Moduli Stabilization in type IIB Orientifolds

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Contents

Zusammenfassung

Diese Arbeit beschäftigt sich mit der Stabilisierung der Modulifelder bei den Kompaktifizierungen der Typ IIB String Theorie auf Orientifolds. Ein konkretes Verfahren für die Konstruktion von Lösungen, bei denen alle Modulifelder fixiert sind, bietet das KKLT-Szenario. Wir untersuchen, auf welche Modelle das Szenario sich anwenden lässt, wenn man auf Näherungen der originalen KKLT-Arbeit verzichtet. Wir finden, dass bei einer Reihe von Modellen, nämlich solchen ohne Komplexe-Struktur-Moduli, die Konstruktion der konsistenten Lösungen im Rahmen des KKLT-Szenarios nicht möglich ist. Die nichtperturbativen Effekte, wie $D3$-Instantonen und Gauginokondensate, sind ein weiterer Bestandteil des KKLT-Szenarios. Sie führen zur Stabilisierung der Kählermoduli. Wir geben Kriterien an für das Erzeugen des Superpotentials infolge der $D3$-Instantonen bei einer Calabi-Yau-Mannigfaltigkeit in Anwesenheit der Flüsse. Weiterhin zeigen wir, dass obwohl die Anwesenheit des nichtperturbativen Superpotentials in den Bewegungsgleichungen mit dem Einschalten aller ISD- und IASD-Flüsse korreliert, das Entscheidungkriterium für das Erzeugen des nichtperturbativen Superpotentials nur von den Flüssen vom Typ $(2,1)$ abhängt. Anschließend diskutieren wir zwei Modelle, bei denen wir alle Modulifelder stabilisieren. Dabei handelt es sich um Calabi-Yau-Orientifolds, die man durch eine Blow-Up-Prozedur aus den $\mathbf{Z}_{6-II}$ und $\mathbf{Z}_2 \times \mathbf{Z}_4$ Orientifolds erhalten hat.

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Chapter 1

Introduction

1.1 Why strings?

Relevant observations for high energy physics usually come from scattering experiments. As possible sources one can either use accelerators built on earth or natural ones like the sun for neutrinos or supernovae for highly energetic protons and electrons. In the seventies a model was constructed which until now is very successful in describing the observed scattering processes. The Standard Model of elementary particles consists of three families of quarks and leptons, which are particles of spin one half. Their interactions are mediated by spin one $SU(3) \times SU(2) \times U(1)$ gauge bosons. Additionally, the Standard Model has a spin zero Higgs boson needed for symmetry breaking. The dynamics is governed by a Lagrangian with 19 free parameters such as gauge and Yukawa couplings.

Every day, we experience the force of gravity. Since it is about 10^{25} smaller than the weakest interaction of the Standard Model, it has no relevance for scattering experiments, but it is essential for the motion of objects at macroscopic scales. The theory which describes gravitational interactions is General Relativity. During the last eighty years since its formulation the theory of General Relativity has been very well confirmed by experiments. Let us mention one of the first experiments and a recent one. In 1917, during a solar eclipse bending of the light rays around the sun was observed. In 2004, the Gravity Probe B Satelite was launched to test the predictions coming from General Relativity. Very precise gyroscopes should measure two effects, the geodetic effect and frame-dragging. The geodetic effect is the amount by which the mass of the Earth warps the local space-time in which it resides. The other effect, called frame-dragging, is the amount by which the rotating Earth drags local space-time around with it. The final results are expected by the end of 2007, but from the already analyzed data it is clear that the geodetic effect is confirmed to a precision of better than 1 percent [1].

Until now we have mentioned very successful models and some of their predictions but we have remained silent about their restrictions and problems. The troubles which we encounter are twofold. On one hand there are restrictions of the theoretical models, like ill-defined regions of applicability and computational difficulties. On the other hand the observations which have been made recently probably do not destroy the established models but demand their modification or enlargement.

Let us first discuss the underlying principles of the theoretical models in more detail. The Standard Model is based on the framework of quantum gauge field theories. Because of the mathematical complexity most of the calculations could be done only in a perturbative regime of the coupling constant. Quarks at energies less than 1 GeV are strongly coupled, which is why their computational description is very limited. Another problematic aspect are the ultra-violet divergences which appear in calculations of scattering amplitudes. Renormalization is the standard way to handle the divergences by introducing some cut-off Λ, behind which the divergences are hidden. The observable physical quantities are required not to depend on Λ, so that the limit $\Lambda \to \infty$ is well defined. If one takes the cut-off to be of the order of a characteristic scale of the theory like the mass of the W and Z bosons, the full theory can be replaced by an effective theory, which consists of the renormalizable theory below the cut-off plus non-renormalizable interaction terms. This suggests that quantum field theories could be regarded as effective theories of some more fundamental theory. Another argument for the Standard Model not to be fundamental, is its arbitrariness. There are too many parameters and it is not clear why we have three generations of quarks and leptons, why the interactions are so weak compared to the Planck scale and so on.

General Relativity on the other hand is a classical theory and it is well defined in the case of weak gravitational fields. In the case of strong fields, for example near black holes, it collapses and produces singularities in space-time. Again, this could be interpreted such that General Relativity is only a low energy limit of a more fundamental theory. We can come to the same conclusion if we observe that classical theories like mechanics or electrodynamics at short distances have been replaced by their quantized versions, which later were recognized to be more fundamental. In the same way it could be expected that the description of strong fields also need a quantum version of General Relativity. However, its quantization seems to be impossible since it is not a renormalizable theory.

So far we have discussed the theoretical deficits of the underlying models. On top, there are observational ones, of which we only mention two. During the last years different experiments confirmed the phenomenon of neutrino oscillations, which implies the neutrinos to be massive [2]. In the Standard Model, the fermions obtain masses from the interaction with the Higgs-boson. This interaction needs fermions of both chiralities, however observations give evidence only for the existence of left-handed neutrinos. The possible mechanisms which would explain this, like for example the seesaw mechanism, are not part of the Standard Model.

Another very interesting recent observation comes from the Wilkinson Microwave Anisotropy Probe (WMAP) satellite. They are consistent with a universe made up of 74% dark energy, 22% dark matter, and 4% ordinary matter. Since only part of the dark matter could be built up by baryons and leptons, most constituents of dark matter are not known [3].

A possible candidate for a theory which overcomes the mentioned deficits is String Theory. It makes the assumption that quantum states (particles) are given by the modes of a quantized one dimensional object, a string. The interactions occur no longer at points, but are smeared out over the worldsheet of the string, and the theory is finite. To be consistent, such a theory needs to live in ten dimensions and to be supersymmetric. We assume that ten dimensions factorize in our four-dimensional space-time and a compact six-dimensional manifold. The nice feature of String Theory is that it has only one free parameter, the string tension, and the gauge theories can be incorporated within a framework of open strings. It incorporates a spin-two state, the graviton, and has General Relativity as its classical limit. It seems that String Theory could unify the Standard Model and General Relativity.

But what about the new observations which do not fit into the established models? In spite of computational difficulties there are a lot of indications that all new observations could be incorporated into the framework of String Theory.

During the last few years it was recognized that String Theory has a large configuration space which means that we have many solutions to describe low energy physics [4]. Their number has been estimated to be of the order 10^{500} [5]. In this context one speaks about the “landscape” of string solutions [6]. One of the reasons for the number to be so large is the freedom of choice for the compact space.

By now, no selection rules for the choice of a unique solution is known. All of them seem to be on the same footing. However, in our study of different aspects of String Theory we should require its solution to be consistent with the physical observations. What are the constraints on the solutions? On one side we should demand that at low energies the solution has the content of the Standard Model. On the other side if we consider only the vacuum solution without any particle states, then it should not have any additional massless fields.

In this thesis we analyze the low energy description of String Theory and look for the consistent vacuum solutions in four space-time dimensions. Since it is not obvious that such solutions exist at all, one of the first steps in analyzing different mechanisms in String Theory should be an explicit construction of such solutions. A compact space of a given topology can be described by a certain number of shape and form parameters. In four-dimensional space-time, these parameters appear as additional scalar fields. If String Theory gives no restriction to the values of these parameters, then they could take any value in four dimensions without changing the potential energy of the vacuum state. The appearance of an additional massless field would correspond to a fifth force and contradict our observations. Within String Theory, there are some mechanisms which could give masses to these scalar fields and so far produce consistent solutions. Our goal is to analyze the methods to construct such solutions and to present concrete examples.

1.2 From strings to the low energy effective action

In the rest of this chapter we give an introduction to the main text. It is obvious that such an introduction can not be very detailed and exhaustive. Since the subject of string theory and already of string compactifications is huge, we will try to take a path in this field to the point where the results of this thesis apply.

The calculations that we perform in the main text are not based on the string theory action, but on its low energy pendant, namely supergravity action. In this section we try to give a sketchy justification for the low energy action and explain why this type of description is sufficient. We mainly follow the text book of Polchinski [7] and restrict ourselves to the case without fermions.

The action of a moving string or Polyakov action[1] is

$$S = \frac{1}{4\pi\alpha'}\int_M d^2\sigma g^{1/2} g^{ab} G_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu \ . \tag{1.2.1}$$

The prefactor $\frac{1}{4\pi\alpha'}$ is the tension of the string, which has the dimensions energy per unit length. α' is the Regge slope[2] of the string, $\sigma = (\sigma^0, \sigma^1)$ parameterizes the area swept by the string, $g_{ab}(\sigma)$ is the metric of the string world-sheet. The functions X^μ define a map from the string world-sheet into the physical space-time, and finally $G_{\mu\nu}$ is a metric of the target space. Here we do not make any assumption about the dimension D of the target space.

This action has a set of symmetries, which should remain after quantization procedure: D-dimensional Poincaré invariance, reparametrization invariance and invariance under local rescaling of the world-sheet metric or Weyl-invariance.

After quantizing the string we find that the spectrum contains a state with negative mass, a so called tachyon[3]; a massless state, which transforms as a 2-tensor under $SO(D-2)$, and infinitely many massive states. The mass squared of the lightest massive state is $1/\alpha'$. The reducible representation of the massless state decomposes into a symmetric traceless tensor, an antisymmetric tensor and a scalar.

A consistent quantization demands the number of dimensions of the target space to be $D = 26$. If we include fermions the number changes to $D = 10$.

The action (1.2.1) is known in field theory as a non-linear sigma model. We expand the path-integral around a classical solution: $X^\mu(\sigma) = x_0^\mu + Y^\mu(\sigma)$, where Y^μ are the quantum fluctuations at a chosen point x_0^μ. The integrand of the action is given by

$$G_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu = \left(G_{\mu\nu}(x_0) + G_{\mu\nu,\omega}(x_0)Y^\omega + \frac{1}{2}G_{\mu\nu,\omega\rho}(x_0)Y^\omega Y^\rho + \dots\right)\partial_a Y^\mu \partial_b Y^\nu \ . \tag{1.2.2}$$

[1] The action of a string which is proportional to the area swept by a string is the Nambu-Goto action. The Nambu-Goto and Polyakov actions are equivalent, but the latter one is more suitable for quantization.

[2] The Regge slope is defined to be the maximum possible angular momentum per unit energy squared.

[3] Tachyons are not present in the consistent string theory with fermions.

The coupling constants $G_{\mu\nu,\omega}(x_0)$ and so on in the expansion involve derivatives of the metric at the point x_0. In a target space with curvature radius R_c, the coupling constant $G_{\mu\nu,\omega}(x_0)$ is of order R_c^{-1}, and therefore the full dimensionless coupling constant is of order $\alpha'^{1/2}R_c^{-1}$. If the radius of curvature R_c is much bigger than the characteristic length scale of the string, then the coupling constant is small and we can use perturbation theory. Additionally, if the limit $\alpha'^{1/2}R_c^{-1} \ll 1$ is fulfilled, no massive string states are created and we can use the low energy effective field theory.

To derive a connection to supergravity, we have to consider the Weyl invariance in detail. Without further explanation we say that Weyl invariance on a curved world-sheet necessarily implies vanishing of the renormalization group beta functions. To calculate the beta functions one can use the background field method, by which one picks a vacuum expectation value - in our case it would be for the irreducible representations of the massless 2-tensor, $G_{\mu\nu}, B_{\mu\nu}, \phi$ - and makes an expansion around it. This needs to generalize eq. (1.2.1) by adding backgrounds of other massless string states, $B_{\mu\nu}$ and ϕ. This leads to

$$\beta^G_{\mu\nu} = \alpha' R_{\mu\nu} + 2\alpha'\nabla_\mu\nabla_\nu\phi - \frac{\alpha'}{4}H_{\mu\lambda\omega}H_\nu{}^{\lambda\omega} + \mathcal{O}(\alpha'^2) \ , \tag{1.2.3}$$

$$\beta^B_{\mu\nu} = -\frac{\alpha'}{2}\nabla^\omega H_{\omega\mu\nu} + \alpha'\nabla^\omega\phi H_{\omega\mu\nu} + \mathcal{O}(\alpha'^2) \ , \tag{1.2.4}$$

$$\beta^\phi = \frac{D-26}{6} - \frac{\alpha'}{2}\nabla^2\phi + \alpha'\nabla_\omega\nabla^\omega\phi - \frac{\alpha'}{24}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + \mathcal{O}(\alpha'^2) \ . \tag{1.2.5}$$

Setting the beta functions[4] to zero gives us a set of equations of motion, which we can also obtain from a space-time action of the form

$$S = \frac{1}{2\kappa_0^2}\int d^Dx(-G)^{1/2}e^{-2\phi}\left(-\frac{2(D-26)}{3\alpha'} + R - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + 4\partial_\mu\phi\partial^\mu\phi + \mathcal{O}(\alpha'^2)\right) \ . \tag{1.2.6}$$

The normalization constant κ_0 is not determined by the field equations and has no physical significance, since it can be absorbed by ϕ. In the redefined action with a canonical Einstein-Hilbert term, κ_0 should be the observed gravitational coupling constant.

The upshot of this section is that the string action, which describes a two-dimensional field theory, in some limit can be replaced by a field theory living in a D-dimensional space-time.

1.3 Effective type IIB string theory, compactifications, moduli problem

If we add fermions to the bosonic string theory, then there are five possible consistent theories after the quantization. In this thesis we only consider one of them, namely of type

[4]The authors of [8] call them three beta functionals, since there is continuously infinite number of couplings.

IIB. The Latin number II means that we have two generators of supersymmetry in ten dimensions, and B is in contrast to the type IIA theory. The difference is that in the type IIB there are two gravitinos (fermionic fields with additional vector index) with the same and in the type IIA with the opposite chirality .

Let us describe the massless bosonic spectrum of the type IIB theory. On the one hand we have massless bosonic fields in the so called NS-NS-sector, g_{MN}, B_{MN}, ϕ, and, additionally, in the R-R sector C, C_{MN}, C_{MNPQ} with the corresponding field strengths F_M, F_{MNP} and F_{MNPQR}. In the following, we will use the language of differential forms and the lower indices of fields will correspond to the degree of the form. The tree level effective action of type IIB string theory is

$$\begin{aligned} S_{IIB} &= \frac{1}{2{\kappa_0}^2}\int d^{10}x\sqrt{-g_s}\left(e^{-2\phi}\left(\mathcal{R}_s+4(\nabla\phi)^2\right)-\frac{F_{(1)}^2}{2}-\frac{1}{2\cdot 3!}G_{(3)}\cdot\bar{G}_{(3)}-\frac{\widetilde{F}_{(5)}^2}{4\cdot 5!}\right) \\ &\quad +\frac{1}{8i{\kappa_0}^2}\int e^{\phi}C_{(4)}\wedge G_{(3)}\wedge\bar{G}_{(3)}+\text{fermionic terms}\ , \end{aligned} \tag{1.3.7}$$

where g_s denotes the metric and R_s the scalar curvature of the target space. The subscript s indicates the use of the string frame. $G_{(3)}$ is a combined three form

$$G_{(3)} = F_{(3)} + iSH_{(3)}\ , \tag{1.3.8}$$

where S is the axion-dilaton field $S = iC_{(0)} + e^{-\phi}$. Additionally, the condition $\widetilde{F}_{(5)} = *\widetilde{F}_{(5)}$ must be imposed by hand. $\widetilde{F}_{(5)}$ is given by

$$\widetilde{F}_{(5)} = F_{(5)} - \frac{1}{2}C_{(2)}\wedge H_{(3)} + \frac{1}{2}B_{(2)}\wedge F_{(3)}\ , \tag{1.3.9}$$

with $H_{(3)} = dB_{(2)}$.

The Lagrangian (1.3.7) is formulated on a ten-dimensional manifold with Lorentzian signature. To make a connection to the observed physics, one assumes that the ten-dimensional space-time has a product structure

$$\mathcal{M}_{1,9} = \mathcal{M}_{1,3}\times\mathcal{M}_6\ , \tag{1.3.10}$$

where $\mathcal{M}_{1,3}$ corresponds to the observed four-dimensional space-time and $\mathcal{M}_6$ is a compact manifold.

We look for a vacuum solution which conserves supersymmetry. A supersymmetric ground state is a ground state $|\Omega>$ which is annihilated by the supersymmetry generator Q. This is equivalent to the statement $<\Omega|\{Q,U\}|\Omega>= 0$ for any field operator U, since Q is hermitian and acts on a ket-vector in the same way as on a bra-vector.

The anti-commutator $\{Q,U\}$ is the supersymmetric variation of U. For a bosonic U the vaccum expectation value (VEV) of δU is equal to the VEV of some fermion. In the Poincaré invariant vacuum the VEVs of the fields with specified directions are not allowed, such that VEVs of fermions are zero. Therefore δU is automatically zero. In the case of a fermionic U this is not so, and one has to demand

$$\delta\,(\text{fermionic field}) = 0\ . \tag{1.3.11}$$

There are two types of fermionic fields in the ten dimensional theory: a gravitino ψ_M and a dilatino λ. Let us consider a supersymmetric variation of the gravitino and see what the requirement of supersymmetry means for the topology of the compact part of the ten-dimensional space-time:

$$\delta\psi_M = \nabla_M\epsilon + \Gamma\cdot F\ , \tag{1.3.12}$$

where the last term represents different contractions between NS-NS, R-R fields and gamma matrices. Let us assume that in the vacuum state the VEVs of the RR-fields and H_3 both vanish.

In this case the condition for unbroken supersymmetry is

$$\nabla_i\epsilon = 0, i = 5,\ldots,10 \tag{1.3.13}$$

or

$$[\nabla_i,\nabla_j]\epsilon = R_{ijkl}\Gamma^{kl}\epsilon = 0. \tag{1.3.14}$$

After some manipulation using gamma matrix and Riemann curvature identity one obtains

$$\Gamma^{ik}R_{ik}\epsilon = 0\ . \tag{1.3.15}$$

It means that the condition of a supersymmetric vacuum state implies the Ricci-flatness of the compact manifold. The existence of a covariantly constant spinor ϵ is the condition that the manifold has $SU(3)$ holonomy. In other words if we parallel transport a spinor along any closed loop on the manifold, then the new spinor will be rotated by an $SU(3)$ group element. Manifolds of $SU(N)$ holonomy are called Calabi-Yau manifolds. It can be shown [9] that a manifold with $SU(N)$ holonomy is equivalent to a complex manifold with a closed Kähler form. The Kähler form is a 2-form J which is constructed from the CY-metric contracted with the complex structure tensor $J_m{}^n$

$$J = J_m{}^n g_{np} dx^m \wedge dx^p\ , \tag{1.3.16}$$

where the complex structure tensor $J_m{}^n$ must satisfy:

$$J_m{}^n J_n{}^k = -\delta_m{}^k\ , \tag{1.3.17}$$
$$J_{[i}{}^k{}_{;j]} - J_{[i}{}^p J_{j]}{}^q J_p{}^k{}_{;q} = 0\ . \tag{1.3.18}$$

Topologies of different Calabi-Yau manifolds are classified by their Hodge diamonds. A Hodge diamond consists of all hodge numbers of a given Calabi-Yau. Hodge numbers correspond to dimensions of the Dolbeault-cohomology groups on the Calabi-Yau, or in other words, they correspond to the number of homologically non-equivalent cycles of the

Calabi-Yau and the hodge numbers correspond to the number of parameters which fully describe the geometry of the CY-manifold.

In three complex dimensions every CY-manifold is described by two hodge numbers: $h_{(1,1)}$, $h_{(2,1)}$. All other numbers could be obtained either through some symmetry operations $h_{(p,q)} = h_{(q,p)} = h_{(3-p,3-q)}$ or they are completely fixed $h_{(1,0)} = h_{(2,0)} = 0$, $h_{(0,0)} = h_{(3,0)} = 1$.

The parameters of the CY-manifold are called moduli. In the compactified four-dimensional theories these geometrical moduli appear as scalar fields. From now on we call them moduli fields, denoted z^A in the four-dimensions:

$$S = \int d^4x \frac{1}{2} \left(\mathcal{R}_{(4)} + g_{A\overline{B}} \partial_\mu z^A \partial^\mu \overline{z}^{\overline{B}} \right) \ . \tag{1.3.19}$$

The indices A and B count the moduli fields and $g_{A\overline{B}}$ is the metric on the space of moduli. Since the scalar fields are massless, the vacuum solutions of such a theory have flat directions for every scalar field. For the observed physics this would be equivalent to the appearance of a fifth force. It means that a consistent four dimensional vacuum solution demands the absence of all massless fields. The scalar fields can obtain masses if we allow non-vanishing VEVs of the form fields, which we have neglected so far.

The assumption of non-vanishing VEV for the form fields on a compact space modifies the condition (1.3.13). The connection ∇ gets a torsion. We should mention that on a non-compact part of the target space the VEVs of the form fields are not allowed because of Lorenz symmetry[5]. Solutions of the new equation are, usually, not manifolds of $SU(3)$ holonomy, and in general even not complex. Topological classes of the allowed supersymmetric spaces could be classified with the use of torsion classes of the new connection in eq. (1.3.13) [10].

We will restrict ourselves to the simplest case, so called conformal Calabi-Yau manifolds. In this case we have a warped metric of the form

$$ds_{10}^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \widetilde{g}_{mn} dy^m dy^n \ , \tag{1.3.20}$$

where $\widetilde{g}_{mn}$ is the Calabi-Yau metric and $A(y)$ is a function depending on the coordinates of the compact space.

Integrated equations of motion yield no-go theorem for compactifications to Minkowski or de Sitter spaces [11, 12, 13]. Giddings, Kachru and Polchinski (GKP) showed that the no-go theorem can be evaded by inclusion of some localized sources satisfying a certain BPS bound involving their energy-momentum tensor. The localized sources, which one has considered, are Dp-branes and O-planes. The full tree level action in the Einstein frame has the following form

$$\begin{aligned} S_{\mathrm{IIB}} &= \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left(\mathcal{R} - \frac{\partial_M S \partial^M \bar{S}}{2(\mathrm{Re}S)^2} - \frac{G_{(3)} \cdot \overline{G}_{(3)}}{12 \mathrm{Re}S} - \frac{\widetilde{F}_{(5)}^2}{4 \cdot 5!} \right) \\ &\quad + \frac{1}{8i\kappa_{10}^2} \int \frac{C_{(4)} \wedge G_{(3)} \wedge \overline{G}_{(3)}}{\mathrm{Re}S} + S_{\mathrm{loc}} \ , \end{aligned} \tag{1.3.21}$$

[5] An exception from this statement is a five form if it has non-vanishing components in all directions of the non-compact space. In this case the Lorentz symmetry is not broken.

where $S_{\rm loc}$ is the action of the localized sources.

The equation of motion for the warp factor which one obtains with the metric ansatz (1.3.20) is

$$\widetilde{\nabla}^2 e^{4A} = e^{2A}\frac{G_{mnp}\overline{G}^{mnp}}{12\mathrm{Re}S} + e^{-6A}\left(\partial_m\alpha\partial^m\alpha + \partial_m e^{4A}\partial^m e^{4A}\right) + \frac{\kappa_{10}^2}{2}e^{2A}(T^m_m - T^\mu_\mu)^{\rm loc} \ , \quad (1.3.22)$$

where α is a potential coming from the $\widetilde{F}_5$. Lorentz symmetry and the Bianchi identity forces $\widetilde{F}_{(5)}$ to be of the form

$$\widetilde{F}_{(5)} = (1+*)\left(d\alpha\wedge dx^0\wedge dx^1\wedge dx^2\wedge dx^3\right) \ , \quad (1.3.23)$$

where α is a function on the compact space. The left part of (1.3.22) is a total derivative and vanishes after the integration over the compact manifold. In the absence of localized sources all other terms on the RHS are strictly positive and therefore have to vanish. This produces the no-go theorem mentioned above.

In the case of a p-brane the localized term in eq. (1.3.22) has the form

$$(T^m_m - T^\mu_\mu)^{\rm loc} = (7-p)T_p\delta(\Sigma) \ , \quad (1.3.24)$$

where T_p is the tension of the p-brane and $\delta(\Sigma)$ is a projector along the cycle which is wrapped by the localized object. In string theory, there are objects with negative tension, which can compensate strictly positive terms on the RHS of (1.3.22).

Additionally, the three form fields and localized sources should satisfy the Bianchi identity

$$d\widetilde{F}_{(5)} = H_{(3)}\wedge F_{(3)} + 2\kappa_{10}^2 T_3\rho_3^{\rm loc} \ , \quad (1.3.25)$$

where $\rho_3^{\rm loc}$ is a $D3$ charge density from the localized sources; this includes also contributions of the $D7$-branes and $O3$-planes. The intergrated form of the Bianchi identity is given by

$$\frac{1}{2\kappa_{10}^2 T_3}\int_{\mathcal{M}} H_{(3)}\wedge F_{(3)} + Q_3^{\rm loc} = 0 \quad (1.3.26)$$

and states that the total $D3$ charge from the supergravity background vanishes.

In terms of the potential α the Bianchi identity (1.3.25) becomes

$$\widetilde{\nabla}^2\alpha = ie^{2A}\frac{G_{mnp}(*_6\overline{G}^{mnp})}{12\mathrm{Re}S} + 2e^{-6A}\partial_m\alpha\partial^m e^{4A} + 2\kappa_{10}^2 e^{2A}T_3\rho_3^{\rm loc} \ . \quad (1.3.27)$$

Subtracting (1.3.27) from the equation of motion (1.3.22) we obtain

$$\begin{aligned}\widetilde{\nabla}^2(e^{4A}-\alpha) \ &= \ \frac{e^{2A}}{6\mathrm{Re}S}\left|iG_{(3)} - *_6G_{(3)}\right|^2 + e^{-6A}|\partial(e^{4A}-\alpha)|^2 \\ &\quad +2\kappa_{10}^2 e^{2A}\left(\frac{1}{4}(T^m_m - T^\mu_\mu)^{\rm loc} - T_3\rho_3^{\rm loc}\right) \ . \end{aligned} \quad (1.3.28)$$

This gives us the following constraints

$$
\begin{aligned}
*_6 G_{(3)} &= iG_{(3)} \ , \\
e^{4A} &= \alpha \ , \\
\frac{1}{4}(T^m_m - T^\mu_\mu)^{\rm loc} &= T_3 \rho_3^{\rm loc} \ .
\end{aligned} \tag{1.3.29}
$$

$D3$, $D7$-branes and $O3$-branes satisfy the last constraint automatically. The second equation gives a connection between the warping factor and the potential of the five-form. And finally, the first equation gives us the constraint on allowed types of three-form fluxes. The only types which satisfy this equation are fluxes of type $(2,1)$ and $(3,0)$. We call them imaginary self-dual (ISD) in contrast to $(1,2)$ and $(0,3)$ types which are imaginary antiself-dual (IASD).

Until now we have seen that it is possible to construct consistent supersymmetric solutions in the low energy limit of type IIB string theory. But what are the advantages of non-vanishing VEVs (turning on fluxes) of the form-fields?

One of the main reasons for the investigations of the flux solutions is that turning on fluxes produces mass terms for the moduli fields in the four-dimensional effective theory. The moduli fields which are fixed in this way are dilaton-axion and complex structure moduli. To see how the stabilization works we go to four dimensions.

The form of the four dimensional N=1 supergravity Lagrangian is completely fixed by three functions, the gauge kinetic function, Kähler potential and the superpotential. Only the last two are relevant for the form of the potential. The superpotential is a holomorphic function of the fields Φ^i, while the Kähler potential is a real function, usually written in terms of Φ^i and $\overline{\Phi}^{\bar{i}}$. The potential is given by

$$
V = e^K \left(K^{i\bar{j}} D_i W D_{\bar{j}} \overline{W} - 3W\overline{W} \right) \tag{1.3.30}
$$

with

$$
K^{i\bar{j}} = \left(\frac{\partial^2 K}{\partial \Phi^i \partial \Phi^{\bar{j}}} \right)^{-1} \ , \qquad D_i W = \frac{\partial W}{\partial \Phi^i} + \frac{\partial K}{\partial \Phi^i} W \ . \tag{1.3.31}
$$

The superpotential W and the Kähler potential K can be derived directly by dimensional reduction [13] or other methods [14, 15]

$$
K = -3 \ln(T + \overline{T}) - \ln(S + \overline{S}) + \ln \left(-i \int_{\mathcal{M}} \Omega \wedge \overline{\Omega} \right) \ , \tag{1.3.32}
$$

$$
W = \int_{\mathcal{M}} G_{(3)} \wedge \Omega \ , \tag{1.3.33}
$$

where T is a Kähler modulus, S axion-dilaton and Ω is nowhere vanishing holomorphic $(3,0)$-form on the Calabi-Yau. Here we have assumed only one Kähler modulus. In general, the Kähler potential would have a more complicated form. In the main text we discuss this in greater details.

The requirement for supersymmetric solutions in four dimensions is the vanishing of the Kähler covariant derivatives with respect to all scalar fields. The scalar fields, which we have in four dimensions, are $h_{(2,1)}(\mathcal{M}_6)$ complex structure moduli fields U^i, $h_{(1,1)}(\mathcal{M}_6)$ Kähler moduli fields T^i and an axion-dilaton S; all together $2(h_{(2,1)}+h_{(1,1)}+1)$ real scalar fields.

$$
\begin{aligned}
D_{T^i}W &= \partial_{T^i}W + \partial_{T^i}KW = \partial_{T^i}KW = 0 \quad \longrightarrow \quad \int_{\mathcal{M}} G_{(3)} \wedge \Omega = 0 \quad \longrightarrow \quad G_{(0,3)} = 0 \ , \\
D_{U^i}W &= \partial_{U^i}W + \partial_{U^i}KW = \int_{\mathcal{M}} G_{(3)} \wedge \chi^i = 0 \quad \longrightarrow \quad G_{(1,2)} = 0 \ , \\
D_S W &= \partial_S W + \partial_S KW = \frac{1}{T^i + \overline{T}^i} \int_{\mathcal{M}} \overline{G}_{(3)} \wedge \Omega = 0 \quad \longrightarrow \quad G_{(3,0)} = 0 \ , \qquad (1.3.34)
\end{aligned}
$$

where χ_i is a basis of $(2,1)$-forms on $\mathcal{M}_6$.

The constraints in ten dimensions (1.3.29) are satisfied by the fluxes of type $(2,1)$ and $(0,3)$. In four dimensions the requirement of supersymmetry makes further restrictions on the fluxes by setting the $(0,3)$ flux to zero.

The upshot of the last part of this section is that if we are interested in the four-dimensional supersymmetric solutions with stabilized complex structure moduli and axion-dilaton, then we have to

- choose H_3 and F_3-fluxes such that they satisfy the Bianchi identity (1.3.26),
- compute the Kähler potential K and the superpotential W,
- solve the supersymmetry conditions.

1.4 What is the KKLT scenario?

As we have seen in the previous section, the superpotential generated by fluxes depends on the complex structure moduli and axion-dilaton. Equations (1.3.34) imply that at the supersymmetric vaccum the values of these moduli fields will be fixed. Since the superpotential does not depend on Kähler moduli, the potential still has flat directions, namely the directions corresponding to these moduli fields. Such a solution of the low energy action has two obvious deficits. Besides the massless Kähler moduli fields the vacuum state is supersymmetric and Minkowski. Since we expect that the realistic vacuum would be de Sitter and non-supersymmetric, there should be mechanisms within string theory providing this.

Kachru, Kallosh, Linde and Trivedi (KKLT) proposed in their article [16] a scenario for obtaining consistent de Sitter vacua in string theory (for review see ref. [17]). There are perturbative and non-perturbative effects which can provide mass for the Kähler moduli fields. The perturbative effects are α' and g_s-corrections to the low energy action. The non-perturbative effects are Eucledian $D3$-branes wrapping four-cycles of the compact manifold and gaugino condensates produced by a stack of $D7$-branes.

The effective action of the ten-dimensional superstring theory is a double expansion in g_s and α' ,

$$S = S_{(0)} + \alpha'^3 S_{(3)} + \ldots + \alpha'^n S_{(n)} + \ldots + S_0^{(CS)} + S_{(0)}^{\rm loc} + \alpha'^2 S_{(2)}^{\rm loc} \ , \tag{1.4.35}$$

where S^{CS} are the Chern-Simons terms, $S^{\rm loc}$ the localized p-brane actions, and the expansion in g_s is subsumed. The first loop-corrections for the bulk terms appear in $S_{(3)}$.

These corrections lead to the correction of the four-dimensional supergravity. The Kähler potential receives corrections at every order in the perturbation theory and non-perturbative corrections whereas the superpotential receives only non-perturbative corrections:

$$\begin{aligned} K &= K_0 + K_p + K_{np} \ , \\ W &= W_0 + W_{np} \ . \end{aligned} \tag{1.4.36}$$

Computing the potential from the Kähler derivatives

$$\begin{aligned} D_T W &= \partial_T W_{np} + (\partial_T K_0 + \partial_T K_p + \partial_T K_{np})\,(W_0 + W_{np}) \ , \\ D_S W &= \partial_S W_0 + (\partial_S K_0 + \partial_S K_p + \partial_S K_{np})\,(W_0 + W_{np}) \ , \\ D_U W &= \partial_U (W_0 + W_{np}) + (\partial_U K_0 + \partial_U K_p + \partial_U K_{np})\,(W_0 + W_{np}) \ , \end{aligned} \tag{1.4.37}$$

we obtain at the supersymmetric minimum

$$\begin{aligned} V = -3e^K |W|^2 &= -3(1 + K_0 + K_p + K_{np} + \ldots)(|W_0|^2 + |W_{np}|^2 + W_0 \overline{W}_{np} + \overline{W}_0 W_{np}) \\ &= V_0 + V_{Kp} + V_{W_{np}} + \ldots \ , \end{aligned} \tag{1.4.38}$$

where V_0, V_p, V_{np} are

$$V_0 \sim |W_0|^2, \qquad V_{Kp} \sim |W_0|^2 K_p \ , \qquad V_{W_{np}} \sim |W_{np}|^2 + W_0 \overline{W}_{np} + \overline{W}_0 W_{np} \ . \tag{1.4.39}$$

We see that in the case of $W_0 = 0$ the term V_{Kp} vanishes. In a more general case with $W_0 \neq 0$ but still much smaller than one in suitable units the contribution from the $V_{W_{np}}$ will still dominate the contribution from V_{Kp} if W_0 is of order W_{np} and $W_{np}/K_p \ll 1$. This is the regime of KKLT, in which we can neglect the perturbative corrections.

Let us assume that we have an orientifold with some axion-dilaton, complex structure Kähler moduli. The orientifold action on the spectrum of the fields is introduced to obtain $N = 1$ from the original $N = 2$. For more details see Section (2.1.2).

One makes an assumption that all complex structure moduli and the dilaton-axion modulus are stabilized by ISD fluxes of $(2,1)$- and $(0,3)$-type, where one assumes that amount of $(0,3)$-fluxes is very small. Such a point in the moduli space is not supersymmetric, since $(0,3)$-flux breaks supersymmetry. However, we can see that this point is approximately supersymmetric in the space of complex structure moduli and axion-dilaton modulus so far the contribution from $(0,3)$-fluxes is small. The perturbation of this point coming in the next step from the non-perturbative effects is suppressed by the $e^{-Vol(\Sigma)}$. Σ is the volume of a fours-cycle in the compact space.

Let W_0 be the VEV of the superpotential after stabilizing axion-dilaton and complex structure moduli. The Kähler potential and the superpotential are then of the form

$$K = -3\ln(T+\overline{T}) \ , \tag{1.4.40}$$

$$W = W_0 + Ae^{-aT} \ . \tag{1.4.41}$$

At the supersymmetric minimum $D_T W$ should be zero:

$$D_T W = 0 \longrightarrow W_0 = -Ae^{-aT_{\rm cr}}\left(\frac{a}{3}(T_{\rm cr}+\overline{T}_{\rm cr})+1\right) \ . \tag{1.4.42}$$

To make the equations simpler we assume that the VEV of the axion (the imaginary part of the modulus T) is zero, i.e. $T=\overline{T}$. From eq. (1.4.42) we see that the volume modulus of the compact space is now fixed at some value $T_{\rm cr}$. The supergravity potential at the supersymmetric minimum is then

$$V\Big|_{T=T_{\rm cr}} = -3e^K W^2\Big|_{T=T_{\rm cr}} = -\frac{a^2A^2e^{-2aT_{\rm cr}}}{6T_{\rm cr}} \ . \tag{1.4.43}$$

The supersymmetric point with all moduli fields fixed is an AdS-solution, since the value of the potential at this point is strictly negative. To obtain a dS solution we make an uplift by adding a small number n of anti-$D3$-branes to the setup. The reason why we add space-time filling $\overline{D3}$- and not for example $D3$-branes is that they do not have translational moduli. They are already fixed by the ISD-fluxes. In the warped geometry $\overline{D3}$-branes are driven by energetic reasons to the end of the throat, part of the Calabi-Yau where the warping factor is very large. The contribution to the potential from $\overline{D3}$-branes is

$$V_{\overline{D3}} = \frac{D}{T^3} \ , \tag{1.4.44}$$

where D is proportional to n and the value of the warping factor at the position of the brane. The full potential is then given by

$$V = e^K(K^{T\overline{T}}D_TWD_{\overline{T}}\overline{W}-3W\overline{W})+V_{\overline{D3}} = \frac{aAe^{-aT}}{2T^2}\left(W_0+Ae^{-aT}\left(1+\frac{aT}{3}\right)\right)+\frac{D}{T^3} \tag{1.4.45}$$

The form of the uplift term, AdS- and full potential is given in the figure 1.1, taken from ref. [18].

The potential has a runaway behavior, and its minimum is only a false vacuum. The authors of the KKLT scenario gave arguments that the false vacuum is sufficiently stable to survive during the 10^{10} years of cosmological evolution. Additionally, the life time of the constructed dS vacuum is not longer than the recurrence time $t_r \sim e^{S_0}$, where S_0 is the dS entropy. This is another condition which should by applied to a consistent dS vacuum.

One of the technical details which we did not mention before is the tadpole cancellation condition. It should be provided for the full KKLT scenario, so after inclusion of $\overline{D3}$-branes. It means that at the step of the moduli stabilization with AdS vacuum the tadpole

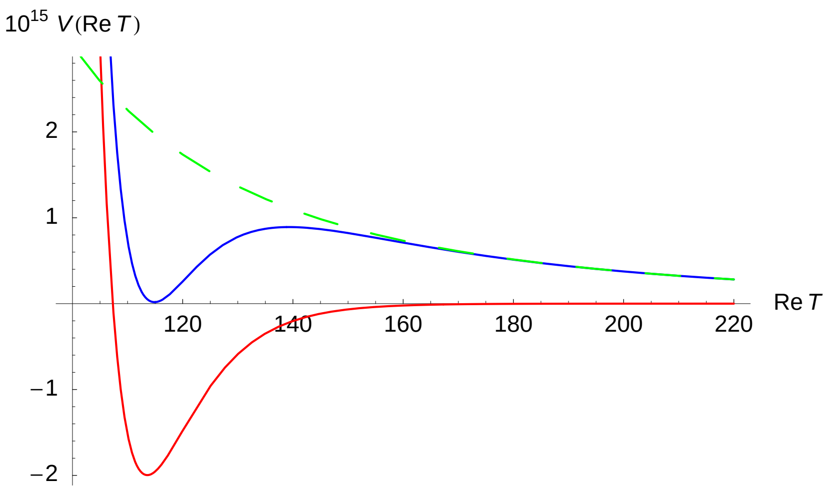

Figure 1.1: Scalar potentials for the model $W_0 = -10^{-4}$, $A = 1$, $a = 0.1$, $D = 3 \times 10^{-9}$ in a KKLT scenario: V_{AdS} in red, $V_{\overline{D3}}$ in green, and V_{total} in blue, taken from ref. [18].

cancellation condition is not fulfilled by the amount of the $\overline{D}3$-branes which one includes for the uplifting.

The other technical detail, which we did not mention is the condition for the presence of the $D3$-brane instantons and gaugino condensation. Here we will just give the criteria and postpone the detailed explanation to chapter 3. As it was shown in [19] the $D3$-brane instantons are present if the four cycle wrapped by the brane satisfies some topological condition. In the original formulation by Witten [19] when the $D3$-brane in question lifts to an $M5$-brane wrapping a divisor with the holomorphic characteristic $\chi(D) = \sum_p (-1)^p h^{(0,p)}(D) = 1$ then the non-perturbative potential will arise. The criteria for the presence of the non-perturbative superpotential coming from the gaugino condensation will be disscussed in section 3.3.

The KKLT scenario attracted a lot of reseach in the last four years. Beside showing the possibility of dS vacua in string theory, the scenario is very flexible (or in other words allows fine tuning). It allows the variation of the extent of supersymmetry breaking and the resulting cosmological constant of the dS minimum in two ways. We can vary the warping of the compactification by changing the flux quanta and the number of the $\overline{D}3$-branes.

1.5 Motivation and structure of the thesis

There are several points where the KKLT procedure could be criticized. The authors of [16] assume that one can construct de Sitter vacua in three “independent” steps. First, one fixes the complex structure moduli and the axion-dilaton by assuming a pure flux potential. In the second step one adds the non-perturbative contribution to the fixed flux superpotential and solves the supersymmetry condition for the Kähler modulus. In the last step, one adds the uplift term. The correct procedure demands to do all steps at the same time. Since this is not obvious, we demonstrate it in a simple example in appendix B.

The several step procedure is actually needed to handle potential minimization in an analytical way. It was argued in the original KKLT work [20] that since the masses of the Kähler moduli are much smaller than those of the complex structure and the axion-dilaton, the position of the latter is perturbed only minimally. Choi et al. [21] considered a model with one Kähler modulus and fixed complex structure moduli. They minimized the potential in one step without integrating out the axion-dilaton field. The exact solution they obtained could be seen to be a saddle point of the potential rather than a stable minimum as obtained by the two step KKLT procedure.

Another ingredient of the KKLT procedure is a non-perturbative superpotential W_{np}. As we mentioned in section 1.3 it is produced either by $D3$-instantons or gaugino condensation on a stack of $D7$-branes. The full expression of W_{np} is not known and there is only a certain criterion, which decides about the instanton contribution to the superpotential. In general, this criterion is not sufficient. It is obtained by counting the number of fermionic zero modes with respect to their chirality on a world-volume of the $D3$-brane. In section 3.1 we give an explanation of its origin following the paper of [19]. Since the original formulation applies to the case of M-/F-theory without fluxes, it is not obvious that the KKLT procedure would work in concrete examples. For the model under consideration one needs to understand the effects of fluxes on the zero mode counting and analyze the topology of four-cycles which are wrapped by the $D3$-branes.

The uplift procedure was criticized, too, insofar as the uplift term needs to be extra fine tuned to obtain a long-living dS vacuum with small cosmological constant, which is probably difficult to achieve. Since we do not touch this subject, we just mention that there are few alternatives to the uplift-procedure [22, 23]. The last and may be the most crucial critical point is the lack of concrete examples.

In this thesis, we discuss the points mentioned above. We find that the KKLT procedure has certain restrictions concerning its applicability. Additionally, we show in which way the procedure should be modified. This will concern the choice of four-cycles responsible for the generation of non-perturbative effects. Finally, we give concrete examples with all moduli fixed following the steps of KKLT.

The next three chapters constitute the main part of the thesis and deal with different aspects of the KKLT scenario. In chapter 2, we discuss flux quantization and moduli stabilization in toroidal type IIB $\mathbf{Z}_N$ and $\mathbf{Z}_N \times \mathbf{Z}_M$-orientifolds, focusing mainly on their toroidal limits. After giving a short introduction of their moduli spaces and effective actions, we study the supersymmetric vacuum structure of these models and derive criteria for the existence of stable minima.

In chapter 3 we discuss the criteria for the presence of the non-perturbative superpotential generated by the Euclidean $D3$-branes wrapping four-cycles in the compact manifold. In the first part of this chapter, we show how the presence of background fluxes of type $(2, 1)$ change Witten's criterion in the case of a Calabi-Yau threefold. As mentioned in the introduction, the topology of the divisor should fulfill some topological constraint, namely its holomorphic characteristic χ should be 1. This geometric criterion comes from the demand to have two fermionic zero modes on the world volume of the D-brane. In the second part of this chapter we study the effect of background fluxes of general Hodge type on the supersymmetry conditions and on the fermionic zero modes on the world-volume of a Euclidean $M5/D3$-brane in M-theory/type IIB string theory. Using the naive supersymmetric variation of the modulino fields to determine the number of zero modes in the presence of a flux of general Hodge type, an inconsistency appears. This inconsistency is resolved by a modification of the supersymmetry variation of the modulinos, which captures the back-reaction of the non-perturbative effects on the background flux and the geometry. In the third part of chapter 3, we give a short overview of the criteria for the presence of the non-perturbative superpotential generated by the gaugino condensation.

In chapter 4, we use the results from the previous chapters to construct models of resolved $\mathbf{Z}_6$ and $\mathbf{Z}_2 \times \mathbf{Z}_4$ orbifolds with all moduli fields stabilized. The results of chapter 2 give us a hint which models we should consider and chapter 3 gives us a tool to decide whether the non-perturbative superpotential is produced.

The conclusions contain the discussion of the results.

Chapter 2

Vacuum structure of orientifolds in the orbifold limits

This chapter is based on the material published in [24] and [25].

2.1 Calabi-Yau orientifolds of type IIB with $D3/D7$–branes

We start with a type IIB compactification on a Calabi-Yau (CY) manifold Y_6. This leads to N=2 supersymmetry in $D = 4$ dimensions. The geometry of the manifold Y_6 is described by $h_{(1,1)}(Y_6)$ Kähler moduli and $h_{(2,1)}(Y_6)$ complex structure moduli. These moduli fields represent scalar components of N=2 hyper– and vector multiplets, respectively. Together with the universal hypermultiplet we have $h_{(1,1)}(Y_6) + 1$ hypermultiplets and $h_{(2,1)}(Y_6)$ vector multiplets.

To arrive at N=1 supersymmetry in $D = 4$ we introduce an orientifold projection $\mathcal{O}$, which produces orientifold $O3$– and $O7$–planes. To cancel tadpoles and to construct models of phenomenological interest we add $D3$– and $D7$–branes. The orientifold projection $\mathcal{O}$ [26, 27]

$$\mathcal{O} = (-1)^{F_L}\ \Omega\ \sigma^\star \tag{2.1.1}$$

acting on the closed type IIB string states is given by a combination of world–sheet parity transformation Ω and a reflection σ in the internal CY space. The CY geometry Y_6 modded out by the additional involution σ is labeled by X_6. To obtain $O3/O7$–planes the action σ must act holomorphically and satisfy

$$\sigma^\star\ \Omega_{(3,0)} = -\Omega_{(3,0)}\ , \tag{2.1.2}$$

with $\Omega_{(3,0)}$ the holomorphic 3–form of the Calabi-Yau manifold X_6.

Due to the holomorphic action of σ, the latter splits the cohomology groups $H^{(p,q)}(Y_6)$ into a direct sum of an even eigenspace $H_+^{(p,q)}(X_6)$ and an odd eigenspace $H_-^{(p,q)}(X_6)$ [27]. Since the Kähler form J is invariant under the orientifold action, it is expanded w.r.t. a basis of $H_+^{(1,1)}(X_6)$. On the other hand, because of (2.1.2), the holomorphic 3–form $\Omega_{(3,0)}$ may be expanded w.r.t. a real symplectic basis $(\alpha_\lambda, \beta^\lambda)$ of $H_-^{(3)}(X_6)$, i.e.

$$J = \sum_{k=1}^{h^{(+)}_{(1,1)}(X_6)} t^k\ \omega_k \quad , \quad \Omega_{(3,0)} = \sum_{\lambda=0}^{h^{(-)}_{(2,1)}(X_6)} X^\lambda \alpha_\lambda - F_\lambda \beta^\lambda \ , \tag{2.1.3}$$

with (X^λ, F_λ) the periods of the original Calabi-Yau Manifold Y_6. Furthermore, in type IIB orientifolds with $D3$– and $D7$–branes, the NS-NS two–form B_2 and the R-R two–form C_2 are odd under the orientifold action $(-1)^{F_L}\Omega$. Hence, they are expanded w.r.t. a basis of the cohomology $H_-^{(1,1)}(X_6)$, i.e.

$$B_2 = \sum_{a=1}^{h^{(-)}_{(1,1)}(X_6)} b^a\ \omega_a \quad , \quad C_2 = \sum_{a=1}^{h^{(-)}_{(1,1)}(X_6)} c^a\ \omega_a \ . \tag{2.1.4}$$

In type IIB orientifolds the fields b^a and c^a give rise to $h^{(-)}_{(1,1)}(X_6)$ complex scalars

$$G^a = i\ c^a - S\ b^a \quad , \quad a = 1, \ldots, h^{(-)}_{(1,1)}(X_6) \tag{2.1.5}$$

of N=1 chiral multiplets [28], whose VEVs eventually should be fixed. Clearly, $D3$– and $D7$–branes may be wrapped only around 4–cycles whose Poincaré dual 2–form is an element of $H_+^2(X_6)$. In addition, there is the dilaton field S:

$$S = e^{-\phi_{10}} + i\ C_0 \tag{2.1.6}$$

with ϕ_{10} the dilaton field and C_0 the Ramond scalar in $D = 10$. The parameter space of S is locally spanned by the coset

$$\mathcal{M}_S = \frac{SU(1,1)}{U(1)} \ . \tag{2.1.7}$$

Furthermore, we have: $e^{-\phi_{10}} = e^{-\phi_4}\ Vol(X_6)^{-1/2}$, with $Vol(X_6)$ the volume of the compactification manifold X_6.

Without D–brane moduli, locally the closed string moduli space $\mathcal{M}$ is a direct product of the complex dilaton field S, the Kähler $\mathcal{M}_K$ and complex structure moduli $\mathcal{M}_{CS}$ [29](see also ref. [30, 31, 32, 28]):

$$\mathcal{M} = \mathcal{M}_S \otimes \mathcal{M}_K \otimes \mathcal{M}_{CS} \ . \tag{2.1.8}$$

To summarize, in addition to the dilaton field S a CY orientifold compactification X_6 has $h^{(+)}_{(1,1)}(X_6)$ Kähler moduli t^k, $h^{(-)}_{(1,1)}(X_6)$ scalars G^a and $h^{-}_{(2,1)}(X_6)$ complex structure moduli u^λ. As shown[1] in table 2.1, under the orientifold action $\mathcal{O}$ the original set of $h_{(1,1)}(Y_6) + 1$ N=2 hypermultiplets and $h_{(2,1)}(Y_6)$ N=2 vectormultiplets is split into a set of N=1 chiral and vectormultiplets.

[1]Here and in the following, where no confusion with the orientifold action Ω may occur we shall use Ω for $\Omega_{(3,0)}$.

1	dilaton	S	chiral multiplet	$\int_{X_6} \Omega \wedge G_3$	ISD 3-form flux G_3
$h^{(-)}_{(2,1)}(X_6)$	CS moduli	u^λ	chiral multiplets	$\int_{X_6} \Omega \wedge G_3$	ISD 3-form flux G_3
$h^{(+)}_{(1,1)}(X_6)$	Kähler moduli	t^k,ρ^k	chiral/ linear multiplets	e^{-T}	D3 instanton gaugino condensation
$h^{(-)}_{(1,1)}(X_6)$	add. moduli	b^a, c^a	chiral multiplets	$\int_{C_4} J \wedge B_2$ $(D_\mu G^a)^2$	calibration massive vector
$h^{(+)}_{(2,1)}(X_6)$	add. vectors	$V_\mu^{\tilde{j}}$	vector multiplets	-	-

Table 2.1: Moduli of Calabi-Yau orientifold X_6 and their stabilization mechanism.

The additional $h^{(+)}_{(2,1)}(X_6)$ vectors (and their magnetic duals) arise from the Ramond 4–form C_4 reduced w.r.t. the cohomology $H^{(3)}_+(X_6)$. Besides the dilaton field S in the Kähler potential for the moduli fields

$$K = -\ln(S+\overline{S}) - 2\ln\left(\frac{1}{6}\int_{X_6} J\wedge J\wedge J\right) - \ln\left(-i\int_{X_6}\Omega\wedge\overline{\Omega}\right) \tag{2.1.9}$$

only the $h^{(+)}_{(1,1)}(X_6)$ invariant Kähler moduli t^k and the $h^{-}_{(2,1)}(X_6)$ invariant complex structure moduli enter explicitly. However, the string theoretical Kähler moduli t^j are not yet scalars of an N=1 chiral multiplet. After defining the proper holomorphic moduli fields T^j (in the string frame[2]) [28]

$$T^j = \frac{3}{4}\ \mathcal{K}_{jkl}\ t^k\ t^l - \frac{3}{8}\ e^{\phi_{10}}\ \mathcal{K}_{jbc}\ \overline{G}^b\ (G+\overline{G})^c + \frac{3}{2}\ i\ \left(\ \rho^j - \frac{1}{2}\ \mathcal{K}_{jbc}\ c^b\ b^c\ \right)\ , \tag{2.1.10}$$

the second term $K_{KM} = -2\ln Vol(X_6) = -2\ln\frac{1}{6}\mathcal{K}_{ijk}t^it^jt^k$ in (2.1.9) may be expressed in terms of the N=1 fields T^j. This way, in the low–energy effective action of type IIB CY orientifolds, the fields G^a do enter the Kähler potential for the Kähler moduli t^k through eliminating the moduli t^k via the definition (2.1.10). By that the Kähler potential K_{KM} for the $h^{(+)}_{(1,1)}(X_6)$ Kähler moduli T^j becomes a complicated function $K_{KM}(S,T^j,G^a)$ depending on the dilaton S, the $h^{(+)}_{(1,1)}(X_6)$ moduli T^j and the $h^{(-)}_{(1,1)}(X_6)$ moduli G^a [28]. In (2.1.10) the axion ρ^j originates from integrating the RR 4–form along the 4–cycle C_j. The full Kähler potential

$$K = -\ln(S+\overline{S}) - 2\ln Vol(X_6) + K_{CS} \tag{2.1.11}$$

[2]In the Einstein frame the Kähler moduli t^k are multiplied with $e^{-\frac{1}{2}\phi_{10}}$. In the Einstein frame the CY volume reads $Vol(X_6) = \frac{1}{6}e^{-\frac{3}{2}\phi_{10}}\ \mathcal{K}_{ijk}t^it^jt^k$.

for the dilaton S, $h^{(+)}_{(1,1)}(X_6)$ Kähler moduli T^k, $h^{(-)}_{(1,1)}(X_6)$ scalars G^a and $h^{(-)}_{(2,1)}(X_6)$ complex structure moduli takes the form [28]:

$$K = -\ln(S+\overline{S}) + K_{KM}(S,T^j,G^a) + K_{CS} \ . \tag{2.1.12}$$

To illustrate the structure of the modified Kähler potential, let us briefly discuss the case $h^{(+)}_{(1,1)}(X_6) = 1 = h^{(-)}_{(1,1)}(X_6)$. The Kähler potential for the single Kähler modulus t is: $K_{KM}(t) = -2\ln t^3$. With the intersection numbers $\mathcal{K}_{ttt} = 6$, $\mathcal{K}_t = 6t^2$ and $\mathcal{K}_{tbb} = 1$ we derive from (2.1.10)

$$T = \frac{9}{2}\, t^2 - \frac{3}{8}\, e^{\phi_{10}}\ \overline{G}\ (G+\overline{G}) + \frac{3}{2}\, i\ \left(\rho - \frac{1}{2}\, c\, b\right)\ ,$$

and the full Kähler potential (2.1.12) becomes:

$$K = -\ln(S+\overline{S}) - 3\ \ln\frac{1}{9}\left[\ T+\overline{T}+\frac{3}{4}\ \frac{(G+\overline{G})^2}{S+\overline{S}}\ \right] + K_{CS} \ . \tag{2.1.13}$$

Before adding background fluxes, in the effective $D=4$ action the fields S, u^λ, t^j, b^a and c^a have flat directions, i.e. no potential is generated for them and their VEVs may assume arbitrary values in their moduli spaces. Fixing these moduli through some F– or D–term potential is the main topic of the chapter 4. In the two last columns of table 2.1 we have shown the different mechanisms how to stabilize these moduli.

2.1.1 Type IIB orientifolds of resolved $\mathbf{Z}_N$– and $\mathbf{Z}_N\times\mathbf{Z}_M$–orbifolds

In chapter 4 we shall investigate moduli stabilization for type IIB orientifold compactifications X_6. We shall discuss orientifolds X_6 of the resolved toroidal orbifolds Y_6

$$Y_6 = T^6/\Gamma\ , \qquad \Gamma = \mathbf{Z}_N\ , \qquad \mathbf{Z}_N\times\mathbf{Z}_M\ , \tag{2.1.14}$$

with orbifold group Γ. To define the orbifold compactification X_6, we must specify the six–torus T^6 and the discrete point group Γ. We will restrict ourselves to orbifolds with Abelian point group without discrete torsion. The point group element θ can then be written as $\theta = \exp[2\pi i(v^1M^{12}+v^2M^{34}+v^3M^{56})]$, where the M^{ij} are the generators of the Cartan sub-algebra and $0\leq|v^i|<1,\ i=1,2,3$. To obtain N=2 supersymmetry, the point group Γ must be a subgroup of $SU(3)$. This gives us $\pm v^1\pm v^2\pm v^3=0$. This condition together with the requirement that Γ must act crystallographically on the lattice specified by T^6 leads to Γ being either $\mathbf{Z}_N$ with $N=3,4,6,7,8,12$ or $\mathbf{Z}_M\times\mathbf{Z}_N$ with N a multiple of M and $N=2,3,4$. $\mathbf{Z}_6$, $\mathbf{Z}_8$ and $\mathbf{Z}_{12}$ have two inequivalent embeddings in $SO(6)$. We will use the standard embeddings, as given e.g. in [33].

In table 2.2, we give a list of possible $\mathbf{Z}_N$ and $\mathbf{Z}_N\times\mathbf{Z}_M$ orbifolds, together with the torus lattices they live on and their Hodge numbers.

The twist elements $\theta,\ldots,\theta^{N-1}$ produce conical singularities. In a small neighborhood around them, the space locally looks like $\mathbf{C}^3/\Gamma$ (isolated singularity) or $\mathbf{C}^2/\Gamma^{(2)}\times\mathbf{C}$ (non–isolated singularity). In ref. [34] these singularities are resolved using the methods of toric

$\mathbf{Z}_N$	Lattice	$h^{\text{untw.}}_{(1,1)}$	$h^{\text{untw.}}_{(2,1)}$	$h^{\text{twist.}}_{(1,1)}$	$h^{\text{twist.}}_{(2,1)}$
$\mathbf{Z}_3$	$SU(3)^3$	9	0	27	0
$\mathbf{Z}_4$	$SU(4)^2$	5	1	20	0
$\mathbf{Z}_4$	$SU(2) \times SU(4) \times SO(5)$	5	1	22	2
$\mathbf{Z}_4$	$SU(2)^2 \times SO(5)^2$	5	1	26	6
$\mathbf{Z}_{6-I}$	$G_2 \times SU(3)^2$	5	0	20	1
$\mathbf{Z}_{6-I}$	$SU(3) \times G_2^2$	5	0	24	5
$\mathbf{Z}_{6-II}$	$SU(2) \times SU(6)$	3	1	22	0
$\mathbf{Z}_{6-II}$	$SU(3) \times SO(8)$	3	1	26	4
$\mathbf{Z}_{6-II}$	$SU(2)^2 \times SU(3) \times SU(3)$	3	1	28	6
$\mathbf{Z}_{6-II}$	$SU(2)^2 \times SU(3) \times G_2$	3	1	32	10
$\mathbf{Z}_7$	$SU(7)$	3	0	21	0
$\mathbf{Z}_{8-I}$	$SU(4) \times SU(4)$	3	0	21	0
$\mathbf{Z}_{8-I}$	$SO(5) \times SO(9)$	3	0	24	3
$\mathbf{Z}_{8-II}$	$SU(2) \times SO(10)$	3	1	24	2
$\mathbf{Z}_{8-II}$	$SO(4) \times SO(9)$	3	1	28	6
$\mathbf{Z}_{12-I}$	E_6	3	0	22	1
$\mathbf{Z}_{12-I}$	$SU(3) \times F_4$	3	0	26	5
$\mathbf{Z}_{12-II}$	$SO(4) \times F_4$	3	1	28	6
$\mathbf{Z}_2 \times \mathbf{Z}_2$	$SU(2)^6$	3	3	48	0
$\mathbf{Z}_2 \times \mathbf{Z}_4$	$SU(2)^2 \times SO(5)^2$	3	1	58	0
$\mathbf{Z}_2 \times \mathbf{Z}_6$	$SU(2)^2 \times SU(3) \times G_2$	3	1	48	2
$\mathbf{Z}_2 \times \mathbf{Z}_{6'}$	$SU(3) \times G_2^2$	3	0	33	0
$\mathbf{Z}_3 \times \mathbf{Z}_3$	$SU(3)^3$	3	0	81	0
$\mathbf{Z}_3 \times \mathbf{Z}_6$	$SU(3) \times G_2^2$	3	0	70	1
$\mathbf{Z}_4 \times \mathbf{Z}_4$	$SO(5)^3$	3	0	87	0
$\mathbf{Z}_6 \times \mathbf{Z}_6$	G_2^3	3	0	81	0

Table 2.2: Twists, lattices and Hodge numbers for $\mathbf{Z}_N$ and $\mathbf{Z}_N \times \mathbf{Z}_M$ orbifolds.

geometry resulting in a smooth Calabi-Yau space Y_6. Afterwards a consistent orientifold action $\mathcal{O}$ is introduced, resulting in the Calabi-Yau orientifold X_6. After resolving the orbifold, three kinds of divisors $\mathcal{D}$ appear, namely E_α, $D_{i\alpha}$, and R_i. The divisors E_α are the exceptional divisors arising from the resolution of an orbifold singularity f_α (or an orbit under the orbifold group), while the divisors $D_{i\alpha}$ denote hyperplanes passsing through fixed points: $D_{i\alpha} = \{z^i = z^i_{fixed,\alpha}\}$. The divisors $R_i = \{z^i = c\}$ for $c \neq z^i_{fixed,\alpha}$ are hyperplanes not passing through a fixed point [34]. As opposed to the $D_{i\alpha}$ they are allowed to move.

Γ	$h^{(+)}_{(1,1)}$	$h^{(-)}_{(1,1)}$	Γ	$h^{(+)}_{(1,1)}$	$h^{(-)}_{(1,1)}$
$\mathbf{Z}_3$	23	13	$\mathbf{Z}_{8-II}$	27	4
$\mathbf{Z}_4$	25	6	$\mathbf{Z}_{8-II}$	31	0
$\mathbf{Z}_4$	27	4	$\mathbf{Z}_{12-I}$	18	6
$\mathbf{Z}_4$	31	0	$\mathbf{Z}_{12-I}$	22	6
$\mathbf{Z}_{6-I}$	19	6	$\mathbf{Z}_{12-II}$	31	0
$\mathbf{Z}_{6-I}$	23	6	$\mathbf{Z}_2 \times \mathbf{Z}_2$	51	0
$\mathbf{Z}_{6-II}$	19	6	$\mathbf{Z}_2 \times \mathbf{Z}_4$	61	0
$\mathbf{Z}_{6-II}$	23	6	$\mathbf{Z}_2 \times \mathbf{Z}_6$	51	0
$\mathbf{Z}_{6-II}$	21	8	$\mathbf{Z}_2 \times \mathbf{Z}_{6'}$	36	0
$\mathbf{Z}_{6-II}$	25	8	$\mathbf{Z}_3 \times \mathbf{Z}_3$	47	37
$\mathbf{Z}_7$	15	9	$\mathbf{Z}_3 \times \mathbf{Z}_6$	51	22
$\mathbf{Z}_{8-I}$	24	5	$\mathbf{Z}_4 \times \mathbf{Z}_4$	90	0
$\mathbf{Z}_{8-I}$	27	0	$\mathbf{Z}_6 \times \mathbf{Z}_6$	84	0

Table 2.3: Hodge numbers $h_{(1,1)}(X_6)$ after the orientifold action

Some divisors E (or divisor orbits under the orbifold group on the T^6) in the geometry of the covering space Y_6 may not be invariant under the orientifold action σ. In this case, a pair of divisors (E_i, E_a), which are eigenstates (with eigenvalues ± 1) under σ may be constructed. To this end, the original number of divisors $h_{(1,1)}(Y_6)$ is split into $h^{(+)}_{(1,1)}(X_6)$ even and $h^{(-)}_{(1,1)}(X_6)$ odd divisors. These numbers are determined for the orientifolds of the resolved orbifolds 2.1.14 in ref. [34] and are displayed in table 2.3.

We choose the orientifold action such that it gives rise to $O3$–planes and $O7$–planes. On the local $\mathbf{C}^3/\Gamma$ patches, an involution, possibly involving the new coordinates associated to the exceptional divisors is chosen, see Section 5 of [34].

Since each $O7$–plane induces -8 units of $D7$–brane charge, we choose to cancel this tadpole locally by placing a stack of 8 coincident $D7$–branes on top of each divisor fixed under the combination of the involution and the scaling action. Each such stack therefore carries an SO(8) gauge group. For the $D3$–brane charge, the case is a bit more involved. The contribution from the $O3$–planes is (in the orientifold quotient X_6 of Y_6)

$$Q_3(O3) = -\frac{1}{4}\, n_{O3} \ ,$$

where n_{O3} denotes the number of $O3$–planes. The $D7$–branes also contribute to the $D3$–tadpole (in the orientifold quotient X_6)

$$Q_3(D7) = -\frac{1}{2} \sum_a \frac{n_{D7,a}\, \chi(\mathcal{D}_a)}{24} \ ,$$

where $n_{D7,a}$ denotes the number of $D7$–branes in the stack located on the divisor $\mathcal{D}_a$. As we have seen, the $\mathcal{D}_a$ can be local D–divisors as well as exceptional divisors E. The last contribution to the $D3$–brane tadpole comes from the $O7$–planes (in the orientifold quotient X_6):

$$Q_3(O7) = -\frac{1}{2}\sum_a \frac{\chi(\mathcal{D}_a)}{6}\ .$$

So the total $D3$–brane charge that must be cancelled is:

$$Q_{3,tot} = -\frac{n_{O3}}{4} - \frac{1}{2}\sum_a \frac{(n_{D7,a}+4)\,\chi(\mathcal{D}_a)}{24}\ . \tag{2.1.15}$$

These are the values for the orientifold quotient X_6, in the double cover Y_6 this value must be multiplied by two (cf. Section 4.3). Because we would like to avoid mobile $D3$–branes, this tadpole will be saturated by 3–form flux G_3.

The formula (2.1.15) for the total $D3$–brane charge $Q_{3,tot}$ differs from the known tadpole equation for the singular orbifold case by the second term. The latter is induced by the curvature of the $D7$–branes which is absent in the singular case. In that case, the number of orientifold $O3$–planes is always 64, i.e. $n_{O3} = 64$, and (2.1.15) boils down to $Q_{3,tot} = -16$ [35]. In the CFT description, this tadpole originates from the total leading divergent contribution of the Klein bottle amplitude $Z_{\mathcal{K}}(1,1)$ of the untwisted orbifold sector. However, there are additional tadpole contributions from other orbifold sectors to be cancelled. More precisely, the tadpole arising from the Klein bottle amplitude $Z_{\mathcal{K}}(1,\theta^k)$ and in addition for even N the $\mathbf{Z}_2$–twisted tadpole related to $Z_{\mathcal{K}}(\theta^{N/2},\theta^k)$ have to be cancelled ($k = 0,\ldots,N-1$). The tadpoles from the sector $(1,1)$ and for even N also from the sector $(1,\theta^{N/2})$ may be cancelled by introducing the right amount of $D3$–brane (or/and 3–form flux) and $D7$–branes, respectively. On the other hand, the divergences of the Klein bottle amplitude $Z_{\mathcal{K}}(1,\theta^k),\ k\neq 0$ or for even N from the combination $Z_{\mathcal{K}}(1,\theta^k)+Z_{\mathcal{K}}(\theta^{N/2},\theta^k),\ k\neq 0, N/2$ can only be cancelled against any of the annulus and Möbius strip contributions in the case that the orbifold group Γ is $\mathbf{Z}_3$, $\mathbf{Z}_{6-I}$, $\mathbf{Z}_{6-II}$, $\mathbf{Z}_7$ or $\mathbf{Z}_{12-I}$ [36] or $\mathbf{Z}_2\times\mathbf{Z}_2, \mathbf{Z}_3\times\mathbf{Z}_3, \mathbf{Z}_6\times\mathbf{Z}_6, \mathbf{Z}_2\times\mathbf{Z}_3, \mathbf{Z}_2\times\mathbf{Z}_6, \mathbf{Z}_2\times\mathbf{Z}_6'$ [37]. Hence singular orbifolds have much more constraining tadpole equations, which are non–trivial to fulfill for all $\mathbf{Z}_N$– and $\mathbf{Z}_N\times\mathbf{Z}_M$ orbifolds. However, if one introduces discrete torsion or vector structure tadpoles from all orbifold sectors may be completely cancelled in all singular orbifold cases [38].

Nevertheless, the orientifolds X_6 constructed geometrically in ref. [34] in the large radius regime from resolved orbifolds Y_6 need not have a CFT counterpart in their orbifold limit, since D–branes (in particular stacks of $D7$ and $O7$–branes) wrapping cycles which vanish in the orbifold limit, give rise to extra non–perturbative states in the orbifold limit.

2.1.2 Type IIB orientifolds of toroidal limits

In the previous section we discussed CY-Orientifolds obtained by resolving orbifolds of $\mathbf{Z}_N$ and $\mathbf{Z}_N\times\mathbf{Z}_M$-type. Now we shall discuss the case of their toroidal limits.

To obtain an N=1 (closed) string spectrum, one introduces an orientifold projection ΩI_n, with Ω describing a reversal of the orientation of the closed string world–sheet and I_n a reflection of n internal coordinates. For ΩI_n to represent a symmetry of the original theory, n has to be an even integer in type IIB. Generically, this projection produces orientifold fixed planes [$O(9-n)$–planes], placed at the orbifold fixed points of T^6/I_n. They have negative tension, which has to be balanced by introducing positive tension objects. Candidates for the latter may be collections of $D(9-n)$–branes and/or non–vanishing three–form fluxes H_3 and C_3. The orbifold group Γ mixes with the orientifold group ΩI_n. As a result, if the group Γ contains $\mathbf{Z}_2$–elements θ, which leave one complex plane fixed, we obtain additional $O(9-|n-4|)$– or $O(3+|n-2|)$–planes from the element $\Omega I_n \theta$.

The geometry of the orbifold X_6 is described by $h_{(1,1)}(X_6)$ Kähler moduli $\mathcal{T}^i$ and $h_{(2,1)}(X_6)$ complex structure moduli $\mathcal{U}^i$, which split into twisted and untwisted moduli. In the following, the dimension of the latter is denoted by $h^{\text{untw.}}_{(1,1)}(X_6)$ and $h^{\text{untw.}}_{(2,1)}(X_6)$, respectively.

Depending on the numbers $h^{\text{untw.}}_{(1,1)}, h^{\text{untw.}}_{(2,1)}$ of untwisted Kähler $\mathcal{T}^i$ and complex structure moduli $\mathcal{U}^j$, the generic (untwisted) moduli spaces $\mathcal{M}_{\mathcal{K}}$, $\mathcal{M}_{\mathcal{CS}}$ appearing in toroidal orbifold compactifications are described by the following six different cosets [39, 40, 41, 42]

$$
\begin{aligned}
h^{\text{untw.}}_{(1,1)} = 3 \ , \ h^{\text{untw.}}_{(2,1)} = 0,1,3 \quad &: \ \mathcal{M}_{\mathcal{K}} = \left(\frac{SU(1,1)}{U(1)}\right)^3 \ , \ \mathcal{M}_{\mathcal{CS}} = \left(\frac{SU(1,1)}{U(1)}\right)^{h^{\text{untw.}}_{(2,1)}} \ , \\
h^{\text{untw.}}_{(1,1)} = 5 \ , \ h^{\text{untw.}}_{(2,1)} = 0,1 \quad &: \ \mathcal{M}_{\mathcal{K}} = \frac{SU(2,2)}{SU(2)\times SU(2)\times U(1)} \times \left(\frac{SU(1,1)}{U(1)}\right) \ , \\
&\quad\ \mathcal{M}_{\mathcal{CS}} = \left(\frac{SU(1,1)}{U(1)}\right)^{h^{\text{untw.}}_{(2,1)}} \ , \\
h^{\text{untw.}}_{(1,1)} = 9 \ , \ h^{\text{untw.}}_{(2,1)} = 0 \quad &: \ \mathcal{M}_{\mathcal{K}} = \frac{SU(3,3)}{SU(3)\times SU(3)\times U(1)} \ .
\end{aligned}
\tag{2.1.16}
$$

The numbers $h^{\text{twist.}}_{(1,1)}, h^{\text{twist.}}_{(2,1)}$ depend both on the orbifold group Γ and the underlying torus lattice T^6 [43]. The corresponding Kähler potentials for the spaces (2.1.16) are known from heterotic string compactifications [40]:

$$
\begin{aligned}
h^{\text{untw.}}_{(1,1)} = 3 \ , \ h^{\text{untw.}}_{(2,1)} = 0,1,3 \quad &: \ K_{\mathcal{K}} = -\sum_{i=1}^{3} \ln(\mathcal{T}^i + \overline{\mathcal{T}}^i) \ , \ K_{\mathcal{CS}} = -\sum_{j=1}^{h^{\text{untw.}}_{(2,1)}} \ln(\mathcal{U}^j + \overline{\mathcal{U}}^j) \ , \\
h^{\text{untw.}}_{(1,1)} = 5 \ , \ h^{\text{untw.}}_{(2,1)} = 0,1 \quad &: \ K_{\mathcal{K}} = -\ln\det(\mathcal{T}^{ij} + \overline{\mathcal{T}}^{ij}) - \ln(\mathcal{T}^5 + \overline{\mathcal{T}}^5) \ , \\
&\quad\ K_{\mathcal{CS}} = -\sum_{j=1}^{h^{\text{untw.}}_{(2,1)}} \ln(\mathcal{U}^j + \overline{\mathcal{U}}^j) \ , \\
h^{\text{untw.}}_{(1,1)} = 9 \ , \ h^{\text{untw.}}_{(2,1)} = 0 \quad &: \ K_{\mathcal{K}} = -\ln\det(\mathcal{T}^{ij} + \overline{\mathcal{T}}^{ij}) \ .
\end{aligned}
\tag{2.1.17}
$$

The parameterization of the moduli fields $\mathcal{T}^i$,$\mathcal{U}^i$ in terms of the data of the torus, i.e. the real metric g and the discrete symmetries of the underlying effective field theory, was elaborated in ref. [24].

There is one important difference when compactifing the heterotic and type IIB string on the same six–manifold X_6. In the heterotic string, the complexification of the Kähler moduli $\mathcal{T}^i$ is achieved through the Neveu–Schwarz antisymmetric tensor B_2, while in the orientfolds we discuss here, this is accomplished with the Ramond 4–form C_4. Moreover, while the string–theoretical moduli fields $\mathcal{T}^i$ define proper complex scalars of chiral N=1 multiplets in $D=4$ heterotic compactifications, they do not enjoy this property in type IIB orientifolds. More precisely, in type IIB the axionic part of the complexified Kähler modulus $\mathcal{T}^i$ is given by some internal component of the Ramond 4–form C_4, i.e. the 4–cycle integral $\int_{C_i} C_4$, while for the heterotic compactification on the same manifold X_6, the Kähler moduli are complexified with some internal part of the NS 2–form B_2, i.e. $\int_{C_j} B_2$, with some 2–cycle C_j. Since $h_{(2,2)}(X_6) = h_{(1,1)}(X_6)$, from the cohomological point of view, there is not much difference, as the 2–form ω_i, which appears in the expansion of B_2, is the Poincaré dual of the 4–cycle C_i. An other peculiarity in type IIB orientifold compactifications with wrapped $D7$–branes is that the Kähler moduli $\mathcal{T}^i$ following from the geometry of the manifold X_6 do not represent scalars of chiral N=1 multiplets in $D=4$. One has to define new moduli T^i, which refer to the underlying effective field theory and lead to the correct effective field theory description. In fact, a quite general formula may be given, which relates the $h_{(1,1)}$ string theoretical moduli fields $\mathcal{T}^i$ to their field–theoretical analogs T^i:

$$T^i = \frac{\partial}{\partial \mathrm{Re}(\mathcal{T}^i)}\ Vol(X_6(\mathcal{T}^j)) + i\ \int_{C_i} C_4\ . \tag{2.1.18}$$

Here, $Vol(X_6(\mathcal{T}^j))$ is the volume (in string units) of the internal manifold X_6 expressed in terms of the Kähler moduli $\mathcal{T}^j$, defined in type IIB on X_6.

As we may see from the list (2.1.16), the complex structure moduli space is much simpler, as this space only consists of factors of $\frac{SU(1,1)}{U(1)}$. Furthermore, in many of the orbifold examples, the complex structure moduli $\mathcal{U}^i$ are fixed through the orbifold twist, i.e. $h^{\text{untw.}}_{(2,1)} = 0$. Only in the case when the orbifold has $\mathbf{Z}_2$–subelements, some $\mathcal{U}^i$ remain unfixed. Except for the twist $\mathbf{Z}_2\times\mathbf{Z}_2$, there may only exist one such $\mathbf{Z}_2$–element in order to preserve N=1 supersymmetry in $D=4$. Hence, for $\mathbf{Z}_2\times\mathbf{Z}_2$ we have $h^{\text{untw.}}_{(2,1)} = 3$, while *all* other orbifolds with $\mathbf{Z}_2$–elements have $h^{\text{untw.}}_{(2,1)} = 1$. On the other hand, in type IIB orientifolds the complex structure moduli $\mathcal{U}^i$ following from the string background X_6 already describe scalars U^i of N=1 chiral multiplets in $D=4$. Hence, we have:

$$U^i = \mathcal{U}^i \quad , \quad i = 1,\ldots,h^{\text{untw.}}_{(2,1)}\ . \tag{2.1.19}$$

2.1.3 Three–form flux G_3 in $\mathbf{Z}_N$ and $\mathbf{Z}_N\times\mathbf{Z}_M$–orbifolds

Let us now give non–vanishing VEVs to some of the (untwisted) flux components H_{ijk} and F_{ijk}, with $F_3 = dC_2$, $H_3 = dB_2$. The two 3–forms F_3, H_3 are organized in the $SL(2,\mathbf{Z})_S$

covariant field:

$$G_3 = F_3 + i\ S\ H_3\ . \tag{2.1.20}$$

On the torus T^6, we would have 20+20 independent internal components for H_{ijk} and F_{ijk}. However, only a portion of them is invariant under the orbifold group Γ. More precisely, of the 20 complex (untwisted) components comprising the flux G_3, only $2h^{\text{untw.}}_{(2,1)}(X_6)+2$ survive the orbifold twist. The orientifold action $\Omega(-1)^{F_L}I_6$ producing $O3$–planes does not give rise to any further restrictions. If the orbifold group Γ contains $\mathbf{Z}_2$–elements θ which leave the j–th complex plane fixed, we also encounter $O7_j$–planes transverse to the j–th plane. Since $I_2^j = I_6\theta$, the orientifold generator $\Omega(-1)^{F_L}I_2^j$ does not put further restrictions on the $2h^{\text{untw.}}_{(2,1)}(X_6)+2$ twist invariant components. Hence, the allowed flux components are most conveniently found in the complex basis, in which the orbifold group Γ acts diagonally. In the following, we shall concentrate on the orientifold/orbifolds $T^6/(\Gamma+\Gamma\Omega I_6)$, with Γ being one of the orbifold twists $\mathbf{Z}_N$ or $\mathbf{Z}_N\times\mathbf{Z}_M$ encountered above. Note that $O7$–planes appear in the case that the orbifold twist Γ is of even order.

The most general 3–form flux G_3 on T^6 has 20 components, which appear in the expansion

$$\frac{1}{(2\pi)^2\alpha'}\ G_3 = \sum_{i=0}^{3}(A^i\omega_{A_i}+B^i\omega_{B_i}) + \sum_{j=1}^{6}(C^j\omega_{C_j}+D^j\omega_{D_j}) \tag{2.1.21}$$

w.r.t. the complex 3–form cohomology $H^3 = H^{(3,0)}\oplus H^{(2,1)}\oplus H^{(1,2)}\oplus H^{(0,3)}$:

$$\begin{aligned}
\omega_{A_0} &= dz^1\wedge dz^2\wedge dz^3 \quad , \quad \omega_{B_0} = d\overline{z}^1\wedge d\overline{z}^2\wedge d\overline{z}^3\ ,\\
\omega_{A_1} &= d\overline{z}^1\wedge dz^2\wedge dz^3 \quad , \quad \omega_{B_1} = dz^1\wedge d\overline{z}^2\wedge d\overline{z}^3\ ,\\
\omega_{A_2} &= dz^1\wedge d\overline{z}^2\wedge dz^3 \quad , \quad \omega_{B_2} = d\overline{z}^1\wedge dz^2\wedge d\overline{z}^3\ ,\\
\omega_{A_3} &= dz^1\wedge dz^2\wedge d\overline{z}^3 \quad , \quad \omega_{B_3} = d\overline{z}^1\wedge d\overline{z}^2\wedge dz^3\ ,\\
\omega_{C_1} &= dz^1\wedge d\overline{z}^1\wedge dz^2 \quad , \quad \omega_{D_1} = dz^1\wedge d\overline{z}^1\wedge d\overline{z}^2\ ,\\
\omega_{C_2} &= dz^1\wedge d\overline{z}^1\wedge dz^3 \quad , \quad \omega_{D_2} = dz^1\wedge d\overline{z}^1\wedge d\overline{z}^3\ ,\\
\omega_{C_3} &= dz^1\wedge dz^2\wedge d\overline{z}^2 \quad , \quad \omega_{D_3} = d\overline{z}^1\wedge dz^2\wedge d\overline{z}^2\ ,\\
\omega_{C_4} &= dz^2\wedge d\overline{z}^2\wedge dz^3 \quad , \quad \omega_{D_4} = dz^2\wedge d\overline{z}^2\wedge d\overline{z}^3\ ,\\
\omega_{C_5} &= dz^1\wedge dz^3\wedge d\overline{z}^3 \quad , \quad \omega_{D_5} = d\overline{z}^1\wedge dz^3\wedge d\overline{z}^3\ ,\\
\omega_{C_6} &= dz^2\wedge dz^3\wedge d\overline{z}^3 \quad , \quad \omega_{D_6} = d\overline{z}^2\wedge dz^3\wedge d\overline{z}^3\ .
\end{aligned} \tag{2.1.22}$$

The ω_{A_i}, ω_{B_i} correspond to flux components with all one–forms coming from different planes, while the ω_{C_i}, ω_{D_i} are flux components with two one–forms coming from the same plane. The latter we have just written down for completeness, as they are projected out in all orbifolds. In the $\mathbf{Z}_2\times\mathbf{Z}_2$ orientifold/orbifold, which allows for the largest number of (untwisted) fluxes [35], all ω_{A_i} and ω_{B_i} remain, whereas in most other orbifolds only ω_{A_0} and ω_{B_0} survive. That the $(0,3)$ and $(3,0)$-flux always survive is quite clear, as the $(3,0)$-flux corresponds to the Calabi-Yau 3-form Ω, which is always present, and the $(0,3)$-flux to its conjugate.

While in the form (2.1.21), the cohomology structure of G_3 is manifest, in order to impose the flux quantization on G_3, i.e.

$$\frac{1}{(2\pi)^2\alpha'}\int_{C_3} F_3 \in n_0\ \mathbf{Z} \quad , \quad \frac{1}{(2\pi)^2\alpha'}\int_{C_3} H_3 \in n_0\ \mathbf{Z}\ , \tag{2.1.23}$$

with some integer n_0 (depending on the orbifold group Γ) to be specified later, one has to transform the forms (2.1.22) into a real basis of the following 20 elements

$$\begin{aligned}
\alpha_0 &= dx^1\wedge dx^2\wedge dx^3 \quad , & \beta^0 &= dy^1\wedge dy^2\wedge dy^3\ ,\\
\alpha_1 &= dy^1\wedge dx^2\wedge dx^3 \quad , & \beta^1 &= -dx^1\wedge dy^2\wedge dy^3\ ,\\
\alpha_2 &= dx^1\wedge dy^2\wedge dx^3 \quad , & \beta^2 &= -dy^1\wedge dx^2\wedge dy^3\ ,\\
\alpha_3 &= dx^1\wedge dx^2\wedge dy^3 \quad , & \beta^3 &= -dy^1\wedge dy^2\wedge dx^3\ ,\\
\gamma_1 &= dx^1\wedge dy^1\wedge dx^2 \quad , & \delta^1 &= -dy^2\wedge dx^3\wedge dy^3\ ,\\
\gamma_2 &= dx^1\wedge dy^1\wedge dx^3 \quad , & \delta^2 &= -dx^2\wedge dy^2\wedge dy^3\ ,\\
\gamma_3 &= dx^1\wedge dx^2\wedge dy^2 \quad , & \delta^3 &= -dy^1\wedge dx^3\wedge dy^3\ ,\\
\gamma_4 &= dx^2\wedge dy^2\wedge dx^3 \quad , & \delta^4 &= -dx^1\wedge dy^1\wedge dy^3\ ,\\
\gamma_5 &= dx^1\wedge dx^3\wedge dy^3 \quad , & \delta^5 &= -dy^1\wedge dx^2\wedge dy^2\ ,\\
\gamma_6 &= dx^2\wedge dx^3\wedge dy^3 \quad , & \delta^6 &= -dx^1\wedge dy^1\wedge dy^2\ ,
\end{aligned} \tag{2.1.24}$$

with the six real periodic coordinates x^i, y^i on the torus T^6, i.e. $x^i \cong x^i+1$ and $y^i \cong y^i+1$. The basis (2.1.24) has the property $\int_{X_6}\alpha_i\wedge\beta^j=\delta_i^j,\ \int_{X_6}\gamma_i\wedge\delta^j=\delta_i^j$, with the choice of orientation $\int_{X_6} dx^1\wedge dx^2\wedge dx^3\wedge dy^1\wedge dy^2\wedge dy^3=1$. In real notation, the flux has the form:

$$\frac{1}{(2\pi)^2\alpha'}G_3=\sum_{i=0}^{3}\left[(a^i+iSc^i)\alpha_i+(b_i+iSd_i)\beta^i\right]+\sum_{j=1}^{6}\left[(e^j+iSg^j)\gamma_j+(f_j+iSh_j)\delta^j\right]\ . \tag{2.1.25}$$

In this basis, the $SL(2,\mathbf{Z})_S$–covariance of G_3 is manifest. The coefficients $a^i,\ b_i,\ e^i,\ f_i$ refer to the Ramond part of G_3, whereas the coefficients $c^i,\ d_i,\ g^i,\ h_i$ refer to the Neveu-Schwarz part.

To pass from the complex basis (2.1.22) to the real basis (2.1.24), one introduces complex structures, i.e. the complex coordinates:

$$dz^j=\sum_{i=1}^{3}\ \rho_i^j\ dx^i+\tau_i^j\ dy^i \quad , \quad j=1,2,3\ . \tag{2.1.26}$$

Most of the parameters ρ_i^j and τ_i^j are fixed through the orbifold twist Γ, with only those remaining undetermined, which correspond to the $\mathbf{Z}_2$–elements of Γ. The latter are eventually fixed through the flux quantization condition (cf. appendix B of ref. [25]). As we shall see in a moment, the specific values of the constants ρ_i^j and τ_i^j are relevant for finding flux solutions.

Let us briefly comment on the integers n_0, introduced in the flux quantization conditions (2.1.23). It has been pointed out in ref. [44], that there are subtleties for toroidal orientifolds due to additional 3–cycles, which are not present in the covering space T^6. If some integers are odd, additional discrete flux has to be turned on in order to meet the quantization rule for those 3–cycles. We may bypass these problems in the $\mathbf{Z}_N$ ($\mathbf{Z}_N \times \mathbf{Z}_M$)–orientifolds, if we choose the quantization numbers to be multiples of $n_0 = 2N$ ($n_0 = 2NM$) and do not allow for discrete flux at the orientifold planes [45, 46, 47]. Note, that for $h^{\text{twist.}}_{(2,1)} \neq 0$, in addition to the untwisted flux components H_{ijk} and F_{ijk} there may be also NS-NS– and R-R–flux components from the twisted sector. We do not consider them here. It is assumed, that their quantization rules freeze the blowing up moduli at the orbifold singularities.

To illustrate the above procedure, we shall discuss the $\mathbf{Z}_{6-II}$ orbifold with the lattice $(SU(2))^2 \times SU(3) \times G_2$ and present the fluxes compatible with the complex structures of this orbifold. We will present only one example, while the other orbifolds are treated in appendix B of ref. [25].

At this level, no supersymmetry conditions are imposed. Imposing further conditions will fix S and the complex structure moduli (in case they are present in the particular orbifold) and/or constrain the coefficients a_i, b_i, c_i, d_i which are real integers.

The $\mathbf{Z}_{6-II}$ orbifold on the lattice $(SU(2))^2 \times SU(3) \times G_2$ is a case with one complex structure modulus $\mathcal{U}^3$ left unfixed, therefore the flux takes the form

$$\frac{1}{(2\pi)^2\alpha'}\, G_3 = A_0\,\omega_{A_0} + A_3\,\omega_{A_3} + B_0\,\omega_{B_0} + B_3\,\omega_{B_3}.$$

With the complex coordinates, worked out in [25],

$$dz^1 = 3^{1/4}(dx^1 + e^{2\pi i/3}dx^2)\ , \qquad dz^2 = dx^3 + \frac{1}{\sqrt{3}}e^{5\pi i/6}dx^4\ , \qquad dz^3 = dx^5 + \frac{1}{\sqrt{3}}e^{-5\pi i/6}dx^6 \tag{2.1.27}$$

the $(3,0)$–form on this orbifold takes the form

$$\begin{aligned}\omega_{A_0} \ =\ & \frac{1}{3}\{3\,\alpha_0 + \sqrt{3}\,e^{5\pi i/6}\,\alpha_1 + 3\,e^{2\pi i/3}\,\alpha_2 \\ & +\mathcal{U}^3\,[\,3\,\alpha_3 - i(\sqrt{3}\,\beta_0 + 3\,e^{\pi i/6}\,\beta_1 + \sqrt{3}\,e^{2\pi i/6}\,\beta_2)] + i\sqrt{3}\beta_3\,\}.\end{aligned} \tag{2.1.28}$$

The one $(2,1)$–form surviving the twist takes the form

$$\begin{aligned}\omega_{A_3} \ =\ & \frac{1}{3}\{3\,\alpha_0 + \sqrt{3}\,e^{5\pi i/6}\,\alpha_1 + 3\,e^{2\pi i/3}\,\alpha_2 \\ & +\overline{\mathcal{U}}^3\,[\,3\,\alpha_3 - i(\sqrt{3}\,\beta_0 + 3\,e^{\pi i/6}\,\beta_1 + \sqrt{3}\,e^{2\pi i/6}\,\beta_2)] + i\sqrt{3}\,\beta_3\,\}.\end{aligned} \tag{2.1.29}$$

ω_{B_0} and ω_{B_3} are the complex conjugates of the above. For the complex coefficients we find

$$A_0 \ =\ \frac{1}{2\,\mathrm{Im}\,\mathcal{U}^3}\,\{\,e^{2\pi i/12}b_0 - ib_2 + iS\,(e^{2\pi i/12}d_0 - i\,d_2)$$

$$
\begin{aligned}
& +\overline{\mathcal{U}}^3 \left[\frac{1}{\sqrt{3}} e^{2\pi i/6} a_0 + \frac{1}{\sqrt{3}} a_2 + iS \, (-\sqrt{3} \, e^{2\pi i/6} \, c_0 + \frac{1}{\sqrt{3}} \, c_2) \right] \Big\} \, , \\
B_0 \;\; = \;\; & \frac{1}{2 \, \mathrm{Im} \mathcal{U}^3} \Big\{ \, e^{-2\pi i/12} b_0 + i b_2 + iS \, (e^{-2\pi i/12} \, d_0 + i \, d_2) \\
& + \mathcal{U}^3 \left[\frac{1}{\sqrt{3}} e^{-2\pi i/6} a_0 + \frac{1}{\sqrt{3}} a_2 + iS \, (\sqrt{3} \, e^{-2\pi i/6} \, c_0 + \frac{1}{\sqrt{3}} \, c_2) \right] \Big\} \, , \\
A_3 \;\; = \;\; & \frac{1}{2 \, \mathrm{Im} \mathcal{U}^3} \Big\{ \, e^{-10\pi i/12} b_0 + i b_2 + iS \, (e^{-10\pi i/12} \, d_0 + i \, d_2) \\
& - \mathcal{U}^3 \left[\frac{1}{\sqrt{3}} e^{2\pi i/6} a_0 + \frac{1}{\sqrt{3}} a_2 + iS \, (\sqrt{3} \, e^{2\pi i/6} \, c_0 + \frac{1}{\sqrt{3}} \, c_2) \right] \Big\} \, , \\
B_3 \;\; = \;\; & \frac{1}{2 \, \mathrm{Im} \mathcal{U}^3} \Big\{ \, e^{10\pi i/12} b_0 - i b_2 + iS \, (e^{10\pi i/12} \, d_0 - i \, d_2) \\
& - \overline{\mathcal{U}}^3 \left[\frac{1}{\sqrt{3}} e^{-2\pi i/6} a_0 + \frac{1}{\sqrt{3}} a_2 + iS \, (\sqrt{3} \, e^{-2\pi i/6} \, c_0 + \frac{1}{\sqrt{3}} \, c_2) \right] \Big\} \, . \qquad (2.1.30)
\end{aligned}
$$

Note that the normalization of the 3–forms is $\int \omega_{A_0} \wedge \omega_{B_0} = 2i \, \mathrm{Im} \mathcal{U}^3$. Expressed in the real 3–forms, the flux takes the form

$$
\begin{aligned}
\frac{1}{(2\pi)^2 \alpha'} G_3 \;\; = \;\; & (a^0 + iS \, c^0) \, \alpha_0 + \frac{1}{3} \, (-a^0 + a^2 - iS \, (c^0 - c^2)) \, \alpha_1 + (a^2 + iS \, c^2) \, \alpha_2 \\
& + (-b_0 + 2 \, b_2 + iS \, (-d_0 + 2 \, d_2)) \, \alpha_3 + (b_0 + iS \, d_0) \, \beta^0 + (b_0 + b_2 \\
& + iS \, (d_0 + d_2)) \, \beta^1 + (b_2 + iS \, d_2) \, \beta^2 + \frac{1}{3} \, (a^0 + 2 \, a^2 + iS \, (c^0 + 2 \, c^2)) \, \beta^3] \, .
\end{aligned}
\qquad (2.1.31)
$$

2.2 Stability of type IIB orientifolds

We will not discuss hier in very great detail the microscopic origin of the non-perturbative superpotential W_{np}, but leave this for the chapter 4. Our main emphasis in this section is the investigation of the vacuum-structure of type IIB orientifold compactifications in their various toroidal orbifold limits. Hence, we simply assume the existence of a non-perturbative superpotential W_{np}, which depends only on the untwisted Kähler moduli T^i. The effects of blowing up the orbifold or the presence of blowing up Kähler moduli will be neglected, respectively postponed to chapter 4. So, $W_{np}(T^i)$ can be viewed as being the truncation of a more complete superpotential that contains all Kähler moduli. Nevertheless, several interesting questions can be addressed within the orbifold framework.

First, in KKLT one assumes that the complex structure moduli are fixed by W_{flux} alone and then are integrated out assuming that they are heavy. In particular, the assumption is made that the flux vacua are still given through 3–form fluxes which are still imaginary self dual (ISD) and are of the Hodge types $G_{(2,1)}$ and $G_{(0,3)}$. We will see however that the inclusion of the additional non-perturbative effects in the superpotential besides the 3–form fluxes has the effect of generic supersymmetric AdS ground-states being described by fluxes which are not anymore ISD with only $G_{(2,1)}$ components, but will rather include also all IASD (imaginary anti self-dual) types as well (see also the discussion in [48]).

The second problem is related to the stability of the obtained supersymmetric vacua. Although stable AdS vacua generically allow for scalar fields with negative (mass)^2, provided the masses still fulfill the Breitenlohner-Freedman bound [49], the KKLT framework only works if all (mass)^2 eigenvalues of the fixed scalar fields are already positive in the AdS ground state. The reason for this stronger requirement is that otherwise, the uplift to a dS vacuum by adding a positive constant to the scalar potential would not work, i.e. would not lead to a stable dS ground state. However in concrete orientifold models, this stability criterion is far from being automatically satisfied, as already observed in [21]. We will discuss in which orbifold compactifications there is a chance to obtain stable AdS ground states with positive scalar (mass)^2.

In this chapter, we investigate the vacuum structure of type IIB orientifold compactifications in their orbifold limits. The discussion is based on the following effective N=1 superpotential

$$W = W_{\text{flux}}(S, U^j) + W_{\text{np}}(T^i) \ , \tag{2.2.32}$$

with:

$$\begin{aligned} W_{\text{flux}}(S, U^j) &= \frac{\lambda}{(2\pi)^2 \alpha'} \int_{X_6} G_3 \wedge \Omega \ , & (2.2.33) \\ W_{\text{np}}(T^i) &= \sum_{i=1}^{h_{(1,1)}(X_6)} g^i \, e^{-h^i T^i} \quad , \quad g^i \in \mathbf{C}, \ h^i \in \mathbf{R}^+ \ . & (2.2.34) \end{aligned}$$

The first term is the perturbative contribution to the superpotential due to non-vanishing 3–form fluxes [14], and it depends on the dilaton field S and, if present, also on the untwisted complex structure moduli U^j (with the normalization $\kappa_{10}^{-2} = \frac{\lambda}{(2\pi)^2\alpha'}$). The second term is of non-perturbative nature and depends on the untwisted Kähler moduli T^i.

The vacua of the effective N=1 supergravity theory are determined by the associated scalar potential [50]

$$V = e^{\kappa_4^2 K} \left(|D_S W|^2 + \sum_{i=1}^{h_{(1,1)}(X_6)} |D_{T^i} W|^2 + \sum_{j=1}^{h_{(2,1)}(X_6)} |D_{U^j} W|^2 - 3\, |W|^2 \right) \ , \tag{2.2.35}$$

with the Kähler potential for the fields S, T^j, U^j. During the process of minimizing V, the following two aspects will become important: first, the supersymmetry conditions $D_{S,T^i,U^j} W = 0$ will imply that generic supersymmetric AdS ground states are described by fluxes which are not anymore ISD with only $G_{(2,1)}$ components, but rather will include $G_{(0,3)}$ and also all IASD (imaginary anti self-dual) types as well. The second issue concerns the stability of the obtained extrema after imposing the supersymmetry conditions. One has to require the absence of any tachyonic scalar fields, i.e. the (mass)^2 of all scalars must be positive. This means that all eigenvalues of the scalar field mass matrix $\frac{\partial^2 V}{\partial \phi_\alpha \partial \bar{\phi}_\beta}$

$(\phi_\alpha, \phi_\beta = S, U^j, T^i)$ must be positive. As we will see, this requirement can be only satisfied by those orbifolds which contain untwisted complex structure moduli U^j. In this way, we derive some severe constraints on which orbifolds can lead to stable vacua. This result is contrasted by the procedure originally applied in KKLT, where first the dilaton field S and the complex structure moduli were integrated out by solving the flux supersymmetry conditions $D_S W_{\text{flux}} = D_{U^j} W_{\text{flux}} = 0$ using ISD $(2,1)$– or $(0,3)$–fluxes, and then plugging the obtained values for S and U^j back into W. This leads to a constant term W_0. However, the integrating-out procedure is in addition only consistent, if the masses of the integrated-out fields S and U^j are heavy compared to the Kähler moduli T^i. Otherwise, the results on the vacuum structure and especially what concerns the stability problems are misleading and cannot be trusted anymore.

This problem has been emphasized and thoroughly discussed recently in ref. [21]. In this chapter, we want to generalize this discussion into several directions. First, we discuss under what conditions stable minima may be found if all moduli are minimized at once without first integrating out the complex structure moduli. This way, in subsection 2.2.4 we find a stable minimum for the case $h^{\text{untw.}}_{(1,1)} = 3$ and $h^{\text{untw.}}_{(2,1)} = 1$. On the other hand, in ref. [21] it is has been proven that this case would not lead to a stable minimum, if the complex structure modulus was integrated out first. Secondly, in subsection 2.2.3, we shall investigate the KKLT scenario in toroidal orbifolds for more than one Kähler modulus and more general Kähler potentials (cf. (2.1.17)) at fixed complex structure modulus. We find, that in those cases no stable minimum is possible generalizing the one Kähler modulus case discussed in [21]. This result rules out all toroidal orbifold limits with only Kähler moduli for a KKLT scenario, as e.g. the $\mathbf{Z}_7$–orbifold. Furthermore, in subsection 2.2.5, we find a more general effective superpotential (compared to the ones discussed in [21]) after integrating out several complex structure moduli. Finally, the conditions and solutions for the extrema are presented.

2.2.1 Supersymmetry conditions

In this subsection, we shall study the supersymmetry (SUSY) conditions for the $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifold, with $h^{\text{untw.}}_{(1,1)} = h^{\text{untw.}}_{(2,1)} = 3$. They read

$$D_i W \equiv \partial_i W + \kappa_4^2 \; W \; \partial_i K = 0 \; , \qquad i = S, U^i, T^i \tag{2.2.36}$$

and allow us to explore the Hodge structure of the flux G_3 in the supersymmetric case. The Kähler potential for the dilaton S and Kähler moduli T^i is given in (2.1.17), while for the complex structure moduli U^i it may be read off from (2.1.17). With the superpotential (2.2.32) the conditions (2.2.36) lead to:

$$\begin{aligned} D_{T^i} W &= 0 \Longrightarrow \frac{\lambda}{(2\pi)^2 \alpha'} \int G_3 \wedge \Omega = -\left(T^i + \bar{T}^i\right) g^i h^i e^{-h^i T^i} - \sum_{j=1}^{3} g^j e^{-h^j T^j} \; , \quad i = 1,2,3 \; , \\ D_S W &= 0 \Longrightarrow \frac{\lambda}{(2\pi)^2 \alpha'} \int \bar{G}_3 \wedge \Omega = -\sum_{i=1}^{3} g^i e^{-h^i T^i} \; , \end{aligned}$$

$$D_{U^i}W \;=\; 0 \Longrightarrow \frac{\lambda}{(2\pi)^2\alpha'}\int G_3\wedge\omega_{Ai} = -\sum_{j=1}^{3} g^j e^{-h^jT^j} \quad , \quad i=1,2,3\ . \tag{2.2.37}$$

After writing G_3 in the complex basis (cf. eq. (2.1.21))

$$\frac{1}{(2\pi)^2\alpha'}\ G_3 = \sum_{i=0}^{3}\left(A^i\omega_{Ai} + B^i\omega_{Bi}\right)\ , \tag{2.2.38}$$

where ω_{A0} is a $(3,0)$-form, ω_{Ai} are $(2,1)$-forms, ω_{B0} is a $(0,3)$-form and ω_{Bi} are $(1,2)$-forms, we obtain from (2.2.37):

$$\begin{aligned}
B^0 &= -\frac{1}{\lambda\prod\limits_{k=1}^{3}\left(U^k+\bar U^k\right)}\left[\left(T^i+\bar T^i\right)g^ih^ie^{-hT^i} + \sum_{j=1}^{3}g^je^{-h^jT^j}\right]\ , \qquad i=1,2,3\ ,\\
A^0 &= -\frac{1}{\lambda}\,\frac{\sum\limits_{i=1}^{3}g^ie^{-h^i\bar T^i}}{\prod\limits_{j=1}^{3}\left(U^j+\bar U^j\right)}\ , \quad B^i = -\frac{1}{\lambda}\,\frac{\sum\limits_{k=1}^{3}g^ke^{-h^kT^k}}{\prod\limits_{j=1}^{3}\left(U^j+\bar U^j\right)}\ , \qquad i=1,2,3\ .
\end{aligned} \tag{2.2.39}$$

Here we have used $\int_{X_6}\omega_{A_0}\wedge\omega_{B_0} = \int_{X_6}\omega_{B_i}\wedge\omega_{A_i} = \prod\limits_{k=1}^{3}(U^k+\overline{U}^k)$. We see that in the presence of the non–perturbative term the $(1,2),(0,3)$ and $(3,0)$–components of the flux are no longer vanishing. Next, with the formula [50]

$$\overline{F}^{\overline{I}} = e^{\kappa_4^2/2\ K}\ K^{\overline{I}J}\ (\partial_J W + \kappa_4^2\ W\ \partial_J K) \tag{2.2.40}$$

we present the F-terms:

$$\begin{aligned}
\bar F^{\bar S} &= \left(S+\bar S\right)^{\frac12}\prod_{i=1}^{3}\left(T^i+\bar T^i\right)^{-\frac12}\prod_{j=1}^{3}\left(U^j+\bar U^j\right)^{-\frac12}\kappa_4^2\left(\frac{\lambda}{(2\pi)^2\alpha'}\int\bar G_3\wedge\Omega + \sum_{k=1}^{3}g^ke^{-h^kT^k}\right)\ ,\\
\bar F^{\bar U^i} &= \left(S+\bar S\right)^{-\frac12}\left(U^i+\bar U^i\right)^{\frac12}\left(U^j+\bar U^j\right)^{-\frac12}\left(U^k+\bar U^k\right)^{-\frac12}\prod_{l=1}^{3}\left(T^l+\bar T^l\right)^{-\frac12}\times\\
&\quad\times\kappa_4^2\left(\frac{\lambda}{(2\pi)^2\alpha'}\int G_3\wedge\omega_{Ai} + \sum_{m=1}^{3}g^me^{-h^mT^m}\right)\ ,\\
\bar F^{\bar T^i} &= \left(S+\bar S\right)^{-\frac12}\left(T^i+\bar T^i\right)^{\frac12}\left(T^j+\bar T^j\right)^{-\frac12}\left(T^k+\bar T^k\right)^{-\frac12}\prod_{l=1}^{3}\left(U^l+\bar U^l\right)^{-\frac12}\times\\
&\quad\times\kappa_4^2\left[W_{\rm flux} + \left(T^i+\bar T^i\right)\ g^ih^i\ e^{-h^iT^i}\right]\ , \quad i\neq j\neq k\ .
\end{aligned} \tag{2.2.41}$$

With [50]

$$V = K_{I\overline{J}}\ F^I\ \overline{F}^{\overline{J}} - 3\ e^{\kappa_4^2\ K}\ \kappa_4^2\ |W|^2 \tag{2.2.42}$$

the potential becomes:

$$\begin{aligned} V &= \kappa_4^2\Big(|S+\bar{S}|\prod_{j=1}^{3}(T^j+\bar{T}^j)\prod_{k=1}^{3}(U^k+\bar{U}^k)\Big)^{-1}\times \\ &\times\Big\{\sum_{i=1}^{3}\Big|W_{\rm flux}+(T^i+\bar{T}^i)\ g^ih^i\ e^{-h^iT^i}\Big|^2+\Bigg|\frac{\lambda}{(2\pi)^2\alpha'}\int\bar{G}_3\wedge\Omega+\sum_{l=1}^{3}\ g^l\ e^{-h^lT^l}\Bigg|^2 \\ &+\sum_{l=1}^{3}\Bigg|\frac{\lambda}{(2\pi)^2\alpha'}\int G_3\wedge\omega_{Al}+\sum_{m=1}^{3}g^m\ e^{-h^mT^m}\Bigg|^2-3\ |W|^2\Big\}\ . \end{aligned} \tag{2.2.43}$$

Using (2.2.38) we can rewrite the potential as:

$$\begin{aligned} V &= \kappa_4^2\left(\left|S+\bar{S}\right|\prod_{i=1}^{3}\left|T^i+\bar{T}^i\right|\prod_{j=1}^{3}\left|U^j+\bar{U}^j\right|\right)^{-1}\Big\{-3\Big|B^0\lambda\prod_{l=1}^{3}(U^l+\bar{U}^l)+\sum_{l=1}^{3}g^le^{-h^lT^l}\Big|^2 \\ &+\sum_{k=1}^{3}\Bigg|\ B^0\lambda\ \prod_{l=1}^{3}(U^l+\bar{U}^l)+\sum_{l=1}^{3}\ g^l\ e^{-h^lT^l}+(T^k+\bar{T}^k)g^kh^ke^{-h^kT^k}\ \Bigg|^2 \\ &+\Bigg|\ \lambda\ \prod_{n=1}^{3}(U^n+\bar{U}^n)\bar{A}^0+\sum_{p=1}^{3}\ g^p\ e^{-h^pT^p}\ \Bigg|^2 \\ &+\sum_{r=1}^{3}\Bigg|\ \lambda\ \prod_{n=1}^{3}(U^n+\bar{U}^n)B^r+\sum_{p=1}^{3}\ g^p\ e^{-h^pT^p}\ \Bigg|^2\Big\}. \end{aligned} \tag{2.2.44}$$

In the supersymmetric case, i.e. $F^S=F^{U^j}=F^{T^i}=0$, the potential reduces to:

$$V_0=-3\ \kappa_4^2\ \frac{\Big|\ B^0\lambda\ \prod_{l=1}^{3}(U^l+\bar{U}^l)+\sum_{l=1}^{3}g^l\ e^{-h^lT^l}\ \Big|^2}{\left|S+\bar{S}\right|\prod_{i=1}^{3}\left|T^i+\bar{T}^i\right|\prod_{j=1}^{3}\left|U^j+\bar{U}^j\right|}\ . \tag{2.2.45}$$

Next, we plug the superpotential (2.2.32) (cf. also [35] for $W_{\rm flux}$)

$$\begin{aligned} W &= \left(a^0+iSc^0\right)U^1U^2U^3-\left\{\left(a^1+iSc^1\right)U^2U^3+\left(a^2+iSc^2\right)U^1U^3+\left(a^3+iSc^3\right)U^1U^2\right\} \\ &-\sum_{i=3}^{3}\left(b_i+iSd_i\right)U^i-\left(b_0+iSd_0\right)+\sum_i g^ie^{-h^iT^i} \end{aligned} \tag{2.2.46}$$

into eq. (2.2.37). The equations become:

$$0 = \bar{U}^1\bar{U}^2\bar{U}^3\left(a^0+iSc^0\right)-\sum_{i\neq j\neq k}\left(a^i+iSc^i\right)\bar{U}^j\bar{U}^k-\left(b_0+iSd_0\right)$$

$$
\begin{aligned}
& - \sum_{i=1}^{3} \left(b_i + iSd_i\right) \bar{U}^i + \sum_{i=1}^{3} g^i e^{-h^i \bar{T}^i} \ , \\
0 \ &= \ U^1U^2U^3 \left(a^0 + iSc^0\right) - \sum_{i\neq j\neq k} \left(a^i + iSc^i\right) U^j U^k - \left(b_0 + iSd_0\right) \\
& - \sum_{i=1}^{3} \left(b_i + iSd_i\right) U^i + \sum_{j=1}^{3} g^j \ e^{-h^j T^j} + g^i h^i \left(T^i + \bar{T}^i\right) e^{-h^i T^i} \ , i = 1,2,3 \ , \\
0 \ &= \ \bar{U}^1U^2U^3 \left(a^0 + iSc_0\right) - \left\{\left(a^1 + iSc^1\right) U^2U^3 + \left(a^2 + iSc^2\right) \bar{U}^1U^3 + \left(a^3 + iSc^3\right) \bar{U}^1U^2\right\} \\
& - \left(b_0 + iSd_0\right) - \left\{\left(b_1 + iSd_1\right) \bar{U}^1 + \left(b_2 + iSd_2\right) U^2 + \left(b_3 + iSd_3\right) U^3\right\} + \sum_{i=1}^{3} g^i e^{-h^i T^i} \ , \\
0 \ &= \ U^1\bar{U}^2U^3 \left(a^0 + iSc^0\right) - \left\{\left(a^1 + iSc^1\right) \bar{U}^2U^3 + \left(a^2 + iSc^2\right) U^1U^3 + \left(a^3 + iSc^3\right) U^1\bar{U}^2\right\} \\
& - \left(b_0 + iSd_o\right) - \left\{\left(b_1 + iSd_1\right) U^1 + \left(b_2 + iSd_2\right) \bar{U}^2 + \left(b_3 + iSd_3\right) U^3\right\} + \sum_{i=1}^{3} g^i e^{-h^i T^i} \ , \\
0 \ &= \ U^1U^2\bar{U}^3 \left(a^0 + iSc^0\right) - \left\{\left(a^1 + iSc^1\right) U^2\bar{U}^3 + \left(a^2 + iSc^2\right) U^1\bar{U}^3 + \left(a^3 + iSc^3\right) U^1U^2\right\} \\
& - \left(b_0 + iSd_0\right) - \left\{\left(b_1 + iSd_1\right) U^1 + \left(b_2 + iSd_2\right) U^2 + \left(b_3 + iSd_3\right) \bar{U}^3\right\} + \sum_{i=1}^{3} g^i e^{-h^i T^i} \ .
\end{aligned}
\tag{2.2.47}
$$

These are the equations to be satisfied at the supersymmetric point of the moduli space.

2.2.2 Orientifolds without complex structure modulus

Let us now discuss the vacuum structure of orientifold compactifications without any complex structure moduli, i.e. $h_{(2,1)} = 0$. So the moduli fields which we want to determine by the supersymmetry conditions are the dilaton S and the Kähler moduli T^i ($i = 1, \ldots, h^{\text{untw.}}_{(1,1)}$). Since ω_{A_0} and ω_{B_0} are the only non-trivial 3-forms, the flux G_3, expressed in the complex basis, reads:

$$
\frac{1}{(2\pi)^2\alpha'} \ G_3 = G_{(3,0)} + G_{(0,3)} = A^0(S) \ \omega_{A_0} + B^0(S) \ \omega_{B_0} \, . \tag{2.2.48}
$$

$B^0(S)$ is a linear function in S with complex coefficients B^0_1, B^0_2:

$$
B^0(S) = B^0_1 - iS \, B^0_2 \, . \tag{2.2.49}
$$

The precise form of the B^0_K ($K = 1,2$) depends on the considered orbifold, as we will discuss in the following. However, the other flux coefficient $A^0(S)$ is not anymore an independent function, but it is given as

$$
A^0(S) = \bar{B}^0_1 + iS \, \bar{B}^0_2 \, . \tag{2.2.50}
$$

The flux superpotential which contains the contribution from the G_3 flux as well as the non-perturbative Kähler moduli dependent term, is given in eq. (2.2.32). Inserting G_3 of eq. (2.2.48), W becomes:

$$W = \lambda\,(B_1^0 - iS\,B_2^0) + \sum_{i=1}^{3} g \sim e^{-h^i T^i}\,. \tag{2.2.51}$$

This superpotential is of the same structure as the superpotential discussed in [21] (see Section 2.2.1 in that paper). The main difference to the superpotential of [21] is that here, the coefficients B_1^0 and B_2^0 have a microscopic explanation in terms of 3–form flux quantum numbers. It follows that these coefficients are integer-valued. Hence the flux quantization will put some additional constraints on the allowed solutions of the supersymmetry equations.

Let us consider in more detail the $\mathbf{Z}_3 \times \mathbf{Z}_3$ orbifold. Here the complex flux coefficients read (see appendix B.7.of ref. [24]):

$$B_1^0 = \frac{1}{\sqrt{3}}\,(i\,a^1 + e^{-5\pi i/6}\,b_1)\,,\quad B_2^0 = \frac{1}{\sqrt{3}}\,(i\,c^1 + e^{-5\pi i/6}\,d_1)\,,\quad a^1, b_1, c^1, d_1 \in \mathbf{Z}\,. \tag{2.2.52}$$

In order to determine the exact form of the flux part of the superpotential, we also need the prefactor λ. For the $\mathbf{Z}_3 \times \mathbf{Z}_3$ orbifold it takes the value $\lambda = i\sqrt{3}$ [47].

Now we may determine the solutions of the two supersymmetry conditions $D_T W = 0$ and $D_S W = 0$. We may essentially follow the procedure outlined in [21]. We shall consider the simplified case where all Kähler moduli T^i are identified, i.e. $T^i = T$, and also $h^i = h$. Now observe that via a field redefinition in T, namely a constant shift in Im T, the coefficient g can always be chosen to be real. Similarly one can shift Im S, such that $i\sqrt{3}B_1^0$ is real. So we choose $b_1 = 0$ in eq. (2.2.52). For simplicity we also choose $i\sqrt{3}B_2^0$ to be real. Taking all this into account, the superpotential (2.2.51) becomes:

$$W = -a^1 + \frac{\sqrt{3}}{2} d_1\ S + 3g\ e^{-hT}\,,\quad a^1, d_1 \in \mathbf{Z}\ . \tag{2.2.53}$$

As in [21], we may restrict the analysis to the case where the moduli S and T are purely real, i.e. $T = t$ and $S = s$. Then the two supersymmetry conditions provide the following two constraints on s and t:

$$a^1 = g\ e^{-ht}(ht+3)\,,\quad \frac{d_1}{a^1} = -\frac{2ht}{\sqrt{3}s(ht+3)}\,. \tag{2.2.54}$$

Since $e^{-ht}(ht+3) \leq 3$, it follows that the first equation has only solutions for integer values of a^1, if the parameter $|g| \geq 1/3$. In fact due to charge quantization, for any given $|g| \geq 1/3$, this equation has a finite number of allowed solutions (for $|g| = 1/3$ the solution occurs at $t = 0$). Specifically, the first equation possesses solutions in t for the following values of the flux a^1:

$$a^1 = 1, \ldots, [g']\,,\quad g' = 3g\ . \tag{2.2.55}$$

Here we have assumed that $g > 0$, otherwise $a^1 < 0$. Finally, after having solved the first constraint in (2.2.54) which fixes the modulus t, the second equation does not put any further conditions on the allowed fluxes, it possesses precisely one solution in s for any given choice of a^1, c^1. Let us assume that g is very large, $|g| >> |a^1|$. Then the supersymmetry condition is solved for very large t. Furthermore, if we insist on weak string coupling, i.e. large s, we have to demand that $|a^1| >> |c^1|$.

As discussed in [21], the above solutions of the two supersymmetry conditions do not correspond to stable supersymmetric vacua, but the supersymmetric point rather is a saddle point with instabilities along the moduli and axionic directions. Hence, we like to proceed to consider orbifolds with more than one Kähler modulus and/or complex structure moduli in order to see whether stable supersymmetric ground states now become possible.

2.2.3 Orientifolds with three Kähler moduli

After having discussed the case of one Kähler modulus in the previous subsection, we now shall move on to the three Kähler moduli case with fixed complex structure. This case captures e.g. the $\mathbf{Z}_7$–orbifold. We start with the following ansatz for the superpotential (2.2.32)

$$W = \alpha_1 + \alpha_2\ S + \sum_{j=1}^{3} g^j e^{-h^j T^j}\ , \tag{2.2.56}$$

with complex coefficients $\alpha_1 = B_1^0$, $\alpha_2 = -iB_2^0$, g^j and $h^j > 0$. With the Kähler potential

$$\kappa_4^2\ K = -\ln(S+\overline{S}) - \sum_{j=1}^{3} \ln(T^j + \overline{T}^j) \tag{2.2.57}$$

for the closed string moduli sector we derive the following F–terms:

$$\begin{aligned} -(S+\overline{S})^{-1/2} \prod_{i=1}^{3}(T^i+\overline{T}^i)^{1/2}\ \overline{F}^S &= \alpha_1 - \alpha_2\overline{S} + \sum_{j=1}^{3} g^j\ e^{-h^j\ T^j}\ , \\ -\frac{(S+\overline{S})^{1/2}\ (T^i+\overline{T}^i)^{1/2}\ (T^k+\overline{T}^k)^{1/2}}{(T^j+\overline{T}^j)^{1/2}}\ \overline{F}^{T^j} &= h^j\ g^j\ (T^j+\overline{T}^j)\ e^{-h^j\ T^j} + \alpha_1 + \alpha_2\ S \\ &\quad + \sum_{j=1}^{3} g^j\ e^{-h^j\ T^j}\quad ,\quad (i,j,k) = \overline{(1,2,3)}\ , \end{aligned} \tag{2.2.58}$$

and similarly for their complex conjugate F^{T^j} and F^S. Demanding $F^S = 0 = F^{T^j}$ leads to the following relations:

$$\alpha_1 = \alpha_2 \left(\overline{S} + \sum_{j=1}^{3} \frac{S+\overline{S}}{h^j\ \left(T^j+\overline{T}^j\right)}\right)\quad ,\quad g^j = -\frac{\alpha_2\ e^{h^j\ T^j}}{h^j}\ \frac{S+\overline{S}}{T^j+\overline{T}^j}\ ,\ j=1,2,3\ , \tag{2.2.59}$$

and their complex conjugate. These relations have to be obeyed at the extremum of the potential. In principle, the point (S_0, T_0^j) of the extremum may be determined from these relations (2.2.59). It is straightforward to calculate the scalar potential $V(S, T^j)$. At the extremum (S_0, T_0^j), its value is given by

$$V_0 = -3\ \frac{|\alpha_2|^2\ (S_0 + \overline{S}_0)}{\prod_{j=1}^{3}(T_0^j + \overline{T}_0^j)}\ . \tag{2.2.60}$$

To determine the kind of extremum, we have to calculate the second derivatives of the potential $V(S, T^j)$ w.r.t. the moduli fields. It is convenient to introduce $S = s_1 + is_2$ and $T^j = t_1^j + it_2^j$. W.r.t. the parameters s_i, t_i^j we find the following identities for the mixed derivatives:

$$\frac{\partial^2 V}{\partial s_1\ \partial t_2^j} = \frac{\partial^2 V}{\partial t_1^j\ \partial s_2} = \frac{\partial^2 V}{\partial s_1\ \partial s_2} = \frac{\partial^2 V}{\partial t_1^k\ \partial t_2^l} = 0\ . \tag{2.2.61}$$

On the other hand, the non–vanishing components of the Hessian $H = \begin{pmatrix} H_1 & 0 \\ 0 & H_2 \end{pmatrix}$ are arranged in a block–form with two 4×4 matrices H_1 and H_2, with their determinants given by:

$$\begin{aligned}
\det H_1 &= -\frac{s_1^2\ |\alpha_2|^8}{512\ (t_1^1 t_1^2 t_1^3)^6} \left(2 + h^1\ h^2\ h^3\ t_1^1\ t_1^2\ t_1^3 - \sum_{j=1}^{3} h^j\ t_1^j \right) \\
&\quad \times \left(5 + 16\ h^1\ h^2\ h^3\ t_1^1\ t_1^2\ t_1^3 + 8\ \sum_{j=1}^{3} h^j\ t_1^j + 6\ \sum_{i \neq j} h^i\ h^j\ t_1^i\ t_1^j \right)\ , \\
\det H_2 &= -\frac{h^1\ h^2\ h^3\ s_1^2\ |\alpha_2|^8}{512\ (t_1^1 t_1^2 t_1^3)^5} \left(27 + 16\ h^1\ h^2\ h^3\ t_1^1\ t_1^2\ t_1^3 - 6\ \sum_{i \neq j} h^i\ h^j\ t_1^i\ t_1^j \right)\ .
\end{aligned} \tag{2.2.62}$$

The latter may become positive in a certain region of the parameter space $h^i\ t_1^i$. In order for H_1 and H_2 to be positive definite, also their subdeterminants have to be positive, i.e. $H_{11} > 0,\ H_{11}H_{22} - H_{12}^2 > 0$ and $\det\left(H_{11}\quad H_{12}\quad H_{13}H_{12}\quad H_{22}\quad H_{23}H_{13}\quad H_{23}\quad H_{33} \right) > 0$. However, we find

$$(H_1)_{11}(H_1)_{22} - (H_1)_{12}^2 = -\frac{1}{32(t_1^1)^4(t_1^2)^2(t_1^3)^2}\ |\alpha_2|^4\ (4 + 5h^1\ t_1^1) < 0\ ,$$

and

$$(H_2)_{11}(H_2)_{22} - (H_2)_{12}^2 = -\frac{3}{32(t_1^1)^3(t_1^2)^2(t_1^3)^2}\ |\alpha_2|^4\ h^1\ < 0$$

and conclude that the extremum (S_0, T_0^j) is no minimum.

Hence a KKLT scenario is not possible in the $\mathbf{Z}_7$–orbifold with only untwisted Kähler moduli. This generalizes the results of [21] for one Kähler modulus to the three Kähler moduli case.

2.2.4 Orientifolds with one untwisted complex structure modulus

Now consider orientifolds with one untwisted complex structure modulus, labelled by U^3. The main issue will be to solve the supersymmetry conditions, taking into account the flux quantization, and to see if in contrast to the previous case there are stable vacua. The relevant 3-forms are the $(3,0)$–form ω_{A_0} and one $(2,1)$–form ω_{A_3} plus their conjugate $(0,3)$ and $(1,2)$–forms ω_{B_0} and ω_{B_3}. In terms of these complex 3–forms, the flux G_3 may be expanded as:

$$\begin{aligned}\frac{1}{(2\pi)^2\alpha'}\,G_3 &= G_{(3,0)}+G_{(2,1)}+G_{(0,3)}+G_{(1,2)}\\ &= A^0(S,U^3)\;\omega_{A_0}+A^3(S,U^3)\;\omega_{A_3}+B^0(S,U^3)\;\omega_{B_0}+B^3(S,U^3)\;\omega_{B_3}\;.\end{aligned} \tag{2.2.63}$$

Now, the complex coefficients take the form

$$B^0(S)=B^0_1(U^3)-iB^0_2(U^3)\;S\,,\quad B^3(S)=B^3_1(U^3)-iB^3_2(U^3)\;S\;, \tag{2.2.64}$$

where the $B^0(U^3)$, $B^3(U^3)$ each contain a constant term and a term linear in U^3. All together they comprise eight real integer valued flux parameters, whose explicit forms depend on the individual orientifold under investigation (see later). Using this 3–form flux, the superpotential (2.2.32) may be written as

$$W=\lambda\;[B^0_1(U^3)-i\;B^0_2(U^3)\;S]+\sum_{i=1}^{3}g^ie^{-h^iT^i}\;, \tag{2.2.65}$$

which for convenience we parameterize as:

$$W=\alpha_0+\alpha_1\;U^3+\alpha_2\;S+\alpha_3\;SU^3+\sum_{i=1}^{3}g^ie^{-h^iT^i}\;,\quad \alpha_i\in\mathbf{R}\;. \tag{2.2.66}$$

In the following, we consider first the situation, where in the first step the complex structure modulus U^3 is integrated out; this leads to an effective superpotential $W_{\rm eff}(S,T)$. In the second step, the supersymmetry conditions $D_TW_{\rm eff}(S,T)=D_SW_{\rm eff}(S,T)=0$ are imposed for the effective superpotential $W_{\rm eff}(S,T)$. As pointed out in ref. [21], this procedure is valid as long as the vacuum has the property that the complex structure moduli U^i are much heavier than the fields S and T_i. Assuming that this assumption indeed holds, we consider the supersymmetry condition for U^3,

$$D_{U^3}W=\alpha_1+\alpha_3S-\frac{\alpha_0+\alpha_1\;U^3+\alpha_2\;S+\alpha_3\;SU^3+\sum\limits_{i=1}^{3}g^ie^{-h^iT^i}}{U^3+\bar U^3}=0\,, \tag{2.2.67}$$

and plug back its solution for U^3 into the superpotential. This results in the following effective superpotential that now depends only on S and T_i (for real U^3):

$$W_{\text{eff}}(S,T) = 2\left(\alpha_0 + \alpha_2\, S + \sum_{i=1}^{3} g^i e^{-h^i T^i}\right). \tag{2.2.68}$$

We see that this effective superpotential is again a linear function in S. In fact, it is precisely of the same structure as the superpotential (2.2.51) of the previous section without complex structure modulus. Hence all conclusions about the vacuum structure with respect to S and T are unchanged. In particular, the supersymmetric stationary points in S and T are not stable ground states with a positive definite moduli mass matrix. This result has already been obtained in [21].

Alternatively, we can also determine the solutions of all supersymmetry conditions $D_{U^3}W = D_SW = D_TW = 0$ at the same time without first integrating out U^3. For simplicity we consider the isotropic case $T := T^1 = T^2 = T^3$, $h_1 = h_2 = h_3$ and real flux parameters α_i. We write the moduli fields as $T = t + i\tau$, $S = s + i\sigma$ and $U^3 = u_3 + i\nu$. To make the calculation clear, we confine ourselves to the supersymmetric point with $\sigma = 0$, $\nu = 0$, $\tau = 0$. The constraints which have to be fulfilled at the supersymmetric point become:

$$\begin{aligned} s &= -\frac{1}{\alpha_2}\left(\alpha_0 + (3+ht)ge^{-ht}\right)\ , \\ u_3 &= -\frac{\alpha_2}{\alpha_3}\left(\frac{\alpha_0 + 3ge^{-ht}}{\alpha_0 + (3+ht)ge^{-ht}}\right)\ , \\ \alpha_1 &= \frac{\alpha_3\left(\alpha_0 + ge^{-ht}(3+ht)\right)^2}{\alpha_2\ \left(\alpha_0 + 3ge^{-ht}\right)}\ . \end{aligned} \tag{2.2.69}$$

Here, s and u_3 are the real parts of the dilaton and complex structure moduli respectively, and should be positive. From the above constraints we see that this excludes some values for α_0, α_2 and α_3. If we allow for t every positive value, the situation is simple. One has two possibilities

$$\alpha_0 \geq 0\ , \quad \alpha_2 < 0\ , \quad \alpha_3 > 0 \tag{2.2.70}$$

and

$$\alpha_0 < -(3+ht)\ g\ e^{-ht}\ , \quad \alpha_2 > 0\ , \quad \alpha_3 < 0\ . \tag{2.2.71}$$

In the same way as in the previous section we compute the potential and then calculate the second derivatives at the supersymmetric point. This means that we plug the constraints (2.2.69) into the matrix of second derivatives. The resulting six-dimensional matrix is of block diagonal form (two blocks 3×3). The condition for the supersymmetric point to be a minimum is that the diagonal blocks should be positive definite. This requirement may be translated into the statement that the determinants associated with all upper–left submatrices are positive. We abbreviate the sub–determinants of the upper block by a_{11},

a_{22}, a_{33} and those of the lower block by a_{44}, a_{55}, a_{66}. They are

$$
\begin{aligned}
a_{11} &= \frac{\alpha_3^3}{8\alpha_2^2 t^3(\alpha_0 + 3g\ e^{-ht})^3}\Big(\alpha_0 + (3+ht)g\ e^{-ht}\Big)^2 \times \\
&\quad \times\Big(2\alpha_0^2 + 2\alpha_0(6+ht)g\ e^{-ht} + g^2 e^{-2ht}(18+6ht+h^2t^2)\Big)\ , \\
a_{22} &= \frac{\alpha_3^4}{64t^6(\alpha_0 + 3g\ e^{-ht})^4}\Big(2\alpha_0^2 + \alpha_0(12+ht)g\ e^{-ht} + g^2e^{-2ht}(18+3ht-h^2t^2)\Big) \times \\
&\quad \times\Big(2\alpha_0^2 + 3\alpha_0(4+ht)g\ e^{-ht} + 3g^2e^{-2ht}(6+3ht+h^2t^2)\Big)\ , \\
a_{33} &= \frac{3\alpha_3^5 h^2 g^2 e^{-2ht}}{512t^9(\alpha_0 + 3g\ e^{-ht})^5}\Big(2\alpha_0^2 + \alpha_0(12+ht)g\ e^{-ht} + g^2e^{-2ht}(18+3ht-h^2t^2)\Big) \times \\
&\quad \times\Big(\alpha_0(1+2ht) + 3g\ e^{-ht}(1+ht)\Big)\Big(2\alpha_0(2+ht) + 3g\ e^{-ht}(4+3ht+h^2t^2)\Big)\ , \\
a_{44} &= \frac{\alpha_3^3}{8\alpha_2^2 t^3(\alpha_0 + 3g\ e^{-ht})^3}\Big(\alpha_0 + (3+ht)g\ e^{-ht}\Big)^2 \times \\
&\quad \times\Big(2\alpha_0^2 + 2\alpha_0(6+ht)g\ e^{-ht} + g^2 e^{-2ht}(18+6ht+h^2t^2)\Big)\ , \\
a_{55} &= \frac{\alpha_3^4}{64t^6(\alpha_0 + 3g\ e^{-ht})^3}\Big(2\alpha_0 + 3(2h+t)g\ e^{-ht}\Big) \times \\
&\quad \times\Big(2\alpha_0^2 + \alpha_0(12+ht)ge^{-ht} + g^2e^{-2ht}(18+3ht+2h^2t^2)\Big)\ , \\
a_{66} &= \frac{3\alpha_3^5 h^3 g^2 e^{-2ht}}{512t^8(\alpha_0 + 3g\ e^{-ht})^4}\Big(4\alpha_0^2 + 2\alpha_0(9+2ht)g\ e^{-ht} + (18+3ht-3h^2t^2)g^2e^{-2ht}\Big) \times \\
&\quad \times\Big(\alpha_0(3+2ht) + (9+6ht+2h^2t^2)g\ e^{-ht}\Big)\ . \qquad (2.2.72)
\end{aligned}
$$

To analyze these minors we have to distinguish the two cases (2.2.70) und (2.2.71).

In the first case (2.2.70), the conditions for the positivity of the minors are

$$
\begin{aligned}
2\alpha_0^2 &\ + \alpha_0(12+ht)g\ e^{-ht} + (18+3ht-h^2t^2)g^2e^{-2ht} > 0 \\
4\alpha_0^2 &\ + 2\alpha_0(9+2ht)g\ e^{-ht} + (18+3ht-3h^2t^2)g^2e^{-2ht} > 0\ . \qquad (2.2.73)
\end{aligned}
$$

In the case for vanishing α_0 we obtain $ht < 3$. In other cases the term α_0^2 is dominant for large t and (2.2.73) is true. For the small t, the values of a_{22}, a_{33} and a_{66} could be negative. However, this depends on the values of g and h.

In the second case (2.2.71), the conditions are the same (2.2.73), with the difference that $\alpha_0 < -(3+ht)ge^{-ht}$. It means all minors are positive for large t as in the previous case.

To conclude, stable minima do exist for orbifolds with one complex structure modulus. In addition, we see that there is a discrepancy between whether we integrate out the complex structure modulus or not. The reason for this discrepancy is that the complex structure modulus is not heavy and therefore is not allowed to be simply integrated out.

Finally, we give an example which falls into class (2.2.70) of the solutions. This example is Z_{6-II} on $(SU(2))^2 \times SU(3) \times G_2$. The superpotential is given by:

$$\begin{aligned} W &= -ie^{-\pi i/6} b_0 + b_2 + S\left(ie^{-\pi i/6} d_0 - d_2\right) \\ & \quad -U^3 \left[\frac{i}{\sqrt{3}} e^{-\pi i/3} a_0 + \frac{i}{\sqrt{3}} a_2 - S\left(\sqrt{3}\, e^{-2\pi i/6} c_0 + \frac{1}{\sqrt{3}} c_2\right)\right] + ge^{-hT} \ . \end{aligned} \tag{2.2.74}$$

We choose $b_0 = d_0 = c_0 = 0$ and $a_2 = -\frac{1}{2} a_0$. Further a_0, b_2, c_2 and d_2 should be positive. In this case, we obtain a superpotential of the form (2.2.66).

2.2.5 Orientifolds with three untwisted complex structure moduli

In the $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifold, we have three untwisted complex structure moduli U^i undetermined. In this case, all ω_{A_i} and ω_{B_i} survive, and the (primitive) 3-form flux takes the following form:

$$\frac{1}{(2\pi)^2 \alpha'}\, G_3 = \sum_{i=0}^{3} [\ A^i(S, U^i)\ \omega_{A_i} + B^i(S, U^i)\ \omega_{B_i}\ ] \ . \tag{2.2.75}$$

The corresponding superpotential (2.2.32) becomes:

$$W = \lambda\ [\ B_1^0(U^i) - i\ B_2^0(U^i)\ S\ ] + \sum_{i=1}^{3} g^i\ e^{-h^i T^i} \ . \tag{2.2.76}$$

The coefficients $B_{1,2}^0$ are determined by 16 integer valued flux quantum numbers. The supersymmetry conditions for this superpotential with seven moduli fields and 16 flux quantum numbers have been given in (2.2.47). However, it is very involved to solve them in a closed form. Therefore, we reduce the number of fields and parameters by setting two of the complex structure moduli equal to each other, e.g. $U^1 = U^2$. Then the superpotential is somewhat simpler and may be parameterized by eight integer valued fluxes α_j $(j = 0, \dots, 7)$ in the following way:

$$W = \alpha_0 + \alpha_1\ U^1 + \alpha_2\ U^3 + \alpha_3\ S + \alpha_4\ SU^1 + \alpha_5\ SU^3 + \alpha_6\ U^1 U^3 + \alpha_7\ SU^1 U^3 + \sum_{i=1}^{3} g^i e^{-h^i T^i} \ . \tag{2.2.77}$$

In this case, the effective Kähler potential is given by (cf. Section 2.1.2):

$$\kappa_4^2 K = -\ln(S + \bar{S}) - 2\ln(U^1 + \bar{U}^1) - \ln(U^3 + \bar{U}^3) - \sum_{i=1}^{3} \ln(T^i + \bar{T}^i) \ . \tag{2.2.78}$$

In order to determine the vacuum structure of this class of models we will first use the integrating out procedure for all three complex structure moduli, assuming that they are heavy compared to S and T^i. Again, the aim of this investigation is to see, whether there are stable supersymmetric vacua with positive definite mass matrix in S and T or not. Hence, we consider the two supersymmetry conditions $D_{U^{1,2}}W = 0$. Their solution becomes (for real U^i):

$$\begin{aligned} U^1 &= \frac{-2\ (\alpha_2+\alpha_5 S)\ \left(\alpha_0+\alpha_3\ S+\sum\limits_{i=1}^{3} g^i\ e^{-h^i T^i}\right)}{\alpha_0\alpha_6+\alpha_1(\alpha_2+\alpha_5 S)+S\big[(\alpha_2\alpha_4+\alpha_0\alpha_7+\alpha_4\alpha_5 S+\alpha_3(\alpha_6+\alpha_7 S)\big]+(\alpha_6+\alpha_7 S)\sum\limits_{j=1}^{3} g^j e^{-h^j T^j}}\ , \\ U^3 &= -\frac{\alpha_0+\alpha_3 S+\sum\limits_{i=1}^{3} g^i\ e^{-h^i T^i}}{\alpha_2+\alpha_5 S}\ . \end{aligned} \tag{2.2.79}$$

We can now insert this solution back into eq. (2.2.77). This way we derive the following effective superpotential:

$$\begin{aligned} W_{\text{eff}}(S,T^i) \;&=\; \Bigg\{2\Big(\alpha_0+\alpha_3 S+\sum_{i=1}^{3} g^i e^{-h^i T^i}\Big)\Big[-\alpha_1(\alpha_2+\alpha_5 S)+\alpha_0(\alpha_6+\alpha_7 S) \\ &\quad +iS(\alpha_2\alpha_4+\alpha_4\alpha_5 S-\alpha_3(\alpha_6+\alpha_7 S))+(\alpha_6+\alpha_7 S)\sum_{k=1}^{3} g^k e^{-h^k T^k}\Big]\Bigg\} \\ &\quad \times\Bigg\{\sum_{j=1}^{3} g^j e^{-h^j T^j}(\alpha_6+\alpha_7 S)+\alpha_0\alpha_6+\alpha_1(\alpha_2+\alpha_5 S) \\ &\quad +S\ \Big[\alpha_2\alpha_4+\alpha_0\alpha_7+\alpha_4\alpha_5 S+\alpha_3(\alpha_6+\alpha_7 S)\Big]\Bigg\}^{-1}\ . \end{aligned} \tag{2.2.80}$$

$$\tag{2.2.81}$$

The numerator is a polynomial of third degree in S and second degree in denominator.

To apply the analysis of [21] requires to compute the ratio $\frac{SW_{\text{eff}}(S)''}{W_{\text{eff}}(S)'}$ and to analyze, if its value is bigger than one. However, this analysis assumes a superpotential of the form $W_{\text{eff}}(S)+\sum\limits_{i=1}^{3} g^i e^{-h^i T^i}$. Obviously, our effective superpotential (2.2.80) is not of this form. This would only be achieved for a special choice of the coefficients α_i. The condition on α_i for the numerator and denominator be divisible without remainder is:

$$(\alpha_1+\alpha_4 S)\ (\alpha_2+\alpha_5 S)=0\ . \tag{2.2.82}$$

After inserting this condition the effective superpotential (2.2.80) becomes

$$W_{\text{eff}} = 2\ \left(\alpha_0+\alpha_3 S+\sum_{i=1}^{3} g^i e^{-h^i T^i}\right)\ . \tag{2.2.83}$$

This is again the already analyzed case of the previous section, in which there is no stable minimum.

2.2.6 Cubic superpotential

We consider the case of three complex structure moduli (U^i) and three Kähler moduli (T^i). To make the calculations simple, we assume $U := U^1 = U^2 = U^3$, $T := T^1 = T^2 = T^3$. The superpotential and the Kähler potential have the following form:

$$W = \alpha_0 + \alpha_1 U + \alpha_2 (U)^2 + \alpha_3 (U)^3 + S(\alpha_4 + \alpha_5 U + \alpha_6 (U)^2 + \alpha_7 (U)^3) + 3ge^{-hT} \ , \quad (2.2.84)$$

$$K = -\ln(S+\bar{S}) - 3\ln(U+\bar{U}) - 3\ln(T+\bar{T}) \ , \quad g, \alpha_i, h \in \mathbf{R} \ , \ h \text{ positive} \ . \quad (2.2.85)$$

We rewrite U and T using the real basis: $U = u + i\nu$, $T = t + i\tau$ and compute the supersymmetry conditions ($D_U W = D_T W = 0$) at the point of vanishing ν and τ:

$$\begin{aligned}
\alpha_0 &= ge^{-ht}(-3+2ht) + u(\alpha_5 s + \alpha_2 u + 2\alpha_6 su + 2\alpha_3 u^2 + 3\alpha_7 su^2)) \ , \\
\alpha_1 &= -\frac{1}{u} 3ghte^{-ht} - \alpha_5 s - u(2\alpha_2 + 2\alpha_6 s + 3\alpha_3 u + \alpha_7 su) \ , \\
\alpha_4 &= -\frac{1}{s}\left(ghte^{-ht} + su(\alpha_5 + \alpha_6 u + \alpha_7 u^2)\right) \ . \qquad (2.2.86)
\end{aligned}$$

As in the previous cases, we compute the scalar potential and its Hessian at the supersymmetric points. This means that we calculate the second derivatives of the potential and eliminate α_0, α_1, α_4 by using (2.2.86). It is irrelevant which of the parameters or fields are eliminated through (2.2.86). We choose this particular combination by the criterion of simplicity of the later analysis.

The Hessian should have positive eigenvalues at the minimum or equivalently, its upper-left submatrices should be positive definite. The determinants of the upper-left submatrices are of the form

$$\begin{aligned}
a_{11} &= a_{44} = \frac{1}{48st^3u^3}\Big(9u^4(4a_3^2 + 8\alpha_3\alpha_7 s + 7\alpha_7^2 s^2) \\
&\quad + u^3(24\alpha_2\alpha_3 + 24\alpha_2\alpha_7 s + 24\alpha_3\alpha_6 s + 60\alpha_6\alpha_7 s^2) \\
&\quad + u^2(4\alpha_2^2 + 8\alpha_2\alpha_6 s + 16\alpha_6^2 s^2 + 18\alpha_5\alpha_7 s^2) + 12\alpha_5\alpha_6 s^2 u + 3\alpha_5^2 s^2\Big) + \mathcal{O}\left(e^{-ht}\right) \ , \\
a_{22} &= a_{55} = \frac{(\alpha_5 + 2\alpha_6 u + 3\alpha_7 u^2)^2}{768t^4u^4} + \mathcal{O}\left(e^{-ht}\right) \ , \\
a_{33} &= \frac{g^2h^2e^{-2ht}(2+ht)(1+2ht)(\alpha_5 + u(2\alpha_6 + 3\alpha_7 u))^4}{8192st^9u^7} + \mathcal{O}\left(e^{-3ht}\right) \ , \\
a_{66} &= \frac{g^2h^3e^{-2ht}(3+2ht)(\alpha_5 + u(2\alpha_6 + 3\alpha_7 u))^4}{8192st^8u^7} + \mathcal{O}\left(e^{-3ht}\right) \ . \qquad (2.2.87)
\end{aligned}$$

For positive flux parameters $\alpha_2, \alpha_3, \alpha_5, \alpha_6, \alpha_7$ and in the region of large t ($t^{-4} >> e^{-ht}$), all sub-determinants are positive. So in the case of three complex structure moduli, there is some region for which there is a supersymmetric minimum.

Chapter 3

Non-perturbative effects in the presence of the fluxes

3.1 $D3$-Instantons in the presence of $G_{(2,1)}$-fluxes

This section is based on the material published in [25].

In this section, we discuss the question for which cases a non-perturbative superpotential from brane instantons is produced. As shown in [19], the necessary two fermionic zero modes for the instanton contribution will be present, if a divisor[1], wrapped by an $M5$-brane in the dual M-theory picture (dual to type IIB which one is considering), has holomorphic Euler characteristic

$$\chi = h_{(0,0)} - h_{(0,1)} + h_{(0,2)} - h_{(0,3)} = 1 \ . \tag{3.1.1}$$

We will reproduce the argumentation given by Witten in [19] for the formula (3.1.1). He starts with the observation that M-theory compactified on a CY-fourfold X produces several multiplets in three dimensions. They are chiral multiplets coming from the complex structure moduli fields, then chiral multiplets coming from the three-form $C_{(3)}$ integrated over three cycles in X and, finally, linear multiplets. The latter contain a real scalar coming from the Kähler form integrated over two-cycles in X and a gauge field coming from $C_{(3)}$ integrated over two cycles. In three dimensions, linear multiplets are dual to chiral multiplets. We know that in the limit of eleven-dimensional supergravity there is no superpotential in three dimensions. Since this limit is obtained from M-theory by scaling up the metric of X, all terms in the superpotential (if they exist) should depend on the Kähler moduli.

At first sight, a gauge field $A_{(1)}$ from the linear multiplet can not produce a superpotential, since its gauge invariant combination contains derivatives $F_{(2)} = dA_{(1)}$. In that case, the scalar field from the dual chiral multiplet would have derivative couplings $d\phi_D = *F_{(2)}$,

[1] A divisor is a formal linear combination of analytical hypersurfaces. An analytical hypersurface is given as a zero locus of a single holomorphic non-zero function. We assume that all our divisors are smooth.

too. However, terms without derivative couplings which are gauge invariant under the transformation $\phi_D \to \phi_D +$ constant could come from the certain kind of instanton, which looks like a magnetic monopole for the $F_{(2)}$-field. Such an instanton would have an interaction proportional to

$$e^{-i\gamma\phi_D} \ , \tag{3.1.2}$$

with γ proportional to the magnetic charge of the instanton.

The three dimensional gauge field $A_{(1)}$ is a mode of $C_{(3)}$, so the instanton is a magnetic source for $C_{(3)}$. In eleven-dimensional supergravity the magnetic source for the $C_{(3)}$ is a five-brane. It means that the non-perturbative contribution to the superpotential comes from the five-brane wrapping a six-cycle D in X, giving what looks like an instanton in three dimensions. The amplitude of such an instanton would be proportional to the volume of the six-manifold e^{V_D}. Combined with the term (3.1.2) the amplitude would be proportional to $e^{V_D+i\phi_D}$.

Let z be the local coordinate of the normal direction of D in X. Then one can argue that the $U(1)$-transformation $z \to e^{i\theta}z$ would be a symmetry for the M-theory action. Let W be the corresponding symmetry generator. Witten analyzed the anomaly of $W$and found that it is given by the alternating sum of Hodge numbers $h_{(0,k)}$ of D

$$\Delta W = \sum_{n=0}^{3}(-1)^n h_{(0,n)} \ , \tag{3.1.3}$$

where $h_{(0,0)}$ and $h_{(0,2)}$ correspond to the number of fermionic zero modes with positive and $h_{(0,1)}$ and $h_{(0,3)}$ to the number with negative chirality with respect to the normal bundle of D inside X. This sum is known as arithmetic genus or holomorphic characteristic $\chi(D, \mathcal{O}_D)$ of D.

In the compactified theory, the superpotential should be anomaly free. The factor $e^{(-V_D+i\phi_D)}$ in the superpotential according to Witten carries charge $-\chi(D, \mathcal{O}_D)$. For the case of $h_{(0,1)} = h_{(0,2)} = h_{(0,3)} = 0$ the holomorphic characteristic χ would be 1, so the measure $d^2\theta$ of the superpotential should be 1. Since the measure has always the same charge, the necessary criterion for anomaly cancellation and generation of the superpotential is $\chi = 1$ or equivalently two[2] unpaired fermionic zero modes.

From the above discussion we see that the requirement of two fermionic zero modes is only a necessary but not a sufficient one for the non-vanishing non-perturbative superpotential. To have the full guarantee one would need to calculate the one loop determinant $g(U)$ of the instanton,

$$W_{np} = g(U)\ e^{V_D+i\phi} \ . \tag{3.1.4}$$

The prefactor with the one loop determinant depends on the complex structure moduli and is in general not known. It is important to mention that in the case of $h_{(0,1)} = h_{(0,2)} = h_{(0,3)} = 0$ the index criterion $\chi = 1$ is a sufficient condition.

[2]In the formula (3.1.3) we see only one unpaired mode. Since the fermions have an additional index corresponding to the transformations in $\mathbf{R}^3$, their number is actually doubled.

In chapter 4, we will discuss the stabilization of all moduli fields in the blown up orbifolds. Since our models are type IIB, the application of Witten's result would demand an F-theory[3] lift. On the other hand Witten's result is derived in the case without fluxes and that is why it is not directly applicable. In the last two years, there was some work [51, 52, 53, 54, 55] on the generalization of the above index criterion first to the case with fluxes and then to the case of type IIB theory. The authors of these papers took a different path. Since the presence of the instantons was reduced to the statement of having two fermionic zero modes on the world-volume of the Euclidean $D3$-brane, it was sufficient to analyze the Dirac equation on the $D3$-brane.

During the rest of this section we build up on the results of [54]. The authors of this paper performed fermionic zero mode counting for some special models. Using the fact that Dirac equations which one uses for the counting procedure are the same in all coordinate patches, we can generalize their results.

3.1.1 Index for the type IIB case

To calculate the number of the zero modes, we have to realize what are the possible 4-cycles wrapped by the $D3$-branes in the compact space. The Hodge numbers $h_{(0,0)}, h_{(0,1)}, h_{(0,2)}$ of the 4-cycle give the number of the zero modes with positive (N_+) and negative (N_-) chirality with respect to the normal bundle of the $D3$-brane. If one takes into account background fluxes, orientifold action and fixing of the κ-symmetry, some of the zero modes could be lifted and the index

$$\chi_{D3} = \frac{1}{2}\left(N_+ - N_-\right)$$

will change. χ_{D3} is not anymore of purely geometrical nature. In the case of type IIB, Bergshoeff et al. [54] showed that only $h_{(0,1)}$ and $h_{(0,2)}$ of N_+ can be lifted by fluxes. Thus, if the topology of the divisor has vanishing $h_{(0,1)}$, $h_{(0,2)}$, we can neglect the effect of the fluxes altogether and concentrate only on the action of the O-planes on the zero mode counting.

The correspondence between zero modes of the Dirac operator on the worldvolume of the 4-cycles and Hodge numbers $h_{(0,0)}, h_{(0,1)}, h_{(0,2)}$ of these cycles becomes apparent by mapping the spinors to $(0,p)$-differential forms.[4] Then fermionic zero modes of the Dirac operator correspond to the harmonic forms by above mapping. Locally we can write the world-volume spinors on the $D3$-brane as

$$\begin{aligned} \epsilon_+ &= \phi|\Omega> + \phi_{\bar{a}}\gamma^{\bar{a}}|\Omega> + \phi_{\overline{ab}}\gamma^{\overline{ab}}|\Omega> , \\ \epsilon_- &= \phi_{\bar{z}}\gamma^{\bar{z}}|\Omega> + \phi_{\overline{az}}\gamma^{\overline{az}}|\Omega> + \phi_{\overline{abz}}\gamma^{\overline{abz}}|\Omega> , \end{aligned} \tag{3.1.5}$$

where ϵ_+ and ϵ_- are states with positive and negative chirality with respect to the normal

[3] The index discussed above is applicable to M-theory. The connection to type IIB theory can be made over F-theory. The fourfold X should be elliptically fibered, with a three-fold as base and two-tori as fibers. For the details see section 3 of [19].

[4] $\phi_{a_1\dots a_N}\gamma^{a_1\dots a_N}|\Omega> \longleftrightarrow \phi_{a_1\dots a_N} dz^{a_1}\dots dz^{a_N}$

bundle $SO(2)$ of the $D3$-brane inside the compact space. a, b are the $D3$-brane worldvolume directions, z is the normal direction to the worldvolume.

Note that ϵ_+ and ϵ_- transform under $SO(4) \times SO(2) \times SO(1,3)$ and the modes ϕ have an additional spinor index which transforms in the $\mathbf{2} \oplus \bar{\mathbf{2}}$ under $SO(1,3)$. Thus, the number of the zero modes given by the Hodge numbers of the 4-cycle has to be doubled.

All modes of ϵ_- have legs in the normal direction to the $D3$-brane. By use of Serre's generalization of the Poincaré duality, these modes can be mapped to those taking values in the worldvolume of $D3$-brane. This duality maps $(0,p)$-forms with values in the bundle $\Omega^{0,p}(X)$ of the 4-cycle X to $(0, 2-p)$-forms with values in $\Omega^{(0,2-p)} \otimes K$, where K is the canonical bundle of the 4-cycle. In the case of the wrapped $D3$-brane, the canonical bundle is equal to the normal bundle, so this duality is realized by multiplying by the covariantly constant 3-form Ω_{abc} and building the Hodge[5] dual.

$$
\begin{aligned}
g^{z\bar{z}}\Omega_{\overline{abz}}\phi_z &= \widetilde{\phi}_{\overline{ab}} \ , \\
g^{z\bar{z}}g^{a\bar{a}}\Omega_{\overline{abz}}\phi_{az} &= \widetilde{\phi}_{\overline{a}} \ , \\
g^{z\bar{z}}g^{a\bar{a}}g^{b\bar{b}}\Omega_{\overline{abz}}\phi_{abz} &= \widetilde{\phi} \ .
\end{aligned} \tag{3.1.7}
$$

This means that the numbers of the modes with positive and negative chirality match. If all zero modes are present, the corresponding index

$$
\chi_{D3} = \frac{1}{2}\ (N_+ - N_-) = \left(\ h^{(+)}_{(0,0)} + h^{(+)}_{(0,1)} + h^{(+)}_{(0,2)}\ \right) - \left(\ h^{(-)}_{(0,0)} + h^{(-)}_{(0,1)} + h^{(-)}_{(0,2)}\ \right) \tag{3.1.8}
$$

will be 0.

3.1.2 Calculation of χ_{D3} for divisors with $h_{(0,0)} = 1$, $h_{(0,1)} = h_{(0,2)} = 0$

As we shall see in the next section, many of the divisors arising in resolved toroidal orbifold models have the Hodge numbers $h_{(0,0)} = 1$, $h_{(0,1)} = h_{(0,2)} = 0$. We will therefore start by calculating the number of zero modes for this especially simple case. We choose $a, \bar{a}, b, \bar{b}$ as holomorphic and antiholomorphic coordinates on the $D3$-brane, which take the values $1, \bar{1}, 2, \bar{2}$. z and $\bar{z}$ should correspond to the transverse directions with values $3, \bar{3}$.

The fermionic states on the $D3$-brane corresponding to $h_{(0,0)}$ are

$$
\epsilon_+ = \phi|\Omega> \ , \qquad \epsilon_- = \phi_{\overline{abz}}\gamma^{\overline{abz}}|\Omega> \ . \tag{3.1.9}
$$

On the brane, some of the modes are pure gauge due to the κ-symmetry. These are the modes which are annihilated by the κ-symmetry projector $(1 - \Gamma_{D3})\theta = 0$, where $\Gamma_{D3} = \sigma_2 \otimes \gamma_5$ with γ_5 four ten dimensional γ-matrices pulled back on the brane. θ

[5]
$$
\begin{aligned}
*\Big(\phi_{\overline{a}_1\ldots\overline{a}_{N-p}}\Omega_{a_1\ldots a_N}dz^{\overline{a}_1} \ \ \ldots \ \ dz^{\overline{a}_{N-p}}dz^{a_1}\ldots dz^{a_N}\Big) \\
= \ \ \epsilon_{\overline{a}_1\ldots\overline{a}_N}\epsilon_{a_1\ldots a_N}\phi^{\overline{a}_1\ldots\overline{a}_{N-p}}\Omega^{a_1\ldots a_N}\ dz^{\overline{N-p+1}}\ldots dz^{\overline{N}}
\end{aligned} \tag{3.1.6}
$$

Note, that in our convention the form is complex conjugated by applying the Hodge star.

corresponds to two 32-component spinors written in the double spinor formalism [56]. Additionally, some of the modes can be projected out by the orientifold action. We have to choose κ-symmetry fixing in such a way that it commutes with the orientifold action [54].

There are three different cases to distinguish for the position of the $O7$-plane: it can be on top of the D3-brane, can intersect it along one direction, or can be parallel to it. We assume that the $O7$-plane fills the non-compact directions.

- Case 1: an O-plane lies on top of a D3-brane

It is convenient to do the calculations in the local coordinate patch. The κ-symmetry fixing condition and the projection through the orientifold action are given by

$$\begin{aligned}(1-\sigma_2\gamma^{1\bar{1}2\bar{2}})\theta &= 0\ ,\\ (1-\sigma_2\gamma^{3\bar{3}})\theta &= 0\ .\end{aligned} \tag{3.1.10}$$

Both conditions written together yield

$$(1-\gamma^{1\bar{1}2\bar{2}3\bar{3}})\theta = 0\ . \tag{3.1.11}$$

Inserting $\theta = \epsilon_+ + \epsilon_-$ shows that ϕ survives this projection and $\phi_{\overline{abz}}$ not. The index is $\chi_{D3} = h_{(0,0)} = 1$.

- Case 2: Intersection with an O-plane along one complex dimension

The O-plane intersects the D-brane along complex direction 1. Then, κ-symmetry fixing condition and the projection through the orientifold action are given by

$$\begin{aligned}(1-\sigma_2\gamma^{1\bar{1}2\bar{2}})\theta &= 0\ ,\\ (1-\sigma_2\gamma^{2\bar{2}})\theta &= 0\ .\end{aligned} \tag{3.1.12}$$

Both conditions written together give

$$(1-\gamma^{1\bar{1}})\theta = 0\ . \tag{3.1.13}$$

ϕ survives this projection, $\phi_{\overline{abz}}$ not. From this it follows $\chi_{D3} = h_{(0,0)} = 1$.

- Case 3: No intersection with an O-plane

It can be the case, when the O-plane is parallel to the $D3$-brane. The orientifold action maps fermions of the brane to the fermions in the mirror brane, so no modes are projected out. There is only the κ-symmetry fixing condition, by which no modes are cut. The modes $\phi|\Omega>$, $\phi_{\overline{abc}}\gamma^{\overline{abc}}|\Omega>$ corresponding to $h_{(0,0)}$ are present and the index is $\chi_{D3} = h_{(0,0)} - h_{(0,0)} = 0$.

By investigating all configurations of the $O7$-plane we obtain a general statement:

Divisors with Hodge numbers $h_{(0,0)} = 1$, $h_{(0,1)} = h_{(0,2)} = 0$ will have the index $\chi_{D3} = 1$ if an $O7$-plane lies on top of them or if it intersects the divisor along one complex dimension. Otherwise, $\chi_{D3} = 0$.

3.1.3 General case: $h_{(0,1)}$, $h_{(0,2)} \neq 0$

As discussed in the last subsection, locally, there are always only three different configurations of the $O7$-plane relative to the divisor in question. It can be on top of it, intersect it in one complex direction, or be parallel to it. a, b are again the coordinates on the $D3$-brane. The projector equations from the fixing of the κ-symmetry and the orientifold action will be as in the previous subsection. The only difference is that the modes $\phi_{\overline{a}}|\Omega>$ and $\phi_{\overline{ab}}\gamma^{\overline{ab}}|\Omega>$ are now present. They can be lifted by fluxes. When turning on fluxes, we assume that they will be of the the most unfavorable form for the presence of zero modes. This would correspond to a general form for the fluxes. We summarize the results of the action of the projector equations in all three cases in the following Table:

	O-plane on top of D3		O-plane intersects D3		O-Plane does not intersect D3	
chirality	+	-	+	-	+	-
$h_{(0,0)}$	ϕ		ϕ		ϕ	$\phi_{\overline{abz}}$
$h_{(0,1)}$		$\phi_{\overline{az}}$	$[\phi_{\overline{a}}]$	$\phi_{\overline{az}}$	$[\phi_{\overline{a}}]$	$\phi_{\overline{az}}$
$h_{(0,2)}$	$[\phi_{\overline{ab}}]$			$\phi_{\overline{z}}$	$[\phi_{\overline{ab}}]$	$\phi_{\overline{z}}$
# of zero modes	$2 - 2h^{(-)}_{(0,1)} + 2[h^{(+)}_{(0,2)}]$		$2 - 2h^{(-)}_{(0,1)} - 2h^{(-)}_{(0,2)} + 2[h^{(+)}_{(0,1)}]$		$2[h^{(+)}_{(0,1)}] + 2[h^{(+)}_{(0,2)}] - 2h^{(-)}_{(0,1)} - 2h^{(-)}_{(0,2)}$	

Table 3.1: Zero modes after fixing κ-symmetry and orientifold projection

In the horizontal line we give the zero modes associated to the Hodge numbers $h_{(0,0)}$, $h_{(0,1)}$, $h_{(0,2)}$. '+' and '-' denote the chirality with respect to the normal bundle of the $D3$-brane. In brackets we put the modes which are in general lifted in the presence of fluxes, and in the last line we give the number of zero modes which are left.

Let us discuss this result first before turning on flux. The first column represents the case where the influence of the orientifold projection is fully felt by the divisor in question. As in the M-theory case discussed by Witten, only one chirality survives for each Hodge number and the index reduces again to the holomorphic Euler characteristic $h_{(0,0)} - h_{(0,1)} + h_{(0,2)}$. The third column corresponds to the case where the influence of the orientifold is not felt at all by the wrapped divisor. Both chiralities survive and cancel each other out. This agrees with the observation that for a compactification on a CY manifold (without orientifold projection), no non-perturbative superpotential is generated. The second column represents an intermediate case. It is obvious that the knowledge of the Hodge numbers is of prime importance to be able to decide whether a divisor contributes to the non-perturbative superpotential. We can see that without turning on flux, we can get a contribution in the first column for $h_{(0,1)} = h_{(0,2)}$. With flux, a contribution is only possible for $h_{(0,1)} = 0$. We get a contribution from the second column for $h_{(0,2)} = 0$ if no flux is turned on, with flux only for $h_{(0,1)} = h_{(0,2)} = 0$. With or without flux, column three never gives a contribution since the number of zero modes is always less than or equal to zero.

In the present work we do not discuss the counting of zero modes for the case with non-vanishing 2-form flux f on the D-brane world-volume Recently, work towards this direction has been accomplished in ref. [57] for the case of heterotic M-theory. The authors have found that world-volume flux does not change the zero mode counting for the case of some particular background fluxes. Those fluxes were chosen such that they do not lift any zero modes. For the case of IIB, Bandos and Sorokin derived the Dirac equation for the $D3$-brane in the presence of worldvolume flux [58]. Its implication for the zero mode counting has to be analyzed [59]. Compared to the case without 2-form flux, there are more complicated conditions on the gauge fixing of the κ-symmetry and an additional field equation for the 2-form flux, which depend on the topology of the Calabi-Yau manifold.

We finish this section by the remark that the formalism described above requires the wrapped 4-cycle to be Kähler. In the models of chapter 4 it is the case because we have to deal with divisors which are hyperplanes in the Calabi-Yau manifolds and hyperplanes of a Kähler manifold are Kähler. This can be also directly observed from the topology of the divisors, which are either products or fibrations of tori and $\mathbb{P}^1$s.

3.2 $D3$-Instantons in the presence of all ISD- and IASD-fluxes

This section is based on the material published in [60].

Recently, there has been a lot of progress in the investigation of KKLT-type models [61]. On the one hand, specific examples of candidate models have been constructed [62, 63]. On the other hand, the generation of a non-perturbative superpotential which may serve to stabilize all Kähler moduli has been investigated in much detail. As we explained in the first part of this chapter the recent research in this line extends the earlier work of Witten [19] by taking into account non-vanishing background fluxes [51, 64, 52, 53, 56, 65] and working out the conditions for the generation of the superpotential directly for type IIB-orientifolds without the detour of analyzing the M/F-theory case first [54, 55, 66]. If $M5/D3$-brane instantons wrapping a divisor in the compactification manifold are the source of a possible non-perturbative superpotential, the analysis involves deriving the Dirac equation in the world-volume of the $M5/D3$-brane and studying the structure of its fermionic zero modes. So far, only the case of the background flux being of Hodge type $(2,2)$ in M/F-theory, or $(2,1)$ in type IIB-theory has been considered.

The present section resolves a seeming puzzle concerning the fermionic zero mode structure in the presence of background fluxes of general Hodge type. As has been shown in [48, 24], the conditions for a supersymmetric background flux obtained from the minimization of the effective four-dimensional superpotential change in the presence of a non-perturbative term. The supersymmetric flux is no longer of Hodge type $(2,2)$ (resp. $(2,1)$ for type IIB), but receives contributions of all Hodge types. We will show that, if one now, guided by this result, plugs a flux of general Hodge type into the zero mode conditions obtained from the Dirac equation, an inconsistency arises: If with $(2,2)$-flux, the conditions for the generation of a superpotential were met, this is no longer the case for general flux.

As we explain in the following, this apparent mismatch disappears after the introduction of a modification of the supersymmetry variation of the modulino, which basically captures the back-reaction of the non-perturbative effects on the background flux and the geometry.

3.2.1 Effective Potential

We first consider the compactification of type IIB theory on a Calabi-Yau threefold. The resulting low energy supergravity action is given by

$$S = \int d^4x \frac{1}{2}\sqrt{-g}\left\{R + g_{A\overline{B}}\partial_\mu z^A \partial^\mu \overline{z}^B\right\} + V_{\text{eff}} + S_{\text{gauge}} \ . \tag{3.2.14}$$

Here, we used a condensed notation: The indices $\{A, B, \cdots\} = \{i, I, \tau\}$ denote both the complex structure moduli $\{i\}$, Kähler moduli $\{I\}$, and the complexified axion-dilaton field τ. S_{gauge} denotes the gauge field dependent part of the action. The effective potential

$$V_{\text{eff}} = \frac{1}{2}e^K\left(g^{AB}D_A W\overline{D_B W} - 3|W|^2\right) \tag{3.2.15}$$

is given in terms of the total superpotential

$$W = W_{flux} + W_{np} \tag{3.2.16}$$

and the Kähler potential K. Here W_{flux} is the flux superpotential [15]

$$W_{flux} = \int G_3 \wedge \Omega_3 \ , \tag{3.2.17}$$

and W_{np} is the superpotential arising from nonperturbative effects. Ω_3 is the holomorphic $(3,0)$-form on the CY space and

$$G_3 = F_3 - \tau H_3 \ , \tag{3.2.18}$$

F_3 and H_3 being the RR and NS field strengths, respectively. The flux superpotential depends only on the complex structure moduli. We assume the nonperturbative superpotential to depend on the Kähler moduli only.

The supersymmetry preserving minima are obtained by solving the equations

$$D_A W = 0 \ . \tag{3.2.19}$$

It is well known that in the absence of a nonperturbative term, $W = W_{flux}$, the condition (3.2.19) requires G_3 to be of type $(2,1)$ and primitive [13]. For $W_{np} \neq 0$, this is no longer true [24], and G_3 acquires non-vanishing $(1,2), (3,0)$ and $(0,3)$ parts:

$$\int G_3 \wedge \chi_i^{(2,1)} + \partial_i K W_{np} = 0 \ ,$$

$$\int G_3 \wedge \Omega_3 \; \partial_I K + D_I W_{np} = 0 \; ,$$
$$\int \overline{G}_3 \wedge \Omega_3 + W_{np} = 0 \; . \tag{3.2.20}$$

The primitivity condition $G_3 \wedge J = 0$, being a D-term condition, remains intact despite W_{np}. Here $\chi_i^{(2,1)}$ is a form of type $(2,1)$.

We can similarly obtain the supersymmetric conditions for M-theory compactification on a Calabi-Yau fourfold. The flux superpotential is now given by [14]

$$W_{flux} = \int G_4 \wedge \Omega_4 \; . \tag{3.2.21}$$

Here, G_4 is the four-form flux present in 11-dim. supergravity theory and Ω_4 is the holomorphic $(4,0)$-form on the CY fourfold. The supersymmetric conditions take the form:

$$\int G_4 \wedge \chi_i^{(3,1)} + \partial_i K W_{np} = 0 \; ,$$
$$\int G_4 \wedge \Omega_4 \; \partial_I K + D_I W_{np} = 0 \; . \tag{3.2.22}$$

In the following subsection, we will show how the above conditions can be derived from the modulino variations.

3.2.2 Spinor Conditions

Now, it is important to remember that the BPS supersymmetric variation of the gravitino is equivalent to solving the supersymmetric conditions in the effective field theory, as discussed in [67] for M-theory on a fourfold, in [68] and in [69] for type IIB on a CY threefold, and also by [70] for the heterotic string. Thus we must modify the spinor conditions accordingly in order to obtain the supersymmetric conditions eq. (3.2.20) in IIB theory and eq. (3.2.22) in M-theory. In what follows, we will first review the spinor conditions in the absence of W_{np}, and then consider the generalization when W_{np} is included.

Let us first consider the situation in IIB theory. This has been worked out in [14]. The supersymmetry variations can be summarized as follows:

$$\begin{aligned}
\kappa\delta\psi_\mu &= \partial_\mu\epsilon - \frac{1}{8}\gamma_\mu\gamma^m\left(\partial_m \ln Z - 4\kappa Z\Gamma^4\partial_m h\right)\epsilon + \frac{1}{16}\kappa\gamma_\mu G\epsilon^*, \\
\kappa\delta\psi_m &= \left(\widetilde{D}_m - \frac{i}{2}Q_m\right)\epsilon + \frac{1}{8}\epsilon\partial_m \ln Z - \frac{1}{16}\kappa\gamma_m G\epsilon^* - \frac{1}{8}\kappa G\gamma_m\epsilon^*, \\
\kappa\delta\lambda^* &= -i\gamma^m P_m^*\epsilon + \frac{i}{4}\kappa\overline{G}\epsilon^*.
\end{aligned} \tag{3.2.23}$$

The first equation is the supersymmetry variation of the four-dimensional gravitino field. Second, $\delta\psi_m$ corresponds to the variation of the internal gravitino. After compactification the internal gravitino degrees of freedom become in the effective 4D field theory the modulino fields, i.e. the fermionic superpartners of the Kähler and complex structure moduli fields. Concretely, the modulino equations which one obtains by dimensional reduction (see appendix) are

$$\begin{aligned}
\delta\phi^i_{\overline{eab}} &= -\frac{1}{8}G^i_{\overline{eab}}\widehat{\xi}^* - \frac{1}{16}g^{a\overline{c}}g_{e\overline{a}}G^i_{\overline{abc}}\widehat{\xi}^* , && i=1,\dots,h^{(2,1)} ,\\
\delta\phi^I_{\overline{e}|\overline{ab}} &= -\frac{1}{16}G_{\overline{eab}}\widehat{\xi}^* , && I=1,\dots,h^{(1,1)} ,\\
\delta\lambda^*_{\overline{abc}} &= \frac{i}{4}\overline{G}_{\overline{abc}}\widehat{\xi}^* ,
\end{aligned} \tag{3.2.24}$$

where $\widehat{\xi}$ is a four dimensional supersymmetry parameter. Finally, $\delta\psi_m$ indeed comprises the supersymmetry variations of all modulinos, namely it leads after compactification to $h_{1,1}+h_{2,1}$ independent spinor equations, which we call modulino equations. Finally, $\delta\lambda^*$ is the supersymmetry variation of the four-dimensional dilatino. In these equations, we use the same notation as [69]. In particular, $G=\frac{1}{6}G_{mnp}\gamma^{mnp}$, Z is the warp factor, $\widetilde{D}_m$ is the covariant derivative with respect to the internal metric, h is related to the RR four-form field, $h=C_{0123}$, and

$$\begin{aligned}
&P_m = f^2\partial_m B\ , \quad Q_m = f^2 \mathrm{Im}\left(B\partial_m B^*\right)\ ,\\
&B = \frac{1+i\tau}{1-i\tau}\ , \quad f^{-2} = 1-BB^*.
\end{aligned} \tag{3.2.25}$$

The conditions (3.2.23) can be solved to show that G_3 is of type $(2,1)$ and primitive.

Clearly, the explicit dependence on the superpotential W_{flux} and its covariant derivatives is not apparent in the modulino variations (3.2.24). We need to make this precise, in order to generalize the above formulae in presence of W_{np}. Since we are interested in the G_3 dependence of the variations, we can as well ignore the effects of warping and the five-form flux, and also set the complexified axion-dilaton field to constant.

It is now easy to introduce the flux superpotential in the above equations. Note that

$$\begin{aligned}
&D_iW_{flux} = \int G_3\wedge\chi_i^{2,1} \Longrightarrow G^i_{\overline{abc}} = \epsilon_{\overline{abc}}D_iW_{flux}\ ,\\
&D_IW_{flux} = \partial_I K\int G_3\wedge\Omega_3 \Longrightarrow G_{\overline{abc}} = \epsilon_{\overline{abc}}\frac{D_IW_{flux}}{\partial_I K}\ .
\end{aligned} \tag{3.2.26}$$

Substituting the above into the modulino variations, we find

$$\begin{aligned}
\delta\phi^i_{\overline{eab}} &= -\frac{1}{8}\epsilon_{\overline{eab}}D_iW_{flux} - \frac{1}{16}g^{a\overline{c}}g_{e\overline{a}}G^i_{\overline{abc}}\widehat{\xi}\ , && i=1\dots h^{(2,1)}\ ,\\
\delta\phi^I_{\overline{e}|\overline{ab}} &= -\frac{1}{16}\epsilon_{\overline{eab}}\frac{D_IW_{flux}}{\partial_I K}\ , && I=1\dots h^{(1,1)}\ .
\end{aligned} \tag{3.2.27}$$

Similarly, using

$$\overline{G}_{\overline{abc}} = -\epsilon_{\overline{abc}}(\tau-\overline{\tau})D_\tau W_{flux}\ , \tag{3.2.28}$$

we find

$$\delta\lambda^*_{\overline{abc}} = -\frac{i}{4}\epsilon_{\overline{abc}}(\tau-\overline{\tau})D_\tau W_{flux}\widehat{\xi}\ . \tag{3.2.29}$$

For covariantly constant spinors, we recover the susy conditions

$$D_i W_{flux} = D_I W_{flux} = D_\tau W_{flux} = 0 \ . \tag{3.2.30}$$

Now, it is easy to generalize the spinor variations in presence of the non-perturbative superpotential. We simply replace W_{flux} by $W = W_{flux} + W_{np}$. The variation equations then become

$$\begin{aligned}
\delta\phi^i_{e\overline{ab}} &= -\frac{1}{8}\epsilon_{\overline{ebc}} D_i W - \frac{1}{16} g^{a\overline{c}} g_{e\overline{a}} G^i_{\overline{abc}} \widehat{\xi} \ , \\
\delta\phi^I_{\overline{e}|\overline{ab}} &= -\frac{1}{16}\epsilon_{\overline{eab}} \frac{D_I W}{\partial_I K} \ , \\
\delta\lambda^*_{\overline{abc}} &= -\frac{i}{4}\epsilon_{\overline{abc}} (\tau - \overline{\tau}) D_\tau W \ .
\end{aligned} \tag{3.2.31}$$

We clearly see that, for covariantly constant spinors, the first of the above equations implies the flux to be primitive and in addition $D_i W$ is zero. The second and third equations then imply that $D_I W$ and $D_\tau W$ are zero respectively. Thus we recover the susy conditions

$$D_i W = D_I W = D_\tau W = 0 \ . \tag{3.2.32}$$

We now proceed to work out the modulino transformations in M-theory in presence of W_{np} in a similar fashion. This has been analyzed in [67]. We will first express the variation equations in terms of the flux superpotential, and then generalize it to the case of $W_{np} \neq 0$. Consider first the internal gravitino variation without W_{np}:

$$\delta\psi_m = \nabla_m \xi + \frac{1}{24}\gamma^{npq} G_{mnpq} \xi \ . \tag{3.2.33}$$

By dimensional reduction we obtain (see appendix)

$$\begin{aligned}
\delta\phi^k_{e\overline{c}} &= \frac{1}{4}\left(G_{eb\overline{cd}} g^{b\overline{d}}\right)^k \widehat{\xi} \ , && k = 1, \ldots, h^{(1,1)} \ , \\
\delta\phi^i_{\overline{eabc}} &= \frac{1}{24} G^I_{\overline{eabc}} \widehat{\xi} \ , && i = 1, \ldots, h^{(3,1)} \ , \\
\delta\phi^I_{\overline{e}|\overline{abc}} &= \frac{1}{24} G_{\overline{eabc}} \widehat{\xi} \ , && I = 1, \ldots, h^{(1,1)} \ .
\end{aligned} \tag{3.2.34}$$

By solving the susy conditions, we get in general $h_{3,1}$ equations for the complex structure moduli and $h_{1,1}$ equations for the Kähler moduli. The same conditions should be reproduced by setting $\delta\phi^i$ and $\delta\phi^I$ to zero. There are $h^{3,1}$ fluxes of type $(1,3)$. The $(0,4)$-flux is a solution of $h^{1,1}$ independent equations. Because of these reasons, it is natural to say that for every $G_{a\overline{bcd}}$ and every $G_{\overline{abcd}}$ (same $G_{\overline{abcd}}$ coming from $h^{1,1}$ equations), the variation of the gravitino should be zero.

There is no I on the r.h.s. This emphasizes the fact that the $h^{1,1}$ supersymmetry conditions are degenerate in the $(0,4)$-flux. Using

$$D_i W_{flux} = \int G_4 \wedge \chi^i_{3,1} = G^i_{e\overline{bcd}} \epsilon^{\overline{ebcd}} \tag{3.2.35}$$

and

$$D_I W_{flux} = \partial_I K \int G_4 \wedge \Omega_4 = \partial_I K G_{\overline{ebcd}} \epsilon^{\overline{ebcd}} \ , \tag{3.2.36}$$

we can immediately rewrite (3.2.34) into

$$\begin{aligned}
\delta\phi^k_{e\bar{c}} &= \frac{1}{4}\left(G_{eb\overline{cd}} g^{b\overline{d}}\right)^k \widehat{\xi} \ , & k &= 1, \ldots, h^{(1,1)} \ , \\
\delta\phi^i_{e\overline{abc}} &= \frac{1}{24}\epsilon_{e\overline{abc}} D_i W_{flux} \widehat{\xi} \ , & i &= 1, \ldots, h^{(3,1)} \ , \\
\delta\phi^I_{\overline{e}|\overline{abc}} &= \frac{1}{24}\epsilon_{\overline{eabc}} \frac{D_I W_{flux}}{\partial_I K} \widehat{\xi} \ , & I &= 1, \ldots, h^{(1,1)} \ .
\end{aligned} \tag{3.2.37}$$

The supersymmetry conditions and the primitivity condition are reproduced by setting $\delta\phi^k$, $\delta\phi^i$, $\delta\phi^I$ to zero.

This gives immediately

$$\begin{aligned}
& g^{a\bar{d}} g^{b\bar{c}} G_{eb\overline{cd}} = 0 \ , \\
& D_i W_{flux} = 0 \ , \ i = 1, \ldots, h^{1,3} \ , \\
& D_I W_{flux} = 0 \ , \ I = 1, \ldots, h^{1,1} \ .
\end{aligned} \tag{3.2.38}$$

These equations correspond to the primitivity conditions on $G_{2,2}$ and the vanishing of $G_{1,3}$ and $G_{0,4}$.

In the next step, we would like to make a proposal for the form of the additional terms of the supersymmetry variation of the modulinos in the presence of the non-perturbative term W_{np}. The supersymmetry conditions which should be reproduced, change to

$$\begin{aligned}
& D_i W = D_i W_{flux} + D_i W_{np} = 0 \ , \\
& D_I W = D_I W_{flux} + D_I W_{np} = 0 \ .
\end{aligned} \tag{3.2.39}$$

From (3.2.37), we immediately see that the variation of the modulinos should be changed to

$$\begin{aligned}
\delta\phi^k_{e\bar{c}} &= \frac{1}{4}\left(G_{eb\overline{cd}} g^{b\overline{d}}\right)^k \widehat{\xi} \ , & k &= 1, \ldots, h^{(1,1)} \ , \\
\delta\phi^i_{e\overline{abc}} &= \frac{1}{24}\epsilon_{e\overline{abc}} D_i W \widehat{\xi} \ , & i &= 1, \ldots, h^{(3,1)} \ , \\
\delta\phi^I_{\overline{e}|\overline{abc}} &= \frac{1}{24}\epsilon_{\overline{eabc}} \frac{D_I W}{\partial_I K} \widehat{\xi} \ , & I &= 1, \ldots, h^{(1,1)} \ .
\end{aligned} \tag{3.2.40}$$

3.2.3 Zero modes from fluxes and non-perturbative superpotential

The non-perturbative superpotential may be generated via gaugino condensation or via instanton effects or both. Here, we will concentrate on the case of instantons. In type IIB theory, they correspond to Euclidean $D3$-branes wrapping divisors of the CY threefold, whereas in M-theory, they come from Euclidean $M5$-branes wrapping divisors of the CY fourfold. It has been pointed out by Witten [19] some time ago that the necessary condition for an $M5$-instanton to generate a superpotential is that the corresponding divisor has holomorphic Euler characteristic equal to one. This provides a stringent condition on the possible CY fourfolds [71]. For type IIB compactification on a Calabi-Yau without the orientifold projection (without flux), the index is always zero and hence no superpotential is generated due to instanton effects [54]. It has been argued recently [72], that the index might change in the presence of flux. An explicit example has been constructed to show that some of the wold-volume fermion zero modes are lifted due to flux [64]. Subsequently, a generalized index formula was derived in M-theory [52, 53], as well as in type IIB theory [54]. However, these results are based on the assumption that the flux is primitive and of type $(2,1)$ in type IIB, or $(2,2)$ respectively in M-theory. As we have already discussed, the supersymmetric flux no longer remains $(2,1)$ (resp. $(2,2)$) in presence of the non-perturbative superpotential. In this section, we will analyze the fermion zero modes on the world-volume of $D3/M5$-branes in the presence of general flux.

3.2.4 General fluxes

The fermionic bilinear terms in the $D3$-brane world-volume action in presence of background flux have been derived in [73, 64] by using the method of gauge completion, and also in [74, 75, 56] from the $M2$-brane world-volume action using T-duality. Upon Euclidean continuation and by an appropriate gauge choice [54], the Lagrangian takes the form

$$L^{D3} = 2\sqrt{\det g}\;\; \theta \left\{ e^{-\phi}\gamma^m \nabla_m + \frac{1}{8}\widetilde{G}_{mn\widehat{p}}\gamma^{mn\widehat{p}} \right\} \theta \ . \tag{3.2.41}$$

Here $m, n, \ldots$ are directions along the brane and $\widehat{p}$ stands for directions transverse to the brane. As always, we turn on the three-form flux only along the directions of the internal manifold. Also for simplicity, we set the flux F_2 due to the world-volume gauge fields to zero. $\widetilde{G}$ is defined to be

$$\widetilde{G}_{mnp} = e^{-\phi} H_{mnp} + iF'_{mnp}\gamma_5 \ , \tag{3.2.42}$$

with $F' = dC_2 - C_0 H_3$. The Dirac equation, obtained from the above action, reads

$$\left\{ e^{-\phi}\gamma^m \nabla_m + \frac{1}{8}\widetilde{G}_{mn\widehat{p}}\gamma^{mn\widehat{p}} \right\} \theta = 0 \ . \tag{3.2.43}$$

Locally, we can express the internal metric as

$$ds^2 = g_{a\bar{b}} dy^a dy^{\bar{b}} + g_{z\bar{z}} dz d\bar{z} \ , \tag{3.2.44}$$

where $a, b, \ldots$ are complex coordinates on the $D3$-brane and $z, \overline{z}$ are directions transverse to the brane. We define the Clifford vacuum to be

$$\gamma^z|\Omega >= \gamma^a|\Omega >= 0 \ . \tag{3.2.45}$$

The spinor θ can be written in terms of positive and negative chirality spinors as $\theta = \epsilon_+ + \epsilon_-$ with

$$\begin{aligned} \epsilon_+ &= \phi|\Omega> + \phi_{\overline{a}}\gamma^{\overline{a}}|\Omega> + \phi_{\overline{ab}}\gamma^{\overline{ab}}|\Omega> , \\ \epsilon_- &= \phi_{\overline{z}}\gamma^{\overline{z}}|\Omega> + \phi_{\overline{az}}\gamma^{\overline{az}}|\Omega> + \phi_{\overline{abz}}\gamma^{\overline{abz}}|\Omega> . \end{aligned} \tag{3.2.46}$$

Substituting this into the Dirac equation, we find

$$\begin{aligned} e^{-\phi}2g^{a\overline{a}}\partial_a\phi_{\overline{a}} + 2ig^{z\overline{z}}g^{a\overline{b'}}g^{b\overline{a'}}G_{abz}\phi_{\overline{a'b'z}} + \frac{1}{2}ig^{z\overline{z}}g^{a\overline{b}}\phi_{\overline{z}}G_{a\overline{b}z} = 0 \ , \\ e^{-\phi}\left(\partial_{\overline{a'}}\phi + 4g^{a\overline{b'}}\partial_a\phi_{\overline{b'a'}}\right) + \frac{1}{2}ig^{z\overline{z}}g^{a\overline{b}}\left(\phi_{\overline{a'z}}\overline{G}_{a\overline{b}z} - 2\phi_{\overline{bz}}\overline{G}_{a\overline{a}z}\right) = 0 \ , \\ e^{-\phi}\partial_{[\overline{a'}}\phi_{\overline{b'}]} + \frac{1}{2}ig^{z\overline{z}}g^{a\overline{b}}\left(\phi_{\overline{a'b'z}}G_{a\overline{b}z} - 4\phi_{\overline{bb'z}}G_{a\overline{a'}z}\right) + \frac{1}{4}ig^{z\overline{z}}\phi_{\overline{z}}G_{\overline{a'b'}z} = 0 \end{aligned} \tag{3.2.47}$$

and

$$\begin{aligned} e^{-\phi}2g^{a\overline{a}}\partial_a\phi_{\overline{az}} + ig^{a\overline{b'}}g^{b\overline{a'}}\phi_{\overline{a'b'}}G_{ab\overline{z}} + \frac{1}{4}ig^{a\overline{b}}\phi G_{a\overline{bz}} = 0 \ , \\ e^{-\phi}\left(\partial_{\overline{a'}}\phi_{\overline{z}} + 4g^{a\overline{b'}}\partial_a\phi_{\overline{b'a'z}}\right) - \frac{1}{4}ig^{a\overline{b}}\left(\phi_{\overline{a'}}\overline{G}_{a\overline{bz}} - 2\phi_{\overline{b}}\overline{G}_{a\overline{a'z}}\right) = 0 \ , \\ e^{-\phi}\partial_{[\overline{a'}}\phi_{\overline{b'}]z} + \frac{1}{4}ig^{a\overline{b}}\left(\phi_{\overline{a'b'}}G_{a\overline{bz}} - 4\phi_{\overline{bb'}}G_{a\overline{a'z}}\right) + \frac{1}{8}i\phi G_{\overline{a'b'z}} = 0 \ . \end{aligned} \tag{3.2.48}$$

We can similarly work out the equations for world-volume $M5$-brane fermions. The fermionic bilinear terms on the $M5$-brane world-volume in the presence of background flux have been derived in [51]. Upon setting the world-volume gauge flux to zero, we have the Dirac equation

$$\gamma^m\nabla_m\theta - \frac{1}{24}\gamma^{\widehat{q}}\gamma^{mnp}G_{mnp\widehat{q}}\theta = 0 \ . \tag{3.2.49}$$

Again, we turn on the fluxes only along the compact directions. Here, $m, n, p, \ldots$ are real indices. A $\widehat{\ }$ indicates the directions transverse to the brane. We denote by $a, b, \ldots$ the holomorphic indices along the brane and by $\bar{a}, \bar{b}, \ldots$ the anti-holomorphic indices; z is the complex coordinate along the normal to the divisor. The spinor θ can be expressed in terms of the Clifford vacuum and the creation operators as

$$\theta = \phi|\Omega> + \phi_{\bar{z}}\gamma^{\bar{z}}|\Omega> + \phi_{\bar{a}\bar{b}}\gamma^{\bar{a}\bar{b}}|\Omega> + \phi_{\bar{z}\bar{a}\bar{b}}\gamma^{\bar{z}\bar{a}\bar{b}}|\Omega> \ . \tag{3.2.50}$$

Plugging this expression for θ into the Dirac equation, we find

$$\begin{aligned} &(\partial_{\bar{c}}\phi + 4g^{b\bar{b}'}\partial_b\phi_{\bar{b}'\bar{c}}) \\ &+\frac{1}{2}\left[4g^{a\bar{a}'}g^{b\bar{b}'}g^{z\bar{z}}(G_{ab\bar{b}'z}\phi_{\bar{z}\bar{a}'\bar{c}} - G_{ab\bar{c}z}\phi_{\bar{z}\bar{a}'\bar{b}'}) + g^{z\bar{z}}g^{a\bar{b}}\phi_{\bar{z}}G_{a\bar{b}\bar{c}z}\right] = 0 \ , \\ &(\partial_{\bar{a}}\phi_{\bar{z}} + 4g^{b\bar{b}'}\partial_b\phi_{\bar{z}\bar{a}\bar{b}'}) \\ &-\frac{1}{4}\left[4g^{a\bar{a}'}g^{b\bar{b}'}(G_{ab\bar{b}'\bar{z}}\phi_{\bar{a}'\bar{c}} - G_{ab\bar{c}\bar{z}}\phi_{\bar{a}'\bar{b}'}) + g^{a\bar{b}}\phi G_{a\bar{b}\bar{c}\bar{z}}\right] = 0 \ , \end{aligned}$$

$$
\begin{aligned}
&\partial_{[\bar{a}}\phi_{\bar{b}\bar{c}]} + \frac{1}{12} g^{z\bar{z}}\phi_{\bar{z}}G_{\bar{a}\bar{b}\bar{c}z} = 0 \ , \\
&\partial_{[\bar{a}}\phi_{\bar{z}\bar{b}\bar{c}]} + \frac{1}{24}\phi G_{\bar{a}\bar{b}\bar{c}\bar{z}} = 0 \ .
\end{aligned} \tag{3.2.51}
$$

These expressions can be simplified a lot using the primitivity condition:

$$
\begin{aligned}
&(\partial_{\bar{c}}\phi + 4g^{b\bar{b}'}\partial_b\phi_{\bar{b}'\bar{c}}) - 2g^{a\bar{a}'}g^{b\bar{b}'}g^{z\bar{z}}G_{ab\bar{c}z}\phi_{\bar{z}\bar{a}'\bar{b}'} = 0 \ , \\
&(\partial_{\bar{a}}\phi_{\bar{z}} + 4g^{b\bar{b}'}\partial_b\phi_{\bar{z}\bar{a}\bar{b}'}) + g^{a\bar{a}'}g^{b\bar{b}'}G_{ab\bar{c}\bar{z}}\phi_{\bar{a}'\bar{b}'} = 0 \ , \\
&\partial_{[\bar{a}}\phi_{\bar{b}\bar{c}]} + \frac{1}{12} g^{z\bar{z}}\phi_{\bar{z}}G_{\bar{a}\bar{b}\bar{c}z} = 0 \ , \\
&\partial_{[\bar{a}}\phi_{\bar{z}\bar{b}\bar{c}]} + \frac{1}{24}\phi G_{\bar{a}\bar{b}\bar{c}\bar{z}} = 0 \ .
\end{aligned} \tag{3.2.52}
$$

The equations are modified due to the $(3,1)$- and $(4,0)$-fluxes, and so is the zero mode counting. To understand this better, we shall turn to the example of compactification on $K3 \times K3$.

3.2.5 Example: $K3 \times K3$

To acquire a better understanding of the above equations, we consider here the example of M/F-theory compactified on $K3_1 \times K3_2$ with background flux [76, 65, 77]. Consider one of the $K3$s (say $K3_2$) to be elliptically fibered. Wrap the $M5$-brane on one of the divisors of the form $K3 \times S$, where S corresponds to the P^1s of the elliptic $K3$. Let z parameterize the direction normal to the brane.

We will now briefly review the case of the flux being of type $(2,2)$ and primitive and then consider the case of general flux. Let us first analyze the case of the flux preserving $N = 2$ supersymmetry. In this case, the $(2,2)$-flux must take the form

$$
G_4 \in H^{1,1}(K3_1) \otimes H^{1,1}(K3_2) \ , \tag{3.2.53}
$$

which implies that the $N = 2$ flux must be a $(1,1)$-form in $K3_2$. Since it is an elliptically fibered $K3$, we have to use the spectral sequence, which tells us that the flux belongs to [78]

$$
H^0(B, R^2\pi_*\mathbf{R}) \oplus H^2(B, \pi_*\mathbf{R}) \ ,
$$

which in simple terms means that the flux has either both legs in the fiber or both in the base. So the $N = 2$ flux is always of the type $G_{a\bar{b}c\bar{d}}$ or $G_{a\bar{b}z\bar{z}}$. Contrarily to this, the flux appearing in the Dirac equation of the brane world-volume is always of type $G_{a\bar{b}c\bar{z}}$ or $G_{a\bar{b}z\bar{c}}$. Thus for $N = 2$ flux, the Dirac equation does not change at all and the zero modes are same as those of the fluxless case.

We now turn our attention to fluxes preserving $N = 1$ supersymmetry. Such a flux is of the form

$$
G_4 \in \left(H^{2,0}(K3_1) \otimes H^{0,2}(K3_2)\right) \oplus \left(H^{0,2}(K3_1) \otimes H^{2,0}(K3_2)\right) \ . \tag{3.2.54}
$$

In addition, it may contain flux of the form as given in eq. (3.2.53). The susy conditions in presence of such a flux have been analyzed in great detail in [76]. It has been realized there, that by an appropriate choice of $(2,2)$ primitive flux, it is in fact possible to lift all the complex structure as well as Kähler moduli except the overall size of the $K3$. It has also been noticed that the fluxes of the type given in eq. (3.2.54) stabilize both the $K3$s at an attractor point [77]. Attractive $K3$ surfaces are completely classified. They are in one-to-one correspondence with the (SL(2,Z) equivalent) matrices

$$Q = \begin{pmatrix} 2a & b \\ b & 2c \end{pmatrix},$$

where a, b and c are integers, and in addition a, c and the the determinant of Q are required to be positive. Two such matrices represent the same $K3$ if they are $SL(2,Z)$ equivalent. It has been shown in ref. [77], that the tadpole cancellation condition puts very strong constraints on the integers a,b and c appearing in the above matrix Q. Thus the $N=1$ solutions are very limited and all of them can be determined.

We now consider $M5$-branes wrapping divisors of the form $K3\times S$ in presence of such a flux. Locally, these fluxes are of the form $G_{ab\overline{cz}}, G_{\overline{abc}z}$. The divisors under consideration have the cohomology

$$H^{1,0}(K3\times P^1) = H^{3,0}(K3\times P^1) = 0\ . \tag{3.2.55}$$

Since $\phi_{\bar z}$ and $\phi_{\bar z\bar a\bar b}$ belong to these cohomology groups, they must be identically zero. We can now clearly see from the Dirac equations that the forms $\phi, \phi_{\overline{ab}}$ are harmonic, and in addition we have

$$g^{a\bar a'}g^{b\bar b'}G_{ab\bar c\bar z}\phi_{\bar a'\bar b'} = 0\ . \tag{3.2.56}$$

This condition lifts the $\phi_{\bar a'\bar b'}$ mode. Hence, we only have massless modes corresponding to $\phi\in H^{0,0}(D)$. Note, that all the spinors also carry an $SO(2,1)$-index, and hence there is a doubling of massless modes. Since $H^{0,0}(D)$ is one-dimensional, we are now left with two fermion zero modes, which is the right number for the instanton to contribute to W_{np}.

We now study the Dirac equations in presence of $(3,1)$- and $(4,0)$-flux. They take the simple form

$$\begin{aligned}
&(\partial_{\bar c}\phi + 4g^{b\bar b'}\partial_b\phi_{\bar b'\bar c}) = 0\,,\\
&g^{a\bar a'}g^{b\bar b'}G_{ab\bar c\bar z}\phi_{\bar a'\bar b'} = 0\,,\\
&\partial_{[\bar a}\phi_{\bar b\bar c]} = 0\,,\\
&\phi G_{\bar a\bar b\bar c\bar z} = 0\,.
\end{aligned} \tag{3.2.57}$$

Again, we find from the above that the forms $\phi, \phi_{\overline{ab}}$ are harmonic. In addition, we find that both zero modes ϕ as well as $\phi_{\overline{ab}}$ must be zero. Thus the presence of $(4,0)$-flux lifts all the zero modes. As a result, we don't have any contribution to W_{np} from the $M5$-instantons.

We have seen in the above that we can choose an appropriate (2,2) flux preserving $N=1$ susy, so that we have the correct number of fermion zero modes to have a non-perturbative superpotential. But once we include a (4,0) flux, as enforced by the non-perturbative term in the susy conditions, all the zero modes are lifted which means that it is not consistent to keep the non-perturbative term. This raises a puzzle which we intend to resolve in the following section.

3.2.6 Inclusion of the non-perturbative superpotential

In the last section, we have seen that a $(4,0)$-component of G lifts all zero modes. On the other hand, the susy conditions tell us that the $(4,0)$-part of G is non-zero in the presence of W_{np}. So there is an apparent mismatch. The resolution of this puzzle seems to be to include W_{np} into the Dirac equation which determines the number of zero modes. Then, $G_{4,0}$ should be balanced against W_{np}, as it is the case for the susy conditions.

The Dirac part of the world-volume action on an $M5$-brane with fluxes has the form [53]:

$$L_f^{M5} = \frac{1}{2}\theta[\widetilde{\gamma}^m\nabla_m + \frac{1}{24}(\gamma^{\widehat{m}\widehat{n}\widehat{p}}\widetilde{\gamma}^q G_{q\widehat{m}\widehat{n}\widehat{p}} - \gamma^{\widehat{q}}\widetilde{\gamma}^{mnp}G_{mnp\widehat{q}})]\theta\,. \tag{3.2.58}$$

For us, it is important to note that the corresponding Dirac equation, whose solutions count the number of fermionic zero modes, is essentially determined by the susy variation of the 11-dimensional gravitino field. This can be seen as follows [54]. The supersymmetry conditions on the bulk, closed string background are given by

$$\delta\psi_M\epsilon = 0\,, \tag{3.2.59}$$

which is the supersymmetry transformation of the 11-dimensional gravitino. This can be translated to the linear part of the Dirac equation from the world-volume action as follows:

$$(1-\Gamma_{M5})\Gamma^\alpha\delta\psi_\alpha\theta = 0\,. \tag{3.2.60}$$

Here, $\delta\psi_\alpha$ is the pull-back of the gravitino variation to the brane via $\delta\psi_\alpha = \delta\psi_M\partial_\alpha x^M$ and $\Gamma_\alpha = \Gamma_N e^N_M\partial_\alpha x^M$. Therefore, one sees that the pull-back of the bulk gravitino equation is equivalent to a solution of the Dirac equation. Furthermore, one has to take into account the constraint from κ-symmetry on the $M5$-brane:

$$(1+\Gamma_{M5})\theta = 0\,. \tag{3.2.61}$$

The number of zero modes is then given by the difference between the numbers of solutions of these two equations.

As we have already stated, we can recover the $M5$-brane world-volume action eq. (3.2.58) by using the explicit expressions for the internal gravitino variations in the absence of W_{np} in eq. (3.2.60). We have already seen in §2 that turning on W_{np} alters the susy equations in the effective potential, as the effective superpotential now is $W = W_{flux}+W_{np}$. This addition should be described by the modulino equations, i.e. $\delta\phi^i = \delta\phi^I = 0$ should be now equivalent to $DW_{flux} + DW_{np} = 0$.

Substituting the expressions for the internal gravitino transformations with the general fluxes in (3.2.58), one obatins

$$\begin{aligned}
&(\partial_{\bar c}\phi + 4g^{b\bar b'}\partial_b\phi_{\bar b'\bar c}) - 2g^{a\bar a'}g^{b\bar b'}g^{z\bar z}G_{ab\bar c z}\phi_{\bar z\bar a'\bar b'} = 0\,,\\
&(\partial_{\bar a}\phi_{\bar z} + 4g^{b\bar b'}\partial_b\phi_{\bar z\bar a\bar b'}) + g^{a\bar a'}g^{b\bar b'}G_{ab\bar c\bar z}\phi_{\bar a'\bar b'} = 0\,,\\
&\partial_{[\bar a}\phi_{\bar b\bar c]} + \frac{1}{12}g^{z\bar z}\phi_{\bar z}G_{\bar a\bar b\bar c z} = 0\,,\\
&\partial_{[\bar a}\phi_{\bar z\bar b\bar c]} + \frac{1}{24}\phi G_{\bar a\bar b\bar c\bar z} = 0\,.
\end{aligned} \tag{3.2.62}$$

This is a set of local equations in the internal space. Every summand of (3.2.62) vanishes separately. This means that the set of equations

$$\begin{aligned}
G_{ab\overline{c}z}\phi^{abz} &= 0\,,\\
G_{ab\overline{cz}}\phi^{ab} &= 0\,,\\
G_{\overline{abc}z}\phi^{z} &= 0\,,\\
G_{\overline{abcz}}\phi &= 0
\end{aligned} \tag{3.2.63}$$

is preventing the ϕ, $\phi_{\overline{a}}$, $\phi_{\overline{ab}}$ and $\phi_{\overline{abz}}$ to be non-trivial zero-modes in the case of general flux G_3. On the other hand G_{mnpq} correspond to the three-dimensional constant scalar fields which one obtains as coefficients by expansion of G_3 in the harmonic basis on CY_4:

$$\begin{aligned}
G_4 \;=\;& G_{abcd}dz^a\wedge dz^b\wedge dz^c\wedge dz^d + \sum_{i=1}^{h^{(3,1)}} G^i_{\overline{a}bcd}\omega^{i\;\overline{a}bcd}\\
&+ \sum_{k=1}^{h^{(2,2)}} G^k_{\overline{ab}cd}\widetilde{\omega}^{k\;\overline{ab}cd} + \sum_{i=1}^{h^{(3,1)}} G^i_{\overline{abc}d}\overline{\omega}^{i\;\overline{abc}d}\\
&+ G_{\overline{abcd}}d\bar z^{\bar a}\wedge d\bar z^{\bar b}\wedge d\bar z^{\bar c}\wedge d\bar z^{\bar d}
\end{aligned} \tag{3.2.64}$$

with $\widetilde{\omega}^k$ being basis elements of $H^{2,2}$. Since $H^{2,0}=0$, they can be expressed in terms of the basis elements ω^I of $H^{1,1}$ as $\widetilde{\omega}^k = \sum_{I,J}\chi^k_{IJ}\omega^I\wedge\omega^J$. The scalar fields G, G^k, G^i are related to the flux superpotential by (3.2.35) and (3.2.36). From the modulino equations (3.2.40) we see that W_{flux} has to be replaced by $W = W_{\text{flux}} + W_{\text{np}}$. This corresponds to the modification of G to

$$\begin{aligned}
\widehat{G}_{2,2} \;&:\; \widehat{G}_{ab\overline{cd}} = G_{ab\overline{cd}}\,,\\
\widehat{G}^i_{1,3} \;&:\; \widehat{G}^i_{a\overline{bcd}} = \epsilon_{a\overline{bcd}}D_iW = G^i_{a\overline{bcd}} + \epsilon_{a\overline{bcd}}D_iW_{np}\,,\\
\widehat{G}_{0,4} \;&:\; \widehat{G}_{\overline{abcd}} = \epsilon_{\overline{abcd}}\frac{D_IW}{\partial_IK} = G_{\overline{abcd}} + \epsilon_{\overline{abcd}}\frac{D_IW_{np}}{\partial_IK}\,.
\end{aligned} \tag{3.2.65}$$

This amounts to modifying the world-volume action (3.2.58) in presence of the nonperturbative superpotential, where we now replace G by $\widehat{G}$. It is than straightforward to see that, using the susy conditions $D_iW = D_IW = 0$, the Dirac equation can be expressed as:

$$(\partial_{\bar c}\phi + 4g^{b\bar b'}\partial_b\phi_{\bar b'\bar c}) = 0\,,$$

$$
\begin{aligned}
&(\partial_{\bar{a}}\phi_{\bar{z}} + 4g^{b\bar{b}'}\partial_b\phi_{\bar{z}\bar{a}\bar{b}'}) + g^{a\bar{a}'}g^{b\bar{b}'}G_{ab\bar{c}\bar{z}}\phi_{\bar{a}'\bar{b}'} = 0\,,\\
&\partial_{[\bar{a}}\phi_{\bar{b}\bar{c}]} = 0\,,\\
&\partial_{[\bar{a}}\phi_{\bar{z}\bar{b}\bar{c}]} = 0\,.
\end{aligned}
\tag{3.2.66}
$$

These conditions are identical to the ones coming from $(2,2)$ primitive flux without W_{np}. The $(4,0)$- and $(3,1)$-parts of the flux are compensated by the nonperturbative term. As a result, we find that the number of fermion zero modes is unaltered. The apparent mismatch of the two answers in the previous section was due to the fact that we had then ignored the back-reaction of the instanton on the background flux and the geometry. Once we take care of this by modifying the fermionic terms accordingly, we obtain the expected result.

For the type IIB Euclidean $D3$-brane, the story is very similar, hence we will be very brief in the following. The Dirac Lagrangian can be written in terms of the type IIB gravitino variation, where in addition also the dilatino variation appears:

$$
L_f^{D3} = \frac{1}{2}e^{-\phi}\sqrt{\det g}\;\bar{\theta}(1-\Gamma_{D3})(\Gamma^\alpha\delta\psi_\alpha - \delta\lambda)\theta\,,
\tag{3.2.67}
$$

where the bulk susy variations are $\delta\psi_m = 0$ and $\delta\lambda = 0$. Substituting the expressions for $\delta\psi_m$ and $\delta\lambda$ without W_{np} into the above equation yields

$$
L^{D3} = 2\sqrt{\det\, g}\;\;\theta\left\{e^{-\phi}\gamma^m\nabla_m + \frac{1}{8}\widetilde{G}_{mn\widehat{p}}\gamma^{mn\widehat{p}}\right\}\theta\,.
\tag{3.2.68}
$$

Once we use the modified expressions for $\delta\phi^k$, $\delta\phi^i$, $\delta\phi^I$ and $\delta\lambda$ in presence of W_{np}, we replace G by

$$
\begin{aligned}
\widehat{G}_{2,1} &: \;\widehat{G}_{ab\bar{c}} = \widetilde{G}_{ab\bar{c}}\,,\\
\widehat{G}^i_{1,2} &: \;\widehat{G}^i_{a\overline{bc}} = \widetilde{G}^i_{a\overline{bc}} + \epsilon_{a\overline{bc}}D_iW_{np}\,,\\
\widehat{G}_{0,3} &: \;\widehat{G}_{\overline{abc}} = \epsilon_{\overline{abc}}\frac{D_IW}{\partial_IK} = \widetilde{G}_{\overline{abc}} + \epsilon_{\overline{abc}}\frac{D_IW_{np}}{\partial_IK}\,,\\
\widehat{G}_{3,0} &: \;\widehat{G}_{\overline{abc}} = \epsilon_{abc}(\bar{\tau}-\tau)D_\tau W \;= \widetilde{G}_{\overline{abc}} + \epsilon_{abc}(\bar{\tau}-\tau)D_IW_{np}\,.
\end{aligned}
\tag{3.2.69}
$$

We can similarly analyze the Dirac equations. As expected, the number of fermion zero modes remains the same as in the case of primitive $(2,1)$-flux without the non-perturbative term.

3.3 Non-perturbative effects from gaugino condensation

Another source for the non-perturbative superpotential is gaugino condensation on a stack of $D7$-branes which fill space-time and wrap four-dimensional divisors. In general compactifications, their presence is required by the tadpole cancellation condition, namely in order to cancel the Ramond charge of the orientifold planes. The open string spectrum on

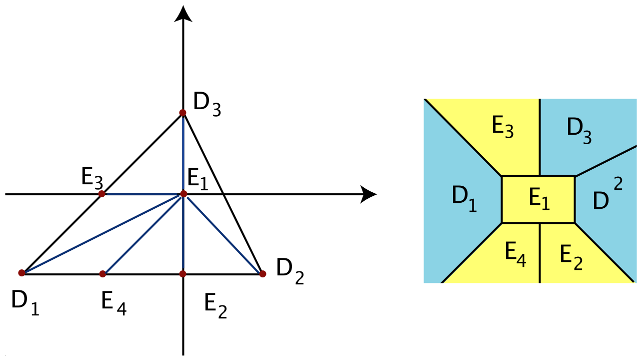

Figure 3.1: Toric diagram of the resolution of $\mathbf{C}^3/\mathbf{Z}_{6-II}$ and dual graph

the $D7$-branes is described by the effective $\mathcal{N}=1$ supersymmetric $SU(N)$ gauge theory with some additional matter fields.

In the $\mathcal{N}$ Yang-Mills theory with gauge group G without any matter, gaugino condensation generates a non-perturbative superpotential

$$W_{\text{np}} \sim \Lambda^3 = e^{-\frac{8\pi^2}{bg^2}} \ , \tag{3.3.70}$$

where b is the β-function coefficient of the corresponding group, and Λ is the dynamical scale of the gauge theory. The gauge coupling is related to the volume V_i of the divisor by

$$\frac{4\pi}{g^2} = V_i \ , \tag{3.3.71}$$

and contribution to the superpotential by gaugino condensation can be expressed as

$$W_{\text{np}} \sim g_i e^{-\frac{2\pi V_i}{b}} . \tag{3.3.72}$$

The open string spectrum on the $D7$-branes consists either of adjoint or bi-fundamental matter. The adjoint scalars correspond to the position of the $D7$s (called position fields in the following) or they are associated to Wilson lines (called Wilson fields in the following). The number of the position and Wilson line fields is given by the topological data of the divisor wrapped by the $D7$-brane, namely $h_{(0,1)}$ for the former and $h_{(0,2)}$ for latter fields. The massless bi-fundamental fields arise as open string states localized at the intersection loci of two $D7$-branes.

One can argue (see ref. [24]) that if all matter fields become heavy, then gaugino condensation is generated. This means that on the one hand the Hodge numbers $h_{(0,1)}$, $h_{(0,2)}$ of the divisor wrapped by the $D7$-brane should vanish and on the other hand the $D7$-branes responsible for gaugino condensation should not intersect each other.

Let us give a simple example. In the case of the resolved singularity of $T^6/\mathbf{Z}_{6-II}$ [34] the toric diagram is presented in figure 3.1. The exceptional divisors E_i have topologies of $\mathbf{P^1} \times \mathbf{P^1}$ and D_1, D_2, D_3 are blow-ups of $\mathbf{P^1} \times \mathbf{P^1}$ in 12, 8, and 9 points, respectively. This means, all of them have $h_{(0,1)} = h_{(0,2)} = 0$ and both mentioned criteria are satisfied either by the triple D_1, D_3, E_2 or by E_3, D_2, E_4. Because of the special choice of the orientifold action in the glued CY, the $D7$-branes are put on the first triple of divisors.

Chapter 4

Fixing all moduli in two Calabi-Yaus

This chapter is based on the material published in [25].

4.1 Stabilization of Kähler moduli associated to the cohomology $H_{+}^{(1,1)}(X_6)$

In the following we shall assume[1] a flux compactification of a type IIB CY orientifold with $h_{(1,1)}^{(-)}(X_6) = 0$ and a general Kähler potential K_{KM} for the $n := h_{(1,1)}^{(+)}(X_6)$ Kähler moduli:

$$K_{KM}(T^1, \ldots, T^n, \overline{T}^1, \ldots, \overline{T}^n) = K(T^1 + \overline{T}^1, \ldots, T^n + \overline{T}^n) \ . \tag{4.1.1}$$

We consider the racetrack superpotential [62]:

$$W = W_0(S,U) + \lambda \sum_{j=1}^{n} \gamma_j(S,U) \ e^{a_j \ T^j} \ . \tag{4.1.2}$$

The first term W_0 of (4.1.2) represents the tree–level flux superpotential ([14])

$$W_0(S,U) = \int_{X_6} G_3 \wedge \Omega \tag{4.1.3}$$

depending on the dilaton S and the $h_{(2,1)}^{(-)}(X_6)$ complex structure moduli U^λ. Since $\Omega \in H_{-}^{(3)}(X_6)$, we also must have $G_3 \in H_{-}^{(3)}(X_6)$ with $2h_{2,1}^{(-)}(X_6) + 2$ complex flux components. In eq. (4.1.2) we assume $W_0 \in \mathbf{C}$, $\gamma_j \in \mathbf{C}$, and $a_j \in \mathbf{R}_{-}$. In addition, $\lambda \in \mathbf{R}$ is a real parameter accounting for a possible so–called Kähler gauge (cf. Section 4.4). The latter may be used to adjust a certain flux value given by W_0 to a given minimum in the Kähler moduli space $(T^1, \ldots, T^n)$, cf. ref. [63]. We do not consider a possible open string moduli dependence of the superpotential [72, 76].

[1] See Ref. [25] for the disscussion of Kähler moduli associated to the cohomology $H_{-}^{(1,1)}(X_6)$

The work of KKLT [61] proposes a mechanism to stabilize all moduli at a small positive cosmological constant. This procedure is accomplished through three steps. One first dynamically fixes the dilaton S and the complex structure moduli U^λ through the tree–level piece W_0 (given in eq. (4.1.3)) of the superpotential. This is accomplished with a generic 3–form flux G_3 with both ISD– and $IASD$–flux components. At the minimum of the scalar potential in the complex structure and dilaton directions, the flux becomes ISD and the potential assumes the value $V_0(S,U^\lambda) = -3e^K|W_0|^2$. The soft masses m_S, m_{U^λ} for the dilaton and complex structure scalars are generically of the order α'/R^3 [79]. In the large radius approximation $\mathrm{Re}(T) \gg 1$, the non–perturbative terms in (4.1.2) only amount to a small exponentially suppressed additional contribution to m_S, m_{U^λ}. According to [80] the latter is negligible. The second step is the addition of the non–perturbative piece to the superpotential (4.1.2), which allows the stabilization of the Kähler moduli T^j at a supersymmetric AdS minimum. The soft masses for the Kähler moduli are much smaller than soft masses m_S and m_{U^λ}. This property allows us to separate the first and second step, i.e. to effectively first integrate out the dilaton and complex structure moduli. Nonetheless, strictly speaking these two steps should be treated at the same time. The stability of AdS vacua in gravity coupled to scalar fields has been investigated in ref. [49]. Stability is guaranteed, if all scalar masses fulfill the Breitenlohner–Freedman (BF) bound [49], i.e. their mass eigenvalues do not fall below a certain minimal bound. The latter is a negative number related to the scalar potential at the minimum. It can be shown in a completely model independent way that all scalars have masses above this bound at any N=1 supersymmetric AdS minimum in supergravity theories (cf. e.g. [81] and Appendix C of ref. [82]). However, the third and final step in the KKLT scenario consists in the addition of one anti $D3$–brane, i.e. a positive contribution to the scalar potential, which lifts the AdS minimum to a dS minimum. The masses for the moduli fields do not change significantly during this process. However stable dS vacua require positive mass eigenvalues. Hence, any negative mass eigenvalue before the uplift is unacceptable since the effect of the anti $D3$–branes on the mass eigenvalues is too small to change a negative mass to positive.

In (4.1.2), the sum of exponentials accounts for $D3$–brane instantons and gaugino condensation on stacks of $D7$–branes. The $D3$–instantons come from wrapping (Euclidean) $D3$–branes on internal 4–cycles C_4^j of the CY orientifold X_6. The latter have the volume $\mathrm{Re}(T^j)$ and lead to the instanton effect $e^{-2\pi T^j}$ in the superpotential, i.e. $a_j = -2\pi$. The gauge coupling on a $D7$–brane which is wrapped on the 4–cycle C_4^j is given by $\mathrm{Re}(T^j)$, cf. eq. (2.1.10). Hence, gaugino condensation on this $D7$–brane yields the effect e^{-T^j/b_a} in the superpotential. *E.g.* for the gauge group $SU(M)$ we have $b_{SU(M)} = \frac{M}{2\pi}$, i.e. $a_j = -\frac{2\pi}{M}$. On the $D7$–brane, $\gamma_j(S,U)$ may comprise one–loop effects and further instanton effects from $D(-1)$–branes: One loop corrections to the gauge coupling give rise to [83]

$$\gamma_j \sim \eta(U^\lambda)^{-2/b_a} \;, \tag{4.1.4}$$

while additional instantons in the $D7$–gauge theory amount to:

$$\gamma_j \sim e^{-S/b_a \int_{C_4^j} F\wedge F} \;. \tag{4.1.5}$$

Supersymmetric vacuum solutions are found by finding the zeros of the F–terms: $\overline{F}^{\overline{M}} = K^{\overline{M}J}\ (\partial_J W + W\ K_J)$. Solutions to the equations $F^M = 0$ give rise to extremal points of the scalar potential. In addition, it has to be verified whether those zeros lead to a stable minimum. Since the matrix $K^{\overline{M}J}$ is positive definite, the zeros $(T_0^1, \ldots, T_0^n)$ in the Kähler moduli space are determined by the n equations:

$$\partial_{T^j} W + W\ K_{T^j} = 0 \quad , \quad j = 1, \ldots, n \tag{4.1.6}$$

following from the requirement of vanishing F–terms. These equations turn into the $h_{(1,1)}^{(+)}(X_6)$ relations

$$\lambda\ \gamma_i = -\left(\prod_{j\neq i}^{n} |a_j|\right)\ e^{-a_i\ T^i}\ W_0\ \left.\frac{K_{T^i}}{\sum\limits_{j=1}^{n} K_{T^j}\ \prod\limits_{\substack{k=1\\k\neq j}}^{n} |a_k| - \prod\limits_{k=1}^{n} |a_k|}\ \right|_{T^l = T_0^l} \quad , \quad i = 1, \ldots, n \tag{4.1.7}$$

to be satisfied at this extremum. Since K_{T^i} and a_i are real, from eq. (4.1.7) we may easily deduce the VEVs t_2^j of the axions at the AdS–minimum. After introducing the phases $W_0 = |W_0|\ e^{i\varphi}$and $\gamma_i = |\gamma_i|\ e^{i\phi_i}$ we obtain $(T^j = t_1^j + it_2^j)$

$$t_2^i = \frac{1}{a_i}\ \left[\ \varphi + \pi\ (1 + \rho^i) - \phi_i\ \right] + \frac{2\pi}{a_i}\ \mathbf{Z} \quad , \quad i = 1, \ldots, n \tag{4.1.8}$$

as VEVs for the axion fields. Above we have introduced the numbers

$$\rho^i = \frac{1}{\pi}\ arg\left(\frac{\lambda\ K_{T^i}}{\sum\limits_{j=1}^{n} K_{T^j}\ \prod\limits_{\substack{k=1\\k\neq j}}^{n} |a_k| - \prod\limits_{k=1}^{n} |a_k|}\right) \in \{0, 1\}\ .$$

For the case that an exponential $e^{a_j T^j}$ accounts for gaugino condensation in an $SU(M)$ gauge group, we have $a_j = -\frac{2\pi}{M}$ and in eq. (4.1.8) the additional shift $\frac{2\pi}{a_j}\ \mathbf{Z}$ becomes $M\ \mathbf{Z}$. The latter becomes trivial, if the Kähler modulus T^j enjoys a discrete shift symmetry, e.g. $T^j \to T^j + 1$. On the other hand, if no such symmetry exists, in eq. (4.1.8) the additional shifts $\frac{2\pi}{a_j}\ \mathbf{Z}$ give rise to an infinite number of extrema obtained from one another by shifts in the axionic directions t_2^j. A useful relation to be satisfied at the extremum is the ratio

$$\frac{\gamma_i}{\gamma_j} = e^{a_j\ T^j - a_i\ T^i}\ \frac{a_j}{a_i}\ \frac{K_{T^i}}{K_{T^j}} \quad , \quad i, j = 1, \ldots, n\ . \tag{4.1.9}$$

The latter equation may be written as

$$\frac{|\gamma_i|}{|\gamma_j|} = e^{i\phi_{ji}}\ e^{a_j\ t_1^j - a_i\ t_1^i}\ \frac{a_2}{a_1}\ \frac{K_{T^1}}{K_{T^2}} \quad , \quad \phi_{ji} = \phi_j - \phi_i + a_j\ t_2^j - a_i\ t_2^i\ . \tag{4.1.10}$$

Since $\phi_{ji} \in \{0, \pi\}$, the directions of the axions t_2^j strongly depend on the signs of the first Kähler derivatives K_{T^j} and the phases ϕ_j of the coefficients γ_j.

Moreover, from the relation (4.1.8) we see that any complex phase of W_0 and γ_i may be absorbed into a redefinition of the axion VEV at the minimum. Hence, in the following we may assume without any restriction:

$$W_0 \in \mathbf{R}^+ \quad , \quad \gamma_j \in \mathbf{R}^+ \ .$$

Finally, at the extremum $(T_0^1, \ldots, T_0^n)$, the scalar potential $\widetilde{V}(T^1, \overline{T}^1, \ldots, T^n, \overline{T}^n)$ assumes the negative value

$$\widetilde{V}_{min} = -3 \ \ e^K \left(\prod_{k=1}^{h^{(+)}_{(1,1)}(X_6)} a_k^2 \right) \left. \frac{|W_0|^2}{\left(\sum\limits_{j=1}^{n} K_{T^j} \prod\limits_{\substack{k=1 \\ k \neq j}}^{n} |a_k| - \prod\limits_{k=1}^{n} |a_k| \right)^2} \right|_{T^l = T_0^l} \tag{4.1.11}$$

4.2 Resolved toroidal orientifolds as candidate models for a KKLT-scenario

In [63], a toroidal orbifold model, namely type IIB string theory compactified on the orientifold of the resolved $T^6/\mathbf{Z}_2 \times \mathbf{Z}_2$, was checked for its suitability as a compactification manifold for the KKLT proposal. Since the F–theory lift of this example is known, Witten's criterion could be checked directly and the results of [63] strongly indicate that in this model, all geometric moduli can be fixed.

The methods to obtain a smooth Calabi-Yau manifold from a toroidal orbifold and to subsequently pass to the corresponding orientifold as described in ref. [34] enable us to explicitly check other toroidal orbifolds for their suitability as candidate models for the KKLT proposal.

The requirement that the scalar mass matrix be positive, places severe constraints on the list of possible models. Those orbifolds without complex structure moduli do not give rise to stable vacua after the uplift to dS space. Thus $\mathbf{Z}_3$, $\mathbf{Z}_7$, $\mathbf{Z}_{8-I}$ on $SU(4)^2$, $\mathbf{Z}_2 \times \mathbf{Z}_{6'}$, $\mathbf{Z}_3 \times \mathbf{Z}_3$, $\mathbf{Z}_4 \times \mathbf{Z}_4$ and $\mathbf{Z}_6 \times \mathbf{Z}_6$ are excluded from the list of possible models given in table 2.2.

Since the stabilization of twisted complex structure moduli via 3–form flux is not well understood yet, the models with $h^{twist.}_{(2,1)}(X_6) \neq 0$ cannot be checked explicitly. Yet considerations regarding the topology of their divisors suggest that they might not be suitable candidate models anyway, as will be explained later on. The only models which are not yet excluded and are directly amenable to our methods are thus $T^6/\mathbf{Z}_4$ on $SU(4)^2$, $T^6/\mathbf{Z}_{6-II}$ on $SU(2) \times SU(6)$, the above mentioned $T^6/\mathbf{Z}_2 \times \mathbf{Z}_2$, and $T^6/\mathbf{Z}_2 \times \mathbf{Z}_4$. The example $T^6/\mathbf{Z}_4$ on $SU(4)^2$ contains five instead of the usual three untwisted Kähler moduli. Since it is

Topology	$O7$ on top	inters. in 1 dim.	no intersection
K3	2/[1]	0	0/[-1]
T^4	0/[-1]	0/[-2]	0/[-3]
$\mathbf{P}^1 \times T^2$	0	1/[0]	0/[-1]
$\mathbf{P}^2$, $\mathbf{F}_n$	1	1	0

Table 4.1: Index χ_{D3} for the four basic topologies

not clear how these two extra non–diagonal untwisted Kähler moduli contribute to the superpotential, this example will not be discussed explicitly.

The question one would like to answer is: Do enough of the divisors of the above models contribute to the non-perturbative superpotential that all Kähler moduli can be fixed? To answer this question, the topologies of the divisors must be studied. In Section 4.3 of [34] it was shown that there are four basic topologies for the divisors of the resolved toroidal orbifolds: The divisors R_i inherited from the covering T^6 have the topology of either **(i)** $K3$ or **(ii)** T^4. The exceptional divisors E_i which arise in the blowing up process can be birationally equivalent to either **(iii)** a rational surface (i.e. $\mathbf{P}^2$ or $\mathbf{F}_n$) or **(iv)** $\mathbf{P}^1 \times T^2$. The same is true for the D–divisors, which correspond to planes fixed at the loci of the fixed points and are linear combinations of the Rs and Es. The rational surfaces have $h_{(1,0)} = h_{(2,0)} = 0$ and therefore $\chi(\mathcal{O}_S) = 1$. Since $h_{(1,0)}$ and $h_{(2,0)}$ are birational invariants, the number of blow–ups which depends on the triangulation of the resolution is irrelevant here. $\mathbf{P}^1 \times T^2$ has $h_{(1,0)} = 1,\ h_{(2,0)} = 0$, T^4 has $h_{(1,0)} = 2,\ h_{(2,0)} = 1$, which both results in $\chi(\mathcal{O}_S) = 0$. $K3$ has $h_{(1,0)} = 0,\ h_{(2,0)} = 1$ and therefore $\chi(\mathcal{O}_S) = 2$.

Since except for $T^6/\mathbf{Z}_2 \times \mathbf{Z}_2$, the F-theory lifts of these models are not known, it must be determined directly in type IIB which divisors contribute to the non–perturbative superpotential. Here, we make use of the index for the Dirac operator on the world–volume of the Euclidean $D3$–brane (3.1.8). The values of the index for the four divisor topologies arising from resolutions of toroidal orbifolds are given in table 4.1. The numbers in square brackets are the values of the index in the case that the corresponding zero modes have been lifted by flux, cf. table 3.1. We see thus that for the case that the $O7$–plane does not intersect the divisor, we never get a contribution, so we better seek an orientifold action which leads to many $O7$–plane solutions. $K3$ can contribute for the case that the $O7$ lies on top of the divisor if the $h^{(+)}_{(2,0)}$ zero modes are lifted by flux. In our set–up, the case that the $O7$ lies on top of the divisor cannot arise, since only the inherited divisors R_i can have the topology of $K3$, and these divisors are never wrapped by $O7$–planes. A divisor with the topology of T^4 can likewise never contribute. $\mathbf{P}^1 \times T^2$ can contribute in case of an intersection with the $O7$–plane in one direction if no zero modes are lifted by flux. The rational surfaces always contribute except if there is no intersection irrespective of the background flux. To summarize: All those models are likely to allow the stabilization of all geometric moduli for which

(i) the fixed points and fixed lines are all in equivalence classes with only one member, giving rise to E and D divisors which are birationally equivalent to rational surfaces and

(ii) an orientifold action exists which gives rise to enough $O7$–plane solutions that each divisor intersects an $O7$–plane in at least one complex dimension.

When these conditions are met, it is likely that all geometric moduli will be stabilized when the full scalar potential is minimized.

Requirements (i) and (ii) are both met by $T^6/\mathbf{Z}_4$ on $SU(4)^2$, $T^6/\mathbf{Z}_{6-II}$ on $SU(2)\times SU(6)$, $T^6/\mathbf{Z}_2\times\mathbf{Z}_2$ and $T^6/\mathbf{Z}_2\times\mathbf{Z}_4$, therefore we expect that all geometric moduli can be stabilized in these cases.

Models with fixed lines without fixed points on them which lie in orbits of length greater than one do not satisfy criterion (i) since the divisors corresponding to these fixed lines have the topology of $\mathbf{P}^1\times T^2$. These are exactly the models with $h^{(2,1)}_{twist.}\neq 0$. Unless an elaborate configuration of O–planes can be chosen such that all these divisors intersect on $O7$–plane along one dimension, these examples in general allow only for a partial stabilization of the geometric moduli via Euclidean $D3$–brane instantons. It should be stressed that examples like these are still not completely hopeless since additional effects might lead to the complete stabilization of all moduli. On the other hand, this survey again confirms the old suspicion that manifolds with the right geometrical properties to allow the stabilization of all Kähler moduli by Euclidean $D3$–brane instantons or gaugino condensates are not very generic.

So far, we discussed the conditions for a contribution to the non–perturbative superpotential from Euclidean $D3$–instantons. Since we cancel the $O7$–tadpole by placing $D7$–branes on top of the $O7$–planes, a gaugino condensate can arise on the world–volume of the $D7$–branes. As mentioned before, for a contribution to the non–perturbative superpotential to arise from a gaugino condensate, we should have

(a) no bi-fundamental matter. This is given when the different divisors on which $D7$–branes are wrapped do not intersect. This condition can be easily checked by inspection of the toric diagram of the resolved patches.

(b) no adjoint matter. This depends on the Hodge numbers of the divisor which is wrapped by the brane. For rational surfaces, i.e. $h_{(1,0)}=h_{(2,0)}=0$, this criterion is fulfilled.

In the following, moduli stabilization will be discussed in detail for the two examples $T^6/\mathbf{Z}_{6-II}$ on $SU(2)\times SU(6)$ and $T^6/\mathbf{Z}_2\times\mathbf{Z}_4$ on $SU(2)^2\times SO(5)^2$. In Section 4.3 stabilization of the dilaton and complex structure moduli through 3–form flux G_3 is discussed and in Section 4.4, the stabilization of the Kähler moduli.

4.3 Complex structure and dilaton stabilization through 3–form flux

For the orbifolds X_6 with $h^{(-)}_{(2,1)}(X_6)=1$ the Kähler potential for the dilaton and complex structure modulus ($U\equiv U^3$) (2.1.11) is:

$$K_0=-\log(S+\bar{S})-\log(U+\bar{U})\ , \tag{4.3.12}$$

while the tree–level superpotential (4.1.3) may be written as

$$W_0 = A + B\ S + U\ (C + D\ S)\ , \tag{4.3.13}$$

with $A, B, C, D \in \mathbf{C}$ to be specified later. With the F–terms

$$\begin{aligned}\overline{F}^{\overline{S}} &= \left(\tfrac{S+\overline{S}}{U+\overline{U}}\right)^{1/2}\ \left[\ -A + B\ \overline{S} - U\ (C - D\ \overline{S})\ \right]\ ,\\ \overline{F}^{\overline{U}} &= \left(\tfrac{U+\overline{U}}{S+\overline{S}}\right)^{1/2}\ \left[\ -A - B\ S + \overline{U}\ (C + D\ S)\ \right]\ ,\end{aligned} \tag{4.3.14}$$

we may cast the scalar potential

$$V = g_{S\overline{S}}\ F^S \overline{F}^{\overline{S}} + g_{U\overline{U}}\ F^U \overline{F}^{\overline{U}} - 3\ e^{K_0}\ |W_0|^2$$

into the form:

$$\begin{aligned} V &= \tfrac{1}{U+\overline{U}}\tfrac{1}{S+\overline{S}}\ \left[\ |\ A - B\ \overline{S} + U\ (C - D\ \overline{S})\ |^2 + |\ A + B\ S - \overline{U}\ (C + D\ S)\ |^2\right.\\ &\qquad \left. -3\ |\ A + B\ S + U\ (C + D\ S)\ |^2\ \right]\ . \end{aligned} \tag{4.3.15}$$

The extremal points in the moduli space (S, U) are determined by the solutions of the equations $F^S, F^U = 0$:

$$s_2 = \frac{i}{2}\ \frac{\overline{B}\ C - B\ \overline{C} - \overline{A}\ D + A\ \overline{D}}{\overline{B}\ D + B\ \overline{D}}\quad ,\quad u_2 = \frac{i}{2}\ \frac{-\overline{B}\ C + B\ \overline{C} - \overline{A}\ D + A\ \overline{D}}{\overline{C}\ D + C\ \overline{D}}\ , \tag{4.3.16}$$

and similarly for the real parts s_1, u_1.

The 3–form flux $G_3 = F_3 + i\ S\ H_3$

$$\begin{aligned}\frac{1}{(2\pi)^2\alpha'}\ G_3 &= \sum_{i=0}^{3}\left[\ (a^i + i\ S\ c^i)\ \alpha_i + (b_i + i\ S\ d_i)\ \beta^i\ \right]\\ &\quad + \sum_{j=1}^{6}\left[(e^j + i\ S\ g^j)\ \gamma_j + (f_j + i\ S\ h_j)\ \delta^j\right]\end{aligned} \tag{4.3.17}$$

entering (4.1.3) is given as linear combination w.r.t. the integer cohomology basis $\{\alpha_i, \beta^i\}_{i=0,\ldots,3}$ and $\{\gamma_j, \delta^j\}_{j=1,\ldots,6}$ [24]. This gives rise to 20 real flux components to be constrained by the respective orbifold group $\mathbf{Z}_N$. This allows to express the complex parameters A, B, C, D through the eight integers $a^0, a^1, b_0, b_1, c^0, c^1, d_0, d_1$. For more details cf. [24]. The F–flatness conditions $F^S, F^U = 0$ force the complex structure to align such, that the flux G_3

$$\begin{aligned}\frac{1}{(2\pi)^2\alpha'}\ G_3 &= \tfrac{i}{2\ \mathrm{Re}(U)}\ \left\{\ \left[\ \overline{A} - \overline{B}\ S + \overline{U}\ (\overline{C} - \overline{D}\ S)\ \right]\ dz^1 \wedge dz^2 \wedge dz^3\right.\\ &\quad - [\ A + B\ S + U\ (C + D\ S)\ ]\ d\overline{z}^1 \wedge d\overline{z}^2 \wedge d\overline{z}^3\\ &\quad + \left[\ A + B\ S - \overline{U}\ (C + D\ S)\ \right]\ d\overline{z}^1 \wedge d\overline{z}^2 \wedge dz^3\\ &\quad \left. - \left[\ \overline{A} - \overline{B}\ S - U\ (\overline{C} - \overline{D}\ S)\ \right]\ dz^1 \wedge dz^2 \wedge d\overline{z}^3\ \right\}\end{aligned} \tag{4.3.18}$$

becomes ISD, i.e. it has only $(2,1)$ and $(0,3)$–components at the extremum. The flux G_3 induces the contribution of

$$N_{flux} = \frac{1}{(2\pi)^4\alpha'^2} \int_{Y_6} F_3 \wedge H_3 \qquad (4.3.19)$$

to the total $D3$–brane charge (2.1.15). Generically, this integral is calculated in the orientifold cover Y_6. Therefore the number N_{flux} has to be twice the negative value of the total $D3$–brane charge (2.1.15), i.e.

$$N_{flux} = -2\ Q_{3,tot} \qquad (4.3.20)$$

to cancel the latter by flux only.

<u>(i) $\mathbf{Z}_{6-II}$ – orbifold on the $SU(2)\times SU(6)$ lattice :</u>

The $\mathbf{Z}_{6-II}$–orbifold has the action $(v^1, v^2, v^3) = (\frac{1}{6}, \frac{1}{3}, -\frac{1}{2})$. The 3–form flux (4.3.17) constrained by the $\mathbf{Z}_{6-II}$–orbifold group becomes:

$$\begin{aligned}\frac{1}{(2\pi)^2\alpha'}\, G_3 = \ & \tfrac{1}{3}\,(a_0 + iSc_0)\,(3\,\alpha_0 + 2\,\beta_3 + \gamma_1 - 2\gamma_2 - 2\,\gamma_3 + \gamma_4 - \delta_5)\\ & +(b_0 + iSd_0)\,(-\alpha_3 + \beta_0 + \gamma_5 - \gamma_6)\\ & +\tfrac{1}{2}\,(b_1 + iSd_1)\,(2\,\beta_1 + \beta_2 + \delta_1 - \delta_2 - 2\,\delta_3 - \delta_4)\\ & +(a_1 + iSc_1)\,(\alpha_1 + \alpha_2 + \beta_3 - \gamma_2 - \gamma_3 - \delta_6)\ . \end{aligned} \qquad (4.3.21)$$

This flux correspond to the flux number:

$$N_{\rm flux} = 2\ b_0\ c_0 + b_1\ (c_0 + 3\ c_1) - 2\ a_0\ d_0 - d_1\ (a_0 + 3\ a_1)\ . \qquad (4.3.22)$$

For the $\mathbf{Z}_{6-II}$ orbifold with $SU(2)\times SU(6)$ lattice the coefficients A, B, C, D entering (4.3.15) become:

$$\begin{aligned} A &= -\frac{\sqrt{3}}{2}\, b_1 + ib_0 \quad , \quad B = -d_0 - \frac{\sqrt{3}\,i}{2} d_1\ ,\\ C &= a_0 + i\left(\frac{a_0}{\sqrt{3}} + \sqrt{3}a_1\right) \quad , \quad D = -\left(\frac{c_0}{\sqrt{3}} + \sqrt{3}c_1\right) + ic_0\ . \end{aligned} \qquad (4.3.23)$$

With this information, eq. (4.3.13) yields the superpotential:

$$\begin{aligned} W_0 &= -\frac{\sqrt{3}}{2}\, b_1 + i\ b_0 - S\ \left(d_0 + \frac{\sqrt{3}i}{2}\ d_1\right)\\ & +U\ \left[\ a_0 + i\left(\frac{a_0}{\sqrt{3}} + \sqrt{3}\ a_1\right)\ \right] - S\ U\ \left(\frac{c_0}{\sqrt{3}} + \sqrt{3}\ c_1 - i\ c_0\right)\ . \end{aligned} \qquad (4.3.24)$$

Since the total $D3$–brane charge in the CY orientifold is $Q_{3,tot} = -22$ (see Section 4.4), we look for fluxes (4.3.21) with $N_{flux} = 44$ on the covering space Y_6. Furthermore, the fields

$(a^0,\ b_0,\ c^0,\ d_0,\ a^1,\ b_1,\ c^1,\ d_1)$	s_1	s_2	u_1	u_2	m_S	m_U
(-5 , 12 , 0 , 2 , -4 , -8 , -1 , 0)	3.15788	5.83333	1.26315	0.0666667	2.18	13.68
(-5 , 10 , 0 , 2 , -3 , -8 , -1 , 0)	3.15788	4.83333	1.26315	0.0666667	2.18	13.68
(-5 , 6 , 0 , 2 , -1 , -8 , -1 , 0)	3.15788	2.83333	1.26315	0.0666667	2.18	13.68
(-5 , 0 , 0 , 2 , 2 , -8 , -1 , 0)	3.15788	-0.166667	1.26315	0.0666667	2.18	13.68
(-5 , -4 , 0 , 2 , 4 , -8 , -1 , 0)	3.15788	-2.16667	1.26315	0.0666667	2.18	13.68
(-5 , -8 , 0 , 2 , 6 , -8 , -1 , 0)	3.15788	-4.16667	1.26315	0.0666667	2.18	13.68
(-5 , -12 , 0 , 2 , 8 , -8 , -1 , 0)	3.15788	-6.16667	1.26315	0.0666667	2.18	13.68
(5 , 10 , 0 , -2 , -7 , 8 , 1 , 0)	3.15788	-5.16667	1.26315	0.0666667	2.18	13.68
(5 , 8 , 0 , -2 , -6 , 8 , 1 , 0)	3.15788	-4.16667	1.26315	0.0666667	2.18	13.68
(5 , 6 , 0 , -2 , -5 , 8 , 1 , 0)	3.15788	-3.16667	1.26315	0.0666667	2.18	13.68
(5 , 2 , 0 , -2 , -3 , 8 , 1 , 0)	3.15788	-1.16667	1.26315	0.0666667	2.18	13.68

Table 4.2: Discrete landscape of supersymmetric AdS minima for $N_{flux} = 44$ $e^{K_0/2}\ |W_0| = 0.34864$ and $V_0 = -0.364644$.

$(a^0, b_0, c^0, d_0, a^1, b_1, c^1, d_1)$	s_1	s_2	u_1	u_2	$-V_0$	$e^{K/2}\ \lvert W_0\rvert$	m_S	m_U
(-2, -7, -1, 2, 3, -8, 0, -2)	3.8092	-0.3	2.11622	0.722222	0.02273	0.087046	1.52	4.91
(0, -10, -1, -1, 5, -3, 1, -2)	3.7944	3.95	1.2648	0.316667	0.278999	0.304958	1.51	13.67
(0, -10, -1, 2, 3, -6, 0, -2)	3.7934	-1.9	2.10745	0.944444	0.296119	0.314176	1.51	4.92
(2, -10, -1, -1, 3, -1, 1, -2)	3.7918	2.65	1.26392	0.55	0.324662	0.328969	1.51	13.67
(0, -10, -1, -1, 6, -6, 1, -2)	3.7296	4.7	1.036	0.305556	1.40124	0.683432	1.51	19.86
(0, -10, -1, -1, 4, 0, 1, -2)	3.7095	3.2	1.5456	0.333333	1.75049	0.763869	1.51	8.87
(5, -9, -1, -1, 0, 3, 1, -2)	3.6575	0.35	1.21918	0.783333	2.64978	0.93982	1.51	13.95

Table 4.3: Supersymmetric AdS minima in the (S, U)–space for $N_{flux} = 44$ and specific $e^{K_0/2}\ |W_0|$

$S = s_1 + is_2$ and $U = u_1 + iu_2$ should be fixed (cf. (4.3.16)) to realistic values. A reasonable value for ReS is $s_1 \sim 3.6$, which corresponds to a string coupling constant $g_{\rm string} \sim 0.27$ at the string scale. Besides, the complex structure modulus U is expected to be around the ρ–point in the fundamental region, with $\rho = \frac{1}{2} + \frac{i}{2}\sqrt{3}$. An additional constraint may be imposed on the tuning parameter $e^{K_0}|W_0|^2$, which should be small to avoid higher order effects in the full non–perturbative superpotential (4.1.2). After a systematic scan in the flux space $(a^0, a^1, b_0, b_1, c^0, c^1, d_0, d_1) \in \mathbf{Z}^8$ we find hundreds of vacua, which meet these criteria. A set of equivalent vacua, differing only in the discrete flux parameters $(a^0, a^1, b_0, b_1, c^0, c^1, d_0, d_1)$, is given in the table 4.2.

Clearly, the axionic VEV s_2 may be shifted back into the fundamental region $s_2 \equiv -0.166667$, while the flux number N_{flux} in (4.3.19) and K_0, W_0 are preserved [84]. Furthermore, in table 4.3 we present a set of supersymmetric AdS minima in the (S, U)–space with different tuning parameters $e^{K_0}|W_0|^2$.

(ii) $\mathbf{Z}_2 \times \mathbf{Z}_4$ – orbifold on the $SU(2)^2 \times SO(5)^2$ lattice :

The $\mathbf{Z}_2 \times \mathbf{Z}_4$–orbifold has the two actions $(v^1, v^2, v^3) = \frac{1}{2}(1, 0, -1)$ and $(w^1, w^2, w^3) = \frac{1}{4}(0, 1, -1)$. The 3–form flux (4.3.17) constrained by the $\mathbf{Z}_2 \times \mathbf{Z}_4$–orbifold group becomes:

$$\begin{aligned} \frac{1}{(2\pi)^2\alpha'} G_3 &= (a_3 + iSc_0)(-\alpha_2 + \alpha_3) + (a_0 + iSc_0)\left(\alpha_0 - \alpha_2 - \frac{1}{2}\beta_1\right) \\ &\quad + (b_2 + iSd_2)\left(\alpha_1 + \frac{1}{2}\beta_0 + \beta_2\right) + (b_3 + iSd_3)\left(\alpha_1 + \frac{1}{2}\beta_0 + \beta_3\right) . \end{aligned} \tag{4.3.25}$$

The coefficients A, B, C, D entering (4.3.15) are given in the case of $\mathbf{Z}_2 \times \mathbf{Z}_4$–orbifold with $SU(2)^2 \times SO(5)^2$ lattice by:

$$\begin{aligned} A &= -\tfrac{1+2}{2}\,(b_2 - i\ b_3) \quad , \quad B = \tfrac{1-i}{2}\,(d_2 - i\ d_3)\ , \\ C &= \tfrac{1+i}{2} a_0 + a_3 \quad , \quad D = \tfrac{-1+i}{2} c_0 + i\ c_3\ . \end{aligned} \tag{4.3.26}$$

Furthermore, the flux number is:

$$N_{\text{flux}} = a_3 d_2 - b_2 c_3 + b_3\ (c_0 + c_3) - (a_0 + a_3)\ d_3\ . \tag{4.3.27}$$

With this information, the superpotential (4.3.13) becomes:

$$\begin{aligned} W_0 &= -\frac{1+i}{2}\ (b_2 - i\ b_3) + S\ \frac{(1-i)}{2}\ (d_2 - i\ d_3) \\ &\quad + U\left(\frac{1+i}{2}\ a_0 + a_3\right) + S\ U\ \left(\frac{-1+i}{2}\ c_0 + i\ c_3\right) . \end{aligned} \tag{4.3.28}$$

We search for fluxes (4.3.25) with $N_{flux} = 52$. We fix the value of the s_1 at 3.24, which corresponds to a string coupling constant $g_{\text{string}} = 0.30$ at the string scale. A set of equivalent vacua, differing only in the discrete flux parameters $(a^0, b_2, c^0, d_2, a^3, b_3, c^3, d_3)$, is given in the table 4.4.

In the next table, we present a set of supersymmetric AdS minima in the (S, U)-space with same tuning parameter $e^{K_0}|W_0|^2$, but different choices for S and U.

(iii) $\mathbf{Z}_4$ – orbifold on the $SU(4)^2$ lattice :

The $\mathbf{Z}_4$–orbifold has the action $(v^1,\ v^2,\ v^3) = (\frac{1}{4},\ \frac{1}{4},\ -\frac{1}{2})$. The 3–form flux (4.3.17) constrained by the $\mathbf{Z}_4$–orbifold group becomes

$$\begin{aligned} \frac{1}{(2\pi)^2\alpha'} G_3 &= (a_0 + iSc_0)\ (\alpha_0 + \alpha_3 + \beta_2 - \gamma_2 - \gamma_3 - \delta_5) \\ &\quad + \frac{1}{2}\ (b_0 + iSd_0)\ (-\alpha_2 + 2\beta_0 + \beta_3 + \gamma_4 + \gamma_5 - \gamma_6 + \delta_3) \\ &\quad + \frac{1}{2}\ (b_1 + iSd_1)\ (\alpha_2 + 2\beta_1 + \beta_3 - \gamma_4 + \gamma_5 - \gamma_6 - 2\delta_2 - \delta_3) \\ &\quad + (a_1 + iSc_1)\ (\alpha_1 + 2\alpha_3 + \beta_2 - \gamma_2 - 2\gamma_3 + \delta_4 - \delta_5 - \delta_6)\ . \end{aligned} \tag{4.3.29}$$

$(a^0,\ b_2,\ c^0,\ d_2,\ a^3,\ b_3,\ c^3,\ d_3)$	s_1	s_2	u_1	u_2	m_S	m_U
(14 , 14 , 0 , 1 , -3 , 14 , -2 , -5)	3.23796	-1.875	0.925131	-1.46429	2.46	30.21
(14 , 15 , 0 , 1 , -3 , 15 , -2 , -5)	3.23796	-2.125	0.925131	-1.53571	2.46	30.21
(-8 , 8 , 4 , 1 , 11 , 8 , -2 , -3)	3.23796	-1.875	0.925131	0.535714	2.46	30.21
(-14 , 14 , 0 , 5 , 11 , 14 , 2 , -1)	3.23796	1.875	0.925131	1.46429	2.46	30.21
(-14 , 15 , 0 , 5 , 11 , 15 , 2 , -1)	3.23796	2.125	0.925131	1.53571	2.46	30.21
(8 , 8 , 4 , 3 , 3 , 8 , -2 , -1)	3.23796	1.875	0.925131	-0.535714	2.46	30.21
(14 , -16 , 0 , -3 , -19 , -16 , -2, -1)	3.23796	6.125	0.925131	0.535714	2.46	30.21
(14 , -15 , 0 , -3 , -19 , -15 , -2 , -1)	3.23796	5.875	0.925131	0.464286	2.46	30.21
(14 , -14, 0, -5, -11, -14, -2, 1)	3.23796	1.875	0.925131	1.46429	2.46	30.21

Table 4.4: Discrete landscape of supersymmetric AdS minima for $N_{flux} = 52$, $e^{K_0/2}\ |W_0| = 0.310374$ and $V_0 = -0.288997$.

$(a^0,\ b_2,\ c^0,\ d_2,\ a^3,\ b_3,\ c^3,\ d_3)$	s_1	s_2	u_1	u_2	m_S	m_U
(-13 , 17 , 3 , 1 , 11 , 17 , -1 , -5)	3.70	-2.85714	1.29518	1.85	1.88	15.42
(-10 , 16 , 3 , 1 , 9 , 16 , -1 , -5)	3.70	-2.14286	1.52375	1.82353	1.88	11.14
(9 , 15 , 1 , 3 , -1 , 15 , -2 , -5)	3.70	-1.14286	1.52375	-2.17647	1.88	11.14
(-14 , 20 , 3 , -1 , 16 , 20 , -2 , -4)	3.70	-6.14286	1.29518	1.15	1.88	15.41
(-9 , 15 , 1 , 5 , 8 , 15 , 1 , -3)	3.70	1.14286	1.52375	2.17647	1.88	11.14
(-4 , 15 , 2 , 2 , 6 , 15 , -1 , -5)	3.70	-1.85714	3.23796	1.625	1.87	2.48
(2 , 20 , 1 , -1 , 13 , 20 , -2 , -3)	3.70	-8.14286	1.52375	-0.176471	1.88	11.14
(-6 , 20 , 2 , -1 , 12 , 20 , -2 , -4)	3.24	-5.875	2.15864	0.583333	2.46	5.56
(-5, 10 , 3 , 2 , 7 , 10 , -1 , -4)	3.24	-0.875	1.61898	1.0625	2.46	9.87
(3 , 11 , 2 , 2 , 3 , 11 , -2 , -4)	3.24	-1.375	2.15864	-0.916667	2.46	5.56
(-2 , 15 , 2 , 3 , 4 , 15 , -1 , -5)	3.24	-0.875	4.31728	1.16667	1.38	2.47
(-6 , 14 , 2 , 4 , 6 , 14 , 0 , -5)	3.24	-0.125	2.15864	2.41667	2.46	5.56

Table 4.5: Supersymmetric AdS minima in the (S,U)–space for $N_{flux} = 52$, $e^{K_0/2}\ |W_0| = 0.310374$ and $V_0 = -0.28900$.

In the case of the $\mathbf{Z}_4$–orbifold with the lattice $SU(4)^2$ the coefficients A, B, C, D entering (4.3.15) are given by:

$$\begin{aligned} A &= -b_1 + i\ b_0 \quad , \qquad B = -d_0 - i\ d_1\ , \\ C &= a_0 + i\ (a_0 + 2a_1) \quad , \quad D = -c_0 - 2c_1 + i\ c_0\ . \end{aligned} \tag{4.3.30}$$

Furthermore, the flux number is:

$$N_{\rm flux} = 2\ [\ (b_0 + b_1)\ c_0 + b_1 c_1 - 2a_1 d_1 - (d_0 + d_1)\ a_0\ ]\ . \tag{4.3.31}$$

With this information the superpotential (4.3.13) becomes:

$$\begin{aligned} W_0 &= -b_1 + i\ b_0 - S\ \ (d_1 + i\ d_0) \\ &\quad +U\ \ [\ a_0 + i\ (a_0 + 2\ a_1)\ ] - S\ U\ \ (c_0 + 2\ c_1 - i\ c_0)\ \ . \end{aligned} \tag{4.3.32}$$

4.4 Kähler moduli stabilization

We consider the racetrack superpotential (4.1.2)

$$W = \widetilde{W}_0 + \sum_{j=1}^{h^{(+)}_{(1,1)}(X_6)} \gamma_j \; e^{a_j \; T^j} \; , \tag{4.4.33}$$

with $\widetilde{W}_0$ related to the tree–level flux superpotential (4.3.13), by $\widetilde{W}_0 = -e^{K_0/2}|W_0|$. The redefined quantity $\widetilde{W}_0$ makes sure, that the minimization procedure w.r.t. the set of Kähler moduli $T^1, \ldots, T^n$ in the Kähler gauge $K_0 \equiv 0$ yields the correct value $-3e^K|\widetilde{W}_0|^2 = -3e^{K_0+K}|W_0|^2$ in the scalar potential (4.1.11). This value accounts for the contribution of the dilaton and complex structure stabilization procedure, which is decoupled and performed in the previous section. Here and in the following K is the Kähler potential (4.1.1) for the $n = h^{(+)}_{(1,1)}(X_6)$ Kähler moduli $T^1, \ldots, T^n$. According to Section 4.1, we may assume $\gamma_j \in \mathbf{R}^+$, i.e. any complex phase of γ_j has been put into the axionic VEVs (4.1.8) of the Kähler moduli T^j. The supersymmetric vacua are given by the equations (4.1.6), i.e. by the critical points of $e^{K/2}W$. These equations fix the real part of the Kähler moduli T^j, i.e. the divisor volumes $\mathrm{Vol}(\mathcal{D}_i)$ of an even divisor $\mathcal{D}_j$:

$$\mathrm{Vol}(\mathcal{D}_j) = \mathrm{Re}(T^j) = \frac{3}{4}\; \mathcal{K}_{ijk}\; t^j\; t^k = \frac{3}{2}\; \frac{\partial}{\partial t^j}\; \mathrm{Vol}(X_6) \; . \tag{4.4.34}$$

To ignore α'–corrections, the Kähler moduli T^j or divisor volumes $\mathrm{Vol}(\mathcal{D}_j)$ should be stabilized at large values, resulting in a large CY volume $\mathrm{Vol}(X_6)$. The F–flatness conditions (4.1.6) roughly give rise to the relations $\widetilde{W}_0 \sim \gamma_j\; e^{a_j\; T^j}$. Hence, a smaller $\widetilde{W}_0$ or larger coefficients γ_j yield larger divisor volumes $\mathrm{Re}T^j$. Hence a small $\widetilde{W}_0$ or large divisor volumes guarantee that α'–corrections may be neglected. In (4.1.2), the exponentials $e^{a_j\; T^j}$ should be small $\sim \mathcal{O}(10^{-4})$, such that multi–instanton processes or multi–wrappings may be neglected. In principle, this means that $\widetilde{W}_0$ should be also of this order $\sim 10^{-4}$ [61]. Furthermore, in ref. [61] it has been argued that due to the smallness of these exponentials a dependence of the coefficients γ_j on the dilaton S and complex structure moduli U^j does not change the critical points of the dilaton and complex structure moduli much, as derived in the previous section, as long as the relative derivatives $\gamma_j^{-1}\; \partial_{S,U^\lambda}\gamma_j$ and $\gamma_j^{-1}\; \partial^2_{S,U^\lambda}\gamma_j$ are not huge.

<u>(i) $\mathbf{Z}_{6-II}$ – orbifold on the $SU(2) \times SU(6)$ lattice :</u>

We consider the resolved $\mathbf{Z}_{6-II}$ orbifold Y_6 on the lattice $SU(2) \times SU(6)$ which has $h_{(1,1)}(Y_6) = 25$.

The configuration of the fixed point set is displayed in figure 4.1 in a schematic way, where each complex coordinate is shown as a coordinate axis and the opposite faces of the resulting cube of length 1 are identified. We see that there are 12 local $\mathbf{C}^3/\mathbf{Z}_{6-II}$ patches

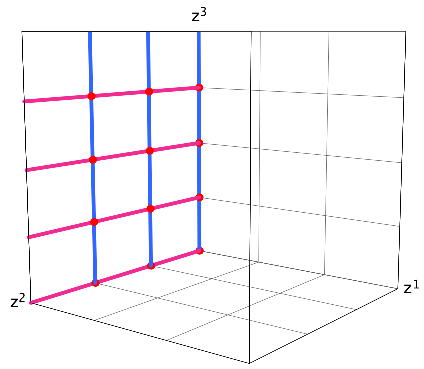

Figure 4.1: Schematic picture of the fixed set configuration of $\mathbf{Z}_{6-II}$ on $SU(2) \times SU(6)$

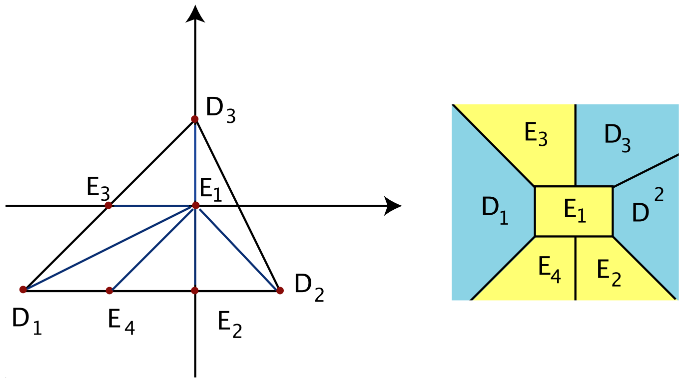

Figure 4.2: Toric diagram of the resolution of $\mathbf{C}^3/\mathbf{Z}_{6-II}$ and dual graph

which each sit at the intersection of two fixed lines, 3 $\mathbf{C}^2/\mathbf{Z}_3$ fixed lines in the z^3 direction originating from the order three element θ^2 and 4 $\mathbf{C}^2/\mathbf{Z}_2$ fixed lines in the z^2 direction originating from the order two element θ^3. The resolution of the $\mathbf{C}^3/\mathbf{Z}_{6-II}$ singularity is described by the toric diagram in figure 4.2. From these two figures, we can read off the exceptional divisors [34], which together with the inherited divisors R_i form a basis for $H^{(1,1)}(X_6)$:

$$R_1 \quad , \quad R_2 \quad , \quad R_3 \quad , \quad E_{1,\beta\gamma} \quad , \quad E_{3,\gamma} \quad , \quad E_{2,\beta} \quad , \quad E_{4,\beta} \ , \tag{4.4.35}$$

with $\beta = 1,2,3$, $\gamma = 1,\ldots,4$. In addition, the orbifold fixed points give rise to the eight divisors $D_1, D_{2,\beta}$ and $D_{3,\gamma}$. The topologies of these divisors were determined in [34]. $E_{1,\beta\gamma}$ is a blow–up of $\mathbf{F}_1$ in two points, while the remaining exceptional divisors $E_{2,\beta}, E_{3,\gamma}, E_{4,\beta}$ are all $\mathbf{P}^1 \times \mathbf{P}^1$. $D_1, D_{2\beta}, D_{3\gamma}$ are blow–ups of $\mathbf{P}^1 \times \mathbf{P}^1$ in 12, 8, and 9 points, respectively. Finally, R_1 is a T^4 and R_2, R_3 are K3 surfaces.

Any orientifold projection $\mathcal{O}$ with $O3$– and $O7$–planes is such that the twelve divisors $E_{1,2\gamma}$, $E_{1,3\gamma}$, $E_{2,2}$, $E_{2,3}$, $E_{4,2}$, and $E_{4,3}$ are not invariant under the orientifold action σ [34]:

$$\begin{aligned} \sigma\ E_{1,2\gamma} &= E_{1,3\gamma} \quad , \quad \sigma\ E_{1,3\gamma} = E_{1,2\gamma} \ , \\ \sigma\ E_{2,2} &= E_{2,3} \quad , \quad \sigma\ E_{2,3} = E_{2,2} \ , \end{aligned}$$

$$\sigma\ E_{4,2} \ = \ E_{4,3} \quad , \quad \sigma\ E_{4,3} = E_{4,2}\ , \tag{4.4.36}$$

and the six pairs of eigenstates $(E,\widetilde{E})$ under σ have to be constructed:

$$\begin{aligned} E_{1,\gamma} &:= \frac{1}{2}\ (\ E_{1,2\gamma}+E_{1,3\gamma}\ ) \quad , \quad \widetilde{E}_{1,\gamma} := \frac{1}{2}\ (\ E_{1,2\gamma}-E_{1,3\gamma}\ )\ ,\\ E_{2} &:= \frac{1}{2}\ (\ E_{2,2}+E_{2,3}\ ) \quad , \quad \widetilde{E}_{2} := \frac{1}{2}\ (\ E_{2,2}-E_{2,3}\ )\ ,\\ E_{4} &:= \frac{1}{2}\ (\ E_{4,2}+E_{4,3}\ ) \quad , \quad \widetilde{E}_{4} := \frac{1}{2}\ (\ E_{4,2}-E_{4,3}\ )\ . \end{aligned} \tag{4.4.37}$$

As a consequence, we have $h^{(-)}_{(1,1)}(X_6)=6$ (cf. also table 2.3). Furthermore, the divisors $D_{2,2}$ and $D_{2,3}$ are mapped to each other under σ and only the combination $D_2=\frac{1}{2}(D_{2,2}+D_{2,3})$ is invariant. To summarize, the orientifold action $\mathcal{O}$ splits the divisors (4.4.35) into the even divisors

$$H^{(4)}_{+}(X_6)\ \ni\ E_{1,1\gamma}\ ,\ E_{1,\gamma}\ ,\ E_{2,1}\ ,\ E_2\ ,\ E_{4,1}\ ,\ E_4\ ,\ E_{3,\gamma}\ ,\ D_1\ ,\ D_2\ ,\ D_{2,1}\ ,\ D_{3,\gamma}\ ,\ R_1,\ ,\ R_2\ ,\ R_3 \tag{4.4.38}$$

and into the odd divisors

$$H^{(4)}_{-}(X_6)\ \ni\ \widetilde{E}_2 \quad , \quad \widetilde{E}_4 \quad , \quad \widetilde{E}_{1,\gamma} \quad , \quad \widetilde{D}_2 \tag{4.4.39}$$

with $\gamma=1,\ldots,4$. We choose the orientifold action σ such that its fixed point set consists of seven $O7$–planes wrapped on the divisors $D_1, D_{3,\gamma}, E_{2,1}$ and E_2. In addition, there are twelve $O3$–planes at $z^2=0\ ,\ z^1\neq 0$. Because of $\chi(D_1)=16$, $\chi(D_{3,\gamma})=13$ and $\chi(E_{2,1})=\chi(E_2)=4$, the total $D3$–brane charge $Q_{3,tot}$ in eq. (2.1.15) is $Q_{3,tot}=-22$. The Poincaré dual 2–forms ω_i of the 19 invariant divisors represent a basis for the Kähler form J:

$$\begin{aligned} J = \quad & r_1\ R_1+r_2\ R_2+r_3\ R_3-t_2\ E_2-t_4\ E_4-t_{2,1}\ E_{2,1}-t_{4,1}\ E_{4,1}\\ & -\sum_{\gamma=1}^{4}(\ t_{1,\gamma}\ E_{1,\gamma}+t_{1,1\gamma}\ E_{1,1\gamma}+t_{3,\gamma}\ E_{3,\gamma}\ )\ , \end{aligned} \tag{4.4.40}$$

with the 19 Kähler coordinates $r_1,\ r_2,\ r_3,\ t_{1,\gamma},\ t_{1,1\gamma},\ t_2,\ t_{2,1},\ t_{3,\gamma},\ t_4,\ t_{4,1}$.

The volume $\mathrm{Vol}(X_6)=\frac{1}{6}\int_{X_6}J\wedge J\wedge J=\frac{1}{6}\ \mathcal{K}_{ijk}\ t^i\ t^j\ t^k$ of the CY orientifold X_6 becomes:

$$\begin{aligned} \mathrm{Vol}(X_6) \ =\ & 3r_1r_2r_3+r_3\left(t_{2,1}t_{4,1}+\frac{1}{2}t_2t_4\right)-\sum_{\gamma=1}^{4}\left(\ t_{1,1\gamma}t_{2,1}t_{4,1}+\frac{1}{4}t_{1,\gamma}t_2t_4\right)\\ & -\frac{1}{2}r_2\sum_{\gamma=1}^{4}t_{3,\gamma}^2-r_3\left(2t_{2,1}^2+\frac{1}{2}\,t_{4,1}^2+t_2^2+\frac{1}{4}t_4^2\right)-\frac{2}{3}\sum_{\gamma=1}^{4}t_{3,\gamma}^3\\ & -\frac{1}{2}\sum_{\gamma=1}^{4}\left(t_{1,1\gamma}^3+\frac{1}{4}t_{1,\gamma}^3\right)+2t_{2,1}^2t_{4,1}+\frac{1}{2}t_2^2t_4-\frac{4}{3}\left(4\,t_{2,1}^3+\frac{1}{2}\,t_{4,1}^3+t_2^3+\frac{1}{8}t_4^3\right) \end{aligned}$$

$$+\sum_{\gamma=1}^{4}\left(2t_{1,1\gamma}t_{2,1}^2+\frac{1}{2}t_{1,1\gamma}t_{3,\gamma}^2+\frac{1}{2}\,t_{1,1\gamma}t_{4,1}^2+\frac{1}{2}t_{1,\gamma}t_2^2+\frac{1}{2}t_{1,\gamma}t_{3,\gamma}^2+\frac{1}{8}t_{1,\gamma}t_4^2\right)\ , \tag{4.4.41}$$

where we plugged the intersection numbers K_{ijk} from the Ref [34].

According to eq. (4.4.34) or (4.4.41), the 19 divisor volumes are derived:

$$\begin{aligned}
\mathrm{Vol}(R_1) &= \frac{9}{2}\,r_2\,r_3 \quad , \quad \mathrm{Vol}(R_2)=\frac{9}{2}\,r_1\,r_3-\frac{3}{4}\,\sum_{\gamma=1}^{4}t_{3,\gamma}^2\ ,\\
\mathrm{Vol}(R_3) &= \frac{9}{2}\,r_1\,r_2+\frac{3}{2}\,t_{2,1}\,t_{4,1}+\frac{3}{4}\,t_2\,t_4-3\,t_{2,1}^2-\frac{3}{4}\,t_{4,1}^2-\frac{3}{2}\,t_2^2-\frac{3}{8}\,t_4^2\ ,\\
\mathrm{Vol}(E_{1,\gamma}) &= -\frac{9}{16}\,t_{1,\gamma}^2+\frac{3}{4}\,t_2^2+\frac{3}{16}\,t_4^2+\frac{3}{4}\,t_{3,\gamma}^2-\frac{3}{8}\,t_2\,t_4\ ,\\
\mathrm{Vol}(E_{1,1\gamma}) &= -\frac{9}{4}\,t_{1,1\gamma}^2+3\,t_{2,1}^2+\frac{3}{4}\,t_{4,1}^2+\frac{3}{4}\,t_{3,\gamma}^2-\frac{3}{2}\,t_{2,1}\,t_{4,1}\ ,\\
\mathrm{Vol}(E_2) &= \frac{3}{4}\,r_3\,t_4-3\,r_3\,t_2+\frac{3}{2}\,t_2\,t_4-6\,t_2^2-\frac{3}{8}\,\sum_{\gamma=1}^{4}(\,t_{1,\gamma}\,t_4-4t_2\,t_{1,\gamma}\,)\ ,\\
\mathrm{Vol}(E_{2,1}) &= \frac{3}{2}\,r_3\,t_{4,1}-6\,r_3\,t_{2,1}+6\,t_{2,1}\,t_{4,1}-24\,t_{2,1}^2-\frac{3}{2}\,\sum_{\gamma=1}^{4}(\,t_{1,1\gamma}\,t_{4,1}-4t_{2,1}\,t_{1,1\gamma}\,)\ ,\\
\mathrm{Vol}(E_{3,\gamma}) &= -\frac{3}{2}\,r_2\,t_{3,\gamma}-3\,t_{3,\gamma}^2+\frac{3}{2}\,t_{1,1\gamma}\,t_{3,\gamma}+\frac{3}{2}\,t_{1,\gamma}\,t_{3\gamma}\ ,\\
\mathrm{Vol}(E_4) &= \frac{3}{4}\,r_3\,t_2-\frac{3}{4}\,r_3\,t_4+\frac{3}{4}\,t_2^2-\frac{3}{4}\,t_4^2+\frac{3}{8}\,\sum_{\gamma=1}^{4}t_{1,\gamma}\,t_4-\frac{3}{8}\,\sum_{\gamma=1}^{4}t_{1,\gamma}\,t_2\ ,\\
\mathrm{Vol}(E_{4,1}) &= \frac{3}{2}\,r_3\,t_{2,1}-\frac{3}{2}\,r_3\,t_{4,1}+3\,t_{2,1}^2-3\,t_{4,1}^2+\frac{3}{2}\,\sum_{\gamma=1}^{4}t_{1,1\gamma}\,t_{4,1}-\frac{3}{2}\,\sum_{\gamma=1}^{4}t_{1,1\gamma}\,t_{2,1}\ .
\end{aligned} \tag{4.4.42}$$

The seven (invariant) planes $D_{i\alpha}$ localized at the fix points are given through the relations [34]:

$$\begin{aligned}
D_1 &= \frac{1}{3}\,(R_1-E_{2,1}-4\,E_{4,1}-2\,E_2-8\,E_{4,2}\,)-\frac{1}{3}\,\sum_{\gamma=1}^{4}(\,3\,E_{3,\gamma}+2\,E_{1,\gamma}+E_{1,1\gamma}\,)\ ,\\
D_2 &= \frac{1}{3}\,(\,R_2-E_2-E_4\,)-\frac{1}{3}\,\sum_{\gamma=1}^{4}E_{1,\gamma}\ ,\ D_{2,1}=\frac{1}{3}\,(\,R_2-E_{2,1}-E_{4,1}\,)-\frac{1}{3}\,\sum_{\gamma=1}^{4}E_{1,1\gamma}\ ,\\
D_{3,\gamma} &= R_3-E_{1,1\gamma}-2\,E_{1,\gamma}-E_{3,\gamma}\ .
\end{aligned} \tag{4.4.43}$$

In the superpotential (4.1.2), we have two sets of contributing divisors $\mathcal{D}_i$: On the seven divisors $\mathcal{D}_{D7}=\{D_1,D_{3,\gamma},E_{2,1},E_2\}$, a stack of one $O7$–plane and eight $D7$–branes is wrapped.

Gaugino condensation takes place in the $SO(8)$ gauge theory. Therefore, we have $a_j = -\frac{2\pi}{6}$ for the set $\mathcal{D}_{D7}$ of divisors contributing in (4.1.2). On the other hand, divisors in the set $\mathcal{D}_{D3} = \{D_2, D_{2,1}, E_{1,\gamma}, E_{1,1\gamma}, E_{3\gamma}, E_4, E_{4,1}\}$ can be wrapped by Euclidean $D3$–branes. Since all D and E divisors intersect one of the divisors carrying an $O7$–plane in at least one complex dimension (cf. figures 4.1 and 4.2), the condition $\chi_{D3} = 1$ for a non–vanishing instanton contribution in the superpotential (4.1.2) is always met for the set $\mathcal{D}_{D3}$ of divisors. In total we have 23 contributing divisors and the superpotential (4.1.2) reads:

$$W = W_0(S,U) + \sum_{\mathcal{D}_i \in \mathcal{D}_{D7}} e^{-2\pi \ \frac{\mathrm{Vol}(\mathcal{D}_i)}{6}} + \sum_{\mathcal{D}_i \in \mathcal{D}_{D3}} e^{-2\pi \ \mathrm{Vol}(\mathcal{D}_i)} \ . \tag{4.4.44}$$

Now we are ready to stabilize all 19 Kähler moduli $r_i, t_{1,\gamma}, t_{1,1\gamma}, t_2, t_{2,1}, t_{3,\gamma}, t_4$ and $t_{4,1}$. The $D3$–brane charge $Q_{3,tot} = -22$ is completely cancelled by the 3–form flux G_3, given in eq. (4.3.21). In fact, in the previous section we have presented critical points for the dilaton and complex structure moduli corresponding to a set of flux solutions, with $N_{flux} = 44$. For a $\widetilde{W}_0 = -0.34864$, corresponding to the critical points of table 4.2, we find the following 23 divisor volumes (measured in string units):

$$\begin{array}{ccc} \mathrm{Vol}(D_1) = 4.92087 \ , & \mathrm{Vol}(D_{2,1}) = 17.1883 \ , & \mathrm{Vol}(D_{2,2}) = 17.9329 \ , \\ \mathrm{Vol}(D_{3,\gamma}) = 35.1656 \ , & \mathrm{Vol}(E_2) = 3.55689 \ , & \mathrm{Vol}(E_4) = 0.710518 \ , \\ \mathrm{Vol}(E_{2,1}) = 4.84171 \ , & \mathrm{Vol}(E_{4,1}) = 0.922548 \ , & \mathrm{Vol}(E_{3,\gamma}) = 1.01315 \ , \\ \mathrm{Vol}(E_{1,1\gamma}) = 1.06872 \ , & \mathrm{Vol}(E_{1,\gamma}) = 0.884484 \ , & \gamma = 1, \ldots, 4 \ , \end{array} \tag{4.4.45}$$

corresponding to the sizes of the nineteen Kähler moduli:

$$\begin{array}{rcl} r_1 & = & 3.04765 \quad , \quad r_2 = 2.91779 \quad , \quad r_3 = 4.53928 \ , \\ t_{1,\gamma} & = & 1.52711 \quad , \quad t_{1,1\gamma} = 0.869367 \quad , \quad t_{3,\gamma} = 0.46524 \quad , \quad \gamma = 1, \ldots, 4 \ , \\ t_{2,1} & = & 0.443261 \quad , \quad t_2 = 0.663503 \quad , \quad t_4 = 0.967525 \quad , \quad t_{4,1} = 0.634432 \ . \end{array} \tag{4.4.46}$$

The divisor volumes give rise to the total volume $\mathrm{Vol}(X_6) = 115.94$. This is large enough, that one–loop (and higher loop) corrections to the Kähler potential are suppressed, with a string–coupling constant $g_{\mathrm{string}} \sim 0.3$ (cf. Section 4.3). Furthermore, the divisor volumes (4.4.45) are large enough to suppress higher order instanton effects (e.g. from multi–wrapped instantons) in (4.4.51), since $e^{-2\pi \mathrm{Vol}(E_2)/6} \sim 0.02$ and even smaller for the other divisors.

The six divisors (4.4.39) or their corresponding cohomology elements give rise to the non–vanishing bulk 2–forms B_2, C_2, with the twelve real scalars b^a, c^a. According to eq. (2.1.5) the latter are combined into the six complex scalars G^a, defined in eq. (2.1.5). Since we did not disscuss the methods for stabilizing $H^{(1,1)}_-$ moduli, we just mention that they are stabilized in this model at

$$b^a = 0 \ .$$

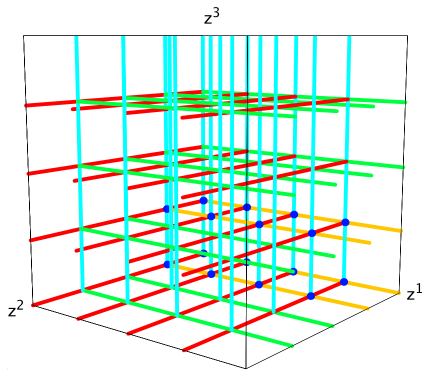

Figure 4.3: Schematic picture of the fixed set configuration of $\mathbf{Z}_2 \times \mathbf{Z}_4$

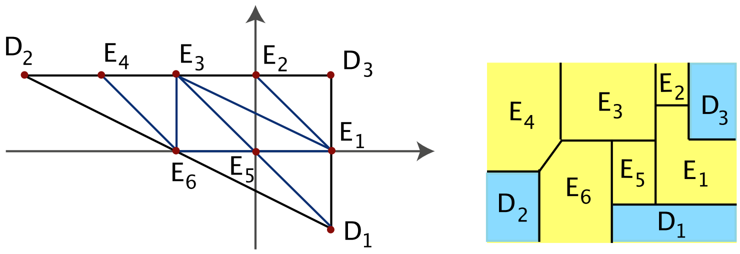

Figure 4.4: Toric diagram of two of the resolutions of $\mathbf{C}^3/\mathbf{Z}_2 \times \mathbf{Z}_4$ and dual graphs

For further details see ref. [25]. Furthermore, in [25] a mechanism has been proposed to also stabilize the fields c^a by turning on the 2–form flux ${}^Y f_2$ from the ambient space Y_6 and also the discussion at the end of Section 3.1. To this end, for the $\mathbf{Z}_6$ orbifold we have stabilized all 27 moduli fields.

(ii) $\mathbf{Z}_2 \times \mathbf{Z}_4$ – orbifold on the $SU(2)^2 \times SO(5)^2$ lattice :

As our second example, we consider the resolved $\mathbf{Z}_2 \times \mathbf{Z}_4$ orbifold Y_6 on the lattice $SU(2)^2 \times SO(5)^2$. This orbifold has $h_{(1,1)}(Y_6) = 61$ Kähler moduli. We summarize here the relevant data from [34].

The configuration of the fixed point set is displayed in figure 4.3 in a schematic way.

There are 16 local $\mathbf{C}^3/\mathbf{Z}_2 \times \mathbf{Z}_4$ patches. The resolution of the $\mathbf{C}^3/\mathbf{Z}_2 \times \mathbf{Z}_4$ singularity is described by the toric diagram in figure 4.4. There are four $\mathbf{C}^2/\mathbf{Z}_4$ fixed lines in the z^1 direction from the order four element θ^2. Furthermore, there are $12 + 12 + (10 - 4) = 30$ $\mathbf{C}^2/\mathbf{Z}_2$ fixed lines from the order two elements: From θ^1, $\theta^1(\theta^2)^2$, and $(\theta^2)^2$ in the z^2, z^3, and z^1 direction, respectively. The intersection points of three $\mathbf{Z}_2$ fixed lines are locally described by the resolved $\mathbf{C}^3/\mathbf{Z}_2 \times \mathbf{Z}_2$ patches.

The resolution is described by the toric diagram in figure 4.5. From these two figures, we can read off the exceptional divisors [34], which together with the inherited divisors R_i

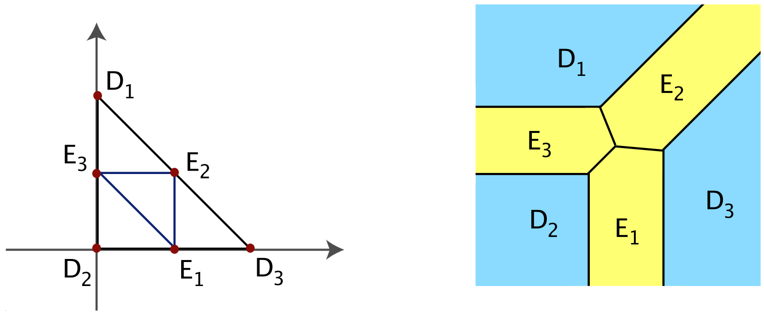

Figure 4.5: Toric diagram of the resolution of $\mathbf{C}^3/\mathbf{Z}_2 \times \mathbf{Z}_2$ and its dual graph

form a basis for $H^{(1,1)}(Y_6)$:

$$R_1 \ , \ R_2 \ , \ R_3 \ , \ E_{1,\alpha\gamma'} \ , \ E_{2,\beta\gamma} \ , \ E_{3,\mu} \ , \ E_{4,\beta\gamma} \ , \ E_{5,\alpha\beta\gamma} \ , \ E_{6,\alpha\beta'}, \tag{4.4.47}$$

with $\alpha = 1,\ldots,4$, $\beta = 1,2$, $\beta' = 1,2,3$, $\gamma = 1,2$, $\gamma' = 1,2,3$ and $\mu = 1,\ldots,10$. The divisors $E_{3,\mu}$, $\mu = 1,2,4,5$ will also be denoted by $E_{3,\beta\gamma}$, $\beta,\gamma = 1,2$. In addition, the orbifold fixed points give rise to the ten divisors [34]:

$$D_{1,\alpha} \quad , \quad D_{2,\beta'} \quad , \quad D_{3,\gamma'} \ . \tag{4.4.48}$$

The topology of these divisors was determined in [34]. The divisors $E_{1,\alpha\gamma}$ and $E_{6,\alpha\beta}$ are blow–ups of $\mathbf{P}^1 \times \mathbf{P}^1$ in 5 points, the divisors $E_{1,\alpha 3}$, $E_{3,\mu}$, $\mu = 3,6,\ldots,10$, $E_{6,\alpha 3}$, and $D_{1\alpha}$ are blow–ups of $\mathbf{P}^1\times\mathbf{P}^1$ in 4 points, the divisors $E_{3,\mu}$, $\mu = 1,2,4,5$ are blow–ups of $\mathbf{P}^1\times\mathbf{P}^1$ in 8 points. The divisors $E_{2,\beta\gamma}$, $E_{4,\beta\gamma}$, $D_{2\beta'}$, and $D_{3\gamma'}$ are $\mathbf{P}^1\times\mathbf{P}^1$, while the divisors $E_{5,\alpha\beta\gamma}$ are $\mathbf{F}_1$, and the R_i are K3 surfaces.

The orientifold projection $\mathcal{O}$ leaves all the divisors (4.4.47) and (4.4.48) invariant, hence $h^{(-)}_{(1,1)}(X_6) = 0$ (cf. also table 2.3). We choose an orientifold action σ such that its fixed point set consists of 14 $O7$–planes wrapped on the divisors $D_{1,\alpha}$, $D_{2\beta'}$, $D_{3,\gamma'}$, and $E_{3,\mu}$, $\mu = 1,2,4,5$. There are no $O3$–planes. Because of $\chi(D_{1\alpha}) = 8$, $\chi(D_{2\beta'}) = \chi(D_{3,\gamma}) = 4$, and $\chi(E_{3,\mu}) = 12$, the total $D3$–brane charge $Q_{3,tot}$ in eq. (2.1.15) is $Q_{3,tot} = -26$. The Poincare dual 2–forms ω_i of the 61 invariant divisors (4.4.47) represent a basis for the Kähler form J:

$$\begin{aligned} J \ &= \ r_1\ R_1 + r_2\ R_2 + r_3\ R_3 - \sum_{\beta,\gamma=1,2}\left(t_{2,\beta\gamma}\ E_{2,\beta\gamma} + t_{4,\beta\gamma}\ E_{4,\beta\gamma} + \sum_{\alpha=1}^{4} t_{5,\alpha\beta\gamma}\ E_{5,\alpha\beta\gamma}\right) \\ &\quad - \sum_{\alpha=1}^{4}\left(\sum_{\gamma=1,2,3} t_{1,\alpha\gamma}\ E_{1,\alpha\gamma} + \sum_{\beta=1,2,3} t_{6,\alpha\beta}\ E_{6,\alpha\beta}\right) - \sum_{\mu=1}^{10} t_{3,\mu}\ E_{3,\mu}\ , \end{aligned} \tag{4.4.49}$$

with the 61 Kähler coordinates $r_i,\ t_{1,\alpha\gamma},\ t_{2,\beta\gamma},\ t_{3,\mu},\ t_{4,\beta\gamma},\ t_{5,\alpha\beta\gamma},\ t_{6,\alpha\beta}$.

Furthermore, the orientifold action changes the linear relations [34] between the divisors D_i and R_i:

$$
\begin{aligned}
R_1 &= 2\,D_{1,\alpha} + \frac{1}{2}\sum_{\gamma=1}^{3} E_{1,\alpha\gamma} + \frac{1}{2}\sum_{\beta,\gamma=1,2} E_{5,\alpha\beta\gamma} + \frac{1}{2}\sum_{\beta=1}^{3} E_{6,\alpha\beta} \quad , \quad \alpha = 1,\ldots,4\ , \\
R_2 &= 4\,D_{2,\beta} + \sum_{\gamma=1,2}\left(\frac{1}{2}\ E_{2,\beta\gamma} + 2\ E_{3,\beta\gamma} + \frac{3}{2}\ E_{4,\beta\gamma}\right) + \frac{1}{2}\sum_{\alpha=1}^{4}\sum_{\gamma=1,2} E_{5,\alpha\beta\gamma} \\
&\quad + \sum_{\alpha=1}^{4} E_{6,\alpha\beta} + E_{3,\mu} \quad , \quad (\beta,\mu) \in \{\ (1,3)\ ,\ (2,6)\ \}\ , \\
R_2 &= 2\,D_{2,3} + \frac{1}{2}\sum_{\alpha=1}^{4} E_{6,\alpha 3} + \frac{1}{2}\sum_{\mu=7}^{10} E_{3,\mu}\ , \\
R_3 &= 4\,D_{3,\gamma} + \sum_{\alpha=1}^{4} E_{1,\alpha\gamma} + \sum_{\beta=1,2}\left(\frac{3}{2}\ E_{2,\beta\gamma} + 2\ E_{3,\beta\gamma} + \frac{1}{2}\ E_{4,\beta\gamma}\right) + \frac{1}{2}\sum_{\alpha=1}^{4}\sum_{\beta=1,2} E_{5,\alpha\beta\gamma} \\
&\quad + E_{3,\mu} \quad , \quad (\gamma,\mu) \in \{\ (1,7)\ ,\ (2,8)\ \}\ , \\
R_3 &= 2\,D_{3,3} + \frac{1}{2}\sum_{\alpha=1}^{4} E_{1,\alpha 3} + \frac{1}{2}\sum_{\mu=3,6,9,10} E_{3,\mu}\ .
\end{aligned} \tag{4.4.50}
$$

With the intersection numbers from ref. [34] and the relation (4.4.34), the divisor volumes $\mathrm{Vol}(E)$ and $Vol(D)$ of the 68 divisors (4.4.47) and (4.4.48) may be calculated. Since the expressions are rather long we do not display them here.

In the superpotential (4.1.2), the 68 divisors split into two sets of contributing divisors $\mathcal{D}_i$: On the 14 divisors $\mathcal{D}_{D7} = \{D_{1,\alpha},\ D_{2,\beta},\ D_{3,\gamma},\ E_{3,1},\ E_{3,2},\ E_{3,4},\ E_{3,5}\}$, a stack of one $O7$–plane and eight $D7$–branes is wrapped. Gaugino condensation takes place in the pure $SO(8)$ gauge theory. Therefore we have again $a_j = -\frac{2\pi}{6}$ for the set $\mathcal{D}_{D7}$ of divisors contributing in (4.1.2). On the other hand, Euclidean $D3$–branes can be wrapped on the divisors in the set $\mathcal{D}_{D3} = \{E_{3,3}, E_{3,6}, E_{3,7}, E_{3,8}, E_{3,9}, E_{3,10}, E_{1,\alpha\gamma}, E_{6,\alpha\beta}, E_{2,\beta\gamma}, E_{4,\beta\gamma}, E_{5,\alpha\beta\gamma}\}$ of the 54 remaining divisors. Since all D and E divisors intersect one of the divisors carrying an $O7$–plane in at least one complex dimension, the condition $\chi_{D3} = 1$ for a non–vanishing instanton contribution in the superpotential (4.1.2) is always met for the set $\mathcal{D}_{D3}$ of divisors (cf. Section 3.1). In total we obtain for full the superpotential (4.1.2):

$$
W = W_0(S,U) + \sum_{\mathcal{D}_i\in\mathcal{D}_{D7}} e^{-2\pi\ \frac{\mathrm{Vol}(\mathcal{D}_i)}{6}} + \sum_{\mathcal{D}_i\in\mathcal{D}_{D3}} e^{-2\pi\ \mathrm{Vol}(\mathcal{D}_i)}\ . \tag{4.4.51}
$$

Now we are ready to stabilize all 61 Kähler moduli $r_i,\ t_{1,\alpha\gamma},\ t_{2,\beta\gamma},\ t_{3,\mu},\ t_{4,\beta\gamma},\ t_{5,\alpha\beta\gamma},\ t_{6,\alpha\beta}$. The $D3$–brane charge $Q_{3,tot} = -26$ is completely cancelled by the 3–form flux G_3, given in eq. (4.3.25). In fact, in the previous section we have presented critical points for the dilaton and complex structure moduli corresponding to a set of flux solutions, with $N_{flux} = 52$.

For a $\widetilde{W}_0 = -0.3104$ corresponding to the critical points of table (4.4) we find the following 68 divisor volumes (measured in string units):

$$\begin{aligned}
\mathrm{Vol}(D_{1,\alpha}) &= 14.00 \quad , \quad \mathrm{Vol}(D_{2,3}) = 5.543 \quad , \quad \mathrm{Vol}(D_{3,3}) = 5.60 \ , \\
\mathrm{Vol}(D_{2,\gamma}) &= 10.30 \quad , \quad \mathrm{Vol}(D_{3,\gamma}) = 11.07 \ , \ \gamma = 1,2 \ , \\
\mathrm{Vol}(E_{5,\alpha\beta\gamma}) &= 1.30 \quad , \quad \mathrm{Vol}(E_{4,\beta\gamma}) = 1.59 \ , \\
\mathrm{Vol}(E_{2,\beta\gamma}) &= 3.30 \ , \ \alpha = 1,\ldots,4 \ , \ \beta,\gamma = 1,2 \ , \\
\mathrm{Vol}(E_{1,\alpha\gamma}) &= 9.82 \quad , \quad \mathrm{Vol}(E_{6,\alpha\gamma}) = 14.72 \ , \ \alpha = 1,\ldots,4 \ , \ \gamma = 1,2 \ , \\
\mathrm{Vol}(E_{1,\alpha 3}) &= 15.23 \quad , \quad \mathrm{Vol}(E_{6,\alpha 3}) = 21.62 \ , \ \alpha = 1,\ldots,4 \ , \\
\mathrm{Vol}(E_{3,\mu}) &= 8.06 \ , \ \mu = 1,2,4,5 \quad , \quad \mathrm{Vol}(E_{3,\mu}) = 27.15 \ , \ \mu = 3,6 \ , \\
\mathrm{Vol}(E_{3,\mu}) &= 19.26 \ , \ \mu = 7,8 \quad , \quad \mathrm{Vol}(E_{3,\mu}) = 27.00 \ , \ \mu = 9,10 \ .
\end{aligned} \tag{4.4.52}$$

corresponding to the sizes of the 61 Kähler moduli:

$$\begin{aligned}
r_1 &= 7.826 \quad , \quad r_2 = 5.410 \quad , \quad r_3 = 4.593 \ , \\
t_{5,\alpha\beta\gamma} &= 0.770 \quad , \quad t_{4,\beta\gamma} = 0.107 \ , \\
t_{2,\beta\gamma} &= 0.239 \ , \ \alpha = 1,\ldots,4 \ , \ \beta,\gamma = 1,2 \ , \\
t_{1,\alpha\gamma} &= 0.858 \quad , \quad t_{6,\alpha\gamma} = 1.860 \ , \ \alpha = 1,\ldots,4 \ , \ \gamma = 1,2 \ , \\
t_{1,\alpha 3} &= 0.686 \quad , \quad t_{6,\alpha 3} = 1.320 \ , \ \alpha = 1,\ldots,4 \ , \\
t_{3,\mu} &= 0.037 \ , \ \mu = 1,2,4,5 \quad , \quad t_{3,\mu} = 0.569 \ , \ \mu = 3,6 \ , \\
t_{3,\mu} &= 0.302 \ , \ \mu = 7,8 \quad , \quad t_{3,\mu} = 0.413 \ , \ \mu = 9,10 \ .
\end{aligned} \tag{4.4.53}$$

The divisor volumes give rise to the total volume $\mathrm{Vol}(X_6) = 229.22$. This is large enough, that one–loop corrections to the Kähler potential are suppressed. Furthermore, the divisor volumes (4.4.52) are large enough to suppress higher order instanton effects in (4.4.51), since e.g. $e^{-2\pi \mathrm{Vol}(E_{5,\alpha\beta\gamma})} \sim 0.0002$ and even smaller for the other divisors. To this end, for the $\mathbf{Z}_2 \times \mathbf{Z}_4$ orbifold we have stabilized all 63 moduli fields.

Chapter 5

Conclusions

This thesis deals with moduli stabilization in type IIB orientifolds à la KKLT, i.e. with tree–level 3–form flux superpotential plus non-perturbative superpotential from $D3$-instantons and/or gaugino condensation.

In chapter 2, the main emphasis of the work was on orientifold compactifications in their various orbifold limits. We showed that it is indeed possible to find stable (i.e. tachyon free), supersymmetric AdS-minima with stabilized dilaton, untwisted Kähler T^i and untwisted complex structure moduli U^j, as long as the geometrical orbifold group still allows for the existence of untwisted complex structure moduli fields. On the other hand, if the orbifold group action already freezes all complex structure moduli, then the scalar potential in the S, T^i-sector is such that the (mass)^2 matrix for these fields contains negative eigenvalues. We also pointed out some problems with the integrating out procedure of complex structure moduli, namely we investigated cases, where integrating out the U^j leads to non-stable AdS-vacua in the remaining S, T^i potential, whereas the minimization of the full S, T^i, U^j potential does not suffer from any instabilities.

The coefficient $\gamma_j(S, U)$ in the non–perturbative superpotential (4.1.2) accounts for gaugino condensation or $D3$–instanton effects. Generically, this weight factor depends both on the dilaton S and the complex structure moduli U^λ. Due to the non–renormalization of the gauge kinetic function beyond one–loop, for gaugino condensation this dependence is fairly well under control perturbatively, cf. eq. (4.1.4). On the other hand, for $D3$–instantons the factor $\gamma_j(S, U)$ represents the one–loop determinant of the instanton solution. The latter is hard to compute directly, except (in)directly in F– or M–theory [19] or through some duality arguments [65, 77]. In chapter 3.1.3 we have presented general results (cf. table 3.1), under which conditions this coefficient γ is non–vanishing in type IIB CY orientifolds. However, it is certainly important to directly calculate the dilaton and complex structure modulus dependence of γ by means of an instanton calculation.

The non-vanishing non-perturbative superpotential implies the presence of all ISD and IASD fluxes. Their presence changes fermionic zero modes counting, which gives a criterion for a generation of non-perturbative superpotential. In the presence of ISD and IASD fluxes all fermionic modes are lifted and non-perturbative superpotential cannot be generated. We demonstrated that this apparent mismatch disappears after the introduction of a modification of the supersymmetry variation, which basically captures the back-reaction of the non-perturbative effects on the background flux and the geometry.

In chapter 4 equipped with results of previous chapters we accomplished to fix all moduli in some examples of resolved orbifolds: $\mathbf{Z}_{6-II}$ on the root lattice of $SU(2) \times SU(6)$ and $\mathbf{Z}_2 \times \mathbf{Z}_4$ on $SU(2)^2 \times SO(5)^2$: We have stabilized for the $\mathbf{Z}_{6-II}$–orbifold all its 27 moduli fields, cf. table 4.2 and (4.4.46) and for the $\mathbf{Z}_2 \times \mathbf{Z}_4$ orbifold all its 63 moduli fields, cf. table 4.4 and (4.4.53).

In most of the existing literature on flux compactifications, one works in the lowest α'–expansion at string tree–level, i.e. in the supergravity approximation. Since at this order in α' and g_{string} the theory has a no–scale structure, which does not fix all Kähler moduli, one adds some effects which eventually allow the stabilization of all moduli fields. As proposed by KKLT [61], one promising possibility is to consider the racetrack superpotential (4.1.2), which we also have used throughout the chapter 4. The critical points found in section 4.4 are inert against corrections in α' and g_{string}, since the coupling constant g_{string} is small and the volume V of the compactification manifold is large. However, to discuss also other choices of minima one must go beyond this approximation and include corrections in both α' and g_{string}. In particular, there are both perturbative corrections to the Kähler potential K to all orders in g_{string} and world–sheet as well as space–time instanton corrections to the Kähler potential K in N=1 CY orientifolds. It is certainly very important to gain control over these corrections.

We have obtained a fairly complete picture of the critical points of type IIB orientifolds of resolved orbifolds. Indeed, as tables 4.2 and 4.4 show, for one orbifold there is a huge number of vacua with the same physical quantities thus giving rise to a landscape of supersymmetric vacuum solutions in the flux space. Throughout this thesis, the fixing of open string moduli is not addressed. This is legitimate as we only discuss $D3$–branes wrapping internal 4–cycles and no space–time filling $D3$–branes. The complete tadpole (2.1.15) originating from the RR 4–form is cancelled by curvature and flux. More general setups would also allow space–time filling $D3$–branes and $D7$–branes away from the orientifold planes. It has been shown in [35, 85], that even an ISD 3–form flux implies stabilization of the $D7$–brane positions and soft–masses for corresponding the open string moduli. Certainly, a thorough discussion of the stabilization of open string moduli would enrich the present picture of the string landscape [66].

Appendix A

Dimensional reduction of $\delta\psi_m$

We demonstrate the dimensional reduction of the supersymmetric variation of the gravitino on CY_4.

Firstly, we write the internal gravitino variation using holomorphic and antiholomorphic indices.

$$
\begin{aligned}
\delta\psi_e &= \left[\nabla_e + \frac{1}{24}\left(3\gamma^{b\overline{cd}}G_{eb\overline{cd}} + \gamma^{\overline{bcd}}G_{e\overline{bcd}}\right)\right]\xi \ , \\
\delta\psi_{\bar{e}} &= \left[\nabla_{\bar{e}} + \frac{1}{24}\left(3\gamma^{b\overline{cd}}G_{\bar{e}b\overline{cd}} + \gamma^{\overline{bcd}}G_{\overline{ebcd}}\right)\right]\xi \ .
\end{aligned} \tag{A.1}
$$

ψ_m is a vector-spinor, where m is an internal vector index which transforms in the $\mathbf{4}\oplus\bar{\mathbf{4}}$ representation of $SU(4)$. The spinor index of the eleven dimensional gravitino transforms in the $\mathbf{32}$ under $SO(1,10)$. After compactification on a CY_4, $SO(1,10)$ is broken to $SU(4)\times SO(2,1)$ and the spinor transforms in the $(\mathbf{1},\mathbf{2})\oplus(\mathbf{4},\mathbf{2})\oplus(\mathbf{6},\mathbf{2})\oplus(\bar{\mathbf{4}},\mathbf{2})\oplus(\bar{\mathbf{1}},\mathbf{2})$. This means that ψ_e can be written as a sum of $(0,p)$-forms with one additional holomorphic or antiholomorphic index.

$$
\psi_e = \phi_e|\Omega> + \phi_{e\bar{a}}\gamma^{\bar{a}}|\Omega> + \phi_{e\overline{ab}}\gamma^{\overline{ab}}|\Omega> + \phi_{e\overline{abc}}\gamma^{\overline{abc}}|\Omega> + \phi_{e\overline{abcd}}\gamma^{\overline{abcd}}|\Omega> \ . \tag{A.2}
$$

Note that ψ_e in (A.2) has an additional spinor index which transforms in the $\mathbf{2}$ of $SO(1,2)$. The rhs. of (A.1) is also such a spinor. ξ can be written as $\xi = \epsilon\otimes\eta$, where η is a covariantly constant spinor on the CY_4 and ϵ a supersymmetry parameter in the non-compact dimensions. We write ξ as

$$
\xi = \widehat{\xi}|\Omega> + \widehat{\xi}_{\overline{abcd}}\gamma^{\overline{abcd}}|\Omega> \ . \tag{A.3}
$$

and should remember that $\widehat{\xi}$ has an additional index which transforms in the $\mathbf{2}$ under $SO(1,2)$. The rhs. of the first equation in (A.1) is then

$$
\delta\psi_e \ = \ \left[\nabla_e + \frac{1}{24}\left(3\gamma^{b\overline{cd}}G_{eb\overline{cd}} + \gamma^{\overline{bcd}}G_{e\overline{bcd}}\right)\right]\xi
$$

$$\begin{aligned} &= \left[\nabla_e + \frac{1}{24}\left(6G_{eb\overline{cd}}g^{b\bar{d}}\gamma^{\bar{c}} + G_{e\overline{bcd}}\gamma^{\overline{bcd}}\right)\right]\xi \\ &= \nabla_e\left(\widehat{\xi}|\Omega> + \widehat{\xi}_{\overline{abcd}}\gamma^{\overline{abcd}}|\Omega>\right) + \frac{1}{4}G_{eb\overline{cd}}g^{b\bar{d}}\widehat{\xi}\gamma^{\bar{c}}|\Omega> + \frac{1}{24}G_{e\overline{bcd}}\widehat{\xi}\gamma^{\overline{bcd}}|\Omega> \quad . \end{aligned} \tag{A.4}$$

The open index e corresponds to a one-form index, which means that we have a collection of $(1,p)$-forms.[1] We compare the forms of the same type on both sides and obtain the following set of equations:

$$\begin{aligned} \delta\left(\phi_{e\bar{a}}\gamma^{\bar{a}}|\Omega>\right) &= \frac{1}{4}G_{eb\overline{cd}}g^{b\bar{d}}\widehat{\xi}\gamma^{\bar{c}}|\Omega> \ , \\ \delta\left(\phi_{e\overline{ab}}\gamma^{\overline{ab}}|\Omega>\right) &= 0 \ , \\ \delta\left(\phi_{e\overline{abc}}\gamma^{\overline{abc}}|\Omega>\right) &= \frac{1}{24}G_{e\overline{bcd}}\widehat{\xi}\gamma^{\overline{bcd}}|\Omega> \quad . \end{aligned} \tag{A.5}$$

These are the only forms from (A.4), which do not vanish on a CY_4.

Let us look at the second equation of (A.1) where the additional index is antiholomorphic. To see this index as a form index we have to make it holomorphic. This can be done by applying Serre's generalization of Poincaré duality

$$\widetilde{\psi}_{abc} = \psi_{\bar{e}}g^{e\bar{e}}\omega_{abce} \ , \tag{A.6}$$

where ω_{abce} is the $(4,0)$-form of the CY_4.

$$\begin{aligned} \delta\widetilde{\psi}_{abc} &= g^{e\bar{e}}\omega_{abce}\left(\nabla_{\bar{e}} + \frac{1}{24}\left(3\gamma^{f\overline{gh}}G_{\bar{e}f\overline{gh}} + \gamma^{\overline{fgh}}G_{\overline{efgh}}\right)\right)\left(\widehat{\xi}|\Omega> + \widehat{\xi}_{\overline{ijkl}}\gamma^{\overline{ijkl}}\right)|\Omega> \\ &= g^{e\bar{e}}\omega_{abce}\left(\nabla_{\bar{e}} + \frac{1}{4}G_{\bar{e}f\overline{gh}}g^{f\bar{h}}\gamma^{\bar{g}} + \frac{1}{24}G_{\overline{efgh}}\gamma^{\overline{fgh}}\right)\left(\widehat{\xi}|\Omega> + \widehat{\xi}_{\overline{ijkl}}\gamma^{\overline{ijkl}}\right)|\Omega> \end{aligned} \tag{A.7}$$

Again, comparing the forms of the same type gives us

$$\begin{aligned} \delta\left(\widetilde{\phi}_{abc\bar{a}}\gamma^{\bar{a}}\right)|\Omega> &= \frac{1}{4}g^{e\bar{e}}\omega_{abce}G_{\bar{e}f\overline{gh}}g^{f\bar{h}}\gamma^{\bar{g}}\widehat{\xi}|\Omega> \ , \\ \delta\left(\widetilde{\phi}_{abc\overline{ab}}\gamma^{\overline{ab}}\right)|\Omega> &= 0 \ , \\ \delta\left(\widetilde{\phi}_{abc\overline{abc}}\gamma^{\overline{abc}}\right)|\Omega> &= \frac{1}{24}g^{e\bar{e}}\omega_{abce}G_{\overline{efgh}}\gamma^{\overline{fgh}}\widehat{\xi}|\Omega> \quad . \end{aligned} \tag{A.8}$$

Eqs. (A.5) and (A.8) can be expanded in the basis of harmonic forms on the CY_4 as follows:

$$\begin{aligned} \delta\left(\phi_i\omega^i_{(1,3)}\right) &= g_i\omega^i_{(1,3)}\widehat{\xi} \\ \delta\left(\phi_I\omega^I_{(1,1)}\right) &= g_I\omega^I_{(1,1)}\widehat{\xi} \end{aligned}$$

[1] We can introduce a second set of gamma matrices, which will commute with the first one, so for example $\phi_{a_1\ldots a_p\bar{a}_1\ldots\bar{a}_q}\widetilde{\gamma}^{a_1}\ldots\widetilde{\gamma}^{a_p}\gamma^{\bar{a}_1}\ldots\gamma^{\bar{a}_q}|\Omega>$ will correspond to a (p,q)-form. Here we will omit the second set of gamma-matrices to make the equations more transparent. A detailed explanation of this formalism is given in Chapter 15 of [9].

$$\begin{aligned} \delta\left(\phi_i \omega^i_{(1,2)}\right) &= 0 \\ \delta\left(\phi_i \omega^i_{(2,3)}\right) &= 0 \end{aligned} \qquad (A.9)$$

where $\omega^I_{(1,1)}$ and $\omega^i_{(1,3)}$ are basis elements of $H^{1,1}(CY_4)$ and $H^{1,3}(CY_4)$ respectively.

If we repeat the calculations for the type IIB case, we obtain an equation for the $(1,2)$-form, another one for the $(2,2)$-form and $(3,0)$-form for the dilatino:

$$\begin{aligned} \delta\left(\phi_{e\overline{ab}}\gamma^{\overline{ab}}|\Omega>\right) &= -\frac{1}{8}G_{e\overline{ab}}\gamma^{\overline{ab}}\widehat{\xi}^*|\Omega> -\frac{1}{16}g^{a\overline{c}}g_{e\overline{a}}G_{\overline{abc}}\gamma^{\overline{ab}}\widehat{\xi}^*|\Omega> , \\ \delta\left(\phi_{\overline{e}|\overline{ab}}\gamma^{\overline{ab}}|\Omega>\right) &= -\frac{1}{16}G_{\overline{abe}}\gamma^{\overline{ab}}\widehat{\xi}^*|\Omega> , \\ \delta\left(\lambda^*_{\overline{abc}}\gamma^{\overline{abc}}|\Omega>\right) &= \frac{i}{4}\overline{G}_{\overline{abc}}\gamma^{\overline{abc}}\widehat{\xi}^*|\Omega> \ . \end{aligned} \qquad (A.10)$$

The second equation corresponds to the $(2,2)$-form[2] after applying Serre's duality and to a $(1,1)$–form by forming the Hodge dual.

These equations (A.10) can be expanded in the basis of harmonic forms on the CY_3 and written then as

$$\begin{aligned} \delta\left(\phi_i\,\omega^i_{(1,2)}\right) &= g_i\,\omega^i_{(1,2)}\widehat{\xi} , & i &= 1,\ldots,h^{(2,1)} , \\ \delta\left(\phi_I\,\omega^I_{(1,1)}\right) &= g_I\,\omega^I_{(1,1)}\widehat{\xi} , & I &= 1,\ldots,h^{(1,1)} , \\ \delta\left(\lambda^{(0,3)}\omega_{(0,3)}\right) &= g^{(0,3)}\omega_{(0,3)}\widehat{\xi} \ . \end{aligned} \qquad (A.11)$$

ϕ_i, ϕ_I and $\lambda^{(0,3)}$ correspond to the 4-dimensional complex structure modulinos, the Kähler modulinos and the dilatino respectively.

Finally, let us rewrite the variation of the modulino fields as it will be needed for our investigation:
For the M-theory case:

$$\begin{aligned} \delta\phi^k_{e\overline{c}} &= \frac{1}{4}\left(G_{eb\overline{cd}}g^{b\overline{d}}\right)^k\widehat{\xi} , & k &= 1,\ldots,h^{(1,1)} , \\ \delta\phi^i_{e\overline{abc}} &= \frac{1}{24}G^i_{e\overline{abc}}\widehat{\xi} , & i &= 1,\ldots,h^{(3,1)} , \\ \delta\phi^I_{\overline{e}|\overline{abc}} &= \frac{1}{24}G_{\overline{eabc}}\widehat{\xi} , & I &= 1,\ldots,h^{(1,1)} \ . \end{aligned} \qquad (A.12)$$

For the type IIB case

$$\begin{aligned} \delta\phi^i_{e\overline{ab}} &= -\frac{1}{8}G^i_{e\overline{ab}}\widehat{\xi}^* - \frac{1}{16}g^{a\overline{c}}g_{e\overline{a}}G^i_{\overline{abc}}\widehat{\xi}^* , & i &= 1,\ldots,h^{(2,1)} , \\ \delta\phi^I_{\overline{e}|\overline{ab}} &= -\frac{1}{16}G_{\overline{eab}}\widehat{\xi}^* , & I &= 1,\ldots,h^{(1,1)} , \\ \delta\lambda^*_{\overline{abc}} &= \frac{i}{4}\overline{G}_{\overline{abc}}\widehat{\xi}^* \ . \end{aligned} \qquad (A.13)$$

[2]Note, that in this notation the holomorphic indices correspond to the holomorphic part of the form and vice versa. The antiholomorphic index $\overline{e}$ has no meaning as form index before applying Serre's duality. That is why we put $|$ there to prevent its mixing with the antiholomorphic indices.

We label the modulinos with the indices k, i, I. Additionally, they have indices from the beginning of the alphabet. Let us briefly comment about this.

A (p,q)-form ν can be expanded in the basis of harmonic (p,q)-forms ω^i: $\nu = \nu_i\,\omega^i$. In the case of a complex manifold, the number of the harmonic forms is given by the corresponding Hodge number $h^{(p,q)}$. On the other hand we can write the form in every local patch as $\nu = \nu_{a_1\ldots a_p\ \overline{a}_1\ldots\overline{a}_q} dz^{a_1} \wedge \ldots \wedge dz^{a_p} \wedge dz^{\overline{a}_1} \wedge \ldots \wedge dz^{\overline{a}_q}$. If $\nu_{a_1\ldots a_p\ \overline{a}_1\ldots\overline{a}_q}$ are constant, they should correspond to the coefficients ν_i. The whole $\nu_{a_1\ldots a_p\ \overline{a}_1\ldots\overline{a}_q}$ in all coordinate patches span a vector space, in which so many $\nu_{a_1\ldots a_p\ \overline{a}_1\ldots\overline{a}_q}$ are linearly dependent by the transition functions that the dimension of this vector space is $h^{(p,q)}$. The linearly independent combinations of $\nu_{a_1\ldots a_p\ \overline{a}_1\ldots\overline{a}_q}$ are then in one to one correspondence to the ν_i.

Appendix B

Example for a false minimum after the integrating-out procedure

Let us consider a superpotential which depends only on the axion-dilaton and a Kähler modulus. Supersymmetric minima (S_0, T_0) are given by solving

$$D_T W(S,T) = \frac{\partial W(S,T)}{\partial T} + W(S,T)\frac{\partial K(S,\overline{S},T,\overline{T})}{\partial T} = 0 \; , \tag{B.1}$$

$$D_S W(S,T) = \frac{\partial W(S,T)}{\partial S} + W(S,T)\frac{\partial K(S,\overline{S},T,\overline{T})}{\partial S} = 0 \tag{B.2}$$

and their complex conjugates. We would like to show, that solutions of these equations are in general not equivalent to the ones of

$$D_T W_{\text{eff}}\Big(S(T,\overline{T}),T\Big) = \frac{dW_{\text{eff}}\Big(S(T,\overline{T}),T\Big)}{dT} + W_{\text{eff}}\Big(S(T,\overline{T}),T\Big)\frac{dK_{\text{eff}}\Big(S(T,\overline{T}),\overline{S}(T,\overline{T}),T,\overline{T}\Big)}{dT} \; , \tag{B.3}$$

where $W_{\text{eff}}\Big(S(T,\overline{T}),T\Big)$ and $K_{\text{eff}}(S,\overline{S},T,\overline{T})$ are obtained by inserting the solution of (B.2) and its complex conjugate into $W(S,T)$ and $K(S,\overline{S},T,\overline{T})$. After taking the derivatives we obtain

$$\begin{aligned} D_T W_{\text{eff}}\Big(S(T,\overline{T}),T\Big) &= \frac{\partial W_{\text{eff}}}{\partial T} + \frac{dW_{\text{eff}}}{dS}\frac{\partial S(T,\overline{T})}{\partial T} \\ &+ W_{\text{eff}}\left(\frac{\partial K_{\text{eff}}}{\partial T} + \frac{dK_{\text{eff}}}{dS}\frac{\partial S(T,\overline{T})}{\partial T} + \frac{dK_{\text{eff}}}{d\overline{S}}\frac{\partial \overline{S}(T,\overline{T})}{\partial T}\right) \; . \end{aligned} \tag{B.4}$$

Since $W_{\text{eff}}(S_0,T_0) = W(S_0,T_0)$ and

$$\left.\frac{\partial W(S,T)}{\partial S}\right|_{S=S_0,T=T_0} = \left.\frac{dW_{\text{eff}}(S(T,\overline{T}),T)}{dS}\right|_{S=S_0,T=T_0} \; ,$$

$$\left.\frac{\partial K(S,\overline{S},T,\overline{T})}{\partial S}\right|_{S=S_0,T=T_0} = \left.\frac{dK_{\text{eff}}(S(T,\overline{T}),\overline{S}(T,\overline{T}),T,\overline{T})}{dS}\right|_{S=S_0,T=T_0} , \tag{B.5}$$

$$\tag{B.6}$$

we obtain

$$D_T W_{\text{eff}}\Big(S(T_0,\overline{T}_0),T_0\Big) = W_{\text{eff}}\left.\frac{\partial K}{\partial \overline{S}}\frac{\partial \overline{S}(T,\overline{T})}{\partial T}\right|_{S=S_0,T=T_0} , \tag{B.7}$$

where we made use of eqs. (B.1) and (B.2).

Since the expression (B.7) is in general not zero, solving the supersymmetry condition of the superpotential after integrating-out one of the fields ($D_T W_{\text{eff}} = 0$) gives not a supersymmetric minimum of the potential.

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