x) \tag{2.30}$$

where $L_2(x)$ is the dilogarithm

$$L_2(x) = - \int_0^x dt\, \frac{\ln(1-t)}{t} \tag{2.31}$$

The expansion of $\phi_{||}^V$ in Gegenbauer polynomials gives

$$\int_0^1 dx\, g(x)\, \phi_{||}^V(x) = -\frac{1}{2} - 3i\pi + \left(\frac{11}{2} - 3i\pi \right) \alpha_1^V - \frac{21}{20}\, \alpha_2^V + \ldots \tag{2.32}$$

Replacing $g(x)$ by $g(1-x)$ leads to a change of sign in front of the odd Gegenbauer coefficients on the right-hand side.

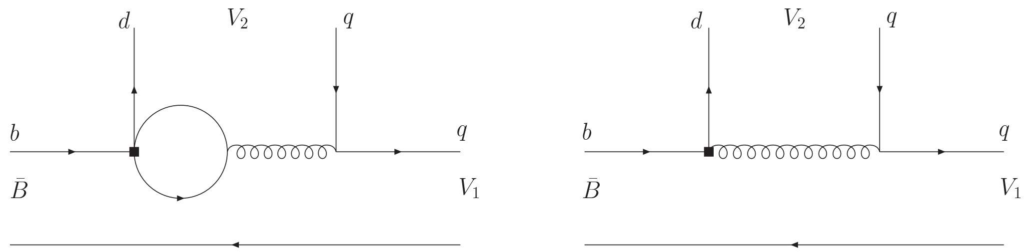

Figure 2.3: Penguin diagrams.

Next, the penguin contributions (Fig. 2.3) are

$$\begin{aligned} P^p_{V,2} &= C_1 \left[\frac{4}{3}\ln\frac{m_b}{\mu} + \frac{2}{3} - G_V(s_p)\right] + C_3 \left[\frac{8}{3}\ln\frac{m_b}{\mu} + \frac{4}{3} - G_V(0) - G_V(1)\right] \\ &\quad + (C_4 + C_6)\left[\frac{20}{3}\ln\frac{m_b}{\mu} - 3G_V(0) - G_V(s_c) - G_V(1)\right] \\ &\quad - 2C^{\rm eff}_{8g}\int_0^1 \frac{dx}{1-x}\,\phi^V_{||}(x)\,, \\ P^{p,\rm EW}_{V,2} &= (C_1 + N_c C_2)\left[\frac{4}{3}\ln\frac{m_b}{\mu} + \frac{2}{3} - G_V(s_p)\right] - 3\,C^{\rm eff}_{7\gamma}\int_0^1 \frac{dx}{1-x}\,\phi^V_{||}(x) \end{aligned} \tag{2.33}$$

where $s_u = 0$ and $s_c = (m_c/m_b)^2$. Small contributions from the electroweak coefficients $C_7, \ldots, C_{10}$ are consistently neglected in $P^p_{V,2}$ within our approximation scheme. Also, the very small corrections from $C_3, \ldots, C_6$ in $P^{p,\rm EW}_{V,2}$ (and in $P^{p,\rm EW}_{V,3}$, see (2.38) below) are omitted for simplicity.

The function $G_V(s)$ is

$$G_V(s) = \int_0^1 dx\, G(s - i\epsilon, 1-x)\,\phi^V_{||}(x)\,, \tag{2.34}$$

$$\begin{aligned} G(s,x) &= -4\int_0^1 du\, u(1-u)\ln[s - u(1-u)x] \\ &= \frac{2(12s + 5x - 3x\ln s)}{9x} - \frac{4\sqrt{4s-x}\,(2s+x)}{3x^{3/2}}\arctan\sqrt{\frac{x}{4s-x}} \end{aligned} \tag{2.35}$$

Expanding in Gegenbauer moments one finds

$$
\begin{aligned}
G_V(s_c) = {} & \frac{5}{3} - \frac{2}{3}\ln s_c + \frac{\alpha_1^V}{2} + \frac{\alpha_2^V}{5} + \frac{4}{3}\left(8 + 9\alpha_1^V + 9\alpha_2^V\right) s_c \\
& + 2(8 + 63\alpha_1^V + 214\alpha_2^V)s_c^2 - 24(9\alpha_1^V + 80\alpha_2^V)s_c^3 + 2880\alpha_2^V s_c^4 \\
& - \frac{2}{3}\sqrt{1-4s_c}\,\Big[1 + 2s_c + 6(4 + 27\alpha_1^V + 78\alpha_2^V)s_c^2 \\
& \quad - 36(9\alpha_1^V + 70\alpha_2^V)s_c^3 + 4320\alpha_2^V s_c^4\Big]\left(2\,\mathrm{arctanh}\sqrt{1-4s_c} - i\pi\right) \\
& + 12 s_c^2\,\Big[1 + 3\alpha_1^V + 6\alpha_2^V - \frac{4}{3}\left(1 + 9\alpha_1^V + 36\alpha_2^V\right) s_c \\
& \quad + 18(\alpha_1^V + 10\alpha_2^V)s_c^2 - 240\alpha_2^V s_c^3\Big]\left(2\,\mathrm{arctanh}\sqrt{1-4s_c} - i\pi\right)^2 + \ldots\,, \\
G_V(0) = {} & \frac{5}{3} + \frac{2i\pi}{3} + \frac{\alpha_1^V}{2} + \frac{\alpha_2^V}{5} + \ldots\,, \\
G_V(1) = {} & \frac{85}{3} - 6\sqrt{3}\,\pi + \frac{4\pi^2}{9} - \left(\frac{155}{2} - 36\sqrt{3}\,\pi + 12\pi^2\right)\alpha_1^V \\
& + \left(\frac{7001}{5} - 504\sqrt{3}\,\pi + 136\pi^2\right)\alpha_2^V + \ldots
\end{aligned}
\tag{2.36}
$$

The function $G_V(s)$ and its expansion in Gegenbauer moments have the same form as $G_K(s)$ in the case of $B \to K\pi$ discussed in [30]. Likewise the integrals proportional to $C_{7\gamma}^{\rm eff}$ and $C_{8g}^{\rm eff}$ in (2.33) are similar to those in $B \to K\pi$. They read

$$
\int_0^1 \frac{dx}{1-x}\,\phi_{||}^V(x) = 3(1 + \alpha_1^V + \alpha_2^V + \ldots)
\tag{2.37}
$$

The twist-3 terms from the penguin diagrams are obtained from the twist-2 terms by the replacement $\phi_{||}^V(x) \to \Phi_v^V(x)$, except for the terms proportional to $C_{7\gamma}^{\rm eff}$ and $C_{8g}^{\rm eff}$. Here the factor of $(1-x)$ in the denominator of the integral in (2.33) is canceled by the twist-3 projection. An important difference between the twist-3 penguin contributions in $B \to V_L V_L$ and $B \to K\pi$ arises from the different properties of the twist-3 wave functions in these two cases. Since $\int_0^1 dx\,\Phi_v^V(x) = 0$ it follows that the contributions from $C_{7\gamma}^{\rm eff}$ and $C_{8g}^{\rm eff}$ vanish in the former case. The same holds for all x-independent constants in the hard-scattering kernel, in particular for the scale and scheme dependent terms. We then find

$$
\begin{aligned}
P_{V,3}^p = {} & -\Big[C_1\,\hat G_V(s_p) + C_3\,(\hat G_V(0) + \hat G_V(1)) + (C_4 + C_6)\,(3\hat G_V(0) + \hat G_V(s_c) + \hat G_V(1))\Big] \\
P_{V,3}^{p,\rm EW} = {} & -(C_1 + N_c C_2)\,\hat G_V(s_p)
\end{aligned}
\tag{2.38}
$$

with

$$
\hat G_V(s) = \int_0^1 dx\,G(s - i\epsilon, 1-x)\,\Phi_v^V(x)
\tag{2.39}
$$

Using the asymptotic form of the wave function $\Phi_v^V(x) = 3(2x-1)$ leads to

$$\begin{aligned}\hat{G}_V(s_c) &= 1 - 36 s_c + 12 s_c \sqrt{1-4s_c}\left(2\,\mathrm{arctanh}\sqrt{1-4s_c} - i\pi\right) \\ &\quad -12 s_c^2 \left(2\,\mathrm{arctanh}\sqrt{1-4s_c} - i\pi\right)^2 , \\ \hat{G}_V(0) &= 1\,, \qquad \hat{G}_V(1) = \frac{4}{3}\pi^2 + 4\sqrt{3}\pi - 35 \end{aligned} \tag{2.40}$$

Finally, we give the electromagnetic penguin contributions $P_{V,n}^{p,\mathrm{EW}}$ ($p = u,\ c$). For intermediate charm, $p = c$, these are calculable in perturbation theory and read

$$P_{V,n}^{c,\mathrm{EW}} = (C_1 + N_c C_2)\left[\frac{4}{3}\ln\frac{m_b}{\mu} + \frac{2}{3} + \frac{4}{3}\ln\frac{m_c}{m_b}\right] - 3C_{7\gamma}^{\mathrm{eff}} \tag{2.41}$$

In the case of the up-quark loop, $p = u$, the amplitude becomes sensitive to additional long-distance dynamics, which is not strictly calculable. Using a suitable hadronic representation of the light-quark loop, we estimate

$$\begin{aligned}P_{V,n}^{u,\mathrm{EW}} &= (C_1 + N_c C_2)\left[\frac{4}{3}\ln\frac{m_b}{\mu} - \frac{10}{9} + \frac{4\pi^2}{3}\sum_{r=\rho,\omega}\frac{f_r^2}{m_V^2 - m_r^2 + i m_r\Gamma_r}\right. \\ &\quad \left. -\frac{2\pi}{3}\frac{m_V^2}{t_c} i + \frac{2}{3}\ln\frac{m_V^2}{m_b^2} + \frac{2}{3}\frac{t_c - m_V^2}{t_c}\ln\frac{t_c - m_V^2}{m_V^2}\right] - 3C_{7\gamma}^{\mathrm{eff}}\end{aligned} \tag{2.42}$$

where $t_c = 4\pi^2(f_\rho^2 + f_\omega^2)$. This point is discussed further in section 2.6.

The authors of [37] factorize the term (2.42) into a short-distance and a long-distance part, separated by a scale ν. The short-distance part is equivalent to (2.41) with m_c replaced by ν. The long-distance part is not considered explicitly in [37]. Our treatment is consistent with the framework of [37], but supplies a concrete model representation for the long-distance contribution of the electromagnetic penguin.

Hard spectator scattering

The hard spectator interactions (Fig. 2.4) determining the coefficients $a_{i,II}$ in (2.27) are governed by the quantities

$$H_{V_1V_2} = \frac{f_B f_{V_1}}{m_B^2 A_0^{B\to V_1}(0)}\int_0^1 \frac{d\xi}{\xi}\Phi_B(\xi)\int_0^1 \frac{dx}{\bar{x}}\phi_{||}^{V_2}(x)\int_0^1 \frac{dy}{\bar{y}}\left[\phi_{||}^{V_1}(y) + r_\perp^{V_1}\frac{\bar{x}}{x}\Phi_v^{V_1}(y)\right] \tag{2.43}$$

$$H'_{V_1V_2} = \frac{f_B f_{V_1}}{m_B^2 A_0^{B\to V_1}(0)}\int_0^1 \frac{d\xi}{\xi}\Phi_B(\xi)\int_0^1 \frac{dx}{x}\phi_{||}^{V_2}(x)\int_0^1 \frac{dy}{\bar{y}}\left[\phi_{||}^{V_1}(y) + r_\perp^{V_1}\frac{x}{\bar{x}}\Phi_v^{V_1}(y)\right] \tag{2.44}$$

Strictly speaking, only the twist-2 components $\phi_{||}$ of the vector meson distribution amplitudes contribute at leading power and are consistently calculable in the present approach. The twist-3 part described by Φ_v leads to terms with logarithmic endpoint singularities, but

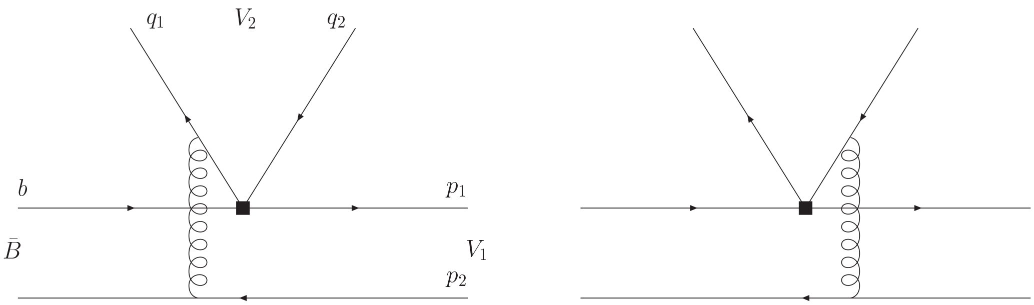

Figure 2.4: Hard spectator diagrams.

these terms are suppressed by one power of Λ_{QCD}/m_b. We shall include them in our analysis using a simple model, in order to estimate the potential impact of power corrections from this source. Following [30], we parametrize the endpoint singularity by

$$X_H = \int_0^1 \frac{dy}{\bar{y}} = \left(1 + \rho_H \, e^{i\phi_H}\right) \ln \frac{m_B}{\Lambda_h} \tag{2.45}$$

The logarithm comes from cutting off the lower range of integration at $\bar{y}_{min} = \Lambda_h/m_B$, and ρ_H, ϕ_H are real model parameters to allow for a complex X_H and a deviation from the default value $\ln(m_B/\Lambda_h)$. The integral over Φ_v in (2.43), (2.44) then becomes

$$\int_0^1 \frac{dy}{\bar{y}} \, \Phi_v(y) = 3(X_H - 2) \tag{2.46}$$

Throughout we use $\mu_h = \sqrt{\Lambda_h \, \mu}$ with $\Lambda_h = 0.5\,\text{GeV}$ as the scale in the spectator-scattering contributions.

2.2.2 $\bar{B} \to V_{1L} V_{2L}$ decay amplitudes

The transition operators $\mathcal{T}^d$, $\mathcal{T}^s$ describe a total of 28 two-body decays of B^-, $\bar{B}_d$ and $\bar{B}_s$ into the charmless vector mesons K^{*+}, K^{*-}, K^{*0}, $\bar{K}^{*0}$, ρ^+, ρ^-, ρ^0, ω and ϕ. There are 15 $\Delta S = 0$ ($b \to d$) and 13 $\Delta S = 1$ ($b \to s$) transitions. In this section we give the expressions for the amplitudes of these processes in terms of the factorization coefficients a_i. All light vector mesons are taken to be longitudinally polarized, that is, $\rho^-\rho^0$ here means $\rho_L^-\rho_L^0$. We use the abreviation

$$A_{V_1V_2} = i\frac{G_F}{\sqrt{2}} m_B^2 \, A_0^{B\to V_1}(m_{V_2}^2) \, f_{V_2} \tag{2.47}$$

suppressing the dependence of $A_{V_1V_2}$ on the B-meson flavour in the notation.

The decay amplitudes, up to the factor (2.47), are conveniently obtained from the transition operator by the following 'bosonization' of the bilinear quark currents in (2.26):

$$
\begin{aligned}
(\bar{u}b)_{V-A} &= \frac{B^-\rho^0 + B^-\omega}{\sqrt{2}} + \bar{B}_d\rho^+ + \bar{B}_s K^{*+} \\
(\bar{d}b)_{V-A} &= B^-\rho^- + \frac{\bar{B}_d\omega - \bar{B}_d\rho^0}{\sqrt{2}} + \bar{B}_s K^{*0} \\
(\bar{s}b)_{V-A} &= B^- K^{*-} + \bar{B}_d \bar{K}^{*0} + \bar{B}_s\phi
\end{aligned} \quad (2.48)
$$

$$
(\bar{d}u)_V = \rho^-, \qquad (\bar{u}d)_V = \rho^+, \qquad (\bar{s}d)_V = \bar{K}^{*0} \quad (2.49)
$$

$$
(\bar{d}s)_V = K^{*0}, \qquad (\bar{s}u)_V = K^{*-}, \qquad (\bar{u}s)_V = K^{*+} \quad (2.50)
$$

$$
(\bar{u}u)_V = \frac{\rho^0 + \omega}{\sqrt{2}}, \qquad (\bar{d}d)_V = \frac{\omega - \rho^0}{\sqrt{2}}, \qquad (\bar{s}s)_V = \phi \quad (2.51)
$$

In our notation the charge of B mesons (light mesons) corresponds to that of particles in the initial (final) state. Note that the axial vector parts of the light-quark currents don't contribute for final-state vector mesons. Insertion of these expressions in $\mathcal{T}^d$ ($\mathcal{T}^s$) from eq. (2.26) generates all $\Delta S = 0$ ($\Delta S = 1$) amplitudes. For a specific process $\bar{B} \to V_1 V_2$ the amplitude is found as the coefficient of $(\bar{B}V_1)V_2$ (and of $(\bar{B}V_2)V_1$ if $V_1 \neq V_2$). This procedure automatically keeps track of all sign and Clebsch-Gordan factors. Note, however, that an extra symmetry factor of 2 has to be included for amplitudes with two identical particles in the final state. From the structure of $\mathcal{T}^{d(s)}$ it follows that the coefficients a_3, a_5 and a_7^p, a_9^p always appear in the combination $a_3 + a_5$ and $a_7^p + a_9^p$, respectively. Representative numerical values for the coefficients a_i can be found in appendix 2.7.

The $\Delta S = 0$ transition amplitudes then read (a summation over $p = u, c$ is understood):

$$
\sqrt{2}\mathcal{A}(B^- \to \rho^-\rho^0) = \left[\lambda_u(a_1 + a_2) + \frac{3}{2}\lambda_p(a_7^p + a_9^p + a_{10}^p)\right] A_{\rho\rho} \quad (2.52)
$$

$$
\begin{aligned}
\sqrt{2}\mathcal{A}(B^- \to \rho^-\omega) &= \left[\lambda_u a_1 + \lambda_p(a_4^p + a_{10}^p)\right] A_{\omega\rho} \\
&\quad + \left[\lambda_u a_2 + \lambda_p\left(a_4^p + 2(a_3 + a_5) + \frac{1}{2}(a_7^p + a_9^p - a_{10}^p)\right)\right] A_{\rho\omega}
\end{aligned} \quad (2.53)
$$

$$
\mathcal{A}(B^- \to \rho^-\phi) = \lambda_p\left[a_3 + a_5 - \frac{1}{2}(a_7^p + a_9^p)\right] A_{\rho\phi} \quad (2.54)
$$

$$
\mathcal{A}(B^- \to K^{*-}K^{*0}) = \lambda_p\left[a_4^p - \frac{1}{2}a_{10}^p\right] A_{K^*K^*} \quad (2.55)
$$

$$
\mathcal{A}(\bar{B}_d \to \rho^0\rho^0) = \left[-\lambda_u a_2 + \lambda_p\left(a_4^p - \frac{3}{2}(a_7^p + a_9^p) - \frac{1}{2}a_{10}^p\right)\right] A_{\rho\rho} \quad (2.56)
$$

$$
\begin{aligned}
\mathcal{A}(\bar{B}_d \to \rho^0\omega) &= \frac{1}{2}\left[\lambda_u a_2 + \lambda_p\left(-a_4^p + \frac{3}{2}(a_7^p + a_9^p) + \frac{1}{2}a_{10}^p\right)\right] A_{\omega\rho} \\
&\quad -\frac{1}{2}\left[\lambda_u a_2 + \lambda_p\left(a_4^p + 2(a_3 + a_5) + \frac{1}{2}(a_7^p + a_9^p - a_{10}^p)\right)\right] A_{\rho\omega}
\end{aligned} \quad (2.57)
$$

$$\mathcal{A}(\bar{B}_d \to \omega\omega) = \left[\lambda_u a_2 + \lambda_p \left(a_4^p + 2(a_3 + a_5) + \frac{1}{2}(a_7^p + a_9^p - a_{10}^p)\right)\right] A_{\omega\omega} \tag{2.58}$$

$$\sqrt{2}\mathcal{A}(\bar{B}_d \to \rho^0\phi) = -\lambda_p \left[a_3 + a_5 - \frac{1}{2}(a_7^p + a_9^p)\right] A_{\rho\phi} \tag{2.59}$$

$$\sqrt{2}\mathcal{A}(\bar{B}_d \to \omega\phi) = \lambda_p \left[a_3 + a_5 - \frac{1}{2}(a_7^p + a_9^p)\right] A_{\omega\phi} \tag{2.60}$$

$$\mathcal{A}(\bar{B}_d \to \rho^+\rho^-) = [\lambda_u a_1 + \lambda_p(a_4^p + a_{10}^p)]\, A_{\rho\rho} \tag{2.61}$$

$$\mathcal{A}(\bar{B}_d \to \bar{K}^{*0}K^{*0}) = \lambda_p \left[a_4^p - \frac{1}{2}a_{10}^p\right] A_{K^*K^*} \tag{2.62}$$

$$\sqrt{2}\mathcal{A}(\bar{B}_s \to K^{*0}\rho^0) = \left[\lambda_u a_2 + \lambda_p \left(-a_4^p + \frac{3}{2}(a_7^p + a_9^p) + \frac{1}{2}a_{10}^p\right)\right] A_{K^*\rho} \tag{2.63}$$

$$\sqrt{2}\mathcal{A}(\bar{B}_s \to K^{*0}\omega) = \left[\lambda_u a_2 + \lambda_p \left(a_4^p + 2(a_3 + a_5) + \frac{1}{2}(a_7^p + a_9^p - a_{10}^p)\right)\right] A_{K^*\omega} \tag{2.64}$$

$$\mathcal{A}(\bar{B}_s \to K^{*0}\phi) = \lambda_p \left[a_4^p - \frac{1}{2}a_{10}^p\right] A_{\phi K^*} + \lambda_p \left[a_3 + a_5 - \frac{1}{2}(a_7^p + a_9^p)\right] A_{K^*\phi} \tag{2.65}$$

$$\mathcal{A}(\bar{B}_s \to K^{*+}\rho^-) = [\lambda_u a_1 + \lambda_p(a_4^p + a_{10}^p)]\, A_{K^*\rho} \tag{2.66}$$

The amplitudes for $\Delta S = 1$ transitions are found to be:

$$\sqrt{2}\mathcal{A}(B^- \to K^{*-}\rho^0) = \left[\lambda'_u a_1 + \lambda'_p(a_4^p + a_{10}^p)\right] A_{\rho K^*} + \left[\lambda'_u a_2 + \frac{3}{2}\lambda'_p(a_7^p + a_9^p)\right] A_{K^*\rho} \tag{2.67}$$

$$\begin{aligned}\sqrt{2}\mathcal{A}(B^- \to K^{*-}\omega) = {} & \left[\lambda'_u a_1 + \lambda'_p(a_4^p + a_{10}^p)\right] A_{\omega K^*} \\ & + \left[\lambda'_u a_2 + \lambda'_p \left(2(a_3 + a_5) + \frac{1}{2}(a_7^p + a_9^p)\right)\right] A_{K^*\omega}\end{aligned} \tag{2.68}$$

$$\mathcal{A}(B^- \to K^{*-}\phi) = \lambda'_p \left[a_4^p + a_3 + a_5 - \frac{1}{2}(a_7^p + a_9^p + a_{10}^p)\right] A_{K^*\phi} \tag{2.69}$$

$$\mathcal{A}(B^- \to \bar{K}^{*0}\rho^-) = \lambda'_p \left[a_4^p - \frac{1}{2}a_{10}^p\right] A_{\rho K^*} \tag{2.70}$$

$$\sqrt{2}\mathcal{A}(\bar{B}_d \to \bar{K}^{*0}\rho^0) = \left[\lambda'_u a_2 + \frac{3}{2}\lambda'_p(a_7^p + a_9^p)\right] A_{K^*\rho} - \lambda'_p \left[a_4^p - \frac{1}{2}a_{10}^p\right] A_{\rho K^*} \tag{2.71}$$

$$\sqrt{2}\mathcal{A}(\bar{B}_d \to \bar{K}^{*0}\omega) = \left[\lambda'_u a_2 + \lambda'_p \left(2(a_3 + a_5) + \frac{1}{2}(a_7^p + a_9^p)\right)\right] A_{K^*\omega} + \lambda'_p \left[a_4^p - \frac{1}{2}a_{10}^p\right] A_{\omega K^*} \quad (2.72)$$

$$\mathcal{A}(\bar{B}_d \to \bar{K}^{*0}\phi) = \lambda'_p \left[a_4^p + a_3 + a_5 - \frac{1}{2}(a_7^p + a_9^p + a_{10}^p)\right] A_{K^*\phi} \quad (2.73)$$

$$\mathcal{A}(\bar{B}_d \to K^{*-}\rho^+) = \left[\lambda'_u a_1 + \lambda'_p(a_4^p + a_{10}^p)\right] A_{\rho K^*} \quad (2.74)$$

$$\sqrt{2}\mathcal{A}(\bar{B}_s \to \rho^0\phi) = \left[\lambda'_u a_2 + \frac{3}{2}\lambda'_p(a_7^p + a_9^p)\right] A_{\phi\rho} \quad (2.75)$$

$$\sqrt{2}\mathcal{A}(\bar{B}_s \to \omega\phi) = \left[\lambda'_u a_2 + \lambda'_p \left(2(a_3 + a_5) + \frac{1}{2}(a_7^p + a_9^p)\right)\right] A_{\phi\omega} \quad (2.76)$$

$$\mathcal{A}(\bar{B}_s \to \phi\phi) = 2\lambda'_p \left[a_4^p + a_3 + a_5 - \frac{1}{2}(a_7^p + a_9^p + a_{10}^p)\right] A_{\phi\phi} \quad (2.77)$$

$$\mathcal{A}(\bar{B}_s \to K^{*+}K^{*-}) = \left[\lambda'_u a_1 + \lambda'_p(a_4^p + a_{10}^p)\right] A_{K^*K^*} \quad (2.78)$$

$$\mathcal{A}(\bar{B}_s \to \bar{K}^{*0}K^{*0}) = \lambda'_p \left[a_4^p - \frac{1}{2}a_{10}^p\right] A_{K^*K^*} \quad (2.79)$$

2.2.3 Weak annihilation amplitudes

The decay mechanism of weak annihilation (Fig. 2.5) gives contributions to the amplitudes for $B \to V_L V_L$ decays that are supressed by Λ_{QCD}/m_b. These power corrections are not calculable in the usual factorization framework. This is indicated by end-point singularities from the integrals over light-cone momentum fractions in a hard-scattering ansatz. We shall use the model of [30], which is based on this ansatz together with a cut-off procedure, to estimate the impact of annihilation effects on the leading decay amplitudes. Following the notation of [30] we write

$$\langle V_{1L}V_{2L}|\mathcal{H}_{\text{eff}}^{\Delta S=0}|\bar{B}\rangle_{\text{ann}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \, \langle V_{1L}V_{2L}|\mathcal{T}_p^{\text{ann},d}|\bar{B}\rangle \quad (2.80)$$

with $\lambda_p = V_{pb}V_{pd}^*$ and

$$\mathcal{T}_p^{\text{ann},d} = \delta_{up}\,(\delta_{rd}\, b_1\, \sigma_u^u + \delta_{ru}\, b_2\, \sigma_d^u) + b_3\, \sigma_d^r + \delta_{rd}\, b_4\, \text{tr}(\sigma) + \frac{3}{2}\, b_3^{\text{EW}}\, e_r\, \sigma_d^r + \frac{3}{2}\, \delta_{rd}\, b_4^{\text{EW}}\, \text{tr}(Q\,\sigma) \quad (2.81)$$

Here the index $r = u,\, d,\, s$ denotes the flavour of the spectator quark in the B meson and $Q = \text{diag}(2/3, -1/3, -1/3)$. The corresponding formulas for $\Delta S = 1$ transitions are

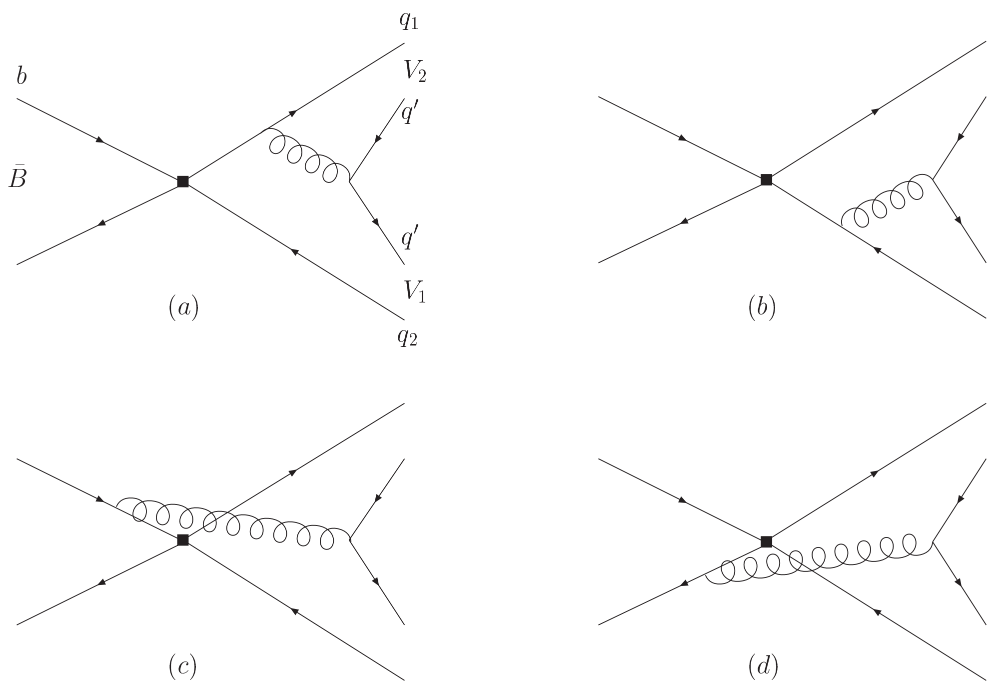

Figure 2.5: Annihilation diagrams.

obtained by interchanging the labels $d \leftrightarrow s$ in the expressions (2.80), (2.81) for $\Delta S = 0$. In particular, λ_p is then replaced by $\lambda'_p = V_{pb}V^*_{ps}$.

The operators

$$\sigma^{q_2}_{q_1} = \sum_{q'=u,d,s} (\bar{q}' q_2) \times (\bar{q}_1 q') \tag{2.82}$$

encode the valence quarks of the final state mesons. Matrix elements of the product of currents in (2.82) are defined as

$$\langle V_{1L} V_{2L} | j_1 \times j_2 | \bar{B} \rangle \equiv i c f_B f_{V_1} f_{V_2} \tag{2.83}$$

where $c = 0,\, 1,\, \pm 1/\sqrt{2}$, etc., is the appropriate Clebsch-Gordan coefficient and symmetry factor, relating the currents j_1 and j_2 to the mesons V_1, V_2. The coefficients b_i are given by

$$\begin{aligned}
b_1 &= \frac{C_F}{N_c^2} C_1 A_1^i\,, \qquad b_3 = \frac{C_F}{N_c^2}\left[C_3 A_1^i + C_5 (A_3^i + A_3^f) + N_c C_6 A_3^f\right],\\
b_2 &= \frac{C_F}{N_c^2} C_2 A_1^i\,, \qquad b_4 = \frac{C_F}{N_c^2}\left[C_4 A_1^i + C_6 A_2^i\right],\\
b_3^{\rm EW} &= \frac{C_F}{N_c^2}\left[C_9 A_1^i + C_7 (A_3^i + A_3^f) + N_c C_8 A_3^f\right],\\
b_4^{\rm EW} &= \frac{C_F}{N_c^2}\left[C_{10} A_1^i + C_8 A_2^i\right]
\end{aligned} \tag{2.84}$$

and correspond to current–current annihilation (b_1, b_2), penguin annihilation (b_3, b_4), and electroweak penguin annihilation $(b_3^{\rm EW}, b_4^{\rm EW})$. These coefficients depend on the final-state mesons, $b_i = b_i(V_1V_2)$, but this dependence will be left implicit in the following. Finally, the $A_k^{i,f}$ read

$$\begin{aligned}
A_1^i &= \pi\alpha_s \int_0^1 dxdy\, \left\{ \phi_{||}^{V_1}(y)\phi_{||}^{V_2}(x) \left[\frac{1}{y(1-\bar{y}x)} + \frac{1}{y\bar{x}^2}\right] - \Phi_v^{V_1}(y)\Phi_v^{V_2}(x) r_\perp^{V_1} r_\perp^{V_2} \frac{2}{y\bar{x}} \right\}, \\
A_1^f &= 0, \\
A_2^i &= \pi\alpha_s \int_0^1 dxdy\, \left\{ \phi_{||}^{V_1}(y)\phi_{||}^{V_2}(x) \left[\frac{1}{\bar{x}(1-\bar{y}x)} + \frac{1}{y^2\bar{x}}\right] - \Phi_v^{V_1}(y)\Phi_v^{V_2}(x) r_\perp^{V_1} r_\perp^{V_2} \frac{2}{y\bar{x}} \right\}, \\
A_2^f &= 0, \\
A_3^i &= \pi\alpha_s \int_0^1 dxdy\, \left\{ \phi_{||}^{V_1}(y)\Phi_v^{V_2}(x) r_\perp^{V_2} \frac{2x}{y\bar{x}(1-\bar{y}x)} + \phi_{||}^{V_2}(x)\Phi_v^{V_1}(y) r_\perp^{V_1} \frac{2\bar{y}}{y\bar{x}(1-\bar{y}x)} \right\}, \\
A_3^f &= \pi\alpha_s \int_0^1 dxdy\, \left\{ -\phi_{||}^{V_1}(y)\Phi_v^{V_2}(x) r_\perp^{V_2} \frac{2(1+y)}{y^2\bar{x}} + \phi_{||}^{V_2}(x)\Phi_v^{V_1}(y) r_\perp^{V_1} \frac{2(1+\bar{x})}{y\bar{x}^2} \right\}
\end{aligned} \tag{2.85}$$

The superscript i (f) denotes gluon emission from the initial- (final-)state quarks, as shown in Fig. 2.5 (c) and (d) ((a) and (b)). The subscript k indicates the Dirac structure of the four-quark operators, $\Gamma_1 \otimes \Gamma_2 = (V-A)\otimes(V-A)$ $(k=1)$, $(V-A)\otimes(V+A)$ $(k=2)$, $(-2)(S-P)\otimes(S+P)$ $(k=3)$. The various quantities in (2.84) will be evaluated at the scale $\mu_h = \sqrt{\Lambda_h\,\mu}$, similarly to the spectator-interaction terms.

For the numerical estimate of weak annihilation the IR-divergent quantities $A_k^{i,f}$ in (2.85) will be parametrized by

$$X_A = \int_0^1 \frac{dx}{x} = \left(1 + \rho_A\, e^{i\phi_A}\right) \ln\frac{m_B}{\Lambda_h} \tag{2.86}$$

The quantity X_A is the cut-off regulated integral $\int_{\Lambda_h/m_B}^1 dx/x = \ln(m_B/\Lambda_h)$, with scale $\Lambda_h = 0.5\,\text{GeV}$, modified by a phenomenological magnitude ρ_A and phase ϕ_A [30]. Using $SU(3)$ flavour symmetry and the asymptotic forms of the meson wavefunctions $\phi_{||}$ and Φ_v, one finds $A_1^i = A_2^i$, $A_3^i = 0$ and

$$\begin{aligned}
A_1^i &\approx \pi\alpha_s \left[18\left(X_A - 4 + \frac{\pi^2}{3}\right) + 18(r_\perp^V)^2\left(X_A - 2\right)^2\right] \\
A_3^f &\approx -36\pi\alpha_s r_\perp^V \left(2X_A^2 - 5X_A + 2\right)
\end{aligned} \tag{2.87}$$

We next give the results for the annihilation amplitudes in terms of the coefficients b_i, where we define

$$B_{V_1V_2} = i\frac{G_F}{\sqrt{2}}\, f_B f_{V_1} f_{V_2} \tag{2.88}$$

The decay constant f_B depends on the flavour of the decaying B meson, even though this is not made explicit in the notation for the $B_{V_1V_2}$. The following expressions can be efficiently obtained with a procedure similar to the one described at the beginning of sec. 2.2.2. Typical numerical values for the coefficients b_i are given in appendix 2.7. For the channels with $\Delta S = 0$ the annihilation contributions read

$$\mathcal{A}_{\rm ann}(B^- \to \rho^- \rho^0) = 0 \tag{2.89}$$

$$\sqrt{2}\mathcal{A}_{\rm ann}(B^- \to \rho^- \omega) = \left[\lambda_u \, 2b_2 + (\lambda_u + \lambda_c)\, 2(b_3 + b_3^{\rm EW})\right] B_{\rho\omega} \tag{2.90}$$

$$\mathcal{A}_{\rm ann}(B^- \to \rho^- \phi) = 0 \tag{2.91}$$

$$\mathcal{A}_{\rm ann}(B^- \to K^{*-} K^{*0}) = \left[\lambda_u \, b_2 + (\lambda_u + \lambda_c)\, (b_3 + b_3^{\rm EW})\right] B_{K^* K^*} \tag{2.92}$$

$$\mathcal{A}_{\rm ann}(\bar{B}_d \to \rho^0 \rho^0) = \left[\lambda_u \, b_1 + (\lambda_u + \lambda_c) \left(b_3 + 2b_4 - \frac{1}{2}b_3^{\rm EW} + \frac{1}{2}b_4^{\rm EW}\right)\right] B_{\rho\rho} \tag{2.93}$$

$$\mathcal{A}_{\rm ann}(\bar{B}_d \to \rho^0 \omega) = \left[\lambda_u \, b_1 + (\lambda_u + \lambda_c) \left(-b_3 + \frac{1}{2}b_3^{\rm EW} + \frac{3}{2}b_4^{\rm EW}\right)\right] B_{\rho\omega} \tag{2.94}$$

$$\mathcal{A}_{\rm ann}(\bar{B}_d \to \omega \omega) = \left[\lambda_u \, b_1 + (\lambda_u + \lambda_c) \left(b_3 + 2b_4 - \frac{1}{2}b_3^{\rm EW} + \frac{1}{2}b_4^{\rm EW}\right)\right] B_{\omega\omega} \tag{2.95}$$

$$\mathcal{A}_{\rm ann}(\bar{B}_d \to \rho^0 \phi) = 0 \tag{2.96}$$

$$\mathcal{A}_{\rm ann}(\bar{B}_d \to \omega \phi) = 0 \tag{2.97}$$

$$\mathcal{A}_{\rm ann}(\bar{B}_d \to \rho^+ \rho^-) = \left[\lambda_u \, b_1 + (\lambda_u + \lambda_c) \left(b_3 + 2b_4 - \frac{1}{2}b_3^{\rm EW} + \frac{1}{2}b_4^{\rm EW}\right)\right] B_{\rho\rho} \tag{2.98}$$

$$\mathcal{A}_{\rm ann}(\bar{B}_d \to \bar{K}^{*0} K^{*0}) = (\lambda_u + \lambda_c) \left(b_3 + 2b_4 - \frac{1}{2}b_3^{\rm EW} - b_4^{\rm EW}\right) B_{K^* K^*} \tag{2.99}$$

$$\sqrt{2}\mathcal{A}_{\rm ann}(\bar{B}_s \to K^{*0} \rho^0) = (\lambda_u + \lambda_c) \left(-b_3 + \frac{1}{2}b_3^{\rm EW}\right) B_{K^* \rho} \tag{2.100}$$

$$\sqrt{2}\mathcal{A}_{\rm ann}(\bar{B}_s \to K^{*0} \omega) = (\lambda_u + \lambda_c) \left(b_3 - \frac{1}{2}b_3^{\rm EW}\right) B_{K^* \omega} \tag{2.101}$$

$$\mathcal{A}_{\rm ann}(\bar{B}_s \to K^{*0} \phi) = (\lambda_u + \lambda_c) \left(b_3 - \frac{1}{2}b_3^{\rm EW}\right) B_{K^* \phi} \tag{2.102}$$

$$\mathcal{A}_{\rm ann}(\bar{B}_s \to K^{*+} \rho^-) = (\lambda_u + \lambda_c) \left(b_3 - \frac{1}{2}b_3^{\rm EW}\right) B_{K^* \rho} \tag{2.103}$$

In addition there are two $\Delta S = 0$ decay modes that proceed only through annihilation diagrams:

$$\mathcal{A}_{\rm ann}(\bar{B}_d \to K^{*+} K^{*-}) = \left[\lambda_u \, b_1 + (\lambda_u + \lambda_c) \left(2b_4 + \frac{1}{2}b_4^{\rm EW}\right)\right] B_{K^* K^*} \tag{2.104}$$

$$\mathcal{A}_{\rm ann}(\bar{B}_d \to \phi\phi) = (\lambda_u + \lambda_c) \left(2b_4 - b_4^{\rm EW}\right) B_{\phi\phi} \quad (2.105)$$

For the annihilation amplitudes with $\Delta S = 1$ we obtain:

$$\sqrt{2}\mathcal{A}_{\rm ann}(B^- \to K^{*-}\rho^0) = \left[\lambda'_u\, b_2 + (\lambda'_u + \lambda'_c) \left(b_3 + b_3^{\rm EW}\right)\right] B_{K^*\rho} \quad (2.106)$$

$$\sqrt{2}\mathcal{A}_{\rm ann}(B^- \to K^{*-}\omega) = \left[\lambda'_u\, b_2 + (\lambda'_u + \lambda'_c) \left(b_3 + b_3^{\rm EW}\right)\right] B_{K^*\omega} \quad (2.107)$$

$$\mathcal{A}_{\rm ann}(B^- \to K^{*-}\phi) = \left[\lambda'_u\, b_2 + (\lambda'_u + \lambda'_c) \left(b_3 + b_3^{\rm EW}\right)\right] B_{K^*\phi} \quad (2.108)$$

$$\mathcal{A}_{\rm ann}(B^- \to \bar{K}^{*0}\rho^-) = \left[\lambda'_u\, b_2 + (\lambda'_u + \lambda'_c) \left(b_3 + b_3^{\rm EW}\right)\right] B_{K^*\rho} \quad (2.109)$$

$$\sqrt{2}\mathcal{A}_{\rm ann}(\bar{B}_d \to \bar{K}^{*0}\rho^0) = (\lambda'_u + \lambda'_c) \left(-b_3 + \frac{1}{2}b_3^{\rm EW}\right) B_{K^*\rho} \quad (2.110)$$

$$\sqrt{2}\mathcal{A}_{\rm ann}(\bar{B}_d \to \bar{K}^{*0}\omega) = (\lambda'_u + \lambda'_c) \left(b_3 - \frac{1}{2}b_3^{\rm EW}\right) B_{K^*\omega} \quad (2.111)$$

$$\mathcal{A}_{\rm ann}(\bar{B}_d \to \bar{K}^{*0}\phi) = (\lambda'_u + \lambda'_c) \left(b_3 - \frac{1}{2}b_3^{\rm EW}\right) B_{K^*\phi} \quad (2.112)$$

$$\mathcal{A}_{\rm ann}(\bar{B}_d \to K^{*-}\rho^+) = (\lambda'_u + \lambda'_c) \left(b_3 - \frac{1}{2}b_3^{\rm EW}\right) B_{K^*\rho} \quad (2.113)$$

$$\mathcal{A}_{\rm ann}(\bar{B}_s \to \rho^0\phi) = 0 \quad (2.114)$$

$$\mathcal{A}_{\rm ann}(\bar{B}_s \to \omega\phi) = 0 \quad (2.115)$$

$$\mathcal{A}_{\rm ann}(\bar{B}_s \to \phi\phi) = (\lambda'_u + \lambda'_c) \left(2b_3 + 2b_4 - b_3^{\rm EW} - b_4^{\rm EW}\right) B_{\phi\phi} \quad (2.116)$$

$$\mathcal{A}_{\rm ann}(\bar{B}_s \to K^{*+}K^{*-}) = \left[\lambda'_u\, b_1 + (\lambda'_u + \lambda'_c) \left(b_3 + 2b_4 - \frac{1}{2}b_3^{\rm EW} + \frac{1}{2}b_4^{\rm EW}\right)\right] B_{K^*K^*} \quad (2.117)$$

$$\mathcal{A}_{\rm ann}(\bar{B}_s \to \bar{K}^{*0}K^{*0}) = (\lambda'_u + \lambda'_c) \left(b_3 + 2b_4 - \frac{1}{2}b_3^{\rm EW} - b_4^{\rm EW}\right) B_{K^*K^*} \quad (2.118)$$

In the case of $\Delta S = 1$ transitions there are four pure annihilation modes. Their amplitudes have the form:

$$\mathcal{A}_{\rm ann}(\bar{B}_s \to \rho^+\rho^-) = \left[\lambda'_u\, b_1 + (\lambda'_u + \lambda'_c) \left(2b_4 + \frac{1}{2}b_4^{\rm EW}\right)\right] B_{\rho\rho} \quad (2.119)$$

$$\mathcal{A}_{\rm ann}(\bar{B}_s \to \rho^0\rho^0) = \left[\lambda'_u\, b_1 + (\lambda'_u + \lambda'_c) \left(2b_4 + \frac{1}{2}b_4^{\rm EW}\right)\right] B_{\rho\rho} \quad (2.120)$$

$$\mathcal{A}_{\rm ann}(\bar{B}_s \to \omega\omega) = \left[\lambda'_u\, b_1 + (\lambda'_u + \lambda'_c) \left(2b_4 + \frac{1}{2}b_4^{\rm EW}\right)\right] B_{\omega\omega} \quad (2.121)$$

$$\mathcal{A}_{\rm ann}(\bar{B}_s \to \rho^0\omega) = \left[\lambda'_u\, b_1 + (\lambda'_u + \lambda'_c) \frac{3}{2}b_4^{\rm EW}\right] B_{\rho\omega} \quad (2.122)$$

Table 2.1: Experimental results [55] for CP-averaged branching ratios and longitudinal polarization fractions f_L of $\bar{B} \to VV$ decays. Here the B meson is either a B^- or a $\bar{B}_d$. The branching ratios for $\bar{B} \to V_L V_L$ have been obtained as $f_L\, B(\bar{B} \to VV)$. Also shown are the direct CP asymmetries $A_{CP} \equiv -C \equiv (B(\bar{B} \to \bar{f}) - B(B \to f))/(B(\bar{B} \to \bar{f}) + B(B \to f))$. The label ($L$) indicates that A_{CP} refers to vector mesons with longitudinal polarization only.

VV	$B(\bar{B} \to VV)/10^{-6}$	f_L	$B(\bar{B} \to V_L V_L)/10^{-6}$	A_{CP}
$\rho^+\rho^-$	$24.2^{+3.1}_{-3.2}$	$0.978^{+0.025}_{-0.022}$	23.7 ± 3.2	0.06 ± 0.13 (L)
$\rho^0\rho^0$	$0.73^{+0.27}_{-0.28}$	$0.75^{+0.12}_{-0.15}$	0.55 ± 0.22	-0.2 ± 0.9 (L)
$\rho^-\rho^0$	$24.0^{+1.9}_{-2.0}$	0.950 ± 0.016	22.8 ± 1.9	-0.051 ± 0.054
$\rho^-\omega$	15.9 ± 2.1	0.90 ± 0.06	14.3 ± 2.1	-0.20 ± 0.09
$\bar{K}^{*0}K^{*0}$	0.81 ± 0.23	$0.80^{+0.12}_{-0.13}$	0.65 ± 0.21	—
$K^{*-}K^{*0}$	1.2 ± 0.5	$0.75^{+0.16}_{-0.26}$	0.9 ± 0.45	—
$\bar{K}^{*0}\rho^0$	3.4 ± 1.0	0.57 ± 0.12	1.9 ± 0.7	0.09 ± 0.19
$K^{*-}\rho^+$	< 12	—	< 12	—
$K^{*-}\rho^0$	< 6.1	—	< 6.1	$0.20^{+0.32}_{-0.29}$
$\bar{K}^{*0}\rho^-$	9.2 ± 1.5	0.48 ± 0.08	4.4 ± 1.0	-0.01 ± 0.16
$\bar{K}^{*0}\phi$	9.8 ± 0.7	0.480 ± 0.030	4.7 ± 0.5	-0.01 ± 0.05
$K^{*-}\phi$	10.0 ± 1.1	0.50 ± 0.05	5.0 ± 0.7	-0.01 ± 0.08
$\bar{K}^{*0}\omega$	2.0 ± 0.5	0.70 ± 0.13	1.4 ± 0.44	0.45 ± 0.25

Because of similarities in the flavour structure of $\mathcal{T}_p^{d(s)}$ and $\mathcal{T}_p^{ann,d(s)}$, in all amplitudes the coefficient b_3 appears together with the factorization coefficient a_4^p in the combination $a_4^p + B_{V_1V_2}/A_{V_1V_2} b_3$. This has been noted before in the context of PP and PV final states [37].

2.3 Experimental results and input parameters

Available data on the decays of B^- and $\bar{B}_d$ mesons into a pair of light vector mesons are displayed in Tables 2.1 and 2.2. The results are from [55] unless indicated otherwise. CP averaging is understood for branching ratios and f_L. No data are available yet on $\bar{B}_s \to VV$ decays.

Table 2.3 collects the input parameters used in our analysis. The values of m_V, Γ_V, m_B, $|V_{us}|$ and $|V_{cb}|$ have been taken from [57] and are compatible with [58], except for $\Gamma_V(K^*)$. They have only small uncertainties, which we neglect. Our choice for $|V_{ub}/V_{cb}|$

Table 2.2: Experimental results for further $B \to VV$ decays [55]. Quoted are the CP-averaged branching fractions in units of 10^{-6}. The B meson is either a B^- or a $\bar{B}_d$.

$K^{*-}\omega$	$\rho^-\phi$	$K^{*+}K^{*-}$	$\rho^0\omega$
< 7.4	< 3.0	< 2.0	< 1.6
$\omega\omega$	$\rho^0\phi$	$\omega\phi$	$\phi\phi$
< 4.0	< 0.33	< 1.2	< 0.2

is compatible with the exclusive determinations quoted in [57]. We prefer those over the inclusive values since we use V_{ub} in exclusive processes where the form factors rely on similar theoretical methods (light-cone QCD sum rules, lattice) as in the exclusive extraction of $|V_{ub}|$. The lifetimes of B^- and $\bar{B}_d$ are also from [57]. On the other hand, the lifetime of $\bar{B}_s$ is put equal to $\tau_{\bar{B}_d}$, which is expected theoretically to hold to very high accuracy. The value of $\tau_{\bar{B}_s}$ from [57] is compatible with this, but is still affected by a larger error.

The number for $\sin 2\beta$ is the average of [55] from CP violation in $b \to c\bar{c}s$ modes. The angle γ corresponds to the result of global CKM fits [26, 59]. The standard model parameters $\Lambda^{(5)}_{\overline{\text{MS}}}$, m_b, m_c, m_t and M_W are the same as in [30]. Changes in these values have been small in comparison with the relevant uncertainties. The quark masses are running $\overline{\text{MS}}$-masses.

The decay constants f_V can be determined from data on $V \to l^+l^-$ and $\tau \to V\nu$. We use the values quoted in [37]. The transverse decay constants $f_V^\perp$ need to be computed theoretically, for instance with QCD sum rules. The results we use for $f_V^\perp$ have been compiled in [60]. The $B \to V$ form factors are from QCD sum rules on the light cone [61]. These results do not yet incorporate some improvements in the treatment of $SU(3)$ breaking that has been achieved in the meantime (see comments in sec. 2.3 of [62]). The uncertainties on the form factors in Table 2.3 are taken to be somewhat larger than reported in [61]. The Gegenbauer coefficients $\alpha^V_{1,2}$ are still rather uncertain. We adopt numbers of the typical size found in QCD sum rule calculations [61,63] and allow for sizable uncertainties. The range of numbers for the B-meson decay constants is representative of results from recent unquenched lattice simulations (see sec. 2.4 in [62] for a review and detailed references). The parameter λ_B is not well known at present. We shall consider here the generous range already used in [30]. No attempt is made to account for $SU(3)$ breaking in this quantity.

Table 2.3: Input parameters for $B \to V_L V_L$ decays. Here B_q stands for either B^- or $\bar{B}_d$. The values of the scale dependent quantities $f_V^\perp = f_V^\perp(\mu)$ are given for $\mu = 1\,\text{GeV}$. The scale dependence of $\alpha_{1,2}^V$ is neglected.

Light vector mesons						
V	m_V/MeV	Γ_V/MeV	f_V/MeV	$f_V^\perp$/MeV	α_1^V	α_2^V
ρ	776	149	209	165 ± 9	0	0.1 ± 0.3
ω	783	8	187	151 ± 9	0	0.1 ± 0.3
K^*	894	51	218	185 ± 10	0.1 ± 0.1	0.1 ± 0.3
ϕ	1019	4	221	186 ± 9	0	0.1 ± 0.3

B mesons				
B	m_B/GeV	τ_B/ps	f_B/MeV	λ_B/MeV
B^-	5.28	1.64	200 ± 30	350 ± 150
$\bar{B}_d$	5.28	1.53	200 ± 30	350 ± 150
$\bar{B}_s$	5.37	1.53	230 ± 30	350 ± 150

Form factors				
$A_0^{B_q\to\rho}(0)$	$A_0^{B_q\to\omega}(0)$	$A_0^{B_q\to K^*}(0)$	$A_0^{B_s\to K^*}(0)$	$A_0^{B_s\to\phi}(0)$
0.30 ± 0.04	0.28 ± 0.05	0.37 ± 0.05	0.36 ± 0.05	0.47 ± 0.06

SM parameters				
$\Lambda_{\overline{\text{MS}}}^{(5)}$/MeV	$m_b(m_b)$/GeV	$m_c(m_b)$/GeV	$m_t(m_t)$/GeV	M_W/GeV
225	4.2	1.3 ± 0.2	167	80.4
$\lvert V_{us}\rvert$	$\lvert V_{cb}\rvert$	$\lvert V_{ub}/V_{cb}\rvert$	γ	$\sin 2\beta$
0.226	0.0416	0.09 ± 0.01	$(67 \pm 12)^\circ$	0.673 ± 0.023

2.4 Phenomenological analysis

2.4.1 $B \to V_L V_L$ branching fractions

The branching fraction of a decay $\bar{B} \to V_{1L} V_{2L}$ is obtained from the corresponding amplitude $\mathcal{A}$ as

$$B(\bar{B} \to V_{1L} V_{2L}) = S\,\frac{\tau_B}{16\pi m_B}\,|\mathcal{A}(\bar{B} \to V_{1L} V_{2L})|^2 \qquad (2.123)$$

Here S is a symmetry factor with $S = 1/2$ if V_1 and V_2 are identical and $S = 1$ otherwise.

Predictions of CP averaged branching ratios are compiled in Table 2.4 and Table 2.5 for strangeness-conserving and strangeness-changing $\bar{B} \to V_{1L} V_{2L}$ decays, respectively. Absolute branching fractions have in general sizable uncertainties from hadronic input

Table 2.4: CP-averaged branching fractions for $B \to V_L V_L$ decays with $\Delta S = 0$. The sensitivity to variations in the input parameters according to Table 2.3 is displayed where the upper (lower) entry corresponds to the larger (smaller) value of the parameter. The renormalization scale μ is varied between $2m_b$ and $m_b/2$. The model parameters $X_{A,H}(\rho_{A,H}, \phi_{A,H})$ from power corrections are varied within the range given by $0 \le \rho_{A,H} \le 1$ and $0 \le \phi_{A,H} \le 2\pi$. Here upper (lower) entries refer to positive (negative) $\mathrm{Im}X_{A,H}$. The appropriate units for each mode are given in square brackets.

mode	central	A_0	α_2^V	λ_B	f_B	μ	X_A	X_H	$\left\lvert\frac{V_{ub}}{V_{cb}}\right\rvert$	γ
$B^- \to \rho^- \rho^0$	$17.5[10^{-6}]$	$^{+4.5}_{-4.0}$	$^{+1.9}_{-1.4}$	$^{-0.9}_{+2.5}$	$^{+0.5}_{-0.5}$	$^{+0.1}_{-0.0}$	–	$^{+1.5}_{-1.4}$	$^{+4.1}_{-3.6}$	$^{-0.2}_{+0.2}$
$B^- \to \rho^- \omega$	$15.5[10^{-6}]$	$^{+5.6}_{-4.7}$	$^{+1.4}_{-0.9}$	$^{-0.8}_{+2.0}$	$^{+0.3}_{-0.3}$	$^{+0.3}_{-0.3}$	$^{-1.2}_{+0.9}$	$^{+1.2}_{-1.1}$	$^{+3.4}_{-3.0}$	$^{-0.8}_{+0.7}$
$B^- \to \rho^- \phi$	$6.0[10^{-9}]$	$^{+0.9}_{-0.8}$	$^{+3.7}_{-2.2}$	$^{-1.5}_{+4.8}$	$^{+0.9}_{-0.8}$	$^{-1.1}_{+3.4}$	–	$^{+2.8}_{-2.2}$	$^{+0.0}_{-0.0}$	$^{+1.1}_{-1.0}$
$B^- \to K^{*-} K^{*0}$	$2.7[10^{-7}]$	$^{+0.9}_{-0.8}$	$^{-0.8}_{+0.9}$	$^{+0.2}_{-0.4}$	$^{-0.1}_{+0.1}$	$^{-0.4}_{+0.5}$	$^{-2.5}_{+3.9}$	$^{-0.3}_{+0.3}$	$^{+0.0}_{-0.0}$	$^{+0.5}_{-0.5}$
$\bar{B}_d \to \rho^0 \rho^0$	$3.3[10^{-7}]$	$^{+0.3}_{-0.3}$	$^{+4.3}_{-1.6}$	$^{-1.3}_{+6.1}$	$^{+0.7}_{-0.6}$	$^{-0.0}_{+0.7}$	$^{+2.2}_{-1.7}$	$^{-2.2}_{+3.7}$	$^{+0.8}_{-0.7}$	$^{+0.3}_{-0.3}$
$\bar{B}_d \to \rho^0 \omega$	$8.0[10^{-8}]$	$^{+3.1}_{-2.5}$	$^{-3.3}_{+3.9}$	$^{+1.1}_{-2.4}$	$^{-0.4}_{+0.5}$	$^{-1.4}_{+1.4}$	$^{-2.7}_{+22.5}$	$^{-1.6}_{+1.8}$	$^{+0.4}_{-0.3}$	$^{+2.3}_{-2.1}$
$\bar{B}_d \to \omega \omega$	$5.0[10^{-7}]$	$^{+0.7}_{-0.6}$	$^{+3.7}_{-1.7}$	$^{-1.4}_{+5.4}$	$^{+1.0}_{-0.9}$	$^{-0.2}_{+1.0}$	$^{-1.3}_{+2.1}$	$^{-2.3}_{+3.3}$	$^{+0.9}_{-0.8}$	$^{-0.4}_{+0.4}$
$\bar{B}_d \to \rho^0 \phi$	$2.8[10^{-9}]$	$^{+0.4}_{-0.4}$	$^{+1.7}_{-1.0}$	$^{-0.7}_{+2.2}$	$^{+0.4}_{-0.4}$	$^{-0.5}_{+1.6}$	–	$^{-1.0}_{+1.3}$	$^{+0.0}_{-0.0}$	$^{+0.5}_{-0.5}$
$\bar{B}_d \to \omega \phi$	$2.4[10^{-9}]$	$^{+0.5}_{-0.4}$	$^{+1.4}_{-0.9}$	$^{-0.6}_{+1.8}$	$^{+0.3}_{-0.3}$	$^{-0.4}_{+1.2}$	–	$^{-0.9}_{+1.1}$	$^{+0.0}_{-0.0}$	$^{+0.4}_{-0.4}$
$\bar{B}_d \to \rho^+ \rho^-$	$25.8[10^{-6}]$	$^{+7.6}_{-6.6}$	$^{-2.0}_{+1.6}$	$^{+1.0}_{-2.4}$	$^{-0.3}_{+0.3}$	$^{-0.3}_{+0.1}$	$^{+2.4}_{-1.8}$	$^{-1.5}_{+1.5}$	$^{+5.8}_{-5.2}$	$^{-0.9}_{+0.8}$
$\bar{B}_d \to \bar{K}^{*0} K^{*0}$	$3.2[10^{-7}]$	$^{+0.9}_{-0.8}$	$^{-0.9}_{+0.9}$	$^{+0.2}_{-0.4}$	$^{+0.0}_{-0.0}$	$^{-0.7}_{+0.9}$	$^{-2.5}_{+3.0}$	$^{-0.3}_{+0.4}$	$^{+0.0}_{-0.0}$	$^{+0.5}_{-0.4}$
$\bar{B}_s \to K^{*0} \rho^0$	$5.6[10^{-7}]$	$^{+0.4}_{-0.4}$	$^{+6.7}_{-2.7}$	$^{-2.1}_{+8.8}$	$^{+1.2}_{-1.0}$	$^{-0.0}_{+1.1}$	$^{-0.4}_{+1.5}$	$^{-4.2}_{+7.5}$	$^{+1.3}_{-1.2}$	$^{+0.4}_{-0.4}$
$\bar{B}_s \to K^{*0} \omega$	$6.5[10^{-7}]$	$^{+0.9}_{-0.7}$	$^{+5.5}_{-2.3}$	$^{-1.9}_{+7.6}$	$^{+1.0}_{-0.9}$	$^{-0.0}_{+0.9}$	$^{-0.9}_{+1.6}$	$^{-3.9}_{+6.6}$	$^{+1.2}_{-1.1}$	$^{-0.5}_{+0.4}$
$\bar{B}_s \to K^{*0} \phi$	$3.4[10^{-7}]$	$^{+1.1}_{-1.0}$	$^{-1.3}_{+1.4}$	$^{+0.3}_{-0.7}$	$^{-0.2}_{+0.2}$	$^{-0.6}_{+0.6}$	$^{-3.3}_{+5.1}$	$^{-0.7}_{+0.8}$	$^{-0.0}_{+0.0}$	$^{+0.5}_{-0.5}$
$\bar{B}_s \to K^{*+} \rho^-$	$37.2[10^{-6}]$	$^{+11.8}_{-10.2}$	$^{-2.9}_{+2.3}$	$^{+1.4}_{-3.3}$	$^{-0.6}_{+0.6}$	$^{+0.0}_{-0.3}$	$^{-0.2}_{+0.4}$	$^{-2.6}_{+2.7}$	$^{+8.4}_{-7.5}$	$^{-1.2}_{+1.1}$
$\bar{B}_d \to K^{*+} K^{*-}$	$2.9[10^{-8}]$	–	–	–	$^{+0.9}_{-0.8}$	$^{-1.0}_{+2.2}$	$^{-2.8}_{+19.1}$	–	$^{+0.5}_{-0.4}$	$^{-0.4}_{+0.3}$
$\bar{B}_d \to \phi \phi$	$2.5[10^{-9}]$	–	–	–	$^{+0.8}_{-0.7}$	$^{-1.3}_{+3.4}$	$^{+16.7}_{-2.4}$	–	$^{+0.0}_{-0.0}$	$^{+0.5}_{-0.4}$

Table 2.5: CP-averaged branching fractions for $B \to V_L V_L$ decays with $\Delta S = 1$. The sensitivity to variations in the input parameters according to Table 2.3 is displayed where the upper (lower) entry corresponds to the larger (smaller) value of the parameter. The renormalization scale μ is varied between $2m_b$ and $m_b/2$. The model parameters $X_{A,H}(\rho_{A,H}, \phi_{A,H})$ from power corrections are varied within the range given by $0 \leq \rho_{A,H} \leq 1$ and $0 \leq \phi_{A,H} \leq 2\pi$. Here upper (lower) entries refer to positive (negative) Im$X_{A,H}$. The appropriate units for each mode are given in square brackets.

mode	central	A_0	α_1^V	α_2^V	λ_B	f_B	μ	X_A	X_H	γ
$B^- \to K^{*-}\rho^0$	$3.4[10^{-6}]$	$^{+1.0}_{-0.9}$	$^{-0.1}_{+0.1}$	$^{-0.6}_{+0.7}$	$^{+0.1}_{-0.2}$	$^{-0.1}_{+0.1}$	$^{-0.2}_{+0.1}$	$^{-2.5}_{+3.7}$	$^{-0.1}_{+0.1}$	$^{+0.7}_{-0.7}$
$B^- \to K^{*-}\omega$	$1.7[10^{-6}]$	$^{+0.8}_{-0.6}$	$^{-0.1}_{+0.1}$	$^{-0.4}_{+0.6}$	$^{+0.2}_{-0.3}$	$^{-0.1}_{+0.1}$	$^{-0.1}_{+0.0}$	$^{-1.0}_{+2.4}$	$^{-0.2}_{+0.3}$	$^{+0.5}_{-0.4}$
$B^- \to K^{*-}\phi$	$4.1[10^{-6}]$	$^{+1.5}_{-1.3}$	$^{-0.1}_{+0.1}$	$^{-1.7}_{+2.0}$	$^{+0.5}_{-1.2}$	$^{-0.3}_{+0.3}$	$^{-0.7}_{+0.5}$	$^{-4.1}_{+7.8}$	$^{-0.9}_{+1.0}$	$^{-0.0}_{+0.0}$
$B^- \to \bar{K}^{*0}\rho^-$	$3.3[10^{-6}]$	$^{+1.1}_{-1.0}$	$^{-0.3}_{+0.3}$	$^{-1.1}_{+1.2}$	$^{+0.3}_{-0.6}$	$^{-0.2}_{+0.2}$	$^{-0.4}_{+0.3}$	$^{-3.3}_{+6.5}$	$^{-0.4}_{+0.4}$	$^{-0.0}_{+0.0}$
$\bar{B}_d \to \bar{K}^{*0}\rho^0$	$5.0[10^{-7}]$	$^{+1.7}_{-1.4}$	$^{-0.6}_{+0.6}$	$^{-1.7}_{+2.5}$	$^{+0.2}_{-0.2}$	$^{-0.3}_{+0.3}$	$^{-1.1}_{+0.9}$	$^{-4.7}_{+24.2}$	$^{-0.2}_{+0.3}$	$^{-0.2}_{+0.2}$
$\bar{B}_d \to \bar{K}^{*0}\omega$	$1.4[10^{-6}]$	$^{+0.8}_{-0.6}$	$^{-0.2}_{+0.2}$	$^{-0.7}_{+0.8}$	$^{+0.3}_{-0.6}$	$^{-0.2}_{+0.2}$	$^{-0.1}_{+0.0}$	$^{-1.3}_{+2.7}$	$^{-0.4}_{+0.5}$	$^{+0.0}_{-0.0}$
$\bar{B}_d \to \bar{K}^{*0}\phi$	$3.7[10^{-6}]$	$^{+1.4}_{-1.2}$	$^{-0.1}_{+0.1}$	$^{-1.6}_{+1.8}$	$^{+0.5}_{-1.1}$	$^{-0.3}_{+0.3}$	$^{-0.6}_{+0.4}$	$^{-3.7}_{+7.5}$	$^{+0.9}_{-0.8}$	$^{-0.0}_{+0.0}$
$\bar{B}_d \to K^{*-}\rho^+$	$3.0[10^{-6}]$	$^{+1.0}_{-0.8}$	$^{-0.2}_{+0.2}$	$^{-0.7}_{+0.8}$	$^{+0.1}_{-0.3}$	$^{-0.1}_{+0.1}$	$^{-0.3}_{+0.2}$	$^{-1.8}_{+5.4}$	$^{-0.1}_{+0.1}$	$^{+0.8}_{-0.7}$
$\bar{B}_s \to \rho^0\phi$	$5.9[10^{-7}]$	$^{+1.8}_{-1.5}$	–	$^{-0.5}_{+0.5}$	$^{+0.4}_{-0.6}$	$^{-0.1}_{+0.1}$	$^{+0.1}_{-0.0}$	–	$^{-0.6}_{+0.9}$	$^{+0.5}_{-0.4}$
$\bar{B}_s \to \omega\phi$	$4.4[10^{-8}]$	$^{+1.3}_{-0.7}$	–	$^{+5.1}_{-0.0}$	$^{-0.0}_{+8.4}$	$^{+0.4}_{-0.0}$	$^{-0.1}_{+4.0}$	–	$^{-1.8}_{+11.3}$	$^{+0.5}_{-0.4}$
$\bar{B}_s \to \phi\phi$	$15.5[10^{-6}]$	$^{+5.0}_{-4.3}$	–	$^{-5.8}_{+6.5}$	$^{+1.6}_{-3.6}$	$^{-0.7}_{+0.7}$	$^{-3.1}_{+3.3}$	$^{-14.4}_{+20.2}$	$^{-3.2}_{+3.6}$	$^{-0.1}_{+0.1}$
$\bar{B}_s \to K^{*+}K^{*-}$	$5.9[10^{-6}]$	$^{+1.7}_{-1.5}$	$^{-0.3}_{+0.3}$	$^{-1.3}_{+1.4}$	$^{+0.2}_{-0.4}$	$^{+0.0}_{-0.0}$	$^{-0.9}_{+1.1}$	$^{-3.8}_{+6.5}$	$^{-0.3}_{+0.3}$	$^{+1.5}_{-1.3}$
$\bar{B}_s \to \bar{K}^{*0}K^{*0}$	$6.2[10^{-6}]$	$^{+1.9}_{-1.7}$	$^{-0.3}_{+0.3}$	$^{-1.9}_{+2.0}$	$^{+0.4}_{-1.0}$	$^{-0.1}_{+0.1}$	$^{-1.2}_{+1.5}$	$^{-5.6}_{+7.5}$	$^{-0.7}_{+0.7}$	$^{-0.0}_{+0.0}$
$\bar{B}_s \to \rho^+\rho^-$	$1.0[10^{-7}]$	–	–	–	–	$^{+0.3}_{-0.2}$	$^{-0.5}_{+1.4}$	$^{+6.7}_{-1.0}$	–	$^{+0.0}_{-0.0}$
$\bar{B}_s \to \rho^0\rho^0$	$5.1[10^{-8}]$	–	–	–	–	$^{+1.4}_{-1.2}$	$^{-2.7}_{+7.0}$	$^{+33.7}_{-5.0}$	–	$^{+0.2}_{-0.2}$
$\bar{B}_s \to \omega\omega$	$3.2[10^{-8}]$	–	–	–	–	$^{+0.9}_{-0.8}$	$^{-1.7}_{+4.5}$	$^{-3.2}_{+21.6}$	–	$^{+0.1}_{-0.1}$
$\bar{B}_s \to \rho^0\omega$	$1.5[10^{-9}]$	–	–	–	–	$^{+0.4}_{-0.4}$	$^{-0.5}_{+1.1}$	$^{+9.8}_{-1.4}$	–	$^{-0.1}_{+0.1}$

quantities, for instance from $B \to V_L$ form factors. Taking ratios or other combinations of suitable branching fractions can eliminate part of the uncertainties and lead to theoretically cleaner observables. In spite of this it is still interesting to present the theory expectations for the branching fractions, which can be directly confronted with experimental data. In addition, we use Table 2.4 and 2.5 to display in detail the sensitivity of the results on the most important input parameters.

In Fig. 2.6 we compare theory and experiment for $\bar{B} \to V_L V_L$ branching fractions, for

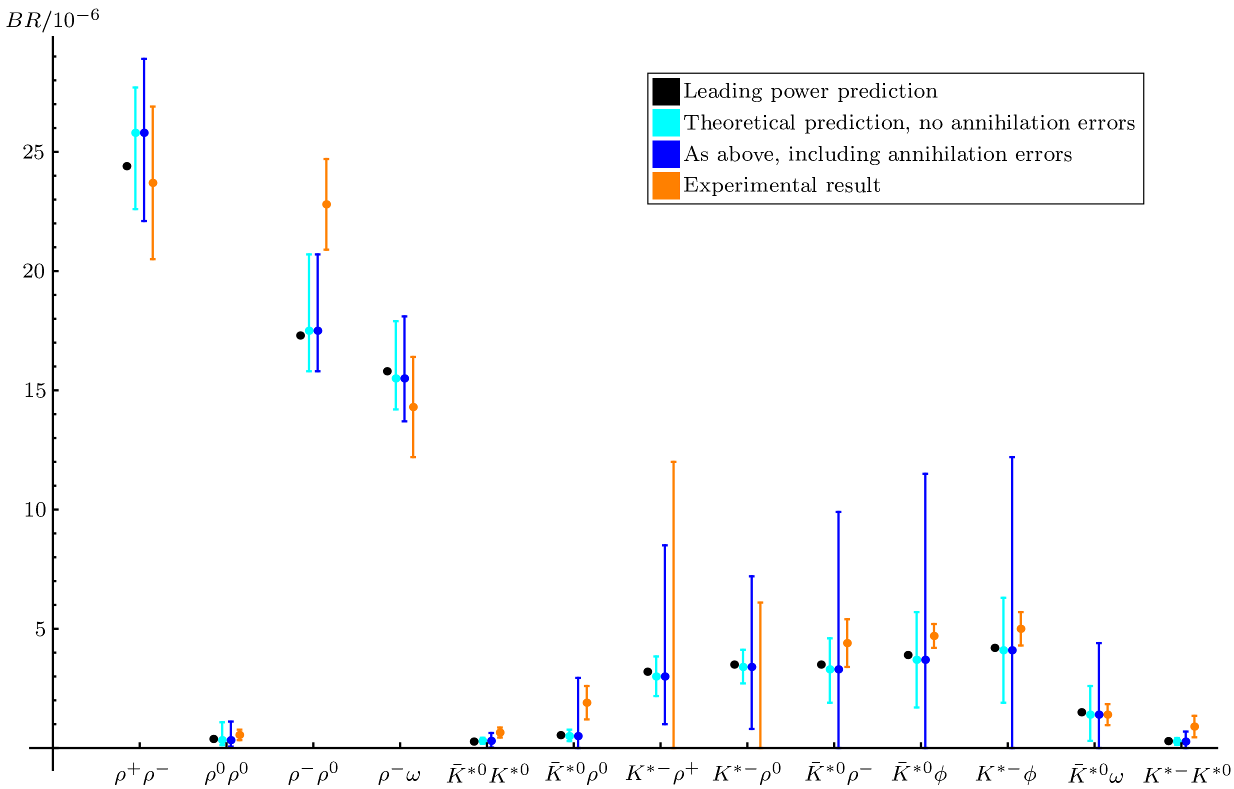

Figure 2.6: Comparison between theory predictions [dots (black), left bar (cyan), middle bar (marine blue)] and experimental results [right bar (orange)] for $\bar{B}_d \to V_L V_L$ modes, for which measurements are available. The theoretical error bars display the hadronic errors without [left (cyan)] and with [middle (marine blue)] the model-dependent error estimate for annihilation topologies. The form-factor uncertainties are not included in the error bars. The black dots are the central values of the theory predictions where all power corrections have been omitted. From experiment only upper limits are known for the two $K^{*-}\rho$ channels.

which measurements are available. For this comparison the form factors have been fixed to their central values. In the present discussion we will assume that the Standard Model is valid. Under this assumption the comparison with experimental data will serve as a test of the theory of QCD interactions in hadronic weak decays. It should be kept in mind that possible deviations between predictions and measurements may in principle indicate the existence of New Physics. In order to disentangle New Physics from QCD effects it is important to consider observables with very little hadronic uncertainty. We will discuss several examples for this in the following sections. For the moment we stay with the discussion of the theoretically less clean absolute branching fractions for the purpose of testing the method of QCD factorization, under the (provisional) assumption that physics beyond the Standard Model is absent.

Fig. 2.6 shows good agreement of theory and experiment within errors. An exception is $B^- \to \rho^- \rho$, where a recent measurement of the branching ratio [64] is somewhat high with respect to the expectation from theory.

In the other two ρ-meson channels, $\bar{B}_d \to \rho^+ \rho^-$ and $\bar{B}_d \to \rho^0 \rho^0$, theory and experiment agree very well, as it has also been found in [38]. The $\rho^0 \rho^0$ channel is a colour-suppressed mode and comes with large uncertainties. Hard spectator scattering plays an important role and therefore the sensitivity to the poorly known parameter λ_B is large. Still the experimental result can be accounted for naturally with default values of the hadronic parameters.

The penguin modes $\bar{B}_d \to \bar{K}^{*0} K^{*0}$ and $\bar{B}_d \to \bar{K}^{*0} \rho^0$ tend to have relatively small predicted branching ratios, which however stretch into the range of measured values within errors. The compatibility is better for $\bar{B}_d \to \bar{K}^{*0} K^{*0}$ than for $\bar{B}_d \to \bar{K}^{*0} \rho^0$. At the same time the latter mode is also seen to be very sensitive to the annihilation contributions.

It is interesting to note that the central values of the experimental and theoretical results are particularly close for the penguin decays $B^- \to \bar{K}^{*0} \rho^-$, $\bar{B}_d \to \bar{K}^{*0} \phi$, $B^- \to K^{*-} \phi$, $\bar{B} \to \bar{K}^{*0} \omega$. On the other hand, the dependence on weak annihilation is very strong. The huge variations from these effects shown in Fig. 2.6 suggest that, at least for these channels, the annihilation model used by us is likely to overestimate the related uncertainty.

Further branching ratio predictions and information on the various error sources for all 34 $\bar{B} \to V_L V_L$ decays can be obtained from Tables 2.4 and 2.5.

Our results include estimates of some effects that are suppressed by a factor of Λ_{QCD}/m_b. These corrections are weak annihilation and the effects proportional to $r_\perp^V$ (see eq. (2.28)). Terms at this order are not calculable in QCD factorization. They have still been included as model estimates in order to permit us to assess the sensitivity of factorization predictions on potentially important power corrections. Weak annihilation is the most prominent example. For the default choice of input parameters the impact of power corrections on the predicted branching ratios is in general small. This can already be seen from Fig. 2.6, where central results with all power corrections omitted are indicated by the black dots. They differ very little from the central theory predictions that include such effects. To make these statements more quantitative, we list the differences between the central values for all $\Delta S = 0$ branching ratios without and including power corrections, $BR(\text{no power corr.})/BR(\text{default}) - 1$, in the order of appearance in Table 2.4, in %:

$$-1,\ +2,\ -6,\ +6,\ +15,\ +7,\ -27,\ -6,\ -6,\ -5,\ -16,\ -12,\ -9,\ +6,\ +1,\ -100,\ -100 \tag{2.124}$$

The same information for the $\Delta S = 1$ decays of Table 2.5 reads

$$+5,\ +5,\ +3,\ +7,\ +9,\ +12,\ +6,\ +7,\ +2,\ +0,\ -8,\ -17,\ -19,\ -100,\ -100,\ -100,\ -100 \quad (2.125)$$

The deviation is -100% for the six pure annihilation decays, which have no leading-power contribution. In all other cases the impact of the default power corrections is rather moderate or indeed very small, notably for the dominant decay channels.

We finally comment on the impact of the long-distance electromagnetic penguin correction defined in (2.42) and discussed in appendix 2.6. This contribution affects only decays with the emission of ρ^0, ω or ϕ, where it enters through the coefficient $a_7^u + a_9^u$. The long-distance effects are sizable, on the scale of this coefficient, for ρ^0 and ω, but much less in the case of ϕ. Since the long-distance terms are of order $\alpha = 1/129$ their overall contribution is in general very small. This is particularly true for the $\Delta S = 1$ decays where the up-quark sector is also CKM suppressed. For the $\Delta S = 0$ transitions the absence of the term in (2.42) would change branching ratios at the level of a few percent at most and below the size of most of the other uncertainties. The situation is similar for the direct CP asymmetries in the $\Delta S = 0$ modes with the exception of $\rho^-\rho^0$, $\rho^-\phi$, $\rho^0\omega$, $\rho^0\phi$ and $\omega\phi$, where the impact is relatively large. However, in any case, the direct CP asymmetry is very uncertain for $\rho^0\omega$ and it is very small for the remaining channels.

2.4.2 Direct CP violation in $B \to V_L V_L$

Direct CP asymmetries require the presence of a strong as well as a weak phase difference between two interfering amplitudes. In the heavy-quark limit this phase difference arises at order α_s. It is therefore parametrically suppressed and at the same time sensitive to uncalculable power corrections. This makes it difficult to obtain accurate predictions for direct CP violation. At present the most precisely measured direct CP asymmetry in B decays is $A_{CP}(B \to K^+\pi^-) = -0.097 \pm 0.012$ [55]. The corresponding strong phase difference is small ($\sim 15°$) [65], but has a sign opposite to the $\mathcal{O}(\alpha_s)$ result in the heavy-quark limit. This may indicate the importance of Λ_{QCD}/m_b corrections.

In Tables 2.6 and 2.7 we show estimates of direct CP asymmetries for the decays under discussion.

The values have large uncertainties, as anticipated. Most of the asymmetries are small or moderate, but there can be exceptions. Large asymmetries may occur when the interfering amplitudes have comparable magnitude and a substantial strong relative phase. Examples are the $\Delta S = 0$ decays with a colour suppressed tree contribution ($\sim a_2$), as $\bar{B}_d \to \rho^0\rho^0$, $\omega\omega$ or $\bar{B}_s \to K^{*0}\rho^0$, $K^{*0}\omega$. Despite the α_s factor the strong phase difference can here be naturally more sizeable. Generically, a decay amplitude of the form

$$A(\bar{B} \to M_1 M_2) \sim e^{-i\gamma} - p e^{i\phi} \quad (2.126)$$

with p, ϕ, γ real, leads to the direct CP asymmetry

$$A_{CP} = \frac{2p \sin\phi\, \sin\gamma}{1 + p^2 - 2p\cos\phi\, \cos\gamma} \quad (2.127)$$

Table 2.6: CP asymmetries for $B \to V_L V_L$ decays with $\Delta S = 0$, defined as $A_{CP} \equiv (B(\bar{B} \to \bar{f}) - B(B \to f))/(B(\bar{B} \to \bar{f}) + B(B \to f))$. The sensitivity to variations in the input parameters according to Table 2.3 is displayed where the upper (lower) entry corresponds to the larger (smaller) value of the parameter. The renormalization scale μ is varied between $2m_b$ and $m_b/2$. The model parameters $X_{A,H}(\rho_{A,H}, \phi_{A,H})$ from power corrections are varied within the range given by $0 \le \rho_{A,H} \le 1$ and $0 \le \phi_{A,H} \le 2\pi$. The appropriate units for each mode are given in square brackets. We refrain from quoting estimates of CP asymmetries for pure annihilation modes.

mode	central	m_c	α_2^V	λ_B	f_B	μ	X_A	X_H	$\left\lvert\frac{V_{ub}}{V_{cb}}\right\rvert$	γ
$B^- \to \rho^- \rho^0$	$-2.6[10^{-4}]$	+0.8 −0.7	+0.2 −0.3	+0.1 −0.2	−0.0 +0.0	−5.6 +7.4	—	+3.7 −3.7	+0.3 −0.3	−0.2 +0.3
$B^- \to \rho^- \omega$	$-9.3[10^{-2}]$	+2.4 −2.0	+1.0 −1.0	−0.4 +0.8	+0.2 −0.2	+1.4 −2.2	+22.4 −21.6	+3.1 −3.1	+0.8 −1.0	−1.1 +1.4
$B^- \to \rho^- \phi$	$-1.2[10^{-2}]$	−0.0 +0.0	+0.2 −0.3	−0.2 +0.3	+0.1 −0.1	−1.1 +1.1	—	+0.2 −0.4	−0.1 +0.1	+0.1 −0.1
$B^- \to K^{*-} K^{*0}$	$-1.0[10^{-1}]$	−1.0 +0.9	−0.0 +0.1	+0.0 −0.1	+0.1 −0.1	+0.1 −0.2	+8.9 −7.9	+0.1 −0.1	−0.1 +0.1	+0.1 −0.1
$\bar{B}_d \to \rho^0 \rho^0$	$+5.3[10^{-1}]$	−0.7 +0.4	−2.7 +3.8	+2.2 −2.8	−0.7 +0.8	−1.0 +0.7	+4.3 −5.8	+3.6 −2.8	−0.5 +0.6	−0.2 +0.0
$\bar{B}_d \to \rho^0 \omega$	$+7.8[10^{-2}]$	−17.4 +14.9	+13.9 −5.3	−2.0 +7.4	+6.7 −6.1	−5.3 +10.8	+92.2 −107.8	+10.2 −10.2	+0.5 −0.6	−1.3 +1.6
$\bar{B}_d \to \omega \omega$	$-4.5[10^{-1}]$	+0.5 −0.4	+1.9 −2.4	−1.5 +2.1	+0.6 −0.8	+0.7 −0.5	+5.0 −2.7	+2.7 −2.8	+0.3 −0.3	−0.7 +0.8
$\bar{B}_d \to \rho^0 \phi$	$-1.2[10^{-2}]$	−0.0 +0.0	+0.2 −0.3	−0.2 +0.3	+0.1 −0.1	−1.1 +1.1	—	+0.2 −0.4	−0.1 +0.1	+0.1 −0.1
$\bar{B}_d \to \omega \phi$	$-1.2[10^{-2}]$	−0.0 +0.0	+0.2 −0.3	−0.2 +0.3	+0.1 −0.1	−1.1 +1.1	—	+0.3 −0.4	−0.1 +0.1	+0.1 −0.1
$\bar{B}_d \to \rho^+ \rho^-$	$-3.7[10^{-2}]$	+1.4 −1.2	+0.0 −0.0	+0.1 −0.2	−0.0 +0.0	+0.3 −0.3	+10.7 −10.5	+0.2 −0.2	+0.3 −0.4	−0.4 +0.5
$\bar{B}_d \to \bar{K}^{*0} K^{*0}$	$-1.5[10^{-1}]$	−0.7 +0.6	−0.2 +0.2	+0.0 −0.1	−0.0 +0.0	+0.2 −0.4	+0.9 −2.2	+0.1 −0.1	−0.2 +0.2	+0.1 −0.0
$\bar{B}_s \to K^{*0} \rho^0$	$+4.3[10^{-1}]$	−0.7 +0.4	−2.1 +3.4	+1.7 −2.1	−0.6 +0.7	−0.7 +0.3	+4.5 −8.2	+4.2 −2.5	−0.4 +0.5	+0.0 −0.2
$\bar{B}_s \to K^{*0} \omega$	$-5.2[10^{-1}]$	+0.5 −0.4	+2.4 −2.9	−1.7 +2.5	+0.6 −0.7	+0.9 −0.5	+6.7 −3.6	+3.9 −3.7	+0.3 −0.3	−0.8 +0.9
$\bar{B}_s \to K^{*0} \phi$	$-2.0[10^{-1}]$	−0.8 +0.7	−0.5 +0.3	+0.1 −0.3	−0.1 +0.1	+0.3 −0.7	+2.3 −7.6	+0.3 −0.3	−0.2 +0.2	+0.1 −0.1
$\bar{B}_s \to K^{*+} \rho^-$	$-3.9[10^{-2}]$	+1.5 −1.2	+0.0 −0.0	+0.1 −0.2	−0.0 +0.0	+0.3 −0.4	+13.6 −13.5	+0.2 −0.2	+0.4 −0.4	−0.4 +0.5

For $\sin\gamma \approx 0.92$, a value $p = \mathcal{O}(1)$ and a substantial phase ϕ give a large asymmetry. In the case of $\bar{B}_d \to \rho_L^0 \rho_L^0$ the central values $p = 0.36$, $\phi = 49°$ give $A_{CP} = 53\%$.

2.4.3 Sensitivity to ω-ϕ mixing

In the other sections of this paper the vector mesons ϕ and ω are always implemented as pure $s\bar{s}$ and $(u\bar{u} + d\bar{d})/\sqrt{2}$-states, respectively. Here we investigate the sensitivity of our results to the deviation from this case of ideal mixing. We assume that other effects with Zweig-rule suppression are negligibly small. We neglect, for example, Zweig-rule forbidden matrix elements of the type $\langle \phi(s\bar{s}) | (\bar{u}b)_{V-A} | B^- \rangle$.

Table 2.7: CP asymmetries for $B \to V_L V_L$ decays with $\Delta S = 1$ (see caption of Table 2.6 for details).

mode	central	m_c	α_1^V	α_2^V	λ_B	μ	X_A	X_H	$\frac{V_{ub}}{V_{cb}}$	γ
$B^- \to K^{*-}\rho^0$	$2.9[10^{-1}]$	$^{-0.7}_{+0.6}$	$^{-0.1}_{+0.1}$	$^{+0.5}_{-0.4}$	$^{-0.1}_{+0.3}$	$^{-0.1}_{+0.3}$	$^{+7.0}_{-9.0}$	$^{+0.8}_{-0.9}$	$^{+0.3}_{-0.3}$	$^{-0.4}_{+0.3}$
$B^- \to K^{*-}\omega$	$4.9[10^{-1}]$	$^{-1.1}_{+0.9}$	$^{-0.0}_{+0.0}$	$^{+1.4}_{-1.1}$	$^{-0.5}_{+1.1}$	$^{-0.5}_{+1.4}$	$^{+5.1}_{-13.3}$	$^{+1.9}_{-2.2}$	$^{+0.2}_{-0.3}$	$^{-0.8}_{+0.9}$
$B^- \to K^{*-}\phi$	$5.4[10^{-3}]$	$^{+5.8}_{-5.0}$	$^{+0.1}_{-0.1}$	$^{-0.5}_{+0.0}$	$^{-0.2}_{+0.5}$	$^{-0.5}_{+0.7}$	$^{+994.6}_{-1005.4}$	$^{+0.9}_{-0.8}$	$^{+0.6}_{-0.6}$	$^{+0.4}_{-0.6}$
$B^- \to \bar{K}^{*0}\rho^-$	$6.0[10^{-3}]$	$^{+5.2}_{-4.5}$	$^{+1.3}_{-1.1}$	$^{+0.2}_{-0.5}$	$^{-0.1}_{+0.4}$	$^{-0.6}_{+1.1}$	$^{+994.0}_{-1006.0}$	$^{+0.5}_{-0.5}$	$^{+0.7}_{-0.7}$	$^{+0.5}_{-0.7}$
$\bar{B}_d \to \bar{K}^{*0}\rho^0$	$-3.7[10^{-1}]$	$^{+0.3}_{-0.2}$	$^{-0.3}_{+0.3}$	$^{-2.6}_{+1.4}$	$^{+0.7}_{-1.6}$	$^{-0.3}_{+0.0}$	$^{+13.7}_{-6.3}$	$^{+2.1}_{-1.8}$	$^{-0.3}_{+0.3}$	$^{-0.4}_{+0.5}$
$\bar{B}_d \to \bar{K}^{*0}\omega$	$2.1[10^{-1}]$	$^{-0.1}_{+0.1}$	$^{+0.2}_{-0.2}$	$^{+2.0}_{-0.8}$	$^{-0.5}_{+2.0}$	$^{-0.1}_{+0.6}$	$^{+7.9}_{-11.0}$	$^{+1.5}_{-1.5}$	$^{+0.2}_{-0.2}$	$^{+0.1}_{-0.2}$
$\bar{B}_d \to \bar{K}^{*0}\phi$	$1.1[10^{-2}]$	$^{+0.4}_{-0.4}$	$^{+0.0}_{-0.0}$	$^{+0.3}_{-0.2}$	$^{-0.1}_{+0.3}$	$^{-0.2}_{+0.4}$	$^{+98.9}_{-101.1}$	$^{+0.2}_{-0.1}$	$^{+0.1}_{-0.1}$	$^{+0.1}_{-0.1}$
$\bar{B}_d \to K^{*-}\rho^+$	$3.3[10^{-1}]$	$^{-1.2}_{+1.1}$	$^{-0.2}_{+0.2}$	$^{+0.7}_{-0.5}$	$^{-0.0}_{+0.1}$	$^{-0.0}_{+0.1}$	$^{+6.7}_{-13.2}$	$^{+0.1}_{-0.1}$	$^{+0.2}_{-0.3}$	$^{-0.5}_{+0.6}$
$\bar{B}_s \to \rho^0\phi$	$3.0[10^{-1}]$	$^{+0.0}_{-0.0}$	—	$^{+0.2}_{-0.2}$	$^{-0.2}_{+0.3}$	$^{-0.4}_{+0.8}$	—	$^{+2.5}_{-2.8}$	$^{+0.3}_{-0.3}$	$^{+0.0}_{-0.1}$
$\bar{B}_s \to \omega\phi$	$9.0[10^{-1}]$	A_0:$^{+0.8}_{-1.9}$	—	$^{-7.0}_{+0.8}$	$^{+0.9}_{-8.0}$	$^{+0.0}_{-3.4}$	—	$^{+1.0}_{-18.0}$	$^{-0.5}_{+0.4}$	$^{-0.3}_{+0.0}$
$\bar{B}_s \to \phi\phi$	$9.7[10^{-3}]$	$^{+4.0}_{-3.4}$	—	$^{+2.4}_{-1.6}$	$^{-0.6}_{+1.8}$	$^{-1.5}_{+3.1}$	$^{+30.8}_{-6.8}$	$^{+1.5}_{-1.2}$	$^{+1.1}_{-1.1}$	$^{+0.7}_{-1.1}$
$\bar{B}_s \to K^{*+}K^{*-}$	$2.6[10^{-1}]$	$^{-1.0}_{+0.9}$	$^{-0.2}_{+0.1}$	$^{+0.5}_{-0.3}$	$^{-0.0}_{+0.1}$	$^{+0.2}_{-0.3}$	$^{+7.0}_{-9.0}$	$^{+0.1}_{-0.1}$	$^{+0.2}_{-0.2}$	$^{-0.4}_{+0.4}$
$\bar{B}_s \to \bar{K}^{*0}K^{*0}$	$8.7[10^{-3}]$	$^{+3.3}_{-2.9}$	$^{+0.9}_{-0.9}$	$^{+1.6}_{-1.2}$	$^{-0.3}_{+0.9}$	$^{-1.2}_{+2.3}$	$^{+19.7}_{-4.9}$	$^{+0.6}_{-0.6}$	$^{+1.0}_{-1.0}$	$^{+0.6}_{-1.0}$

Mixing can be introduced by the following parametrization:

$$\phi(1020) = s\bar{s}\cos\theta + \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}\sin\theta \tag{2.128}$$

$$\omega(782) = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}\cos\theta - s\bar{s}\sin\theta. \tag{2.129}$$

The ideal mixing angle in this parametrization is $\theta = 0$. According to sum-rules quadratic in meson masses [57], the mixing angle can be estimated to be $\theta = 3.4^\circ$. The results of varying the mixing angle up to $\theta = 6.8^\circ$ are shown in Table 2.8. For most branching fractions the effect of a nonvanishing mixing angle $\theta \approx 3.4^\circ$ is very small, in particular for the important modes $B^- \to \rho^-\omega$, $B^- \to K^{*-}\phi$, $\bar{B}_s \to \phi\phi$. On the other hand, the modes $\bar{B}_d \to \rho^0\phi$, $\bar{B}_d \to \omega\phi$, $\bar{B}_s \to \omega\phi$, $\bar{B}_s \to \rho^0\omega$ have a significant dependence on deviations from ideal mixing. The largest effect is observed for $B^- \to \rho^-\phi$. In this case $B^- \to \rho^-\omega$ feeds into the former channel through mixing with a more than three orders of magnitude higher branching ratio compared to $B^- \to \rho^-\phi(s\bar{s})$, which overcompensates the small mixing angle:

$$B(B^- \to \rho^-\phi)_{\text{mix}} \approx \sin^2\theta\, B(B^- \to \rho^-\omega) \tag{2.130}$$

Table 2.8: Dependence of $B \to V_L V_L$ branching fractions on ω-ϕ mixing. The variation of the branching fractions is given for two values of the mixing angle θ. The upper (lower) value corresponds to $\theta = 6.8°$ ($\theta = 3.4°$).

mode	default value	deviation	mode	default value	deviation
$B^- \to \rho^- \omega$	$15.5[10^{-6}]$	$^{-0.2}_{-0.0}$	$B^- \to K^{*-}\omega$	$1.7[10^{-6}]$	$^{-0.4}_{-0.2}$
$B^- \to \rho^- \phi$	$6.0[10^{-9}]$	$^{+207.5}_{+49.8}$	$B^- \to K^{*-}\phi$	$4.1[10^{-6}]$	$^{+0.4}_{+0.2}$
$\bar{B}_d \to \rho^0 \omega$	$8.0[10^{-8}]$	$^{+0.2}_{+0.1}$	$\bar{B}_d \to \bar{K}^{*0}\omega$	$1.4[10^{-6}]$	$^{-0.5}_{-0.3}$
$\bar{B}_d \to \omega\omega$	$5.0[10^{-7}]$	$^{-0.1}_{-0.0}$	$\bar{B}_d \to \bar{K}^{*0}\phi$	$3.7[10^{-6}]$	$^{+0.5}_{+0.3}$
$\bar{B}_d \to \rho^0 \phi$	$2.8[10^{-9}]$	$^{-1.9}_{-1.2}$	$\bar{B}_s \to \rho^0 \phi$	$5.9[10^{-7}]$	$^{-0.1}_{-0.0}$
$\bar{B}_d \to \omega\phi$	$2.4[10^{-9}]$	$^{+10.0}_{+1.7}$	$\bar{B}_s \to \omega\phi$	$4.4[10^{-8}]$	$^{+28.7}_{+4.6}$
$\bar{B}_s \to K^{*0}\omega$	$6.5[10^{-7}]$	$^{-0.6}_{-0.3}$	$\bar{B}_s \to \phi\phi$	$15.5[10^{-6}]$	$^{-0.3}_{-0.0}$
$\bar{B}_s \to K^{*0}\phi$	$3.4[10^{-7}]$	$^{+0.6}_{+0.3}$	$\bar{B}_s \to \omega\omega$	$3.2[10^{-8}]$	$^{+1.6}_{+0.3}$
$\bar{B}_d \to \phi\phi$	$2.5[10^{-9}]$	$^{-0.4}_{-0.3}$	$\bar{B}_s \to \rho^0 \omega$	$1.5[10^{-9}]$	$^{+11.1}_{+3.5}$

A recent discussion of hadronic B decays, mostly with charm in the final state, for which ω-ϕ mixing has a large impact can be found in [66]. Their estimate of $B^- \to \rho^- \phi$ is compatible with ours.

2.4.4 Unitarity triangle from CP violation in $B_d \to \rho_L^+ \rho_L^-$

Determination of $\bar{\rho}$, $\bar{\eta}$, γ and α

The time dependent CP asymmetry in $B_d \to \rho_L^+ \rho_L^-$ is given by

$$\mathcal{A}_{CP,\rho}(t) = \frac{\Gamma(\bar{B}_d(t) \to \rho_L^+ \rho_L^-) - \Gamma(B_d(t) \to \rho_L^+ \rho_L^-)}{\Gamma(\bar{B}_d(t) \to \rho_L^+ \rho_L^-) + \Gamma(B_d(t) \to \rho_L^+ \rho_L^-)} = S_\rho \sin(\Delta m_d\, t) - C_\rho \cos(\Delta m_d\, t) \tag{2.131}$$

The parameters S_ρ and C_ρ have been measured to be

$$S_\rho = -0.05 \pm 0.17 \qquad C_\rho = -0.06 \pm 0.13 \tag{2.132}$$

as quoted by [55], based on results of BaBar [67] and Belle [68]. Together with the experimentally well determined quantity $\sin 2\beta$ from CP violation in $B \to \psi K^0$ decays, the parameter S_ρ can be used to fix the CKM unitarity triangle. The value of $\sin 2\beta$ from Table 2.3 implies $\beta = (21.1 \pm 0.9)°$ or

$$\tau \equiv \cot\beta = 2.58 \pm 0.12 \tag{2.133}$$

In terms of the improved Wolfenstein parameters $\bar{\rho}$ and $\bar{\eta}$ [40] the unitarity triangle is then determined by

$$\bar{\rho} = 1 - \tau\bar{\eta} \tag{2.134}$$

$$\bar{\eta} = \frac{1}{(1+\tau^2)S_\rho}\Bigg[(1+\tau S_\rho)(1+r_\rho\cos\phi_\rho) - \sqrt{(1-S_\rho^2)(1+r_\rho\cos\phi_\rho)^2 - S_\rho(1+\tau^2)(S_\rho+\sin 2\beta)r_\rho^2\sin^2\phi_\rho}\Bigg] \quad (2.135)$$

These formulas have been derived in [43, 69] for $B \to \pi^+\pi^-$, but they apply to the case of $B \to \rho_L^+\rho_L^-$ as well. The parameters r_ρ and ϕ_ρ are hadronic quantities. They are defined here through

$$r_\rho e^{i\phi_\rho} = -\frac{a_4^c + a_{10}^c + r_A^\rho\left(b_3 + 2b_4 - \frac{1}{2}b_3^{\rm EW} + \frac{1}{2}b_4^{\rm EW}\right)}{a_1 + a_4^u + a_{10}^u + r_A^\rho\left(b_1 + b_3 + 2b_4 - \frac{1}{2}b_3^{\rm EW} + \frac{1}{2}b_4^{\rm EW}\right)} \quad (2.136)$$

where all coefficients a_i, b_i refer to the $\rho_L^+\rho_L^-$ final state and

$$r_A^\rho \equiv \frac{B_{\rho\rho}}{A_{\rho\rho}} = \frac{f_B f_\rho}{m_B^2 A_0^{B\to\rho}(0)} \approx 5\cdot 10^{-3} \quad (2.137)$$

The real quantities r_ρ and ϕ_ρ are the magnitude and phase of the penguin-to-tree amplitude ratio in $\bar{B} \to \rho_L^+\rho_L^-$. They are independent of CKM parameters. Numerically we find

$$r_\rho = 0.038 \pm 0.005\,(\mu,\,\alpha_2^\rho)\ {}^{+0.019}_{-0.026}\,(\rho_A,\,\phi_A) \quad (2.138)$$

$$\phi_\rho = 0.23 \pm 0.09\,(m_c,\,\alpha_2^\rho)\ {}^{+0.74}_{-0.73}\,(\rho_A,\,\phi_A) \quad (2.139)$$

$$r_\rho\cos\phi_\rho = 0.037 \pm 0.005\,(\mu,\,\alpha_2^\rho)\ {}^{+0.018}_{-0.026}\,(\rho_A,\,\phi_A) \quad (2.140)$$

The first error is from the uncertainties in the input parameters A_0, α_2^ρ, $f_\rho^\perp$, λ_B, f_B, m_c and a variation of the renormalization scale μ between $m_b/2$ and $2m_b$ around its default value $\mu = m_b$. The dominant sources of uncertainty are indicated in brackets. The second error reflects the sensitivity to the parameters ρ_A, ϕ_A, ρ_H and ϕ_H used to model power corrections from weak annihilation (A) and in the spectator scattering amplitude (H). We have used $0 \le \rho_{A,H} \le 1$, $0 \le \phi_{A,H} \le 2\pi$. The second error is entirely determined by weak annihilation.

The phase ϕ_ρ is parametrically suppressed since it arises only at order α_s or Λ_{QCD}/m_b. Its precise value is rather uncertain, in particular due to the model dependence of power corrections, which may compete numerically with the calculable $\mathcal{O}(\alpha_s)$ term. Fortunately the dependence of $\bar{\eta}$ in (2.135) on ϕ_ρ is very weak [43, 69]. In addition, r_ρ is a small parameter, even smaller than the corresponding quantity r_π in $\bar{B}_d \to \pi^+\pi^-$. The smaller size of the penguin contribution in the case of vector mesons as compared to pseudoscalars has been pointed out before in the context of QCD factorization [37, 70]. The formulation in (2.135) makes it particularly transparent to analyze the impact of a small penguin correction on the determination of the unitarity triangle. To linear order in r_ρ, eq. (2.135) implies the simple relation

$$\bar{\eta} = \frac{1 + \tau S_\rho - \sqrt{1-S_\rho^2}}{(1+\tau^2)S_\rho}\,(1 + r_\rho\cos\phi_\rho) \quad (2.141)$$

In this approximation $\bar{\eta}$ and $\bar{\rho}$ depend only on the real part of the penguin-to-tree ratio. As can be seen from (2.135), second order corrections in r_ρ are further suppressed by $\sin^2\phi_\rho$.

With $\bar{\eta}$ and $\bar{\rho}$ also the CKM angles γ and α can be computed:

$$\gamma = \arctan\frac{\bar{\eta}}{1-\tau\bar{\eta}}, \qquad \alpha = \pi - \beta - \gamma \tag{2.142}$$

It is instructive to write down the expressions for small values of S_ρ, which are suggested by the data in (2.132). To first order in both S_ρ and r_ρ we find

$$\gamma = \arctan\tau + \frac{S_\rho}{2} + \tau\, r_\rho \cos\phi_\rho \tag{2.143}$$

$$\alpha = \frac{\pi}{2} - \frac{S_\rho}{2} - \tau\, r_\rho \cos\phi_\rho \tag{2.144}$$

For $S_\rho = 0$ and in the absence of a penguin contribution one has $\alpha = 90°$ and $\gamma = (68.5 \pm 1.0)°$. Non-zero values of the observable S_ρ and the theoretical quantity $r_\rho \cos\phi_\rho$ then compete in shifting γ and α away from these lowest-order approximations.

Evaluation of the exact formulas (2.135) and (2.142) gives

$$\bar{\eta} = 0.346 \pm 0.013\,(\tau)\; \pm 0.012\,(S_\rho)\; \pm 0.008\,(r_\rho\cos\phi_\rho) \tag{2.145}$$

$$\gamma = 72.8° \pm 1.2°\,(\tau)\; \pm 5.1°\,(S_\rho)\; \pm 3.2°\,(r_\rho\cos\phi_\rho) \tag{2.146}$$

Nearly identical results are obtained for γ when the first order expression (2.143) is employed. The approximations (2.143) and (2.144) work to very good accuracy in the relevant range of S_ρ and r_ρ. This greatly facilitates the determination of γ and α and the analysis of errors, which can simply be read off from (2.143) and (2.144).

The calculation of γ in [38] using the longitudinal part of the time dependent CP-asymmetry in the $\rho^+\rho^-$-system and β as input yields a similar result for the hadronic error of $\pm 3°$.

The determination of γ in (2.146) is considerably more precise at present than measurements using $B \to DK$ tree-level decays. Belle has found [71]

$$\gamma = (78.4^{+10.8}_{-11.6}(stat) \pm 3.6(sys) \pm 8.9(model))° \tag{2.147}$$

and a recent analysis from BaBar [72] gives, among other intervals for γ, at 68% confidence level

$$80.9° < \gamma < 99.1° \tag{2.148}$$

Up to 2σ both results are compatible with (2.146).

Bounds on UT parameters

Useful information on the angle γ can also be obtained in the form of a lower bound, which is even less sensitive to theory input than the result in (2.146). It relies only on the conservative condition that $r\cos\phi \geq 0$, which holds in the heavy-quark limit. This bound has been derived in [43, 69]. Further discussions may also be found in [73, 74]. The bound is valid as long as $S > -\sin 2\beta$ and reads

$$\gamma > \frac{\pi}{2} - \arctan \frac{S - \tau(1 - \sqrt{1 - S^2})}{\tau S + 1 - \sqrt{1 - S^2}} \tag{2.149}$$

The constraint (2.149) can be evaluated using CP violation in $B \to \pi^+\pi^-$ ($S = S_\pi$) or in $B \to \rho_L^+ \rho_L^-$ ($S = S_\rho$). The derivation of (2.149) is identical for both cases. In fact, since the (positive) penguin correction $r_\rho \cos\phi_\rho$ is smaller than $r_\pi \cos\phi_\pi$, the bound is expected to be more stringent using S_ρ instead of S_π. This expectation is indeed bourne out by the experimental result $S_\rho > S_\pi = -0.61 \pm 0.08$ (see sec. 2.4.7), which implies that S_ρ gives the better constraint. Qualitatively, these features can also be understood from the approximate relation (2.143).

To linear order in S the bound (2.149) becomes

$$\gamma > \arctan \tau + \frac{S}{2} \tag{2.150}$$

in agreement with (2.143).

Using $S = S_\rho = -0.05$ (central), -0.22 (1σ), -0.39 (2σ), we obtain from (2.149), respectively

$$\gamma > 67^\circ\,, \quad 62^\circ\,, \quad 57^\circ \tag{2.151}$$

The linear approximation (2.150) gives practically identical results. We remark that the relevant values of S_ρ fulfill the condition $S_\rho > -\sin 2\beta$, under which the bound can be applied.

The penguin correction is expected to shift the numbers in (2.151) by approximately $+6^\circ$ to yield the actual value of γ. The bound is therefore quite stringent. Within a Standard Model interpretation it eliminates already a sizable fraction of the allowed range from the direct measurements in (2.147) and (2.148).

Bounds similar to the one for γ can also be derived for $\bar\eta$ and $\bar\rho$ [43, 69]. The lower bound for $\bar\eta$ is given by the right-hand side of (2.141) with r_ρ put to zero, the upper bound on $\bar\rho$ then follows from $\bar\rho = 1 - \tau\bar\eta$. With the same input for S_ρ as in (2.151) we find

$$\bar\eta > 0.338\,, \quad 0.326\,, \quad 0.314 \tag{2.152}$$

and

$$\bar\rho < 0.143\,, \quad 0.172\,, \quad 0.203 \tag{2.153}$$

Precision determination of $|V_{ub}|$ from $\sin 2\beta$ and S_ρ

The preceding analysis has a further interesting application regarding the determination of $|V_{ub}|$ from $\sin 2\beta$ and S_ρ. The value of $|V_{ub}|$ determined in this way may be affected by New Physics entering CP violation in $B_d \to \psi K_S$ and $B_d \to \rho_L^+ \rho_L^-$. The presence of non-standard contributions can be revealed by comparing the extracted value of $|V_{ub}|$

with the result for $|V_{ub}|$ from an independent method. An important example is the direct determination of $|V_{ub}|$ from semileptonic, exclusive or inclusive, $b \to ul\nu$ decays, which are most likely independent of physics beyond the Standard Model. It is clear that the usefulness of such a New Physics test will depend on how precisely $|V_{ub}|$ can be determined. We will show that $\sin 2\beta$ and S_ρ offer a particularly clean and accurate determination of $|V_{ub}|$.

The magnitude of $|V_{ub}|$ is proportional to $R_b \equiv \sqrt{\bar\rho^2 + \bar\eta^2}$. Using the exact formulas in (2.134) and (2.135), we expand R_b^2 in S_ρ and r_ρ. This is motivated by the smallness of the theoretical parameter r_ρ and the empirical observation that also S_ρ is small, as we have discussed in section 2.4.4. Treating S_ρ and r_ρ as small quantities of the same order we find

$$R_b = \sqrt{\bar\rho^2 + \bar\eta^2} = \frac{1}{\sqrt{1+\tau^2}} \cdot \left[1 + \frac{1}{2}\left(\frac{S_\rho}{2} + \tau r_\rho \cos\phi_\rho\right)^2 + \frac{r_\rho}{2}\left(\frac{S_\rho}{2} + \tau r_\rho \cos\phi_\rho\right)\left(S_\rho \cos\phi_\rho + 2\tau r_\rho \sin^2\phi_\rho\right)\right] \tag{2.154}$$

where we have neglected terms of the fourth order. Through terms of third order in S_ρ and r_ρ (S_ρ^3, $S_\rho^2 r_\rho$, $S_\rho r_\rho^2$, r_ρ^3) eq. (2.154) is exact.

The basic features of (2.154) are easy to understand from the geometry of the unitarity triangle. If $S_\rho = r_\rho = 0$ then $\alpha = \pi/2$. In this case $R_b = \sin\beta \equiv 1/\sqrt{1+\tau^2}$, which gives the leading term in (2.154). Because $\sin\beta$ is the minimum value that R_b can take for fixed β, first order corrections in r_ρ and S_ρ are absent and the second-order term is strictly positive. The protection of (2.154) from first-order corrections in S_ρ and r_ρ is the basis for a precise determination of V_{ub}.

The quantity $S_\rho/2 + \tau r_\rho \cos\phi_\rho$ appeared already in (2.143), (2.144). For $S_\rho < 0$ there is a further cancellation in this term with the penguin shift $\tau r_\rho \cos\phi_\rho$. Taking $S_\rho = -0.05 \pm 0.17$ (2.132), $\tau = 2.58 \pm 0.12$ (2.133) and the conservative range $r_\rho \cos\phi_\rho = 0.04 \pm 0.03$ we have

$$\frac{S_\rho}{2} + \tau r_\rho \cos\phi_\rho = 0.078 \pm 0.115 \tag{2.155}$$

The range of $r_\rho \cos\phi_\rho$ covers the result obtained from the QCD analysis in sec. 2.4.4. As we will show in sec. 2.4.6, $r_\rho \cos\phi_\rho$ can also be determined by independent experimental information on the penguin mode $\bar B_d \to \bar K_L^{*0} K_L^{*0}$, which confirms the values employed here.

Through second order in S_ρ and r_ρ the correction factor relative to the lowest-order result in (2.154) reads, using (2.155),

$$1 + \frac{1}{2}\left(\frac{S_\rho}{2} + \tau r_\rho \cos\phi_\rho\right)^2 = 1.003^{+0.014}_{-0.003} \tag{2.156}$$

We remark that the lower limit of 1 for this factor is an absolute bound. The third-order term in (2.154) is less than about $0.2 r_\rho S_\rho/2 \lesssim 0.2 \cdot 0.04 \cdot 0.1 \lesssim 0.001$ and thus completely negligible.

Using $\sin\beta = 0.361 \pm 0.015$ from Table 2.3 we obtain

$$R_b = \sin\beta \left[1 + \frac{1}{2}\left(\frac{S_\rho}{2} + \tau r_\rho \cos\phi_\rho\right)^2\right] = 0.362 \pm 0.016^{+0.005}_{-0.001} \qquad (2.157)$$

From [57] we have

$$\lambda = |V_{us}| = 0.226 \pm 0.002 \qquad |V_{cb}| = 0.0416 \pm 0.0006 \qquad (2.158)$$

This implies

$$|V_{ub}| \equiv \left|\frac{V_{cb}V_{cd}}{V_{ud}}\right| R_b = \frac{\lambda}{1-\frac{\lambda^2}{2}} R_b\, |V_{cb}| = (3.49^{+0.15}_{-0.14}(R_b) \pm 0.05(V_{cb}) \pm 0.03(V_{us})) \cdot 10^{-3} \qquad (2.159)$$

The uncertainty is dominated by the error in β $((\pm 0.14) \cdot 10^{-3})$, followed by the error in the correction from S_ρ, r_ρ $((^{+0.05}_{-0.02}) \cdot 10^{-3})$ and the error in V_{cb}. Adding errors in quadrature the final result reads

$$|V_{ub}| = (3.49 \pm 0.16) \cdot 10^{-3} \qquad (2.160)$$

It corresponds to a ratio $|V_{ub}/V_{cb}| = 0.084 \pm 0.004$, in agreement with Table 2.3. The value in (2.160) should be compared with the direct measurements of $|V_{ub}|$ in $b \to ul\nu$ transitions. An analysis of inclusive decays at NNLO is given in [75]

$$|V_{ub}| = (4.30 \pm 0.24^{+0.26+0.28}_{-0.20-0.27}) \cdot 10^{-3} \qquad E_l > 2.0 \text{ GeV} \quad \text{BABAR} \qquad (2.161)$$

$$|V_{ub}| = (3.96 \pm 0.19^{+0.20+0.26}_{-0.13-0.24}) \cdot 10^{-3} \qquad M_X < 1.55 \text{ GeV} \quad \text{BABAR} \qquad (2.162)$$

$$|V_{ub}| = (3.84 \pm 0.33^{+0.21+0.26}_{-0.13-0.22}) \cdot 10^{-3} \qquad P_+ < 0.66\text{GeV} \quad \text{BELLE} \qquad (2.163)$$

The exclusive determination from $B \to \pi l\nu$ decays has been investigated in [76] with the result

$$|V_{ub}| = (3.50 \pm 0.24) \cdot 10^{-3} \qquad (2.164)$$

Related discussions, in the context of QCD sum rules, can be found for instance in [77] and [78]. Using an average of data from lattice QCD, [79] quotes for the determination from exclusive decays

$$|V_{ub}| = (3.54 \pm 0.40) \cdot 10^{-3} \qquad (2.165)$$

The results for exclusive determinations (2.160), (2.164) and (2.165) are in very good agreement with each other, the inclusive determination (2.161, 2.162, 2.163) agrees within 2σ. They provide us with a test of Standard Model CP violation in $B \to \psi K_S$ and $B \to \rho^+\rho^-$ (2.160) against the $|V_{ub}|$ determination from tree-level, semileptonic $b \to ul\nu$ decays (2.161, 2.162, 2.163), (2.164) and (2.165).

Numbers for $|V_{ub}|$ very similar to (2.160) have been obtained from global fits of the unitarity triangle performed by the CKMfitter [26] and UTfit [59] collaborations, which quote

$$|V_{ub}| = (3.53^{+0.15}_{-0.14}) \cdot 10^{-3} \qquad \text{(CKMfitter)} \qquad (2.166)$$

$$|V_{ub}| = (3.64 \pm 0.11) \cdot 10^{-3} \qquad \text{(UTfit)} \qquad (2.167)$$

While such global fit results summarize our current overall knowledge of quark-mixing parameters, they do not exhibit explicitly the individual pieces of information that determine this knowledge. We emphasize here that the precise result in (2.160) can be obtained from $\sin 2\beta$ and S_ρ alone, with only very moderate requirements on the accuracy of the penguin contribution $\sim r_\rho$ from theory. The representation proposed in (2.157) makes this statement particularly transparent.

The result in (2.160) is currently the most precise determination of $|V_{ub}|$. Since the error is dominated by the uncertainty in $\sin\beta$, an even higher precision will be achieved by a more accurate measurement of $\sin\beta$ as it is expected at the upcoming LHC experiments. For instance, with a determination of $\sin\beta$ to 1%, the error in (2.160) would shrink to $\pm 0.08 \cdot 10^{-3}$, corresponding to a precision of 2% for $|V_{ub}|$.

Constraint on New Physics phase in $B_d - \bar{B}_d$ mixing

The preceding analyses rely on the assumption of a Standard Model phase in $B_d - \bar{B}_d$ mixing. We would like to examine the effect of a small New Physics phase entering only in $B_d - \bar{B}_d$-meson mixing [80,81]. In this scenario the New Physics phase shall not violate unitarity of the Standard Model CKM matrix. The modified mixing phase $\beta + \Omega$, with the New Physics contribution Ω, enters the analysis in the determination of $\tau = \cot(\beta + \Omega)$ from $\bar{B}_d \to J/\psi K_s$ and through mixing-induced CP violation in $\bar{B}_d \to \rho^+\rho^-$. The relation (2.134) for $\bar{\rho}$ depends now on Ω:

$$\bar{\rho} = \frac{(1 - \tau\bar{\eta}) - (\tau + \bar{\eta})\tan\Omega}{1 - \tau\tan\Omega} \tag{2.168}$$

The measurement of S_ρ determines $\bar{\eta}$ up to the mixing phase Ω. The new relation for $\bar{\eta}$ reads

$$\bar{\eta} = \frac{[(1 + \tau S_\rho)\cos\Omega + (S_\rho + \tau)\sin\Omega]\,(1 + r_\rho\cos\phi_\rho)(\cos\Omega - \tau\sin\Omega)}{(1 + \tau^2)\,(S_\rho + \sin(2\Omega))} - \tag{2.169}$$

$$\frac{|\cos\Omega - \tau\sin\Omega|}{(1 + \tau^2)\,(S_\rho + \sin(2\Omega))} \cdot \Big\{\left(1 - S_\rho^2\right)\left(1 + r_\rho^2 + 2r_\rho\cos\phi_\rho\right)(\cos\Omega - \tau\sin\Omega)^2 \tag{2.170}$$

$$-r_\rho^2\sin^2\phi_\rho\,[(1 + \tau S_\rho)\cos\Omega + (S_\rho + \tau)\sin\Omega]^2\Big\}^{1/2} \tag{2.171}$$

Analogously to (2.141) we expand (2.169) in r_ρ and S_ρ:

$$\bar{\eta} = \frac{\tau(1 - \tau\tan\Omega)}{1 + \tau^2} + \frac{(1 - \tau\tan\Omega)\left(\tau r_\rho\cos\phi_\rho + \frac{S_\rho}{2}(1 - \tau\tan\Omega)\right)}{1 + \tau^2} + \mathcal{O}(r_\rho^2, r_\rho S_\rho, S_\rho^2) \tag{2.172}$$

We note that Ω enters the leading term in the expansion with an enhancement of $\tau \approx 2.6$. The first order term is suppressed relative to the leading order term by a factor of ~ 0.1.

For R_b we find the following expression:

$$
\begin{aligned}
R_b =& \frac{1}{4\left(\tau^2+1\right)|\sin(2\Omega)+S_\rho|}\left\{\left[-4|\cos\Omega-\tau\sin\Omega|\right.\right. \\
&\left[\left(1+r_\rho\cos\phi_\rho\right)^2\left(S_\rho(\tau\cos\Omega+\sin\Omega)+\tau\sin\Omega+\cos(\Omega)\right)^2\right. \\
&\left.-\left(\left(\tau^2+1\right)S_\rho+2\tau\right)\left(2r_\rho\cos\phi_\rho+r_\rho^2+1\right)\left(S_\rho+\sin(2\Omega)\right)\right]^{1/2} \\
&+\left(1+r_\rho\cos\phi_\rho\right) \\
&\left.\left[-4\tau^2\sin\Omega\left(S_\rho\cos\Omega+\sin\Omega\right)+4\tau S_\rho\cos(2\Omega)+4\cos\Omega\left(S_\rho\sin\Omega+\cos\Omega\right)\right]\right]^2 \\
&+\frac{1}{(\cos\Omega-\tau\sin\Omega)^2}\left(2(\tau\cos\Omega+\sin\Omega)|\cos\Omega-\tau\sin\Omega|\right. \\
&\left[r_\rho^2\left(S_\rho\left(S_\rho\left(2\cos\left(2\phi_\rho\right)(\tau\cos(\Omega)+\sin\Omega)^2+\left(\tau^2-1\right)\cos(2\Omega)-3\tau^2\right.\right.\right.\right. \\
&\left.\left.+2\tau\sin(2\Omega)-3\right)-4\sin^2\phi_\rho\left(\left(\tau^2+1\right)\sin(2\Omega)+2\tau\right)\right) \\
&\left.+2\cos\left(2\phi_\rho\right)(\tau\sin\Omega+\cos\Omega)^2-\left(\tau^2-1\right)\cos(2\Omega)+\tau^2-6\tau\sin(2\Omega)+1\right) \\
&\left.-8r_\rho\left(S_\rho^2-1\right)\cos\phi_\rho(\cos\Omega-\tau\sin\Omega)^2-4\left(S_\rho^2-1\right)(\cos\Omega-\tau\sin\Omega)^2\right]^{1/2} \\
&+2r_\rho\cos\phi_\rho\left(\left(\tau^2-1\right)\sin(2\Omega)-2\tau\cos(2\Omega)\right) \\
&\left(S_\rho(\tau\cos\Omega+\sin\Omega)+\tau\sin\Omega+\cos\Omega\right)+4S_\rho(\cos\Omega-\tau\sin\Omega)^3 \\
&\left.\left.-4(\tau\cos\Omega-\sin\Omega)(\cos\Omega-\tau\sin\Omega)^2\right)^2\right\}^{1/2} \qquad (2.173)
\end{aligned}
$$

In the expanded form one can see, that R_b is not in a minimum:

$$
R_b = \frac{|1-\tau\tan\Omega|}{\sqrt{1+\tau^2}}\left[1-\tan\Omega\left(\frac{\tau r_\rho\cos\phi_\rho}{1-\tau\tan\Omega}+\frac{S_\rho}{2}\right)\right]+\mathcal{O}(r_\rho^2, r_\rho S_\rho, S_\rho^2) \qquad (2.174)
$$

Again we have an enhancement of the dependence on Ω by τ. For R_b the error from S_ρ and $r_\rho\cos\phi_\rho$ is much less important than the error from τ (2.133), in contrast to the case of $\bar\eta$ (2.145).

Relating R_b to $|V_{ub}|$ as in (2.159), taking $|V_{ub}|$, S_ρ, τ from experiment and r_ρ, ϕ_ρ from QCD factorization, the angle Ω can be extracted. This is visualised in figure 2.7. In general the solution is not unique. Here we assume that the new phase Ω is small, neglecting discrete ambiguities. Such ambiguities may be eliminated with additional measurements. In particular, a second solution with large Ω would imply a negative sign of $\cos(2(\beta+\Omega))$, which is disfavoured by experiment [82]. A more general discussion on the New Physics aspects of this analysis can be found in [81].

If one disregards solutions with $|\Omega| > \arctan(1/\tau) \approx 21^\circ$ and uses the exclusive determination (2.165) of $|V_{ub}|$, then Ω can be determined with an accuracy of few degrees:

$$
\Omega = \left(-0.28^{-0.8}_{+0.9}(\tau)\,^{+0.2}_{-0.1}(S_\rho)\,^{+0.1}_{-0.1}(r_\rho)\,^{+0.3}_{-0.3}(V_{cb})\,^{-2.5}_{+2.5}(V_{ub})\right)^\circ \qquad (2.175)
$$

One may note the very small impact of $S_\rho = -0.05 \pm 0.17$ and $r_\rho = 0.04 \pm 0.03$. Combining the errors in (2.175) in quadrature one finds $\Omega = (-0.28 \pm 2.7)^\circ$.

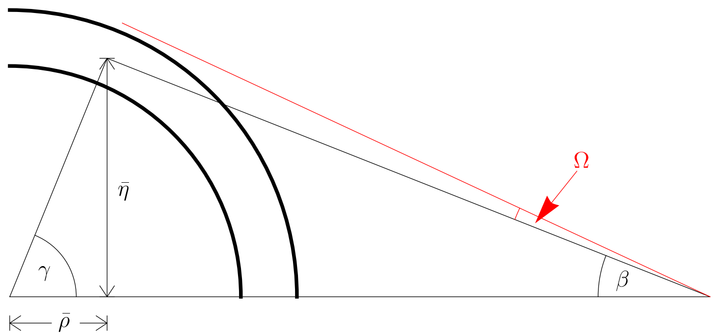

Figure 2.7: Extraction of new physics phase Ω from independent determination of V_{ub}. The circular band is the 1σ error band of V_{ub}. R_b is determined by β, the measurement of S_ρ and the hadronic parameters r_ρ and ϕ_ρ from QCD factorization. The comparison with $\tau = \cot(\beta + \Omega)$ constrains Ω.

2.4.5 Extracting r_ρ from $B^- \to \bar{K}_L^{*0}\rho_L^-$

The precision of CKM angles extracted from CP violation in $B \to \rho_L^+\rho_L^-$ is ultimately limited by our knowledge of the penguin parameters r_ρ and, to a lesser extent, ϕ_ρ. Since r_ρ is small, a very moderate accuracy in this quantity is sufficient to obtain a small theoretical error for CKM parameters. In [83] it has been proposed to constrain the penguin parameter r_ρ using the penguin dominated decay $B^- \to \bar{K}_L^{*0}\rho_L^-$. We will discuss this method in the context of our analysis, comment on the benefits and limitations, present an updated numerical evaluation, and compare with the theory results of sec. 2.4.4.

The main idea of [83] is to determine the penguin amplitude from the pure-penguin process $B^- \to \bar{K}_L^{*0}\rho_L^-$ through

$$B(B^- \to \bar{K}_L^{*0}\rho_L^-) = \frac{\tau_{B_u} G_F^2\, |\lambda_c'|^2}{32\pi m_B}\, |a_c(\rho K^*)|^2 \tag{2.176}$$

Here we defined $a_p(\rho K^*) \equiv a_p(B^- \to \bar{K}_L^{*0}\rho_L^-)$ as the coefficient of $(iG_F/\sqrt{2})\lambda_p$ in the amplitude for $B^- \to \bar{K}_L^{*0}\rho_L^-$, eqs. (2.70) and (2.109). They correspond to the charm- and up-quark penguin amplitudes for this process. Since $a_u(\rho K^*)$ and $a_c(\rho K^*)$ are of comparable size, and the up-quark amplitude is strongly CKM suppressed, the charm penguin completely dominates the branching ratio (2.176). The penguin amplitude $a_c(\rho K^*)$ can be related to the (similarly normalized) penguin amplitude $a_c(\rho)$ in $\bar{B}_d \to \rho^+\rho^-$ by introducing the factor

$$|\kappa| = \left|\frac{a_c(\rho K^*)}{a_c(\rho)}\right| \approx 0.84 \tag{2.177}$$

To lowest order (in α_s and Λ/m_b) this factor would be given by $\kappa = f_K^*/f_\rho = 1.04$. Including QCD corrections this value is reduced to $|\kappa| = 1.01$, and further to $|\kappa| = 0.97$ by the effects of electroweak penguins. The estimate in (2.177) includes also the weak annihilation terms with default model parameters. Annihilation contributions are thus seen to be potentially important. These observations agree with the discussion in [83]. In that paper the ratio of the penguin amplitudes in $B^- \to \bar{K}_L^{*0}\rho_L^-$ and $\bar{B}_d \to \rho^+\rho^-$ has been parametrized in terms of a factor F, which is related to κ through $|\kappa| = \sqrt{F} f_{K^*}/f_\rho$. In [83] a rather wide range for F is assumed, $0.3 < F < 1.5$. We will use the same range, which corresponds to $|\kappa| = 0.93 \pm 0.36$.

For a given value of $|\kappa|$ the penguin parameters r_ρ, ϕ_ρ are then constrained by the ratio

$$\frac{B(B^- \to \bar{K}_L^{*0}\rho_L^-)}{B(\bar{B} \to \rho_L^+\rho_L^-)} = \frac{\tau_{B_u}}{\tau_{B_d}} \left|\frac{V_{cs}}{V_{cd}}\right|^2 \frac{|\kappa|^2\, r_\rho^2}{\bar\rho^2 + \bar\eta^2 + r_\rho^2 + 2\bar\rho\, r_\rho \cos\phi_\rho} \tag{2.178}$$

where CP averaged rates are understood. Using $\tau = \cot\beta$, S_ρ, C_ρ and the ratio of branching fractions in (2.178) as experimental inputs, the four quantities $\bar\rho$, $\bar\eta$, r_ρ and ϕ_ρ can be determined as functions of $|\kappa|$. A discrete ambiguity in the sign of $\cos\phi_\rho$ can be resolved using the heavy-quark limit. The suppression of ϕ_ρ in this limit singles out the solution with $\cos\phi_\rho > 0$. A similar use of the qualitative result $\cos\phi_\rho > 0$ from factorization has been made in [43, 69]. Further details on the extraction of $\bar\rho$, $\bar\eta$, r_ρ, ϕ_ρ are discussed in 2.4.6 in the context of a similar analysis. The results for the present method are collected in Table 2.9. Combining and symmetrizing errors we obtain from Table 2.9

Table 2.9: CKM and penguin parameters extracted from $\tau = \cot\beta = 2.58 \pm 0.12$, $S_\rho = -0.05 \pm 0.17$, $C_\rho = -0.06 \pm 0.13$ and $b = B(B^- \to \bar{K}_L^{*0}\rho_L^-)/B(\bar{B} \to \rho_L^+\rho_L^-) = 0.186 \pm 0.049$. The penguin correction factor is taken to be $|\kappa| = 0.93 \pm 0.36$.

	central	τ	S_ρ	C_ρ	b	$\lvert\kappa\rvert$
$\bar\rho$	0.107	$^{-0.009}_{+0.011}$	$^{-0.030}_{+0.030}$	$^{-0.002}_{+0.023}$	$^{-0.005}_{+0.005}$	$^{+0.010}_{-0.023}$
$\bar\eta$	0.346	$^{-0.012}_{+0.013}$	$^{+0.012}_{-0.012}$	$^{+0.001}_{-0.011}$	$^{+0.002}_{-0.002}$	$^{-0.004}_{+0.009}$
γ[deg]	72.8	$^{+0.9}_{-1.0}$	$^{+5.0}_{-5.1}$	$^{+0.3}_{-4.7}$	$^{+0.8}_{-0.9}$	$^{-1.8}_{+3.9}$
r_ρ	0.039	$^{-0.002}_{+0.002}$	$^{-0.000}_{+0.000}$	$^{+0.000}_{-0.001}$	$^{+0.005}_{-0.006}$	$^{-0.011}_{+0.026}$
ϕ_ρ	−0.32	$^{+0.00}_{-0.00}$	$^{+0.01}_{-0.01}$	$^{+0.69}_{-1.15}$	$^{+0.04}_{-0.05}$	$^{-0.13}_{+0.13}$

$$\gamma = (72.8 \pm 6.9)^\circ \qquad \alpha = \pi - \beta - \gamma = (86.1 \pm 6.9)^\circ \tag{2.179}$$

where S_ρ is the largest source of uncertainty. The results agree very well with those in (2.146). Eq. (2.179) is an update of the results quoted in [83]. We have checked that we obtain the numbers given in that paper if we use the same input.

A disadvantage of the method just described is that the charm-penguin amplitudes in $B^- \to \bar{K}_L^{*0}\rho_L^-$ and in $\bar{B}_d \to \rho_L^+\rho_L^-$ are not related in full QCD by $SU(3)$ flavour symmetry alone. The $SU(3)$ argument relating $a_c(\rho K^*)$ and $a_c(\rho)$ strictly holds only to leading order in the heavy-quark limit. At the level of power corrections from weak annihilation these penguin amplitudes are not related by $SU(3)$. This can be seen from eqs. (2.98) and (2.109), which show that the QCD annihilation penguins are determined by the coefficient b_3 for $a_c(\rho K^*)$, but by b_3+2b_4 instead for $a_c(\rho)$. This difference has been discussed in [83]. In order to account for the corresponding $SU(3)$ breaking, a rather generous correction factor κ (2.177) has been allowed for. While this is certainly a valid procedure, it is somewhat against the spirit of using experimental data to constrain the penguin in $\bar{B}_d \to \rho_L^+\rho_L^-$. An unexpectedly large penguin annihilation effect in this channel, beyond the available model estimates, would not necessarily be indicated by the $B^- \to \bar{K}_L^{*0}\rho_L^-$ rate, not even in the $SU(3)$ limit. In this respect, the method of [83] amounts to the standard analysis of CP violation in $\bar{B}_d \to \rho_L^+\rho_L^-$ with input on r_ρ from factorization calculations, which are validated by comparing similar theory results on the penguin mode $B^- \to \bar{K}_L^{*0}\rho_L^-$ with data. Indeed, QCD factorization works very well for $B^- \to \bar{K}_L^{*0}\rho_L^-$ with little room for sizable power corrections. Correspondingly, the values for r_ρ and the angle γ determined in Table 2.9 are very close to the values found in the factorization analysis, eqs. (2.138) and (2.146). Nevertheless, an independent control of penguin annihilation corrections in $\bar{B}_d \to \rho_L^+\rho_L^-$, which is not guaranteed by $B^- \to \bar{K}_L^{*0}\rho_L^-$, would be very desirable. A variant of the method in [83] that can provide this control will be discussed in the following section.

2.4.6 Extracting r_ρ from $B_d \to \bar{K}_L^{*0}K_L^{*0}$

In this section we propose a method to constrain the penguin parameter r_ρ in $\bar{B}_d \to \rho_L^+\rho_L^-$ (2.136) using $SU(3)$ flavour symmetry and data on the penguin decay $\bar{B}_d \to \bar{K}_L^{*0}K_L^{*0}$. This approach shares the basic idea with the method discussed in section 2.4.5. An important difference is that now, unlike the case of section 2.4.5, the penguin process exhibits an exact $SU(3)$ relation to the penguin amplitude of $\bar{B}_d \to \rho_L^+\rho_L^-$. Because this relation extends beyond the heavy-quark limit, the method offers an independent control of all power corrections, in particular those from weak annihilation topologies. We show that a precise determination of the unitarity triangle is possible, already with present data on $\bar{B}_d \to \bar{K}_L^{*0}K_L^{*0}$. Since the penguin decay $\bar{B}_d \to \bar{K}_L^{*0}K_L^{*0}$ is a $\Delta S = 0$ transition, the up-quark sector of the amplitude does not have the same CKM suppression as for the $\Delta S = 1$ process $B^- \to \bar{K}_L^{*0}\rho_L^-$. We will find that it is still sufficiently well constrained.

The $SU(3)$ relation between the relevant penguin amplitudes can be demonstrated as follows. The penguin contribution for $\bar{B}_d \to \rho_L^+\rho_L^-$ is given by the component of the amplitude proportional to $\lambda_c = V_{cb}V_{cd}^*$. The corresponding part of the effective Hamiltonian (2.1) has the form

$$\mathcal{H}_{\rm QCDP,c} = \frac{G_F}{\sqrt{2}}\left(C_1\, Q_1^c + C_2\, Q_2^c + \sum_{i=3,\ldots,6} C_i\, Q_i + C_{8g}\, Q_{8g}\right) + \text{h.c.} \tag{2.180}$$

where we have neglected higher-order electroweak effects. The operators Q_i are defined in (2.2). The Hamiltonian in (2.180) gives rise to the QCD penguin amplitude in the charm sector of both $\bar{B}_d \to \rho_L^+\rho_L^-$ and $\bar{B}_d \to \bar{K}_L^{*0}K_L^{*0}$. To prove the symmetry relation we note that all operators entering (2.180) are invariant under $SU(2)$ rotations of the doublet (u,s)

of quark flavours, the V-spin subgroup of flavour $SU(3)$. The initial state, a $\bar{B}_d$ meson in both cases, is likewise a V-spin singlet. The final states $\rho^+\rho^-$ and $\bar{K}^{*0}K^{*0}$ are transformed into each other by interchanging u and s quarks, which represents a particular V-spin rotation. In the V-spin symmetry limit, therefore, the relation

$$\langle \bar{K}^{*0}_L K^{*0}_L | \mathcal{H}_{\rm QCDP,c} | \bar{B}_d \rangle = \langle \rho^+_L \rho^-_L | \mathcal{H}_{\rm QCDP,c} | \bar{B}_d \rangle \tag{2.181}$$

holds as an identity in QCD. As a consequence, the QCD penguin amplitudes proportional to λ_c in $\bar{B}_d \to \rho^+_L\rho^-_L$ and $\bar{B}_d \to \bar{K}^{*0}_L K^{*0}_L$ have the same form, including the weak annihilation contributions. This can be seen from (2.61), (2.62) and (2.98), (2.99).

In practice V-spin is broken because the masses of up and strange quarks are not the same. This source of V-spin breaking can be expected to be of the typical size of flavour $SU(3)$ breaking effects, roughly 20-30%. It is possible to estimate the required correction to the V-spin limit using factorization. We will give a more quantitative treatment below. Electroweak effects also violate V-spin symmetry. They are similar to isospin breaking and likely to be much smaller than the $SU(3)$-breaking effects due to the strange-quark mass. For example, the relative importance of (standard) electroweak penguins is governed by the ratio $a^c_{10}/a^c_4 \approx 0.03$. This is safely negligible in comparison with the dominant V-spin breaking effects. Contributions from electroweak penguin annihilation are very small and can also be neglected.

We next turn to the phenomenological implications of the flavour symmetry relation (2.181). Denoting by $a_p(K^*) \equiv a_p(\bar{B}_d \to \bar{K}^{*0}_L K^{*0}_L)$ the coefficient of $(iG_F/\sqrt{2})\lambda_p$ in the amplitude for $\bar{B}_d \to \bar{K}^{*0}_L K^{*0}_L$, eqs. (2.62) and (2.99), the CP averaged branching fraction may be written as

$$B(\bar{B}_d \to \bar{K}^{*0}_L K^{*0}_L) = \frac{\tau_{B_d} G_F^2\, |\lambda_c|^2}{32\pi m_B}\left[f_0 |a_c(K^*)|^2 + 2 f_1 \,{\rm Re}\, a^*_c(K^*)\,\Delta(K^*) + f_2 |\Delta(K^*)|^2 \right] \tag{2.182}$$

$$\Delta(K^*) = a_c(K^*) - a_u(K^*) \tag{2.183}$$

where the functions f_i depend only on CKM parameters. Expressed in terms of $\tau = \cot\beta$ from (2.133) and

$$\sigma \equiv \cot\gamma \tag{2.184}$$

they read

$$f_0(\sigma,\tau) = \frac{1+\tau^2}{(\sigma+\tau)^2} = \frac{|\lambda_t|^2}{|\lambda_c|^2} \tag{2.185}$$

$$f_1(\sigma,\tau) = \frac{\sigma\tau - 1}{(\sigma+\tau)^2} = \frac{{\rm Re}\,\lambda^*_t\lambda_u}{|\lambda_c|^2} \tag{2.186}$$

$$f_2(\sigma,\tau) = \frac{1+\sigma^2}{(\sigma+\tau)^2} = \frac{|\lambda_u|^2}{|\lambda_c|^2} \tag{2.187}$$

In the region of interest for a Standard Model test,

$$\sigma = 0.447 \pm 0.253, \qquad \tau = 2.58 \pm 0.12 \tag{2.188}$$

there is a clear hierarchy among the CKM factors

$$f_0 = 0.836^{+0.159}_{-0.124}, \qquad f_1/f_0 = 0.020 \pm 0.085, \qquad f_2/f_0 = 0.157^{+0.040}_{-0.024} \tag{2.189}$$

implying $|f_1| \ll f_0$ and $f_2 \ll f_0$. The second inequality is a consequence of the fact that numerically $|V_{ub}/V_{td}|^2 \ll 1$. The first inequality arises because $f_1 \sim \cos\alpha$ and the angle α is close to 90°. A similar feature holds for the decay $B \to \rho\gamma$, where it leads to a suppression of hadronic uncertainties [84,85]. The dominance of the f_0 term in (2.182) is reinforced by the hadronic factors since the difference $|\Delta(K^*)|$ is systematically smaller than $|a_c(K^*)|$. This difference is a next-to-leading order effect in QCD factorization, whereas a_c is present at leading order. In addition, several terms cancel in the difference $a_c - a_u$. First, the NLO hard spectator corrections are identical in the c- and u-sector and drop out, which eliminates the uncertainty due to λ_B. Spectator effects in $a_c - a_u$ can only come from penguin diagrams at NNLO ($\mathcal{O}(\alpha_s^2)$), which are very small [86]. Second, also weak annihilation effects cancel in general, in particular those that are taken into account in our model estimate (coefficients b_i). The only exception would be more complicated $b\bar{d} \to s\bar{d}d\bar{s}$ annihilation topologies involving charm and up-quark loops. These are both power and Zweig rule suppressed and not expected to give a significant contribution. Numerically we find

$$|a_c(K^*) - a_u(K^*)|/\mathrm{GeV}^3 = 0.021^{+0.003}_{-0.003}\,(A_0)\ \ {}^{-0.003}_{+0.004}\,(\alpha_2^V)\ \ {}^{+0.006}_{-0.006}\,(m_c)\ \ {}^{-0.004}_{+0.007}\,(\mu) \tag{2.190}$$

Normalized to the central value of $|a_c(K^*)| = 0.084$ we then have

$$\frac{|a_c(K^*) - a_u(K^*)|}{|a_c(K^*)|} = 0.25^{+0.12}_{-0.10} \tag{2.191}$$

Together with the CKM factors from (2.189) we estimate a relative suppression of the third term in (2.182) by $0.010\,(+0.012, -0.007)$ with respect to the first term. For the second term we estimate a relative size of at most 0.009 ± 0.043, neglecting the phase between a_c and Δ. In this case the potential magnitude of the correction depends strongly on the CKM suppression due to f_1, which can be checked after the CKM factors have been determined at the end of the analysis.

Because of the smallness of the f_1, f_2 terms, the first term in (2.182) determines the branching fraction to very good approximation. In the $SU(3)$ limit, and up to negligible corrections from electroweak penguins, $a_c(K^*)$ is equal to the penguin amplitude $a_c(\rho)$ in $\bar{B}_d \to \rho_L^+ \rho_L^-$ (in a corresponding normalization). Introducing the $SU(3)$ factor

$$|\xi| = \left|\frac{a_c(K^*)}{a_c(\rho)}\right| \approx 1.28 \tag{2.192}$$

we obtain the ratio of CP averaged branching fractions

$$\frac{B(\bar{B}_d \to \bar{K}_L^{*0} K_L^{*0})}{B(\bar{B}_d \to \rho_L^+ \rho_L^-)} = \frac{((1-\bar{\rho})^2 + \bar{\eta}^2)\,|\xi|^2\, r_\rho^2}{\bar{\rho}^2 + \bar{\eta}^2 + r_\rho^2 + 2\bar{\rho}\, r_\rho \cos\phi_\rho} \tag{2.193}$$

This result constrains the penguin parameter r_ρ in $\bar{B}_d \to \rho_L^+ \rho_L^-$. The four variables $\bar{\rho}$, $\bar{\eta}$, r_ρ and ϕ_ρ may now be determined from the four measurements of $\tau = (1-\bar{\rho})/\bar{\eta}$, $S_\rho(\bar{\rho}, \bar{\eta}, r_\rho, \phi_\rho)$, $C_\rho(\bar{\rho}, \bar{\eta}, r_\rho, \phi_\rho)$ and $b \equiv B(\bar{B}_d \to \bar{K}_L^{*0} K_L^{*0})/B(\bar{B}_d \to \rho_L^+ \rho_L^-)$. This analysis then depends on a single theoretical parameter, the $SU(3)$ factor $|\xi| = 1.28 \pm 0.14$, where we adopt the estimate in (2.192) and assign a 50% error on the magnitude of $SU(3)$ breaking. The quantity $\xi = a_c(K^*)/a_c(\rho)$ is real to very good approximation, $\xi \approx |\xi|$.

The expressions for S and C in terms of $\bar{\rho}$, $\bar{\eta}$, r and ϕ are identical to the case of $\bar{B}_d \to \pi^+\pi^-$ discussed in [43, 69]. They read

$$S = \frac{2\bar{\eta}[\bar{\rho}^2 + \bar{\eta}^2 - r^2 - \bar{\rho}(1-r^2) + (\bar{\rho}^2 + \bar{\eta}^2 - 1) r\cos\phi]}{((1-\bar{\rho})^2 + \bar{\eta}^2)(\bar{\rho}^2 + \bar{\eta}^2 + r^2 + 2r\bar{\rho}\cos\phi)} \tag{2.194}$$

$$C = \frac{2r\bar{\eta}\,\sin\phi}{\bar{\rho}^2 + \bar{\eta}^2 + r^2 + 2r\bar{\rho}\cos\phi} \tag{2.195}$$

We remark that discrete ambiguities in the determination of $\bar{\rho}$, $\bar{\eta}$, r and ϕ can be avoided using other constraints on the unitarity triangle, which exclude $\bar{\rho}$, $\bar{\eta}$ far outside the region allowed in the Standard Model [43, 69]. A discrete ambiguity in the sign of $\cos\phi_\rho$ can be resolved by the heavy-quark limit, which favours the solution with $\cos\phi_\rho > 0$. As pointed out in a similar context in [83], the discrete choice is still less restrictive in practice, because the second solution has $\cos\phi_\rho < -0.8$, which is in fact far smaller than zero.

The result of this analysis is given in Table 2.10, where we have also summarized the experimental input from Table 2.1 and section 2.4.4. The output values $\bar{\rho}$, $\bar{\eta}$, r_ρ and ϕ_ρ are

Table 2.10: CKM and penguin parameters extracted from $\tau = \cot\beta = 2.58 \pm 0.12$, $S_\rho = -0.05 \pm 0.17$, $C_\rho = -0.06 \pm 0.13$ and $b = B(\bar{B}_d \to \bar{K}_L^{*0} K_L^{*0})/B(\bar{B}_d \to \rho_L^+ \rho_L^-) = 0.027 \pm 0.009$. The $SU(3)$ breaking parameter is taken to be $|\xi| = 1.28 \pm 0.14$.

	central	τ	S_ρ	C_ρ	b	$\lvert\xi\rvert$
$\bar{\rho}$	0.097	$^{-0.009}_{+0.010}$	$^{-0.029}_{+0.029}$	$^{-0.001}_{+0.013}$	$^{-0.007}_{+0.008}$	$^{+0.008}_{-0.012}$
$\bar{\eta}$	0.350	$^{-0.012}_{+0.013}$	$^{+0.011}_{-0.011}$	$^{+0.000}_{-0.005}$	$^{+0.003}_{-0.003}$	$^{-0.003}_{+0.005}$
γ[deg]	74.5	$^{+0.9}_{-0.9}$	$^{+4.8}_{-4.9}$	$^{+0.2}_{-2.2}$	$^{+1.2}_{-1.4}$	$^{-1.4}_{+2.1}$
r_ρ	0.050	$^{-0.002}_{+0.002}$	$^{-0.002}_{+0.002}$	$^{+0.000}_{-0.000}$	$^{+0.008}_{-0.009}$	$^{-0.009}_{+0.014}$
ϕ_ρ	-0.25	$^{+0.00}_{-0.00}$	$^{+0.00}_{-0.00}$	$^{+0.54}_{-0.63}$	$^{+0.03}_{-0.05}$	$^{-0.05}_{+0.05}$

shown together with their sensitivity to the relevant input quantities. From Table 2.10 we draw the following conclusions:

a) The errors on the CKM quantities $\bar{\rho}$, $\bar{\eta}$ and γ are rather small. They are dominated by the uncertainty in S_ρ ($\bar{\eta}$ is sensitive also to $\tau = \cot\beta$).

b) The error from the $SU(3)$ factor $|\xi|$ is smaller than the errors from the experimental quantities τ, S_ρ, b, which may still be improved by future measurements.

c) The penguin parameter is obtained as $r_\rho = 0.050 \pm 0.015$. The central value is somewhat larger than the theoretical number in (2.138), but both results are compatible within errors. This confirms the expected smallness of r_ρ, which is the basis for a precise extraction of CKM quantities.

d) The phase ϕ_ρ is seen to be strongly dependent on C_ρ, but essentially uncorrelated with the remaining parameters and input quantities. In particular $\bar\rho$, $\bar\eta$ and γ are almost unaffected by the value of C_ρ within the measured range. This behaviour is in agreement with the general expectation discussed in 2.4.4. The error on ϕ_ρ is completely dominated by the error on C_ρ. The sign of ϕ_ρ is opposite to the central standard model value from factorization in (2.139). If higher-order perturbative corrections cannot account for this change in sign, and we assume it is not due to New Physics, this would mean that power corrections give an important contribution to the strong phase. A similar situation is known to occur for the direct CP asymmetries in $\bar B_d \to \pi^+\pi^-$ and $\bar B_d \to \pi^+ K^-$. However, within uncertainties the numbers for ϕ_ρ in (2.139) and Table 2.10 are fully consistent with each other. The result for ϕ_ρ in Table 2.10 confirms the prediction of a suppressed phase in the heavy quark limit.

Combining the errors in Table 2.10 we find for the CKM angles

$$\gamma = (74.5 \pm 5.6)^\circ \qquad \alpha = \pi - \beta - \gamma = (84.3 \pm 5.7)^\circ \tag{2.196}$$

where the uncertainty is dominated by the experimental error in S_ρ. The result is in very good agreement with (2.146). It is already rather accurate at present. From Table 2.10 we see that a precision of $\pm 1^\circ$ for γ and α from this method should be possible.

Finally, we remark that the approximations leading to (2.193) may be cross-checked using the extracted value of γ or $\sigma = \cot\gamma = 0.25 \pm 0.10$, and $\tau = \cot\beta = 2.58 \pm 0.12$. Varying also the hadronic input parameters, the relative importance of the correction terms in (2.182) is then smaller than $\pm 3\%$. (For the default parameter set and $\sigma = 0.25$, $\tau = 2.58$, the correction is -0.9%.) The corresponding change in (2.193) could be absorbed in a modification of $|\xi|$ by less than $\pm 1.9\%$, which is entirely negligible.

2.4.7 Unitarity triangle from $B_d \to \pi^+\pi^-$ and $B_d \to \bar K^0 K^0$

The analysis of section 2.4.6 made use of CP violation in $\bar B_d \to \rho_L^+ \rho_L^-$, a measurement of $\sin 2\beta$, and the $\bar B_d \to \bar K_L^{*0} K_L^{*0}$ branching fraction to obtain an accurate determination of the unitarity triangle. The decay $\bar B_d \to \bar K_L^{*0} K_L^{*0}$ served to fix the penguin-to-tree ratio in $\bar B_d \to \rho_L^+ \rho_L^-$ based on $SU(3)$ flavour symmetry.

The same analysis may also be performed with the VV-modes replaced by their pseudoscalar counterparts, that is, using CP violation in $\bar B_d \to \pi^+\pi^-$ and constraining the penguin parameter r_π with $\bar B_d \to \bar K^0 K^0$ and $SU(3)$ symmetry. The formulas of section 2.4.6 apply with obvious substitutions. Related discussions can be found in [43, 69].

Using form factor estimates based on [87]

$$f_+^{B\to\pi}(0) = 0.258 \pm 0.031 \qquad f_+^{B\to K}(0) = 0.304 \pm 0.042 \tag{2.197}$$

we find from QCD factorization [30, 37]

$$|\zeta| \equiv \left|\frac{a_c(K)}{a_c(\pi)}\right| = 1.46 \pm 0.23 \tag{2.198}$$

Again we have assigned a generous 50% uncertainty on the total amount of $SU(3)$ breaking. With experimental input from [55] we obtain the results displayed in Table 2.11.

Table 2.11: CKM and penguin parameters extracted from $\tau = \cot\beta = 2.58 \pm 0.12$, $S_\pi = -0.65 \pm 0.07$, $C_\pi = -0.38 \pm 0.06$ and $b \equiv B(\bar{B}_d \to \bar{K}^0 K^0)/B(\bar{B}_d \to \pi^+\pi^-) = 0.186 \pm 0.040$. The $SU(3)$ breaking parameter is taken to be $|\zeta| = 1.46 \pm 0.23$.

	central	τ	S_π	C_π	b	$\vert\zeta\vert$
$\bar{\rho}$	0.185	$^{-0.012}_{+0.013}$	$^{-0.018}_{+0.021}$	$^{-0.013}_{+0.018}$	$^{-0.015}_{+0.019}$	$^{+0.023}_{-0.027}$
$\bar{\eta}$	0.316	$^{-0.010}_{+0.010}$	$^{+0.007}_{-0.008}$	$^{+0.005}_{-0.007}$	$^{+0.006}_{-0.007}$	$^{-0.009}_{+0.010}$
γ[deg]	59.6	$^{+0.9}_{-0.9}$	$^{+3.1}_{-3.5}$	$^{+2.1}_{-3.0}$	$^{+2.5}_{-3.1}$	$^{-3.8}_{+4.5}$
r_π	0.147	$^{-0.008}_{+0.008}$	$^{-0.006}_{+0.008}$	$^{-0.002}_{+0.002}$	$^{+0.015}_{-0.017}$	$^{-0.020}_{+0.028}$
ϕ_π	−0.89	$^{+0.01}_{-0.01}$	$^{+0.02}_{-0.03}$	$^{+0.17}_{-0.21}$	$^{+0.09}_{-0.14}$	$^{-0.17}_{+0.15}$

Combining errors we obtain from Table 2.11

$$\gamma = (59.6 \pm 6.6)^\circ \tag{2.199}$$

This value is lower than the result in (2.196) but it remains consistent at the level of roughly 2σ. One possible source of this discrepancy is the rather large value of $C_\pi = -0.38 \pm 0.06$, representing the simple average of the BaBar [88] and Belle [89] results

$$C_\pi = -0.25 \pm 0.08 \pm 0.02 \quad \text{(BaBar)} \qquad C_\pi = -0.55 \pm 0.08 \pm 0.05 \quad \text{(Belle)} \tag{2.200}$$

These results are not in very good agreement. With a smaller $|C_\pi|$, favoured by QCD factorization and the BaBar measurement, the extracted value for γ would increase somewhat. For example, with $C_\pi = -0.1$ we obtain $\gamma = 65.4^\circ$, keeping all other inputs fixed. Particularly important for the resulting γ is the value of S_π. If it were 2σ lower in absolute magnitude, at $S_\pi = -0.51$, the central value of γ would shift to $\gamma = 65.5^\circ$. The uncertainties in b and $|\zeta|$ also have a relatively large impact. This is because of the larger size of the penguin contribution r_π in comparison with r_ρ. Note that the error on γ from the uncertainty in b is almost twice as large in Table 2.11 than in Table 2.10, even though b is known with an accuracy of 22% in the former case and only to 33% in the latter. Thus, because of the larger size of the penguin amplitude, and also because of the experimental situation of C_π, which is still not entirely resolved, the determination of the unitarity triangle from $\bar{B} \to \pi^+\pi^-$ and $\bar{B} \to \bar{K}^0 K^0$ appears to be somewhat less precise than the determination from $\bar{B} \to \rho_L^+\rho_L^-$ and $\bar{B} \to \bar{K}_L^{*0} K_L^{*0}$.

2.4.8 CP violation in $B_s \to \phi_L\phi_L$

The decay $\bar{B}_s \to \phi\phi$ is a pure $b \to s$ penguin transition and thus of considerable interest as a New Physics probe. Possible hints of deviations from the Standard Model in CP violation in the $b \to s$ penguin process $\bar{B}_d \to \phi K_S$, and similar modes, have so far remained inconclusive. A detailed experimental study of $\bar{B}_s \to \phi\phi$ will become possible with the LHC [62]. In the Standard Model CP violation in $\bar{B}_s \to \phi\phi$ is small. Any nonzero effect in excess of the Standard Model contribution will signal the presence of New Physics. Based on our next-to-leading order results we shall investigate the size and uncertainty of CP violation in the Standard Model, which ultimately limits the sensitivity to New Physics.

The time dependent CP asymmetry in $\bar{B}_s \to \phi_L\phi_L$ decays is defined by

$$\mathcal{A}_{CP,\phi}(t) = \frac{\Gamma(\bar{B}_s(t) \to \phi_L\phi_L) - \Gamma(B_s(t) \to \phi_L\phi_L)}{\Gamma(\bar{B}_s(t) \to \phi_L\phi_L) + \Gamma(B_s(t) \to \phi_L\phi_L)} = S_\phi \sin(\Delta m_s\, t) - C_\phi \cos(\Delta m_s\, t) \tag{2.201}$$

Here we have neglected the effects of a nonzero width difference $\Delta\Gamma_{B_s}$, which would modify the time dependence of the CP asymmetry. This can be taken into account in extracting S_ϕ and C_ϕ, but would not change the following discussion of these parameters.

For a generic B decay into a CP self-conjugate final state f one has

$$S = \frac{2\,\mathrm{Im}\xi}{1+|\xi|^2}, \qquad C = \frac{1-|\xi|^2}{1+|\xi|^2}, \qquad \xi = -\frac{M_{12}^*}{|M_{12}|}\frac{A(\bar{B} \to f)}{A(B \to f)} \tag{2.202}$$

where $M_{12} = \langle B|\mathcal{H}_{\Delta B=2}|\bar{B}\rangle$ is the B–$\bar{B}$ mixing amplitude. We use the phase convention $CP|\bar{B}\rangle = -|B\rangle$. The CP violation parameters S_ϕ and C_ϕ then become

$$S_\phi = 2\lambda^2\eta\,\mathrm{Re}\frac{a_c(\phi)-a_u(\phi)}{a_c(\phi)}\,, \qquad C_\phi = 2\lambda^2\eta\,\mathrm{Im}\frac{a_c(\phi)-a_u(\phi)}{a_c(\phi)} \tag{2.203}$$

where $a_p(\phi)$, $p = u, c$, is the coefficient of $(iG_F/\sqrt{2})\lambda'_p$ in the $\bar{B}_s \to \phi_L\phi_L$ amplitude (2.77) and (2.116). It can be seen from (2.203) that S_ϕ and C_ϕ depend on the same CKM quantity but on different hadronic parameters. A measurement of C_ϕ is therefore only of limited use in controlling hadronic uncertainties in S_ϕ.

The hadronic parameters in (2.203) depend on the difference between the penguin amplitudes from the charm and the up-quark sector. This difference is calculable in factorization. It has the further advantage that the leading annihilation corrections related to the parameters b_i (2.116) cancel in $a_c(\phi) - a_u(\phi)$. A similar cancellation occurs for the hard-spectator scattering contributions in the NLO approximation. We then find

$$|a_c(\phi) - a_u(\phi)|/\mathrm{GeV}^3 = 0.057^{+0.007}_{-0.007}\,(A_0) \quad {}^{-0.008}_{+0.010}\,(\alpha_2^V) \quad {}^{+0.016}_{-0.015}\,(m_c) \quad {}^{-0.012}_{+0.021}\,(\mu) \tag{2.204}$$

where we show the dominant parametric uncertainties and their origin (in brackets). In contrast to the difference $a_c(\phi) - a_u(\phi)$, the absolute value of $a_c(\phi)$ depends on the annihilation contributions b_i. Rather than aiming for an accurate theoretical prediction, it therefore appears more reliable to extract $|a_c(\phi)|$ from experiment. Neglecting the very small up-quark contribution, we may write

$$B(\bar{B}_s \to \phi_L \phi_L) = \frac{\tau_{B_s} G_F^2 \, |\lambda'_c|^2}{64\pi m_{B_s}} \, |a_c(\phi)|^2 \tag{2.205}$$

A recent measurement for the decay $\bar{B}_s \to \phi\phi$ has been reported by the CDF collaboration, as well as a first measurement of the longitudinal branching fraction

$$B(\bar{B}_s \to \phi\phi) = (2.40 \pm 0.21(\text{stat}) \pm 0.86(\text{syst})) \cdot 10^{-5} \qquad [90] \tag{2.206}$$

$$f_0 = 0.348 \pm 0.041(\text{stat}) \pm 0.021(\text{syst}) \qquad [91] \tag{2.207}$$

This gives

$$|a_c(\phi)| = (0.132 \pm 0.027)\,\text{GeV}^3 \left[\frac{B(\bar{B}_s \to \phi_L \phi_L)}{8.4 \cdot 10^{-6}}\right]^{1/2} \left[\frac{1.47\,\text{ps}}{\tau_{B_s}}\right]^{1/2} \tag{2.208}$$

By the time CP violation in $\bar{B}_s \to \phi\phi$ will be studied at the LHC, the branching fraction will be known with good precision and the number in (2.208) can be easily updated.

The quantity S_ϕ is predicted to be small and positive in the Standard Model. With our default parameter set and $\eta = 0.36$ we obtain

$$S_\phi(\text{default}) \approx 0.01 \tag{2.209}$$

From the discussion above we conclude that the Standard Model upper limit can be written as

$$S_\phi \lesssim 2\lambda^2 \eta \, \frac{|a_c(\phi) - a_u(\phi)|}{|a_c(\phi)|} \lesssim \lambda^2 \eta \left[\frac{B(\bar{B}_s \to \phi_L \phi_L)}{8.4 \cdot 10^{-6}}\right]^{-1/2} \tag{2.210}$$

A similar limit holds for the absolute value of C_ϕ. For $\eta \lesssim 0.4$ we have

$$S_\phi \lesssim 0.02 \qquad |C_\phi| \lesssim 0.02 \tag{2.211}$$

A rescaling for the value of the branching fraction can be done using (2.210). Measurements in excess of these Standard Model limits would constitute evidence for New Physics. The expected sensitivity of LHCb after five years of data taking is $\sigma(S_\phi) \approx 0.05$ [92]. Improvements to values of 0.01 or 0.02 with the anticipated LHCb upgrade appear possible [62]. This should allow us to exploit the full New Physics potential of S_ϕ and to detect non-standard effects in $b \to s$ penguins at the few percent level.

2.5 Comparison with the literature

In this section we comment briefly on the existing literature related to the subject of the present thesis [70, 93].

We re-emphasize that factorization calculations for charmless two-body B decays, in particular $B \to V_L V_L$, are useful for flavour physics analyses such as the determination of CKM parameters. This has also been stressed in [38] and earlier in [30, 37, 43, 69]. The most comprehensive study of $B \to VV$ decays has been presented in [38] with an emphasis on total branching fractions and polarization observables, for instance the longitudinal polarization fractions f_L. More recently these processes were considered in [93] in an extended study, following the analysis of [38]. We do not discuss transverse polarization here but rather concentrate on the decays with longitudinal vector mesons $B \to V_L V_L$. These are calculable in QCD in the heavy-quark limit and thus of special interest for phenomenological applications in flavour physics. We list the detailed results for the $B \to V_L V_L$ amplitudes in explicit terms. The corresponding results of [38] can be reconstructed from similar formulas given for $B \to PV$ decays in [37]. Our main results are consistent with [38]. A minor difference with (the original version of) [38] are the expressions for penguin annihilation $A_3^i \approx 0$ and A_3^f (2.87). The final expressions in [38] give incorrectly, due to a relative sign change, a nonvanishing A_3^i and $A_3^f \approx 0$, even though the basic formulas agree with (2.85). The difference leads to a reduced sensitivity to penguin annihilation in penguin-dominated $B \to V_L V_L$ decays. It does not play a role for $\bar{B} \to \rho^+\rho^-$, because the corresponding decay amplitude is tree dominated and color allowed, so no deviations from modelling of power-suppressed contributions are expected. This point has previously been noted in [93]. There is consensus on the expressions (2.85), (2.87) within the annihilation model used [70, 93] . Another difference with [37, 38] is the treatment of electromagnetic penguin matrix elements contributing to $a^u_{7,9}$, where we have proposed an explicit model for the long-distance contributions to these $\mathcal{O}(\alpha)$ terms.

The article [70] concentrates on transverse polarization and therefore does not report the complete expressions for the amplitudes with longitudinal vector mesons. Where a comparison is possible we agree with the results of [70], except for two minor discrepancies. One is the detailed form of the integrand of the annihilation parameter A_3^f. However, the final result for A_3^f coincides with ours. Another difference is the annihilation part of $B^- \to K^{*-}\phi$, which should read $b_3 + b_3^{\mathrm{EW}}$ instead of $b_3 - b_3^{\mathrm{EW}}/2$. Both issues are inconsequential.

$B \to VV$ decays have been studied within QCD factorization also in [93, 94]. These papers address various VV channels, especially penguin dominated modes such as ϕK^*. Some of them investigate the impact of New Physics scenarios [95–97], [93] extends the analysis to VA and AA modes as well. In comparison, the present paper, while concentrating on $B \to V_L V_L$, gives complete NLO results for all channels, a detailed analysis of uncertainties and applications for precision tests of flavour physics. The authors of [93] employ $m_c(m_b) = 0.91$ GeV, smaller than the value used here and in [38]. We find that the error due to m_c for the longitudinal amplitude is small compared to other experimental input, also for penguin dominated decays.

2.6 Long-distance electromagnetic penguins

For B decays with neutral vector mesons ρ^0, ω or ϕ, the operators $Q^p_{1,2}$ have electromagnetic penguin-type matrix elements where the photon from the $p\bar{p}$ loop is transformed into one of these mesons. The photon virtuality $k^2 = m_V^2$ is then small. While the penguin loop may still be considered short-distance dominated for $p = c$ due to the charm-quark mass, the matrix element becomes sensitive to long-distance hadronic physics for $p = u$. This can be seen from the perturbative result for the case of the charm quark (2.41), which diverges in the limit $m_c \to 0$. The up-quark contribution is thus not strictly calculable. Since this situation arises only in a small electromagnetic correction, it is not a serious problem for practical purposes. In fact, additional dependence on long-distance hadronic physics is to be expected when electromagnetic radiative corrections to hadronic B decays are considered. Still the penguin matrix element under discussion contributes within our approximation scheme of including leading electroweak effects. We shall therefore give an estimate of its size using available information on the long-distance dynamics of the up-quark loop. Apart from obtaining a numerical evaluation of the effect, the long-distance electromagnetic penguin is also interesting for conceptual reasons.

The up-quark loop is closely related to the vacuum polarization function $\Pi(k^2)$, where the UV subtraction is given by the standard renormalization prescription of the weak hamiltonian. We thus write the penguin matrix element, needed at low photon virtuality $k^2 = m_V^2$, as the matrix element evaluated at $k^2 = m_b^2$ plus a remainder proportional to the difference $\Pi(k^2) - \Pi(m_b^2)$. The contribution of the electromagnetic up-quark penguin to the coefficients $a^u_{7,9}$ then takes the form

$$\Delta a^u_{7,9} = \frac{\alpha}{9\pi}(C_1 + N_c C_2)\left[\frac{4}{3}\ln\frac{m_b}{\mu} - \frac{4}{9} - \frac{2\pi}{3}i - \frac{8\pi^2}{N_c}(\Pi(m_V^2) - \Pi(m_b^2))\right] \tag{2.212}$$

The correlator $\Pi(k^2)$ is defined through

$$\Pi_{\mu\nu}(k) = i\int d^4x\, e^{ik\cdot x}\,\langle 0|T j_\mu(x) j_\nu(0)|0\rangle \equiv (k_\mu k_\nu - k^2 g_{\mu\nu})\Pi(k^2) \tag{2.213}$$

where $j_\mu = \bar{u}\gamma_\mu u$. The first terms in the square brackets of (2.212) come from the perturbative evaluation of the matrix element at $k^2 = m_b^2$ and carry the appropriate scale and scheme dependence. The remainder depending on Π may be computed to lowest (one-loop) order, which reproduces the perturbative result for the matrix element. We shall treat $\Pi(k^2) - \Pi(m_b^2)$ as the full hadronic correlator, which includes the nonperturbative hadronic physics relevant at low k^2. This procedure assumes a factorization of the soft hadronic correlator form the remaining parts of the diagram, which is not strictly justified. We adopt this additional assumption to obtain a rough estimate of the long-distance sensitive penguin contributon. A similar method has been proposed and applied in the context of $b \to s(d)e^+e^-$ decays in [98].

The function $\Pi(k^2)$ obeys the dispersion relation

$$\Pi(k^2) = \frac{1}{\pi}\int_0^\infty dt\,\frac{\mathrm{Im}\Pi(t)}{t - k^2 - i\epsilon} \tag{2.214}$$

In this form the dispersion relation needs one subtraction, but the subtraction constant cancels in $\Pi(k^2)-\Pi(m_b^2)$. In principle $\text{Im}\Pi(t)$ could be determined experimentally. Instead, for simplicity, we choose a convenient ansatz that should capture the essential features of the true hadronic quantity $\text{Im}\Pi(t)$. We write $\text{Im}\Pi$ as the sum of a resonance and a continuum contribution

$$\text{Im}\Pi = \text{Im}\Pi_r + \text{Im}\Pi_c \tag{2.215}$$

where

$$\text{Im}\Pi_r(t) = \sum_{r=\rho,\omega} \frac{1}{2} \frac{f_r^2 m_r \Gamma_r}{(t-m_r^2)^2 + m_r^2\Gamma_r^2} \tag{2.216}$$

$$\text{Im}\Pi_c(t) = \frac{t}{4\pi t_c}\Theta(t_c - t) + \frac{1}{4\pi}\Theta(t-t_c), \qquad t_c = 4\pi^2(f_\rho^2 + f_\omega^2), \quad t>0 \tag{2.217}$$

The asymptotic QCD result fixes $\text{Im}\Pi_c$ to $N_c/(12\pi) = 1/(4\pi)$ at large t. Imposing quark-hadron duality for the integral of $\text{Im}\Pi(t)$ up to (at least) $t = t_c$ determines the value of $t_c = 4\pi^2(f_\rho^2 + f_\omega^2) \approx 3.1\,\text{GeV}^2$. The factor 1/2 in (2.216) is an isospin factor coming from the overlap of ρ^0 and ω with the $\bar{u}\gamma_\mu u$ current. Determining Π in (2.212) with the help of (2.214) and (2.215), treating the resonances as narrow and taking the heavy-quark limit $t_c \ll m_b$, we finally obtain (2.42). Concerning the factor in square brackets in (2.42), two limiting cases are worth noting. If $k^2 = m_V^2 \to 0$, we recover an expression similar to (2.41) where the light-quark mass under the logarithm is replaced by the hadronic scale $\sqrt{t_c}$. In the limit $k^2 = m_V^2 \to t_c$ the same terms appear, and in addition the perturbative imaginary part $-2\pi i/3$.

2.7 Coefficients a_i, b_i

In the following Table we quote the central values of the coefficients a_i as defined in (2.26) for two final-state ρ-mesons (2.3). The default value used for the model of power-suppressed hard-spectator contributions is $X_H = \ln \frac{m_B}{\Lambda_h}$ and for the renormalization scale it is $\mu = 4.2$GeV.

a_1	a_2	$a_3 + a_5$	a_4^u	a_4^c
$0.991 + 0.020i$	$0.177 - 0.084i$	$0.002 - 0.001i$	$-0.025 - 0.016i$	$-0.033 - 0.009i$
$(a_7^u + a_9^u)/\alpha$	$(a_7^u - a_9^u)/\alpha$	$(a_7^c + a_9^c)/\alpha$	a_{10}^u/α	a_{10}^c/α
$-1.84 - 0.54i$	$1.15 + 0.02i$	$-1.10 - 0.02i$	$-0.17 + 0.09i$	$-0.17 + 0.09i$

The central values of the coefficients b_i as defined in (2.81) for two final-state ρ-mesons (2.3) are given below. The default value used for the model of power-suppressed annihilation contributions is $X_A = \ln \frac{m_B}{\Lambda_h}$ and for the renormalization scale it is $\mu = 4.2$GeV. Here $r_A = B_{\rho\rho}/A_{\rho\rho}$.

$r_A b_1$	$r_A b_2$	$r_A b_3$	$r_A b_4$	$r_A b_3^{EW}/\alpha$	$r_A b_4^{EW}/\alpha$
0.029	-0.011	0.003	-0.003	-0.035	0.013

Chapter 3

Exclusive Baryonic B_s Decays

In the following we constrain ourselves to the discussion of $B_s \to p\bar{p}$ decays. Due to the flavour structure the decay amplitude is proportional to $\langle 0|\bar{s}\Gamma b|\bar{B}_s\rangle$, i.e. the relevant Feynman diagrams are annihilation topologies. There are three main classes of perturbative contributions that dominantly contribute to this decay channel, as shown in Fig. 3.1. For

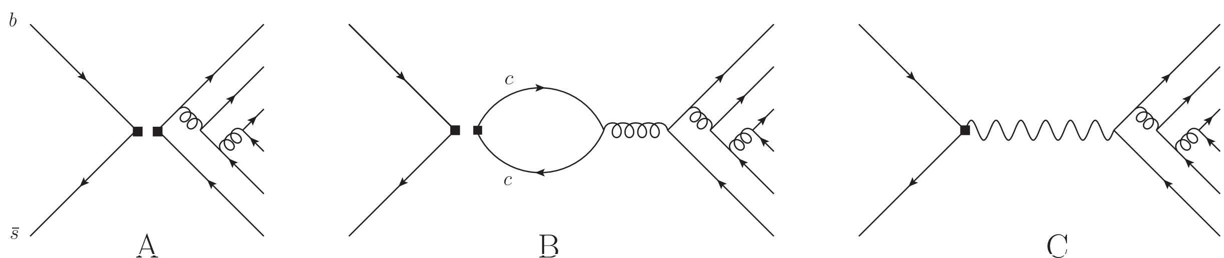

Figure 3.1: Classes of dominant Feynman diagrams for the decay $B_s \to p\bar{p}$. Each class is suppressed by a different mechanism. Class A: Double CKM suppression, $\mathcal{O}(0.01)$. Class B: loop, α_s and colour suppression, $\mathcal{O}(0.01)$. Class C: electromagnetic suppression, $\mathcal{O}(0.01)$. One representative QCD contribution is picked for each class.

the purpose of this thesis we will restrict ourselves to the discussion of class A diagrams.

3.1 Preliminaries

The effective weak Hamiltonian for charmless hadronic B decays, with change in strangeness, $(\Delta S = 1)$, is given by (2.1) and (2.2) and the replacements $\lambda_p \to \lambda'_p = V_{pb}V^*_{ps}$ and $d \to s$. Here we regard only the contributions of the large Wilson coefficients C_1 and C_2:

$$\mathcal{H}_{\text{eff}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda'_p \Big(C_1\, Q_1^p + C_2\, Q_2^p \Big) + \text{h.c.} \tag{3.1}$$

The operators Q_i read correspondingly

$$Q_1^p = (\bar{p}b)_{V-A}(\bar{s}p)_{V-A}\,, \qquad Q_2^p = (\bar{p}_i b_j)_{V-A}(\bar{s}_j p_i)_{V-A} \tag{3.2}$$

Here i, j are colour indices. All diagrammatic expressions are performed in Feynman gauge. Analogously to the discussion of vector mesons in the final state, we need the projector of the B-meson, as given in (2.8), and of the outgoing proton and antiproton, which will be discussed in the following section.

3.1.1 Nucleon distribution amplitudes

In order to classify the distribution amplitudes by twist, the expansion is expressed in terms of the light-like momentum p,

$$p_\mu = P_\mu - z_\mu \frac{M^2}{2p\cdot z}, \tag{3.3}$$

where P is the nucleon momentum and z is a vector with $z^2 = 0$. We choose z and a coordinate system, so that

$$p\cdot z \sim 1, \qquad p^\mu = p_+ n_+^\mu, \qquad z^\mu = z_- n_-^\mu, \tag{3.4}$$

where $p_+ = \sqrt{2}|\mathbf{P}|$ is the large scale in the process, e.g. $z^\mu = n_-^\mu / n_- \cdot p$. The nucleon spinor $N_u(P,\lambda)$ is decomposed into a "large" $N_u^+ \sim \sqrt{p_+}$ and a "small" $N_u^- \sim 1/\sqrt{p_+}$ component:

$$N_u^+(P,\lambda) = \frac{\not{p}\not{z}}{2p\cdot z} N_u(P,\lambda) \tag{3.5}$$

$$N_u^-(P,\lambda) = \frac{\not{z}\not{p}}{2p\cdot z} N_u(P,\lambda) \tag{3.6}$$

The index u of N indicates that this is the spinor of a particle. It has momentum P and helicity λ. In the following sections we suppress the index u. For the spinor of an antiparticle we use N_v.

Using the equation of motion, $\not{P} N_u(P) = M N_u(P)$, we arrive at the following equations for projected spinors:

$$\not{p} N_u^+(P) = 0, \qquad \not{p} N_u^-(P) = M N_u^+(P) \tag{3.7}$$

Analogously for the spinor of an antiparticle:

$$\not{p} N_v^+(P) = 0, \qquad \not{p} N_v^-(P) = -M N_v^+(P) \tag{3.8}$$

In [99] the distribution amplitudes are computed on the light cone, which is useful for determinations with light cone sum rules. The twist expansion up to twist 6 is quoted in the appendix (A.6). Here we compute the decay amplitude with general 4-momenta and only then take the light cone limit, i.e. we set perpendicular momentum components of quarks to 0. Therefore we need to consider the matrix element

$$\langle 0|\varepsilon^{ijk}[0, z_1 - z_3]u^i_\alpha(z_1 - z_3)[0, z_2 - z_3]u^j_\beta(z_2 - z_3)d^k_\gamma(0)|p(P)\rangle = \sum_k \int \mathrm{d}^4p_1\,\mathrm{d}^4p_2\,\mathrm{d}^4p_3\delta^{(4)}(p_1 + p_2 + p_3 - P) \, e^{-i(p_1\cdot(z_1-z_3)+p_2\cdot(z_2-z_3)+P\cdot z_3)}\tilde{F}_k(p_1, p_2, p_3)\Gamma_k(p_1, p_2, p_3), \quad (3.9)$$

where $z_i - z_3$ is not necessarily on the light cone and $\Gamma_k(p_1, p_2, p_3)$ are Dirac structures. Several Wilson lines are inserted, in order to make the matrix element gauge invariant:

$$[x, y] \equiv P\left\{\exp\left[ig_s \int_0^1 ds\,(y - x)^\mu A^a_\mu(sy + (1 - s)x)t^a\right]\right\} \quad (3.10)$$

The Wilson lines are omitted for brevity in the following. The general matrix element can be found by translation

$$\langle 0|\varepsilon^{ijk}u^i_\alpha(z_1)u^j_\beta(z_2)d^k_\gamma(z_3)|p(P)\rangle = e^{-iP\cdot z_3}\langle 0|\varepsilon^{ijk}u^i_\alpha(z_1 - z_3)u^j_\beta(z_2 - z_3)d^k_\gamma(0)|p(P)\rangle \quad (3.11)$$

There are 2 independent spatial Lorentz vectors, $z_1 - z_3$ and $z_2 - z_3$, in opposition to 1, as used in [99] and in [100], where the matrix element $\langle 0|\varepsilon^{ijk}u^i_\alpha(a_1x)u^j_\beta(a_2x)d^k_\gamma(a_3x)|p(P)\rangle$ (a_i are real numbers) is considered off the light cone, i.e. $x^2 \neq 0$. We need the most general decomposition for (3.11), which transforms like a Dirac spinor under Lorentz transformations, is invariant under parity transformations and is symmetric under the simultaneous transformation $z_1 \leftrightarrow z_2$, $\alpha \leftrightarrow \beta$. The matrices with indices α, β have one factor of $C = -i\gamma^2\gamma^0$, in order to preserve Lorentz covariance, as can be seen from (A.3). Parity invariance induces either a γ_5 or a totally antisymmetric tensor to cancel the minus from $\gamma^0C\gamma^0 = -C$. All possible invariant terms are listed in appendix (A.9). The number of terms can be reduced by the equation of motion, $\not{P}N_u(P) = MN_u(P)$, and spinor identities

(A.10). Therefore the distribution amplitude can be written as

$$
\begin{aligned}
&\langle 0|u^i_\alpha(z_{13})u^j_\beta(z_{23})d^k_\gamma(0)|p(P)\rangle = \\
=&\frac{\varepsilon^{ijk}}{4N_c!}\Big[\tilde{S}_1 MC_{\alpha\beta}\left(\gamma_5 N\right)_\gamma + M^2 C_{\alpha\beta}\left(\tilde{S}_2^{\not z_{12}}\gamma_5 N\right)_\gamma + \tilde{S}_3 M^3 C_{\alpha\beta}\left(i\sigma_{z_{13}z_{23}}\gamma_5 N\right)_\gamma \\
&+\tilde{P}_1 M\left(\gamma_5 C\right)_{\alpha\beta}N_\gamma + M^2\left(\gamma_5 C\right)_{\alpha\beta}\left(\tilde{P}_2^{\not z_{12}}N\right)_\gamma + \tilde{P}_3 M^3\left(\gamma_5 C\right)_{\alpha\beta}\left(i\sigma_{z_{13}z_{23}}\gamma_5 N\right)_\gamma \\
&+\tilde{V}_1\left(\not P C\right)_{\alpha\beta}\left(\gamma_5 N\right)_\gamma + M\left(\not P C\right)_{\alpha\beta}\left(\tilde{V}_2^{\not z_{12}}\gamma_5 N\right)_\gamma + \tilde{V}_3 M\left(\gamma_\mu C\right)_{\alpha\beta}\left(\gamma^\mu\gamma_5 N\right)_\gamma \\
&+M^2\left(\tilde{V}_4^{\not z_{12}}C\right)_{\alpha\beta}\left(\gamma_5 N\right)_\gamma + M^2\left(\gamma_\mu C\right)_{\alpha\beta}\left(i\sigma^{\mu\nu}\tilde{V}_{5\nu}^{z_{12}}\gamma_5 N\right)_\gamma \\
&+M^3\left(\tilde{V}_{6a}^{\not z_{12}}C\right)_{\alpha\beta}\left(\not z_{13}\gamma_5 N\right)_\gamma + M^3\left(\tilde{V}_{6b}^{\not z_{12}}C\right)_{\alpha\beta}\left(\not z_{23}\gamma_5 N\right)_\gamma \\
&+\tilde{V}_7 M^2\left(\not P C\right)_{\alpha\beta}\left(i\sigma_{z_{13}z_{23}}\gamma_5 N\right)_\gamma + \tilde{V}_8 M^3\left(\gamma_\mu C\right)_{\alpha\beta}\left(\gamma^\mu i\sigma_{z_{13}z_{23}}\gamma_5 N\right)_\gamma \\
&+M^4\left(\tilde{V}_9^{\not z_{12}}C\right)_{\alpha\beta}\left(i\sigma_{z_{13}z_{23}}\gamma_5 N\right)_\gamma \\
&+\tilde{A}_1\left(\not P\gamma_5 C\right)_{\alpha\beta}N_\gamma + M\left(\not P\gamma_5 C\right)_{\alpha\beta}\left(\tilde{A}_2^{\not z_{12}}N\right)_\gamma + \tilde{A}_3 M\left(\gamma_\mu\gamma_5 C\right)_{\alpha\beta}\left(\gamma^\mu N\right)_\gamma \\
&+M^2\left(\tilde{A}_4^{\not z_{12}}\gamma_5 C\right)_{\alpha\beta}N_\gamma + M^2\left(\gamma_\mu\gamma_5 C\right)_{\alpha\beta}\left(i\sigma^{\mu\nu}\tilde{A}_{5\nu}^{z_{12}}N\right)_\gamma \\
&+M^3\left(\tilde{A}_{6a}^{\not z_{12}}\gamma_5 C\right)_{\alpha\beta}\left(\not z_{13}N\right)_\gamma + M^3\left(\tilde{A}_{6b}^{\not z_{12}}\gamma_5 C\right)_{\alpha\beta}\left(\not z_{23}N\right)_\gamma \\
&+\tilde{A}_7 M^2\left(\not P\gamma_5 C\right)_{\alpha\beta}\left(i\sigma_{z_{13}z_{23}}N\right)_\gamma + \tilde{A}_8 M^3\left(\gamma_\mu\gamma_5 C\right)_{\alpha\beta}\left(\gamma^\mu i\sigma_{z_{13}z_{23}}N\right)_\gamma \\
&+M^4\left(\tilde{A}_9^{\not z_{12}}\gamma_5 C\right)_{\alpha\beta}\left(i\sigma_{z_{13}z_{23}}N\right)_\gamma \\
&+\tilde{T}_1\left(i\sigma_{\mu P}C\right)_{\alpha\beta}\left(\gamma^\mu\gamma_5 N\right)_\gamma + M\left(\tilde{T}_2^{z_{12}\mu}i\sigma_{\mu P}C\right)_{\alpha\beta}\left(\gamma_5 N\right)_\gamma + \tilde{T}_3 M\left(\sigma_{\mu\nu}C\right)_{\alpha\beta}\left(\sigma^{\mu\nu}\gamma_5 N\right)_\gamma \\
&+M\left(\sigma_{\mu P}C\right)_{\alpha\beta}\left(\sigma^{\mu\rho}\tilde{T}_{4\rho}^{z_{12}}\gamma_5 N\right)_\gamma + M^2\left(\tilde{T}_5^{z_{12}\nu}i\sigma_{\mu\nu}C\right)_{\alpha\beta}\left(\gamma^\mu\gamma_5 N\right)_\gamma \\
&+M^2\left(\tilde{T}_{6a}^{z_{12}\mu}i\sigma_{\mu P}C\right)_{\alpha\beta}\left(\not z_{13}\gamma_5 N\right)_\gamma + M^2\left(\tilde{T}_{6b}^{z_{12}\mu}i\sigma_{\mu P}C\right)_{\alpha\beta}\left(\not z_{23}\gamma_5 N\right)_\gamma \\
&+M^2\left(\sigma_{\mu\nu}C\right)_{\alpha\beta}\left(\sigma^{\mu\nu}\tilde{T}_7^{\not z_{12}}\gamma_5 N\right)_\gamma + \tilde{T}_8 M^3\left(\sigma_{\mu\nu}C\right)_{\alpha\beta}\left(\sigma^{\mu\nu}i\sigma_{z_{13}z_{23}}\gamma_5 N\right)_\gamma \\
&+M^3\left(\tilde{T}_{9a}^{z_{12}\nu}\sigma_{\mu\nu}C\right)_{\alpha\beta}\left(\sigma^{\mu z_{13}}\gamma_5 N\right)_\gamma + M^3\left(\tilde{T}_{9b}^{z_{12}\nu}\sigma_{\mu\nu}C\right)_{\alpha\beta}\left(\sigma^{\mu z_{23}}\gamma_5 N\right)_\gamma \\
&+\tilde{T}_{10}M^3\left(i\sigma_{z_{13}z_{23}}C\right)_{\alpha\beta}\left(\gamma_5 N\right)_\gamma + M^4\left(i\sigma_{z_{13}z_{23}}C\right)_{\alpha\beta}\left(\tilde{T}_{11}^{\not z_{12}}\gamma_5 N\right)_\gamma \\
&+\tilde{T}_{12}M^5\left(\sigma_{z_{13}z_{23}}C\right)_{\alpha\beta}\left(\sigma_{z_{13}z_{23}}\gamma_5 N\right)_\gamma\Big], \qquad (3.12)
\end{aligned}
$$

where we use the shortcuts $\tilde{F}_i^{z_{12}\mu} \equiv \frac{1}{2}(\tilde{F}_{i,1}z_{13}^\mu + \tilde{F}_{i,2}z_{23}^\mu)$, $\tilde{F}_i^{\not z_{12}} \equiv \tilde{F}_i^{z_{12}\mu}\gamma_\mu$, $\tilde{F}_i = \tilde{S}, \tilde{P}, \tilde{V}, \tilde{A}, \tilde{T}$ and as notation for contracted Lorentz indices $\sigma_{P\nu} = P^\mu\sigma_{\mu\nu}$. Momentum dependence, spin s and spinor index u of $N_u^s(P)$ are suppressed in the notation. Note that we distinguish the space dependent form of a distribution amplitude from the momentum dependent form only by its arguments:

$$\tilde{F}_i \equiv \tilde{F}_i(z_1, z_2, z_3) =$$
$$= \int \mathrm{d}^4p_1\, \mathrm{d}^4p_2\, \mathrm{d}^4p_3\; \delta^{(4)}(p_1+p_2+p_3-P)e^{-i(p_1\cdot(z_1-z_3)+p_2\cdot(z_2-z_3)+P\cdot z_3)}\tilde{F}_k(p_1,p_2,p_3). \quad (3.13)$$

Terms with a factor of z_{13} or z_{23} can be expressed by

$$z_{i3\nu} \to i\frac{\partial}{\partial p_i^\nu} = i\left(\frac{\bar{p}_\nu}{p\cdot\bar{p}}\frac{\partial}{\partial u_i} + \frac{\partial}{\partial p_{i\perp}^\nu} + \dots\right), \quad (3.14)$$

where we used the definition (3.39) for $i = 1, 2$, defined $\bar{p}$ as $\bar{p}^0 = p^0, \bar{p}^i = -p^i$ and neglected terms of second power in Λ_{QCD}/m_b, similar to the procedure of [30]. The derivatives act on the hard scattering kernel. After applying the derivative and taking the heavy quark limit $p_{i\perp}/m_b \sim \Lambda_{QCD}/m_b \to 0$, the suppressed components can be integrated out. Correspondingly we substitute $\tilde{F}_i \to \bar{F}_i$ (up to a normalization factor) and define

$$\bar{F}_k(a_1z, a_2z, a_3z) = \int \mathcal{D}u\; \bar{F}_k(u_1,u_2,u_3)e^{-ip\cdot z\sum_j u_j a_j} \quad (3.15)$$

with light cone momentum fractions $u_i = n_-\cdot p_i/n_-\cdot p$ and the integration measure

$$\int \mathcal{D}u = \int_0^1 \mathrm{d}u_1 \int_0^1 \mathrm{d}u_2 \int_0^1 \mathrm{d}u_3\; \delta(1-u_1-u_2-u_3). \quad (3.16)$$

In the following we will relate the $\bar{F}_j(u_i)$ to distribution amplitudes $F_j(u_i)$ that are constructed directly from (3.11) on the light cone, as it is done in [99]. In order to do so, we expand (3.12) in powers and take the light cone limit. The following identities can be used to disentangle the power dependence and to identify the higher twist distribution amplitudes:

$$\gamma_\mu\otimes\gamma^\mu = \frac{1}{p\cdot z}\not{p}\otimes\not{z} + \frac{1}{p\cdot z}\not{z}\otimes\not{p} + \gamma_\perp\otimes\gamma^\perp, \qquad \not{\bar{p}}N_u(P) = \frac{2p\cdot\bar{p}}{M}N_u^-(P) \quad (3.17)$$

Up to twist 4 we can make the following identifications with distribution amplitudes F_i that result from an expansion of the left hand of (3.12) on the light cone (A.6):

$$\begin{aligned}
&\bar{V}_1 = V_1, && \bar{V}_1 - p\cdot z\left(a_{13}\bar{V}_{2,1} + a_{23}\bar{V}_{2,2}\right) - 2\bar{V}_3 = V_2, && 2\bar{V}_3 = V_3\\
&\bar{A}_1 = A_1, && \bar{A}_1 + p\cdot z\left(a_{13}\bar{A}_{2,1} + a_{23}\bar{A}_{2,2}\right) + 2\bar{A}_3 = A_2, && 2\bar{A}_3 = A_3\\
&\bar{T}_1 = T_1, && \bar{T}_1 - 4\bar{T}_3 - p\cdot z\left(a_{13}\bar{T}_{4,1} + a_{23}\bar{T}_{4,2}\right) = T_2, && 2\bar{T}_3 = T_7
\end{aligned}$$
$$\bar{T}_1 - \frac{p\cdot z}{2}\left(a_{13}\bar{T}_{2,1} + a_{23}\bar{T}_{2,2}\right) - 2\bar{T}_3 - \frac{p\cdot z}{2}\left(a_{13}\bar{T}_{4,1} + a_{23}\bar{T}_{4,2}\right) = T_3$$
$$\bar{S}_1 = S_1, \qquad \bar{P}_1 = P_1 \quad (3.18)$$

Distribution amplitudes that are accompanied by factors of $a_{i3} = a_i - a_3$ can be rewritten by

$$\begin{aligned}(a_i - a_3)\bar{F} &= \int \mathcal{D}u\ \bar{F}(u_1,u_2,u_3)\left(\frac{i}{p\cdot z}\partial_{u_i}\right)e^{-ip\cdot z\sum_j u_j a_j} \\ &= -\frac{i}{p\cdot z}\int \mathcal{D}u\ \bar{F}'(u_1,u_2,u_3)e^{-ip\cdot z\sum_j u_j a_j}\end{aligned} \tag{3.19}$$

Eventual surface terms are included in the definition of $\bar{F}'(u_i)$. In order to obtain an expression for $\bar{F}(u_i)$ we use the normalization conditions $\langle 0|\varepsilon^{ijk}D_\lambda u^i_\alpha(0)u^j_\beta(0)d^k_\gamma(0)|p(P)\rangle$ and $\langle 0|\varepsilon^{ijk}u^i_\alpha(0)D_\lambda u^j_\beta(0)d^k_\gamma(0)|p(P)\rangle$ for both expansions of $\langle 0|u^i_\alpha(z_{13})u^j_\beta(z_{23})d^k_\gamma(0)|p(P)\rangle$ on the light cone. At leading fock state we have to take the derivative

$$\frac{\partial}{\partial(a_i z^\mu)} = \frac{\bar{z}_\mu}{\bar{z}\cdot z}\frac{\partial}{\partial a_i} \tag{3.20}$$

of the expressions of (3.18) on both sides. In the case of $\bar{V}_{2,1}$ we get

$$\bar{V}_{2,1} = (-i)\int \mathcal{D}u\ u_1\left[V_1(u_i) - V_2(u_i) - 2V_3(u_i)\right]e^{-ip\cdot z\sum_i u_i a_i} \tag{3.21}$$

The distribution amplitude is therefore given by

$$V_{2,1}(u_i) = u_1\left[V_1(u_i) - V_2(u_i) - V_3(u_i)\right], \tag{3.22}$$

where we defined $V_{2,1}(u_i) = i\bar{V}_{2,1}(u_i)$. Analogously the other distribution amplitudes read

$$\begin{aligned}V_{2,2}(u_i) &= u_2\left[V_1(u_i) - V_2(u_i) - V_3(u_i)\right] \\ A_{2,1}(u_i) &= u_1\left[A_2(u_i) - A_1(u_i) - A_3(u_i)\right] \\ A_{2,2}(u_i) &= u_2\left[A_2(u_i) - A_1(u_i) - A_3(u_i)\right] \\ T_{2,1}(u_i) &= u_1\left[T_1(u_i) + T_2(u_i) - 2T_3(u_i)\right] \\ T_{2,2}(u_i) &= u_2\left[T_1(u_i) + T_2(u_i) - 2T_3(u_i)\right] \\ T_{4,1}(u_i) &= u_1\left[T_1(u_i) - T_2(u_i) - 2T_7(u_i)\right] \\ T_{4,2}(u_i) &= u_2\left[T_1(u_i) - T_2(u_i) - 2T_7(u_i)\right]\end{aligned} \tag{3.23}$$

Terms with two factors of z_{i3} in (3.12) are not relevant at twist 4.

In the projector we substitute the perpendicular derivative by

$$\frac{\partial}{\partial p^\nu_{i\perp}} \to \frac{2p_{i\perp\nu}}{p^2_{i\perp}} \tag{3.24}$$

and average over the perpendicular subspace, which produces the same result at first power of $p^\nu_{i\perp}$, [30].

For the leading contributions to $\bar{B}_s \to p\bar{p}$ it is necessary to include power suppressed terms up to first order in M/m_b. The twist expansion reads up to this power

$$
\begin{aligned}
&\langle 0|u^i_\alpha(z_1)u^j_\beta(z_2)d^k_\gamma(z_3)|p(P)\rangle = \\
=&\frac{\varepsilon^{ijk}}{4N_c!}\Big[S_1 M C_{\alpha\beta}\left(\gamma_5 N_u^+\right)_\gamma + P_1 M\left(\gamma_5 C\right)_{\alpha\beta} N^+_{u\gamma} \\
&+ V_1\left(\not{p}C\right)_{\alpha\beta}\left(\gamma_5 N_u^+\right)_\gamma + V_2\left(\not{p}C\right)_{\alpha\beta}\left(\gamma_5 N_u^-\right)_\gamma + V_3\frac{M}{2}\left(\gamma_\perp C\right)_{\alpha\beta}\left(\gamma^\perp\gamma_5 N_u^+\right)_\gamma \\
&+ V_{2,1}\frac{M}{2}\left(\not{p}C\right)_{\alpha\beta}\left(\not{\partial}_{p_{1\perp}}\gamma_5 N_u^+\right)_\gamma + V_{2,2}\frac{M}{2}\left(\not{p}C\right)_{\alpha\beta}\left(\not{\partial}_{p_{2\perp}}\gamma_5 N_u^+\right)_\gamma \\
&+ A_1\left(\not{p}\gamma_5 C\right)_{\alpha\beta} N^+_{u\gamma} + A_2\left(\not{p}\gamma_5 C\right)_{\alpha\beta} N^-_{u\gamma} + A_3\frac{M}{2}\left(\gamma_\perp\gamma_5 C\right)_{\alpha\beta}\left(\gamma^\perp N_u^+\right)_\gamma \\
&+ A_{2,1}\frac{M}{2}\left(\not{p}\gamma_5 C\right)_{\alpha\beta}\left(\not{\partial}_{p_{1\perp}} N_u^+\right)_\gamma + A_{2,2}\frac{M}{2}\left(\not{p}\gamma_5 C\right)_{\alpha\beta}\left(\not{\partial}_{p_{2\perp}} N_u^+\right)_\gamma \\
&+ T_1\left(i\sigma_{\perp p}C\right)_{\alpha\beta}\left(\gamma^\perp\gamma_5 N_u^+\right)_\gamma + T_2\left(i\sigma_{\perp p}C\right)_{\alpha\beta}\left(\gamma^\perp\gamma_5 N_u^-\right)_\gamma \\
&+ T_3\frac{M}{p\cdot z}\left(i\sigma_{pz}C\right)_{\alpha\beta}\left(\gamma_5 N_u^+\right)_\gamma + T_7\frac{M}{2}\left(\sigma_{\perp\perp'}C\right)_{\alpha\beta}\left(\sigma^{\perp\perp'}\gamma_5 N_u^+\right)_\gamma \\
&+ T_{2,1}\frac{M}{2}\partial^\mu_{p_{1\perp}}\left(i\sigma_{\mu p}C\right)_{\alpha\beta}\left(\gamma_5 N_u^+\right)_\gamma + T_{2,2}\frac{M}{2}\partial^\mu_{p_{2\perp}}\left(i\sigma_{\mu p}C\right)_{\alpha\beta}\left(\gamma_5 N_u^+\right)_\gamma \\
&+ T_{4,1}\frac{M}{2}\partial_{p_{1\perp}\mu}\left(\sigma_{\perp p}C\right)_{\alpha\beta}\left(\sigma^{\perp\mu}\gamma_5 N_u^+\right)_\gamma + T_{4,2}\frac{M}{2}\partial_{p_{2\perp}\mu}\left(\sigma_{\perp p}C\right)_{\alpha\beta}\left(\sigma^{\perp\mu}\gamma_5 N_u^+\right)_\gamma\Big], \qquad (3.25)
\end{aligned}
$$

where $z^\mu = \bar{p}^\mu/p\cdot\bar{p}$. $\partial^\mu_{p_{1\perp}}$ and $\partial^\mu_{p_{2\perp}}$ act on the hard scattering kernel. The momentum dependence and spin of the nucleon spinor, N_u, is suppressed in the notation. Further we denote $\sigma_{pz} = \sigma^{\mu\nu}p_\mu z_\nu$, $\gamma_\perp\gamma^\perp = \gamma^\mu g^\perp_{\mu\nu}\gamma^\nu$, $g^\perp_{\mu\nu} = g_{\mu\nu} - (p_\mu z_\nu + z_\mu p_\nu)/p\cdot z$, $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu,\gamma_\nu]$ and use the charge conjugation matrix $C = -i\gamma^2\gamma^0$.

We need the nucleon projector for an outgoing proton and an outgoing antiproton. The corresponding projector for an outgoing proton p with momentum P can be found by

$$
\begin{aligned}
&\langle p(P)|\bar{u}^i_\alpha(z_1)\bar{u}^j_\beta(z_2)\bar{d}^k_\gamma(z_3)|0\rangle = \\
&= -\gamma^0_{\beta\rho}\gamma^0_{\lambda\alpha}\gamma^0_{\delta\gamma}\left[\langle 0|u^i_\lambda(z_1)u^j_\rho(z_2)d^k_\delta(z_3)|p(P)\rangle\right]^\dagger \qquad (3.26)
\end{aligned}
$$

The gauge factors can be inserted above, using $[x,y]^\dagger = [y,x]$. Association of the colour index i to q, $\bar{q}$ denotes transformation under $SU(3)$, $SU(\bar{3})$, respectively. The explicit expression is given below:

$$
\begin{aligned}
&\langle p(P)|\bar{u}^i_\alpha(z_1)\bar{u}^j_\beta(z_2)\bar{d}^k_\gamma(z_3)|0\rangle = \\
=&\frac{\varepsilon^{ijk}}{4N_c!}\Big[S_1^* M C_{\beta\alpha}\left(\bar{N}_u^+\gamma_5\right)_\gamma + P_1^* M\left(C\gamma_5\right)_{\beta\alpha}\bar{N}^+_{u\gamma} \\
&+V_1^*\left(C\not{p}\right)_{\beta\alpha}\left(\bar{N}_u^+\gamma_5\right)_\gamma + V_2^*\left(C\not{p}\right)_{\beta\alpha}\left(\bar{N}_u^-\gamma_5\right)_\gamma + V_3^*\frac{M}{2}\left(C\gamma_\perp\right)_{\beta\alpha}\left(\bar{N}_u^+\gamma_5\gamma^\perp\right)_\gamma \\
&+V_{2,1}^*\frac{M}{2}\left(C\not{p}\right)_{\beta\alpha}\left(\bar{N}_u^+\gamma_5\not{\partial}_{p_{1\perp}}\right)_\gamma + V_{2,2}^*\frac{M}{2}\left(C\not{p}\right)_{\beta\alpha}\left(\bar{N}_u^+\gamma_5\not{\partial}_{p_{2\perp}}\right)_\gamma \\
&+A_1^*\left(C\gamma_5\not{p}\right)_{\beta\alpha}\bar{N}^+_{u\gamma} + A_2^*\left(C\gamma_5\not{p}\right)_{\beta\alpha}\bar{N}^-_{u\gamma} + A_3^*\frac{M}{2}\left(C\gamma_5\gamma_\perp\right)_{\beta\alpha}\left(\bar{N}_u^+\gamma^\perp\right)_\gamma \\
&+A_{2,1}^*\frac{M}{2}\left(C\gamma_5\not{p}\right)_{\beta\alpha}\left(\bar{N}_u^+\not{\partial}_{p_{1\perp}}\right)_\gamma + A_{2,2}^*\frac{M}{2}\left(C\gamma_5\not{p}\right)_{\beta\alpha}\left(\bar{N}_u^+\not{\partial}_{p_{2\perp}}\right)_\gamma \\
&-T_1^*\left(Ci\sigma_{\perp p}\right)_{\beta\alpha}\left(\bar{N}_u^+\gamma_5\gamma^\perp\right)_\gamma - T_2^*\left(Ci\sigma_{\perp p}\right)_{\beta\alpha}\left(\bar{N}_u^-\gamma_5\gamma^\perp\right)_\gamma \\
&-T_3^*\frac{M}{p\cdot z}\left(Ci\sigma_{pz}\right)_{\beta\alpha}\left(\bar{N}_u^+\gamma_5\right)_\gamma + T_7^*\frac{M}{2}\left(C\sigma_{\perp\perp'}\right)_{\beta\alpha}\left(\bar{N}_u^+\gamma_5\sigma^{\perp\perp'}\right)_\gamma \\
&-T_{2,1}^*\frac{M}{2}\partial^\mu_{p_{1\perp}}\left(Ci\sigma_{\mu p}\right)_{\beta\alpha}\left(\bar{N}_u^+\gamma_5\right)_\gamma - T_{2,2}^*\frac{M}{2}\partial^\mu_{p_{2\perp}}\left(Ci\sigma_{\mu p}\right)_{\beta\alpha}\left(\bar{N}_u^+\gamma_5\right)_\gamma \\
&+T_{4,1}^*\frac{M}{2}\partial_{p_{1\perp}\mu}\left(C\sigma_{\perp p}\right)_{\beta\alpha}\left(\bar{N}_u^+\gamma_5\sigma^{\perp\mu}\right)_\gamma + T_{4,2}^*\frac{M}{2}\partial_{p_{2\perp}\mu}\left(C\sigma_{\perp p}\right)_{\beta\alpha}\left(\bar{N}_u^+\gamma_5\sigma^{\perp\mu}\right)_\gamma\Big]
\end{aligned}
\tag{3.27}
$$

The distribution amplitudes, $F_j = V_i, A_i, T_i, S_i, P_i$, are given by

$$
F^*(a_i p\cdot z,\mu) = \int \mathcal{D}x\, e^{ip\cdot z\sum_i a_i x_i} F(x_i,\mu), \tag{3.28}
$$

where the real functions F_i depend on the longitudinal momentum fractions, x_i, $0 < x_i < 1$, $\sum_i x_i = 1$, of the quarks inside the proton and the renormalization scale μ. The distribution amplitudes V_i, T_i are symmetric under exchange of the u-quark momenta (x_1, x_2): $F(x_1,x_2,x_3) = F(x_2,x_1,x_3)$. A_i, S_i, P_i are antisymmetric under $x_1 \leftrightarrow x_2$. The distribution amplitudes F_i relate to the ones used in [101], F_{CS}, by $F_{CS} = f_N F_i$. $f_N = (5.3 \pm 0.5)\cdot 10^{-3}$ is the nucleon decay constant.

Invariance of the QCD Hamiltonian under charge conjugation implies that $\langle\bar{p}(P)|u^i_\alpha(z_1)u^j_\beta(z_2)d^k_\gamma(z_3)|0\rangle$ transforms as a tensor product of spinors:

$$
\langle\bar{p}(P)|u^i_\alpha(z_1)u^j_\beta(z_2)d^k_\gamma(z_3)|0\rangle = -C_{\beta\rho}C_{\lambda\alpha}C_{\gamma\tau}\langle p(P)|\bar{u}^i_\lambda(z_1)\bar{u}^j_\rho(z_2)\bar{d}^k_\tau(z_3)|0\rangle, \tag{3.29}
$$

where we have used the spinor representation as given in the appendix (A.1) and choose phase 1 for the charge conjugation of a proton: $U(C)|p\rangle = |\bar{p}\rangle$. Therefore the explicit expression is given by:

$$
\begin{aligned}
&\langle \bar{p}(P)|u^i_\alpha(z_1)u^j_\beta(z_2)d^k_\gamma(z_3)|0\rangle = \\
&=\frac{\varepsilon^{ijk}}{4N_c!}\Big[S_1^* M C_{\beta\alpha}\left(\gamma_5 N_v^+\right)_\gamma + P_1^* M\left(\gamma_5 C\right)_{\beta\alpha} N^+_{v\gamma} \\
&\quad + V_1^*\left(\not{p}C\right)_{\beta\alpha}\left(\gamma_5 N_v^+\right)_\gamma + V_2^*\left(\not{p}C\right)_{\beta\alpha}\left(\gamma_5 N_v^-\right)_\gamma - V_3^*\frac{M}{2}\left(\gamma_\perp C\right)_{\beta\alpha}\left(\gamma^\perp\gamma_5 N_v^+\right)_\gamma \\
&\quad - V_{2,1}^*\frac{M}{2}\left(\not{p}C\right)_{\beta\alpha}\left(\not{\partial}_{p_{1\perp}}\gamma_5 N_v^+\right)_\gamma - V_{2,2}^*\frac{M}{2}\left(\not{p}C\right)_{\beta\alpha}\left(\not{\partial}_{p_{2\perp}}\gamma_5 N_v^+\right)_\gamma \\
&\quad + A_1^*\left(\gamma_5\not{p}C\right)_{\beta\alpha} N^+_{v\gamma} + A_2^*\left(\gamma_5\not{p}C\right)_{\beta\alpha} N^-_{v\gamma} - A_3^*\frac{M}{2}\left(\gamma_5\gamma_\perp C\right)_{\beta\alpha}\left(\gamma^\perp N_v^+\right)_\gamma \\
&\quad - A_{2,1}^*\frac{M}{2}\left(\gamma_5\not{p}C\right)_{\beta\alpha}\left(\not{\partial}_{p_{1\perp}} N_v^+\right)_\gamma - A_{2,2}^*\frac{M}{2}\left(\gamma_5\not{p}C\right)_{\beta\alpha}\left(\not{\partial}_{p_{2\perp}} N_v^+\right)_\gamma \\
&\quad + T_1^*\left(i\sigma_{\perp p}C\right)_{\beta\alpha}\left(\gamma^\perp\gamma_5 N_v^+\right)_\gamma + T_2^*\left(i\sigma_{\perp p}C\right)_{\beta\alpha}\left(\gamma^\perp\gamma_5 N_v^-\right)_\gamma \\
&\quad - T_3^*\frac{M}{p\cdot z}\left(i\sigma_{pz}C\right)_{\beta\alpha}\left(\gamma_5 N_v^+\right)_\gamma - T_7^*\frac{M}{2}\left(\sigma_{\perp\perp'}C\right)_{\beta\alpha}\left(\sigma^{\perp\perp'}\gamma_5 N_v^+\right)_\gamma \\
&\quad - T_{2,1}^*\frac{M}{2}\partial^\mu_{p_{1\perp}}\left(i\sigma_{\mu p}C\right)_{\beta\alpha}\left(\gamma_5 N_v^+\right)_\gamma - T_{2,2}^*\frac{M}{2}\partial^\mu_{p_{2\perp}}\left(i\sigma_{\mu p}C\right)_{\beta\alpha}\left(\gamma_5 N_v^+\right)_\gamma \\
&\quad -T_{4,1}^*\frac{M}{2}\partial_{p_{1\perp}\mu}\left(\sigma_{\perp p}C\right)_{\beta\alpha}\left(\sigma^{\perp\mu}\gamma_5 N_v^+\right)_\gamma - T_{4,2}^*\frac{M}{2}\partial_{p_{2\perp}\mu}\left(\sigma_{\perp p}C\right)_{\beta\alpha}\left(\sigma^{\perp\mu}\gamma_5 N_v^+\right)_\gamma\Big]
\end{aligned}
\tag{3.30}
$$

For convenience we quote the twist classification of the various distribution amplitudes from [99] in Table 3.1. Due to the twist 4 contributions for perpendicular momentum

Table 3.1: Twist classification of nucleon distribution amplitudes [99].

	twist 3	twist 4	twist 5	twist 6
vector	V_1	V_2 , V_3	V_4 , V_5	V_6
pseudo-vector	A_1	A_2 , A_3	A_4 , A_5	A_6
tensor	T_1	T_2 , T_3 , T_7	T_4 , T_5 , T_8	T_6
scalar		S_1	S_2	
pseudo-scalar		P_1	P_2	

components, the leading twist distribution amplitudes also contribute at subleading power.

3.1.2 Computer assisted calculation

The calculation of the decay amplitude requires suitable computer programs due to the large amount of terms to be calculated. For the proton form factor there are roughly $1.4 \cdot 10^6$

terms to be treated that contain traces with typically 16 Dirac matrices (the number of matrices varies between 12 an 18). There are considerably more terms in the calculation of the decay amplitude of $B_s \to p\bar{p}$, due to the additional B-meson distribution amplitude and a higher number of diagrams. A necessary requirement for a computer program is to cope with the large amount of terms and the limited amount of time and main memory. The existing Mathematica programs seem to be unsuitable for that task, due to the long calculation time for traces over 12 or more Dirac matrices in combination with γ_5 and the large amount of required memory. A program that is designed to cope with large expressions is FORM [102]. It fulfills the requirements about speed and memory usage, but is also much less used than Mathematica and has a smaller number of developers. Although it is used as calculatory tool in a considerable amount of papers, it cannot be excluded that bugs may cause wrong results.[1] Therefore it is appropriate to crosscheck parts of the calculation with another computer program.

For that purpose we developed a Mathematica program (referred to as Trddim) that is not limited by the amount of main memory (just disk memory) and that is considerably faster than other Mathematica programs. This tool allowed to verify the form factor calculation and parts of the the decay amplitude of $B_s \to p\bar{p}$. The program has features that reduce the amount of time needed for the calculation of a single trace as well as the handling of medium to large sized expressions that contain Dirac matrices. In addition it expands products of sums that contain Dirac expressions in small portions and saves the results on the hard drive. From there the Dirac matrices can be treated as required, i.e. user defined functions can be applied to the expressions. The long calculation time and problems of real computing environments made it necessary to implement the feature to resume interrupted calculations. In addition it is possible to use FORM for most computing intensive tasks out of Mathematica. The FORM code for each task is generated by Trddim as needed, the conversion of formats is done by a script in Python. That makes it possible to compare the results within the same framework. The main features of the Mathematica-part of Trddim are parallelized. Although the program aims at reducing the calculation time of large expressions, also single traces are calculated considerably faster, as shown in Table 3.2.

3.2 Proton form factor

The calculation of the decay $\bar{B}_s \to p\bar{p}$ has elements that are similar to the calculation of the timelike proton form factor. In fact the helicity projected form factor $\langle p\bar{p}|\bar{u}\gamma^\mu P_L u|0\rangle$, where $P_L = \frac{1}{2}(1-\gamma_5)$, is part of the calculation. In the following we present the proton form factors in the spacelike region, in order to be easily comparable to other calculations. The timelike formulas can be obtained from the spacelike by crossing symmetry.

[1] There is a report about fixed bugs under `http://www.nikhef.nl/~form/history/history.html`.

Table 3.2: Comparison of different Dirac algebra programs. Trddim is referred to as pure Mathematica implementation ("Trddim Math") and as partial FORM implementation, that is used out of Mathematica ("Trddim FORM"). No parallelization is used in the computation above. The first line represents 1 trace over 12 Dirac matrices and γ_5. Analogous assignments for the other lines. For one entry the computing time could not be evaluated due to insufficient memory, which is indicated by "–".

	FeynCalc [103]	Tracer [104]	Trddim Math	Trddim FORM
1 trace (12,G5) $[s]$	515	113	9	0.4
720 traces (10,G5) $[s]$	10^5	4256	130	0.85
720 traces (12) $[s]$	–	462	251	0.91

3.2.1 Vector form factor

Here we will discuss the electromagnetic, spacelike proton form factor,

$$\langle p(P')|e_u\bar{u}\gamma^\mu u|p(P)\rangle + \langle p(P')|e_d\bar{d}\gamma^\mu d|p(P)\rangle = \\ = \bar{N}(P')\left(\gamma^\mu F_1(Q^2) + i\frac{\sigma^{\mu\lambda}q_\lambda}{2M}F_2(Q^2)\right)N(P), \quad (3.31)$$

including powers of $\mathcal{O}(M/Q)$. At leading power the form factor was computed in [101,105]. Discussions of perturbative calculations that include the baryonic form factor can be found in [106,107]. We use the notations $q = P' - P$, $Q^2 = -q^2$, $Q = \sqrt{Q^2}$ and the nucleon mass M. The connection to the experimentally measured electric and magnetic form factors is given by

$$G_M^p(Q^2) = F_1^p(Q^2) + F_2^p(Q^2), \quad (3.32)$$

$$G_E^p(Q^2) = F_1^p(Q^2) - \frac{Q^2}{4M^2}F_2^p(Q^2) \quad (3.33)$$

They are normalized by $G_E^p(0) = 1$ and $G_M^p(0) = \mu_p$. The experimental data for the magnetic form factor can be described empirically by the dipole formula [108]:

$$G_M^p(Q^2) = \frac{\mu_p}{\left(1 + \frac{Q^2}{\mu_0^2}\right)^2}, \quad (3.34)$$

where $\mu_0^2 = 0.71\ \text{GeV}^2$.

Structure of the calculation

The expansion of the form factor in α_s can be written as

$$F_1(Q^2) \sim A(Q^2) + \left(\frac{\alpha_s(Q^2)}{\pi}\right)\frac{B(Q^2)}{Q^2} + \left(\frac{\alpha_s(Q^2)}{\pi}\right)^2\frac{C}{Q^4} + \dots \quad (3.35)$$

The first two terms in this expansion are power suppressed with respect to the latter one. Estimates [109,110] suggest $A(Q^2) \lesssim 1/Q^6$ and $B(Q^2) \lesssim 1/Q^4$. The helicity changing form factor $F_2(Q^2)$ has a similar expansion as F_1, but is additionally suppressed by M/Q. The first power correction in C is formally leading in power compared to the terms containing A and B, which can be seen by using

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \log \frac{Q^2}{\Lambda^2}} \tag{3.36}$$

and counting $\log Q/M \approx 2.4$ of the order of $\sqrt{Q/M} \approx 2.4$ for Q in the range of $m_{B_s} = 5.4$ Gev.

In the range of moderate energies $Q^2 \sim 4$ Gev^2 there is considerable consensus that the distribution amplitudes deviate strongly from their asymptotic shape. An indication to that is the opposite sign prediction for the neutron form factor compared to the experimental result [105, 111]. The authors in [112] revisit the issue of inadequate fits for the form factors at medium Q^2, if asymptotic distribution amplitudes, $F_i \sim x_1 x_2 x_3$, are used and suggest to use empirical distribution amplitudes, that take into account a 'mean' of perpendicular components. In [113] the authors use a diquark model, which matches the experiment in case of the magnetic form factor rather well. The applicability of the model is limited to situations where the W-boson couples mainly to the d-quark and not to the diquark formed by the u-quarks. Therefore the model is of limited use for our purposes. For a review about the diquark structure of hadrons be referred to [114]. An approach that uses dispersion relations is taken in [115]. For further developments in the medium and high energy range see [100], where the LCSR approach for baryons is developed and other approaches are reviewed. For a recent discussion of the form factor within LCSR see [110]. Here we will discuss the proton form factor in perturbation theory on the light cone at next-to-leading power, which may serve as an estimate of subleading contributions. This correction is also part of the leading contribution for the decay $B_s \to p\bar{p}$.

At $\mathcal{O}(\alpha_s^2)$ the relevant Feynman diagrams with external coupling to a u-quark are given in Fig. 3.2. Non-abelian diagrams do not contribute at this order due to the colour structure.

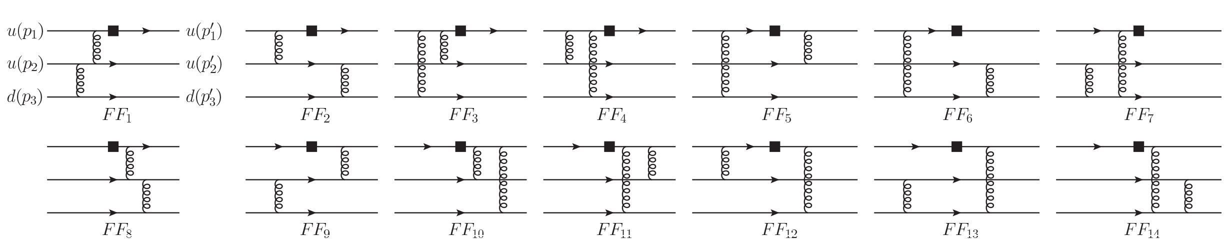

Figure 3.2: Feynman diagrams with photon coupling to u-quark at $\mathcal{O}(\alpha_s^2)$ for the proton form factor. The diagrams with photon coupling to the second, third fermion line can be obtained by exchanging the momentum index 1 with 2, swapping d_3 with u_1, respectively.

The calculation of the diagrams in Fig. 3.2 at $\mathcal{O}(\alpha_s^2)$ has the following structure:

$$\begin{aligned}
&\langle p(P')|\bar{u}(0)\gamma^\mu u(0)|p(P)\rangle|_{\alpha_s^2} = \\
&= -\alpha_s^2 16\pi^2 \int \mathrm{d}^4 z_l \langle p(P')|\bar{u}(0)_\alpha \bar{u}(z_1)_\beta \bar{d}(z_2)_\gamma|0\rangle \\
&\quad (\Gamma_1)_{\alpha\rho}(\Gamma_2)_{\beta\sigma}(\Gamma_3)_{\gamma\tau}\langle 0|u(0)_\rho u(z_3)_\sigma d(z_4)_\tau|p(P)\rangle = \\
&= -\alpha_s^2 16\pi^2 \sum_{m,n} \int \frac{\mathrm{d}^4 P_i'}{(16\pi^3)^2}\frac{\mathrm{d}^4 P_j}{(16\pi^3)^2}\tilde{F}^n(P_i')\tilde{F}^m(P_j) \\
&\quad \mathrm{tr}\left[M^n(p')\Gamma_1(p_i', p_j)M^m(p)\Gamma_2^T(p_i', p_j)\right]\bar{N}_u^{\pm,\mp}(P')\Gamma_3(p_i', p_j)N_u^{\pm,\mp}(P) + \mathcal{O}\left((M/Q)^2\right),
\end{aligned} \tag{3.37}$$

where Γ_k is the Dirac structure of the quark line with momentum index k, $i, j = 1, 2$, $l = 1, 2, 3, 4$, and ^{T} indicates transposition. Colour has been left implicit. In the last step we have used (3.3). If perpendicular momenta do not contribute explicitly at the regarded power in the hard scattering kernel, they are neglected and can be integrated over explicitly. Schematically we can express the matrix element by the distribution amplitudes of (A.6) and (3.27):

$$\int \frac{\mathcal{D}v\, \mathrm{d}^2 k_1^\perp\, \mathrm{d}^2 k_2^\perp}{(16\pi^3)^2}\tilde{F}(v, k_1^\perp, k_2^\perp) = \int \mathcal{D}v F(v_i) \tag{3.38}$$

The momenta of the quarks, which are taken to be on-shell and massless, can be written as a sum of longitudinal and perpendicular components:

$$\begin{aligned}
p_1 &= u_1 p + p_{1\perp} - \frac{p_{1\perp}^2}{2u_1 p_+}n_- \\
p_2 &= u_2 p + p_{2\perp} - \frac{p_{2\perp}^2}{2u_2 p_+}n_- \\
p_3 &= (1 - u_1 - u_2)p - p_{1\perp} - p_{2\perp} - \frac{(p_{1\perp} + p_{2\perp})^2}{2(1 - u_1 - u_2)p_+}n_-,
\end{aligned} \tag{3.39}$$

where $p = p_+ n_+$. Analogously for $p^{'}$. Conventionally we assign the momentum fractions v_i for outgoing protons, where appropriate. The perpendicular momenta are of $\mathcal{O}(\Lambda_{QCD})$. The expansion (3.39) introduces an off-shellness of the proton of $\mathcal{O}(\Lambda_{QCD}^2)$:

$$(p_1 + p_2 + p_3)^2 = -\frac{u_2\bar{u}_2 p_{1\perp}^2 + u_1\bar{u}_1 p_{2\perp}^2 + 2u_1 u_2 p_{1\perp}\cdot p_{2\perp}}{u_1 u_2(1 - u_1 - u_2)}, \tag{3.40}$$

where $\bar{u}_i = 1 - u_i$. The off-shellness is small compared to the proton mass.

The dependence of $F^m(p_j)$ on the perpendicular components is a function of $p_{1\perp}^2$, $p_{1\perp}\cdot p_{2\perp}$ and $p_{2\perp}^2$, due to rotational invariance in the perpendicular subspace. Therefore the integral over momenta eliminates single powers of $p_{i\perp}$, $i = 1, 2$. The procedure is compatible with the way of calculating derivatives of perpendicular momenta (3.24) up to the first power. After the trace in (3.37) is evaluated and the equations of motion (3.7, A.4) are applied the leading and subleading term can be identified. The perpendicular momentum components are suppressed and can be integrated over. We are left with the distribution amplitudes on the light cone $F(x_i)$, as defined on the r.h.s. of (3.28). The final term of (3.37) then simplifies to

$$\begin{aligned}&\langle p(P')|\bar{u}(0)\gamma^\mu u(0)|p(P)\rangle|_{\alpha_s^2} = \\ &= -\alpha_s^2 16\pi^2 \sum_{m,n} \int \mathcal{D}v\,\mathcal{D}u F^n(v_i) F^m(u_j) \\ &\left[f_0^{m,n}(v_i,u_j)\bar{N}_u^+(P')\Gamma_0^{m,n} N_u^+(P) + \frac{M}{Q^2} f_1^{m,n}(v_i,u_j)\bar{N}_u^+(P')\Gamma_1^{m,n} N_u^+(P)\right] + \mathcal{O}\left((M/Q)^2\right),\end{aligned} \tag{3.41}$$

where $\Gamma_j^{m,n}$ can be either γ^μ or $(P'+P)^\mu$.

In order to obtain the full amplitude, we have to add the diagrams 15-28, where the photon couples to the line with momentum index 2. We assign the numbers 15-28 to the diagrams that are obtained from the diagrams in 3.2 by swapping lines with momentum indices 1 and 2. These contributions can be obtained from those of diagrams 1-14, by using symmetry properties of distribution amplitudes at the integrand level. We observe that in the projectors (3.25), (3.27), (3.30) there are only terms symmetric or antisymmetric in momentum indices 1 and 2. In particular the terms with derivatives of perpendicular momentum components can be written as

$$F_{i,1}(u_1,u_2,u_3)\partial^\mu_{p_1,\perp} + F_{i,2}(u_1,u_2,u_3)\partial^\mu_{p_2,\perp} = \hat{F}(u_1,u_2,u_3)\left(u_1\partial^\mu_{p_1,\perp} + u_2\partial^\mu_{p_2,\perp}\right), \tag{3.42}$$

where $\hat{F}(u_1,u_2,u_3)$ is either symmetric or antisymmetric in 1 and 2 and the other factor is always symmetric. This expression acts on the sum of diagrams $D_i(1,2,3)$ with photon couplings to the line with momentum index i, $i=1,2$. For each proton (variables u_i and v_i, respectively) the expression for the diagrams $D_1(1,2,3)$ can be split up in a symmetric, $D_1^s(1,2,3)$, and an antisymmetric, $D_1^a(1,2,3)$ part in (1,2). If (3.42) is symmetric, denoted by $S(1,2,3)$, we obtain

$$\begin{aligned}&S(1,2,3)\left(D_1(1,2,3)+D_2(1,2,3)\right) = \\ &S(1,2,3)\left(D_1(1,2,3)+D_1(2,1,3)\right) = \\ &S(1,2,3)\left(D_1^s(1,2,3)+D_2^s(1,2,3)\right)\end{aligned} \tag{3.43}$$

For antisymmetric (3.42), denoted by $A(1,2,3)$, we obtain

$$\begin{aligned}&A(1,2,3)\left(D_1(1,2,3)+D_2(1,2,3)\right) = \\ &A(1,2,3)\left(D_1(1,2,3)+D_1(2,1,3)\right) = \\ &A(1,2,3)\left(D_1^a(1,2,3)+D_2^a(1,2,3)\right)\end{aligned} \tag{3.44}$$

Several checks of the calculation were taken. The Ward identity is explicitly recovered, which checks most parts of the calculation: the developed computer program, the correctness of diagrams and the projectors. In order to explicitly check the compatibility of projectors for outgoing and incoming protons we calculated the timelike form factors and related them by crossing symmetry. Furthermore the cancellation of perpendicular components at subleading power checks the developed program.

Results for leading power

In the following we provide the twist 3 contributions. At leading power only V_1, A_1 and T_1 contribute. Listed below are the integrands of (3.41) for the upper row of diagrams in Fig. 3.2:

$$\begin{aligned}
FF_1 :&\quad \frac{(V_1^u - A_1^u)(V_1^v - A_1^v) + 4T_1^u T_1^v}{4\bar{u}_1^2 u_3 \bar{v}_1^2 v_3} \\
FF_2 :&\quad 0 \\
FF_3 :&\quad -\frac{T_1^u T_1^v}{u_2 \bar{u}_2 u_3 \bar{v}_1 v_2 v_3} \\
FF_4 :&\quad -\frac{(V_1^u - A_1^u)(V_1^v - A_1^v)}{4u_2 u_3 \bar{u}_3 \bar{v}_1 v_2 v_3} \\
FF_5 :&\quad \frac{(V_1^u + A_1^u)(V_1^v + A_1^v)}{4u_2 \bar{u}_2 u_3 v_2 v_3 \bar{v}_3} \\
FF_6 :&\quad 0 \\
FF_7 :&\quad \frac{(V_1^u - A_1^u)(V_1^v - A_1^v) + 4T_1^u T_1^v}{4\bar{u}_1^2 u_2 \bar{v}_1^2 v_2}
\end{aligned} \tag{3.45}$$

F_i^v, F_i^u $(F = V, A, T)$ abbreviate $F_i(v_1, v_2, v_3)$, $F_i(u_1, u_2, u_3)$, respectively and we use $\bar{x} = 1 - x$. The colour factor

$$\frac{(N_c + 1)^2}{4N_c^2 N_c!} \tag{3.46}$$

and the factor

$$\alpha_s^2 16\pi^2 \frac{1}{Q^4} \bar{N}_u^+(P')\gamma^\mu N_u^+(P) \tag{3.47}$$

have been omitted in FF_{1-7}. The contributions of diagrams 8-14 can be obtained from 1-7 by exchanging $u \leftrightarrow v$. The contributions of diagrams 15-28 with weak vertex at the u-quark line and momentum index 2 can be obtained by exchanging momentum indices $1 \leftrightarrow 2$ in diagrams 1-14.

The explicit expressions for the upper row of diagrams in Fig. 3.2 with weak vertex at the d-quark line are:

$$\begin{aligned}
FF_{29} :&\quad \frac{V_1^u V_1^v + A_1^u A_1^v}{2u_2 \bar{u}_3^2 v_2 \bar{v}_3^2} \\
FF_{30} :&\quad 0 \\
FF_{31} :&\quad -\frac{(V_1^u - A_1^u)(V_1^v - A_1^v)}{4u_1 \bar{u}_1 u_2 v_1 v_2 \bar{v}_3}
\end{aligned}$$

$$
\begin{aligned}
FF_{32}: &\quad -\frac{\left(V_1^u + A_1^u\right)\left(V_1^v + A_1^v\right)}{4u_1 u_2 \bar{u}_2 v_1 v_2 \bar{v}_3} \\
FF_{33}: &\quad \frac{T_1^u T_1^v}{u_1 \bar{u}_1 u_2 v_1 v_2 \bar{v}_2} \\
FF_{34}: &\quad 0 \\
FF_{35}: &\quad \frac{V_1^u V_1^v + A_1^u A_1^v}{2u_1 \bar{u}_3^2 v_1 \bar{v}_3^2}
\end{aligned}
\tag{3.48}
$$

The colour factor (3.46) and the factor (3.47) have been omitted in FF_{29-35}. Diagrams 36-42 are given by exchanging $u \leftrightarrow v$ in 29-35. The expressions of (3.45) and (3.48) agree with those given in [101,107], except for the expression of diagram 10 in their enumeration, which lacks an exchange of momentum indices $1 \leftrightarrow 2$ and has a wrong position of a square. We agree with the subset of expressions that is given in [116]. This includes the expression that disagrees with the two former papers. In the literature there are calculations that differ by a global factor of 2, e.g. the results of [101, 117] are a factor of 2 higher than those of [118, 119]. Our result is higher by a factor of 2 than [101, 117]. The discrepancy in the overall factor with and among the other papers is unresolved [118–120]. Recently a calculation based on (A.6) was done [121] that claims agreement with [118, 119]. Our discrepancy with [121] originates partly from their normalization of the proton colour amplitude to 1, which is incompatible to (A.6). This corresponds to a change of $1/N_c!$ in (3.25) to $1/\sqrt{N_c!}$. The other part appears to be a missing symmetry factor of 24 for the Wick contractions.

The conformal expansion for baryons [52, 122] provides a systematic framework to expand distribution amplitudes, as described in the meson case (2.13). The explicit terms are given in [99] and provided for reference in appendix A.7. After integrating over momentum fractions the result for the two terms of (3.31) reads

$$
\begin{aligned}
&\langle p(P')|\bar{u}\gamma^\mu u|p(P)\rangle = \\
&= \alpha_s^2 16\pi^2 \frac{(N_c+1)^2}{4N_c^2 N_c!} \frac{1}{Q^4} \bar{N}^+(P')\gamma^\mu N^+(P) \\
&\quad \frac{100}{9}\left[81(\phi_3^0)^2 + 54\phi_3^0(\phi_3^- + \phi_3^+) + 43(\phi_3^-)^2 - 36\phi_3^-\phi_3^+ + 63(\phi_3^+)^2\right. \\
&\qquad \left. + \mathcal{O}(M/Q)\right] + \mathcal{O}(\alpha_s^3),
\end{aligned}
\tag{3.49}
$$

$$
\begin{aligned}
&\langle p(P')|\bar{d}\gamma^\mu d|p(P)\rangle = \\
&= \alpha_s^2 16\pi^2 \frac{(N_c+1)^2}{4N_c^2 N_c!} \frac{1}{Q^4} \bar{N}^+(P')\gamma^\mu N^+(P) \\
&\quad \frac{50}{9}\left[324(\phi_3^0)^2 - 36\phi_3^0(4\phi_3^- + 3\phi_3^+) + 13(\phi_3^-)^2 + 18\phi_3^-\phi_3^+ + 225(\phi_3^+)^2\right. \\
&\qquad \left. + \mathcal{O}(M/Q)\right] + \mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{3.50}
$$

Therefore the electromagnetic form factor at leading power is given by

$$\sum_{q=u,d} \langle p(P')|e_q \bar{q}\gamma^\mu q|p(P)\rangle = \tag{3.51}$$

$$\begin{aligned}
=&\alpha_s^2 16\pi^2 \frac{(N_c+1)^2}{4N_c^2 N_c!}\frac{1}{Q^4}\bar{N}^+(P')\gamma^\mu N^+(P) \\
&\frac{50}{9}\left[12\phi_3^0(10\phi_3^- + 9\phi_3^+) + 53(\phi_3^-)^2 - 54\phi_3^-\phi_3^+ + 9(\phi_3^+)^2\right. \\
&\left. +\mathcal{O}(M/Q)\right] + \mathcal{O}(\alpha_s^3)
\end{aligned} \tag{3.52}$$

The next-to-leading power diverges, when integrating over the momentum fractions of the quarks. The full result is given in the appendix (A.11) and to the best of our knowledge not available in the literature. As required by Ward identity, terms that involve $(P'^\mu - P^\mu)$ cancel, when we choose a unique basis in terms of distribution amplitudes (e.g. a basis that is not symmetric under $u_i \leftrightarrow v_i$, see (3.55)).

Regularization procedure

As regularization procedure for divergent expressions we cut off the integration boundaries (3.16) as follows:

$$\int_0^1 \mathrm{d}u_1 \int_0^1 \mathrm{d}u_2 \int_0^1 \mathrm{d}u_3\ \delta(1-u_1-u_2-u_3) \to \int_\varepsilon^1 \mathrm{d}u_1 \int_\varepsilon^1 \mathrm{d}u_2 \int_\varepsilon^1 \mathrm{d}u_3\ \delta(1-u_1-u_2-u_3), \tag{3.53}$$

where $0 < \varepsilon < 1/3$. For finite terms we let $\varepsilon \to 0$, for divergent terms the ε-dependence is kept.

There are divergences in the integrand of (A.11) for $u_2 \to 0$, $u_2 \to 1-u_1$, $u_2 \to 1$, $u_1 \to 0$, $u_1 \to 1$, $v_2 \to 0$, $v_2 \to 1-v_1$, $v_1 \to 0$ and $v_1 \to 1$. They arise when 1 or 2 (anti-) quarks become soft. In order to disentangle the divergent part of the integral from the finite contribution, we write the integrand of (A.11) as a sum of 2 Laurent series around the divergent points plus a holomorphic function. This has to be done for the momentum fraction sets for both (anti-) protons, i.e. (u_1,u_2) and (v_1,v_2). The series expansion for an integrand $f(v_1, v_2)$ has the following general form:

$$
\begin{aligned}
f(v_1, v_2) &= \\
&= \sum_{i,j}^{-1} a^0_{ij} v_2^i v_1^j + \sum_{i,j}^{-1} a^1_{ij} v_2^i (1-v_1)^j + \sum_{i}^{-1} h^a_i(v_1) v_2^i \\
&\quad + \sum_{i,j}^{-1} b^0_{ij} (1-v_1-v_2)^i v_1^j + \sum_{i,j}^{-1} b^1_{ij} (1-v_1-v_2)^i (1-v_1)^j + \sum_{i}^{-1} h^b_i(v_1)(1-v_1-v_2)^i \\
&\quad + \sum_{i,j}^{-1} c^0_{ij} (1-v_2)^i v_1^j + \sum_{i}^{-1} h^c_i(v_1)(1-v_2)^i \\
&\quad + \sum_{i,j}^{-1} d^0_{ij} (v_1+v_2)^i v_1^j + \sum_{i}^{-1} h^d_i(v_1)(v_1+v_2)^i \\
&\quad + \sum_{i}^{-1} a^{h_2}_i(v_2) v_1^i + \sum_{i}^{-1} b^{h_2}_i(v_2)(1-v_1)^i \\
&\quad + h_{12}(v_1, v_2),
\end{aligned}
\tag{3.54}
$$

where i and j are negative integers and $h^a_i(v_1)$, $h^b_i(v_1)$, $h^c_i(v_1)$, $h^d_i(v_1)$, $a^{h_2}_i(v_2)$, $b^{h_2}_i(v_2)$, $h_{12}(v_1, v_2)$ are holomorphic on $[0,1]$ and $[0,1]^2$ for h_{12}. An analogous expansion applies for u_1 and u_2. The terms in this expansion result from an expansion around the divergences of v_2 ("rows"), whose v_1-dependent coefficients are expanded around the divergences in v_1 ("columns"). The finite integrals of the expansion, with exception of $h_{12}(v_1, v_2)$, can be solved analytically, since they are simpler due to the applied expansion. In order to solve the integral over $h_{12}(v_1, v_2)$ we use that it is finite and expand it in ε with the help of the Leibniz rule. The order in ε is determined by the maximum degree of divergence that is produced by the integral over (u_1,u_2). Higher orders in ε than the degree of the divergence do not contribute, since the corresponding terms make the whole integral Taylor expandable in ε and therefore vanish explicitly. For the integration over (u_1,u_2) an analogous procedure holds. It was checked explicitly that the procedure does not depend on the order of integration.

There are 6 cases of 1 or 2 quarks becoming soft, which are easily identified with divergent terms in the expansion. Table 3.3 provides an overview over the correspondence between physical circumstances and terms in (3.54).

Results for subleading power

In order to make the divergent behavior of (A.11) more explicit, we provide the power of the most singular divergence in the integrand for the coefficients in the conformal expansion (A.7, A.8) in Table 3.4. The divergences are extracted for each u_i and v_i, separately, i.e. cancellations of divergent behavior through the other variables are not taken into account.

Table 3.3: Overview of physical interpretation (soft, hard quark) to divergent behavior according to (3.54). The limits are given in the form of brackets that correspond to (v_1,v_2,v_3) and indicate soft (0,1) and hard "–" momentum fractions.

term	$a^0_{ij} v_2^i v_1^j$	$a^1_{ij} v_2^i (1-v_1)^j$	$h^a_i(v_1) v_2^i$
interpretation	(0,0,1)	(1,0,0)	(–,0,–)
term	$b^0_{ij}(1-v_1-v_2)^i v_1^j$	$b^1_{ij}(1-v_1-v_2)^i(1-v_1)^j$	$h^b_i(v_1)(1-v_1-v_2)^i$
interpretation	(0,1,0)	(1,0,0)	(–,–,0)
term	$c^0_{ij}(1-v_2)^i v_1^j$		$h^c_i(v_1)(1-v_2)^i$
interpretation	(0,1,0)		(0,1,0)
term	$d^0_{ij}(v_1+v_2)^i v_1^j$		$h^d_i(v_1)(v_1+v_2)^i$
interpretation	(0,0,1)		(0,0,1)
term	$a^{h_2}_i(v_2) v_1^i$	$b^{h_2}_i(v_2)(1-v_1)^i$	
interpretation	(0,–,–)	(1,0,0)	

As a basis for the integrand we choose

$$\begin{aligned} &V_1(u_i)V_1(v_i), \quad V_1(u_i)V_2(v_i), \quad V_1(u_i)V_3(v_i), \\ &A_1(u_i)A_1(v_i), \quad A_1(u_i)A_2(v_i), \quad A_1(u_i)A_3(v_i), \\ &T_1(u_i)T_1(v_i), \quad T_1(u_i)T_3(v_i), \quad T_1(u_i)T_7(v_i) \end{aligned} \tag{3.55}$$

There are linear and triple logarithmic divergences in (A.11). The integration over twist 3 distribution amplitudes provides only logarithmic contributions. All terms that are proportional to $(1-u_1)^{-2}$ are multiplied by one power of u_2 and thus do not produce a linear divergence. Linear divergences stem from the integration over twist 4 distribution amplitudes. Specifically, the linear divergences in the terms that multiply $\phi_3^0\phi_4^0$ and $\phi_3^0\phi_4^+$ stem from terms proportional to v_3^{-2}. These divergences are produced from derivatives of perpendicular components. The divergence pattern associated with the distribution amplitudes is shown in Table 3.5. Generally twist 3 distribution amplitudes multiply terms of similar degree as twist 4 distribution amplitudes. In contrast to twist 4 distribution amplitudes there is a better suppression of these divergences, due to the asymptotic wave function. This leads to less singular terms in almost all monomials.

Table 3.4: Singular behavior of the next-to-leading power part of the u-form factor (A.11). Displayed are the most singular powers in the expansion (3.54) of the conformal expansion. The table entries denote the negative exponents (i, j) or i, where appropriate. The shown monomials have the lowest total degree $-i-j$. There are no monomials with lower single degrees $-i$ or $-j$. The momentum fractions u_i are associated with twist 3 distribution amplitudes, shown in the upper half of the table. Finite terms are indicated by "—". Expansion coefficients with no singular terms are left out.

	$(\phi_3^0)^2$	$\phi_3^0\phi_4^0$	$\phi_3^0\psi_4^0$	$\phi_3^0\phi_4^-$	$\phi_3^0\phi_4^+$	$\phi_3^0\psi_4^-$	$\phi_3^0\psi_4^+$
Expansion in momentum components u_i^j:							
u_2^{-i}	1	1	1	1	1	1	1
u_3^{-i}	1	1	1	—	1	1	1
$\bar{u}_2^{-i}$	2	2	2	2	2	2	2
u_1^{-i}	1	1	1	1	1	1	1
$\bar{u}_1^{-i}$	2	2	2	2	2	2	2
Expansion in momentum components v_i^j:							
$v_2^{-i}\bar{v}_1^{-j}$	—	$(1,1)$	$(1,1)$	$(1,1)$	$(1,1)$	$(1,1)$	$(1,1)$
v_2^{-i}	1	1	1	1	1	1	1
$v_3^{-i}v_1^{-j}$	—	$(1,1)$	$(1,1)$	—	$(1,1)$	$(1,1)$	$(1,1)$
$v_3^{-i}\bar{v}_1^{-j}$	—	$(1,1)$	$(1,1)$	—	$(1,1)$	$(1,1)$	$(1,1)$
v_3^{-i}	1	2	—	—	2	1	1
$\bar{v}_2^{-i}v_1^{-j}$	—	$(1,1)$	$(1,1)$	$(1,1)$	$(1,1)$	$(1,1)$	$(1,1)$
$\bar{v}_2^{-i}$	2	2	2	2	2	2	2
v_1^{-i}	1	1	1	1	1	1	1
$\bar{v}_1^{-i}$	2	2	2	2	2	2	2

Table 3.5: Displayed are the most singular powers in the expansion (3.54) for the functions that multiply the indicated distribution amplitudes of the power suppressed u-form factor (A.11). The table entries denote the negative exponents (i, j) or i, where appropriate. Finite terms are indicated by "—". Expansion coefficients with no singular terms are left out. The shown monomials have the lowest total degree $-i-j$. There are no monomials with lower single degrees $-i$ or $-j$. We used the short notation $F_i^x \equiv F_i(x_1, x_2, x_3)$.

	$V_1^u V_1^v$	$V_1^u V_2^v$	$V_1^u V_3^v$	$A_1^u A_1^v$	$A_1^u A_2^v$	$A_1^u A_3^v$	$T_1^u T_1^v$	$T_1^u T_3^v$	$T_1^u T_7^v$
Expansion in momentum components u_i^j:									
$u_2^{-i} u_1^{-j}$	$(1,1)$	$(1,1)$	$(1,1)$	—	—	—	$(1,1)$	$(1,1)$	$(1,1)$
$u_2^{-i} \bar{u}_1^{-j}$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$
$u_3^{-i} u_1^{-j}$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(1,2)$	$(1,2)$	$(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$
$u_3^{-i} \bar{u}_1^{-j}$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(1,2)$	$(1,2)$	$(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$
$\bar{u}_2^{-i} u_1^{-j}$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$
$\bar{u}_3^{-i} u_1^{-j}$	—	—	—	—	—	—	$(1,1)$	$(1,1)$	$(1,1)$
Expansion in momentum components v_i^j:									
$v_2^{-i} v_1^{-j}$	$(1,1)$	—	—	—	—	—	$(1,1)$	—	—
$v_2^{-i} \bar{v}_1^{-j}$	$(2,1)$, $(1,2)$	$(1,2)$	$(1,2)$	$(2,1)$, $(1,2)$	$(1,2)$	$(1,2)$	$(2,1)$, $(1,2)$	$(1,2)$	$(1,2)$
$v_3^{-i} v_1^{-j}$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(1,2)$	$(1,2)$	$(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$
$v_3^{-i} \bar{v}_1^{-j}$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(1,2)$	$(1,2)$	$(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$
$\bar{v}_2^{-i} v_1^{-j}$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$	$(2,1)$, $(1,2)$
$\bar{v}_3^{-i} v_1^{-j}$	—	—	—	—	—	—	$(1,1)$	—	—

For the subleading power part of the u-form factor (A.11) we obtain

$$
\begin{aligned}
&\langle p(P')|\bar{u}\gamma^\mu u|p(P)\rangle|_{\text{NLP}} = \\
&= -\alpha_s^2 320\pi^2 \frac{(N_c+1)^2}{4N_c^2 N_c!}\frac{M}{Q^6}\left(P'^\mu + P^\mu\right)\bar{N}^+(P')N^+(P) \\
&\quad \Big[90\left(\phi_3^0\right)^2(5+2\log\varepsilon) \\
&\quad - 12\phi_3^0\phi_4^0 \\
&\qquad \left(\frac{6}{\varepsilon} + 18\log^3\varepsilon + 162\log^2\varepsilon + \log\varepsilon\left(36\log 2 - 3\pi^2 + 543\right) + 102\log 2 + 540 - 7\pi^2\right) \\
&\quad - 3\phi_3^0\psi_4^0 \\
&\qquad \left(48\log^3\varepsilon + 402\log^2\varepsilon + \log\varepsilon\left(72\log 2 - 8\pi^2 + 1200\right) + 180\log 2 - 23\pi^2 + 1266\right) \\
&\quad - 12\phi_3^0\phi_4^- \left(12\log^3\varepsilon + 108\log^2\varepsilon + \log\varepsilon\left(24\log 2 - 2\pi^2 + 356\right) + 60\log 2 - 5\pi^2 + 406\right) \\
&\quad - 6\phi_3^0\phi_4^+ \\
&\qquad \left(\frac{12}{\varepsilon} + 36\log^3\varepsilon + 324\log^2\varepsilon + \log\varepsilon\left(72\log 2 - 6\pi^2 + 1126\right) + 204\log 2 - 14\pi^2 + 1145\right) \\
&\quad + \phi_3^0\psi_4^- \\
&\qquad \left(144\log^3\varepsilon + 1290\log^2\varepsilon + \log\varepsilon\left(360\log 2 - 24\pi^2 + 4352\right) + 900\log 2 - 51\pi^2 + 5068\right) \\
&\quad - 3\phi_3^0\psi_4^+ \\
&\qquad \left(48\log^3\varepsilon + 402\log^2\varepsilon + \log\varepsilon\left(72\log 2 - 8\pi^2 + 1180\right) + 180\log 2 - 23\pi^2 + 1256\right)\Big]
\end{aligned}
\tag{3.56}
$$

The result for the d form factor at next-to-leading power is given in appendix (A.12). This leads to the divergent structures for the coefficients in the conformal expansion of the distribution amplitudes as shown in Table 3.6. There are 16 terms in a minimal basis in terms of distribution amplitudes. Therefore we do not display the divergent structures for the distribution amplitudes here.

The regulated contributions to the next-to-leading power part of the d-form factor are:

$$
\begin{aligned}
&\langle p(P')|\bar{d}\gamma^\mu d|p(P)\rangle|_{\text{NLP}} = \\
&=\alpha_s^2 1280\pi^2 \frac{(N_c+1)^2}{4N_c^2 N_c!}\frac{M}{Q^6}\left(P'^\mu + P^\mu\right)\bar{N}^+(P')N^+(P) \\
&\quad \Big[\left(\phi_3^0\right)^2 45\,(1-\log\varepsilon) \\
&\quad + \phi_3^0\phi_4^0 9\left(41 + 29\log\varepsilon + 4\log^2\varepsilon\right) \\
&\quad + \phi_3^0\psi_4^0 45\left(15 + 11\log\varepsilon + 2\log^2\varepsilon\right) \\
&\quad - \phi_3^0\phi_4^- 2\left(119 + 97\log\varepsilon + 18\log^2\varepsilon\right) \\
&\quad + \phi_3^0\phi_4^+ 6\left(79 + 41\log\varepsilon + 6\log^2\varepsilon\right) \\
&\quad + \phi_3^0\psi_4^- \left(113 + 103\log\varepsilon + 18\log^2\varepsilon\right)
\end{aligned}
$$

$$+\phi_3^0\psi_4^+ 15\left(59 + 37\log\varepsilon + 6\log^2\varepsilon\right)\Big] \tag{3.57}$$

Table 3.6: Singular behavior of the next-to-leading power part of the d-form factor (A.12). Displayed are the most singular powers in the expansion (3.54) of the conformal expansion. The table entries denote the negative exponents (i, j) or i, where appropriate. The shown monomials have the lowest total degree $-i-j$. There are no monomials with lower single degrees $-i$ or $-j$. The momentum fractions u_i are associated with twist 3 distribution amplitudes, shown in the upper half of the table. Finite terms are indicated by "—". Expansion coefficients with no singular terms are left out.

	$(\phi_3^0)^2$	$\phi_3^0\phi_4^0$	$\phi_3^0\psi_4^0$	$\phi_3^0\phi_4^-$	$\phi_3^0\phi_4^+$	$\phi_3^0\psi_4^-$	$\phi_3^0\psi_4^+$
Expansion in momentum components u_i^j:							
u_2^{-i}	1	1	1	1	1	1	1
$\bar{u}_2^{-i}$	1	1	1	1	1	1	1
$\bar{u}_3^{-i}$	1	1	1	1	1	1	1
u_1^{-i}	1	1	1	1	1	1	1
$\bar{u}_1^{-i}$	1	1	1	1	1	1	1
Expansion in momentum components v_i^j:							
v_2^{-i}	—	1	1	1	1	1	1
$\bar{v}_2^{-i}$	1	1	1	1	1	1	1
$\bar{v}_3^{-i}$	—	1	1	1	1	1	1
v_1^{-i}	—	1	1	1	1	1	1
$\bar{v}_1^{-i}$	1	1	1	1	1	1	1

The electromagnetic form factor at subleading power in Λ_{QCD}/m_b and leading order in α_s therefore reads:

$$\sum_{q=u,d} \langle p(P')|e_q\bar{q}\gamma^\mu q|p(P)\rangle|_{\text{NLP}} =$$

$$= -\alpha_s^2\frac{640}{3}\pi^2\frac{(N_c+1)^2}{4N_c^2N_c!}\frac{M}{Q^6}\left(P'^\mu + P^\mu\right)\bar{N}^+(P')N^+(P)$$

$$\Big[\left(\phi_3^0\right)^2 90\left(6+\log\varepsilon\right)$$

$$+\phi_3^0\phi_4^0 6\left(-\frac{12}{\varepsilon} - 957 + 14\pi^2 - 204\log 2 - 999\log\varepsilon + 6\pi^2\log\varepsilon - 72\log 2\log\varepsilon\right.$$

$$\left. - 312\log^2\varepsilon - 36\log^3\varepsilon\right)$$

$$
\begin{aligned}
&+ \phi_3^0\psi_4^0 3\left(-816 + 23\pi^2 - 180\log 2 - 870\log\varepsilon + 8\pi^2\log\varepsilon - 72\log 2\log\varepsilon\right.\\
&\qquad\left. - 342\log^2\varepsilon - 48\log^3\varepsilon\right)\\
&+ \phi_3^0\phi_4^- 4\left(-1337 + 15\pi^2 - 180\log 2 - 1165\log\varepsilon + 6\pi^2\log\varepsilon - 72\log 2\log\varepsilon\right.\\
&\qquad\left. - 342\log^2\varepsilon - 36\log^3\varepsilon\right)\\
&+ \phi_3^0\phi_4^+ 6\left(-\frac{12}{\varepsilon} - 987 + 14\pi^2 - 204\log 2 - 1044\log\varepsilon + 6\pi^2\log\varepsilon - 72\log 2\log\varepsilon\right.\\
&\qquad\left. - 312\log^2\varepsilon - 36\log^3\varepsilon\right)\\
&+ \phi_3^0\psi_4^-\left(5294 - 51\pi^2 + 900\log 2 + 4558\log\varepsilon - 24\pi^2\log\varepsilon + 360\log 2\log\varepsilon\right.\\
&\qquad\left. + 1326\log^2\varepsilon + 144\log^3\varepsilon\right)\\
&+ \phi_3^0\psi_4^+ 3\left(-666 + 23\pi^2 - 180\log 2 - 810\log\varepsilon + 8\pi^2\log\varepsilon - 72\log 2\log\varepsilon\right.\\
&\qquad\left. - 342\log^2\varepsilon - 48\log^3\varepsilon\right)\Big]
\end{aligned}
\tag{3.58}
$$

3.2.2 Axial vector form factor

The spacelike axial vector form factor has 3 independent coefficients:

$$
\begin{aligned}
&\langle p(P')|\bar{f}\gamma^\mu\gamma_5 f|p(P)\rangle = \\
&=\bar{N}(P')\left[\gamma^\mu\gamma_5 F_1^{f5}(Q^2) + i\frac{\sigma^{\mu\lambda}q_\lambda}{2M}\gamma_5 F_2^{f5}(Q^2) + q^\mu\gamma_5 F_3^{f5}(Q^2)\right] N(P),
\end{aligned}
\tag{3.59}
$$

where $q^\mu = P'^\mu - P^\mu$ and f represents the flavours u, d.

Results for leading power

At leading power the diagrams in Fig. 3.2 have the following integrands of $\int \mathcal{D}v\mathcal{D}u$:

$$
\begin{aligned}
FF_1: &\quad \frac{(V_1^u - A_1^u)(V_1^v - A_1^v) + 4T_1^uT_1^v}{4\bar{u}_1^2 u_3\bar{v}_1^2 v_3}\\
FF_2: &\quad 0\\
FF_3: &\quad -\frac{T_1^uT_1^v}{u_2\bar{u}_2 u_3\bar{v}_1 v_2 v_3}\\
FF_4: &\quad -\frac{(V_1^u - A_1^u)(V_1^v - A_1^v)}{4u_2u_3\bar{u}_3\bar{v}_1 v_2 v_3}\\
FF_5: &\quad -\frac{(V_1^u + A_1^u)(V_1^v + A_1^v)}{4u_2\bar{u}_2 u_3 v_2 v_3\bar{v}_3}\\
FF_6: &\quad 0\\
FF_7: &\quad \frac{(V_1^u - A_1^u)(V_1^v - A_1^v) + 4T_1^uT_1^v}{4\bar{u}_1^2 u_2\bar{v}_1^2 v_2}
\end{aligned}
\tag{3.60}
$$

We ommitted the factor

$$\alpha_s^2 16\pi^2 \frac{1}{Q^4} \bar{N}_u^+(P')\gamma^\mu\gamma_5 N_u^+(P) \tag{3.61}$$

and the colour factor (3.46). Apart from the ommitted factors the same comments apply as below (3.45). The explicit expressions for the upper row of diagrams in Fig. 3.2 with weak vertex at the d-quark line are:

$$\begin{aligned}
FF_{29}: &\quad \frac{V_1^u V_1^v + A_1^u A_1^v}{2u_2 v_2 \bar{u}_3^2 \bar{v}_3^2} \\
FF_{30}: &\quad 0 \\
FF_{31}: &\quad -\frac{(V_1^u - A_1^u)(V_1^v - A_1^v)}{4\bar{u}_1 u_1 u_2 v_1 v_2 \bar{v}_3} \\
FF_{32}: &\quad -\frac{(V_1^u + A_1^u)(V_1^v + A_1^v)}{4u_1 u_2 \bar{u}_2 v_1 v_2 \bar{v}_3} \\
FF_{33}: &\quad -\frac{T_1^u T_1^v}{u_1 \bar{u}_1 u_2 v_1 v_2 \bar{v}_2} \\
FF_{34}: &\quad 0 \\
FF_{35}: &\quad \frac{V_1^u V_1^v + A_1^u A_1^v}{2u_1 v_1 \bar{u}_3^2 \bar{v}_3^2}
\end{aligned} \tag{3.62}$$

These expressions have to be multiplied by the same factors as the expressions in (3.60). If we integrate over the momentum fractions and convolve with the distribution amplitudes in (A.7), we get

$$\begin{aligned}
&\langle p(P')|\bar{u}\gamma^\mu\gamma_5 u|p(P)\rangle = \\
&= -\alpha_s^2 16\pi^2 \frac{(N_c+1)^2}{4N_c^2 N_c!} \frac{1}{Q^4} \bar{N}^+(P')\gamma^\mu\gamma_5 N^+(P) \\
&\quad \frac{100}{3}\left[27(\phi_3^0)^2 - 18\phi_3^0\phi_3^- - 13(\phi_3^-)^2 + 12\phi_3^-\phi_3^+ - 21(\phi_3^+)^2\right. \\
&\qquad \left. + \mathcal{O}(M/Q)\right] + \mathcal{O}(\alpha_s^3)
\end{aligned} \tag{3.63}$$

$$\begin{aligned}
&\langle p(P')|\bar{d}\gamma^\mu\gamma_5 d|p(P)\rangle = \\
&= -\alpha_s^2 16\pi^2 \frac{(N_c+1)^2}{4N_c^2 N_c!} \frac{1}{Q^4} \bar{N}^+(P')\gamma^\mu\gamma_5 N^+(P) \\
&\quad 50\left[36(\phi_3^0)^2 - 8\phi_3^0\phi_3^- + 36\phi_3^0\phi_3^+ + \frac{5}{9}(\phi_3^-)^2 - 6\phi_3^-\phi_3^+ - 23(\phi_3^+)^2\right. \\
&\qquad \left. + \mathcal{O}(M/Q)\right] + \mathcal{O}(\alpha_s^3)
\end{aligned} \tag{3.64}$$

The flavour changing axial form factor was calculated in [123,124] for a specific helicity configuration. We confirm the expressions for the diagrams from [123] with a calculation that uses the leading twist part of the nucleon distribution amplitudes (3.25) and (3.27) and that takes into account the different contractions of Dirac indices in a $p \to n$ transition. The relation $g_A = G_M^n$ for the magnetic form factor of the neutron, G_M^n, and asymptotic distribution amplitudes is confirmed.

Results for subleading power

Using (anti-)symmetry of the distribution amplitudes, we arrive at the following integral representation for the u-flavour form factor at next-to-leading power:

$$
\begin{aligned}
&\langle p(P')|\bar{u}\gamma^\mu\gamma_5 u|p(P)\rangle|_{\text{NLP}} = \\
&=\alpha_s^2 16\pi^2 \frac{(N_c+1)^2}{4N_c^2 N_c!}\frac{M}{Q^6}\left(P'^\mu - P^\mu\right)\bar{N}^+(P')\gamma_5 N^+(P)\int \mathcal{D}u \int \mathcal{D}v \\
&\quad \left\{ \frac{V_1^u V_1^v}{16 u_1 \bar{u}_1^2 u_2 \bar{u}_2^2 u_3 v_1 \bar{v}_1^2 v_2 \bar{v}_2^2 v_3} \right. \\
&\quad \Big(u_1 \Big(-\left(-2u_2^2\bar{u}_2 + (5-4u_2)\,u_2^2 v_1 + (-3+(-5+u_2)(-2+u_2)\,u_2)\,v_1^2\right)\bar{v}_1 \\
&\quad + \Big(4u_2(7-5v_1)\,v_1^2 + v_1^2(-7+6v_1) + u_2^2(-7+2v_1(14+3v_1(-7+3v_1))) \\
&\quad +2u_2^3(-3+v_1)(-1+2v_1)\,\bar{v}_1\Big)v_2 + (3+v_1(-7+2(5-2v_1)\,v_1) \\
&\quad +2u_2^2(6+v_1(-21+(25-7v_1)\,v_1)) + 2u_2(-5+v_1(14+v_1(-26+9v_1))) \\
&\quad -u_2^3(5+v_1(-13+3v_1))\,\bar{v}_1\Big)v_2^2 + (-3+6v_1+(-2+u_2)((-5+u_2)\,u_2 \\
&\quad +2(5-2u_2)\,u_2 v_1 + (2+u_2(-8+3u_2))\,v_1^2))\,v_2^3\Big) + u_1^2\Big(u_2^3\Big(1-4v_1+3v_1^2 \\
&\quad +(-2+v_1)(2+v_1(-8+3v_1))\,v_2 + (3+v_1(-14+5v_1))\,v_2^2 + 3v_1v_2^3\Big) \\
&\quad +u_2^2(-2+(-2+v_1)\,v_1(-5+3v_1)+10v_2 - 2v_1(26+v_1(-25+8v_1))\,v_2 \\
&\quad +(-11+v_1(50+v_1(-28+5v_1)))\,v_2^2 + (-3+v_1)(-1+5v_1)\,v_2^3\Big) \\
&\quad +u_2(2+v_1(-7+(12-7v_1)\,v_1) - 7v_2 + 2v_1(14+3v_1(-7+3v_1))\,v_2 \\
&\quad +2(6+v_1(-21+(25-7v_1)\,v_1))\,v_2^2 + (-7+2(9-7v_1)\,v_1)\,v_2^3\Big) \\
&\quad +\bar{v}_1\left(-1+2v_1\bar{v}_1+2v_2+v_1(-5+3v_1)\,v_2+(-2+3v_1)\,v_2^2\right)\bar{v}_2\Big) \\
&\quad +u_1^3\Big(1+v_1(-3+2v_1) - 3v_2 + (-5+v_1)(-2+v_1)\,v_1v_2 + (-2+v_1)(-1+3v_1)\,v_2^2 \\
&\quad +v_1v_2^3 + u_2^2\Big(1-4v_1+3v_1^2+(-2+v_1)(2+v_1(-8+3v_1))\,v_2 + (3+v_1(-14+5v_1))\,v_2^2 \\
&\quad +3v_1v_2^3\Big) + u_2\bar{v}_1\left(-2-(-4+v_1)\,v_1+4v_2+v_1(-10+3v_1)\,v_2+(-1+3v_1)\,v_2^2\right)\bar{v}_2\Big) \\
&\quad -\bar{u}_2\Big(u_2^2\Big(1+v_1(-3+2v_1)-3v_2+(-5+v_1)(-2+v_1)\,v_1v_2+(-2+v_1)(-1+3v_1)\,v_2^2 \\
&\quad +v_1v_2^3\Big) + u_2\Big(-2v_1^2\bar{v}_1 + (5-4v_1)\,v_1^2 v_2 + (-2+v_1)(1+v_1(-2+3v_1))\,v_2^2 \\
&\quad +(2+v_1(-4+3v_1))\,v_2^3\Big) - \bar{v}_1\left(-v_1^2+v_1^2v_2-\bar{v}_1v_2^2\right)\bar{v}_2\Big)\Big) \\
&\quad -\frac{V_1^u V_2^v}{8u_1\bar{u}_1^2 u_2\bar{u}_2^2 u_3 v_1\bar{v}_1^2 v_2\bar{v}_2^2 v_3} \\
&\quad \Big(-\Big(1+u_1(-3+2u_1)-3u_2+(-5+u_1)(-2+u_1)\,u_1u_2+(-2+u_1)(-1+3u_1)\,u_2^2 \\
&\quad +u_1u_2^3\Big)v_1^2\bar{v}_1 + v_1^2\Big(-2u_1^3u_2(-2+v_1)+\bar{u}_2(2-5u_2-2v_1+4u_2v_1)+u_1(-7+6v_1 \\
&\quad +u_2\left(28-21u_2+4u_2^2-2(-5+u_2)(-2+u_2)\,v_1\right)\Big) + u_1^2(5-4v_1+u_2(-21+8u_2
\end{aligned}
$$

$$
\begin{aligned}
&+14v_1-6u_2v_1)))\,v_2+(u_1\,(3-(-5+u_2)\,(-2+u_2)\,u_2-7v_1+u_2\,(28\\
&+u_2\,(-21+4u_2))\,v_1+2\,(5+u_2\,(-26+(21-5u_2)\,u_2))\,v_1^2+(-2+u_2)\,(2\\
&+u_2\,(-8+3u_2))\,v_1^3\big)-\bar{u}_2\,\big(1-2v_1+(-2+v_1)\,\big(u_2-2u_2v_1+(-1+3u_2)\,v_1^2\big)\big)\\
&+u_1^3u_2\,(-1+v_1\,(4+v_1\,(-10+3v_1)))+u_1^2\,((-2+v_1)\,(1+v_1\,(-2+3v_1))\\
&+u_2^2\,(-3+(-2+v_1)\,v_1\,(-4+5v_1))+7u_2\,(1+v_1\,(-3-2\,(-3+v_1)\,v_1)))\big)\,v_2^2\\
&+\big(u_1^3u_2\,(1+v_1\,(-2+3v_1))+u_1\,(-3+6v_1+(-2+u_2)\,((-5+u_2)\,u_2\\
&-2\,(-5+u_2)\,u_2v_1+(2+u_2\,(-8+3u_2))\,v_1^2\big)\big)-\bar{u}_2\,\big(u_2\,(2+v_1\,(-4+3v_1))-\bar{v}_1^2\big)\\
&+u_1^2\,\big(2+v_1\,(-4+3v_1)+u_2^2\,(3+v_1\,(-6+5v_1))-7u_2\,(1-2v_1\bar{v}_1)\big)\big)\,v_2^3\big)\\
&-\frac{V_1^uV_3^v}{4u_1\bar{u}_1^2u_2\bar{u}_2^2u_3v_1\bar{v}_1^2v_2\bar{v}_2^2v_3}\\
&\big(-\big(1+u_1\,(-3+2u_1)-3u_2+(-5+u_1)\,(-2+u_1)\,u_1u_2+(-2+u_1)\,(-1+3u_1)\,u_2^2\\
&+u_1u_2^3\big)\,v_1^2\bar{v}_1+v_1^2\,\big(-2u_1^3u_2\,(-2+v_1)+\bar{u}_2\,(2-5u_2-2v_1+4u_2v_1)+u_1\,(-7+6v_1\\
&+u_2\,\big(28-21u_2+4u_2^2-2\,(-5+u_2)\,(-2+u_2)\,v_1\big)\big)+u_1^2\,(5-4v_1+u_2\,(-21+8u_2\\
&+14v_1-6u_2v_1)))\,v_2+(u_1\,(3-(-5+u_2)\,(-2+u_2)\,u_2-7v_1+u_2\,(28\\
&+u_2\,(-21+4u_2))\,v_1+2\,(5+u_2\,(-26+(21-5u_2)\,u_2))\,v_1^2+(-2+u_2)\,(2\\
&+u_2\,(-8+3u_2))\,v_1^3\big)-\bar{u}_2\,\big(1-2v_1+(-2+v_1)\,\big(u_2-2u_2v_1+(-1+3u_2)\,v_1^2\big)\big)\\
&+u_1^3u_2\,(-1+v_1\,(4+v_1\,(-10+3v_1)))+u_1^2\,((-2+v_1)\,(1+v_1\,(-2+3v_1))\\
&+u_2^2\,(-3+(-2+v_1)\,v_1\,(-4+5v_1))+7u_2\,(1+v_1\,(-3-2\,(-3+v_1)\,v_1)))\big)\,v_2^2\\
&+\big(u_1^3u_2\,(1+v_1\,(-2+3v_1))+u_1\,(-3+6v_1+(-2+u_2)\,((-5+u_2)\,u_2\\
&-2\,(-5+u_2)\,u_2v_1+(2+u_2\,(-8+3u_2))\,v_1^2\big)\big)-\bar{u}_2\,\big(u_2\,(2+v_1\,(-4+3v_1))-\bar{v}_1^2\big)\\
&+u_1^2\,\big(2+v_1\,(-4+3v_1)+u_2^2\,(3+v_1\,(-6+5v_1))-7u_2\,(1-2v_1\bar{v}_1)\big)\big)\,v_2^3\big)\\
&+\frac{A_1^uA_1^v}{16u_1\bar{u}_1^2u_2\bar{u}_2^2u_3\bar{u}_3v_1\bar{v}_1^2v_2\bar{v}_2^2v_3\bar{v}_3}\\
&(u_1-u_2)\,(v_1-v_2)\,\big(u_1^3\,((1-2v_1+u_2\,(-2+u_2-u_2v_1+v_1\,(4+v_1)))\,\bar{v}_1\\
&+\big(-3+u_2^2\,\big(-2+(-2+v_1)^2v_1\big)-v_1\,\big(-6+v_1+v_1^2\big)+2u_2\,(3+v_1\,(-5-2v_1\bar{v}_1))\big)\,v_2\\
&-(-2-v_1+u_2\,(3+v_1\,(7+v_1)+u_2\,(-1+3v_1)))\,\bar{v}_1v_2^2+(-v_1+u_2\,(-1\\
&+v_1\,(4+u_2+v_1)))\,v_2^3\big)-u_1^2\bar{u}_2\,\big(u_2^2\,\big(\bar{v}_1^2+\big(-2+(-2+v_1)^2v_1\big)\,v_2-(-1+3v_1)\,\bar{v}_1v_2^2\\
&+v_1v_2^3\big)+u_2\,(-3-v_1\,(-10+v_1\,(6+v_1))+10v_2+v_1\,(-18+v_1\,(-8+7v_1))\,v_2\\
&-(6+v_1\,(14+3v_1))\,\bar{v}_1v_2^2+(-1+v_1\,(7+3v_1))\,v_2^3\big)-\bar{v}_1\,(-1+2v_1\bar{v}_1+2v_2\\
&+(-5+v_1)\,v_1v_2+(-2+v_1)\,v_2^2\big)\,\bar{v}_2\big)+u_1\,\big(u_2^2\,(4-v_1\,(13+(-10+v_1)\,v_1)-13v_2\\
&-2v_1\,\big(-14+v_1+2v_1^2\big)\,v_2-2\,(-5+v_1\,(1+2v_1\,(1+v_1)))\,v_2^2-(1+2v_1)^2v_2^3\big)\\
&+u_2^3\,\big(-2-v_1\,(-6+v_1\,(3+v_1))+6v_2+2v_1\,(-5-2v_1\bar{v}_1)\,v_2-(3+v_1\,(7+v_1))\,\bar{v}_1v_2^2\\
&+(-1+v_1\,(4+v_1))\,v_2^3\big)+2u_2\bar{v}_1\,(-1+(2-3v_1)\,v_1+2v_2+v_1\,(-7+2v_1)\,v_2\\
&+(-3+2v_1)\,v_2^2\big)\,\bar{v}_2-\big(-3v_1\bar{v}_1+(-3+2\,(5-3v_1)\,v_1)\,v_2+(3+2\,(-3+v_1)\,v_1)\,v_2^2\big)\,\bar{v}_3\big)
\end{aligned}
$$

$$-\bar{u}_2\left(-u_2^2\left(-1+\left(3-2v_1\right)v_1+3v_2+v_1\left(-6+v_1+v_1^2\right)v_2+\left(-2+v_1+v_1^2\right)v_2^2+v_1v_2^3\right)\right.$$
$$+u_2\left(-2v_1\bar{v}_1+\left(-2+\left(7-4v_1\right)v_1\right)v_2+\left(2+\left(-4+v_1\right)v_1\right)v_2^2\right)\bar{v}_3$$
$$-\bar{v}_1\left(-v_1-\bar{v}_1v_2\right)\bar{v}_2\bar{v}_3))$$
$$-\frac{A_1^uA_2^v}{8u_1\bar{u}_1^2u_2\bar{u}_2^2u_3\bar{u}_3v_1\bar{v}_1^2v_2\bar{v}_2^2v_3}$$
$$\left(u_1-u_2\right)\left(v_1-v_2\right)\left(\left(-1+u_1^3u_2+\left(3-2u_2\right)u_2+u_1^2\left(-2+u_2+u_2^2\right)+u_1\left(3\right.\right.\right.$$
$$\left.\left.+u_2\left(-6+u_2+u_2^2\right)\right)\right)v_1\bar{v}_1+\left(-u_1^2\bar{u}_2\left(2+u_2-7\left(1+u_2\right)v_1+2\left(2+u_2\right)v_1^2\right)\right.$$
$$+u_1\left(3-6u_2+u_2^2+u_2^3-5\left(2-4u_2+u_2^3\right)v_1+2\left(3+u_2\left(-6+u_2+u_2^2\right)\right)v_1^2\right)$$
$$\left.+u_1^3u_2\left(1+v_1\left(-5+2v_1\right)\right)+\bar{u}_2\left(-1+\left(3-2v_1\right)v_1+u_2\left(2+v_1\left(-7+4v_1\right)\right)\right)\right)v_2$$
$$+\left(u_1\left(-3-u_2\left(-6+u_2+u_2^2\right)+2\left(3+u_2\left(-6+u_2+u_2^2\right)\right)v_1+\left(-2+\left(-2+u_2\right)^2u_2\right)\right.\right.$$
$$\left.v_1^2\right)+u_1^3u_2\left(-1+v_1\left(2+v_1\right)\right)-u_1^2\bar{u}_2\left(-2-\left(-4+v_1\right)v_1+u_2\left(1+v_1\right)\left(-1+3v_1\right)\right)$$
$$\left.\left.-\bar{u}_2\left(u_2\left(2+\left(-4+v_1\right)v_1\right)-\bar{v}_1^2\right)\right)v_2^2\right)$$
$$+\frac{A_1^uA_3^v}{4u_1\bar{u}_1^2u_2\bar{u}_2^2u_3\bar{u}_3v_1\bar{v}_1^2v_2\bar{v}_2^2v_3}$$
$$\left(u_1-u_2\right)\left(v_1-v_2\right)\left(\left(-1+u_1^3u_2+\left(3-2u_2\right)u_2+u_1^2\left(-2+u_2+u_2^2\right)+u_1\left(3\right.\right.\right.$$
$$\left.\left.+u_2\left(-6+u_2+u_2^2\right)\right)\right)v_1\bar{v}_1+\left(-u_1^2\bar{u}_2\left(2+u_2-7\left(1+u_2\right)v_1+2\left(2+u_2\right)v_1^2\right)\right.$$
$$+u_1\left(3-6u_2+u_2^2+u_2^3-5\left(2-4u_2+u_2^3\right)v_1+2\left(3+u_2\left(-6+u_2+u_2^2\right)\right)v_1^2\right)$$
$$\left.+u_1^3u_2\left(1+v_1\left(-5+2v_1\right)\right)+\bar{u}_2\left(-1+\left(3-2v_1\right)v_1+u_2\left(2+v_1\left(-7+4v_1\right)\right)\right)\right)v_2$$
$$+\left(u_1\left(-3-u_2\left(-6+u_2+u_2^2\right)+2\left(3+u_2\left(-6+u_2+u_2^2\right)\right)v_1+\left(-2+\left(-2+u_2\right)^2u_2\right)v_1^2\right)\right.$$
$$+u_1^3u_2\left(-1+v_1\left(2+v_1\right)\right)-u_1^2\bar{u}_2\left(-2-\left(-4+v_1\right)v_1+u_2\left(1+v_1\right)\left(-1+3v_1\right)\right)$$
$$\left.\left.-\bar{u}_2\left(u_2\left(2+\left(-4+v_1\right)v_1\right)-\bar{v}_1^2\right)\right)v_2^2\right)$$
$$+\frac{T_1^uT_1^v}{4u_1\bar{u}_1^2u_2\bar{u}_2^2u_3v_1\bar{v}_1^2v_2\bar{v}_2^2v_3}$$
$$\left(u_1^2\left(4u_2^2\left(v_1\bar{v}_1+v_2+v_1\left(-10+\left(10-3v_1\right)v_1\right)v_2+\left(-1+v_1\left(10+\left(-6+v_1\right)v_1\right)\right)v_2^2\right.\right.\right.$$
$$\left.+\left(-3+v_1\right)v_1v_2^3\right)-u_2\left(v_1\bar{v}_1^2+v_2-2v_1\left(8+v_1\left(-13+4v_1\right)\right)v_2+2\left(-1+v_1\left(13\right.\right.\right.$$
$$\left.\left.\left.+5\left(-4+v_1\right)v_1\right)\right)v_2^2+\left(1+2v_1\left(-4+5v_1\right)\right)v_2^3\right)+u_2^3\left(-v_2+v_2^3+2v_1^2v_2\left(-5+2v_2\right)\right.$$
$$\left.\left.+v_1^3\left(1+2v_2\right)+v_1\left(-1+2\left(-3+v_2\right)\left(-2+v_2\right)v_2\right)\right)+2\bar{v}_1\left(v_1-v_2\right)^2\bar{v}_2\right)$$
$$+u_1^3\left(v_1^3\left(2+\left(-2+v_2\right)v_2\right)-v_1v_2\left(-4+v_2+2v_2^2\right)+v_1^2\left(-2-v_2+v_2^3\right)+u_2^2\left(-v_2\right.\right.$$
$$\left.+v_2^3+2v_1^2v_2\left(-5+2v_2\right)+v_1^3\left(1+2v_2\right)+v_1\left(-1+2\left(-3+v_2\right)\left(-2+v_2\right)v_2\right)\right)-2v_2^2\bar{v}_2$$
$$\left.+2u_2\bar{v}_1\left(v_1\left(-4+v_2\right)v_2+v_2^2+v_1^2\left(1+v_2\right)\right)\bar{v}_2\right)+u_1\left(-u_2^2\left(v_1\bar{v}_1^2+v_2-2v_1\left(8\right.\right.\right.$$
$$\left.\left.+v_1\left(-13+4v_1\right)\right)v_2+2\left(-1+v_1\left(13+5\left(-4+v_1\right)v_1\right)\right)v_2^2+\left(1+2v_1\left(-4+5v_1\right)\right)v_2^3\right)$$
$$+2u_2^3\bar{v}_1\left(v_1\left(-4+v_2\right)v_2+v_2^2+v_1^2\left(1+v_2\right)\right)\bar{v}_2+4u_2\left(-2v_1\left(-2+v_2\right)v_2^2\right.$$
$$\left.+v_1^3\left(1+v_2\left(-2+3v_2\right)\right)+v_1^2\left(-1+v_2\left(4+v_2\left(-10+3v_2\right)\right)\right)-v_2^2\bar{v}_2\right)$$
$$\left.\left.-v_1v_2\left(v_2+v_1\left(1+v_2\left(-4+\bar{v}_3\right)\right)\right)\right)\right)-u_2\bar{u}_2\left(u_2\left(v_1^3\left(2+\left(-2+v_2\right)v_2\right)-v_1v_2\left(-4+v_2\right.\right.\right.$$

$$
\begin{aligned}
&+2v_2^2\big) + v_1^2\left(-2 - v_2 + v_2^3\right) - 2v_2^2\bar{v}_2\big) + v_1 v_2\left(v_2 + v_1\left(1 + v_2\left(-4 + \bar{v}_3\right)\right)\right)\big)\big) \\
&-\frac{T_1^u T_3^v}{4u_1\bar{u}_1^2 u_2\bar{u}_2^2 u_3 v_1\bar{v}_1^2 v_2\bar{v}_2^2 v_3} \\
&3\left(u_1^3\left(-v_1^2\bar{v}_1 + 2\left(1 + u_2 - v_1\right)v_1^2 v_2 + \left(-1 + v_1\left(2 + \left(-2 + v_1\right)v_1 + 2u_2\left(1\right.\right.\right.\right.\right. \\
&\left.\left.\left.+\left(-4 + v_1\right)v_1\right)\right)\right)v_2^2 + \left(1 + v_1\left(-2 + v_1 + 2u_2 v_1\right)\right)v_2^3\big) \\
&+u_1^2\left(-\left(-1 + u_2\left(-3 + 2u_2\right)\right)v_1^2\bar{v}_1 + v_1^2\left(-1 + 2v_1 + u_2\left(-13 + u_2\left(6 - 4v_1\right) + 6v_1\right)\right)v_2\right. \\
&+\big(1 - v_1 - 2v_1^2 - u_2\left(-3 + v_1\right)\left(1 + 2v_1\left(-2 + 5v_1\right)\right) + 2u_2^2\left(-1 + v_1\left(3 + 2\left(-3\right.\right.\right. \\
&\left.\left.\left.+v_1\right)v_1\right)\right)\big)v_2^2 + \left(-1 + 2v_1 + u_2\left(-3 + 2\left(3 - 5v_1\right)v_1 + u_2\left(2 - 4v_1\bar{v}_1\right)\right)\right)v_2^3\big) \\
&-u_2\bar{u}_2\left(u_2\bar{v}_1\left(-v_1^2 + v_1^2 v_2 - \bar{v}_1 v_2^2\right)\bar{v}_2 + v_1 v_2\left(v_2 + v_1\left(1 + v_2\left(-4 + \bar{v}_3\right)\right)\right)\right) \\
&+u_1\left(u_2^2\left(3v_1^2\bar{v}_1 + v_1^2\left(-13 + 6v_1\right)v_2 - \left(-3 + v_1\right)\left(1 + 2v_1\left(-2 + 5v_1\right)\right)v_2^2\right.\right. \\
&\left.+\left(-3 + 2\left(3 - 5v_1\right)v_1\right)v_2^3\right) + 4u_2\big(-v_1^2\bar{v}_1 - 2\left(-2 + v_1\right)v_1^2 v_2 + \left(-1 + v_1\left(4\right.\right. \\
&\left.\left.+v_1\left(-10 + 3v_1\right)\right)\right)v_2^2 + \left(1 + v_1\left(-2 + 3v_1\right)\right)v_2^3\big) - v_1 v_2\left(v_2 + v_1\left(1 + v_2\left(-4 + \bar{v}_3\right)\right)\right) \\
&\left.\left.+2u_2^3 v_1 v_2\left(v_2 + v_1\left(1 + v_2\left(-4 + \bar{v}_3\right)\right)\right)\right)\right) \\
&-\frac{T_1^u T_7^v}{4u_1\bar{u}_1^2 u_2\bar{u}_2^2 u_3 v_1\bar{v}_1^2 v_2\bar{v}_2^2 v_3} \\
&3\left(u_1^3\left(-v_1^2\bar{v}_1 + 2\left(1 + u_2 - v_1\right)v_1^2 v_2 + \left(-1 + v_1\left(2 + \left(-2 + v_1\right)v_1 + 2u_2\left(1 + \left(-4\right.\right.\right.\right.\right.\right. \\
&\left.\left.\left.\left.+v_1\right)v_1\right)\right)\right)v_2^2 + \left(1 + v_1\left(-2 + v_1 + 2u_2 v_1\right)\right)v_2^3\big) + u_1^2\left(-\left(-1 + u_2\left(-3 + 2u_2\right)\right)v_1^2\bar{v}_1\right. \\
&+v_1^2\left(-1 + 2v_1 + u_2\left(-13 + u_2\left(6 - 4v_1\right) + 6v_1\right)\right)v_2 + \big(1 - v_1 - 2v_1^2 \\
&-u_2\left(-3 + v_1\right)\left(1 + 2v_1\left(-2 + 5v_1\right)\right) + 2u_2^2\left(-1 + v_1\left(3 + 2\left(-3 + v_1\right)v_1\right)\right)\big)v_2^2 \\
&+\left(-1 + 2v_1 + u_2\left(-3 + 2\left(3 - 5v_1\right)v_1 + u_2\left(2 - 4v_1\bar{v}_1\right)\right)\right)v_2^3\big) - u_2\bar{u}_2\big(u_2\bar{v}_1\big(-v_1^2 \\
&+v_1^2 v_2 - \bar{v}_1 v_2^2\big)\bar{v}_2 + v_1 v_2\left(v_2 + v_1\left(1 + v_2\left(-4 + \bar{v}_3\right)\right)\right)\big) + u_1\big(u_2^2\big(3v_1^2\bar{v}_1 + v_1^2\big(-13 \\
&+6v_1\big)v_2 - \left(-3 + v_1\right)\left(1 + 2v_1\left(-2 + 5v_1\right)\right)v_2^2 + \left(-3 + 2\left(3 - 5v_1\right)v_1\right)v_2^3\big) \\
&+4u_2\left(-v_1^2\bar{v}_1 - 2\left(-2 + v_1\right)v_1^2 v_2 + \left(-1 + v_1\left(4 + v_1\left(-10 + 3v_1\right)\right)\right)v_2^2\right. \\
&\left.+\left(1 + v_1\left(-2 + 3v_1\right)\right)v_2^3\right) - v_1 v_2\left(v_2 + v_1\left(1 + v_2\left(-4 + \bar{v}_3\right)\right)\right) \\
&\left.\left.+2u_2^3 v_1 v_2\left(v_2 + v_1\left(1 + v_2\left(-4 + \bar{v}_3\right)\right)\right)\right)\right) \\
&+u_i \leftrightarrow v_i\} + \mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{3.65}
$$

For the regularization of the axial vector form factor, we use the same procedure as for the vector form factor (3.2.1). The integration over the momentum fractions of leading twist distribution amplitudes vanishes, after the conformal expansion (A.7 and A.8) has been employed:

$$
\langle p(P')|\bar{u}\gamma^\mu\gamma_5 u|p(P)\rangle|_{\rm NLP} = 0 + \mathcal{O}(\alpha_s^3) \tag{3.66}
$$

Table 3.7: Singular behavior of the next-to-leading power part of the axial vector d-form factor (3.67). Displayed are the most singular powers in the expansion (3.54) of the conformal expansion. The table entries denote the negative exponents (i, j) or i, where appropriate. The shown monomials have the lowest total degree $-i-j$. There are no monomials with lower single degrees $-i$ or $-j$. The momentum fractions u_i are associated with twist 3 distribution amplitudes, shown in the upper half of the table. Finite terms are indicated by "—". Expansion coefficients with no singular terms are left out.

	$(\phi_3^0)^2$	$\phi_3^0\phi_4^0$	$\phi_3^0\psi_4^0$	$\phi_3^0\phi_4^+$	$\phi_3^0\psi_4^-$	$\phi_3^0\psi_4^+$
Expansion in momentum components u_i^j:						
$\bar{u}_2^{-i}$	1	1	1	1	1	1
$\bar{u}_3^{-i}$	1	1	1	1	1	1
$\bar{u}_1^{-i}$	1	1	1	1	1	1
Expansion in momentum components v_i^j:						
v_2^{-i}	—	—	1	—	1	1
$\bar{v}_3^{-i}$	—	1	—	1	—	—
v_1^{-i}	—	—	1	—	1	1

The d-flavour form factor reads at next-to-leading power:

$$
\begin{aligned}
&\langle p(P')|\bar{d}\gamma^\mu\gamma_5 d|p(P)\rangle|_{\mathrm{NLP}} = \\
&=\alpha_s^2 16\pi^2 \frac{(N_c+1)^2}{4N_c^2 N_c!}\frac{M}{Q^6}\left(P'^\mu - P^\mu\right)\bar{N}^+(P')\gamma_5 N^+(P)\int \mathcal{D}u \int \mathcal{D}v \\
&\quad \Big\{ -\frac{V_1^u V_1^v}{8u_1\bar{u}_1u_2\bar{u}_2\bar{u}_3v_1\bar{v}_1v_2\bar{v}_2\bar{v}_3} \\
&\quad \Big(u_2\left(1-6v_1+4v_1^2+3\left(-2+v_1\right)\bar{v}_1v_2+\left(4-3v_1\right)v_2^2\right)+u_2^2\left(-1-2\left(-2+v_1\right)v_1\right. \\
&\quad \left.+4v_2+\left(-5+v_1\right)v_1v_2+\left(-2+v_1\right)v_2^2\right)+u_1^2\left(-\bar{u}_2+4v_1-3u_2v_1-2v_1^2+u_2v_1^2\right. \\
&\quad \left.-\left(-4+3u_2+v_1\right)\bar{v}_1v_2+\left(-2+u_2+v_1\right)v_2^2\right)-u_1\bar{u}_2\left(-\bar{u}_2+6v_1-3u_2v_1\right. \\
&\quad \left.-4v_1^2+u_2v_1^2-3\left(-2+u_2+v_1\right)\bar{v}_1v_2+\left(-4+u_2+3v_1\right)v_2^2\right)+\bar{v}_1\bar{v}_2\bar{v}_3\Big) \\
&\quad +V_1^uV_2^v\frac{\left(1+u_1^2+\left(-3+u_2\right)u_2-3u_1\bar{u}_2\right)\left(-1+2v_1+2v_2\right)}{4u_1\bar{u}_1u_2\bar{u}_2\bar{u}_3v_1v_2\bar{v}_3} \\
&\quad +V_1^uV_3^v\frac{\left(1+u_1^2+\left(-3+u_2\right)u_2-3u_1\bar{u}_2\right)}{4u_1\bar{u}_1u_2\bar{u}_2\bar{u}_3v_1v_2} - A_1^vV_1^u\frac{\left(v_1-v_2\right)}{4u_1u_2v_1\bar{v}_1v_2\bar{v}_2} \\
&\quad +A_1^vV_2^u\frac{\left(-1+2u_1+2u_2\right)\left(v_1-v_2\right)}{4u_1u_2\bar{u}_3v_1\bar{v}_1v_2\bar{v}_2} + A_1^vV_3^u\frac{\left(v_1-v_2\right)}{4u_1u_2v_1\bar{v}_1v_2\bar{v}_2}
\end{aligned}
$$

$$-A_1^u A_1^v \frac{(u_1-u_2)(v_1-v_2)(\bar{u}_3+\bar{v}_3)}{8u_1u_2\bar{u}_3^2v_1v_2\bar{v}_3^2} + A_1^u A_2^v \frac{(u_1-u_2)(v_1-v_2)(-1+2v_1+2v_2)}{4u_1u_2\bar{u}_3^2v_1v_2\bar{v}_3^2}$$
$$\left. -A_1^u A_3^v \frac{(u_1-u_2)(v_1-v_2)}{4u_1u_2\bar{u}_3^2v_1v_2\bar{v}_3} + u_i \leftrightarrow v_i \right\}$$
$$+\mathcal{O}(\alpha_s^3) \tag{3.67}$$

As in the case of the vector form factor, one can extract the divergent behavior for each integral alone, assuming that the other variables are finite. For illustration purposes the most divergent terms after the conformal expansion of distribution amplitudes are shown in Table 3.7.

The integrand is regulated according to (3.16). The limit, $\varepsilon \to 0$, of the integral is well defined and vanishes:

$$\langle p(P')|\bar{d}\gamma^\mu\gamma_5 d|p(P)\rangle|_{\mathrm{NLP}} = 0 + \mathcal{O}(\alpha_s^3) \tag{3.68}$$

Most higher power corrections that involve light cone distribution amplitudes are not calculable at subleading power. Since this is the case for (3.66) and (3.68) we expect that objects based on the axial vector form factor have rather small hadronic corrections.

3.3 Evaluation of decay amplitude

3.3.1 Outline of factorization scheme

We perform the calculation of matrix elements $\langle p(P')\bar{p}(P)|Q_i(0)|\bar{B}_s\rangle$ at leading power in Λ_{QCD}/m_b. As long as the calculation only involves leading twist nucleon distribution amplitudes, it is formally not necessary to consider higher fock states. Numerical estimates suggest that higher twist coefficients are of comparable size to leading twist coefficients [99]. Expecting a bigger impact on the amplitude of these higher twist terms than subleading fock states, we disregard the latter from our analysis. Because of the flavour structure, annihilation diagrams are the only ones that contribute. If we insert the operators $Q_{7\gamma}$ and Q_{8g}, whose fermion fields can only be contracted with the $\bar{B}_s$-meson, we are left with the matrix element $\langle 0|\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b|\bar{B}_s\rangle$. It vanishes by Lorentz symmetry:

$$\langle 0|\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b|\bar{B}_s\rangle = 0 \tag{3.69}$$

The tree diagram at $\mathcal{O}(\alpha_s^0)$ for the operators $Q_1^p, \cdots Q_{10}$ with implied B-meson projection (2.8) is depicted in Fig. 3.3 and vanishes by the equations of motion. At $\mathcal{O}(\alpha_s)$ we encounter the diagrams of Figs. 3.4 and 3.5. The diagrams at $\mathcal{O}(\alpha_s^2)$ are shown in Figs. 3.6, 3.7 and 3.8. Non-abelian diagrams do not contribute due to the colour structure. In addition to the perturbative expansion we also have a power expansion in Λ_{QCD}/m_b. The leading power can be identified with the help of the leading form factor scaling (3.35):

$$\langle p\bar{p}|\mathcal{H}_{\mathrm{eff}}^{\Delta S=1}|\bar{B}_s\rangle \sim G_F V_{ub} V_{us}^* f_B f_N^2 \frac{M}{m_b^3} \tag{3.70}$$

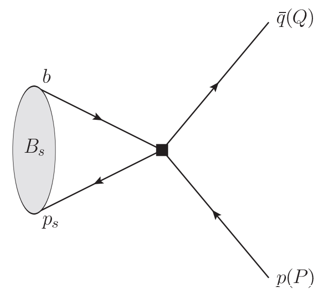

Figure 3.3: Tree diagram at $\mathcal{O}(\alpha_s^0)$. Shaded area indicates B-meson projection.

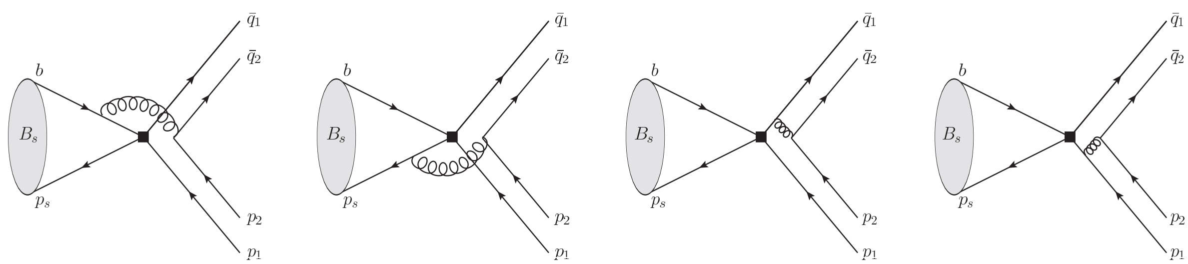

Figure 3.4: Tree diagrams at $\mathcal{O}(\alpha_s)$. Shaded area indicates B-meson projection.

The decay is suppressed by an additional power in M/m_b, due to the conservation of angular momentum and momentum: either the final state is in s-wave and the helicity of the (anti-) proton is flipped or it is in p-wave. The p-wave is suppressed, because one power of $p_\perp/m_b$ has to be picked up to generate the corresponding angular distribution. In order to estimate the diagrams in Fig. 3.4, we project onto the B-meson and identify the leading power expressions. The expressions for each diagram, from left to right, read:

$$-i2\pi\alpha_s \frac{f_B}{m_b^2}\frac{1}{N_c}\frac{1}{u_2\bar{v}_2+v_2}$$

$$\bar{q}(q_1)t^a\gamma^\mu(1-\gamma_5)p(p_1)\left[\frac{1}{v_2}\bar{q}(q_2)t^a\gamma_\mu(1-\gamma_5)p(p_2)+\frac{1}{u_2}\bar{q}(q_2)t^a\gamma_\mu(1+\gamma_5)p(p_2)\right] \tag{3.71}$$

$$-i2\pi\alpha_s \frac{f_B}{m_b^2}\frac{1}{N_c}\frac{1}{u_2 v_2}$$

$$\bar{q}(q_1)t^a\gamma^\mu(1-\gamma_5)p(p_1)\left[\frac{1}{u_2}\bar{q}(q_2)t^a\gamma_\mu(1-\gamma_5)p(p_2)+\frac{1}{v_2}\bar{q}(q_2)t^a\gamma_\mu(1+\gamma_5)p(p_2)\right] \tag{3.72}$$

$$i4\pi\alpha_s \frac{f_B}{m_b^2}\frac{1}{u_2 v_2}\bar{q}(q_1)t^a\gamma^\mu(1-\gamma_5)p(p_1)\bar{q}(q_2)t^a\gamma_\mu p(p_2) \tag{3.73}$$

$$-i4\pi\alpha_s \frac{f_B}{m_b^2}\frac{1}{u_2 v_2(u_2-u_1)}\bar{q}(q_1)t^a\gamma^\mu(1-\gamma_5)p(p_1)\bar{q}(q_2)t^a\gamma_\mu p(p_2) \tag{3.74}$$

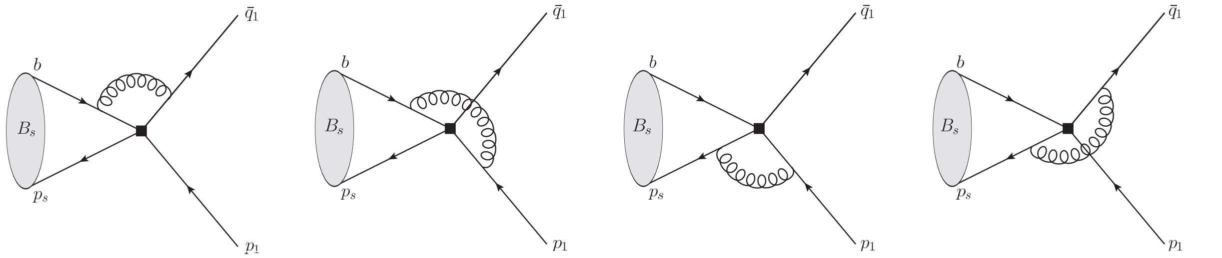

Figure 3.5: One loop diagrams at $\mathcal{O}(\alpha_s)$. Shaded area indicates B-meson projection.

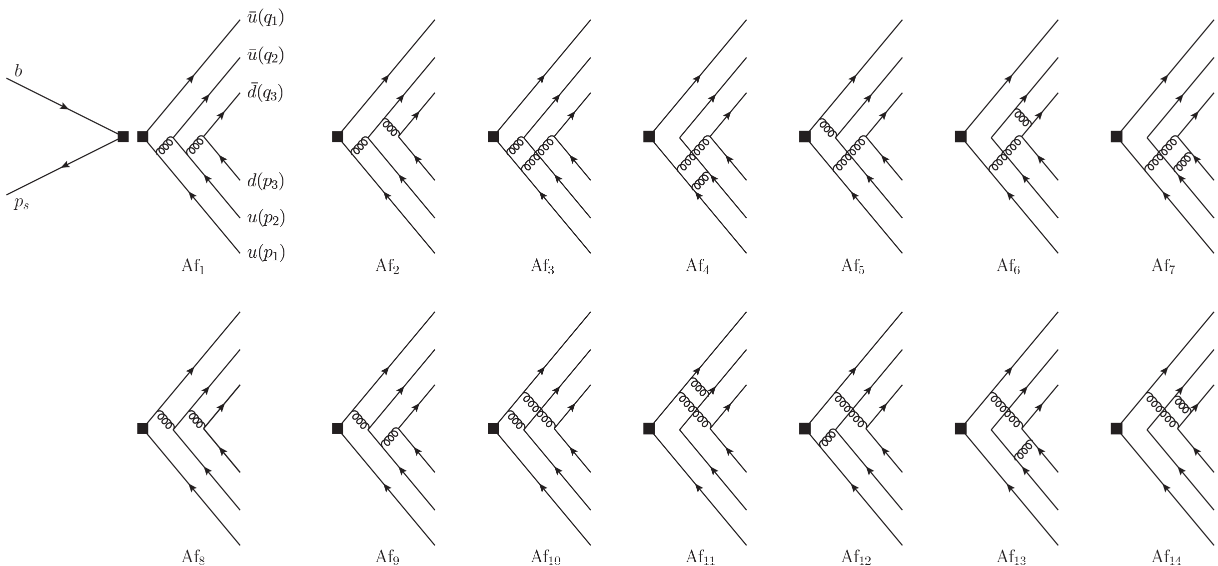

Figure 3.6: Tree diagrams at $\mathcal{O}(\alpha_s^2)$ that include only gluon couplings to final state fermion lines.

We assigned the momentum fractions $q_i = v_i q$ and $p_i = u_i p$ for the leading power momenta. Q_2 does not contribute for the first two diagrams, Q_1 does not contribute for the last two diagrams. We see from (3.71-3.74) that these diagrams do not vanish by equations of motion. Compared to the hard contribution (3.70), the suppression for the protons to pick up two soft quarks was estimated to be $\mathcal{O}((\Lambda_{QCD}/m_b)^2)$ [109, 110]. Furthermore in the expressions (3.71-3.74) there is only one gluon propagator and 1 quark propagator and therefore only terms with momentum fraction x_i up to $1/x_i^3$. At $\mathcal{O}(\alpha_s^2)$ there are 2 gluon and 2 quark propagators and therefore powers up to $1/x_i^6$. Similar to [109] we use the asymptotic distribution amplitude $\Phi_{\rm as}(x_i) = 120 x_1 x_2 x_3$ to estimate the size of the typical

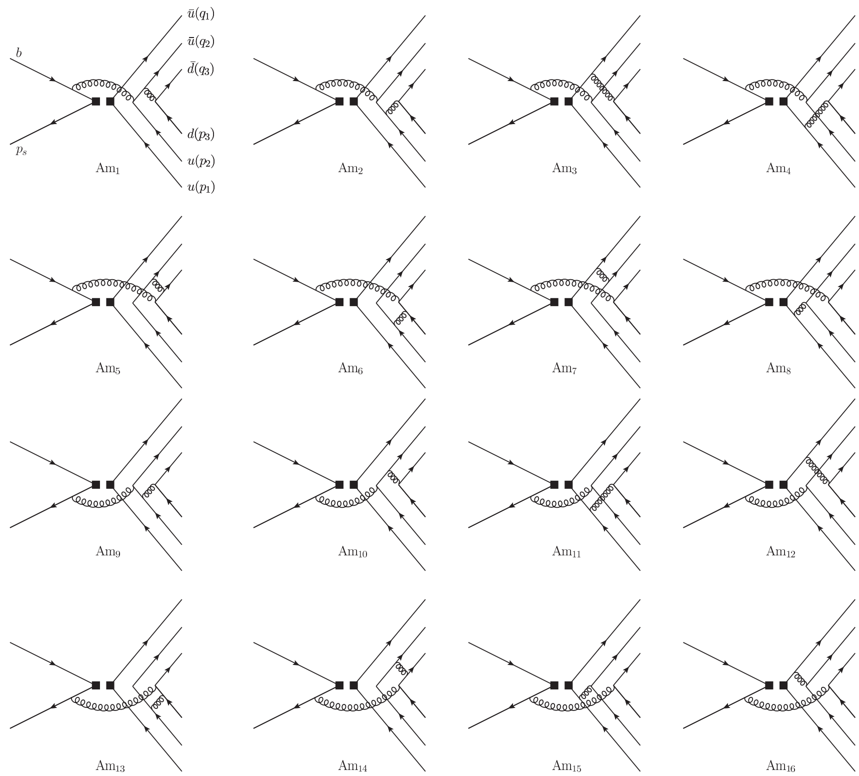

Figure 3.7: Tree diagrams at $\mathcal{O}(\alpha_s^2)$ that include only gluon couplings with exactly one coupling to an incoming fermion line.

integrals over momentum fractions:

$$\left(\int \mathcal{D}x \, \frac{\Phi_{\rm as}(x_i)}{x_1 x_2}\right)\left(\int \mathcal{D}x \, \frac{\Phi_{\rm as}(x_i)}{x_1}\right) \sim 10^2 \tag{3.75}$$

$$\left(\int \mathcal{D}x \, \frac{\Phi_{\rm as}(x_i)}{x_1 x_2 x_3}\right)^2 \sim 10^3 \tag{3.76}$$

Expressions (3.72) and (3.74) have logarithmic singularities. We estimate them using a cut-off at Λ_{QCD}/m_b. When we integrate over the asymptotic distribution amplitude and the explicit momentum fractions of (3.71-3.74), we get values below 10^2 with the highest value of 65 for (3.72). The (anti-) protons in the diagrams of Fig. 3.5 have to pick up 2 soft quarks and are suppressed by at least $(\Lambda_{QCD}/m_b)^2$. We estimate them to be smaller than the contributions of Fig. 3.4. Due to the additional factor α_s/π for the hard contribution, the soft contribution can play a sizeable role.

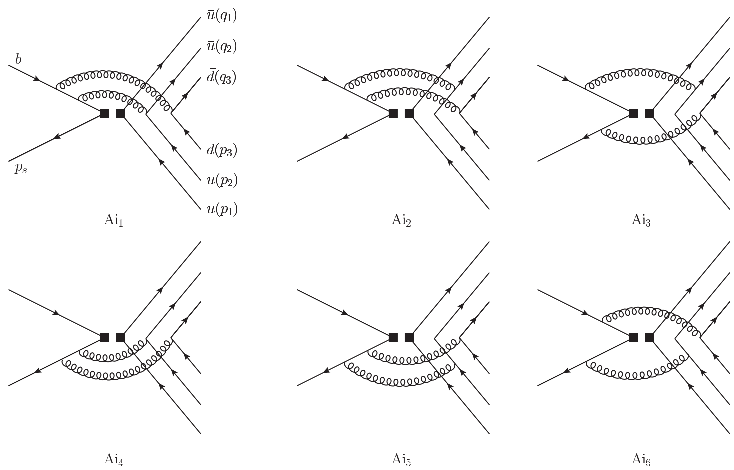

Figure 3.8: Tree diagrams at $\mathcal{O}(\alpha_s^2)$ that include only gluon couplings with exactly two couplings to an initial fermion line and 3 (anti-) quark contractions per (anti-) proton.

The hard contribution to the amplitude at $\mathcal{O}(\alpha_s^2)$ can be represented as follows:

$$\begin{aligned}
&\langle p(P')\bar{p}(P)|\mathcal{H}_{\text{eff}}^{\Delta S=1}|\bar{B}_s\rangle = \\
=&\frac{G_F}{\sqrt{2}}\lambda_u \left[\left(-C_1\frac{(N_c+1)^2}{4N_c^2 N_c!} - C_2\frac{N_c+1}{4N_c^2 N_c!}\right)\mathcal{T}_i \right.\\
&\left. + C_1\frac{(N_c+1)^2}{4N_c^3 N_c!}\mathcal{T}_m + \left(C_1\frac{N_c+1}{4N_c^3 N_c!} + C_2\frac{(N_c+1)^2}{4N_c^2 N_c!}\right)\mathcal{T}_f\right] \qquad (3.77)
\end{aligned}$$

Feynman diagrams at order α_s^2 contribute at power $\mathcal{O}(M/m_b)$. At leading twist in the proton and antiproton distribution amplitude there is no contribution at the integrand level. That is consistent with the expectation of power suppression (3.70). The leading contribution results from taking the proton distribution amplitude at twist 4 and the antiproton distribution amplitude at twist 3 and distribution amplitudes with exchanged twist assignments. At leading power the decay amplitudes do not depend on the spectator quark momentum. In order to show that, we use that the form for the B-meson distribution amplitude for general z has the same Dirac structure as the one on the light cone (2.8), i.e. $z^2 = 0$. Instead of the integration over $\phi_{B2}(\xi)$, we have to integrate over $\psi_{B2}(p_s)$, which depends on the full momentum of the spectator quark. Subsequently we express the momentum of the spectator quark in the integrand as

$$p_s^\mu = p_s^0(p+q)^\mu/m_b + p_{s\perp}^\mu + p_s^3(p-q)^\mu/m_b, \qquad (3.78)$$

apply the equations of motion and average over the momenta of the perpendicular subspace independently for each hadron, similar to the case of the proton form factor as described in 3.2.1. The dependence on all perpendicular momenta and the spectator quark momentum drops out as expected. Therefore we can perform the integration over $\psi_{B2}(p_s)$, which vanishes.

In the next subsection we supply the amplitudes for the Feynman diagrams with coupling to the first u-quark line. The expressions $\mathcal{T}_i$ in (3.77) can be obtained by summing over Ai and following the prescription in (3.43) and (3.44). The analogous holds for the expressions $\mathcal{T}_m$ and $\mathcal{T}_f$.

3.3.2 Feynman diagrams Ai

The diagrams with two gluon couplings to quark lines from incoming fermions are displayed in Fig. 3.8. Apart from the factors separated out in (3.77), the diagrams Ai_1, …, Ai_6, folded with the proton-, antiproton- and B_s-meson projector (3.27, 3.30, 2.8), can be expressed at leading power as follows:

Ai_1 :

$$
\begin{aligned}
& i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4}\int \mathcal{D}v \int \mathcal{D}u \\
& \left\{\frac{1}{4u_2u_3\left(-1+u_1v_1\right)v_2v_3\left(-1+u_1\bar{v}_3+u_2\bar{v}_3\right)}\right. \\
& \left[\bar{N}_u^+(P')N_v^+(P)\left(V_1^uV_2^v2v_2+V_1^vV_2^u2\left(-1+u_1v_1+u_2v_1\right)-V_1^uV_{21}^v\bar{v}_1+V_1^vV_{21}^u\bar{v}_1-V_1^uV_{22}^v\bar{v}_1\right.\right. \\
& +V_1^vV_{22}^u\bar{v}_1+V_1^uV_3^v\left(1-v_1^2-v_2-v_1\left(\bar{u}_3-\bar{v}_2\right)\right)+V_1^vV_3^u\left(\bar{u}_1-u_2\right)+A_{21}^vV_1^u\bar{v}_1-A_{21}^uV_1^v\bar{v}_1 \\
& +A_{22}^vV_1^u\bar{v}_1-A_{22}^uV_1^v\bar{v}_1+A_3^vV_1^u\left(1+v_1^2-v_1\left(1+\bar{u}_3-v_2\right)-v_2\right)-A_3^uV_1^vu_3+A_1^vV_{21}^u\bar{v}_1 \\
& -A_1^uV_{21}^v\bar{v}_1+A_1^vV_{22}^u\bar{v}_1-A_1^uV_{22}^v\bar{v}_1+A_1^vV_3^u\left(\bar{u}_1-u_2\right)+A_1^uV_3^v\left(-v_1^2+v_1\left(1+\bar{u}_3-v_2\right)-\bar{v}_2\right) \\
& +A_1^uA_2^v2v_2+A_1^vA_2^u2\left(-1+u_1v_1+u_2v_1\right)+A_1^uA_{21}^v\bar{v}_1-A_1^vA_{21}^u\bar{v}_1+A_1^uA_{22}^v\bar{v}_1-A_1^vA_{22}^u\bar{v}_1 \\
& +A_1^uA_3^v\left(v_1^2+v_1\left(-u_3+v_2\right)-\bar{v}_2\right)-A_1^vA_3^uu_3-T_1^uT_{21}^v2v_3+T_1^vT_{21}^u2v_3+T_1^uT_3^v2v_1v_3 \\
& -T_1^vT_3^u2u_1v_3-T_1^uT_{41}^v2v_3+T_1^vT_{41}^u2v_3+T_1^uT_7^v2v_1v_3-T_1^vT_7^u2u_1v_3-S_1^vT_1^u2v_1v_3 \\
& \left.+S_1^uT_1^v2\left(-2\bar{u}_2+u_1\left(1+\bar{v}_3\right)\right)+P_1^vT_1^u2v_1v_3-P_1^uT_1^v2\left(-2\bar{u}_2+u_1\left(1+\bar{v}_3\right)\right)\right) \\
& +\bar{N}_u^+(P')\gamma_5N_v^+(P)\left(-\left(V_1^uV_{21}^v\bar{v}_1\right)-V_1^vV_{21}^u\bar{v}_1-V_1^uV_{22}^v\bar{v}_1-V_1^vV_{22}^u\bar{v}_1\right. \\
& +V_1^uV_3^v\left(-v_1^2+v_1\left(1+\bar{u}_3-v_2\right)-\bar{v}_2\right)-V_1^vV_3^uu_3+A_2^vV_1^u2v_2 \\
& -A_2^uV_1^v2\left(-1+u_1v_1+u_2v_1\right)+A_{21}^vV_1^u\bar{v}_1+A_{21}^uV_1^v\bar{v}_1+A_{22}^vV_1^u\bar{v}_1+A_{22}^uV_1^v\bar{v}_1 \\
& +A_3^vV_1^u\left(v_1^2+v_1\left(-u_3+v_2\right)-\bar{v}_2\right)+A_3^uV_1^v\left(\bar{u}_1-u_2\right)-A_1^vV_2^u2\left(-1+u_1v_1+u_2v_1\right) \\
& +A_1^uV_2^v2v_2-A_1^vV_{21}^u\bar{v}_1-A_1^uV_{21}^v\bar{v}_1-A_1^vV_{22}^u\bar{v}_1-A_1^uV_{22}^v\bar{v}_1-A_1^vV_3^uu_3 \\
& +A_1^uV_3^v\left(1-v_1^2-v_2-v_1\left(\bar{u}_3-\bar{v}_2\right)\right)+A_1^uA_{21}^v\bar{v}_1+A_1^vA_{21}^u\bar{v}_1+A_1^uA_{22}^v\bar{v}_1+A_1^vA_{22}^u\bar{v}_1 \\
& +A_1^uA_3^v\left(1+v_1^2-v_1\left(1+\bar{u}_3-v_2\right)-v_2\right)+A_1^vA_3^u\left(\bar{u}_1-u_2\right)+T_1^uT_{21}^v2v_3+T_1^vT_{21}^u2v_3 \\
& -T_1^uT_3^v2v_1v_3-T_1^vT_3^u2u_1v_3+T_1^uT_{41}^v2v_3+T_1^vT_{41}^u2v_3-T_1^uT_7^v2v_1v_3-T_1^vT_7^u2u_1v_3 \\
& \left.\left.\left.+S_1^vT_1^u2v_1v_3+S_1^uT_1^v2\left(-2\bar{u}_2+u_1\left(1+\bar{v}_3\right)\right)-P_1^vT_1^u2v_1v_3-P_1^uT_1^v2\left(-2\bar{u}_2+u_1\left(1+\bar{v}_3\right)\right)\right)\right]\right\} \\
& +\mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{3.79}
$$

Ai_2 :

$$
\begin{aligned}
& i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4} \int \mathcal{D}v \int \mathcal{D}u \\
& \Big\{ \frac{1}{4u_2u_3\left(-1+u_1v_1\right)v_2\left(-v_2-u_2\bar{v}_2\right)v_3} \big[\bar{N}^+(P')N^+(P)\left(V_1^uV_2^v 2v_1v_2 + V_1^vV_2^u 2u_2\right. \\
& -V_1^uV_{21}^v v_2 + V_1^vV_{21}^u v_2 + V_1^uV_{22}^v\bar{v}_1 - V_1^vV_{22}^u\bar{v}_1 + V_1^uV_3^v\left(\bar{v}_1 - v_2\right) \\
& +V_1^vV_3^u\left(-u_2-u_2v_1-\bar{v}_1-u_1v_2\right) - A_{21}^vV_1^uv_2 + A_{21}^uV_1^vv_2 - A_{22}^vV_1^u\bar{v}_1 + A_{22}^uV_1^v\bar{v}_1 \\
& -A_3^vV_1^uv_3 + A_3^uV_1^v\left(u_2\left(-1+v_1\right)+\bar{v}_1-u_1v_2\right) - A_1^vV_{21}^uv_2 + A_1^uV_{21}^vv_2 - A_1^vV_{22}^u\bar{v}_1 + A_1^uV_{22}^v\bar{v}_1 \\
& +A_1^vV_3^u\left(u_2-u_2v_1-\bar{v}_1+u_1v_2\right) + A_1^uV_3^v\left(\bar{v}_1-v_2\right) + A_1^uA_2^v 2v_1v_2 + A_1^vA_2^u 2u_2 + A_1^uA_{21}^vv_2 \\
& -A_1^vA_{21}^uv_2 - A_1^uA_{22}^v\bar{v}_1 + A_1^vA_{22}^u\bar{v}_1 - A_1^uA_3^vv_3 + A_1^vA_3^u\left(u_2+u_2v_1+\bar{v}_1+u_1v_2\right) \\
& +T_1^uT_3^v\left(2-2\bar{u}_2v_1\right) + T_1^vT_3^u\left(-2+2\bar{u}_2v_1\right) + T_1^uT_7^v\left(2-2\bar{u}_2v_1\right) + T_1^vT_7^u\left(-2+2\bar{u}_2v_1\right) \\
& +S_1^vT_1^u 2\left(u_2v_1-\bar{v}_1+2v_2\right) + S_1^uT_1^v\left(2-2\bar{u}_2v_1\right) - P_1^vT_1^u 2\left(u_2v_1-\bar{v}_1+2v_2\right) \\
& +P_1^uT_1^v\left(-2+2\bar{u}_2v_1\right)\big) \\
& +\bar{N}^+(P')\gamma_5N^+(P)\left(V_1^uV_{21}^vv_2 + V_1^vV_{21}^uv_2 + V_1^uV_{22}^v\bar{v}_1 + V_1^vV_{22}^u\bar{v}_1\right. \\
& +V_1^uV_3^v\left(\bar{v}_1-v_2\right) + V_1^vV_3^u\left(u_2\left(-1+v_1\right)+\bar{v}_1-u_1v_2\right) + A_2^vV_1^u 2v_1v_2 - A_2^uV_1^v 2u_2 \\
& +A_{21}^vV_1^uv_2 + A_{21}^uV_1^vv_2 - A_{22}^vV_1^u\bar{v}_1 - A_{22}^uV_1^v\bar{v}_1 - A_3^vV_1^uv_3 \\
& +A_3^uV_1^v\left(-u_2-u_2v_1-\bar{v}_1-u_1v_2\right) - A_1^vV_2^u 2u_2 + A_1^uV_2^v 2v_1v_2 - A_1^vV_{21}^uv_2 - A_1^uV_{21}^vv_2 \\
& +A_1^vV_{22}^u\bar{v}_1 + A_1^uV_{22}^v\bar{v}_1 + A_1^vV_3^u\left(u_2+u_2v_1+\bar{v}_1+u_1v_2\right) + A_1^uV_3^v\left(\bar{v}_1-v_2\right) - A_1^uA_{21}^vv_2 \\
& -A_1^vA_{21}^uv_2 - A_1^uA_{22}^v\bar{v}_1 - A_1^vA_{22}^u\bar{v}_1 - A_1^uA_3^vv_3 + A_1^vA_3^u\left(u_2-u_2v_1-\bar{v}_1+u_1v_2\right) \\
& +T_1^uT_3^v\left(-2+2\bar{u}_2v_1\right) + T_1^vT_3^u\left(-2+2\bar{u}_2v_1\right) + T_1^uT_7^v\left(-2+2\bar{u}_2v_1\right) + T_1^vT_7^u\left(-2+2\bar{u}_2v_1\right) \\
& -S_1^vT_1^u 2\left(u_2v_1-\bar{v}_1+2v_2\right) + S_1^uT_1^v\left(2-2\bar{u}_2v_1\right) + P_1^vT_1^u 2\left(u_2v_1-\bar{v}_1+2v_2\right) \\
& +P_1^uT_1^v\left(-2+2\bar{u}_2v_1\right)\big)\big]\Big\} \\
& +\mathcal{O}(\alpha_s^3)
\end{aligned} \tag{3.80}
$$

Ai_3 :

$$
\begin{aligned}
& i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4} \int \mathcal{D}v \int \mathcal{D}u \\
& \Big\{ \frac{1}{4u_2u_3^2v_2\left(-v_2-u_2\bar{v}_2\right)v_3^2} \\
& \big[\bar{N}^+(P')N^+(P)\left(-\left(V_1^vV_2^u 2u_3v_2\right) + V_1^uV_{21}^vv_2 - V_1^vV_{21}^uv_2 + V_1^uV_{22}^vv_2 - V_1^vV_{22}^uv_2\right. \\
& +V_1^uV_3^v\left(-u_3+2v_2-u_1v_2-u_2v_2-v_1v_2-v_2^2\right) + V_1^vV_3^u\left(\bar{u}_1-u_2\right) + A_{21}^vV_1^uv_2 - A_{21}^uV_1^vv_2 \\
& +A_{22}^vV_1^uv_2 - A_{22}^uV_1^vv_2 + A_3^vV_1^u\left(1-v_1v_2-v_2^2-u_1\bar{v}_2-u_2\bar{v}_2\right) - A_3^uV_1^vu_3 + A_1^vV_{21}^uv_2 \\
& -A_1^uV_{21}^vv_2 + A_1^vV_{22}^uv_2 - A_1^uV_{22}^vv_2 + A_1^vV_3^u\left(\bar{u}_1-u_2\right) \\
& +A_1^uV_3^v\left(-u_3-u_1v_2-u_2v_2+v_1v_2+v_2^2\right) - A_1^vA_2^u 2u_3v_2 - A_1^uA_{21}^vv_2 + A_1^vA_{21}^uv_2 - A_1^uA_{22}^vv_2
\end{aligned}
$$

$$
\begin{aligned}
&+A_1^vA_{22}^uv_2+A_1^uA_3^v\left(1-2v_2+v_1v_2+v_2^2-u_1\bar{v}_2-u_2\bar{v}_2\right)-A_1^vA_3^uu_3-T_1^uT_{22}^v2u_3\\
&+T_1^vT_{22}^u2u_3-T_1^uT_3^v2u_3\bar{v}_2+T_1^vT_3^u2\bar{u}_2u_3-T_1^uT_{42}^v2u_3+T_1^vT_{42}^u2u_3-T_1^uT_7^v2u_3\bar{v}_2\\
&+T_1^vT_7^u2\bar{u}_2u_3+S_1^vT_1^u2u_3\bar{v}_2-S_1^uT_1^v2\left(1+u_2\right)u_3-P_1^vT_1^u2u_3\bar{v}_2+P_1^uT_1^v2\left(1+u_2\right)u_3)\\
&+\bar{N}^+(P')\gamma_5N^+(P)\left(-\left(V_1^uV_{21}^vv_2\right)-V_1^vV_{21}^uv_2-V_1^uV_{22}^vv_2-V_1^vV_{22}^uv_2\right.\\
&+V_1^uV_3^v\left(-u_3-u_1v_2-u_2v_2+v_1v_2+v_2^2\right)-V_1^vV_3^uu_3+A_2^uV_1^v2u_3v_2-A_{21}^vV_1^uv_2\\
&-A_{21}^uV_1^vv_2-A_{22}^vV_1^uv_2-A_{22}^uV_1^vv_2+A_3^vV_1^u\left(1-2v_2+v_1v_2+v_2^2-u_1\bar{v}_2-u_2\bar{v}_2\right)\\
&+A_3^uV_1^v\left(\bar{u}_1-u_2\right)+A_1^vV_2^u2u_3v_2+A_1^vV_{21}^uv_2+A_1^uV_{21}^vv_2+A_1^vV_{22}^uv_2+A_1^uV_{22}^vv_2-A_1^vV_3^uu_3\\
&+A_1^uV_3^v\left(-u_3+2v_2-u_1v_2-u_2v_2-v_1v_2-v_2^2\right)+A_1^uA_{21}^vv_2\\
&+A_1^vA_{21}^uv_2+A_1^uA_{22}^vv_2+A_1^vA_{22}^uv_2+A_1^uA_3^v\left(1-v_1v_2-v_2^2-u_1\bar{v}_2-u_2\bar{v}_2\right)\\
&+A_1^vA_3^u\left(\bar{u}_1-u_2\right)+T_1^uT_{22}^v2u_3+T_1^vT_{22}^u2u_3+T_1^uT_3^v2u_3\bar{v}_2+T_1^vT_3^u2\bar{u}_2u_3+T_1^uT_{42}^v2u_3\\
&+T_1^vT_{42}^u2u_3+T_1^uT_7^v2u_3\bar{v}_2+T_1^vT_7^u2\bar{u}_2u_3-S_1^vT_1^u2u_3\bar{v}_2-S_1^uT_1^v2\left(1+u_2\right)u_3+P_1^vT_1^u2u_3\bar{v}_2\\
&\left.+P_1^uT_1^v2\left(1+u_2\right)u_3)\right]\Big\}\\
&+\mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{3.81}
$$

Ai_4 :

$$
\begin{aligned}
&i\alpha_s^216\pi^2f_B\frac{M}{m_b^4}\int\mathcal{D}v\int\mathcal{D}u\\
&\Bigg\{\frac{1}{4u_2^2u_3v_2^2v_3\left(-1+u_1\bar{v}_3+u_2\bar{v}_3\right)}\\
&\Big[\bar{N}^+(P')N^+(P)\left(-\left(V_1^uV_2^v2u_2\bar{v}_3\right)+V_1^vV_2^u2u_2+V_1^uV_{21}^vu_2-V_1^vV_{21}^uu_2+V_1^uV_{22}^vu_2-V_1^vV_{22}^uu_2\right.\\
&+V_1^vV_3^uu_2\left(-2+\bar{u}_3+\bar{v}_3\right)+A_{21}^vV_1^uu_2-A_{21}^uV_1^vu_2+A_{22}^vV_1^uu_2-A_{22}^uV_1^vu_2\\
&+A_3^uV_1^vu_2\left(\bar{u}_3-v_1-v_2\right)+A_1^vV_{21}^uu_2-A_1^uV_{21}^vu_2+A_1^vV_{22}^uu_2-A_1^uV_{22}^vu_2\\
&+A_1^vV_3^uu_2\left(-u_1-u_2+\bar{v}_3\right)-A_1^uA_2^v2u_2\bar{v}_3+A_1^vA_2^u2u_2-A_1^uA_{21}^vu_2+A_1^vA_{21}^uu_2-A_1^uA_{22}^vu_2\\
&+A_1^vA_{22}^uu_2-A_1^vA_3^uu_2\left(-2+\bar{u}_3+\bar{v}_3\right)-T_1^uT_{22}^v2v_3+T_1^vT_{22}^u2v_3+T_1^uT_3^v2v_2v_3-T_1^vT_3^u2u_2v_3\\
&-T_1^uT_{42}^v2v_3+T_1^vT_{42}^u2v_3+T_1^uT_7^v2v_2v_3-T_1^vT_7^u2u_2v_3+S_1^vT_1^u2v_2v_3+S_1^uT_1^v2u_2v_3\\
&\left.-P_1^vT_1^u2v_2v_3-P_1^uT_1^v2u_2v_3\right)\\
&+\bar{N}^+(P')\gamma_5N^+(P)\left(-\left(V_1^uV_{21}^vu_2\right)-V_1^vV_{21}^uu_2-V_1^uV_{22}^vu_2-V_1^vV_{22}^uu_2\right.\\
&+V_1^vV_3^uu_2\left(\bar{u}_3-v_1-v_2\right)-A_2^vV_1^u2u_2\bar{v}_3-A_2^uV_1^v2u_2-A_{21}^vV_1^uu_2-A_{21}^uV_1^vu_2-A_{22}^vV_1^uu_2\\
&-A_{22}^uV_1^vu_2+A_3^uV_1^vu_2\left(-2+\bar{u}_3+\bar{v}_3\right)-A_1^vV_2^u2u_2-A_1^uV_2^v2u_2\bar{v}_3+A_1^vV_{21}^uu_2+A_1^uV_{21}^vu_2\\
&+A_1^vV_{22}^uu_2+A_1^uV_{22}^vu_2-A_1^vV_3^uu_2\left(-2+\bar{u}_3+\bar{v}_3\right)+A_1^uA_{21}^vu_2+A_1^vA_{21}^uu_2+A_1^uA_{22}^vu_2\\
&+A_1^vA_{22}^uu_2+A_1^vA_3^uu_2\left(-u_1-u_2+\bar{v}_3\right)+T_1^uT_{22}^v2v_3+T_1^vT_{22}^u2v_3-T_1^uT_3^v2v_2v_3-T_1^vT_3^u2u_2v_3\\
&+T_1^uT_{42}^v2v_3+T_1^vT_{42}^u2v_3-T_1^uT_7^v2v_2v_3-T_1^vT_7^u2u_2v_3-S_1^vT_1^u2v_2v_3+S_1^uT_1^v2u_2v_3\\
&\left.+P_1^vT_1^u2v_2v_3-P_1^uT_1^v2u_2v_3)\right]\Bigg\}\\
&+\mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{3.82}
$$

Ai_5 :

$$
\begin{aligned}
& i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4} \int \mathcal{D}v \int \mathcal{D}u \\
& \Big\{ \frac{1}{4\bar{u}_1 u_2 u_3^2 \bar{v}_1 v_2 v_3^2} \\
& \big[\bar{N}^+(P')N^+(P) \left(-\left(V_1^u V_2^v 2\bar{u}_1 v_3\right) - V_1^u V_{21}^v \bar{u}_1 + V_1^v V_{21}^u \bar{u}_1 - V_1^u V_{22}^v \bar{u}_1 + V_1^v V_{22}^u \bar{u}_1\right. \\
& -V_1^v V_3^u \bar{u}_1 \left(-2 + \bar{u}_3 + \bar{v}_3\right) + A_{21}^v V_1^u \bar{u}_1 - A_{21}^u V_1^v \bar{u}_1 + A_{22}^v V_1^u \bar{u}_1 - A_{22}^u V_1^v \bar{u}_1 \\
& +A_3^u V_1^v \bar{u}_1 \left(\bar{u}_3 - v_1 - v_2\right) + A_1^v V_{21}^u \bar{u}_1 - A_1^u V_{21}^v \bar{u}_1 + A_1^v V_{22}^u \bar{u}_1 - A_1^u V_{22}^v \bar{u}_1 \\
& -A_1^v V_3^u \bar{u}_1 \left(\bar{u}_3 - v_1 - v_2\right) - A_1^u A_2^v 2\bar{u}_1 v_3 + A_1^u A_{21}^v \bar{u}_1 - A_1^v A_{21}^u \bar{u}_1 + A_1^u A_{22}^v \bar{u}_1 - A_1^v A_{22}^u \bar{u}_1 \\
& +A_1^v A_3^u \bar{u}_1 \left(-2 + \bar{u}_3 + \bar{v}_3\right) - T_1^u T_{21}^v 2u_3 + T_1^v T_{21}^u 2u_3 - T_1^u T_3^v 2u_3 \bar{v}_1 + T_1^v T_3^u 2\bar{u}_1 u_3 - T_1^u T_{41}^v 2u_3 \\
& +T_1^v T_{41}^u 2u_3 - T_1^u T_7^v 2u_3 \bar{v}_1 + T_1^v T_7^u 2\bar{u}_1 u_3 - S_1^v T_1^u 2u_3 \bar{v}_1 - S_1^u T_1^v 2\bar{u}_1 u_3 + P_1^v T_1^u 2u_3 \bar{v}_1 \\
& \left.+P_1^u T_1^v 2\bar{u}_1 u_3\right) \\
& +\bar{N}^+(P')\gamma_5 N^+(P) \left(-\left(V_1^u V_{21}^v \bar{u}_1\right) - V_1^v V_{21}^u \bar{u}_1 - V_1^u V_{22}^v \bar{u}_1 - V_1^v V_{22}^u \bar{u}_1\right. \\
& +V_1^v V_3^u \bar{u}_1 \left(\bar{u}_3 - v_1 - v_2\right) - A_2^v V_1^u 2\bar{u}_1 v_3 + A_{21}^v V_1^u \bar{u}_1 + A_{21}^u V_1^v \bar{u}_1 + A_{22}^v V_1^u \bar{u}_1 + A_{22}^u V_1^v \bar{u}_1 \\
& -A_3^u V_1^v \bar{u}_1 \left(-2 + \bar{u}_3 + \bar{v}_3\right) - A_1^u V_2^v 2\bar{u}_1 v_3 - A_1^v V_{21}^u \bar{u}_1 - A_1^u V_{21}^v \bar{u}_1 - A_1^v V_{22}^u \bar{u}_1 - A_1^u V_{22}^v \bar{u}_1 \\
& +A_1^v V_3^u \bar{u}_1 \left(-2 + \bar{u}_3 + \bar{v}_3\right) + A_1^u A_{21}^v \bar{u}_1 + A_1^v A_{21}^u \bar{u}_1 + A_1^u A_{22}^v \bar{u}_1 + A_1^v A_{22}^u \bar{u}_1 \\
& -A_1^v A_3^u \bar{u}_1 \left(\bar{u}_3 - v_1 - v_2\right) + T_1^u T_{21}^v 2u_3 + T_1^v T_{21}^u 2u_3 + T_1^u T_3^v 2u_3 \bar{v}_1 + T_1^v T_3^u 2\bar{u}_1 u_3 \\
& +T_1^u T_{41}^v 2u_3 + T_1^v T_{41}^u 2u_3 + T_1^u T_7^v 2u_3 \bar{v}_1 + T_1^v T_7^u 2\bar{u}_1 u_3 + S_1^v T_1^u 2u_3 \bar{v}_1 - S_1^u T_1^v 2\bar{u}_1 u_3 \\
& \left.\left.-P_1^v T_1^u 2u_3 \bar{v}_1 + P_1^u T_1^v 2\bar{u}_1 u_3\right)\big]\Big\} \\
& + \mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{3.83}
$$

Ai_6 :

$$
\begin{aligned}
& i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4} \int \mathcal{D}v \int \mathcal{D}u \\
& \Big\{ \frac{1}{4\bar{u}_1 u_2^2 u_3 \bar{v}_1 v_2^2 v_3} \\
& \big[\bar{N}^+(P')N^+(P) \left(V_1^v V_2^u 2\bar{u}_1 u_2 - V_1^u V_{21}^v u_2 + V_1^v V_{21}^u u_2 + V_1^u V_{22}^v \bar{u}_1 - V_1^v V_{22}^u \bar{u}_1\right. \\
& +V_1^u V_3^v \left(-u_2 \bar{v}_1 - \bar{u}_1 v_2\right) - A_{21}^v V_1^u u_2 + A_{21}^u V_1^v u_2 - A_{22}^v V_1^u \bar{u}_1 + A_{22}^u V_1^v \bar{u}_1 \\
& +A_3^v V_1^u \left(-u_2 \bar{v}_1 + v_2 - u_1 v_2\right) - A_1^v V_{21}^u u_2 + A_1^u V_{21}^v u_2 - A_1^v V_{22}^u \bar{u}_1 + A_1^u V_{22}^v \bar{u}_1 \\
& +A_1^u V_3^v \left(u_2 - u_2 v_1 - \bar{u}_1 v_2\right) + A_1^v A_2^u 2\bar{u}_1 u_2 + A_1^u A_{21}^v u_2 - A_1^v A_{21}^u u_2 - A_1^u A_{22}^v \bar{u}_1 + A_1^v A_{22}^u \bar{u}_1 \\
& +A_1^u A_3^v \left(u_2 - u_2 v_1 + v_2 - u_1 v_2\right) - T_1^u T_3^v 2\bar{u}_1 v_2 + T_1^v T_3^u 2\bar{u}_1 v_2 - T_1^u T_7^v 2\bar{u}_1 v_2 + T_1^v T_7^u 2\bar{u}_1 v_2 \\
& \left.+S_1^v T_1^u 2\bar{u}_1 v_2 + S_1^u T_1^v 2\bar{u}_1 v_2 - P_1^v T_1^u 2\bar{u}_1 v_2 - P_1^u T_1^v 2\bar{u}_1 v_2\right) \\
& +\bar{N}^+(P')\gamma_5 N^+(P) \left(V_1^u V_{21}^v u_2 + V_1^v V_{21}^u u_2 + V_1^u V_{22}^v \bar{u}_1 + V_1^v V_{22}^u \bar{u}_1\right. \\
& +V_1^u V_3^v \left(u_2 - u_2 v_1 - \bar{u}_1 v_2\right) - A_2^u V_1^v 2\bar{u}_1 u_2 + A_{21}^v V_1^u u_2 + A_{21}^u V_1^v u_2 - A_{22}^v V_1^u \bar{u}_1 - A_{22}^u V_1^v \bar{u}_1
\end{aligned}
$$

$$+A_3^vV_1^u\left(u_2-u_2v_1+v_2-u_1v_2\right)-A_1^vV_2^u2\bar{u}_1u_2-A_1^vV_{21}^uu_2-A_1^uV_{21}^vu_2+A_1^vV_{22}^u\bar{u}_1$$
$$+A_1^uV_{22}^v\bar{u}_1+A_1^uV_3^v\left(-u_2\bar{v}_1-\bar{u}_1v_2\right)-A_1^uA_{21}^vu_2-A_1^vA_{21}^uu_2-A_1^uA_{22}^v\bar{u}_1-A_1^vA_{22}^u\bar{u}_1$$
$$+A_1^uA_3^v\left(-u_2\bar{v}_1+v_2-u_1v_2\right)+T_1^uT_3^v2\bar{u}_1v_2+T_1^vT_3^u2\bar{u}_1v_2+T_1^uT_7^v2\bar{u}_1v_2+T_1^vT_7^u2\bar{u}_1v_2$$
$$-S_1^vT_1^u2\bar{u}_1v_2+S_1^uT_1^v2\bar{u}_1v_2+P_1^vT_1^u2\bar{u}_1v_2-P_1^uT_1^v2\bar{u}_1v_2)]\}$$
$$+\mathcal{O}(\alpha_s^3) \qquad (3.84)$$

3.3.3 Feynman diagrams Am

The abelian diagrams with exactly one gluon coupling to fermion lines of incoming partons are displayed in Fig. 3.7. After the amplitude for each diagram is expressed by independent Dirac structures and the leading power is identified, we can express the diagrams by 4 dimensional integrations. The factors of (3.77) are left out.

Am_1 :

$$i\alpha_s^2 16\pi^2 f_B\frac{M}{m_b^4}\int\mathcal{D}v\int\mathcal{D}u$$
$$\Big\{\frac{1}{4\bar{u}_1u_3^2\left(-1+u_1v_1\right)\bar{v}_1^2v_3}$$
$$\big[\bar{N}^+(P')N^+(P)\left(V_1^uV_2^v2\bar{v}_1^2+V_1^uV_{21}^v\bar{v}_1-V_1^vV_{21}^u\bar{v}_1-V_1^vV_{22}^u\bar{v}_1\right.$$
$$+V_1^uV_3^v\left(-1-\left(-3+\bar{u}_3\right)v_1-v_1^2\right)-V_1^vV_3^uu_3+A_{21}^vV_1^u\bar{v}_1-A_{21}^uV_1^v\bar{v}_1-A_{22}^uV_1^v\bar{v}_1$$
$$+A_3^vV_1^u\left(1-\left(1+\bar{u}_3\right)v_1+v_1^2\right)-A_3^uV_1^vu_3+A_1^vV_{21}^u\bar{v}_1-A_1^uV_{21}^v\bar{v}_1+A_1^vV_{22}^u\bar{v}_1$$
$$+A_1^vV_3^u\left(\bar{u}_1-u_2\right)+A_1^uV_3^v\left(-1+\left(1+\bar{u}_3\right)v_1-v_1^2\right)+A_1^uA_2^v2\bar{v}_1^2-A_1^uA_{21}^v\bar{v}_1+A_1^vA_{21}^u\bar{v}_1$$
$$+A_1^vA_{22}^u\bar{v}_1+A_1^uA_3^v\left(1+\left(-3+\bar{u}_3\right)v_1+v_1^2\right)+A_1^vA_3^u\left(\bar{u}_1-u_2\right)+T_1^uT_3^v2u_3v_1-T_1^vT_3^u2u_3v_1$$
$$+T_1^uT_7^v2u_3v_1-T_1^vT_7^u2u_3v_1+S_1^vT_1^u2u_3v_1+S_1^uT_1^v2u_3\left(-2+v_1\right)-P_1^vT_1^u2u_3v_1$$
$$\left.-P_1^uT_1^v2u_3\left(-2+v_1\right)\right)$$
$$+\bar{N}^+(P')\gamma_5N^+(P)\left(-\left(V_1^uV_{21}^v\bar{v}_1\right)-V_1^vV_{21}^u\bar{v}_1-V_1^vV_{22}^u\bar{v}_1+V_1^uV_3^v\left(-1+\left(1+\bar{u}_3\right)v_1-v_1^2\right)\right.$$
$$-V_1^vV_3^uu_3+A_2^vV_1^u2\bar{v}_1^2-A_{21}^vV_1^u\bar{v}_1-A_{21}^uV_1^v\bar{v}_1-A_{22}^uV_1^v\bar{v}_1+A_3^vV_1^u\left(1+\left(-3+\bar{u}_3\right)v_1+v_1^2\right)$$
$$-A_3^uV_1^vu_3+A_1^uV_2^v2\bar{v}_1^2+A_1^vV_{21}^u\bar{v}_1+A_1^uV_{21}^v\bar{v}_1+A_1^vV_{22}^u\bar{v}_1+A_1^vV_3^u\left(\bar{u}_1-u_2\right)$$
$$+A_1^uV_3^v\left(-1-\left(-3+\bar{u}_3\right)v_1-v_1^2\right)+A_1^uA_{21}^v\bar{v}_1+A_1^vA_{21}^u\bar{v}_1+A_1^vA_{22}^u\bar{v}_1$$
$$+A_1^uA_3^v\left(1-\left(1+\bar{u}_3\right)v_1+v_1^2\right)+A_1^vA_3^u\left(\bar{u}_1-u_2\right)-T_1^uT_3^v2u_3v_1-T_1^vT_3^u2u_3v_1$$
$$-T_1^uT_7^v2u_3v_1-T_1^vT_7^u2u_3v_1-S_1^vT_1^u2u_3v_1+S_1^uT_1^v2u_3\left(-2+v_1\right)+P_1^vT_1^u2u_3v_1$$
$$\left.-P_1^uT_1^v2u_3\left(-2+v_1\right)\right)\big]\Big\}$$
$$+\mathcal{O}(\alpha_s^3) \qquad (3.85)$$

Am_2 :

$$i\alpha_s^2 16\pi^2 f_B\frac{M}{m_b^4}\int\mathcal{D}v\int\mathcal{D}u$$

$$
\begin{aligned}
&\Big\{\frac{1}{4\bar{u}_1^2 u_3 \left(-1+u_1 v_1\right) \bar{v}_1 v_3^2} \\
&\big[\bar{N}^+(P')N^+(P)\left(V_1^v V_2^u 2\bar{u}_1\bar{v}_1 - V_1^u V_{21}^v v_2 + V_1^v V_{21}^u v_2 - V_1^u V_{22}^v \bar{v}_1 - V_1^u V_3^v v_3\right. \\
&+V_1^v V_3^u \left(-\bar{v}_1 - u_1\left(-2+2v_1+v_2\right)\right) - A_{21}^v V_1^u v_2 + A_{21}^u V_1^v v_2 - A_{22}^v V_1^u \bar{v}_1 - A_3^v V_1^u v_3 \\
&+A_3^u V_1^v \left(\bar{v}_1 - u_1 v_2\right) - A_1^v V_{21}^u v_2 + A_1^u V_{21}^v v_2 + A_1^u V_{22}^v \bar{v}_1 + A_1^v V_3^u \left(-\bar{v}_1 + u_1 v_2\right) \\
&+A_1^u V_3^v \left(\bar{v}_1 - v_2\right) + A_1^v A_2^u 2\bar{u}_1\bar{v}_1 + A_1^u A_{21}^v v_2 - A_1^v A_{21}^u v_2 + A_1^u A_{22}^v \bar{v}_1 + A_1^u A_3^v \left(\bar{v}_1 - v_2\right) \\
&+A_1^v A_3^u \left(\bar{v}_1 + u_1\left(-2+2v_1+v_2\right)\right) + T_1^u T_{21}^v 2v_3 - T_1^v T_{21}^u 2v_3 - T_1^u T_3^v 2v_1 v_3 + T_1^v T_3^u 2u_1 v_3 \\
&+T_1^u T_{41}^v 2v_3 - T_1^v T_{41}^u 2v_3 - T_1^u T_7^v 2v_1 v_3 + T_1^v T_7^u 2u_1 v_3 + S_1^v T_1^u 2\left(-2+v_1\right) v_3 + S_1^u T_1^v 2u_1 v_3 \\
&\left.-P_1^v T_1^u 2\left(-2+v_1\right) v_3 - P_1^u T_1^v 2u_1 v_3\right) \\
&+\bar{N}^+(P')\gamma_5 N^+(P)\left(V_1^u V_{21}^v v_2 + V_1^v V_{21}^u v_2 + V_1^u V_{22}^v \bar{v}_1 + V_1^u V_3^v \left(\bar{v}_1 - v_2\right) + V_1^v V_3^u \left(\bar{v}_1 - u_1 v_2\right)\right. \\
&+A_2^u V_1^v 2\left(u_1 - u_1 v_1 - \bar{v}_1\right) + A_{21}^v V_1^u v_2 + A_{21}^u V_1^v v_2 + A_{22}^v V_1^u \bar{v}_1 + A_3^v V_1^u \left(\bar{v}_1 - v_2\right) \\
&+A_3^u V_1^v \left(-\bar{v}_1 - u_1\left(-2+2v_1+v_2\right)\right) + A_1^v V_2^u 2\left(u_1 - u_1 v_1 - \bar{v}_1\right) - A_1^v V_{21}^u v_2 - A_1^u V_{21}^v v_2 \\
&-A_1^u V_{22}^v \bar{v}_1 + A_1^v V_3^u \left(\bar{v}_1 + u_1\left(-2+2v_1+v_2\right)\right) - A_1^u V_3^v v_3 - A_1^u A_{21}^v v_2 - A_1^v A_{21}^u v_2 - A_1^u A_{22}^v \bar{v}_1 \\
&-A_1^u A_3^v v_3 + A_1^v A_3^u \left(-\bar{v}_1 + u_1 v_2\right) - T_1^u T_{21}^v 2v_3 - T_1^v T_{21}^u 2v_3 + T_1^u T_3^v 2v_1 v_3 + T_1^v T_3^u 2u_1 v_3 \\
&-T_1^u T_{41}^v 2v_3 - T_1^v T_{41}^u 2v_3 + T_1^u T_7^v 2v_1 v_3 + T_1^v T_7^u 2u_1 v_3 - S_1^v T_1^u 2\left(-2+v_1\right) v_3 + S_1^u T_1^v 2u_1 v_3 \\
&\left.+P_1^v T_1^u 2\left(-2+v_1\right) v_3 - P_1^u T_1^v 2u_1 v_3\right)\big]\Big\} \\
&+\mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{3.86}
$$

Am_3 :

$$
\begin{aligned}
&i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4}\int \mathcal{D}v \int \mathcal{D}u \\
&\Big\{\frac{1}{4u_2 u_3^2 v_2 \bar{v}_2 \left(-v_2 - u_2\bar{v}_2\right) v_3} \\
&\big[\bar{N}^+(P')N^+(P)\left(V_1^u V_2^v 2v_2\bar{v}_2 - V_1^v V_{21}^u v_2 + V_1^u V_{22}^v v_2 - V_1^v V_{22}^u v_2 - V_1^u V_3^v \left(\bar{u}_3 - \bar{v}_2\right)\bar{v}_2\right. \\
&-V_1^v V_3^u u_3 + A_{21}^u V_1^v v_2 - A_{22}^v V_1^u v_2 + A_{22}^u V_1^v v_2 + A_3^v V_1^u \left(-u_3 - v_2\right)\bar{v}_2 + A_3^u V_1^v \left(\bar{u}_1 - u_2\right) \\
&-A_1^v V_{21}^u v_2 - A_1^v V_{22}^u v_2 + A_1^u V_{22}^v v_2 - A_1^v V_3^u u_3 - A_1^u V_3^v \left(-u_3 - v_2\right)\bar{v}_2 + A_1^u A_2^v 2v_2\bar{v}_2 \\
&+A_1^v A_{21}^u v_2 - A_1^u A_{22}^v v_2 + A_1^v A_{22}^u v_2 + A_1^u A_3^v \left(\bar{u}_3 - \bar{v}_2\right)\bar{v}_2 + A_1^v A_3^u \left(\bar{u}_1 - u_2\right) + T_1^u T_{22}^v 2u_3 \\
&-T_1^v T_{22}^u 2u_3 + T_1^u T_3^v 2u_3\bar{v}_2 - T_1^v T_3^u 2\bar{u}_2 u_3 + T_1^u T_{42}^v 2u_3 - T_1^v T_{42}^u 2u_3 + T_1^u T_7^v 2u_3\bar{v}_2 \\
&\left.-T_1^v T_7^u 2\bar{u}_2 u_3 - S_1^v T_1^u 2u_3\bar{v}_2 + S_1^u T_1^v 2\left(1+u_2\right) u_3 + P_1^v T_1^u 2u_3\bar{v}_2 - P_1^u T_1^v 2\left(1+u_2\right) u_3\right) \\
&+\bar{N}^+(P')\gamma_5 N^+(P)\left(V_1^v V_{21}^u v_2 + V_1^u V_{22}^v v_2 + V_1^v V_{22}^u v_2 - V_1^u V_3^v \left(-u_3 - v_2\right)\bar{v}_2\right. \\
&+V_1^v V_3^u \left(\bar{u}_1 - u_2\right) + A_2^v V_1^u 2v_2\bar{v}_2 - A_{21}^u V_1^v v_2 - A_{22}^v V_1^u v_2 - A_{22}^u V_1^v v_2 + A_3^v V_1^u \left(\bar{u}_3 - \bar{v}_2\right)\bar{v}_2 \\
&-A_3^u V_1^v u_3 + A_1^u V_2^v 2v_2\bar{v}_2 + A_1^v V_{21}^u v_2 + A_1^v V_{22}^u v_2 + A_1^u V_{22}^v v_2 + A_1^v V_3^u \left(\bar{u}_1 - u_2\right) \\
&-A_1^u V_3^v \left(\bar{u}_3 - \bar{v}_2\right)\bar{v}_2 - A_1^v A_{21}^u v_2 - A_1^u A_{22}^v v_2 - A_1^v A_{22}^u v_2 + A_1^u A_3^v \left(-u_3 - v_2\right)\bar{v}_2 - A_1^v A_3^u u_3 \\
&-T_1^u T_{22}^v 2u_3 - T_1^v T_{22}^u 2u_3 - T_1^u T_3^v 2u_3\bar{v}_2 - T_1^v T_3^u 2\bar{u}_2 u_3 - T_1^u T_{42}^v 2u_3 - T_1^v T_{42}^u 2u_3 \\
&-T_1^u T_7^v 2u_3\bar{v}_2 - T_1^v T_7^u 2\bar{u}_2 u_3 + S_1^v T_1^u 2u_3\bar{v}_2 + S_1^u T_1^v 2\left(1+u_2\right) u_3 - P_1^v T_1^u 2u_3\bar{v}_2
\end{aligned}
$$

$$-P_1^uT_1^v2\,(1+u_2)\,u_3)]\}$$
$$+\,\mathcal{O}(\alpha_s^3) \tag{3.87}$$

Am_4 :

$$i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4}\int \mathcal{D}v\int \mathcal{D}u$$
$$\Big\{\frac{1}{4u_2\bar{u}_2u_3v_2\,(-v_2-u_2\bar{v}_2)\,v_3^2}$$
$$\big[\bar{N}^+(P')N^+(P)\,(V_1^vV_2^u2\bar{u}_2v_2-V_1^uV_{21}^vv_2-V_1^uV_{22}^v\bar{v}_1+V_1^vV_{22}^u\bar{v}_1-V_1^uV_3^vv_3$$
$$-V_1^vV_3^u\bar{u}_2\,(-\bar{v}_1+2v_2)+A_{21}^vV_1^uv_2+A_{22}^vV_1^u\bar{v}_1-A_{22}^uV_1^v\bar{v}_1+A_3^vV_1^u\,(\bar{v}_1-v_2)$$
$$+A_3^uV_1^v\,(u_2-u_2v_1-\bar{v}_1)-A_1^uV_{21}^vv_2+A_1^vV_{22}^u\bar{v}_1-A_1^uV_{22}^v\bar{v}_1+A_1^vV_3^u\bar{u}_2\bar{v}_1-A_1^uV_3^vv_3$$
$$+A_1^vA_2^u2\bar{u}_2v_2+A_1^uA_{21}^vv_2+A_1^uA_{22}^v\bar{v}_1-A_1^vA_{22}^u\bar{v}_1+A_1^uA_3^v\,(\bar{v}_1-v_2)+A_1^vA_3^u\bar{u}_2\,(-\bar{v}_1+2v_2)$$
$$-T_1^uT_3^v2\bar{u}_2v_3+T_1^vT_3^u2\bar{u}_2v_3-T_1^uT_7^v2\bar{u}_2v_3+T_1^vT_7^u2\bar{u}_2v_3+S_1^vT_1^u2\,(1+u_2)\,v_3-S_1^uT_1^v2\bar{u}_2v_3$$
$$-P_1^vT_1^u2\,(1+u_2)\,v_3+P_1^uT_1^v2\bar{u}_2v_3)$$
$$+\bar{N}^+(P')\gamma_5N^+(P)\,(-\,(V_1^uV_{21}^vv_2)-V_1^uV_{22}^v\bar{v}_1-V_1^vV_{22}^u\bar{v}_1-V_1^uV_3^vv_3$$
$$+V_1^vV_3^u\,(u_2-u_2v_1-\bar{v}_1)-A_2^uV_1^v2\bar{u}_2v_2+A_{21}^vV_1^uv_2+A_{22}^vV_1^u\bar{v}_1+A_{22}^uV_1^v\bar{v}_1$$
$$+A_3^vV_1^u\,(\bar{v}_1-v_2)-A_3^uV_1^v\bar{u}_2\,(-\bar{v}_1+2v_2)-A_1^vV_2^u2\bar{u}_2v_2-A_1^uV_{21}^vv_2-A_1^vV_{22}^u\bar{v}_1-A_1^uV_{22}^v\bar{v}_1$$
$$+A_1^vV_3^u\bar{u}_2\,(-\bar{v}_1+2v_2)-A_1^uV_3^vv_3+A_1^uA_{21}^vv_2+A_1^uA_{22}^v\bar{v}_1+A_1^vA_{22}^u\bar{v}_1+A_1^uA_3^v\,(\bar{v}_1-v_2)$$
$$+A_1^vA_3^u\bar{u}_2\bar{v}_1+T_1^uT_3^v2\bar{u}_2v_3+T_1^vT_3^u2\bar{u}_2v_3+T_1^uT_7^v2\bar{u}_2v_3+T_1^vT_7^u2\bar{u}_2v_3-S_1^vT_1^u2\,(1+u_2)\,v_3$$
$$-S_1^uT_1^v2\bar{u}_2v_3+P_1^vT_1^u2\,(1+u_2)\,v_3+P_1^uT_1^v2\bar{u}_2v_3)]\Big\}$$
$$+\,\mathcal{O}(\alpha_s^3) \tag{3.88}$$

Am_5 :

$$i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4}\int \mathcal{D}v\int \mathcal{D}u$$
$$\Big\{\frac{1}{4\bar{u}_1u_2^2\,(-1+u_1v_1)\,\bar{v}_1^3}\,\big[\bar{N}^+(P')N^+(P)\,\big(-\big(V_1^uV_2^v\bar{v}_1^2\big)+V_1^vV_2^u\,(u_2-u_2v_1)$$
$$+V_1^uV_3^v\,\big(1+(-2+u_2)\,v_1+v_1^2\big)-V_1^vV_3^uu_2-A_2^vV_1^u\bar{v}_1^2-A_2^uV_1^vu_2\bar{v}_1$$
$$+A_3^vV_1^u\,\big(-1+(2+u_2)\,v_1-v_1^2\big)-A_3^uV_1^vu_2-A_1^vV_2^uu_2\bar{v}_1-A_1^uV_2^v\bar{v}_1^2+A_1^vV_3^uu_2$$
$$+A_1^uV_3^v\,\big(1-(2+u_2)\,v_1+v_1^2\big)-A_1^uA_2^v\bar{v}_1^2+A_1^vA_2^u\,(u_2-u_2v_1)$$
$$+A_1^uA_3^v\,\big(-1+2v_1-u_2v_1-v_1^2\big)+A_1^vA_3^uu_2+T_1^uT_{21}^v\,(2-2v_1)+T_1^vT_{22}^u\,(2-2v_1)$$
$$+T_1^uT_3^v2\,\big(1+(-2+u_2)\,v_1+v_1^2\big)-T_1^vT_3^u2u_2+T_1^uT_{41}^v\,(2-2v_1)+T_1^vT_{42}^u\,(2-2v_1)$$
$$+T_1^uT_7^v2\,\big(1+(-2+u_2)\,v_1+v_1^2\big)-T_1^vT_7^u2u_2+S_1^vT_1^u\,\big(-2+2\,(2+u_2)\,v_1-2v_1^2\big)$$
$$-S_1^uT_1^v2u_2+P_1^vT_1^u2\,\big(1-(2+u_2)\,v_1+v_1^2\big)+P_1^uT_1^v2u_2\big)$$

$$
\begin{aligned}
&+\bar{N}^{+}(P')\gamma_5 N^{+}(P)\left(-\left(V_1^u V_2^v \bar{v}_1^2\right)+V_1^v V_2^u\left(u_2-u_2 v_1\right)\right.\\
&+V_1^u V_3^v\left(1-\left(2+u_2\right)v_1+v_1^2\right)-V_1^v V_3^u u_2-A_2^v V_1^u \bar{v}_1^2-A_2^u V_1^v u_2 \bar{v}_1\\
&+A_3^v V_1^u\left(-1+2v_1-u_2 v_1-v_1^2\right)-A_3^u V_1^v u_2-A_1^v V_2^u u_2 \bar{v}_1-A_1^u V_2^v \bar{v}_1^2+A_1^v V_3^u u_2\\
&+A_1^u V_3^v\left(1+\left(-2+u_2\right)v_1+v_1^2\right)-A_1^u A_2^v \bar{v}_1^2+A_1^v A_2^u\left(u_2-u_2 v_1\right)\\
&+A_1^u A_3^v\left(-1+\left(2+u_2\right)v_1-v_1^2\right)+A_1^v A_3^u u_2-T_1^u T_{21}^v 2\bar{v}_1+T_1^v T_{22}^u\left(2-2v_1\right)\\
&-T_1^u T_3^v 2\left(1+\left(-2+u_2\right)v_1+v_1^2\right)-T_1^v T_3^u 2u_2-T_1^u T_{41}^v 2\bar{v}_1+T_1^v T_{42}^u\left(2-2v_1\right)\\
&-T_1^u T_7^v 2\left(1+\left(-2+u_2\right)v_1+v_1^2\right)-T_1^v T_7^u 2u_2+S_1^v T_1^u 2\left(1-\left(2+u_2\right)v_1+v_1^2\right)-S_1^u T_1^v 2u_2\\
&\left.+P_1^v T_1^u\left(-2+2\left(2+u_2\right)v_1-2v_1^2\right)+P_1^u T_1^v 2u_2\right)\Big]\Big\}\\
&+\mathcal{O}(\alpha_s^3) \qquad (3.89)
\end{aligned}
$$

Am_6 :

$$
\begin{aligned}
&i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4}\int \mathcal{D}v \int \mathcal{D}u\\
&\left\{-\frac{1}{4\bar{u}_1^2 u_2\left(-1+u_1 v_1\right)\bar{v}_1 v_2^2}\right.\\
&\Big[\bar{N}^{+}(P')N^{+}(P)\left(-\left(V_1^u V_2^v \bar{v}_1 v_2\right)+V_1^v V_2^u \bar{u}_1 \bar{v}_1-V_1^u V_{21}^v v_2+V_1^v V_{21}^u v_2+V_1^u V_3^v v_2\right.\\
&+V_1^v V_3^u\left(-\bar{v}_1+u_1 v_3\right)+A_2^v V_1^u\left(v_2-v_1 v_2\right)+A_2^u V_1^v \bar{u}_1 \bar{v}_1-A_{21}^v V_1^u v_2+A_{21}^u V_1^v v_2+A_3^v V_1^u v_2\\
&+A_3^u V_1^v\left(\bar{v}_1+u_1\left(-1+v_1-v_2\right)\right)+A_1^v V_2^u \bar{u}_1 \bar{v}_1+A_1^u V_2^v\left(v_2-v_1 v_2\right)-A_1^v V_{21}^u v_2+A_1^u V_{21}^v v_2\\
&+A_1^v V_3^u\left(-\bar{v}_1+u_1\left(1-v_1+v_2\right)\right)-A_1^u V_3^v v_2-A_1^u A_2^v \bar{v}_1 v_2+A_1^v A_2^u \bar{u}_1 \bar{v}_1+A_1^u A_{21}^v v_2\\
&-A_1^v A_{21}^u v_2-A_1^u A_3^v v_2+A_1^v A_3^u\left(\bar{v}_1-u_1 v_3\right)-T_1^u T_{21}^v 2v_2-T_1^v T_{21}^u 2v_3-T_1^u T_{22}^v 2\bar{v}_1\\
&+T_1^u T_3^v 2v_2+T_1^v T_3^u 2\left(-\bar{v}_1+u_1 v_3\right)-T_1^u T_{41}^v 2v_2-T_1^v T_{41}^u 2v_3-T_1^u T_{42}^v 2\bar{v}_1+T_1^u T_7^v 2v_2\\
&+T_1^v T_7^u 2\left(-\bar{v}_1+u_1 v_3\right)+S_1^v T_1^u 2v_2+S_1^u T_1^v\left(2-2v_1+2u_1\left(-\bar{v}_1-v_2\right)\right)-P_1^v T_1^u 2v_2\\
&\left.+P_1^u T_1^v 2\left(-\bar{v}_1+u_1\left(1-v_1+v_2\right)\right)\right)\\
&+\bar{N}^{+}(P')\gamma_5 N^{+}(P)\left(V_1^u V_2^v\left(v_2-v_1 v_2\right)+V_1^v V_2^u\left(u_1-u_1 v_1-\bar{v}_1\right)+V_1^u V_{21}^v v_2+V_1^v V_{21}^u v_2\right.\\
&-V_1^u V_3^v v_2+V_1^v V_3^u\left(\bar{v}_1+u_1\left(-1+v_1-v_2\right)\right)-A_2^v V_1^u \bar{v}_1 v_2+A_2^u V_1^v\left(u_1-u_1 v_1-\bar{v}_1\right)\\
&+A_{21}^v V_1^u v_2+A_{21}^u V_1^v v_2-A_3^v V_1^u v_2+A_3^u V_1^v\left(-\bar{v}_1+u_1 v_3\right)+A_1^v V_2^u\left(u_1-u_1 v_1-\bar{v}_1\right)\\
&-A_1^u V_2^v \bar{v}_1 v_2-A_1^v V_{21}^u v_2-A_1^u V_{21}^v v_2+A_1^v V_3^u\left(\bar{v}_1-u_1 v_3\right)+A_1^u V_3^v v_2+A_1^u A_2^v\left(v_2-v_1 v_2\right)\\
&+A_1^v A_2^u\left(u_1-u_1 v_1-\bar{v}_1\right)-A_1^u A_{21}^v v_2-A_1^v A_{21}^u v_2+A_1^u A_3^v v_2\\
&+A_1^v A_3^u\left(-\bar{v}_1+u_1\left(1-v_1+v_2\right)\right)+T_1^u T_{21}^v 2v_2-T_1^v T_{21}^u 2v_3+T_1^u T_{22}^v\left(2-2v_1\right)-T_1^u T_3^v 2v_2\\
&+T_1^v T_3^u 2\left(-\bar{v}_1+u_1 v_3\right)+T_1^u T_{41}^v 2v_2-T_1^v T_{41}^u 2v_3+T_1^u T_{42}^v\left(2-2v_1\right)-T_1^u T_7^v 2v_2\\
&+T_1^v T_7^u 2\left(-\bar{v}_1+u_1 v_3\right)-S_1^v T_1^u 2v_2+S_1^u T_1^v\left(2-2v_1+2u_1\left(-\bar{v}_1-v_2\right)\right)+P_1^v T_1^u 2v_2\\
&\left.+P_1^u T_1^v 2\left(-\bar{v}_1+u_1\left(1-v_1+v_2\right)\right)\right)\Big]\Big\}\\
&+\mathcal{O}(\alpha_s^3) \qquad (3.90)
\end{aligned}
$$

Am_7 :

$$
\begin{aligned}
& i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4}\int \mathcal{D}v \int \mathcal{D}u \\
& \Big\{ -\frac{1}{4u_2^2 u_3 v_2 \left(v_1^2+v_1\left(-1+2v_2\right)-v_2\bar{v}_2\right)\left(-1+u_1\bar{v}_3+u_2\bar{v}_3\right)} \\
& \big[\bar{N}^+(P')N^+(P)\left(V_1^u V_2^v 2u_2\bar{v}_3 - V_1^v V_2^u 2u_2 + V_1^u V_{21}^v\left(-u_2-v_3\right)\right. \\
& +V_1^v V_{21}^u u_2 + V_1^u V_{22}^v\left(-u_2-v_3\right) + V_1^v V_{22}^u\left(u_2+\bar{v}_1-v_2\right) + V_1^u V_3^v\left(-v_1^2-2v_1v_2-v_2^2+\bar{v}_3\right) \\
& +V_1^v V_3^u u_2u_3 + A_{21}^v V_1^u\left(-u_2+\bar{v}_1-v_2\right) + A_{21}^u V_1^v u_2 + A_{22}^v V_1^u\left(-u_2+\bar{v}_1-v_2\right) \\
& +A_{22}^u V_1^v\left(u_2-v_3\right) + A_3^v V_1^u\left(v_1^2+v_1\left(-1+2v_2\right)-v_2\bar{v}_2\right) + A_3^u V_1^v u_2u_3 - A_1^v V_{21}^u u_2 \\
& +A_1^u V_{21}^v\left(u_2-v_3\right) + A_1^v V_{22}^u\left(-u_2+\bar{v}_1-v_2\right) + A_1^u V_{22}^v\left(u_2-v_3\right) - A_1^v V_3^u u_2u_3 \\
& +A_1^u V_3^v\left(-v_1^2-2v_1v_2-v_2^2+\bar{v}_3\right) + A_1^u A_2^v 2u_2\bar{v}_3 - A_1^v A_2^u 2u_2 + A_1^u A_{21}^v\left(u_2+\bar{v}_1-v_2\right) \\
& -A_1^v A_{21}^u u_2 + A_1^u A_{22}^v\left(u_2+\bar{v}_1-v_2\right) + A_1^v A_{22}^u\left(-u_2-v_3\right) + A_1^u A_3^v\left(v_1^2+v_1\left(-1+2v_2\right)-v_2\bar{v}_2\right) \\
& -A_1^v A_3^u u_2u_3 + S_1^v T_1^u 4\left(v_1^2+v_1\left(-1+2v_2\right)-v_2\bar{v}_2\right) - P_1^v T_1^u 4\left(v_1^2+v_1\left(-1+2v_2\right)-v_2\bar{v}_2\right)\big) \\
& +\bar{N}^+(P')\gamma_5 N^+(P)\left(V_1^u V_{21}^v\left(u_2-v_3\right) + V_1^v V_{21}^u u_2 + V_1^u V_{22}^v\left(u_2-v_3\right) + V_1^v V_{22}^u\left(u_2-v_3\right)\right. \\
& +V_1^u V_3^v\left(-v_1^2-2v_1v_2-v_2^2+\bar{v}_3\right) + V_1^v V_3^u u_2u_3 + A_2^v V_1^u 2u_2\bar{v}_3 + A_2^u V_1^v 2u_2 \\
& +A_{21}^v V_1^u\left(u_2+\bar{v}_1-v_2\right) + A_{21}^u V_1^v u_2 + A_{22}^v V_1^u\left(u_2+\bar{v}_1-v_2\right) + A_{22}^u V_1^v\left(u_2+\bar{v}_1-v_2\right) \\
& +A_3^v V_1^u\left(v_1^2+v_1\left(-1+2v_2\right)-v_2\bar{v}_2\right) + A_3^u V_1^v u_2u_3 + A_1^v V_2^u 2u_2 + A_1^u V_2^v 2u_2\bar{v}_3 - A_1^v V_{21}^u u_2 \\
& +A_1^u V_{21}^v\left(-u_2-v_3\right) + A_1^v V_{22}^u\left(-u_2-v_3\right) + A_1^u V_{22}^v\left(-u_2-v_3\right) - A_1^v V_3^u u_2u_3 \\
& +A_1^u V_3^v\left(-v_1^2-2v_1v_2-v_2^2+\bar{v}_3\right) + A_1^u A_{21}^v\left(-u_2+\bar{v}_1-v_2\right) - A_1^v A_{21}^u u_2 \\
& +A_1^u A_{22}^v\left(-u_2+\bar{v}_1-v_2\right) + A_1^v A_{22}^u\left(-u_2+\bar{v}_1-v_2\right) + A_1^u A_3^v\left(v_1^2+v_1\left(-1+2v_2\right)-v_2\bar{v}_2\right) \\
& -A_1^v A_3^u u_2u_3 - S_1^v T_1^u 4\left(v_1^2+v_1\left(-1+2v_2\right)-v_2\bar{v}_2\right) \\
& \left.+P_1^v T_1^u 4\left(v_1^2+v_1\left(-1+2v_2\right)-v_2\bar{v}_2\right)\right)\big]\Big\} \\
& +\mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{3.91}
$$

Am_8 :

$$
\begin{aligned}
& i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4}\int \mathcal{D}v \int \mathcal{D}u \\
& \Big\{ \frac{1}{4u_2u_3\bar{u}_3 v_2^2 v_3\left(-1+u_1\bar{v}_3+u_2\bar{v}_3\right)}\big[\bar{N}^+(P')N^+(P)\left(-\left(V_1^u V_2^v 2v_2\right) + V_1^v V_2^u 2\bar{u}_3 v_2\right. \\
& +V_1^u V_{21}^v v_2 - V_1^v V_{21}^u \bar{v}_1 + V_1^u V_{22}^v \bar{v}_1 - V_1^v V_{22}^u \bar{v}_1 + V_1^u V_3^v u_3v_2 + V_1^v V_3^u \bar{u}_3v_3 - A_{21}^v V_1^u v_2 \\
& +A_{21}^u V_1^v \bar{v}_1 - A_{22}^v V_1^u \bar{v}_1 + A_{22}^u V_1^v \bar{v}_1 + A_3^v V_1^u u_3v_2 - A_3^u V_1^v \bar{u}_3v_3 - A_1^v V_{21}^u \bar{v}_1 + A_1^u V_{21}^v v_2 \\
& -A_1^v V_{22}^u \bar{v}_1 + A_1^u V_{22}^v \bar{v}_1 + A_1^v V_3^u \bar{u}_3v_3 - A_1^u V_3^v u_3v_2 - A_1^u A_2^v 2v_2 + A_1^v A_2^u 2\bar{u}_3v_2 - A_1^u A_{21}^v v_2 \\
& +A_1^v A_{21}^u \bar{v}_1 - A_1^u A_{22}^v \bar{v}_1 + A_1^v A_{22}^u \bar{v}_1 - A_1^u A_3^v u_3v_2 - A_1^v A_3^u \bar{u}_3v_3 - S_1^u T_1^v 4\bar{u}_3v_3 + P_1^u T_1^v 4\bar{u}_3v_3\big)
\end{aligned}
$$

$$
\begin{aligned}
&+\bar{N}^+(P')\gamma_5 N^+(P)\,(V_1^u V_{21}^v v_2 + V_1^v V_{21}^u \bar{v}_1 + V_1^u V_{22}^v \bar{v}_1 + V_1^v V_{22}^u \bar{v}_1 - V_1^u V_3^v u_3 v_2 - V_1^v V_3^u \bar{u}_3 v_3\\
&-A_2^v V_1^u 2 v_2 - A_2^u V_1^v 2\bar{u}_3 v_2 - A_{21}^v V_1^u v_2 - A_{21}^u V_1^v \bar{v}_1 - A_{22}^v V_1^u \bar{v}_1 - A_{22}^u V_1^v \bar{v}_1 - A_3^v V_1^u u_3 v_2\\
&+A_3^u V_1^v \bar{u}_3 v_3 - A_1^v V_2^u 2\bar{u}_3 v_2 - A_1^u V_2^v 2 v_2 + A_1^v V_{21}^u \bar{v}_1 + A_1^u V_{21}^v v_2\\
&+A_1^v V_{22}^u \bar{v}_1 + A_1^u V_{22}^v \bar{v}_1 - A_1^v V_3^u \bar{u}_3 v_3 + A_1^u V_3^v u_3 v_2 - A_1^u A_{21}^v v_2 - A_1^v A_{21}^u \bar{v}_1 - A_1^u A_{22}^v \bar{v}_1\\
&-A_1^v A_{22}^u \bar{v}_1 + A_1^u A_3^v u_3 v_2 + A_1^v A_3^u \bar{u}_3 v_3 - S_1^u T_1^v 4\bar{u}_3 v_3 + P_1^u T_1^v 4\bar{u}_3 v_3)]\}\\
&+\mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{3.92}
$$

Am_9 :

$$
\begin{aligned}
&i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4}\int \mathcal{D}v \int \mathcal{D}u\\
&\Big\{-\frac{1}{4\bar{u}_1^2 u_3 \bar{v}_1^2 v_3^2}\\
&\big[\bar{N}^+(P')N^+(P)\,(V_1^v V_2^u\,(2-2u_1) - V_1^u V_{21}^v + V_1^v V_{21}^u - V_1^u V_{22}^v\\
&+V_1^v V_3^u\,(-2+u_1+\bar{v}_3) - A_{21}^v V_1^u + A_{21}^u V_1^v - A_{22}^v V_1^u + A_3^u V_1^v\,(-u_1+\bar{v}_3) - A_1^v V_{21}^u + A_1^u V_{21}^v\\
&+A_1^u V_{22}^v + A_1^v V_3^u\,(u_1-v_1-v_2) + A_1^v A_2^u\,(2-2u_1) + A_1^u A_{21}^v - A_1^v A_{21}^u + A_1^u A_{22}^v\\
&+A_1^v A_3^u\,(2-u_1-v_1-v_2) + T_1^u T_3^v 2v_3 - T_1^v T_3^u 2v_3 + T_1^u T_7^v 2v_3 - T_1^v T_7^u 2v_3 - S_1^v T_1^u 2v_3\\
&-S_1^u T_1^v 2v_3 + P_1^v T_1^u 2v_3 + P_1^u T_1^v 2v_3)\\
&+\bar{N}^+(P')\gamma_5 N^+(P)\,(V_1^u V_{21}^v + V_1^v V_{21}^u + V_1^u V_{22}^v + V_1^v V_3^u\,(-u_1+\bar{v}_3) - A_2^u V_1^v 2\bar{u}_1 + A_{21}^v V_1^u\\
&+A_{21}^u V_1^v + A_{22}^v V_1^u + A_3^u V_1^v\,(-2+u_1+\bar{v}_3) - A_1^v V_2^u 2\bar{u}_1 - A_1^v V_{21}^u - A_1^u V_{21}^v\\
&-A_1^u V_{22}^v + A_1^v V_3^u\,(2-u_1-v_1-v_2) - A_1^u A_{21}^v - A_1^v A_{21}^u - A_1^u A_{22}^v\\
&+A_1^v A_3^u\,(u_1-v_1-v_2) - T_1^u T_3^v 2v_3 - T_1^v T_3^u 2v_3 - T_1^u T_7^v 2v_3 - T_1^v T_7^u 2v_3 + S_1^v T_1^u 2v_3\\
&-S_1^u T_1^v 2v_3 - P_1^v T_1^u 2v_3 + P_1^u T_1^v 2v_3)]\Big\}\\
&+\mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{3.93}
$$

Am_{10} :

$$
\begin{aligned}
&i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4}\int \mathcal{D}v \int \mathcal{D}u\\
&\Big\{\frac{1}{4\bar{u}_1^2 u_3^2 \bar{v}_1^3 v_3}\\
&\big[\bar{N}^+(P')N^+(P)\,(V_1^u V_2^v 2\,(u_1-u_1v_1-\bar{v}_1) - V_1^u V_{21}^v u_2 + V_1^v V_{21}^u u_2 + V_1^v V_{22}^u \bar{u}_1\\
&-V_1^u V_3^v\,(-2+2u_1+u_2)\,\bar{v}_1 - A_{21}^v V_1^u u_2 + A_{21}^u V_1^v u_2 + A_{22}^u V_1^v \bar{u}_1 - A_3^v V_1^u u_2 \bar{v}_1 - A_1^v V_{21}^u u_2\\
&+A_1^u V_{21}^v u_2 - A_1^v V_{22}^u \bar{u}_1 + A_1^u V_3^v\,(u_2-u_2v_1) + A_1^u A_2^v 2\,(u_1-u_1v_1-\bar{v}_1) + A_1^u A_{21}^v u_2\\
&-A_1^v A_{21}^u u_2 - A_1^v A_{22}^u \bar{u}_1 + A_1^u A_3^v\,(-2+2u_1+u_2)\,\bar{v}_1 + T_1^u T_{21}^v 2u_3 - T_1^v T_{21}^u 2u_3 + T_1^u T_3^v 2u_3\bar{v}_1\\
&-T_1^v T_3^u 2\bar{u}_1 u_3 + T_1^u T_{41}^v 2u_3 - T_1^v T_{41}^u 2u_3 + T_1^u T_7^v 2u_3\bar{v}_1 - T_1^v T_7^u 2\bar{u}_1 u_3 + S_1^v T_1^u 2u_3\bar{v}_1
\end{aligned}
$$

$$
\begin{aligned}
&+S_1^uT_1^v2\bar{u}_1u_3-P_1^vT_1^u2u_3\bar{v}_1-P_1^uT_1^v2\bar{u}_1u_3)\\
&+\bar{N}^+(P')\gamma_5N^+(P)\left(V_1^uV_{21}^vu_2+V_1^vV_{21}^uu_2+V_1^vV_{22}^u\bar{u}_1+V_1^uV_3^v\left(u_2-u_2v_1\right)\right.\\
&+A_2^vV_1^u2\left(u_1-u_1v_1-\bar{v}_1\right)+A_{21}^vV_1^uu_2+A_{21}^uV_1^vu_2+A_{22}^uV_1^v\bar{u}_1+A_3^vV_1^u\left(-2+2u_1+u_2\right)\bar{v}_1\\
&+A_1^uV_2^v2\left(u_1-u_1v_1-\bar{v}_1\right)-A_1^vV_{21}^uu_2-A_1^uV_{21}^vu_2-A_1^vV_{22}^u\bar{u}_1-A_1^uV_3^v\left(-2+2u_1+u_2\right)\bar{v}_1\\
&-A_1^uA_{21}^vu_2-A_1^vA_{21}^uu_2-A_1^vA_{22}^u\bar{u}_1-A_1^uA_3^vu_2\bar{v}_1-T_1^uT_{21}^v2u_3-T_1^vT_{21}^u2u_3-T_1^uT_3^v2u_3\bar{v}_1\\
&-T_1^vT_3^u2\bar{u}_1u_3-T_1^uT_{41}^v2u_3-T_1^vT_{41}^u2u_3-T_1^uT_7^v2u_3\bar{v}_1-T_1^vT_7^u2\bar{u}_1u_3-S_1^vT_1^u2u_3\bar{v}_1\\
&\left.+S_1^uT_1^v2\bar{u}_1u_3+P_1^vT_1^u2u_3\bar{v}_1-P_1^uT_1^v2\bar{u}_1u_3)\right]\Big\}\\
&+\mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{3.94}
$$

Am_{11} :

$$
\begin{aligned}
&i\alpha_s^216\pi^2f_B\frac{M}{m_b^4}\int\mathcal{D}v\int\mathcal{D}u\\
&\Big\{\frac{1}{4u_2^2\bar{u}_2u_3v_2^2v_3^2}\\
&\Big[\bar{N}^+(P')N^+(P)\left(-\left(V_1^vV_2^u2u_2\bar{u}_2\right)+V_1^uV_{21}^vu_2+V_1^uV_{22}^vu_2-V_1^vV_{22}^uu_2\right.\\
&-V_1^vV_3^uu_2\left(-2+u_2+\bar{v}_3\right)-A_{21}^vV_1^uu_2-A_{22}^vV_1^uu_2+A_{22}^uV_1^vu_2+A_3^uV_1^vu_2\left(-u_2+\bar{v}_3\right)\\
&+A_1^uV_{21}^vu_2-A_1^vV_{22}^uu_2+A_1^uV_{22}^vu_2+A_1^vV_3^uu_2\left(u_2-v_1-v_2\right)-A_1^vA_2^u2u_2\bar{u}_2-A_1^uA_{21}^vu_2\\
&-A_1^uA_{22}^vu_2+A_1^vA_{22}^uu_2+A_1^vA_3^uu_2\left(-2+u_2+\bar{v}_3\right)+T_1^uT_{22}^v2v_3-T_1^vT_{22}^u2v_3-T_1^uT_3^v2v_2v_3\\
&+T_1^vT_3^u2u_2v_3+T_1^uT_{42}^v2v_3-T_1^vT_{42}^u2v_3-T_1^uT_7^v2v_2v_3+T_1^vT_7^u2u_2v_3-S_1^vT_1^u2v_2v_3\\
&\left.-S_1^uT_1^v2u_2v_3+P_1^vT_1^u2v_2v_3+P_1^uT_1^v2u_2v_3\right)\\
&+\bar{N}^+(P')\gamma_5N^+(P)\left(V_1^uV_{21}^vu_2+V_1^uV_{22}^vu_2+V_1^vV_{22}^uu_2+V_1^vV_3^uu_2\left(-u_2+\bar{v}_3\right)+A_2^uV_1^v2u_2\bar{u}_2\right.\\
&-A_{21}^vV_1^uu_2-A_{22}^vV_1^uu_2-A_{22}^uV_1^vu_2-A_3^uV_1^vu_2\left(-2+u_2+\bar{v}_3\right)+A_1^vV_2^u2u_2\bar{u}_2+A_1^uV_{21}^vu_2\\
&+A_1^vV_{22}^uu_2+A_1^uV_{22}^vu_2+A_1^vV_3^uu_2\left(-2+u_2+\bar{v}_3\right)-A_1^uA_{21}^vu_2-A_1^uA_{22}^vu_2-A_1^vA_{22}^uu_2\\
&+A_1^vA_3^uu_2\left(u_2-v_1-v_2\right)-T_1^uT_{22}^v2v_3-T_1^vT_{22}^u2v_3+T_1^uT_3^v2v_2v_3+T_1^vT_3^u2u_2v_3-T_1^uT_{42}^v2v_3\\
&-T_1^vT_{42}^u2v_3+T_1^uT_7^v2v_2v_3+T_1^vT_7^u2u_2v_3+S_1^vT_1^u2v_2v_3-S_1^uT_1^v2u_2v_3-P_1^vT_1^u2v_2v_3\\
&\left.+P_1^uT_1^v2u_2v_3)\right]\Big\}\\
&+\mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{3.95}
$$

Am_{12} :

$$
\begin{aligned}
&i\alpha_s^216\pi^2f_B\frac{M}{m_b^4}\int\mathcal{D}v\int\mathcal{D}u\\
&\Big\{\frac{1}{4u_2^2u_3^2v_2^2\bar{v}_2v_3}\\
&\Big[\bar{N}^+(P')N^+(P)\left(-V_1^uV_2^v2u_2\bar{v}_2+V_1^vV_{21}^uu_2-V_1^uV_{22}^v\bar{u}_1+V_1^vV_{22}^u\bar{u}_1\right.
\end{aligned}
$$

$$+V_1^uV_3^v\left(u_2+v_2-u_1v_2-2u_2v_2\right)-A_{21}^uV_1^vu_2+A_{22}^vV_1^u\bar{u}_1-A_{22}^uV_1^v\bar{u}_1+A_3^vV_1^u\left(u_2-\bar{u}_1v_2\right)$$
$$+A_1^vV_{21}^uu_2+A_1^vV_{22}^u\bar{u}_1-A_1^uV_{22}^v\bar{u}_1+A_1^uV_3^v\left(-u_2+v_2-u_1v_2\right)-A_1^uA_2^v2u_2\bar{v}_2-A_1^vA_{21}^uu_2$$
$$+A_1^uA_{22}^v\bar{u}_1-A_1^vA_{22}^u\bar{u}_1+A_1^uA_3^v\left(-\bar{u}_1v_2+u_2\left(-1+2v_2\right)\right)+T_1^uT_3^v2u_3v_2-T_1^vT_3^u2u_3v_2$$
$$+T_1^uT_7^v2u_3v_2-T_1^vT_7^u2u_3v_2-S_1^vT_1^u2u_3v_2-S_1^uT_1^v2u_3v_2+P_1^vT_1^u2u_3v_2+P_1^uT_1^v2u_3v_2)$$
$$+\bar{N}^+(P')\gamma_5N^+(P)\left(-\left(V_1^vV_{21}^uu_2\right)-V_1^uV_{22}^v\bar{u}_1-V_1^vV_{22}^u\bar{u}_1+V_1^uV_3^v\left(-u_2+v_2-u_1v_2\right)\right.$$
$$-A_2^vV_1^u2u_2\bar{v}_2+A_{21}^uV_1^vu_2+A_{22}^vV_1^u\bar{u}_1+A_{22}^uV_1^v\bar{u}_1+A_3^vV_1^u\left(-\bar{u}_1v_2+u_2\left(-1+2v_2\right)\right)$$
$$-A_1^uV_2^v2u_2\bar{v}_2-A_1^vV_{21}^uu_2-A_1^vV_{22}^u\bar{u}_1-A_1^uV_{22}^v\bar{u}_1+A_1^uV_3^v\left(u_2+v_2-u_1v_2-2u_2v_2\right)$$
$$+A_1^vA_{21}^uu_2+A_1^uA_{22}^v\bar{u}_1+A_1^vA_{22}^u\bar{u}_1+A_1^uA_3^v\left(u_2-\bar{u}_1v_2\right)-T_1^uT_3^v2u_3v_2-T_1^vT_3^u2u_3v_2$$
$$\left.-T_1^uT_7^v2u_3v_2-T_1^vT_7^u2u_3v_2+S_1^vT_1^u2u_3v_2-S_1^uT_1^v2u_3v_2-P_1^vT_1^u2u_3v_2+P_1^uT_1^v2u_3v_2\right)]\}$$
$$+\mathcal{O}(\alpha_s^3) \tag{3.96}$$

Am_{13} :

$$i\alpha_s^2 16\pi^2 f_B\frac{M}{m_b^4}\int\mathcal{D}v\int\mathcal{D}u$$
$$\left\{\frac{1}{4\bar{u}_1^2u_2\bar{v}_1^2v_2^2}\right.$$
$$\left[\bar{N}^+(P')N^+(P)\left(-\left(V_1^uV_2^vv_2\right)+V_1^vV_2^u\bar{u}_1+V_1^vV_3^u\left(u_1-\bar{v}_2\right)+A_2^vV_1^uv_2+A_2^uV_1^v\bar{u}_1\right.\right.$$
$$+A_3^uV_1^v\left(\bar{u}_1+v_2\right)+A_1^vV_2^u\bar{u}_1+A_1^uV_2^vv_2+A_1^vV_3^u\left(-\bar{u}_1-v_2\right)-A_1^uA_2^vv_2+A_1^vA_2^u\bar{u}_1$$
$$+A_1^vA_3^u\left(\bar{u}_1-v_2\right)-T_1^vT_{21}^u2-T_1^uT_{22}^v2+T_1^vT_3^u2\left(u_1-\bar{v}_2\right)-T_1^vT_{41}^u2$$
$$\left.-T_1^uT_{42}^v2+T_1^vT_7^u2\left(u_1-\bar{v}_2\right)+S_1^uT_1^v\left(2-2u_1+2v_2\right)+P_1^uT_1^v2\left(-\bar{u}_1-v_2\right)\right)$$
$$+\bar{N}^+(P')\gamma_5N^+(P)\left(V_1^uV_2^vv_2-V_1^vV_2^u\bar{u}_1+V_1^vV_3^u\left(\bar{u}_1+v_2\right)-A_2^vV_1^uv_2-A_2^uV_1^v\bar{u}_1\right.$$
$$+A_3^uV_1^v\left(u_1-\bar{v}_2\right)-A_1^vV_2^u\bar{u}_1-A_1^uV_2^vv_2+A_1^vV_3^u\left(\bar{u}_1-v_2\right)+A_1^uA_2^vv_2-A_1^vA_2^u\bar{u}_1$$
$$+A_1^vA_3^u\left(-\bar{u}_1-v_2\right)-T_1^vT_{21}^u2+T_1^uT_{22}^v2+T_1^vT_3^u2\left(u_1-\bar{v}_2\right)-T_1^vT_{41}^u2+T_1^uT_{42}^v2$$
$$\left.\left.+T_1^vT_7^u2\left(u_1-\bar{v}_2\right)+S_1^uT_1^v\left(2-2u_1+2v_2\right)+P_1^uT_1^v2\left(-\bar{u}_1-v_2\right)\right)\right]\}$$
$$+\mathcal{O}(\alpha_s^3) \tag{3.97}$$

Am_{14} :

$$i\alpha_s^2 16\pi^2 f_B\frac{M}{m_b^4}\int\mathcal{D}v\int\mathcal{D}u$$
$$\left\{-\frac{1}{4\bar{u}_1^2u_2^2\bar{v}_1^3v_2}\right.$$
$$\left[\bar{N}^+(P')N^+(P)\left(V_1^uV_2^v\left(u_1-u_1v_1-\bar{v}_1\right)+V_1^vV_2^u\left(u_2-u_1u_2\right)-V_1^uV_{21}^vu_2+V_1^vV_{21}^uu_2\right.\right.$$
$$+V_1^uV_3^vu_3\bar{v}_1+A_2^vV_1^u\left(u_1-u_1v_1-\bar{v}_1\right)-A_2^uV_1^v\bar{u}_1u_2-A_{21}^vV_1^uu_2+A_{21}^uV_1^vu_2$$
$$+A_3^vV_1^u\left(-\bar{u}_1-u_2\right)\bar{v}_1-A_1^vV_2^u\bar{u}_1u_2+A_1^uV_2^v\left(u_1-u_1v_1-\bar{v}_1\right)-A_1^vV_{21}^uu_2+A_1^uV_{21}^vu_2$$

$$
\begin{aligned}
&-A_1^u V_3^v\left(-\bar{u}_1-u_2\right)\bar{v}_1+A_1^u A_2^v\left(u_1-u_1 v_1-\bar{v}_1\right)+A_1^v A_2^u\left(u_2-u_1 u_2\right)+A_1^u A_{21}^v u_2\\
&-A_1^v A_{21}^u u_2-A_1^u A_3^v u_3\bar{v}_1+T_1^u T_{21}^v 2u_3+T_1^v T_{21}^u 2u_2+T_1^v T_{22}^u\left(2-2u_1\right)+T_1^u T_3^v 2u_3\bar{v}_1\\
&+T_1^u T_{41}^v 2u_3+T_1^v T_{41}^u 2u_2+T_1^v T_{42}^u\left(2-2u_1\right)+T_1^u T_7^v 2u_3\bar{v}_1-S_1^v T_1^u 2\left(\bar{u}_1+u_2\right)\bar{v}_1\\
&+P_1^v T_1^u 2\left(\bar{u}_1+u_2\right)\bar{v}_1)\\
&+\bar{N}^+(P')\gamma_5 N^+(P)\left(V_1^u V_2^v\left(u_1-u_1 v_1-\bar{v}_1\right)+V_1^v V_2^u\left(u_2-u_1 u_2\right)+V_1^u V_{21}^v u_2+V_1^v V_{21}^u u_2\right.\\
&-V_1^u V_3^v\left(-\bar{u}_1-u_2\right)\bar{v}_1+A_2^v V_1^u\left(u_1-u_1 v_1-\bar{v}_1\right)-A_2^u V_1^v\bar{u}_1 u_2+A_{21}^v V_1^u u_2+A_{21}^u V_1^v u_2\\
&-A_3^v V_1^u u_3\bar{v}_1-A_1^v V_2^u\bar{u}_1 u_2+A_1^u V_2^v\left(u_1-u_1 v_1-\bar{v}_1\right)-A_1^v V_{21}^u u_2-A_1^u V_{21}^v u_2+A_1^u V_3^v u_3\bar{v}_1\\
&+A_1^u A_2^v\left(u_1-u_1 v_1-\bar{v}_1\right)+A_1^v A_2^u\left(u_2-u_1 u_2\right)-A_1^u A_{21}^v u_2-A_1^v A_{21}^u u_2-A_1^u A_3^v\left(\bar{u}_1+u_2\right)\bar{v}_1\\
&-T_1^u T_{21}^v 2u_3+T_1^v T_{21}^u 2u_2+T_1^v T_{22}^u\left(2-2u_1\right)-T_1^u T_3^v 2u_3\bar{v}_1-T_1^u T_{41}^v 2u_3+T_1^v T_{41}^u 2u_2\\
&+T_1^v T_{42}^u\left(2-2u_1\right)-T_1^u T_7^v 2u_3\bar{v}_1+S_1^v T_1^u 2\left(\bar{u}_1+u_2\right)\bar{v}_1-P_1^v T_1^u 2\left(\bar{u}_1+u_2\right)\bar{v}_1)]\}\\
&+\mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{3.98}
$$

Am_{15} :

$$
\begin{aligned}
&i\alpha_s^2 16\pi^2 f_B\frac{M}{m_b^4}\int\mathcal{D}v\int\mathcal{D}u\\
&\left\{\frac{1}{4u_2 u_3^2\bar{u}_3 v_2^2 v_3^2}\right.\\
&\left(\bar{N}^+(P')N^+(P)\left(V_1^v V_2^u 2u_3 v_2-V_1^u V_{21}^v v_2+V_1^v V_{21}^u\left(\bar{u}_1-u_2+v_2\right)-V_1^u V_{22}^v\left(u_3+v_2\right)\right.\right.\\
&+V_1^v V_{22}^u\left(\bar{u}_1-u_2+v_2\right)-V_1^u V_3^v v_2 v_3+V_1^v V_3^u\left(u_1^2+u_1\left(-1+2u_2\right)-u_2\bar{u}_2\right)-A_{21}^v V_1^u v_2\\
&+A_{21}^u V_1^v\left(\bar{u}_3-\bar{v}_2\right)+A_{22}^v V_1^u\left(\bar{u}_1-u_2-v_2\right)+A_{22}^u V_1^v\left(\bar{u}_3-\bar{v}_2\right)-A_3^v V_1^u v_2 v_3\\
&+A_3^u V_1^v\left(-u_1^2-2u_1 u_2-u_2^2+\bar{u}_3\right)+A_1^v V_{21}^u\left(\bar{u}_1-u_2-v_2\right)+A_1^u V_{21}^v v_2\\
&+A_1^v V_{22}^u\left(\bar{u}_1-u_2-v_2\right)+A_1^u V_{22}^v\left(\bar{u}_3-\bar{v}_2\right)+A_1^v V_3^u\left(u_1^2+u_1\left(-1+2u_2\right)-u_2\bar{u}_2\right)\\
&+A_1^u V_3^v v_2 v_3+A_1^v A_2^u 2u_3 v_2+A_1^u A_{21}^v v_2+A_1^v A_{21}^u\left(-u_3-v_2\right)+A_1^u A_{22}^v\left(\bar{u}_1-u_2+v_2\right)\\
&-A_1^v A_{22}^u\left(u_3+v_2\right)+A_1^u A_3^v v_2 v_3+A_1^v A_3^u\left(-u_1^2-2u_1 u_2-u_2^2+\bar{u}_3\right)\\
&-S_1^u T_1^v 4\left(u_1^2+u_1\left(-1+2u_2\right)-u_2\bar{u}_2\right)+P_1^u T_1^v 4\left(u_1^2+u_1\left(-1+2u_2\right)-u_2\bar{u}_2\right)\Big)\\
&+\bar{N}^+(P')\gamma_5 N^+(P)\left(V_1^u V_{21}^v v_2+V_1^v V_{21}^u\left(\bar{u}_3-\bar{v}_2\right)+V_1^u V_{22}^v\left(\bar{u}_3-\bar{v}_2\right)+V_1^v V_{22}^u\left(\bar{u}_3-\bar{v}_2\right)\right.\\
&+V_1^u V_3^v v_2 v_3+V_1^v V_3^u\left(-u_1^2-2u_1 u_2-u_2^2+\bar{u}_3\right)-A_2^v V_1^u 2u_3 v_2+A_{21}^v V_1^u v_2\\
&+A_{21}^u V_1^v\left(\bar{u}_1-u_2+v_2\right)+A_{22}^v V_1^u\left(\bar{u}_1-u_2+v_2\right)+A_{22}^u V_1^v\left(\bar{u}_1-u_2+v_2\right)+A_3^v V_1^u v_2 v_3\\
&+A_3^u V_1^v\left(u_1^2+u_1\left(-1+2u_2\right)-u_2\bar{u}_2\right)-A_1^v V_2^u 2u_3 v_2+A_1^v V_{21}^u\left(-u_3-v_2\right)-A_1^u V_{21}^v v_2\\
&-A_1^v V_{22}^u\left(u_3+v_2\right)+A_1^u V_{22}^v\left(-u_3-v_2\right)+A_1^v V_3^u\left(-u_1^2-2u_1 u_2-u_2^2+\bar{u}_3\right)-A_1^u V_3^v v_2 v_3\\
&-A_1^u A_{21}^v v_2+A_1^v A_{21}^u\left(\bar{u}_1-u_2-v_2\right)+A_1^u A_{22}^v\left(\bar{u}_1-u_2-v_2\right)+A_1^v A_{22}^u\left(\bar{u}_1-u_2-v_2\right)\\
&-A_1^u A_3^v v_2 v_3+A_1^v A_3^u\left(u_1^2+u_1\left(-1+2u_2\right)-u_2\bar{u}_2\right)-S_1^u T_1^v 4\left(u_1^2+u_1\left(-1+2u_2\right)-u_2\bar{u}_2\right)\\
&\left.\left.\left.+P_1^u T_1^v 4\left(u_1^2+u_1\left(-1+2u_2\right)-u_2\bar{u}_2\right)\right)\right)\right\}\\
&+\mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{3.99}
$$

Am_{16} :

$$
\begin{aligned}
& i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4}\int \mathcal{D}v \int \mathcal{D}u \\
& \Big\{\frac{1}{4u_2^2u_3^2v_2v_3^2\bar{v}_3} \\
& \big[\bar{N}^+(P')N^+(P)\,(V_1^uV_2^v 2u_2v_3 + V_1^uV_{21}^v\bar{u}_1 - V_1^vV_{21}^u u_2 + V_1^uV_{22}^v\bar{u}_1 - V_1^vV_{22}^u\bar{u}_1 - V_1^uV_3^v u_3\bar{v}_3 \\
& -V_1^vV_3^u u_2v_3 - A_{21}^vV_1^u\bar{u}_1 + A_{21}^uV_1^v u_2 - A_{22}^vV_1^u\bar{u}_1 + A_{22}^uV_1^v\bar{u}_1 + A_3^vV_1^u u_3\bar{v}_3 - A_3^uV_1^v u_2v_3 \\
& -A_1^vV_{21}^u u_2 + A_1^uV_{21}^v\bar{u}_1 - A_1^vV_{22}^u\bar{u}_1 + A_1^uV_{22}^v\bar{u}_1 + A_1^vV_3^u u_2v_3 - A_1^uV_3^v u_3\bar{v}_3 + A_1^uA_2^v 2u_2v_3 \\
& -A_1^uA_{21}^v\bar{u}_1 + A_1^vA_{21}^u u_2 - A_1^uA_{22}^v\bar{u}_1 + A_1^vA_{22}^u\bar{u}_1 + A_1^uA_3^v u_3\bar{v}_3 + A_1^vA_3^u u_2v_3 + S_1^vT_1^u 4u_3\bar{v}_3 \\
& -P_1^vT_1^u 4u_3\bar{v}_3) \\
& +\bar{N}^+(P')\gamma_5N^+(P)\,(V_1^uV_{21}^v\bar{u}_1 + V_1^vV_{21}^u u_2 + V_1^uV_{22}^v\bar{u}_1 + V_1^vV_{22}^u\bar{u}_1 - V_1^uV_3^v u_3\bar{v}_3 - V_1^vV_3^u u_2v_3 \\
& +A_2^vV_1^u 2u_2v_3 - A_{21}^vV_1^u\bar{u}_1 - A_{21}^uV_1^v u_2 - A_{22}^vV_1^u\bar{u}_1 - A_{22}^uV_1^v\bar{u}_1 + A_3^vV_1^u u_3\bar{v}_3 - A_3^uV_1^v u_2v_3 \\
& +A_1^uV_2^v 2u_2v_3 + A_1^vV_{21}^u u_2 + A_1^uV_{21}^v\bar{u}_1 + A_1^vV_{22}^u\bar{u}_1 \\
& +A_1^uV_{22}^v\bar{u}_1 + A_1^vV_3^u u_2v_3 - A_1^uV_3^v u_3\bar{v}_3 - A_1^uA_{21}^v\bar{u}_1 - A_1^vA_{21}^u u_2 - A_1^uA_{22}^v\bar{u}_1 - A_1^vA_{22}^u\bar{u}_1 \\
& +A_1^uA_3^v u_3\bar{v}_3 + A_1^vA_3^u u_2v_3 - S_1^vT_1^u 4u_3\bar{v}_3 + P_1^vT_1^u 4u_3\bar{v}_3)\big]\Big\} \\
& +\mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{3.100}
$$

3.3.4 Feynman diagrams Af

The abelian diagrams without gluon coupling to fermion lines of incoming partons are displayed in Fig. 3.6. The leading power of the folded expressions can be written as follows:

Af_1 :

$$
\begin{aligned}
& i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4}\int \mathcal{D}v \int \mathcal{D}u \\
& \Big\{-\frac{1}{4\bar{u}_1^2u_3\bar{v}_1^2v_3^2}\,\big[\bar{N}^+(P')N^+(P)\,(V_1^vV_2^u 2\bar{u}_1\bar{v}_1 - V_1^uV_{21}^v v_2 + V_1^vV_{21}^u\bar{v}_1 - V_1^uV_{22}^v\bar{v}_1 - V_1^uV_3^v v_3 \\
& +V_1^vV_3^u\,(u_1 - u_1v_1 - \bar{v}_1) - A_{21}^vV_1^u v_2 + A_{21}^uV_1^v\bar{v}_1 - A_{22}^vV_1^u\bar{v}_1 - A_3^vV_1^u v_3 + A_3^uV_1^v\bar{u}_1\bar{v}_1 \\
& -A_1^vV_{21}^u\bar{v}_1 + A_1^uV_{21}^v v_2 + A_1^uV_{22}^v\bar{v}_1 + A_1^vV_3^u\,(u_1 - u_1v_1 - \bar{v}_1) \\
& +A_1^uV_3^v\,(\bar{v}_1 - v_2) + A_1^vA_2^u 2\bar{u}_1\bar{v}_1 + A_1^uA_{21}^v v_2 - A_1^vA_{21}^u\bar{v}_1 + A_1^uA_{22}^v\bar{v}_1 + A_1^uA_3^v\,(\bar{v}_1 - v_2) \\
& +A_1^vA_3^u\bar{u}_1\bar{v}_1 + T_1^uT_{21}^v 2v_3 - T_1^uT_3^v 2v_1v_3 + T_1^uT_{41}^v 2v_3 - T_1^uT_7^v 2v_1v_3 + S_1^vT_1^u 2\,(-2 + v_1)\,v_3 \\
& -P_1^vT_1^u 2\,(-2 + v_1)\,v_3) \\
& +\bar{N}^+(P')\gamma_5N^+(P)\,(V_1^uV_{21}^v v_2 + V_1^vV_{21}^u\bar{v}_1 + V_1^uV_{22}^v\bar{v}_1 + V_1^uV_3^v\,(\bar{v}_1 - v_2) + V_1^vV_3^u\bar{u}_1\bar{v}_1 \\
& +A_2^uV_1^v 2\,(u_1 - u_1v_1 - \bar{v}_1) + A_{21}^vV_1^u v_2 + A_{21}^uV_1^v\bar{v}_1 + A_{22}^vV_1^u\bar{v}_1 + A_3^vV_1^u\,(\bar{v}_1 - v_2) \\
& +A_3^uV_1^v\,(u_1 - u_1v_1 - \bar{v}_1) + A_1^vV_2^u 2\,(u_1 - u_1v_1 - \bar{v}_1) - A_1^vV_{21}^u\bar{v}_1 - A_1^uV_{21}^v v_2 - A_1^uV_{22}^v\bar{v}_1
\end{aligned}
$$

$$
\begin{aligned}
&+A_1^vV_3^u\bar{u}_1\bar{v}_1 - A_1^uV_3^v v_3 - A_1^uA_{21}^v v_2 - A_1^vA_{21}^u\bar{v}_1 - A_1^uA_{22}^v\bar{v}_1 - A_1^uA_3^v v_3 \\
&+A_1^vA_3^u\left(u_1 - u_1v_1 - \bar{v}_1\right) - T_1^uT_{21}^v 2v_3 + T_1^uT_3^v 2v_1v_3 - T_1^uT_{41}^v 2v_3 + T_1^uT_7^v 2v_1v_3 \\
&-S_1^vT_1^u 2\left(-2+v_1\right)v_3 + P_1^vT_1^u 2\left(-2+v_1\right)v_3)\big]\Big\} \\
&+\mathcal{O}(\alpha_s^3)
\end{aligned} \tag{3.101}
$$

Af_2 :

$$
\begin{aligned}
&i\alpha_s^2 16\pi^2 f_B\frac{M}{m_b^4}\int\mathcal{D}v\int\mathcal{D}u \\
&\Big\{\frac{1}{4\bar{u}_1u_3^2\bar{v}_1^2v_3} \\
&\big[\bar{N}^+(P')N^+(P)\left(-V_1^uV_2^v 2\bar{v}_1 + V_1^uV_{21}^v(-1) + V_1^vV_{21}^u + V_1^vV_{22}^u + V_1^uV_3^v\bar{v}_1\right. \\
&+V_1^vV_3^u\left(\bar{u}_1-u_2\right) - A_{21}^vV_1^u + A_{21}^uV_1^v + A_{22}^uV_1^v - A_3^vV_1^u\bar{v}_1 + A_3^uV_1^v\left(\bar{u}_1-u_2\right) \\
&-A_1^vV_{21}^u + A_1^uV_{21}^v - A_1^vV_{22}^u - A_1^vV_3^u u_3 + A_1^uV_3^v\bar{v}_1 - A_1^uA_2^v 2\bar{v}_1 + A_1^uA_{21}^v \\
&\left.-A_1^vA_{21}^u - A_1^vA_{22}^u - A_1^uA_3^v\bar{v}_1 - A_1^vA_3^u u_3 + S_1^uT_1^v 4u_3 - P_1^uT_1^v 4u_3\right) \\
&+\bar{N}^+(P')\gamma_5N^+(P)\left(V_1^uV_{21}^v + V_1^vV_{21}^u + V_1^vV_{22}^u + V_1^uV_3^v\bar{v}_1 + V_1^vV_3^u\left(\bar{u}_1-u_2\right) - A_2^vV_1^u 2\bar{v}_1\right. \\
&+A_{21}^vV_1^u + A_{21}^uV_1^v + A_{22}^uV_1^v - A_3^vV_1^u\bar{v}_1 + A_3^uV_1^v\left(\bar{u}_1-u_2\right) - A_1^uV_2^v 2\bar{v}_1 - A_1^vV_{21}^u \\
&-A_1^uV_{21}^v - A_1^vV_{22}^u - A_1^vV_3^u u_3 + A_1^uV_3^v\bar{v}_1 - A_1^uA_{21}^v - A_1^vA_{21}^u - A_1^vA_{22}^u - A_1^uA_3^v\bar{v}_1 - A_1^vA_3^u u_3 \\
&\left.+S_1^uT_1^v 4u_3 - P_1^uT_1^v 4u_3\right)\big]\Big\} \\
&+\mathcal{O}(\alpha_s^3)
\end{aligned} \tag{3.102}
$$

Af_3 :

$$
\begin{aligned}
&i\alpha_s^2 16\pi^2 f_B\frac{M}{m_b^4}\int\mathcal{D}v\int\mathcal{D}u \\
&\Big\{-\frac{1}{4u_2\bar{u}_2u_3\bar{v}_1v_2v_3^2} \\
&\big[\bar{N}^+(P')N^+(P)\left(V_1^vV_2^u 2\bar{u}_2\bar{v}_1 - V_1^uV_{21}^v\bar{v}_1 - V_1^uV_{22}^v\bar{v}_1 + V_1^vV_{22}^u\bar{v}_1 - V_1^uV_3^v\bar{v}_1v_3\right. \\
&+V_1^vV_3^u\left(u_2 - u_2v_1 - \bar{v}_1\right) + A_{21}^vV_1^u\bar{v}_1 + A_{22}^vV_1^u\bar{v}_1 - A_{22}^uV_1^v\bar{v}_1 + A_3^vV_1^u\bar{v}_1v_3 \\
&+A_3^uV_1^v\left(u_2 - u_2v_1 - \bar{v}_1\right) - A_1^uV_{21}^v\bar{v}_1 + A_1^vV_{22}^u\bar{v}_1 - A_1^uV_{22}^v\bar{v}_1 + A_1^vV_3^u\bar{u}_2\bar{v}_1 - A_1^uV_3^v\bar{v}_1v_3 \\
&+A_1^vA_2^u 2\bar{u}_2\bar{v}_1 + A_1^uA_{21}^v\bar{v}_1 + A_1^uA_{22}^v\bar{v}_1 - A_1^vA_{22}^u\bar{v}_1 + A_1^uA_3^v\bar{v}_1v_3 + A_1^vA_3^u\bar{u}_2\bar{v}_1 - T_1^uT_{21}^v 2v_3 \\
&\left.+T_1^uT_3^v 2v_1v_3 - T_1^uT_{41}^v 2v_3 + T_1^uT_7^v 2v_1v_3 - S_1^vT_1^u 2\left(-2+v_1\right)v_3 + P_1^vT_1^u 2\left(-2+v_1\right)v_3\right) \\
&+\bar{N}^+(P')\gamma_5N^+(P)\left(-\left(V_1^uV_{21}^v\bar{v}_1\right) - V_1^uV_{22}^v\bar{v}_1 - V_1^vV_{22}^u\bar{v}_1 - V_1^uV_3^v\bar{v}_1v_3\right. \\
&+V_1^vV_3^u\left(u_2 - u_2v_1 - \bar{v}_1\right) + A_2^uV_1^v 2\left(u_2 - u_2v_1 - \bar{v}_1\right) + A_{21}^vV_1^u\bar{v}_1 + A_{22}^vV_1^u\bar{v}_1 + A_{22}^uV_1^v\bar{v}_1 \\
&+A_3^vV_1^u\bar{v}_1v_3 + A_3^uV_1^v\left(u_2 - u_2v_1 - \bar{v}_1\right) + A_1^vV_2^u 2\left(u_2 - u_2v_1 - \bar{v}_1\right) - A_1^uV_{21}^v\bar{v}_1 - A_1^vV_{22}^u\bar{v}_1 \\
&-A_1^uV_{22}^v\bar{v}_1 + A_1^vV_3^u\bar{u}_2\bar{v}_1 - A_1^uV_3^v\bar{v}_1v_3 + A_1^uA_{21}^v\bar{v}_1 + A_1^uA_{22}^v\bar{v}_1 + A_1^vA_{22}^u\bar{v}_1 + A_1^uA_3^v\bar{v}_1v_3
\end{aligned}
$$

$$+A_1^vA_3^u\bar{u}_2\bar{v}_1+T_1^uT_{21}^v2v_3-T_1^uT_3^v2v_1v_3+T_1^uT_{41}^v2v_3-T_1^uT_7^v2v_1v_3+S_1^vT_1^u2\left(-2+v_1\right)v_3$$
$$-P_1^vT_1^u2\left(-2+v_1\right)v_3)]\}$$
$$+\mathcal{O}(\alpha_s^3) \tag{3.103}$$

Af_4 :

$$i\alpha_s^2 16\pi^2 f_B\frac{M}{m_b^4}\int\mathcal{D}v\int\mathcal{D}u$$
$$\left\{\frac{1}{4u_2\left(u_1^2+u_1\left(-1+2u_2\right)-u_2\bar{u}_2\right)\bar{v}_1v_2^2v_3}\right.$$
$$\left[\bar{N}^+(P')N^+(P)\left(-\left(V_1^uV_2^v2\bar{v}_1v_2\right)-V_1^uV_{21}^vv_2-V_1^vV_{21}^u\bar{v}_1+V_1^uV_{22}^v\bar{v}_1-V_1^vV_{22}^u\bar{v}_1+V_1^uV_3^vv_2\right.\right.$$
$$+V_1^vV_3^u\bar{u}_3\bar{v}_1-A_{21}^vV_1^uv_2+A_{21}^uV_1^v\bar{v}_1-A_{22}^vV_1^u\bar{v}_1+A_{22}^uV_1^v\bar{v}_1+A_3^vV_1^uv_2-A_3^uV_1^v\bar{u}_3\bar{v}_1$$
$$-A_1^vV_{21}^u\bar{v}_1+A_1^uV_{21}^vv_2-A_1^vV_{22}^u\bar{v}_1+A_1^uV_{22}^v\bar{v}_1+A_1^vV_3^u\bar{u}_3\bar{v}_1-A_1^uV_3^vv_2-A_1^uA_2^v2\bar{v}_1v_2$$
$$+A_1^uA_{21}^vv_2+A_1^vA_{21}^u\bar{v}_1-A_1^uA_{22}^v\bar{v}_1+A_1^vA_{22}^u\bar{v}_1-A_1^uA_3^vv_2-A_1^vA_3^u\bar{u}_3\bar{v}_1-S_1^uT_1^v4\bar{u}_3\bar{v}_1$$
$$+P_1^uT_1^v4\bar{u}_3\bar{v}_1)$$
$$+\bar{N}^+(P')\gamma_5N^+(P)\left(V_1^uV_{21}^vv_2+V_1^vV_{21}^u\bar{v}_1+V_1^uV_{22}^v\bar{v}_1+V_1^vV_{22}^u\bar{v}_1-V_1^uV_3^vv_2-V_1^vV_3^u\bar{u}_3\bar{v}_1\right.$$
$$-A_2^vV_1^u2\bar{v}_1v_2+A_{21}^vV_1^uv_2-A_{21}^uV_1^v\bar{v}_1-A_{22}^vV_1^u\bar{v}_1-A_{22}^uV_1^v\bar{v}_1-A_3^vV_1^uv_2+A_3^uV_1^v\bar{u}_3\bar{v}_1$$
$$-A_1^uV_2^v2\bar{v}_1v_2+A_1^vV_{21}^u\bar{v}_1-A_1^uV_{21}^vv_2+A_1^vV_{22}^u\bar{v}_1+A_1^uV_{22}^v\bar{v}_1-A_1^vV_3^u\bar{u}_3\bar{v}_1+A_1^uV_3^vv_2$$
$$-A_1^uA_{21}^vv_2-A_1^vA_{21}^u\bar{v}_1-A_1^uA_{22}^v\bar{v}_1-A_1^vA_{22}^u\bar{v}_1+A_1^uA_3^vv_2+A_1^vA_3^u\bar{u}_3\bar{v}_1-S_1^uT_1^v4\bar{u}_3\bar{v}_1$$
$$+P_1^uT_1^v4\bar{u}_3\bar{v}_1)]\}$$
$$+\mathcal{O}(\alpha_s^3) \tag{3.104}$$

Af_5 :

$$i\alpha_s^2 16\pi^2 f_B\frac{M}{m_b^4}\int\mathcal{D}v\int\mathcal{D}u$$
$$\left\{\frac{1}{4u_2^2\bar{u}_2u_3v_2v_3^2\bar{v}_3}\right.$$
$$\left[\bar{N}^+(P')N^+(P)\left(V_1^vV_2^u2u_2\bar{u}_2+V_1^uV_{21}^v\left(-u_2+\bar{v}_1-v_2\right)+V_1^uV_{22}^v\left(-u_2+\bar{v}_1-v_2\right)\right.\right.$$
$$+V_1^vV_{22}^u\left(u_2-v_3\right)+V_1^uV_3^v\left(v_1^2+v_1\left(-1+2v_2\right)-v_2\bar{v}_2\right)-V_1^vV_3^uu_2\bar{u}_2+A_{21}^vV_1^u\left(u_2-v_3\right)$$
$$+A_{22}^vV_1^u\left(u_2-v_3\right)+A_{22}^uV_1^v\left(-u_2+\bar{v}_1-v_2\right)+A_3^vV_1^u\left(-v_1^2-2v_1v_2-v_2^2+\bar{v}_3\right)$$
$$-A_3^uV_1^vu_2\bar{u}_2+A_1^uV_{21}^v\left(-u_2+\bar{v}_1-v_2\right)+A_1^vV_{22}^u\left(u_2-v_3\right)+A_1^uV_{22}^v\left(-u_2+\bar{v}_1-v_2\right)$$
$$+A_1^vV_3^uu_2\bar{u}_2+A_1^uV_3^v\left(v_1^2+v_1\left(-1+2v_2\right)-v_2\bar{v}_2\right)+A_1^vA_2^u2u_2\bar{u}_2+A_1^uA_{21}^v\left(u_2-v_3\right)$$
$$+A_1^uA_{22}^v\left(u_2-v_3\right)+A_1^vA_{22}^u\left(-u_2+\bar{v}_1-v_2\right)+A_1^uA_3^v\left(-v_1^2-2v_1v_2-v_2^2+\bar{v}_3\right)$$
$$+A_1^vA_3^uu_2\bar{u}_2-S_1^vT_1^u4\left(v_1^2+v_1\left(-1+2v_2\right)-v_2\bar{v}_2\right)+P_1^vT_1^u4\left(v_1^2+v_1\left(-1+2v_2\right)-v_2\bar{v}_2\right)\Big)$$
$$+\bar{N}^+(P')\gamma_5N^+(P)\left(V_1^uV_{21}^v\left(-u_2+\bar{v}_1-v_2\right)+V_1^uV_{22}^v\left(-u_2+\bar{v}_1-v_2\right)\right.$$

$$+V_1^vV_{22}^u\left(-u_2+\bar v_1-v_2\right)+V_1^uV_3^v\left(v_1^2+v_1\left(-1+2v_2\right)-v_2\bar v_2\right)-V_1^vV_3^uu_2\bar u_2-A_2^uV_1^v2u_2\bar u_2$$
$$+A_{21}^vV_1^u\left(u_2-v_3\right)+A_{22}^vV_1^u\left(u_2-v_3\right)+A_{22}^uV_1^v\left(u_2-v_3\right)+A_3^vV_1^u\left(-v_1^2-2v_1v_2-v_2^2+\bar v_3\right)$$
$$-A_3^uV_1^vu_2\bar u_2-A_1^vV_2^u2u_2\bar u_2+A_1^uV_{21}^v\left(-u_2+\bar v_1-v_2\right)+A_1^vV_{22}^u\left(-u_2+\bar v_1-v_2\right)$$
$$+A_1^uV_{22}^v\left(-u_2+\bar v_1-v_2\right)+A_1^vV_3^uu_2\bar u_2+A_1^uV_3^v\left(v_1^2+v_1\left(-1+2v_2\right)-v_2\bar v_2\right)$$
$$+A_1^uA_{21}^v\left(u_2-v_3\right)+A_1^uA_{22}^v\left(u_2-v_3\right)+A_1^vA_{22}^u\left(u_2-v_3\right)+A_1^uA_3^v\left(-v_1^2-2v_1v_2-v_2^2+\bar v_3\right)$$
$$+A_1^vA_3^uu_2\bar u_2+S_1^vT_1^u4\left(v_1^2+v_1\left(-1+2v_2\right)-v_2\bar v_2\right)$$
$$-P_1^vT_1^u4\left(v_1^2+v_1\left(-1+2v_2\right)-v_2\bar v_2\right)\big)\big]\Big\}$$
$$+\mathcal{O}(\alpha_s^3) \tag{3.105}$$

Af_6 :

$$i\alpha_s^2 16\pi^2 f_B\frac{M}{m_b^4}\int\mathcal{D}v\int\mathcal{D}u$$
$$\Big\{-\frac{1}{4\bar u_1u_2^2\bar v_1^2v_2}$$
$$\big[\bar N^+(P')N^+(P)\left(-V_1^uV_2^v\bar v_1+V_1^vV_2^uu_2+V_1^uV_3^v\bar v_1-V_1^vV_3^uu_2-A_2^vV_1^u\bar v_1-A_2^uV_1^vu_2\right.$$
$$-A_3^vV_1^u\bar v_1-A_3^uV_1^vu_2-A_1^vV_2^uu_2-A_1^uV_2^v\bar v_1+A_1^vV_3^uu_2+A_1^uV_3^v\bar v_1-A_1^uA_2^v\bar v_1+A_1^vA_2^uu_2$$
$$-A_1^uA_3^v\bar v_1+A_1^vA_3^uu_2+T_1^uT_{21}^v2+T_1^vT_{22}^u2+T_1^uT_3^v\left(2-2v_1\right)-T_1^vT_3^u2u_2+T_1^uT_{41}^v2$$
$$+T_1^vT_{42}^u2+T_1^uT_7^v\left(2-2v_1\right)-T_1^vT_7^u2u_2-S_1^vT_1^u2\bar v_1-S_1^uT_1^v2u_2+P_1^vT_1^u\left(2-2v_1\right)$$
$$\left.+P_1^uT_1^v2u_2\right)$$
$$+\bar N^+(P')\gamma_5N^+(P)\left(-V_1^uV_2^v\bar v_1+V_1^vV_2^uu_2+V_1^uV_3^v\bar v_1-V_1^vV_3^uu_2-A_2^vV_1^u\bar v_1-A_2^uV_1^vu_2\right.$$
$$-A_3^vV_1^u\bar v_1-A_3^uV_1^vu_2-A_1^vV_2^uu_2-A_1^uV_2^v\bar v_1+A_1^vV_3^uu_2+A_1^uV_3^v\bar v_1-A_1^uA_2^v\bar v_1+A_1^vA_2^uu_2$$
$$-A_1^uA_3^v\bar v_1+A_1^vA_3^uu_2-T_1^uT_{21}^v2+T_1^vT_{22}^u2-T_1^uT_3^v2\bar v_1-T_1^vT_3^u2u_2-T_1^uT_{41}^v2$$
$$\left.+T_1^vT_{42}^u2-T_1^uT_7^v2\bar v_1-T_1^vT_7^u2u_2+S_1^vT_1^u\left(2-2v_1\right)-S_1^uT_1^v2u_2-P_1^vT_1^u2\bar v_1+P_1^uT_1^v2u_2\right)\big]\Big\}$$
$$+\mathcal{O}(\alpha_s^3) \tag{3.106}$$

Af_7 :

$$i\alpha_s^2 16\pi^2 f_B\frac{M}{m_b^4}\int\mathcal{D}v\int\mathcal{D}u$$
$$\Big\{\frac{1}{4\bar u_1^2u_2\bar v_1^2v_2^2}$$
$$\big[\bar N^+(P')N^+(P)\left(-V_1^uV_2^v\bar v_1v_2+V_1^vV_2^u\bar u_1\bar v_1-V_1^uV_{21}^vv_2+V_1^uV_3^vv_2+V_1^vV_3^u\left(u_1-u_1v_1-\bar v_1\right)\right.$$
$$+A_2^vV_1^u\left(v_2-v_1v_2\right)+A_2^uV_1^v\bar u_1\bar v_1-A_{21}^vV_1^uv_2+A_3^vV_1^uv_2+A_3^uV_1^v\bar u_1\bar v_1+A_1^vV_2^u\bar u_1\bar v_1$$
$$+A_1^uV_2^v\left(v_2-v_1v_2\right)+A_1^uV_{21}^vv_2+A_1^vV_3^u\left(u_1-u_1v_1-\bar v_1\right)-A_1^uV_3^vv_2-A_1^uA_2^v\bar v_1v_2$$
$$+A_1^vA_2^u\bar u_1\bar v_1+A_1^uA_{21}^vv_2-A_1^uA_3^vv_2+A_1^vA_3^u\bar u_1\bar v_1-T_1^uT_{21}^v2v_2-T_1^vT_{21}^u2\bar v_1-T_1^uT_{22}^v2\bar v_1$$

$$+T_1^uT_3^v2v_2+T_1^vT_3^u2\left(u_1-u_1v_1-\bar{v}_1\right)-T_1^uT_{41}^v2v_2-T_1^vT_{41}^u2\bar{v}_1-T_1^uT_{42}^v2\bar{v}_1+T_1^uT_7^v2v_2$$
$$+T_1^vT_7^u2\left(u_1-u_1v_1-\bar{v}_1\right)+S_1^vT_1^u2v_2+S_1^uT_1^v2\bar{u}_1\bar{v}_1-P_1^vT_1^u2v_2$$
$$+P_1^uT_1^v2\left(u_1-u_1v_1-\bar{v}_1\right))$$
$$+\bar{N}^+(P')\gamma_5N^+(P)\left(V_1^uV_2^v\left(v_2-v_1v_2\right)+V_1^vV_2^u\left(u_1-u_1v_1-\bar{v}_1\right)+V_1^uV_{21}^vv_2-V_1^uV_3^vv_2\right.$$
$$+V_1^vV_3^u\bar{u}_1\bar{v}_1-A_2^vV_1^u\bar{v}_1v_2+A_2^uV_1^v\left(u_1-u_1v_1-\bar{v}_1\right)+A_{21}^vV_1^uv_2-A_3^vV_1^uv_2$$
$$+A_3^uV_1^v\left(u_1-u_1v_1-\bar{v}_1\right)+A_1^vV_2^u\left(u_1-u_1v_1-\bar{v}_1\right)-A_1^uV_2^v\bar{v}_1v_2-A_1^uV_{21}^vv_2+A_1^vV_3^u\bar{u}_1\bar{v}_1$$
$$+A_1^uV_3^vv_2+A_1^uA_2^v\left(v_2-v_1v_2\right)+A_1^vA_2^u\left(u_1-u_1v_1-\bar{v}_1\right)-A_1^uA_{21}^vv_2+A_1^uA_3^vv_2$$
$$+A_1^vA_3^u\left(u_1-u_1v_1-\bar{v}_1\right)+T_1^uT_{21}^v2v_2-T_1^vT_{21}^u2\bar{v}_1+T_1^uT_{22}^v\left(2-2v_1\right)-T_1^uT_3^v2v_2$$
$$+T_1^vT_3^u2\left(u_1-u_1v_1-\bar{v}_1\right)+T_1^uT_{41}^v2v_2-T_1^vT_{41}^u2\bar{v}_1+T_1^uT_{42}^v\left(2-2v_1\right)-T_1^uT_7^v2v_2$$
$$+T_1^vT_7^u2\left(u_1-u_1v_1-\bar{v}_1\right)-S_1^vT_1^u2v_2+S_1^uT_1^v2\bar{u}_1\bar{v}_1+P_1^vT_1^u2v_2$$
$$+P_1^uT_1^v2\left(u_1-u_1v_1-\bar{v}_1\right))]\}$$
$$+\mathcal{O}(\alpha_s^3) \tag{3.107}$$

Af_8 :

$$i\alpha_s^2 16\pi^2 f_B\frac{M}{m_b^4}\int\mathcal{D}v\int\mathcal{D}u$$
$$\left\{-\frac{1}{4\bar{u}_1^2u_3^2\bar{v}_1^2v_3}\right.$$
$$\left[\bar{N}^+(P')N^+(P)\left(V_1^uV_2^v2\left(u_1-u_1v_1-\bar{v}_1\right)-V_1^uV_{21}^v\bar{u}_1+V_1^vV_{21}^uu_2+V_1^vV_{22}^u\bar{u}_1+V_1^uV_3^v\bar{u}_1\bar{v}_1\right.\right.$$
$$+V_1^vV_3^u\left(\bar{u}_1-u_2\right)-A_{21}^vV_1^u\bar{u}_1+A_{21}^uV_1^vu_2+A_{22}^uV_1^v\bar{u}_1+A_3^vV_1^u\left(u_1-u_1v_1-\bar{v}_1\right)$$
$$+A_3^uV_1^v\left(\bar{u}_1-u_2\right)-A_1^vV_{21}^uu_2+A_1^uV_{21}^v\bar{u}_1-A_1^vV_{22}^u\bar{u}_1-A_1^vV_3^uu_3+A_1^uV_3^v\bar{u}_1\bar{v}_1$$
$$+A_1^uA_2^v2\left(u_1-u_1v_1-\bar{v}_1\right)+A_1^uA_{21}^v\bar{u}_1-A_1^vA_{21}^uu_2-A_1^vA_{22}^u\bar{u}_1+A_1^uA_3^v\left(u_1-u_1v_1-\bar{v}_1\right)$$
$$-A_1^vA_3^uu_3-T_1^vT_{21}^u2u_3+T_1^vT_3^u2u_1u_3-T_1^vT_{41}^u2u_3+T_1^vT_7^u2u_1u_3-S_1^uT_1^v2\left(-2+u_1\right)u_3$$
$$+P_1^uT_1^v2\left(-2+u_1\right)u_3)$$
$$+\bar{N}^+(P')\gamma_5N^+(P)\left(V_1^uV_{21}^v\bar{u}_1+V_1^vV_{21}^uu_2+V_1^vV_{22}^u\bar{u}_1+V_1^uV_3^v\bar{u}_1\bar{v}_1+V_1^vV_3^u\left(\bar{u}_1-u_2\right)\right.$$
$$+A_2^vV_1^u2\left(u_1-u_1v_1-\bar{v}_1\right)+A_{21}^vV_1^u\bar{u}_1+A_{21}^uV_1^vu_2+A_{22}^uV_1^v\bar{u}_1+A_3^vV_1^u\left(u_1-u_1v_1-\bar{v}_1\right)$$
$$+A_3^uV_1^v\left(\bar{u}_1-u_2\right)+A_1^uV_2^v2\left(u_1-u_1v_1-\bar{v}_1\right)-A_1^vV_{21}^uu_2-A_1^uV_{21}^v\bar{u}_1-A_1^vV_{22}^u\bar{u}_1$$
$$-A_1^vV_3^uu_3+A_1^uV_3^v\bar{u}_1\bar{v}_1-A_1^uA_{21}^v\bar{u}_1-A_1^vA_{21}^uu_2-A_1^vA_{22}^u\bar{u}_1+A_1^uA_3^v\left(u_1-u_1v_1-\bar{v}_1\right)$$
$$-A_1^vA_3^uu_3-T_1^vT_{21}^u2u_3+T_1^vT_3^u2u_1u_3-T_1^vT_{41}^u2u_3+T_1^vT_7^u2u_1u_3-S_1^uT_1^v2\left(-2+u_1\right)u_3$$
$$+P_1^uT_1^v2\left(-2+u_1\right)u_3)]\}$$
$$+\mathcal{O}(\alpha_s^3) \tag{3.108}$$

Af_9 :

$$i\alpha_s^2 16\pi^2 f_B\frac{M}{m_b^4}\int\mathcal{D}v\int\mathcal{D}u$$

$$\Big\{\frac{1}{4\bar{u}_1^2 u_3 \bar{v}_1 v_3^2}$$
$$\big[\bar{N}^+(P')N^+(P)\left(V_1^v V_2^u (2-2u_1) + V_1^u V_{21}^v(-1) + V_1^v V_{21}^u + V_1^u V_{22}^v(-1) - V_1^u V_3^v v_3\right.$$
$$-V_1^v V_3^u \bar{u}_1 - A_{21}^v V_1^u + A_{21}^u V_1^v - A_{22}^v V_1^u - A_3^v V_1^u v_3 + A_3^u V_1^v \bar{u}_1 - A_1^v V_{21}^u$$
$$+A_1^u V_{21}^v + A_1^u V_{22}^v - A_1^v V_3^u \bar{u}_1 + A_1^u V_3^v (\bar{v}_1 - v_2) + A_1^v A_2^u (2-2u_1) + A_1^u A_{21}^v - A_1^v A_{21}^u$$
$$\left.+A_1^u A_{22}^v + A_1^u A_3^v (\bar{v}_1 - v_2) + A_1^v A_3^u \bar{u}_1 - S_1^v T_1^u 4v_3 + P_1^v T_1^u 4v_3\right)$$
$$+\bar{N}^+(P')\gamma_5 N^+(P)\left(V_1^u V_{21}^v + V_1^v V_{21}^u + V_1^u V_{22}^v + V_1^u V_3^v (\bar{v}_1 - v_2) + V_1^v V_3^u \bar{u}_1 - A_2^u V_1^v 2\bar{u}_1\right.$$
$$+A_{21}^v V_1^u + A_{21}^u V_1^v + A_{22}^v V_1^u + A_3^v V_1^u (\bar{v}_1 - v_2) - A_3^u V_1^v \bar{u}_1 - A_1^v V_2^u 2\bar{u}_1 - A_1^v V_{21}^u$$
$$-A_1^u V_{21}^v - A_1^u V_{22}^v + A_1^v V_3^u \bar{u}_1 - A_1^u V_3^v v_3 - A_1^u A_{21}^v - A_1^v A_{21}^u - A_1^u A_{22}^v$$
$$\left.-A_1^u A_3^v v_3 - A_1^v A_3^u \bar{u}_1 + S_1^v T_1^u 4v_3 - P_1^v T_1^u 4v_3\right)\big]\Big\}$$
$$+\,\mathcal{O}(\alpha_s^3) \tag{3.109}$$

Af_{10} :

$$i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4}\int \mathcal{D}v \int \mathcal{D}u$$
$$\Big\{-\frac{1}{4\bar{u}_1 u_2 u_3^2 v_2 \bar{v}_2 v_3}\big[\bar{N}^+(P')N^+(P)\left(V_1^u V_2^v 2(u_1 - u_1 v_2 - \bar{v}_2) + V_1^v V_{21}^u \bar{u}_1 - V_1^u V_{22}^v \bar{u}_1\right.$$
$$+V_1^v V_{22}^u \bar{u}_1 + V_1^u V_3^v \bar{u}_1 \bar{v}_2 + V_1^v V_3^u \bar{u}_1 u_3 - A_{21}^u V_1^v \bar{u}_1 + A_{22}^v V_1^u \bar{u}_1 - A_{22}^u V_1^v \bar{u}_1 + A_3^v V_1^u \bar{u}_1 \bar{v}_2$$
$$-A_3^u V_1^v \bar{u}_1 u_3 + A_1^v V_{21}^u \bar{u}_1 + A_1^v V_{22}^u \bar{u}_1 - A_1^u V_{22}^v \bar{u}_1 + A_1^v V_3^u \bar{u}_1 u_3 + A_1^u V_3^v (u_1 - u_1 v_2 - \bar{v}_2)$$
$$+A_1^u A_2^v 2(u_1 - u_1 v_2 - \bar{v}_2) - A_1^v A_{21}^u \bar{u}_1 + A_1^u A_{22}^v \bar{u}_1 - A_1^v A_{22}^u \bar{u}_1 + A_1^u A_3^v (u_1 - u_1 v_2 - \bar{v}_2)$$
$$-A_1^v A_3^u \bar{u}_1 u_3 + T_1^v T_{21}^u 2u_3 - T_1^v T_3^u 2u_1 u_3 + T_1^v T_{41}^u 2u_3 - T_1^v T_7^u 2u_1 u_3 + S_1^u T_1^v 2(-2+u_1) u_3$$
$$\left.-P_1^u T_1^v 2(-2+u_1) u_3\right)$$
$$+\bar{N}^+(P')\gamma_5 N^+(P)\left(-\left(V_1^v V_{21}^u \bar{u}_1\right) - V_1^u V_{22}^v \bar{u}_1 - V_1^v V_{22}^u \bar{u}_1\right.$$
$$+V_1^u V_3^v (u_1 - u_1 v_2 - \bar{v}_2) - V_1^v V_3^u \bar{u}_1 u_3 + A_2^v V_1^u 2(u_1 - u_1 v_2 - \bar{v}_2) + A_{21}^u V_1^v \bar{u}_1$$
$$+A_{22}^v V_1^u \bar{u}_1 + A_{22}^u V_1^v \bar{u}_1 + A_3^v V_1^u (u_1 - u_1 v_2 - \bar{v}_2) + A_3^u V_1^v \bar{u}_1 u_3 + A_1^u V_2^v 2(u_1 - u_1 v_2 - \bar{v}_2)$$
$$-A_1^v V_{21}^u \bar{u}_1 - A_1^v V_{22}^u \bar{u}_1 - A_1^u V_{22}^v \bar{u}_1 - A_1^v V_3^u \bar{u}_1 u_3 + A_1^u V_3^v \bar{u}_1 \bar{v}_2 + A_1^v A_{21}^u \bar{u}_1 + A_1^u A_{22}^v \bar{u}_1$$
$$+A_1^v A_{22}^u \bar{u}_1 + A_1^u A_3^v \bar{u}_1 \bar{v}_2 + A_1^v A_3^u \bar{u}_1 u_3 + T_1^v T_{21}^u 2u_3 - T_1^v T_3^u 2u_1 u_3 + T_1^v T_{41}^u 2u_3 - T_1^v T_7^u 2u_1 u_3$$
$$\left.+S_1^u T_1^v 2(-2+u_1) u_3 - P_1^u T_1^v 2(-2+u_1) u_3\right)\big]\Big\}$$
$$+\,\mathcal{O}(\alpha_s^3) \tag{3.110}$$

Af_{11} :

$$i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4}\int \mathcal{D}v \int \mathcal{D}u$$
$$\Big\{\frac{1}{4\bar{u}_1 u_2^2 u_3 v_2 \left(v_1^2 + v_1(-1+2v_2) - v_2 \bar{v}_2\right)}$$

$$\left[\bar{N}^+(P')N^+(P)\left(V_1^vV_2^u2\bar{u}_1u_2+V_1^uV_{21}^v\bar{u}_1+V_1^vV_{21}^uu_2+V_1^uV_{22}^v\bar{u}_1-V_1^vV_{22}^u\bar{u}_1-V_1^uV_3^v\bar{u}_1\bar{v}_3\right.\right.$$
$$-V_1^vV_3^uu_2-A_{21}^vV_1^u\bar{u}_1+A_{21}^uV_1^vu_2-A_{22}^vV_1^u\bar{u}_1+A_{22}^uV_1^v\bar{u}_1+A_3^vV_1^u\bar{u}_1\bar{v}_3-A_3^uV_1^vu_2$$
$$-A_1^vV_{21}^uu_2+A_1^uV_{21}^v\bar{u}_1-A_1^vV_{22}^u\bar{u}_1+A_1^uV_{22}^v\bar{u}_1+A_1^vV_3^uu_2-A_1^uV_3^v\bar{u}_1\bar{v}_3+A_1^vA_2^u2\bar{u}_1u_2$$
$$-A_1^uA_{21}^v\bar{u}_1-A_1^vA_{21}^uu_2-A_1^uA_{22}^v\bar{u}_1+A_1^vA_{22}^u\bar{u}_1+A_1^uA_3^v\bar{u}_1\bar{v}_3+A_1^vA_3^uu_2$$
$$+S_1^vT_1^u4\bar{u}_1\bar{v}_3-P_1^vT_1^u4\bar{u}_1\bar{v}_3)$$
$$+\bar{N}^+(P')\gamma_5N^+(P)\left(V_1^uV_{21}^v\bar{u}_1+V_1^vV_{21}^uu_2+V_1^uV_{22}^v\bar{u}_1+V_1^vV_{22}^u\bar{u}_1-V_1^uV_3^v\bar{u}_1\bar{v}_3\right.$$
$$-V_1^vV_3^uu_2-A_2^uV_1^v2\bar{u}_1u_2-A_{21}^vV_1^u\bar{u}_1+A_{21}^uV_1^vu_2-A_{22}^vV_1^u\bar{u}_1-A_{22}^uV_1^v\bar{u}_1+A_3^vV_1^u\bar{u}_1\bar{v}_3$$
$$-A_3^uV_1^vu_2-A_1^vV_2^u2\bar{u}_1u_2-A_1^vV_{21}^uu_2+A_1^uV_{21}^v\bar{u}_1+A_1^vV_{22}^u\bar{u}_1+A_1^uV_{22}^v\bar{u}_1+A_1^vV_3^uu_2$$
$$-A_1^uV_3^v\bar{u}_1\bar{v}_3-A_1^uA_{21}^v\bar{u}_1-A_1^vA_{21}^uu_2-A_1^uA_{22}^v\bar{u}_1-A_1^vA_{22}^u\bar{u}_1+A_1^uA_3^v\bar{u}_1\bar{v}_3+A_1^vA_3^uu_2$$
$$\left.\left.-S_1^vT_1^u4\bar{u}_1\bar{v}_3+P_1^vT_1^u4\bar{u}_1\bar{v}_3)\right]\right\}$$
$$+\mathcal{O}(\alpha_s^3) \tag{3.111}$$

Af_{12} :

$$i\alpha_s^2 16\pi^2 f_B\frac{M}{m_b^4}\int\mathcal{D}v\int\mathcal{D}u$$
$$\left\{\frac{1}{4u_2u_3^2\bar{u}_3v_2^2\bar{v}_2v_3}\right.$$
$$\left[\bar{N}^+(P')N^+(P)\left(-\left(V_1^uV_2^v2v_2\bar{v}_2\right)+V_1^vV_{21}^u\left(\bar{u}_3-\bar{v}_2\right)+V_1^uV_{22}^v\left(\bar{u}_1-u_2-v_2\right)\right.\right.$$
$$+V_1^vV_{22}^u\left(\bar{u}_3-\bar{v}_2\right)+V_1^uV_3^vv_2\bar{v}_2+V_1^vV_3^u\left(-u_1^2-2u_1u_2-u_2^2+\bar{u}_3\right)+A_{21}^uV_1^v\left(\bar{u}_1-u_2-v_2\right)$$
$$+A_{22}^vV_1^u\left(\bar{u}_3-\bar{v}_2\right)+A_{22}^uV_1^v\left(\bar{u}_1-u_2-v_2\right)+A_3^vV_1^uv_2\bar{v}_2$$
$$+A_3^uV_1^v\left(u_1^2+u_1\left(-1+2u_2\right)-u_2\bar{u}_2\right)+A_1^vV_{21}^u\left(\bar{u}_3-\bar{v}_2\right)+A_1^vV_{22}^u\left(\bar{u}_3-\bar{v}_2\right)$$
$$+A_1^uV_{22}^v\left(\bar{u}_1-u_2-v_2\right)+A_1^vV_3^u\left(-u_1^2-2u_1u_2-u_2^2+\bar{u}_3\right)-A_1^uV_3^vv_2\bar{v}_2-A_1^uA_2^v2v_2\bar{v}_2$$
$$+A_1^vA_{21}^u\left(\bar{u}_1-u_2-v_2\right)+A_1^uA_{22}^v\left(\bar{u}_3-\bar{v}_2\right)+A_1^vA_{22}^u\left(\bar{u}_1-u_2-v_2\right)-A_1^uA_3^vv_2\bar{v}_2$$
$$+A_1^vA_3^u\left(u_1^2+u_1\left(-1+2u_2\right)-u_2\bar{u}_2\right)+S_1^uT_1^v4\left(u_1^2+u_1\left(-1+2u_2\right)-u_2\bar{u}_2\right)$$
$$\left.-P_1^uT_1^v4\left(u_1^2+u_1\left(-1+2u_2\right)-u_2\bar{u}_2\right)\right)$$
$$+\bar{N}^+(P')\gamma_5N^+(P)\left(V_1^vV_{21}^u\left(\bar{u}_1-u_2-v_2\right)+V_1^uV_{22}^v\left(\bar{u}_1-u_2-v_2\right)+V_1^vV_{22}^u\left(\bar{u}_1-u_2-v_2\right)\right.$$
$$-V_1^uV_3^vv_2\bar{v}_2+V_1^vV_3^u\left(u_1^2+u_1\left(-1+2u_2\right)-u_2\bar{u}_2\right)-A_2^vV_1^u2v_2\bar{v}_2+A_{21}^uV_1^v\left(\bar{u}_3-\bar{v}_2\right)$$
$$+A_{22}^vV_1^u\left(\bar{u}_3-\bar{v}_2\right)+A_{22}^uV_1^v\left(\bar{u}_3-\bar{v}_2\right)-A_3^vV_1^uv_2\bar{v}_2+A_3^uV_1^v\left(-u_1^2-2u_1u_2-u_2^2+\bar{u}_3\right)$$
$$-A_1^uV_2^v2v_2\bar{v}_2+A_1^vV_{21}^u\left(\bar{u}_1-u_2-v_2\right)+A_1^vV_{22}^u\left(\bar{u}_1-u_2-v_2\right)+A_1^uV_{22}^v\left(\bar{u}_1-u_2-v_2\right)$$
$$+A_1^vV_3^u\left(u_1^2+u_1\left(-1+2u_2\right)-u_2\bar{u}_2\right)+A_1^uV_3^vv_2\bar{v}_2+A_1^vA_{21}^u\left(\bar{u}_3-\bar{v}_2\right)+A_1^uA_{22}^v\left(\bar{u}_3-\bar{v}_2\right)$$
$$+A_1^vA_{22}^u\left(\bar{u}_3-\bar{v}_2\right)+A_1^uA_3^vv_2\bar{v}_2+A_1^vA_3^u\left(-u_1^2-2u_1u_2-u_2^2+\bar{u}_3\right)$$
$$\left.\left.\left.+S_1^uT_1^v4\left(u_1^2+u_1\left(-1+2u_2\right)-u_2\bar{u}_2\right)-P_1^uT_1^v4\left(u_1^2+u_1\left(-1+2u_2\right)-u_2\bar{u}_2\right)\right)\right]\right\}$$
$$+\mathcal{O}(\alpha_s^3) \tag{3.112}$$

Af_{13} :

$$i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4}\int \mathcal{D}v \int \mathcal{D}u$$
$$\Big\{-\frac{1}{4\bar{u}_1^2 u_2 \bar{v}_1 v_2^2}$$
$$\big[\bar{N}^+(P')N^+(P)\,(-V_1^uV_2^v v_2 + V_1^vV_2^u\bar{u}_1 + V_1^uV_3^v v_2 - V_1^vV_3^u\bar{u}_1 + A_2^vV_1^u v_2 + A_2^uV_1^v\bar{u}_1$$
$$+A_3^vV_1^u v_2 + A_3^uV_1^v\bar{u}_1 + A_1^vV_2^u\bar{u}_1 + A_1^uV_2^v v_2 - A_1^vV_3^u\bar{u}_1 - A_1^uV_3^v v_2 - A_1^uA_2^v v_2 + A_1^vA_2^u\bar{u}_1$$
$$-A_1^uA_3^v v_2 + A_1^vA_3^u\bar{u}_1 - T_1^vT_{21}^u 2 - T_1^uT_{22}^v 2 + T_1^uT_3^v 2v_2 - T_1^vT_3^u 2\bar{u}_1 - T_1^vT_{41}^u 2$$
$$+T_1^uT_{42}^v(-2) + T_1^uT_7^v 2v_2 - T_1^vT_7^u 2\bar{u}_1 + S_1^vT_1^u 2v_2 + S_1^uT_1^v\,(2-2u_1) - P_1^vT_1^u 2v_2$$
$$-P_1^uT_1^v 2\bar{u}_1)$$
$$+\bar{N}^+(P')\gamma_5N^+(P)\,(V_1^uV_2^v v_2 - V_1^vV_2^u\bar{u}_1 - V_1^uV_3^v v_2 + V_1^vV_3^u\bar{u}_1 - A_2^vV_1^u v_2 - A_2^uV_1^v\bar{u}_1$$
$$-A_3^vV_1^u v_2 - A_3^uV_1^v\bar{u}_1 - A_1^vV_2^u\bar{u}_1 - A_1^uV_2^v v_2 + A_1^vV_3^u\bar{u}_1 + A_1^uV_3^v v_2 + A_1^uA_2^v v_2 - A_1^vA_2^u\bar{u}_1$$
$$+A_1^uA_3^v v_2 - A_1^vA_3^u\bar{u}_1 + T_1^vT_{21}^u(-2) + T_1^uT_{22}^v 2 - T_1^uT_3^v 2v_2 - T_1^vT_3^u 2\bar{u}_1 - T_1^vT_{41}^u 2$$
$$+T_1^uT_{42}^v 2 - T_1^uT_7^v 2v_2 - T_1^vT_7^u 2\bar{u}_1 - S_1^vT_1^u 2v_2 + S_1^uT_1^v\,(2-2u_1) + P_1^vT_1^u 2v_2 - P_1^uT_1^v 2\bar{u}_1)\big]\Big\}$$
$$+\,\mathcal{O}(\alpha_s^3) \tag{3.113}$$

Af_{14} :

$$i\alpha_s^2 16\pi^2 f_B \frac{M}{m_b^4}\int \mathcal{D}v \int \mathcal{D}u$$
$$\Big\{\frac{1}{4\bar{u}_1^2 u_2^2 \bar{v}_1^2 v_2}$$
$$\big[\bar{N}^+(P')N^+(P)\,(V_1^uV_2^v\,(u_1 - u_1v_1 - \bar{v}_1) + V_1^vV_2^u\,(u_2 - u_1u_2) + V_1^vV_{21}^u u_2 + V_1^uV_3^v\bar{u}_1\bar{v}_1$$
$$-V_1^vV_3^u u_2 + A_2^vV_1^u\,(u_1 - u_1v_1 - \bar{v}_1) - A_2^uV_1^v\bar{u}_1u_2 + A_{21}^uV_1^v u_2 + A_3^vV_1^u\,(u_1 - u_1v_1 - \bar{v}_1)$$
$$-A_3^uV_1^v u_2 - A_1^vV_2^u\bar{u}_1u_2 + A_1^uV_2^v\,(u_1 - u_1v_1 - \bar{v}_1) - A_1^vV_{21}^u u_2 + A_1^vV_3^u u_2 + A_1^uV_3^v\bar{u}_1\bar{v}_1$$
$$+A_1^uA_2^v\,(u_1 - u_1v_1 - \bar{v}_1) + A_1^vA_2^u\,(u_2 - u_1u_2) - A_1^vA_{21}^u u_2 + A_1^uA_3^v\,(u_1 - u_1v_1 - \bar{v}_1)$$
$$+A_1^vA_3^u u_2 + T_1^uT_{21}^v\,(2-2u_1) + T_1^vT_{21}^u 2u_2 + T_1^vT_{22}^u\,(2-2u_1) + T_1^uT_3^v 2\bar{u}_1\bar{v}_1 - T_1^vT_3^u 2u_2$$
$$+T_1^uT_{41}^v\,(2-2u_1) + T_1^vT_{41}^u 2u_2 + T_1^vT_{42}^u\,(2-2u_1) + T_1^uT_7^v 2\bar{u}_1\bar{v}_1 - T_1^vT_7^u 2u_2$$
$$+S_1^vT_1^u 2\,(u_1 - u_1v_1 - \bar{v}_1) - S_1^uT_1^v 2u_2 + P_1^vT_1^u 2\bar{u}_1\bar{v}_1 + P_1^uT_1^v 2u_2)$$
$$+\bar{N}^+(P')\gamma_5N^+(P)\,(V_1^uV_2^v\,(u_1 - u_1v_1 - \bar{v}_1) + V_1^vV_2^u\,(u_2 - u_1u_2) + V_1^vV_{21}^u u_2 + V_1^uV_3^v\bar{u}_1\bar{v}_1$$
$$-V_1^vV_3^u u_2 + A_2^vV_1^u\,(u_1 - u_1v_1 - \bar{v}_1) - A_2^uV_1^v\bar{u}_1u_2 + A_{21}^uV_1^v u_2 + A_3^vV_1^u\,(u_1 - u_1v_1 - \bar{v}_1)$$
$$-A_3^uV_1^v u_2 - A_1^vV_2^u\bar{u}_1u_2 + A_1^uV_2^v\,(u_1 - u_1v_1 - \bar{v}_1) - A_1^vV_{21}^u u_2 + A_1^vV_3^u u_2 + A_1^uV_3^v\bar{u}_1\bar{v}_1$$
$$+A_1^uA_2^v\,(u_1 - u_1v_1 - \bar{v}_1) + A_1^vA_2^u\,(u_2 - u_1u_2) - A_1^vA_{21}^u u_2 + A_1^uA_3^v\,(u_1 - u_1v_1 - \bar{v}_1)$$
$$+A_1^vA_3^u u_2 - T_1^uT_{21}^v 2\bar{u}_1 + T_1^vT_{21}^u 2u_2 + T_1^vT_{22}^u\,(2-2u_1) + T_1^uT_3^v 2\,(u_1 - u_1v_1 - \bar{v}_1)$$
$$-T_1^vT_3^u 2u_2 - T_1^uT_{41}^v 2\bar{u}_1 + T_1^vT_{41}^u 2u_2 + T_1^vT_{42}^u\,(2-2u_1) + T_1^uT_7^v 2\,(u_1 - u_1v_1 - \bar{v}_1)$$

$$-T_1^v T_7^u 2u_2 + S_1^v T_1^u 2\bar{u}_1\bar{v}_1 - S_1^u T_1^v 2u_2 + P_1^v T_1^u 2\left(u_1 - u_1 v_1 - \bar{v}_1\right) + P_1^u T_1^v 2u_2)]\}$$
$$+ \mathcal{O}(\alpha_s^3) \tag{3.114}$$

3.3.5 Regularization of the decay amplitude

The regularization of $\mathcal{T}_{i,m,f}$ is not as straightforward as for the case of the form factor, since the involved integrals are not suitable to be solved analytically in an automatized fashion. Automatization is necessary, because the integrand is either too complicated or too long for manual computation (an expanded form would involve 37216, 53958, 3453 terms, respectively). Unfortunately the powerful apparatus to perform and simplify integrals of higher order corrections in the perturbative expansion cannot be applied (directly) in this case, since the present integrals are not dimensionally regulated. Some of the features can still be maintained, at the cost of a tremendous complication.

In order to tackle the explicit integration we apply the ideas of *sector decomposition* [125], which are formulated for IR-divergent integrals over Feynman parameters of the type

$$G_l = \int_0^1 \mathrm{d}^{N-1}t \, \frac{\mathcal{U}_l^{N-(L+1)D/2}}{\mathcal{F}_l^{N-LD/2}} \tag{3.115}$$

and regulated by the dimension, i.e. $D = 4 - 2\varepsilon$. By construction the functions $\mathcal{F}_l$ and $\mathcal{U}_l$ are homogeneous. Since the divergences in our case stem from soft quarks, the integrands share the feature that divergences stem from small momentum components of at least one quark. Sector decomposition ensures that subsequently introduced parameters let the integrand diverge only for small values. In our case the regularization is provided by a cut-off and the integrands are fractions of not necessarily homogeneous polynomials, which makes it necessary to modify the procedure. Below we regard N momentum components. The following modifications make it possible to disentangle overlapping divergences for a cut-off procedure as defined in (3.53):

- The fraction of polynomials in the momentum components $u_1, \ldots, u_N$ is made homogeneous by the repeated multiplication of terms with lower degree by $u_1 + \cdots + u_N$.

- The primary sectors are generated by

$$\int_\varepsilon^1 \mathrm{d}^N u = \sum_{l=1}^N \int_\varepsilon^1 \mathrm{d}^N u \prod_{j=1, j\neq l}^N \theta\left(u_l \geq u_j \geq \varepsilon\right), \tag{3.116}$$

where $\theta\left(a \geq b\right) = \begin{cases} 1 & , a \geq b \\ 0 & , a < b \end{cases}$. The following substitution,

$$u_j \to \begin{cases} u_l u_j & , j \neq l \\ u_l & , j = l \end{cases}, \tag{3.117}$$

changes the lower integration boundary of the inner integrals to ε/u_l and the upper boundary from u_l to 1.

- As a next step, the order of integration needs to be changed so that the integral over u_l is integrated first. The integration boundaries depend on u_l and need to be changed appropriately. For integration boundaries that contain products of a special form this can be brought into a closed form, see (A.13). An eventual fractional term that is introduced in the integrand needs to be factored out.

- The integration over the δ-distribution only has non-zero contributions for a part of the integration region. Therefore the integration over u_l gets replaced by a θ-function:

$$\int \mathrm{d}u_l\, \delta\left(u_l - \frac{1}{1+f(u_{j,j\neq l})}\right) \to \theta\left(\phi(u_1,\ldots,u_N,\varepsilon) \leq \frac{1}{1+f(u_{j,j\neq l})}\right), \qquad (3.118)$$

  where $\phi(u_{j,j\neq l},\varepsilon)$ is the lower boundary condition for variable u_l and $f(u_{j,j\neq l})$ is a fraction of polynomials with rational powers of the arguments. The condition that involves a sum is translated into modified integration boundaries at the end of the process by solving the system of inequalities for each integration variable.

- From here the iterative process starts by finding r variables, $u_{\alpha_1},\ldots,u_{\alpha_r}$, that let numerator or denominator vanish, when they vanish. r sectors are generated by

$$\prod_{j=1}^{r} \theta\left(1 \geq u_{\alpha_j} \geq 0\right) = \sum_{k=1}^{r} \prod_{j=1,j\neq k}^{r} \theta\left(u_{\alpha_k} \geq u_{\alpha_j} \geq 0\right) \qquad (3.119)$$

  In this step no change is needed.

- Before carrying out the substitution, the order of the integrals needs to be changed, so that the integral over u_{α_k} is carried out after the integrals over u_{α_j}, $j \neq k$. If necessary, the upper integration boundary is changed to 1 by substitution, which makes subsequent changes of integration orders easier. After that we need to apply the substitution

$$u_{\alpha_j} \to \begin{cases} u_{\alpha_k} u_{\alpha_j} & ,j \neq l \\ u_{\alpha_k} & ,j = l \end{cases} \qquad (3.120)$$

  also to the integration limits. The last step and this step are iterated as long as variables that meet the criteria of the last step, can be found.

- The condition of step 4 needs to be translated into appropriate integration boundaries, by solving the system of inequalities for the integration variables for each variable.

The above process is implemented in Mathematica. In order to solve the system of inequalities in the last step the implementation relies on the Mathematica function *Reduce*, which heavily depends on the order and form of input for a successful solution of the system of inequalities. It is not clear from the beginning, which form of input needs to be supplied. Therefore 4 processes are started in parallel that follow different solution algorithms. This part of the process can fail for complicated expressions, as occur in the result of the calculation for the form factors and especially the decay amplitude.

As an example for a typical integral that needs to be regulated we pick

$$\int_{\varepsilon}^{1} \mathrm{d}u_1 \int_{\varepsilon}^{1} \mathrm{d}u_2 \int_{\varepsilon}^{1} \mathrm{d}u_3 \int_{\varepsilon}^{1} \mathrm{d}v_1 \int_{\varepsilon}^{1} \mathrm{d}v_2 \int_{\varepsilon}^{1} \mathrm{d}v_3 \; \delta(1-u_1-u_2-u_3)\delta(1-v_1-v_2-v_3)$$
$$\frac{u_1^2 u_2 v_1^3}{u_3^2 v_3^2 \left(u_1^2 + u_1 \left(u_2 \left(2v_2 + v_3\right) + u_3\right) + \left(u_2 + u_3\right) v_2 \left(u_2 \left(v_2 + v_3\right) + u_3\right)\right)} \tag{3.121}$$

We follow the first 4 steps of the above description separately for each variable set, u_i and v_i. Then we apply the following parts of the description for the remaining 4 variables together. The result is a sum of integrals that manifestly do not contain overlapping divergences. In the above example we obtain 158 integrals. For illustration we provide one of these integrals:

$$\int_{\frac{\epsilon}{1-2\epsilon}}^{1} \mathrm{d}u_2 \int_{\frac{(u_2+1)\epsilon}{u_2-u_2\epsilon}}^{1} \mathrm{d}u_3 \int_{\frac{\epsilon}{1-2\epsilon}}^{1} \mathrm{d}v_2 \int_{\frac{(v_2+1)\epsilon}{v_2-v_2\epsilon}}^{1} \mathrm{d}v_3 \; \frac{\theta\left(\frac{1}{3}-\varepsilon\right)}{u_3^2 v_2 v_3^2}$$
$$1\Big/ \Big[\left(1+u_3u_2+u_2\right)^2 \left(1+v_3v_2+v_2\right)^2$$
$$\Big(\left(1+v_3v_2+v_2\right)^2 + \left(1+u_3\right) u_2^2 v_2 \left(u_3v_2 + u_3v_2v_3 + u_3 + v_2 + v_2v_3\right)$$
$$+u_2 \left(1+v_3v_2+v_2\right)\left(u_3\left(1+v_2\left(v_3+2\right)\right) + v_2\left(2+v_3\right)\right)\Big)\Big] \tag{3.122}$$

The integration region of these integrals is limited by construction to the cube $\varepsilon < x_i < 1$ in the integration variables x_i. In principle the integration boundaries can be the fraction of any polynomial with rational exponents. The method also produces integrals for finite ε, $a < \varepsilon < b$, which do not contribute in the limit $\varepsilon \to 0$.

From this point the original method of sector decomposition suggests to Taylor-expand the finite part of the integrand in the integration variables up to the degree of the divergence and to integrate the divergent part explicitly. The finite part of the integral is typically too complex to be solved analytically and needs to be integrated numerically. The numerical precision of these integrations tends to be rather poor. Therefore we propose to partially integrate the expression for each variable, starting from the innermost, v_3 in the upper case, up to the degree of divergence in the innermost variable. The divergence of v_3 is then encoded in v_2 and ε. The other partial integrations can be carried out in a similar manner. Typically stem functions can be found for all integrands. Before carrying out the final integration we can extract the divergent behavior by a Laurent expansion of the integrand in ε. The coefficients of this expansion consist of finite integrals that can be Taylor-expanded with the help of Leibniz rule. The remaining integrations are free of ε and can be integrated numerically or, if possible, analytically. It remains for future work to carry out the regularization of the complete amplitude.

Chapter 4

Conclusions

The path to understand most aspects of the CKM matrix necessarily leads to the field of B-decays, which is a very active field of experimental and theoretical physics. With the start of the LHC a new era in B-physics opens and three new experiments will enhance the precision in B_d decays significantly and have excellent opportunities to measure B_s-decays, which are prominent candidates to signal New Physics.

In this thesis we present a study of B decays to selected mesonic and baryonic final states. The first part contains an examination of B-decays to longitudinal vector mesons, $B \to V_L V_L$, based on first principles and a systematic expansion in powers of Λ_{QCD}/m_b:

- We give explicit formulas for the complete set of $\Delta S = 0$ and $\Delta S = 1$ decay amplitudes of $\bar{B} \to V_L V_L$ at NLO in QCD factorization. Estimates of power corrections from weak annihilation are included to study the sensitivity to effects of this kind in phenomenological applications. The set of decays considered comprises 17 $\Delta S = 0$ and 17 $\Delta S = 1$ channels, including 2 and 4 pure annihilation modes, respectively.

- The agreement with the available measured branching ratios of $\bar{B}_d$, B^- decays into $\rho^+\rho^-$, $\rho^0\rho^0$, $\rho^-\omega$, $\bar{K}^{*0}K^{*0}$, $\bar{K}^{*0}\rho^0$, $K^{*-}\rho^+$, $K^{*-}\rho^0$, $\bar{K}^{*0}\rho^-$, $\bar{K}^{*0}\phi$, $K^{*-}\phi$ and $\bar{K}^{*0}\omega$ is very good, within current uncertainties and with the central values used for $\bar{B} \to V_L$ form factors. This also holds, if the error of annihilation modes is neglected. The deviation of $\rho^-\rho^0$ is below 1 σ. We note that QCD factorization works well in particular for the penguin modes and for the three $\rho\rho$ channels and their characteristic hierarchy of branching fractions. Our hadronic input is based on the available literature. No tuning of parameters has been done to improve the fit with data.

- Long distance electromagnetic penguin effects are taken into account and found to be small.

- The deviation from ideal mixing in the ω-ϕ system is found to have a small effect on most decay modes that have these particles in the final state. The impact is very large for $B^- \to \rho^-\phi$.

- We use mixing-induced CP violation in $\bar{B}_d \to \rho_L^+ \rho_L^-$, measured by S_ρ, to extract the parameters of the unitarity triangle. Together with $\sin 2\beta$ the current measurements of S_ρ imply (2.146)
$$\gamma = (72.8 \pm 6.1)^\circ \tag{4.1}$$
where the error is dominated by S_ρ. This analysis benefits from the small penguin-to-tree ratio for vector modes $r_\rho = 0.038 \pm 0.024$ (2.138), which leads to a residual theory uncertainty in γ of $\pm 3^\circ$.

- We propose a method to relate the penguin contribution in $\bar{B}_d \to \rho_L^+ \rho_L^-$ to the decay $\bar{B}_d \to \bar{K}_L^{*0} K_L^{*0}$ based on the V-spin subgroup of flavour $SU(3)$. This makes it possible to constrain the uncertainties due to penguin power corrections, especially from annihilation topologies, and provides us with a check on the penguin-to-tree ratio calculated in QCD fatorization. The absolute value determined by the V-spin method, $r_\rho = 0.050 \pm 0.015$, is consistent with the calculation in QCD factorization. The resulting angle
$$\gamma = (74.5 \pm 5.3)^\circ \tag{4.2}$$
has a residual theory error of $\pm 2^\circ$.

- We point out that within the standard model $\sin 2\beta$ and the CP violation parameter S_ρ in $\bar{B} \to \rho_L^+ \rho_L^-$ determine to
$$|V_{ub}| = (3.49 \pm 0.16) \cdot 10^{-3} \tag{4.3}$$
where the error is at present still entirely dominated by $\sin 2\beta$. Hadronic uncertainties enter only at second order in S_ρ and the penguin parameter r_ρ and are below 2%. Possible New Physics affecting the $B_d - \bar{B}_d$ mixing phase can be constrained by comparing the above value of $|V_{ub}|$ with direct determinations from exclusive or inclusive $b \to u l \nu$ decays.

- In future measurements $\bar{B}_s \to \phi\phi$ will provide tests for New Physics. We present a bound on S_ϕ and C_ϕ, which will further improve when more data are available.

The second part of this thesis is devoted to a QCD factorization study with baryonic final states, in particular $B_s \to p\bar{p}$.

- The nucleon projector at twist 4 is determined and perpendicular distribution amplitudes are related to the light cone distribution amplitudes.

- The spacelike proton vector and axial vector form factor are calculated at subleading power in Λ_{QCD}/m_b. The convolution integrals are regularized by a cut-off regulator. We find that the vector u-form factor diverges linearly and the d-vector form factor double logarithmically. The axial vector form factor turns out to be calculable at subleading power in Λ_{QCD}/m_b. In particular both flavour conserving form factors, u and d, vanish. Several checks are taken to exclude calculation errors: the Ward identity was recovered explicitly, the amplitudes for the timelike form factors were calculated and related by crossing symmetry to the spacelike form factors and the perpendicular components canceled explicitly.

- We developed a computer program for Mathematica that can treat large expressions with Dirac matrices. This opens the possibility to independently check large expressions with Dirac matrices against results from FORM.

- The class of annihilation diagrams is analyzed for the decay $B_s \to p\bar{p}$. The order of contributing diagrams is estimated and the leading hard contribution is calculated at the level of integrals.

- We outline a modified procedure of sector decomposition for the regularization of large expressions and discuss its technical shortfalls.

In the mesonic decays QCD factorization proves to be a valuable tool to extract CKM parameters. The rich phenomenology can be exploited in numerous ways and delivers competitive results. The study in baryonic decays from first principles is the first step towards the subleading power. It reveals exciting results that can reduce uncertainties in hadronic physics substantially.

Appendix A

Exclusive Decays with Baryons

A.1 Spinor properties

If not mentioned otherwise the following definitions are used in chapter 3. We use the spinor representation of [126]:

$$u^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\xi^s \\ \sqrt{p\cdot\bar\sigma}\xi^s \end{pmatrix}, \qquad v^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\xi^{-s} \\ -\sqrt{p\cdot\bar\sigma}\xi^{-s} \end{pmatrix}, \tag{A.1}$$

$$\xi^s = (\xi(\uparrow), \xi(\downarrow)), \qquad \xi^{-s} = -i\sigma^2 (\xi^s)^*,$$

together with the Weyl representation of the Dirac algebra. With the choice of the charge conjugation matrix $C = -i\gamma^2\gamma^0$, this leads to the spinor identities

$$u^s(p) = C\left(\bar v^s(p)\right)^T, \quad v^s(p) = C\left(\bar u^s(p)\right)^T \tag{A.2}$$

Other obvious identities are:

$$CC = -1, \qquad (\gamma^\mu)^T = C\gamma^\mu C, \qquad \left(e^{-\frac{i}{2}\omega_{\mu\nu}S^{\mu\nu}}\right)^T = -Ce^{\frac{i}{2}\omega_{\mu\nu}S^{\mu\nu}}C, \tag{A.3}$$

where $S^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$. Therefore $C_{\alpha\beta}$ is invariant under Lorentz transformations.

The following relations can be useful in the simplification of Dirac structures in the case of an incoming and outgoing proton:

$$\begin{aligned}
\bar N_u^-(P')\gamma^\mu N_u^+(P) + \bar N_u^+(P')\gamma^\mu N_u^-(P) &= 2\frac{M}{Q^2}\left(P'+P\right)^\mu \bar N_u^+(P')N_u^+(P) \\
\bar N_u^-(P')\gamma^\mu N_u^+(P) - \bar N_u^+(P')\gamma^\mu N_u^-(P) &= -2\frac{M}{Q^2}\left(P'-P\right)^\mu \bar N_u^+(P')N_u^+(P) \\
\bar N_u^-(P')\gamma^\mu\gamma_5 N_u^+(P) + \bar N_u^+(P')\gamma^\mu\gamma_5 N_u^-(P) &= -2\frac{M}{Q^2}\left(P'-P\right)^\mu \bar N_u^+(P')\gamma_5 N_u^+(P) \\
\bar N_u^-(P')\gamma^\mu\gamma_5 N_u^+(P) - \bar N_u^+(P')\gamma^\mu\gamma_5 N_u^-(P) &= 2\frac{M}{Q^2}\left(P'+P\right)^\mu \bar N_u^+(P')\gamma_5 N_u^+(P),
\end{aligned} \tag{A.4}$$

where $Q^2 = -(P'-P)^2$ and terms of $\mathcal{O}\left((M/Q)^2\right)$ were neglected. The corresponding formula for an outgoing proton with momentum P' and an outgoing antiproton with momentum P can be obtained by replacing $Q^2 \to s$, $P'+P \to -(P'-P)$, $-(P'-P) \to P'+P$, $N_u^+(P) \to N_v^+(P)$.

A.2 Proton light cone projector

A.2.1 Expansion on the light cone

The following expressions are listed for reference from [99]. With the definitions (3.3), (3.4), (3.5) and (3.6) the 3-particle Fock state projector on the light cone (a_1, a_2, a_3 are real numbers) for a proton p with momentum P is given by

$$
\begin{aligned}
&\langle 0|\varepsilon^{ijk} u_\alpha^i(a_1 z) u_\beta^j(a_2 z) d_\gamma^k(a_3 z)|p(P)\rangle = \\
=&\frac{1}{4}\Big[\mathcal{S}_1 M C_{\alpha\beta}\left(\gamma_5 N\right)_\gamma + \mathcal{S}_2 M^2 C_{\alpha\beta}\left(\not{z}\gamma_5 N\right)_\gamma + \mathcal{P}_1 M\left(\gamma_5 C\right)_{\alpha\beta} N_\gamma + \mathcal{P}_2 M^2\left(\gamma_5 C\right)_{\alpha\beta}\left(\not{z} N\right)_\gamma \\
&+ \mathcal{V}_1\left(\not{P} C\right)_{\alpha\beta}\left(\gamma_5 N\right)_\gamma + \mathcal{V}_2 M\left(\not{P} C\right)_{\alpha\beta}\left(\not{z}\gamma_5 N\right)_\gamma + \mathcal{V}_3 M\left(\gamma_\mu C\right)_{\alpha\beta}\left(\gamma^\mu\gamma_5 N\right)_\gamma \\
&+ \mathcal{V}_4 M^2\left(\not{z} C\right)_{\alpha\beta}\left(\gamma_5 N\right)_\gamma + \mathcal{V}_5 M^2\left(\gamma_\mu C\right)_{\alpha\beta}\left(i\sigma^{\mu\nu} z_\nu\gamma_5 N\right)_\gamma + \mathcal{V}_6 M^3\left(\not{z} C\right)_{\alpha\beta}\left(\not{z}\gamma_5 N\right)_\gamma \\
&+ \mathcal{A}_1\left(\not{P}\gamma_5 C\right)_{\alpha\beta} N_\gamma + \mathcal{A}_2 M\left(\not{P}\gamma_5 C\right)_{\alpha\beta}\left(\not{z} N\right)_\gamma + \mathcal{A}_3 M\left(\gamma_\mu\gamma_5 C\right)_{\alpha\beta}\left(\gamma^\mu N\right)_\gamma \\
&+ \mathcal{A}_4 M^2\left(\not{z}\gamma_5 C\right)_{\alpha\beta} N_\gamma + \mathcal{A}_5 M^2\left(\gamma_\mu\gamma_5 C\right)_{\alpha\beta}\left(i\sigma^{\mu\nu} z_\nu N\right)_\gamma + \mathcal{A}_6 M^3\left(\not{z}\gamma_5 C\right)_{\alpha\beta}\left(\not{z} N\right)_\gamma \\
&+ \mathcal{T}_1\left(P^\nu i\sigma_{\mu\nu} C\right)_{\alpha\beta}\left(\gamma^\mu\gamma_5 N\right)_\gamma + \mathcal{T}_2 M\left(z^\mu P^\nu i\sigma_{\mu\nu} C\right)_{\alpha\beta}\left(\gamma_5 N\right)_\gamma \\
&+ \mathcal{T}_3 M\left(\sigma_{\mu\nu} C\right)_{\alpha\beta}\left(\sigma^{\mu\nu}\gamma_5 N\right)_\gamma + \mathcal{T}_4 M\left(P^\nu\sigma_{\mu\nu} C\right)_{\alpha\beta}\left(\sigma^{\mu\rho} z_\rho\gamma_5 N\right)_\gamma \\
&+ \mathcal{T}_5 M^2\left(z^\nu i\sigma_{\mu\nu} C\right)_{\alpha\beta}\left(\gamma^\mu\gamma_5 N\right)_\gamma + \mathcal{T}_6 M^2\left(z^\mu P^\nu i\sigma_{\mu\nu} C\right)_{\alpha\beta}\left(\not{z}\gamma_5 N\right)_\gamma \\
&+ \mathcal{T}_7 M^2\left(\sigma_{\mu\nu} C\right)_{\alpha\beta}\left(\sigma^{\mu\nu}\not{z}\gamma_5 N\right)_\gamma + \mathcal{T}_8 M^3\left(z^\nu\sigma_{\mu\nu} C\right)_{\alpha\beta}\left(\sigma^{\mu\rho} z_\rho\gamma_5 N\right)_\gamma\Big],
\end{aligned}
\tag{A.5}
$$

where the notation described below (3.25) has been used.

The twist expansion is then given by

$$
\begin{aligned}
&\langle 0|\varepsilon^{ijk}u^i_\alpha(a_1 z)u^j_\beta(a_2 z)d^k_\gamma(a_3 z)|p(P)\rangle = \\
=&\frac{1}{4}\Big[S_1 MC_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma + S_2 MC_{\alpha\beta}\left(\gamma_5 N^-\right)_\gamma + P_1 M\left(\gamma_5 C\right)_{\alpha\beta}N^+_\gamma + P_2 M\left(\gamma_5 C\right)_{\alpha\beta}N^-_\gamma \\
&+ V_1\left(\not{p}C\right)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma + V_2\left(\not{p}C\right)_{\alpha\beta}\left(\gamma_5 N^-\right)_\gamma + V_3\frac{M}{2}\left(\gamma_\perp C\right)_{\alpha\beta}\left(\gamma^\perp\gamma_5 N^+\right)_\gamma \\
&+ V_4\frac{M}{2}\left(\gamma_\perp C\right)_{\alpha\beta}\left(\gamma^\perp\gamma_5 N^-\right)_\gamma + V_5\frac{M^2}{2p\cdot z}\left(\not{z}C\right)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma + V_6\frac{M^2}{2p\cdot z}\left(\not{z}C\right)_{\alpha\beta}\left(\gamma_5 N^-\right)_\gamma \\
&+ A_1\left(\not{p}\gamma_5 C\right)_{\alpha\beta}N^+_\gamma + A_2\left(\not{p}\gamma_5 C\right)_{\alpha\beta}N^-_\gamma + A_3\frac{M}{2}\left(\gamma_\perp\gamma_5 C\right)_{\alpha\beta}\left(\gamma^\perp N^+\right)_\gamma \\
&+ A_4\frac{M}{2}\left(\gamma_\perp\gamma_5 C\right)_{\alpha\beta}\left(\gamma^\perp N^-\right)_\gamma + A_5\frac{M^2}{2p\cdot z}\left(\not{z}\gamma_5 C\right)_{\alpha\beta}N^+_\gamma + A_6\frac{M^2}{2p\cdot z}\left(\not{z}\gamma_5 C\right)_{\alpha\beta}N^-_\gamma \\
&+ T_1\left(i\sigma_{\perp p}C\right)_{\alpha\beta}\left(\gamma^\perp\gamma_5 N^+\right)_\gamma + T_2\left(i\sigma_{\perp p}C\right)_{\alpha\beta}\left(\gamma^\perp\gamma_5 N^-\right)_\gamma \\
&+ T_3\frac{M}{p\cdot z}\left(i\sigma_{pz}C\right)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma + T_4\frac{M}{p\cdot z}\left(i\sigma_{zp}C\right)_{\alpha\beta}\left(\gamma_5 N^-\right)_\gamma \\
&+ T_5\frac{M^2}{2p\cdot z}\left(i\sigma_{\perp z}C\right)_{\alpha\beta}\left(\gamma^\perp\gamma_5 N^+\right)_\gamma + T_6\frac{M^2}{2p\cdot z}\left(i\sigma_{\perp z}C\right)_{\alpha\beta}\left(\gamma^\perp\gamma_5 N^-\right)_\gamma \\
&+ T_7\frac{M}{2}\left(\sigma_{\perp\perp'}C\right)_{\alpha\beta}\left(\sigma^{\perp\perp'}\gamma_5 N^+\right)_\gamma + T_8\frac{M}{2}\left(\sigma_{\perp\perp'}C\right)_{\alpha\beta}\left(\sigma^{\perp\perp'}\gamma_5 N^-\right)_\gamma\Big]\,,
\end{aligned}
\tag{A.6}
$$

where the notation described below (3.25) has been used.

In the following we list the conformal expansion of distribution amplitudes. The leading terms are:

$$
\begin{aligned}
V_1(x_i,\mu) &= 120x_1x_2x_3\left[\phi_3^0(\mu)+\phi_3^+(\mu)(1-3x_3)\right]\,, \\
A_1(x_i,\mu) &= 120x_1x_2x_3(x_2-x_1)\phi_3^-(\mu)\,, \\
T_1(x_i,\mu) &= 120x_1x_2x_3\left[\phi_3^0(\mu)+\frac{1}{2}\left(\phi_3^- - \phi_3^+\right)(\mu)(1-3x_3)\right]
\end{aligned}
\tag{A.7}
$$

The subleading terms are:

$$
\begin{aligned}
V_2(x_i,\mu) =& 24x_1x_2\left[\phi_4^0(\mu)+\phi_4^+(\mu)(1-5x_3)\right] \\
A_2(x_i,\mu) =& 24x_1x_2(x_2-x_1)\phi_4^-(\mu) \\
T_2(x_i,\mu) =& 24x_1x_2\left[\xi_4^0(\mu)+\xi_4^+(\mu)(1-5x_3)\right] \\
V_3(x_i,\mu) =& 12x_3\left[\psi_4^0(\mu)(1-x_3)+\psi_4^-(\mu)(x_1^2+x_2^2-x_3(1-x_3))+\psi_4^+(\mu)(1-x_3-10x_1x_2)\right] \\
A_3(x_i,\mu) =& 12x_3(x_2-x_1)\left[\left(\psi_4^0(\mu)+\psi_4^+(\mu)\right)+\psi_4^-(\mu)(1-2x_3)\right] \\
T_3(x_i,\mu) =& 6x_3\left[(\xi_4^0+\phi_4^0+\psi_4^0)(\mu)(1-x_3)+(\xi_4^-+\phi_4^--\psi_4^-)(\mu)(x_1^2+x_2^2-x_3(1-x_3))\right. \\
&\left.+(\xi_4^++\phi_4^++\psi_4^+)(\mu)(1-x_3-10x_1x_2)\right] \\
T_7(x_i,\mu) =& 6x_3\left[(-\xi_4^0+\phi_4^0+\psi_4^0)(\mu)(1-x_3)+(-\xi_4^-+\phi_4^--\psi_4^-)(\mu)(x_1^2+x_2^2-x_3(1-x_3))\right. \\
&\left.+(-\xi_4^++\phi_4^++\psi_4^+)(\mu)(1-x_3-10x_1x_2)\right]
\end{aligned}
$$

$$
\begin{aligned}
S_1(x_i,\mu) =& 6x_3(x_2-x_1)\left[(\xi_4^0+\phi_4^0+\psi_4^0+\xi_4^+ +\phi_4^+ +\psi_4^+)(\mu)+(\xi_4^- +\phi_4^- -\psi_4^-)(\mu)(1-2x_3)\right]\\
P_1(x_i,\mu) =& 6x_3(x_2-x_1)\left[(\xi_4^0-\phi_4^0-\psi_4^0+\xi_4^+ -\phi_4^+ -\psi_4^+)(\mu)+(\xi_4^- -\phi_4^- +\psi_4^-)(\mu)(1-2x_3)\right]
\end{aligned} \tag{A.8}
$$

A.2.2 Decomposition of twist 4

In the following all suitable terms, that transform as a Dirac spinor and are invariant under parity transformation are written up for the matrix element $\langle 0|\epsilon^{ijk}u_i(z_{13})u_j(z_{23})d_k(0)|p(P)\rangle$. The resulting structure is in the form $(\Gamma_1)_{\alpha\beta}\otimes(\Gamma_2)_\gamma$. $\not{P}N(P)=MN(P)$ is used to reduce the number of dependent terms. The terms are written up in the form $\Gamma_1\otimes\Gamma_2^1,\Gamma_2^2,\ldots$. Brackets $[A,B,\ldots]$ are used to denote the insertion of exactly one of A, B, ...

$$
\begin{aligned}
C \quad\otimes\quad & \gamma_5 N,\quad [\not{z}_{13},\not{z}_{23}]\gamma_5 N,\quad \epsilon_{z_{13}z_{23}P\nu}\gamma^\nu N,\quad \epsilon_{z_{13}[z_{23},P]\mu\nu}\sigma^{\mu\nu}N,\quad \epsilon_{z_{23}P\mu\nu}\sigma^{\mu\nu}N,\\
\gamma^\mu C \quad\otimes\quad & [z_{13\mu},z_{23\mu},P_\mu]\gamma_5 N,\quad [z_{13\mu},z_{23\mu},P_\mu]\not{z}_{13}\gamma_5 N,\quad [z_{13\mu},z_{23\mu},P_\mu]\not{z}_{23}\gamma_5 N,\quad \gamma_\mu\gamma_5 N,\\
& \epsilon_{\mu z_{13}z_{23}P}N,\quad \epsilon_{\mu z_{13}z_{23}P}[\not{z}_{13},\not{z}_{23}]N,\quad \epsilon_{\mu z_{13}[z_{23},P]\nu}\gamma^\nu N,\quad \epsilon_{\mu z_{23}P\nu}\gamma^\nu N,\\
& \epsilon_{\mu z_{13}z_{23}P}\sigma_{z_{13}z_{23}}N,\quad \epsilon_{\mu z_{13}[z_{23},P]\rho}\sigma^{\rho[z_{13},z_{23}]}N,\\
& \epsilon_{\mu z_{23}P\rho}\sigma^{\rho[z_{13},z_{23}]}N,\quad \epsilon_{\mu[z_{13},z_{23},P]\rho\sigma}\sigma^{\rho\sigma}N,\quad \epsilon_{z_{13}z_{23}P\rho}\sigma^\rho{}_\mu N,\\
\sigma^{\mu\nu}C \quad\otimes\quad & z_{13\mu}[z_{23\nu},P_\nu]\gamma_5 N,\quad z_{23\mu}P_\nu\gamma_5 N,\quad z_{13\mu}[z_{23\nu},P_\nu]\not{z}_{13}\gamma_5 N,\quad z_{13\mu}[z_{23\nu},P_\nu]\not{z}_{23}\gamma_5 N,\\
& z_{23\mu}P_\nu[\not{z}_{13},\not{z}_{23}]\gamma_5 N,\quad [z_{13\nu},z_{23\nu},P_\nu]\gamma_\mu\gamma_5 N,\\
& \epsilon_{\mu\nu z_{13}[z_{23},P]}N,\quad \epsilon_{\mu\nu z_{23}P}N,\quad \epsilon_{\mu\nu z_{13}[z_{23},P]}\not{z}_{13}N,\quad \epsilon_{\mu\nu z_{13}[z_{23},P]}\not{z}_{23}N,\\
& \epsilon_{\mu z_{13}z_{23}P}\gamma_\nu N,\quad \epsilon_{\mu\nu z_{23}P}[\not{z}_{13},\not{z}_{23}]N,\quad \epsilon_{\mu\nu[z_{13},z_{23},P]\sigma}\gamma^\sigma N,\quad \epsilon_{\mu\nu z_{13}[z_{23},P]}\sigma_{z_{13}z_{23}}N,\\
& \epsilon_{\mu\nu z_{23}P}\sigma_{z_{13}z_{23}}N,\quad \epsilon_{\mu\nu[z_{13},z_{23},P]\rho}\sigma^{\rho[z_{13},z_{23}]}N,\quad \epsilon_{\mu\nu\rho\sigma}\sigma^{\rho\sigma}N,\\
& \epsilon_{\mu z_{13}z_{23}P}\sigma_{\nu[z_{13},z_{23}]}N,\quad \epsilon_{\mu z_{13}[z_{23},P]\rho}\sigma^\rho{}_\nu N,\quad \epsilon_{\mu z_{23}P\rho}\sigma^\rho{}_\nu N,\\
\gamma^\mu\gamma_5 C \quad\otimes\quad & [z_{13\mu},z_{23\mu},P_\mu]N,\quad [z_{13\mu},z_{23\mu},P_\mu]\not{z}_{13}N,\quad [z_{13\mu},z_{23\mu},P_\mu]\not{z}_{23}N,\quad \gamma_\mu N,\\
& [z_{13\mu},z_{23\mu},P_\mu]\sigma_{z_{13}z_{23}}N,\quad \sigma_{\mu[z_{13},z_{23}]}N,\\
& \epsilon_{\mu z_{13}[z_{23},P]\nu}\gamma^\nu\gamma_5 N,\quad \epsilon_{\mu z_{23}P\nu}\gamma^\nu\gamma_5 N,\quad \epsilon_{\mu z_{13}z_{23}P}\gamma_5 N,\\
\gamma_5 C \quad\otimes\quad & N,\quad [\not{z}_{13},\not{z}_{23}]N,\quad \sigma_{z_{13}z_{23}}N,\quad \epsilon_{z_{13}z_{23}P\nu}\gamma^\nu\gamma_5 N,
\end{aligned} \tag{A.9}
$$

where the notation $\epsilon_{P\mu\nu\rho}\equiv P^\sigma\epsilon_{\sigma\mu\nu\rho}$ is used. The following identities for the convention $\epsilon^{0123}=-1$ are helpful in the determination of independent terms:

$$
\begin{aligned}
\gamma^\mu\gamma^\nu\gamma^\rho =& g^{\nu\rho}\gamma^\mu-g^{\mu\rho}\gamma^\nu+g^{\mu\nu}\gamma^\rho-i\epsilon^{\mu\nu\rho\sigma}\gamma_\sigma\gamma_5,\qquad \sigma^{\mu\nu}\gamma_5=-\frac{i}{2}\epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma},\\
\gamma^{[\mu}\gamma^\nu\gamma^\rho\gamma^{\sigma]} =& i\epsilon^{\mu\nu\rho\sigma}\gamma_5
\end{aligned} \tag{A.10}
$$

Square brackets for indices denote the total antisymmetric part of the corresponding tensor.

A.3 Proton form factor

For reference we provide the explicit integral representation of the proton form factor at subleading power:

$$
\begin{aligned}
&\langle p(P')|\bar{u}\gamma^\mu u|p(P)\rangle|_{\mathrm{NLP}} = \\
=&\alpha_s^2 16\pi^2 \frac{(N_c+1)^2}{4N_c^2 N_c!}\frac{M}{Q^6}\left(P'^\mu + P^\mu\right)\bar{N}^+(P')N^+(P)\int \mathcal{D}u \int \mathcal{D}v \\
&\Big\{-\frac{V_1^u V_1^v}{16u_1^2\bar{u}_1^2u_2^2\bar{u}_2^2u_3^2v_1^2\bar{v}_1^2v_2^2\bar{v}_2^2v_3^2}\Big[\Big(-v_3\left(\left(\left((v_2-4)\,v_2+2\right)v_1^4-(3v_2-4)\left(3v_2^2-1\right)v_1^3\right.\right.\\
&+\left(v_2\left(v_2\left(v_2\left(v_2+12\right)-18\right)+2\right)+2\right)v_1^2+v_2\left(4v_2^2-\bar{v}_2\right)\bar{v}_2v_1+2v_2^2\bar{v}_2^2\Big)u_2^3\\
&+\Big(\left(v_2\left((v_2-8)\,v_2+9\right)-4\right)v_1^5+\left(v_2\left(v_2\left(v_2\left(2v_2-1\right)+13\right)-22\right)+12\right)v_1^4\\
&+\left(v_2\left(v_2\left(v_2\left(-v_2\bar{v}_2-14\right)+11\right)+13\right)-12\right)v_1^3+\left(v_2\left(v_2\left(v_2\left(8v_2-5\right)-16\right)+8\right)+4\right)\bar{v}_2v_1^2\\
&+v_2\left(v_2\left(9v_2-4\right)-4\right)\bar{v}_2^2v_1+4v_2^2\bar{v}_2^3\Big)u_2^2+\Big(\left(v_2\left((9-4v_2)\,v_2-8\right)+2\right)v_1^5\\
&+\left(2v_2\left(v_2\left((11-4v_2)\,v_2-13\right)+10\right)-6\right)v_1^4\\
&+\left(v_2\left(v_2\left(v_2\left(-4v_2^2+22v_2-29\right)+17\right)-13\right)+6\right)v_1^3-\left(9v_2^4-17v_2^3+4v_2+2\right)\bar{v}_2v_1^2\\
&-v_2\left(8v_2^2-4v_2-3\right)\bar{v}_2^2v_1-2v_2^2\bar{v}_2^3\Big)u_2-2v_1v_2v_3\bar{v}_1^2\bar{v}_2^2\bar{v}_3\Big)u_1^5\\
&+\Big(-2v_3\left(\left(\left((v_2-4)\,v_2+2\right)v_1^4-(3v_2-4)\left(3v_2^2-1\right)v_1^3\right.\\
&+\left(v_2\left(v_2\left(v_2\left(v_2+12\right)-18\right)+2\right)+2\right)v_1^2+v_2\left(4v_2^2-\bar{v}_2\right)\bar{v}_2v_1+2v_2^2\bar{v}_2^2\Big)u_2^4\\
&-\Big(\left(v_2\left(8v_2^2+v_2-22\right)+14\right)v_1^5+\left(v_2\left(v_2\left(4v_2\left(4v_2-21\right)+55\right)+53\right)-42\right)v_1^4\\
&+\left(v_2\left(v_2\left(v_2\left(8v_2^2-84v_2+231\right)-163\right)-33\right)+42\right)v_1^3\\
&-\left(v_2\left(v_2\left(v_2\left(v_2+56\right)-107\right)+19\right)+14\right)\bar{v}_2v_1^2-v_2\left(v_2\left(22v_2-9\right)-7\right)\bar{v}_2^2v_1-14v_2^2\bar{v}_2^3\Big)u_2^3\\
&+\Big(\left(v_2\left(v_2\left(7v_2+13\right)-26\right)+16\right)v_1^5+\left(v_2\left(v_2\left(v_2\left(14v_2-55\right)+2\right)+67\right)-48\right)v_1^4\\
&+\left(v_2\left(v_2\left(v_2\left(v_2\left(7v_2-55\right)+135\right)-82\right)-43\right)+48\right)v_1^3\\
&-\left(v_2\left(v_2\left(v_2\left(13v_2+15\right)-67\right)+27\right)+16\right)\bar{v}_2v_1^2\\
&-v_2\left(v_2\left(26v_2-15\right)-13\right)\bar{v}_2^2v_1-16v_2^2\bar{v}_2^3\Big)u_2^2+\Big(\left(v_2\left(v_2\left(9v_2-22\right)+20\right)-6\right)v_1^5\\
&+\left(v_2\left(v_2\left(v_2\left(18v_2-53\right)+67\right)-52\right)+18\right)v_1^4\\
&+\left(v_2\left(v_2\left((v_2-3)\,v_2\left(9v_2-26\right)-51\right)+36\right)-18\right)v_1^3\\
&+\left(v_2\left(v_2\left(v_2\left(22v_2-45\right)+6\right)+10\right)+6\right)\bar{v}_2v_1^2\\
&-4v_2\left(5v_2+2\right)\bar{v}_2^3v_1+6v_2^2\bar{v}_2^3\Big)u_2+6v_1v_2v_3\bar{v}_1^2\bar{v}_2^2\bar{v}_3\Big)u_1^4\\
&+\Big(\Big(\left(6-u_2\left((u_2-3)\,u_2\left(8u_2-9\right)+5\right)\right)v_1^3+\left(u_2\left(u_2\left(u_2\left(-u_2\bar{u}_2-14\right)+11\right)+13\right)-12\right)v_1^2\\
&+\left(u_2\left(u_2\left(u_2\left(-4u_2^2+22u_2-29\right)+17\right)-13\right)+6\right)v_1+2u_2\left((u_2-5)\,u_2+3\right)\bar{u}_2^2\Big)v_2^5\\
&-\Big(2\left(u_2\left((u_2-3)\,u_2\left(8u_2-9\right)+5\right)-6\right)v_1^4\\
&+\left(u_2\left(u_2\left(u_2\left(8u_2^2-84u_2+231\right)-163\right)-33\right)+42\right)v_1^3\\
&+\left(u_2\left(u_2\left(u_2\left((55-7u_2)\,u_2-135\right)+82\right)+43\right)-48\right)v_1^2\\
&+\left(u_2\left(u_2\left(51-(u_2-3)\,u_2\left(9u_2-26\right)\right)-36\right)+18\right)v_1+6u_2\left((u_2-5)\,u_2+3\right)\bar{u}_2^2\Big)v_2^4
\end{aligned}
$$

$$
\begin{aligned}
&+\left((6-v_1\left((v_1-3)\,v_1\,(8v_1-9)+5\right))\,u_2^5\right.\\
&+\left(v_1\left(v_1\left(v_1\left(-8v_1^2+84v_1-231\right)+163\right)+33\right)-42\right)u_2^4\\
&+\left(v_1\left(v_1\left(v_1\left(33\left(v_1-7\right)v_1+476\right)-289\right)-55\right)+84\right)u_2^3\\
&+\left(v_1\left(v_1\left(v_1\left(\left(163-27v_1\right)v_1-289\right)+162\right)+36\right)-66\right)u_2^2\\
&+\left(v_1\left(v_1\left(v_1\left(\left(33-5v_1\right)v_1-55\right)+36\right)-26\right)+18\right)u_2+6v_1\left(\left(v_1-5\right)v_1+3\right)\bar{v}_1^2\Big)\,v_2^3\\
&-\Big(\left(u_2\left(u_2\left(u_2\left(-u_2\bar{u}_2-14\right)+11\right)+13\right)-12\right)v_1^4\\
&+\left(u_2\left(u_2\left(u_2\left(8\left(u_2-7\right)u_2+121\right)-71\right)-30\right)+36\right)v_1^3\\
&+\left(u_2\left(u_2\left(u_2\left(\left(107-19u_2\right)u_2-168\right)+91\right)+6\right)-30\right)v_1^2\\
&+\left(u_2\left(u_2\left(u_2\left(u_2\left(3u_2-19\right)+36\right)-35\right)+10\right)+6\right)v_1+2u_2\left(\left(u_2-5\right)u_2+3\right)\bar{u}_2^2\Big)\,\bar{v}_1v_2^2\\
&-v_1\left(\left(u_2\left(u_2\left(u_2\left(4u_2^2-22u_2+29\right)-17\right)+13\right)-6\right)v_1^2\right.\\
&-\left(u_2-2\right)u_2\left(-\left(u_2-6\right)u_2\bar{u}_2-5\right)v_1+6v_1+u_2\left(\left(u_2-4\right)\left(u_2-2\right)u_2-7\right)\bar{u}_2\Big)\,\bar{v}_1^2v_2\\
&-2u_2\left(\left(u_2-5\right)u_2+3\right)v_1^2\bar{u}_2^2\bar{v}_1^3\Big)\,u_1^3-\Big(\Big(\left(v_1\left(\left(v_1-8\right)v_1+9\right)-4\right)u_2^4\\
&+\left(v_1\left(v_1\left(8v_1+5\right)-17\right)+12\right)u_2^3+\left(\left(16-19v_1\right)v_1^2-10\right)u_2^2\\
&+\left(\left(v_1-2\right)v_1\left(3v_1-2\right)+2\right)u_2+2v_1\bar{v}_1^2\Big)\,v_2^5+\Big(2\left(u_2\left(u_2\left(u_2\left(u_2+8\right)-19\right)+3\right)+2\right)v_1^4\\
&-\left(u_2\left(u_2\left(u_2\left(u_2+56\right)-107\right)+19\right)+14\right)v_1^3\\
&+\left(u_2\left(u_2\left(u_2\left(13u_2+15\right)-67\right)+27\right)+16\right)v_1^2-\left(u_2\left(u_2\left(u_2\left(22u_2-45\right)+6\right)+10\right)+6\right)v_1\\
&-6u_2\left(2\left(u_2-2\right)u_2+1\right)\bar{u}_2\Big)\,v_2^4+\Big(\left(v_1\left(v_1\left(v_1\left(-v_1\bar{v}_1-14\right)+11\right)+13\right)-12\right)u_2^4\\
&+\left(v_1\left(v_1\left(v_1\left(8\left(v_1-7\right)v_1+121\right)-71\right)-30\right)+36\right)u_2^3\\
&+\left(v_1\left(v_1\left(v_1\left(\left(107-19v_1\right)v_1-168\right)+91\right)+6\right)-30\right)u_2^2\\
&+\left(v_1\left(v_1\left(v_1\left(v_1\left(3v_1-19\right)+36\right)-35\right)+10\right)+6\right)u_2+2v_1\left(\left(v_1-5\right)v_1+3\right)\bar{v}_1^2\Big)\,v_2^3\\
&+\bar{v}_1\Big(\left(v_1\left(v_1\left(v_1\left(8v_1-5\right)-16\right)+8\right)+4\right)u_2^4-\left(v_1\left(v_1\left(5v_1\left(v_1+4\right)-51\right)+19\right)+12\right)u_2^3\\
&+\left(v_1\left(v_1\left(\left(51-16v_1\right)v_1-40\right)+16\right)+10\right)u_2^2+\left(v_1\left(v_1\left(v_1\left(8v_1-19\right)+16\right)-8\right)-2\right)u_2\\
&-2v_1\left(2\left(v_1-2\right)v_1+1\right)\bar{v}_1\Big)\,v_2^2+v_1\bar{v}_1^2\Big(\left(v_1\left(9v_1-4\right)-4\right)u_2^4+\left(\left(11-17v_1\right)v_1+9\right)u_2^3\\
&-6\left(v_1+1\right)u_2^2+\left(4v_1^2-2v_1+2\right)u_2-2v_1\bar{v}_1\Big)\,v_2-2u_2\left(2\left(u_2-2\right)u_2+1\right)v_1^2\bar{u}_2\bar{v}_1^3\Big)\,\bar{u}_2u_1^2\\
&-u_2\Big(\Big(\left(v_2\left(v_2\left(4v_2-9\right)+8\right)-2\right)v_1^5+\left(2v_2\left(v_2\left(v_2\left(4v_2-11\right)+13\right)-10\right)+6\right)v_1^4\\
&+\left(v_2\left(v_2\left(v_2\left(4v_2^2-22v_2+29\right)-17\right)+13\right)-6\right)v_1^3+\left(9v_2^4-17v_2^3+4v_2+2\right)\bar{v}_2v_1^2\\
&+v_2\left(8v_2^2-4v_2-3\right)\bar{v}_2^2v_1+2v_2^2\bar{v}_2^3\Big)\,u_2^2-\Big(\Big(\left(v_2-2\right)^2v_2-2\Big)\,v_1^5\\
&+\left(v_2\left(v_2\left(v_2\left(2v_2-9\right)+15\right)-12\right)+6\right)v_1^4+\left(\left(v_2-2\right)v_2\left(-\left(v_2-6\right)v_2\bar{v}_2-5\right)-6\right)v_1^3\\
&+\left(\left(v_2\left(\left(15-4v_2\right)v_2-17\right)+4\right)v_2^2+2\right)v_1^2+2v_2\bar{v}_2^2\left(-2v_2\bar{v}_2-1\right)v_1+2v_2^2\bar{v}_2^3\Big)\,u_2\\
&+v_1v_2\left(\left(v_2-3\right)v_1^2+\left(v_2^2+2\right)v_1+\left(2-3v_2\right)v_2\right)v_3\bar{v}_1\bar{v}_2\Big)\,\bar{u}_2^2u_1+2u_2^2v_1v_2v_3\bar{u}_2^3\bar{v}_1^2\bar{v}_2^2\bar{v}_3\Big]\\
&-\frac{V_1^uV_2^v}{8u_1^2\bar{u}_1^2u_2^2\bar{u}_2^2u_3^2v_1\bar{v}_1^2v_2\bar{v}_2^2v_3^2}\Big[\Big(-\Big(\left(v_1\left(7v_1-8\right)+3\right)v_2^3+\left(v_1\left(v_1\left(7v_1-30\right)+26\right)-7\right)v_2^2\\
&-2\left(v_1-2\right)v_1\left(4v_1-5\right)v_2+4v_2-v_1\left(3v_1-4\right)\bar{v}_1\Big)\,v_3u_2^2-2\left(2v_1v_2^3+\left(v_1\left(4v_1-7\right)+1\right)v_2^2\right.\\
&-\left(v_1\left(2v_1-5\right)+1\right)\bar{v}_1v_2-v_1\bar{v}_1\Big)\,\bar{v}_1\bar{v}_2u_2-2v_1v_2v_3\bar{v}_1\bar{v}_2\left(\bar{v}_3-2\right)\Big)\,u_1^5
\end{aligned}
$$

$$
\begin{aligned}
&+\Big(-2\left(\left(v_1\left(20v_1-27\right)+10\right)v_2^3+\left(v_1\left(20v_1^2-78v_1+71\right)-19\right)v_2^2\right.\\
&+\left(v_1\left(\left(71-27v_1\right)v_1-50\right)+9\right)v_2-v_1\left(10v_1-9\right)\bar{v}_1\Big)v_3u_2^3+\Big(\left(3\left(34-25v_1\right)v_1-38\right)v_2^4\\
&+\left(v_1\left(25\left(19-6v_1\right)v_1-417\right)+114\right)v_2^3+\left(v_1\left(553-25v_1\left(v_1\left(3v_1-19\right)+34\right)\right)-114\right)v_2^2\\
&-\left(v_1\left(3v_1\left(34v_1-105\right)+238\right)-38\right)\bar{v}_1v_2+38v_1\bar{v}_1^3\Big)u_2^2+\Big(\left(v_1\left(31v_1-42\right)+12\right)v_2^4\\
&+\left(v_1\left(v_1\left(62v_1-199\right)+175\right)-40\right)v_2^3+\left(v_1\left(v_1\left(v_1\left(31v_1-199\right)+362\right)-237\right)+44\right)v_2^2\\
&+\left(v_1\left(7v_1\left(6v_1-19\right)+104\right)-16\right)\bar{v}_1v_2+4v_1\left(3v_1-4\right)\bar{v}_1^2\Big)u_2+6v_1v_2v_3\bar{v}_1\bar{v}_2\left(\bar{v}_3-2\right)\Big)u_1^4\\
&+\Big(-2\left(\left(v_1\left(20v_1-27\right)+10\right)v_2^3+\left(v_1\left(20v_1^2-78v_1+71\right)-19\right)v_2^2\right.\\
&+\left(v_1\left(\left(71-27v_1\right)v_1-50\right)+9\right)v_2-v_1\left(10v_1-9\right)\bar{v}_1\Big)v_3u_2^4+\Big(\left(v_1\left(161v_1-220\right)+83\right)v_2^3\\
&+\left(v_1\left(v_1\left(161v_1-626\right)+574\right)-157\right)v_2^2+2\left(v_1\left(\left(287-110v_1\right)v_1-202\right)+37\right)v_2\\
&-v_1\left(83v_1-74\right)\bar{v}_1\Big)v_3u_2^3+\Big(\left(v_1\left(171v_1-242\right)+93\right)v_2^4\\
&+\left(v_1\left(v_1\left(342v_1-1079\right)+965\right)-272\right)v_2^3\\
&+\left(v_1\left(v_1\left(v_1\left(171v_1-1079\right)+1922\right)-1257\right)+265\right)v_2^2\\
&+\left(v_1\left(v_1\left(242v_1-723\right)+534\right)-86\right)\bar{v}_1v_2+v_1\left(93v_1-86\right)\bar{v}_1^2\Big)u_2^2\\
&+\Big(\left(\left(76-51v_1\right)v_1-27\right)v_2^4+\left(82-3v_1\left(v_1\left(34v_1-109\right)+101\right)\right)v_2^3\\
&+\left(3v_1\left(v_1\left(\left(109-17v_1\right)v_1-198\right)+133\right)-83\right)v_2^2\\
&-\left(v_1\left(v_1\left(76v_1-227\right)+172\right)-28\right)\bar{v}_1v_2-v_1\left(27v_1-28\right)\bar{v}_1^2\Big)u_2\\
&-6v_1v_2v_3\bar{v}_1\bar{v}_2\left(\bar{v}_3-2\right)\Big)u_1^3-\bar{u}_2\Big(-\Big(\left(v_1\left(7v_1-8\right)+3\right)v_2^3+\left(v_1\left(v_1\left(7v_1-30\right)+26\right)-7\right)v_2^2\\
&-2\left(v_1-2\right)v_1\left(4v_1-5\right)v_2+4v_2-v_1\left(3v_1-4\right)\bar{v}_1\Big)v_3u_2^4+\Big(\left(-68v_1^2+94v_1-35\right)v_2^4\\
&+2\left(v_1\left(\left(215-68v_1\right)v_1-190\right)+52\right)v_2^3+\left(2v_1\left(v_1\left(\left(215-34v_1\right)v_1-384\right)+250\right)\right.\\
&\left.-103\right)v_2^2-2\left(v_1\left(v_1\left(47v_1-143\right)+107\right)-17\right)\bar{v}_1v_2-v_1\left(35v_1-34\right)\bar{v}_1^2\Big)u_2^3\\
&+\Big(\left(v_1\left(103v_1-148\right)+58\right)v_2^4+\left(v_1\left(v_1\left(206v_1-649\right)+585\right)-168\right)v_2^3\\
&+\left(v_1\left(v_1\left(v_1\left(103v_1-649\right)+1154\right)-757\right)+162\right)v_2^2\\
&+\left(v_1\left(v_1\left(148v_1-437\right)+320\right)-52\right)\bar{v}_1v_2+2v_1\left(29v_1-26\right)\bar{v}_1^2\Big)u_2^2\\
&+\Big(\left(\left(40-23v_1\right)v_1-18\right)v_2^4+\left(v_1\left(\left(149-46v_1\right)v_1-153\right)+52\right)v_2^3\\
&+\left(v_1\left(v_1\left(\left(149-23v_1\right)v_1-274\right)+197\right)-50\right)v_2^2-\left(v_1\left(v_1\left(40v_1-113\right)+84\right)-16\right)\bar{v}_1v_2\\
&-2v_1\left(9v_1-8\right)\bar{v}_1^2\Big)u_2-2v_1v_2v_3\bar{v}_1\bar{v}_2\left(\bar{v}_3-2\right)\Big)u_1^2\\
&-u_2\Big(\Big(-4v_1\bar{v}_1u_2^2+\left(\left(34-23v_1\right)v_1-12\right)u_2+v_1^2-4v_1+3\Big)v_2^4\\
&+\Big(-2\left(v_1\left(4v_1-9\right)+1\right)\bar{v}_1u_2^2+\left(v_1\left(\left(147-46v_1\right)v_1-135\right)+36\right)u_2\\
&-\left(v_1\left(2v_1-5\right)+8\right)\bar{v}_1\Big)v_2^3+\Big(-2\left(v_1\left(v_1\left(2v_1-11\right)+13\right)-2\right)\bar{v}_1u_2^2\\
&+\left(v_1\left(v_1\left(\left(147-23v_1\right)v_1-266\right)+177\right)-36\right)u_2-\left(\left(v_1-4\right)\left(v_1-2\right)v_1-7\right)\bar{v}_1\Big)v_2^2\\
&+\bar{v}_1\Big(-2\left(2\left(v_1-3\right)v_1+1\right)\bar{v}_1u_2^2+\left(v_1\left(\left(101-34v_1\right)v_1-76\right)+12\right)u_2\\
&+v_1\left(v_1\left(4v_1-9\right)+6\right)-2\Big)v_2-v_1\left(-3v_1+2u_2\left(u_2+6v_1-6\right)+2\right)\bar{v}_1^2\Big)\bar{u}_2^2u_1\\
&+2u_2^2v_1v_2v_3\bar{u}_2^3\bar{v}_1\bar{v}_2\left(\bar{v}_3-2\right)\Big]
\end{aligned}
$$

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\begin{aligned}
&+\frac{V_1^u V_3^v}{4u_1^2\bar{u}_1^2u_2^2\bar{u}_2^2u_3^2v_1\bar{v}_1^2v_2\bar{v}_2^2v_3^2}\left[\left(\left(\left(u_2\left(5u_2-8\right)+4\right)v_1-3\bar{u}_2^2\right)v_2^3+\left(\left(2v_1\left(5v_1-8\right)+5\right)u_2^2\right.\right.\right.\\
&+\left(\left(29-16v_1\right)v_1-11\right)u_2+v_1\left(8v_1-15\right)+6\right)v_2^2+\left(\left(\left(v_1-2\right)v_1\left(5v_1-6\right)-2\right)u_2^2\right.\\
&+\left(v_1\left(8v_1-21\right)+5\right)\bar{v}_1u_2-\left(v_1\left(4v_1-11\right)+3\right)\bar{v}_1\right)v_2\\
&-v_1\left(-2u_2-3v_1\bar{u}_2+3\right)\bar{u}_2\bar{v}_1\right)\bar{v}_1\bar{v}_2u_1^5+\left(\left(-\bar{u}_2\left(3-2u_2\right)^2\right.\right.\\
&+\left(u_2\left(u_2\left(8u_2-27\right)+29\right)-12\right)v_1^2+\left(u_2\left(\left(44-13u_2\right)u_2-50\right)+21\right)v_1\right)v_2^4\\
&+\left(\left(v_1\left(v_1\left(16v_1-51\right)+48\right)-11\right)u_2^3+2\left(v_1\left(\left(86-27v_1\right)v_1-82\right)+22\right)u_2^2\right.\\
&-2\left(v_1\left(29v_1-67\right)+30\right)\bar{v}_1u_2+3\left(v_1\left(8v_1-19\right)+9\right)\bar{v}_1\right)v_2^3\\
&+\left(\left(v_1\left(v_1\left(v_1\left(8v_1-51\right)+92\right)-60\right)+10\right)u_2^3\right.\\
&+\left(v_1\left(v_1\left(\left(172-27v_1\right)v_1-310\right)+206\right)-40\right)u_2^2\\
&-\left(v_1\left(v_1\left(29v_1-163\right)+199\right)-57\right)\bar{v}_1u_2+3\left(v_1\left(v_1\left(4v_1-23\right)+29\right)-9\right)\bar{v}_1\right)v_2^2\\
&+\bar{v}_1\left(-\left(v_1\left(13v_1-22\right)+3\right)\bar{v}_1u_2^3+2\left(v_1\left(-22v_1^2+60v_1-43\right)+6\right)u_2^2\right.\\
&-2\left(v_1\left(25v_1-47\right)+9\right)\bar{v}_1u_2+3\left(7\left(v_1-2\right)v_1+3\right)\bar{v}_1\right)v_2\\
&-v_1\bar{u}_2\left(\left(u_2-3\right)u_2\left(4v_1-3\right)-9\bar{v}_1\right)\bar{v}_1^2\right)u_1^4+\left(\left(\left(u_2\left(u_2\left(u_2\left(8u_2-31\right)+45\right)-33\right)\right.\right.\right.\\
&+12)v_1^2+u_2\left(u_2\left(\left(49-13u_2\right)u_2-71\right)+55\right)v_1-21v_1\\
&+\left(u_2-2\right)u_2\left(4u_2^2-6u_2+11\right)+9\right)v_2^4+\left(\left(v_1\left(v_1\left(16v_1-51\right)+48\right)-11\right)u_2^4\right.\\
&+2\left(v_1\left(\left(98-31v_1\right)v_1-90\right)+19\right)u_2^3+\left(v_1\left(90v_1^2-286v_1+265\right)-63\right)u_2^2\\
&+\left(v_1\left(66v_1-151\right)+63\right)\bar{v}_1u_2-3\left(v_1\left(8v_1-19\right)+9\right)\bar{v}_1\right)v_2^3\\
&+\left(\left(v_1\left(v_1\left(v_1\left(8v_1-51\right)+92\right)-60\right)+10\right)u_2^4+\left(v_1\left(v_1\left(\left(196-31v_1\right)v_1-350\right)+223\right)\right.\right.\\
&-34)u_2^3+\left(v_1\left(v_1\left(v_1\left(45v_1-286\right)+514\right)-333\right)+57\right)u_2^2\\
&+\left(v_1\left(v_1\left(33v_1-184\right)+222\right)-60\right)\bar{v}_1u_2-3\left(v_1\left(v_1\left(4v_1-23\right)+29\right)-9\right)\bar{v}_1\right)v_2^2\\
&+\bar{v}_1\left(-\left(v_1\left(13v_1-22\right)+3\right)\bar{v}_1u_2^4+\left(v_1\left(49v_1-82\right)+10\right)\bar{v}_1u_2^3\right.\\
&+\left(v_1\left(v_1\left(71v_1-194\right)+139\right)-17\right)u_2^2+\left(v_1\left(55v_1-104\right)+19\right)\bar{v}_1u_2\\
&-3\left(7\left(v_1-2\right)v_1+3\right)\bar{v}_1\right)v_2-v_1\left(-9v_1+u_2\left(13v_1+u_2\left(-10v_1+u_2\left(4v_1-3\right)+7\right)\right.\right.\\
&-10)+9)\bar{u}_2\bar{v}_1^2\right)u_1^3-\left(\left(\left(u_2\left(u_2\left(u_2\left(5u_2-22\right)+23\right)-7\right)+4\right)v_1^2\right.\right.\\
&+u_2\left(u_2\left(4\left(9-2u_2\right)u_2-35\right)+9\right)v_1-7v_1+u_2\left(3u_2-1\right)\left(\left(u_2-4\right)u_2+2\right)+3\right)v_2^4\\
&+\left(-\left(2v_1-1\right)\left(5v_1-8\right)\bar{v}_1u_2^4+\left(v_1\left(\left(141-44v_1\right)v_1-135\right)+36\right)u_2^3\right.\\
&+\left(v_1\left(v_1\left(46v_1-145\right)+130\right)-27\right)u_2^2+\left(v_1\left(14v_1-29\right)+5\right)\bar{v}_1u_2\\
&-\left(v_1\left(8v_1-19\right)+9\right)\bar{v}_1\right)v_2^3+\left(\left(v_1\left(5v_1-21\right)+7\right)\bar{v}_1^2u_2^4+\left(v_1\left(v_1\left(\left(141-22v_1\right)v_1-256\right)\right.\right.\right.\\
&+171)-33)u_2^3+\left(v_1\left(v_1\left(v_1\left(23v_1-145\right)+258\right)-162\right)+24\right)u_2^2\\
&+\left(v_1\left(v_1\left(7v_1-36\right)+38\right)-4\right)\bar{v}_1u_2-\left(v_1\left(v_1\left(4v_1-23\right)+29\right)-9\right)\bar{v}_1\right)v_2^2\\
&+\bar{v}_1\left(\left(v_1\left(v_1\left(8v_1-21\right)+14\right)-2\right)u_2^4+\left(10-9v_1\left(v_1\left(4v_1-11\right)+8\right)\right)u_2^3\right.\\
&-\left(5v_1\left(7v_1-12\right)+7\right)\bar{v}_1u_2^2+\left(v_1\left(9v_1-16\right)+1\right)\bar{v}_1u_2-\left(7\left(v_1-2\right)v_1+3\right)\bar{v}_1\right)v_2\\
&+v_1\left(-2u_2^4+10u_2^3-7u_2^2+u_2+\left(u_2\left(3u_2-1\right)\left(\left(u_2-4\right)u_2+2\right)+3\right)v_1-3\right)\bar{v}_1^2\right)\bar{u}_2u_1^2\\
&-u_2\bar{u}_2^2\bar{v}_1\bar{v}_2\left(\left(-v_2+u_2\left(8v_2u_2-6u_2-13v_2+9\right)+2\right)v_1^3+\left(2\left(u_2\left(8u_2-13\right)-1\right)v_2^2\right.\right.
\end{aligned}
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\begin{aligned}
&+(47-29u_2)\,u_2v_2+5v_2+u_2\,(11u_2-17)-4)\,v_1^2-\big((u_2\,(8u_2-13)-1)\,v_2^2\\
&+(34-21u_2)\,u_2v_2+4v_2+u_2\,(5u_2-8)-2)\,\bar{v}_2v_1\\
&+v_2\,(-2v_2+u_2\,(6v_2u_2-5u_2-9v_2+8)+2)\,\bar{v}_2)\,u_1+u_2^2v_3\bar{u}_2^3\bar{v}_1\bar{v}_2\,\big((4v_2-3)\,v_1^2\\
&+(4\,(v_2-2)\,v_2+3)\,v_1+3v_2\bar{v}_2)]\\
&-\frac{A_1^uA_1^v}{16u_1^2\bar{u}_1^2u_2^2\bar{u}_2^2u_3^2\bar{u}_3v_1^2\bar{v}_1^2v_2^2\bar{v}_2^2v_3^2\bar{v}_3}\,(u_1-u_2)\,(v_1-v_2)\,\big[\big(\big(((v_1\,(3v_1-4)+2)\,u_2^3\\
&+(v_1\,(v_1\,(3v_1-16)+11)-4)\,u_2^2+(v_1\,((11-4v_1)\,v_1-8)+2)\,u_2+2v_1\bar{v}_1^2\big)\,v_2^5\\
&-\big(2\,(u_2\,(3u_2-4)+2)\,v_1^3+((u_2-8)\,u_2\,(4u_2-3)-10)\,v_1^2\\
&+(u_2\,((22-9u_2)\,u_2-18)+6)\,v_1+6u_2\bar{u}_2^2\big)\,\bar{v}_1v_2^4\\
&+\big((v_1-2)\,(v_1\,(v_1\,(4v_1-9)+9)-3)\,u_2^3+(v_1\,(v_1\,((v_1-11)\,v_1\,(3v_1-8)-71)+39)\\
&-12)\,u_2^2+(v_1\,(v_1\,(v_1\,(-4\,(v_1-8)\,v_1-65)+55)-25)+6)\,u_2\\
&+2v_1\,((v_1-5)\,v_1+3)\,\bar{v}_1^2\big)\,v_2^3-\big((u_2\,(u_2\,(3u_2-16)+11)-4)\,v_1^4\\
&+(u_2\,(10u_2-31)\,\bar{u}_2+12)\,v_1^3+(u_2\,(u_2\,(17u_2-30)+24)-10)\,v_1^2\\
&+(u_2\,((16-11u_2)\,u_2-8)+2)\,v_1+2u_2\bar{u}_2^2\big)\,\bar{v}_1v_2^2-v_1\bar{v}_1^2\,\big(-(4v_1-3)\,\bar{v}_1u_2^3\\
&+((12-11v_1)\,v_1-4)\,u_2^2+(1-8v_1\bar{v}_1)\,u_2+2v_1\bar{v}_1\big)\,v_2-2u_2v_1^2\bar{u}_2^2\bar{v}_1^3\big)\,u_1^5\\
&-\bar{u}_2\,\big(2\,\big((v_1\,(3v_1-4)+2)\,v_2^5-(v_1\,(4v_1-9)+6)\,\bar{v}_1v_2^4\\
&+(v_1-2)\,(v_1\,(v_1\,(4v_1-9)+9)-3)\,v_2^3-(v_1\,(v_1\,(v_1\,(3v_1-10)+17)-11)+2)\,\bar{v}_1v_2^2\\
&+v_1\,(4v_1-3)\,\bar{v}_1^3v_2-2v_1^2\bar{v}_1^3\big)\,u_2^3+\big(((v_1-8)\,v_1\,(4v_1-3)-10)\,v_2^5\\
&-\big(v_1\,\big(8v_1^2-76v_1+43\big)-30\big)\,\bar{v}_1v_2^4+(2v_1-3)\,(v_1\,(v_1\,(v_1\,(2v_1-39)+31)-21)\\
&+10)\,v_2^3+(v_1\,(v_1\,(7v_1\,(5v_1-12)+51)-33)+10)\,\bar{v}_1v_2^2+v_1\,(v_1\,(24v_1-25)+9)\,\bar{v}_1^2v_2\\
&+10v_1^2\bar{v}_1^3\big)\,u_2^2+\big((v_1\,((22-9v_1)\,v_1-18)+6)\,v_2^5+(v_1\,(v_1\,(18v_1-43)+40)-18)\,\bar{v}_1v_2^4\\
&+(v_1\,(v_1\,(v_1\,((61-9v_1)\,v_1-122)+117)-66)+18)\,v_2^3\\
&-(v_1\,(v_1\,(v_1\,(22v_1-61)+56)-24)+6)\,\bar{v}_1v_2^2+2v_1\,(9v_1-2)\,\bar{v}_1^3v_2-6v_1^2\bar{v}_1^3\big)\,u_2\\
&-6v_1v_2v_3\bar{v}_1^2\bar{v}_2^2\bar{v}_3\big)\,u_1^4+\big(\big(((u_2-2)\,(u_2\,(u_2\,(4u_2-9)+9)-3)\,v_1^3\\
&+(u_2\,(u_2\,((u_2-11)\,u_2\,(3u_2-8)-71)+39)-12)\,v_1^2+(u_2\,(u_2\,(u_2\,(-4\,(u_2-8)\,u_2-65)\\
&+55)-25)+6)\,v_1+2u_2\,((u_2-5)\,u_2+3)\,\bar{u}_2^2\big)\,v_2^5\\
&-\big(2\,(u_2-2)\,(u_2\,(u_2\,(4u_2-9)+9)-3)\,v_1^3\\
&+(2u_2-3)\,(u_2\,(u_2\,(u_2\,(2u_2-39)+31)-21)+10)\,v_1^2\\
&+(u_2\,(u_2\,(u_2\,((61-9u_2)\,u_2-122)+117)-66)+18)\,v_1+6u_2\,((u_2-5)\,u_2+3)\,\bar{u}_2^2\big)\,\bar{v}_1v_2^4\\
&+\big((v_1-2)\,(v_1\,(v_1\,(4v_1-9)+9)-3)\,u_2^5+(v_1\,(v_1\,(v_1\,(4\,(v_1-23)\,v_1+213)-189)\\
&+125)-42)\,u_2^4+(v_1\,(v_1\,(v_1\,((213-17v_1)\,v_1-440)+367)-241)+84)\,u_2^3\\
&+(v_1\,(v_1\,(v_1\,(27\,(v_1-7)\,v_1+367)-346)+218)-66)\,u_2^2\\
&+(v_1\,(v_1\,(v_1\,((125-21v_1)\,v_1-241)+218)-98)+18)\,u_2+6v_1\,((v_1-5)\,v_1+3)\,\bar{v}_1^2\big)\,v_2^3\\
&-\big((u_2\,(u_2\,((u_2-11)\,u_2\,(3u_2-8)-71)+39)-12)\,v_1^4
\end{aligned}
$$

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\begin{aligned}
&+\left(u_2\left(u_2\left(u_2\left(2\left(52-5u_2\right)u_2-213\right)+181\right)-110\right)+36\right)v_1^3\\
&+\left(u_2\left(u_2\left(u_2\left(17\left(u_2-5\right)u_2+154\right)-165\right)+108\right)-30\right)v_1^2\\
&+\left(u_2\left(u_2\left(u_2\left(-11\left(u_2-5\right)u_2-96\right)+85\right)-38\right)+6\right)v_1+2u_2\left(\left(u_2-5\right)u_2+3\right)\bar{u}_2^2\big)\bar{v}_1v_2^2\\
&-v_1\Big(\left(u_2\left(u_2\left(2u_2-11\right)\left(u_2\left(2u_2-5\right)+5\right)+25\right)-6\right)v_1^2\\
&+\left(u_2\left(u_2\left(u_2\left(\left(39-7u_2\right)u_2-76\right)+73\right)-34\right)+6\right)v_1-u_2\Big(3\left(u_2-2\right)^2u_2-5\Big)\bar{u}_2\Big)\bar{v}_1^2v_2\\
&-2u_2\left(\left(u_2-5\right)u_2+3\right)v_1^2\bar{u}_2^2\bar{v}_1^3\Big)u_1^3\\
&-\Big(\Big(\left(v_1\left(v_1\left(3v_1-16\right)+11\right)-4\right)u_2^4+\left(v_1\left(10v_1-31\right)\bar{v}_1+12\right)u_2^3\\
&+\left(v_1\left(v_1\left(17v_1-30\right)+24\right)-10\right)u_2^2+\left(v_1\left(\left(16-11v_1\right)v_1-8\right)+2\right)u_2+2v_1\bar{v}_1^2\Big)v_2^5\\
&-\Big(2\left(u_2\left(u_2\left(u_2\left(3u_2-10\right)+17\right)-11\right)+2\right)v_1^3\\
&+\left(u_2\left(u_2\left(7\left(12-5u_2\right)u_2-51\right)+33\right)-10\right)v_1^2\\
&+\left(u_2\left(u_2\left(u_2\left(22u_2-61\right)+56\right)-24\right)+6\right)v_1+6u_2\left(u_2\left(-2\left(u_2-3\right)u_2-5\right)+1\right)\right)\bar{v}_1v_2^4\\
&+\Big(\left(v_1\left(v_1\left(\left(v_1-11\right)v_1\left(3v_1-8\right)-71\right)+39\right)-12\right)u_2^4\\
&+\left(v_1\left(v_1\left(v_1\left(2\left(52-5v_1\right)v_1-213\right)+181\right)-110\right)+36\right)u_2^3\\
&+\left(v_1\left(v_1\left(v_1\left(17\left(v_1-5\right)v_1+154\right)-165\right)+108\right)-30\right)u_2^2\\
&+\left(v_1\left(v_1\left(v_1\left(-11\left(v_1-5\right)v_1-96\right)+85\right)-38\right)+6\right)u_2+2v_1\left(\left(v_1-5\right)v_1+3\right)\bar{v}_1^2\Big)v_2^3\\
&+\bar{v}_1\Big(\left(v_1\left(v_1\left(v_1\left(16v_1-41\right)+30\right)-16\right)+4\right)u_2^4\\
&+\left(v_1\left(v_1\left(\left(104-41v_1\right)v_1-77\right)+41\right)-12\right)u_2^3\\
&+\left(v_1\left(v_1\left(v_1\left(30v_1-77\right)+88\right)-44\right)+10\right)u_2^2\\
&+\left(v_1\left(v_1\left(\left(41-16v_1\right)v_1-44\right)+16\right)-2\right)u_2-2v_1\left(2\left(v_1-2\right)v_1+1\right)\bar{v}_1\right)v_2^2\\
&+v_1\Big(\left(u_2\left(u_2\left(u_2\left(11u_2-31\right)+24\right)-8\right)+2\right)v_1^2\\
&+\left(u_2\left(u_2\left(\left(35-12u_2\right)u_2-38\right)+14\right)-2\right)v_1+u_2\left(u_2\left(u_2\left(4u_2-9\right)+8\right)-2\right)\right)\bar{v}_1^2v_2\\
&-2u_2\left(2\left(u_2-2\right)u_2+1\right)v_1^2\bar{u}_2\bar{v}_1^3\Big)\bar{u}_2u_1^2-u_2\bar{u}_2^2\Big(\Big(\left(v_1\left(v_1\left(4v_1-11\right)+8\right)-2\right)v_2^5\\
&-2\Big(\left(3-2v_1\right)^2v_1-3\Big)\bar{v}_1v_2^4+\left(v_1\left(v_1\left(2v_1-11\right)\left(v_1\left(2v_1-5\right)+5\right)+25\right)-6\right)v_2^3\\
&+\left(v_1\left(v_1\left(v_1\left(11v_1-31\right)+24\right)-8\right)+2\right)\bar{v}_1v_2^2+v_1\bar{v}_1^2\left(1-8v_1\bar{v}_1\right)v_2+2v_1^2\bar{v}_1^3\Big)u_2^2\\
&+\Big(\left(v_1\left(\left(12-7v_1\right)v_1-8\right)+2\right)v_2^5+\left(v_1\left(v_1\left(14v_1-25\right)+22\right)-6\right)\bar{v}_1v_2^4\\
&+\left(v_1\left(v_1\left(v_1\left(\left(39-7v_1\right)v_1-76\right)+73\right)-34\right)+6\right)v_2^3-\left(v_1\left(v_1\left(v_1\left(12v_1-35\right)+38\right)\right.\right.\\
&\left.\left.-14\right)+2\right)\bar{v}_1v_2^2-2v_1\Big(4v_1^2-6v_1+1\Big)\bar{v}_1^2v_2-2v_1^2\bar{v}_1^3\Big)u_2\\
&-v_1v_2\left(3v_2v_1-v_1-v_2+2\right)v_3\bar{v}_1\bar{v}_2\bar{v}_3\big)u_1+2u_2^2v_1v_2v_3\bar{u}_2^3\bar{v}_1^2\bar{v}_2^2\bar{v}_3\Big]\\
&+\frac{A_1^uA_2^v}{8u_1^2\bar{u}_1^2u_2^2\bar{u}_2^2u_3^2\bar{u}_3v_1\bar{v}_1^2v_2\bar{v}_2^2v_3^2}\left(u_1-u_2\right)\left(v_1-v_2\right)\Big[\Big(\Big(\left(v_1\left(3v_1-8\right)+3\right)v_2^3\\
&+\left(v_1-3\right)\left(3\left(v_1-2\right)v_1+2\right)v_2^2+\left(4v_1\left(2v_1-3\right)+3\right)\bar{v}_1v_2+3v_1\bar{v}_1^2\Big)u_2^2\\
&-2\left(-v_2+v_1\left(v_2\left(2v_1+2v_2-1\right)-1\right)+1\right)\bar{v}_1\bar{v}_2u_2-2v_1v_2v_3\bar{v}_1\bar{v}_2\big)u_1^5\\
&-\Big(2\left(\left(1-v_1\left(2v_1+3\right)\right)v_2^3+\left(v_1\left(2v_1\bar{v}_1+3\right)+1\right)v_2^2+\Big(3v_1^2-5\Big)\bar{v}_1v_2+\left(v_1+3\right)\bar{v}_1^2\Big)u_2^2\\
&+\Big(\left(12-v_1\left(5v_1+8\right)\right)v_2^3-\left(v_1\left(v_1\left(5v_1+17\right)-63\right)+40\right)v_2^2+\left(v_1\left(8v_1-55\right)+44\right)\bar{v}_1v_2
\end{aligned}
$$

$$+4\left(3v_1-4\right)\bar{v}_1^2\big)\,u_2-6v_1v_2v_3\bar{v}_1\bar{v}_2\big)\,\bar{u}_2u_1^4+\Big(\Big(\big(u_2\left(u_2\left(\left(7-4u_2\right)u_2+3\right)-13\right)+6\big)\,v_1^2$$
$$-2\left(u_2\left(u_2\left(u_2\left(3u_2+8\right)-25\right)+8\right)+3\right)v_1+u_2\left(u_2\left(u_2\left(2u_2+23\right)-55\right)+27\right)\big)\,v_2^3$$
$$+\big(\left(u_2\left(u_2\left(\left(7-4u_2\right)u_2+3\right)-13\right)+6\right)v_1^3$$
$$+\left(u_2\left(u_2\left(u_2\left(4u_2-71\right)+121\right)-25\right)-18\right)v_1^2$$
$$+2\left(u_2\left(u_2\left(3u_2\left(u_2+23\right)-149\right)+61\right)+6\right)v_1+2u_2\left(u_2\left(\left(u_2-44\right)u_2+88\right)-41\right)\big)\,v_2^2$$
$$+\big(2\left(u_2\left(u_2\left(u_2\left(3u_2+8\right)-25\right)+8\right)+3\right)v_1^2-2\left(u_2\left(u_2\left(61u_2-124\right)+53\right)+3\right)v_1$$
$$+u_2\left(83-u_2\left(2u_2-17\right)\left(5u_2-11\right)\right)\big)\,\bar{v}_1v_2+u_2\big(6u_2\left(\left(u_2-7\right)u_2+11\right)$$
$$+u_2\left(u_2\left(2u_2+23\right)-55\right)v_1+27v_1-28\big)\,\bar{v}_1^2\Big)\,u_1^3$$
$$-\Big(\Big(\left(v_1\left(3v_1-8\right)+3\right)u_2^4+\left(2\left(v_1-5\right)v_1+13\right)u_2^3+\left(5v_1\left(v_1+8\right)-42\right)u_2^2$$
$$+\left(18-v_1\left(7v_1+12\right)\right)u_2-2v_1\bar{v}_1\Big)\,v_2^3+\big(\left(v_1-3\right)\left(3\left(v_1-2\right)v_1+2\right)u_2^4$$
$$+\left(v_1\left(2\left(v_1-18\right)v_1+77\right)-48\right)u_2^3+\left(v_1\left(5v_1\left(v_1+17\right)-221\right)+128\right)u_2^2$$
$$-\left(v_1\left(v_1\left(7v_1+19\right)-79\right)+52\right)u_2-2\left(v_1-2\right)v_1\bar{v}_1\big)\,v_2^2$$
$$+\big(2\left(u_2\left(u_2\left(u_2\left(4u_2+5\right)-20\right)+6\right)+1\right)v_1^2-\big(u_2\left(u_2\left(u_2\left(12u_2+67\right)-181\right)+67\right)$$
$$+2\big)\,v_1+u_2\left(u_2\left(3u_2\left(u_2+19\right)-130\right)+50\right)\big)\,\bar{v}_1v_2+u_2\big(2\left(9v_1-8\right)$$
$$+u_2\left(-42v_1+u_2\left(\left(3u_2+13\right)v_1-22\right)+44\right)\big)\,\bar{v}_1^2\Big)\,\bar{u}_2u_1^2$$
$$+u_2\Big(-2\left(-v_2+v_1\left(v_2\left(2v_1+2v_2-1\right)-1\right)+1\right)\bar{v}_1\bar{v}_2u_2^2+\Big(\left(v_1\left(3v_1+10\right)-12\right)v_2^3$$
$$+\left(v_1\left(v_1\left(3v_1+19\right)-59\right)+36\right)v_2^2-\left(v_1-4\right)\left(10v_1-9\right)\bar{v}_1v_2+12\bar{v}_1^3\Big)\,u_2$$
$$+\left(3v_1+3\left(v_1+1\right)v_2-2\right)v_3\bar{v}_1\bar{v}_2\Big)\,\bar{u}_2^2u_1+2u_2^2v_1v_2v_3\bar{u}_2^3\bar{v}_1\bar{v}_2\Big]$$
$$-\frac{A_1^uA_3^v}{4u_1^2\bar{u}_1^2u_2^2\bar{u}_2^2u_3^2\bar{u}_3v_1\bar{v}_1^2v_2\bar{v}_2^2v_3^2}\left(u_1-u_2\right)\left(v_1-v_2\right)\Big[\Big(\left(2v_1+u_2\left(-4v_1-3u_2\bar{v}_1+6\right)-3\right)v_2^2$$
$$+\big(\left(u_2\left(3u_2-4\right)+2\right)v_1^2+\left(15-7u_2\right)u_2v_1-8v_1+u_2\left(5u_2-11\right)+6\big)\,v_2$$
$$-\left(-2u_2-3v_1\bar{u}_2+3\right)\bar{u}_2\bar{v}_1\Big)\,\bar{v}_1\bar{v}_2u_1^5-\Big(\Big(\big(v_1\left(8v_1-17\right)+8\big)\,v_2^3$$
$$+\left(2v_1-3\right)\left(4\left(v_1-3\right)v_1+7\right)v_2^2+\left(v_1\left(17v_1-33\right)+18\right)\bar{v}_1v_2+\left(8v_1-5\right)\bar{v}_1^2\Big)\,u_2^2$$
$$-\big(\left(11v_1-18\right)v_2^2-11\left(v_1-3\right)\bar{v}_1v_2+3\left(6v_1-5\right)\bar{v}_1\big)\,\bar{v}_1\bar{v}_2u_2$$
$$-3\left(2v_2v_1-3v_1-3v_2+3\right)v_3\bar{v}_1\bar{v}_2\Big)\,\bar{u}_2u_1^4+\Big(\Big(\big(v_1\left(8v_1-17\right)+8\big)\,v_2^3$$
$$+\left(2v_1-3\right)\left(4\left(v_1-3\right)v_1+7\right)v_2^2+\left(v_1\left(17v_1-33\right)+18\right)\bar{v}_1v_2+\left(8v_1-5\right)\bar{v}_1^2\Big)\,u_2^4$$
$$+\big(\left(\left(69-29v_1\right)v_1-38\right)v_2^3+\left(v_1\left(\left(151-29v_1\right)v_1-228\right)+104\right)v_2^2$$
$$-\left(3v_1\left(23v_1-53\right)+94\right)\bar{v}_1v_2-2\left(19v_1-14\right)\bar{v}_1^2\big)\,u_2^3+\big(\left(v_1\left(39v_1-101\right)+61\right)v_2^3$$
$$+\left(v_1\left(v_1\left(39v_1-229\right)+364\right)-173\right)v_2^2+\left(v_1\left(101v_1-263\right)+163\right)\bar{v}_1v_2$$
$$+\left(61v_1-51\right)\bar{v}_1^2\big)\,u_2^2-\big(-40v_1^2+77v_1+5\left(5v_1-8\right)v_2^2-\left(25v_1-77\right)v_2\bar{v}_1-37\big)\,\bar{v}_1\bar{v}_2u_2$$
$$-3\left(2v_2v_1-3v_1-3v_2+3\right)v_3\bar{v}_1\bar{v}_2\Big)\,u_1^3-\Big(\big(-3\bar{v}_1v_2^2+v_1\left(3v_1-7\right)v_2+5v_2+\left(5-3v_1\right)v_1$$
$$-2\big)\,\bar{v}_1\bar{v}_2u_2^4+\big(\left(8\left(5-2v_1\right)v_1-23\right)v_2^3+\left(v_1\left(\left(89-16v_1\right)v_1-138\right)+64\right)v_2^2$$
$$-\left(40v_1^2-98v_1+59\right)\bar{v}_1v_2-\left(23v_1-18\right)\bar{v}_1^2\big)\,u_2^3+\big(-38v_1^2+71v_1+\left(23v_1-38\right)v_2^2$$
$$-\left(23v_1-71\right)v_2\bar{v}_1-33\big)\,\bar{v}_1\bar{v}_2u_2^2-\big(-20v_1^2+39v_1+\left(13v_1-20\right)v_2^2-13\left(v_1-3\right)v_2\bar{v}_1$$

$$
\begin{aligned}
&-19)\,\bar{v}_1\bar{v}_2u_2-(2v_2v_1-3v_1-3v_2+3)\,v_3\bar{v}_1\bar{v}_2)\,\bar{u}_2u_1^2\\
&-u_2\left((3v_1+u_2\,(-9v_1+u_2\,(4v_1-6)+15)-4)\,v_2^2\right.\\
&-(3v_1+u_2\,(-9v_1+u_2\,(4v_1-11)+29)-8)\,\bar{v}_1v_2+(u_2\,(-15v_1+u_2\,(6v_1-5)+14)\\
&\left.-4\bar{v}_1)\,\bar{v}_1)\,\bar{u}_2^2\bar{v}_1\bar{v}_2u_1+u_2^2\,(-3v_2+v_1\,(2v_2-3)+3)\,v_3\bar{u}_2^3\bar{v}_1\bar{v}_2\right]\\
&-\frac{T_1^uT_1^v}{4u_1^2\bar{u}_1^2u_2^2\bar{u}_2^2u_3v_1^2\bar{v}_1^2v_2^2\bar{v}_2^2v_3}\left[\left(\left(\left(\left(6v_1^2-9v_1+4\right)u_2^3+2\,(-v_1\,(3v_1-7)\,\bar{v}_1-2)\,u_2^2\right.\right.\right.\right.\\
&+(v_1\,((14-9v_1)\,v_1-10)+2)\,u_2+2v_1\,(1-2v_1\bar{v}_1))\,v_2^4\\
&+\left(2\,(3v_1\,(v_1\,(2v_1-5)+4)-4)\,u_2^3+(8-2v_1\,(3\,(v_1-5)\,v_1+17)\,\bar{v}_1)\,u_2^2\right.\\
&+(v_1\,(v_1\,((32-9v_1)\,v_1-31)+18)-4)\,u_2+2v_1\,(v_1\,(2\,(v_1-3)\,v_1+3)-1))\,v_2^3\\
&+\left((3v_1\,(2\,(v_1-3)\,(v_1-2)\,v_1-5)+4)\,u_2^3-4\,(1-v_1\,(v_1\,(5v_1-11)+5)\,\bar{v}_1)\,u_2^2\right.\\
&\left.+(2v_1-1)\,(v_1\,(v_1\,(7v_1-12)+4)-2)\,u_2+2\,(3-2v_1)\,v_1^3\right)v_2^2\\
&+v_1^2\,(u_2\,(10v_1+u_2\,(-14v_1+3u_2\,(3v_1-5)+20)-8)-2v_1)\,\bar{v}_1v_2\\
&\left.+2u_2v_1^2\,(1-2u_2\bar{u}_2)\,\bar{v}_1^2\right)u_1^4+\left(\left(2\,(3u_2\,(u_2\,(2u_2-5)+4)-4)\,v_1^3\right.\right.\\
&+2\,(4-u_2\,(3\,(u_2-5)\,u_2+17)\,\bar{u}_2)\,v_1^2+(u_2\,(u_2\,((32-9u_2)\,u_2-31)+18)-4)\,v_1\\
&+2u_2\,(u_2\,(2\,(u_2-3)\,u_2+3)-1))\,v_2^4+2\left((3v_1\,(v_1\,(2v_1-5)+4)-4)\,u_2^4\right.\\
&+2\,(v_1\,(v_1\,(v_1\,(3v_1-22)+36)-21)+6)\,u_2^3+(v_1\,(v_1\,(3\,(24-5v_1)\,v_1-97)+37)-6)\,u_2^2\\
&+(v_1\,(v_1\,(6v_1\,(2v_1-7)+37)-15)+2)\,u_2+2v_1\,(v_1\,(-2\,(v_1-3)\,v_1-3)+1))\,v_2^3\\
&+\left((3v_1\,(2\,(v_1-3)\,(v_1-2)\,v_1-5)+4)\,u_2^4-4\,(v_1\,(v_1\,(9\,(v_1-4)\,v_1+38)-13)+3)\,u_2^3\right.\\
&+(v_1\,(2v_1\,(v_1\,(32v_1-97)+88)-43)+6)\,u_2^2-2\,(v_1\,(v_1\,(v_1\,(17v_1-37)+22)-6)+1)\,u_2\\
&\left.+4v_1^3\,(2v_1-3)\right)v_2^2+v_1^2\,(4v_1+u_2\,(-18v_1+u_2\,(31v_1+u_2\,(-32v_1+3u_2\,(3v_1-5)+52)\\
&\left.-43)+12))\,\bar{v}_1v_2+2u_2\,(u_2\,(2\,(u_2-3)\,u_2+3)-1)\,v_1^2\bar{v}_1^2\right)u_1^3+\left(\left(2\,(3-2u_2)\,u_2^3\right.\right.\\
&+(3u_2\,(2\,(u_2-3)\,(u_2-2)\,u_2-5)+4)\,v_1^3+(2u_2-1)\,(u_2\,(u_2\,(7u_2-12)+4)-2)\,v_1\\
&\left.-4v_1^2\,(1-u_2\,(u_2\,(5u_2-11)+5)\,\bar{u}_2)\right)v_2^4+\left((3u_2\,(2\,(u_2-3)\,(u_2-2)\,u_2-5)+4)\,v_1^4\right.\\
&-4\,(u_2\,(u_2\,(9\,(u_2-4)\,u_2+38)-13)+3)\,v_1^3+(u_2\,(2u_2\,(u_2\,(32u_2-97)+88)-43)+6)\,v_1^2\\
&\left.-2\,(u_2\,(u_2\,(u_2\,(17u_2-37)+22)-6)+1)\,v_1+4u_2^3\,(2u_2-3)\right)v_2^3\\
&+\left(-4\,(1-v_1\,(v_1\,(5v_1-11)+5)\,\bar{v}_1)\,u_2^4+(v_1\,(2v_1\,(v_1\,(32v_1-97)+88)-43)+6)\,u_2^3\right.\\
&-8v_1\,(2v_1\,(v_1\,(4v_1-11)+9)-3)\,u_2^2+v_1\,(5v_1-2)\,(v_1\,(4v_1-7)+2)\,u_2\\
&\left.+2\,(3-2v_1)\,v_1^3\right)v_2^2-v_1^2\,(2v_1+u_2\,(-8v_1+u_2\,(4\,(5v_1-6)\\
&\left.+u_2\,(-31v_1+2u_2\,(7v_1-10)+43))+4))\,\bar{v}_1v_2-2u_2^3\,(2u_2-3)\,v_1^2\bar{v}_1^2\right)u_1^2\\
&+u_2^2\left((v_1\,(5\,(4-3v_1)\,v_1-8)+u_2\,(v_1\,(v_1\,(9v_1-14)+10)-2))\,v_2^4\right.\\
&+(v_1\,(v_1\,((52-15v_1)\,v_1-43)+12)+u_2\,(v_1\,(v_1\,(v_1\,(9v_1-32)+31)-18)+4))\,v_2^3\\
&+(v_1\,(5v_1-2)\,(v_1\,(4v_1-7)+2)+u_2\,(v_1\,(v_1\,((31-14v_1)\,v_1-20)+8)-2))\,v_2^2\\
&\left.-2v_1^2\,(-4v_1+u_2\,(5v_1-4)+2)\,\bar{v}_1v_2-2u_2v_1^2\bar{v}_1^2\right)\bar{u}_2u_1\\
&\left.-2u_2^2v_1v_2v_3\bar{u}_2^2\left((1-2v_2\bar{v}_2)\,v_1^2-2v_2^2v_1+v_2^2\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{T_1^u T_3^v}{4u_1^2\bar{u}_1^2u_2^2\bar{u}_2^2u_3v_1\bar{v}_1^2v_2\bar{v}_2^2v_3}\left[\left(\left(3\left(6u_2^2-9u_2+4\right)v_1^2+2\left(17-14u_2\right)u_2v_1\right.\right.\right.\\
&-14v_1+3u_2\left(4u_2-5\right)+6\right)v_2^3+\left(2\left(v_1\left(v_1\left(9v_1-32\right)+31\right)-10\right)u_2^2\right.\\
&+\left(v_1\left(9\left(10-3v_1\right)v_1-68\right)+21\right)u_2+4\left(-v_1\left(3v_1-7\right)\bar{v}_1-2\right)\right)v_2^2\\
&+2\left(\left(v_1\left(\left(31-14v_1\right)v_1-20\right)+4\right)u_2^2+\left(v_1\left(17\left(v_1-2\right)v_1+16\right)-3\right)u_2\right.\\
&+v_1\left(-7\left(v_1-2\right)v_1-6\right)+1\right)v_2-v_1\left(6v_1+u_2\left(-15v_1+4u_2\left(3v_1-2\right)+6\right)-2\right)\bar{v}_1\right)u_1^4\\
&+\left(\left(6\left(3u_2\left(u_2\left(2u_2-5\right)+4\right)-4\right)v_2^2+4\left(u_2\left(4\left(8-3u_2\right)u_2-23\right)+7\right)v_2\right.\right.\\
&+3\left(u_2\left(u_2\left(6u_2-17\right)+13\right)-4\right)\right)v_1^3+\left(6\left(3u_2\left(u_2\left(2u_2-5\right)+4\right)-4\right)v_2^3\right.\\
&+2\left(u_2\left(\left(163-66u_2\right)u_2-123\right)+40\right)v_2^2+u_2\left(u_2\left(114u_2-295\right)+193\right)v_2-56v_2\\
&+u_2\left(5\left(17-6u_2\right)u_2-57\right)+16\right)v_1^2+\left(4\left(u_2\left(4\left(8-3u_2\right)u_2-23\right)+7\right)v_2^3\right.\\
&+\left(u_2\left(u_2\left(114u_2-295\right)+193\right)-56\right)v_2^2-4\left(u_2-2\right)\left(u_2\left(18u_2-11\right)+3\right)v_2\\
&+2\left(u_2\left(u_2\left(6u_2-17\right)+9\right)-2\right)\right)v_1-v_2\left(-12v_2+u_2\left(39v_2\right.\right.\\
&\left.\left.+u_2\left(6u_2-17\right)\left(3v_2-2\right)-18\right)+4\right)\bar{v}_2\right)u_1^3\\
&+\left(\left(3\left(3u_2\left(2\left(u_2-3\right)\left(u_2-2\right)u_2-5\right)+4\right)v_2^2+2u_2\left(29-2u_2\left(u_2\left(7u_2-32\right)+38\right)\right)v_2\right.\right.\\
&-14v_2+3u_2\left(u_2\left(u_2\left(4u_2-17\right)+20\right)-8\right)+6\right)v_1^3\\
&+\left(3\left(3u_2\left(2\left(u_2-3\right)\left(u_2-2\right)u_2-5\right)+4\right)v_2^3-2\left(u_2\left(u_2\left(u_2\left(32u_2-163\right)+196\right)\right.\right.\right.\\
&\left.\left.-78\right)+20\right)v_2^2+u_2\left(u_2\left(u_2\left(62u_2-295\right)+352\right)-125\right)v_2+28v_2\\
&+u_2\left(36-5u_2\left(u_2\left(4u_2-17\right)+20\right)\right)-8\right)v_1^2+\left(-2\left(u_2\left(2u_2\left(u_2\left(7u_2-32\right)+38\right)\right.\right.\right.\\
&\left.\left.-29\right)+7\right)v_2^3+\left(u_2\left(u_2\left(u_2\left(62u_2-295\right)+352\right)-125\right)+28\right)v_2^2\\
&-4\left(u_2\left(u_2\left(u_2\left(10u_2-47\right)+56\right)-17\right)+3\right)v_2+2u_2\left(u_2\left(u_2\left(4u_2-17\right)+20\right)-6\right)+2\right)v_1\\
&-v_2\left(6v_2+u_2\left(-24v_2+u_2\left(u_2\left(4u_2-17\right)+20\right)\left(3v_2-2\right)+12\right)-2\right)\bar{v}_2\right)u_1^2\\
&-u_2^2\left(\left(v_1\left(45v_1-58\right)+u_2\left(\left(34-27v_1\right)v_1-15\right)+24\right)v_2^3+\left(v_1\left(3v_1\left(15v_1-52\right)+125\right)\right.\right.\\
&+u_2\left(v_1\left(9\left(10-3v_1\right)v_1-68\right)+21\right)-36\right)v_2^2+\left(v_1\left(\left(125-58v_1\right)v_1-68\right)\right.\\
&+u_2\left(2v_1\left(17\left(v_1-2\right)v_1+16\right)-6\right)+12\right)v_2+3v_1\left(-8v_1+u_2\left(5v_1-2\right)+4\right)\bar{v}_1\right)\bar{u}_2u_1\\
&-2u_2^2\left(\left(v_2\left(6v_2-7\right)+3\right)v_1^2+\left(\left(4-7v_2\right)v_2-1\right)v_1+v_2\left(3v_2-1\right)\right)v_3\bar{u}_2^2\right]\\
&+\frac{T_1^u T_7^v}{4u_1^2\bar{u}_1^2u_2^2\bar{u}_2^2u_3v_1\bar{v}_1^2v_2\bar{v}_2^2v_3}\left[\left(\left(3\left(6u_2^2-9u_2+4\right)v_1^2+2\left(17-14u_2\right)u_2v_1-14v_1\right.\right.\right.\\
&+3u_2\left(4u_2-5\right)+6\right)v_2^3+\left(2\left(v_1\left(v_1\left(9v_1-32\right)+31\right)-10\right)u_2^2\right.\\
&+\left(v_1\left(9\left(10-3v_1\right)v_1-68\right)+21\right)u_2+4\left(-v_1\left(3v_1-7\right)\bar{v}_1-2\right)\right)v_2^2\\
&+2\left(\left(v_1\left(\left(31-14v_1\right)v_1-20\right)+4\right)u_2^2+\left(v_1\left(17\left(v_1-2\right)v_1+16\right)-3\right)u_2\right.\\
&+v_1\left(-7\left(v_1-2\right)v_1-6\right)+1\right)v_2-v_1\left(6v_1+u_2\left(-15v_1+4u_2\left(3v_1-2\right)+6\right)-2\right)\bar{v}_1\right)u_1^4\\
&+\left(\left(6\left(3u_2\left(u_2\left(2u_2-5\right)+4\right)-4\right)v_2^2+4\left(u_2\left(4\left(8-3u_2\right)u_2-23\right)+7\right)v_2\right.\right.\\
&+3\left(u_2\left(u_2\left(6u_2-17\right)+13\right)-4\right)\right)v_1^3+\left(6\left(3u_2\left(u_2\left(2u_2-5\right)+4\right)-4\right)v_2^3\right.\\
&+2\left(u_2\left(\left(163-66u_2\right)u_2-123\right)+40\right)v_2^2+u_2\left(u_2\left(114u_2-295\right)+193\right)v_2\\
&-56v_2+u_2\left(5\left(17-6u_2\right)u_2-57\right)+16\right)v_1^2+\left(4\left(u_2\left(4\left(8-3u_2\right)u_2-23\right)+7\right)v_2^3\right.
\end{aligned}
$$

$$
\begin{aligned}
&+ \left(u_2\left(u_2\left(114u_2-295\right)+193\right)-56\right)v_2^2-4\left(u_2-2\right)\left(u_2\left(18u_2-11\right)+3\right)v_2\\
&+2\left(u_2\left(u_2\left(6u_2-17\right)+9\right)-2\right)\right)v_1-v_2\left(-12v_2+u_2\left(39v_2+u_2\left(6u_2-17\right)\left(3v_2-2\right)\right.\right.\\
&\left.\left.-18\right)+4\right)\bar{v}_2\right)u_1^3+\left(\left(3\left(3u_2\left(2\left(u_2-3\right)\left(u_2-2\right)u_2-5\right)+4\right)v_2^2\right.\right.\\
&+2u_2\left(29-2u_2\left(u_2\left(7u_2-32\right)+38\right)\right)v_2-14v_2\\
&+3u_2\left(u_2\left(u_2\left(4u_2-17\right)+20\right)-8\right)+6\right)v_1^3+\left(3\left(3u_2\left(2\left(u_2-3\right)\left(u_2-2\right)u_2-5\right)+4\right)v_2^3\right.\\
&-2\left(u_2\left(u_2\left(u_2\left(32u_2-163\right)+196\right)-78\right)+20\right)v_2^2\\
&+u_2\left(u_2\left(u_2\left(62u_2-295\right)+352\right)-125\right)v_2+28v_2\\
&+u_2\left(36-5u_2\left(u_2\left(4u_2-17\right)+20\right)\right)-8\right)v_1^2+\left(-2\left(u_2\left(2u_2\left(u_2\left(7u_2-32\right)+38\right)-29\right)\right.\right.\\
&\left.+7\right)v_2^3+\left(u_2\left(u_2\left(u_2\left(62u_2-295\right)+352\right)-125\right)+28\right)v_2^2-4\left(u_2\left(u_2\left(u_2\left(10u_2-47\right)\right.\right.\right.\\
&\left.\left.\left.+56\right)-17\right)+3\right)v_2+2u_2\left(u_2\left(u_2\left(4u_2-17\right)+20\right)-6\right)+2\right)v_1\\
&-v_2\left(6v_2+u_2\left(-24v_2+u_2\left(u_2\left(4u_2-17\right)+20\right)\left(3v_2-2\right)+12\right)-2\right)\bar{v}_2\right)u_1^2\\
&-u_2^2\left(\left(v_1\left(45v_1-58\right)+u_2\left(\left(34-27v_1\right)v_1-15\right)+24\right)v_2^3+\left(v_1\left(3v_1\left(15v_1-52\right)+125\right)\right.\right.\\
&+u_2\left(v_1\left(9\left(10-3v_1\right)v_1-68\right)+21\right)-36\right)v_2^2+\left(v_1\left(\left(125-58v_1\right)v_1-68\right)\right.\\
&+u_2\left(2v_1\left(17\left(v_1-2\right)v_1+16\right)-6\right)+12\right)v_2+3v_1\left(-8v_1+u_2\left(5v_1-2\right)+4\right)\bar{v}_1\right)\bar{u}_2u_1\\
&-2u_2^2\left(\left(v_2\left(6v_2-7\right)+3\right)v_1^2+\left(\left(4-7v_2\right)v_2-1\right)v_1+v_2\left(3v_2-1\right)\right)v_3\bar{u}_2^2\Big]\\
&+(u_i\leftrightarrow v_i)\Big\}+\mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{A.11}
$$

The (anti-)symmetric part under the exchange of indices $1 \leftrightarrow 2$ has been integrated to 0 according to the symmetry (antisymmetry) of the distribution amplitudes $F_i(x_1, x_2, x_3)$ in $x_1 \leftrightarrow x_2$. The result for the d form factor reads:

$$
\begin{aligned}
&\langle p(P')|\bar{d}\gamma^\mu d|p(P)\rangle|_{\rm NLP} =\\
&= -\alpha_s^2 16\pi^2 \frac{(N_c+1)^2}{4N_c^2 N_c!}\frac{M}{Q^6}\bar{N}^+(P')N^+(P)\left(P'^\mu+P^\mu\right)\int\mathcal{D}u\int\mathcal{D}v\\
&\quad\Big\{-\frac{V_1^uV_1^v}{8u_1^2u_2^2v_1^2v_2^2\bar{u}_1\bar{u}_2\bar{u}_3\bar{v}_1\bar{v}_2\bar{v}_3}\Big[u_1^3\left(v_2^3\left(u_2\left(2v_1-1\right)-v_1\right)\left(u_2-\bar{v}_1\right)\right.\\
&\quad\left.-v_2^2\bar{v}_1\left(u_2\left(2v_1-1\right)-v_1\right)\left(u_2-\bar{v}_1\right)+v_1^2v_2\left(u_2\left(2\left(u_2-2\right)v_1-3u_2+5\right)-\bar{v}_1\right)-u_2v_1^2\bar{u}_2\bar{v}_1\right)\\
&\quad-u_1^2\bar{u}_2\left(v_2^3\left(u_2\left(2v_1-1\right)-v_1\right)\left(u_2-\bar{v}_1\right)-v_2^2\bar{v}_1\left(u_2\left(2v_1-1\right)-v_1\right)\left(u_2-\bar{v}_1\right)\right.\\
&\quad\left.+v_1^2v_2\left(u_2\left(2\left(u_2-2\right)v_1-3u_2+5\right)-\bar{v}_1\right)-u_2v_1^2\bar{u}_2\bar{v}_1\right)\\
&\quad+u_2^2u_1\left(v_1^3\left(-\bar{u}_2+\left(2u_2-3\right)v_2^2-4u_2v_2+5v_2\right)-v_1^2\bar{v}_2\left(-\bar{u}_2+\left(2u_2-3\right)v_2^2-4u_2v_2+5v_2\right)\right.\\
&\quad\left.+v_2^2\bar{u}_2\bar{v}_2+v_2^2v_1\left(-4u_2v_2+5u_2+5v_2-6\right)\right)+u_2^2v_1v_2\bar{u}_2\bar{v}_1\bar{v}_2\bar{v}_3\Big]\\
&\quad+\frac{V_1^uV_2^v}{4u_1^2u_2^2v_1v_2\bar{u}_1\bar{u}_2\bar{u}_3\bar{v}_3}\Big[u_1^3\left(\bar{v}_3+u_2\left(v_1+v_2+1\right)\right)+u_1^2\bar{u}_2\left(u_2\left(\bar{v}_3-3\right)-v_1-v_2\right)\\
&\quad-u_2^2\bar{u}_2\bar{v}_3+u_2u_1\left(u_2\left(u_2v_1+\left(u_2+2\right)v_2+u_2+2v_1-3\right)-5v_1-5v_2+1\right)\Big]\\
&\quad+\frac{V_1^uV_3^v}{4u_1^2u_2^2v_1v_2\bar{u}_1\bar{u}_2\bar{u}_3}\Big[-\left(7u_2-4\right)u_1^2\bar{u}_2+4u_2^2\bar{u}_2+\left(5u_2-4\right)u_1^3+u_2\left(u_2\left(5u_2-11\right)+5\right)u_1\Big]\\
&\quad-\frac{A_1^vV_1^u}{4u_1^2u_2^2v_1^2v_2^2\bar{u}_3\bar{v}_1\bar{v}_2\bar{v}_3^2}\left(v_1-v_2\right)\Big[u_1^2\left(u_2\left(v_1v_2\left(1-2v_1\bar{v}_1\right)+v_1^2\bar{v}_1+\left(2v_1-1\right)v_2^3\right.\right.
\end{aligned}
$$

$$
\begin{aligned}
&+ \left(v_1 \left(3v_1 - 2\right) + 1\right) v_2^2\Big) + v_1 v_2 \bar{v}_1 \bar{v}_2 \bar{v}_3\Big) + u_2 u_1 \Big(u_2 \Big(v_1 v_2 \left(1 - 2v_1 \bar{v}_1\right) + v_1^2 \bar{v}_1 + \left(2v_1 - 1\right) v_2^3 \\
&+ \left(v_1 \left(3v_1 - 2\right) + 1\right) v_2^2\Big) - v_1 v_2 \bar{v}_1 \bar{v}_2 \bar{v}_3\Big) + u_2^2 v_1 v_2 \bar{v}_1 \bar{v}_2 \bar{v}_3\Big] \\
&+ \frac{A_2^v V_1^u}{4u_1^2 u_2^2 v_1 v_2 \bar{u}_3 \bar{v}_3} \left(u_1^2 - u_2 u_1 + u_2^2\right) \left(v_1 - v_2\right) \\
&- \frac{A_3^v V_1^u}{2u_1^2 u_2^2 v_1 v_2 \bar{u}_3 \bar{v}_3} \left(u_1^2 - u_2 u_1 + u_2^2\right) \left(v_1 - v_2\right) \\
&+ \frac{A_1^v V_2^u}{4u_1 u_2 v_1^2 v_2^2 \bar{u}_3 \bar{v}_1 \bar{v}_2 \bar{v}_3^2} \left(v_1 - v_2\right) \Big[v_2^3 \left(\bar{u}_3 + \left(u_1 + u_2 + 1\right) v_1\right) \\
&+ v_2^2 \Big(v_1^2 \left(\bar{u}_3 + 2\right) + 4v_1 \bar{u}_3 - u_1 - u_2\Big) + v_1 v_2 \Big(v_1^2 \left(\bar{u}_3 + 1\right) + 4v_1 \bar{u}_3 - 3\bar{u}_3\Big) - v_1^2 \bar{u}_3 \bar{v}_1\Big] \\
&- \frac{A_1^v V_3^u}{4u_1 u_2 v_1^2 v_2^2 \bar{v}_1 \bar{v}_2 \bar{v}_3^2} \left(v_1 - v_2\right) \Big[-4v_2^2 \bar{v}_2 + \left(4 - 5v_2\right) v_1^3 - 2\left(v_2 \left(4v_2 - 5\right) + 2\right) v_1^2 \\
&+ v_2 \left(-5\left(v_2 - 2\right) v_2 - 6\right) v_1\Big] \\
&- \frac{T_1^u T_1^v}{u_1^2 u_2^2 v_1^2 v_2^2 \bar{u}_1 \bar{u}_2 \bar{v}_1 \bar{v}_2} \Big[u_1^2 \Big(2u_2^2 \bar{v}_1 \bar{v}_2 \bar{v}_3 + u_2 \Big(-v_1 \bar{v}_1 + \left(2\left(v_1 - 2\right) v_1 + 1\right) v_2^2 \\
&+ \Big(-4v_1^2 + 6v_1 - 1\Big) v_2\Big) + v_1 v_2 \left(\bar{v}_3 - 2v_1 v_2\right)\Big) + u_1 \Big(u_2^2 \Big(-v_1 \bar{v}_1 + \left(2\left(v_1 - 2\right) v_1 + 1\right) v_2^2 \\
&+ \Big(-4v_1^2 + 6v_1 - 1\Big) v_2\Big) + 2u_2 v_1 v_2 \left(-2v_2 v_1 + 3v_1 + 3v_2 - 4\right) + v_1 v_2 \left(2v_2 v_1 - v_1 - v_2\right)\Big) \\
&+ u_2 v_1 v_2 \left(v_1 \left(2v_2 - 1\right) - v_2\right) \bar{u}_2\Big] \\
&- \frac{T_1^u T_3^v}{u_1^2 u_2^2 v_1 v_2 \bar{u}_1 \bar{u}_2 \bar{v}_1 \bar{v}_2} \left[u_1 \left(-3\left(u_2 - 2\right) u_2 \bar{v}_1 - 3v_2 \left(v_1 - \left(u_2 - 2\right) u_2 \bar{v}_1\right) + 2v_1 + 2v_2 - 1\right)\right. \\
&\left. - u_2 \left(-2v_2 + v_1 \left(3v_2 - 2\right) + 1\right) \bar{u}_2 + u_1^2 \left(3u_2 \left(-v_1 v_2 - v_3\right) + 3v_2 v_1 - 2v_1 - 2v_2 + 1\right)\right] \\
&- \frac{T_1^u T_7^v}{u_1^2 u_2^2 v_1 v_2 \bar{u}_1 \bar{u}_2 \bar{v}_1 \bar{v}_2} \left[u_1 \left(-3\left(u_2 - 2\right) u_2 \bar{v}_1 - 3v_2 \left(v_1 - \left(u_2 - 2\right) u_2 \bar{v}_1\right) + 2v_1 + 2v_2 - 1\right)\right. \\
&\left. - u_2 \left(-2v_2 + v_1 \left(3v_2 - 2\right) + 1\right) \bar{u}_2 + u_1^2 \left(3u_2 \left(-v_1 v_2 - v_3\right) + 3v_2 v_1 - 2v_1 - 2v_2 + 1\right)\right] \\
&- \frac{S_1^v T_1^u}{u_1^2 u_2^2 v_1 v_2 \bar{u}_3 \bar{v}_3} \left(u_1^2 + u_2^2\right) \left(v_1 - v_2\right) \\
&+ \frac{P_1^v T_1^u}{u_1^2 u_2^2 v_1 v_2 \bar{u}_3 \bar{v}_3} \left(u_1^2 + u_2^2\right) \left(v_1 - v_2\right) \\
&- \frac{A_1^u A_1^v}{4u_1^2 u_2^2 v_1^2 v_2^2 \bar{u}_3 \bar{v}_3} \left(u_1 - u_2\right) \left(v_1 - v_2\right) \left[u_1 u_2 v_1 + u_2 v_2 v_1 + u_1 v_2 \left(u_2 + v_1\right)\right] \\
&+ \frac{A_1^u A_2^v}{4u_1^2 u_2^2 v_1 v_2 \bar{u}_3^2 \bar{v}_3^2} \left(u_1 - u_2\right) \left(v_1 - v_2\right) \left[u_1^2 \bar{v}_3 + u_2 u_1 \left(\bar{v}_3 + 1\right) + u_2^2 \bar{v}_3\right] \\
&- \frac{A_1^u A_3^v}{4u_1^2 u_2^2 v_1 v_2 \bar{u}_3^2 \bar{v}_3} \left(u_1 - u_2\right) \left(2u_1 + u_2\right) \left(u_1 + 2u_2\right) \left(v_1 - v_2\right) \\
&+ \left(u_i \leftrightarrow v_i\right)\Big\} + \mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{A.12}
$$

A.4 Integration boundaries in sector decomposition

In the implementation of sector decomposition with finite integration boundaries it is necessary to change the order of integrations. The following rules apply for $r > 0$:

$$\int_a^1 \mathrm{d}x \int_{b/x^r}^1 \mathrm{d}t\, f(x,t) = \begin{cases} \frac{b}{a^r}\int_{a^r}^1 \mathrm{d}t \int_{a/t^{1/r}}^1 \mathrm{d}x\, f\left(x, \frac{b}{a^r}t\right) + \int_{b/a^r}^1 \mathrm{d}t \int_a^1 \mathrm{d}x\, f(x,t)\,, & \text{if} \quad b < a^r \\ \int_b^1 \mathrm{d}t \int_{(b/t)^{1/r}}^1 \mathrm{d}x\, f(x,t), & \text{if} \quad b \geq a^r \end{cases}$$

$$\int_a^1 \mathrm{d}x \int_{bx^r}^1 \mathrm{d}t\, f(x,t) = \begin{cases} b\int_{a^r}^1 \mathrm{d}t \int_{a/t^{1/r}}^1 \mathrm{d}x\, t^{1/r} f\left(t^{1/r}x, bt\right) + \int_b^1 \mathrm{d}t \int_a^1 \mathrm{d}x\, f(x,t)\,, & \text{if} \quad b < 1 \\ \int_b^1 \mathrm{d}t \int_{a(b/t)^{1/r}}^1 \mathrm{d}x\, \left(\frac{t}{b}\right)^{1/r} f\left(\left(\frac{t}{b}\right)^{1/r} x, t\right), & \text{if} \quad b \geq 1 \end{cases} \tag{A.13}$$

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