Using the empirical Bayes method to estimate and evaluate bycatch rates of seabirds from individual fishing vessels

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Minimizing bycatch of seabirds is a major goal of the U.S. National Marine Fisheries Service. In Alaska waters, the bycatch (i.e., inadvertent catches) of seabirds has been an incidental result of demersal groundfish longline fishery operations. Notably, the endangered short-tailed albatross (*Phoebastria albatrus*) has been taken in this groundfish fishery. Bycatch rates of seabirds from individual vessels may be of particular interest because vessels with high bycatch rates may not be functioning effectively with seabird avoidance gears, and there may be a need for suggestions on how to use these avoidance gears more effectively. Therefore, bycatch estimates are usually made on an individual vessel basis and then summed to obtain the total estimate for the entire fleet.

The empirical Bayes (EB) (Efron and Morris, 1975; Casella, 1985) method offers the possibility of improving within-vessel bycatch estimates, with the assumption that the individual vessel bycatch rate of seabirds has a gamma prior distribution. With the resulting Poisson-gamma EB model, it is assumed that each vessel's bycatch of seabirds has a Poisson distribution conditioned on the realized "true" bycatch rate. The basic principle of the EB method comes from the realization that the parameters for the gamma distribution can be estimated from individual vessel bycatches, and that the resulting EB estimators of individual vessel bycatch rates should provide estimates of individual bycatch rates that have smaller total mean squared error (*TMSE*) than the individual vessel bycatch rates estimated independently. The independently estimated individual vessel bycatch rate is simply the bycatch per thousand hooks fished for each vessel. A more complete introduction to the empirical Bayes method as it has been applied to different types of problems is provided by Ver Hoef (1996).

The goal of this note is to clearly describe empirical Bayes estimation and provide a detailed example of its application to the problem of estimating seabird bycatch. It is to be hoped that a better understanding of the theory underlying empirical Bayes methods will lead to more applications in the area of fisheries management.

Materials and methods

General theory

Mathematically, the empirical Bayes (EB) method can be described as a statistical procedure that has clearly defined steps (Carlin and Louis, 2000). Let the prior distribution of a parameter θ (the parameter of greatest interest) be $g(\theta|\eta)$, where the η are unknown parameters, and the sampling distribution for each stratum observation y is $f(y|\theta)$. From the joint distribution defined by $h(y,\theta|\eta)=f(y|\theta)g(\theta|\eta)$, the marginal distribution of the observed y can be derived by integrating out θ: $m(y|\eta)=\int h(y,\theta|\eta)d\theta$. The empirical Bayes method arises from the recognition that η can be estimated from $m(y|\eta)$ by using the marginal maximum likelihood (MML) estimators or related methods. Once $\hat{\eta}$ is estimated, the posterior distribution of θ can be obtained by using the Bayes rule, $p(\theta|y,\hat{\eta})=f(y|\theta)g(\theta|\hat{\eta})/m(y|\hat{\eta})$, and an EB estimate of θ can be made from this posterior distribution.

The Poisson-gamma empirical Bayes model

The Poisson-gamma model is ideal for illustrating how to calculate EB estimators from the general theory because the all the required integrals result in a gamma function. For this model, denote the gamma prior for the seabird bycatch rate of vessel i as $g(\lambda_i|\alpha,\beta)$, and the Poisson sampling distribution as $f(y_i|\lambda_i,\tau_i)$, where y_i is the number of seabirds observed, and τ_i are the number of hooks observed. The joint distribution of y_i and λ_i is then

$$h(y_i,\lambda_i|\alpha,\beta,\tau_i)=\frac{\exp(-\lambda_i\tau_i)(\lambda_i\tau_i)^{y_i}}{y_i!}\times \frac{\lambda_i^{\alpha-1}\exp(-\lambda_i/\beta)}{\Gamma(\alpha)\beta^{\alpha}}$$

$$\text{for } \alpha,\beta,\lambda_i>0, y_i\ge 0.$$

The marginal distribution is calculated by integrating out λ_i:

$$m(y_i|\alpha,\beta,\tau_i)= \frac{\tau_i^{y_i}}{y_i!\Gamma(\alpha)\beta^{\alpha}}\frac{\Gamma(y_i+\alpha)}{(\tau_i+1/\beta)^{y_i+\alpha}}.$$

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The probability for all vessels (i.e., strata) can then be written as

$$P[X_1 = y_1,\ldots,X_n = y_n \mid \tau_1,\ldots,\tau_n] = \prod_{i=1}^{n} \frac{\tau_i^{y_i}}{y_i!\Gamma(\alpha)\beta^{\alpha}} \frac{\Gamma(y_i+\alpha)}{(\tau_i+1/\beta)^{y_i+\alpha}}.$$

The parameters of the gamma prior, $g(\lambda_i \mid \alpha, \beta)$, can then be estimated by maximizing the marginal likelihood given above and arriving at MML estimates $(\hat{\alpha},\hat{\beta})$. Initial estimates for the gamma distribution can be provided by moment estimators (Carlin and Louis, 2000) where: $r_i = m_i/\tau_i$, and $\bar{r}$ and s_r^2 and the sample mean and variance of the$\{r_i\}$, and

$$\hat{\alpha}_0 = \bar{r}^2/(s_r^2 - \bar{r}\sum_{i=1}^{n}(1/\tau_i)/n) \text{ and } \hat{\beta}_0 = \bar{r}/\hat{\alpha}.$$

The gamma prior is the conjugate distribution (Patrick, 1972; Carlin and Louis, 2000) for the Poisson sampling distribution, which means that the posterior distribution is in the same family as the prior distribution for each stratum. The posterior distribution is $p(\lambda_i \mid y_i, \hat{\alpha},\hat{\beta},\tau_i) = f(y_i, \lambda_i \mid \hat{\alpha},\hat{\beta},\tau_i)/g(y_i \mid \hat{\alpha},\hat{\beta},\tau_i)$, which can be verified to have the gamma distribution $g(\lambda_i \mid \alpha' = y_i+\alpha, \beta' = 1/(\tau_i+1/\beta))$. The mean of this posterior distribution, providing estimators for the λ_i, can be calculated from

$$\begin{aligned}\tilde{\lambda}_i &= E(\lambda_i) \\ &= \int_0^{\infty} \lambda_i p(\lambda_i \mid y_i,\hat{\alpha},\hat{\beta},\tau_i)d\lambda_i \\ &= (y_i+\hat{\alpha})/(\tau_i+1/\hat{\beta})\end{aligned}$$

or more simply can be recognized as the product of the parameters of the posterior distribution.

The conventional maximum likelihood (ML) estimator of λ_i for the Poisson strata is $\hat{\lambda}_i = y_i/\tau_i$. The EB estimator of λ_i based on the mean of the posterior distribution can be seen as the weighted average of the ML stratum estimator and the mean of the gamma prior $\hat{\alpha}\hat{\beta}$ and will lie between these two values.

Simulation methods

Simulation was performed on the Poisson-gamma EB model described above. Each replication simulated the seabird bycatch of 50 vessels, and was repeated 1000 times. Each replication assumed that the "true" bycatch rate for each vessel (λ_i) was distributed as an observation from the gamma distribution $g(\lambda_i \mid \alpha=0.603, \beta=0.030)$; whose parameters were estimated in the EB analysis which follows. The number of hooks that were "observed," in thousands, was distributed uniformly as $U(0, \tau_{\max})$, with $\tau_{\max}$ = {200, 500, 1000, 2000, 5000}. For each of these simulations then, τ_{ave} = {100, 250, 500, 1000, 2500}. Finally, the number of "observed" seabirds (y_i) was simulated using the Poisson distribution with $\lambda_i' = \lambda_i\tau_i$, where λ_i and τ_i were previously randomly generated as described.

For each replication, the simulated (y_i, τ_i) were analyzed by using the empirical Bayes method, by first estimating $(\hat{\alpha},\hat{\beta})$ using the MML, and then using these parameters to calculate the EB estimate $\tilde{\lambda}_i = (y_i+\hat{\alpha})/(\tau_i+1/\hat{\beta})$. The ML estimator for each stratum was $\hat{\lambda}_i = y_i/\tau_i$, and the global unstratified (GU) estimator was $\hat{\lambda} = \Sigma y_i/\Sigma\tau_i$. The performance of these estimators was measured by using

$$TMSE = \sum_{i=1}^{50}(X_i - \lambda_i)^2/50,$$

where X_i could be any of $\hat{\lambda}_i$, $\tilde{\lambda}_i$, or $\hat{\lambda}$. The simulation was repeated 1000 times, and the $TMSE$ values were averaged to measure the overall performance of these estimators.

Analysis of bycatch data

In 2002, The North Pacific Longline Association, which has many longline vessel operators as members, voluntarily followed proposed regulations that required the use of effective seabird avoidance gear during fishing operations. These voluntary guidelines were implemented into formal regulations in February 2004.

EB analysis was performed on the bycatch of seabirds from individual longline vessels fishing in the eastern Bering Sea. The data were the annual observed bycatches of seabirds (y_i) and the total number of observed hooks in thousands (τ_i) of individual fishing vessels for 2002 and 2003. The 2002 data were used to fit the EB model, and resulting λ_i estimates were used to predict the λ_i for 2003. As a comparison, a similar analysis was performed on data collected from 1997 and 1998, a time when many vessels did not use bird-avoidance gear and when the bycatch rate of seabirds was much higher than in 2002 and 2003.

Results

Simulation results

All simulations consisted of 1000 replications as described above. When measured by $TMSE$, the EB estimator was clearly superior to both the maximum likelihood (ML) and global unstratified (GU) estimators (Table 1). This was true regardless of whatever value of $\tau_{\max}$ was used in the simulations. The ratio $R = TMSE(\hat{\lambda}_i)/TMSE(\tilde{\lambda}_i)$ increased as $\tau_{\max}$ increased, but the values of $TMSE$ for GU remained constant. Note that the $(\hat{\alpha},\hat{\beta})$ appeared biased when $\tau_{\max}$ = 5000 (Table 1).

Table 1

Simulation results from the Poisson-gamma empirical Bayes (EB) model with the assumption of 50 vessels (strata), and with random sample sizes (observed number of 1000 hooks) distributed as $U(0, \tau_{\max})$ and replicated 1000 times. The gamma distribution prior was assumed to have parameters α=0.603, β=0.030 , as were estimated from the seabird bycatch analysis, and the average $\hat{\alpha}$ and $\hat{\beta}$ were calculated from the 1000 replications. The seabird bycatch rate for vessel i, λ_i, was estimated by the maximum likelihood (ML) estimator $\hat{\lambda}_i$, the empirical Bayes (EB) estimator $\tilde{\lambda}_i$, and the global unstratified (GU) estimator $\hat{\lambda}$. Performance of these estimators was measured by the total mean squared error (*TMSE*) averaged over the 1000 replications, and R was defined as the ratio of *TMSE* values calculated for ML and EB estimates.

Sample size $\tau_{\max}$	$TMSE(\hat{\lambda}_i)$ ML	$TMSE(\tilde{\lambda}_i)$ EB	$TMSE(\hat{\lambda}_i)$ GU	$R = \frac{TMSE(\hat{\lambda}_i)}{TMSE(\tilde{\lambda}_i)}$	Average $\hat{\alpha}$	Average $\hat{\beta}$
200	0.001390	0.000189	0.000542	7.4	0.712	0.029
500	0.001440	0.000103	0.000531	14.0	0.655	0.029
1000	0.001000	0.000063	0.000541	15.9	0.638	0.030
2000	0.000935	0.000038	0.000532	24.6	0.621	0.031
5000	0.000906	0.000018	0.000537	50.3	0.473	0.041

Results from fitting the Poisson-gamma model

The bycatch per thousand hooks in 1997–98, when bird avoidance gear was not as common, was 0.085 birds, compared with 0.013 birds in 2002–03 when seabird avoidance gear was voluntarily employed. Thus, the bycatch rate for all seabirds was reduced in 2002–03 to 15% of the 1997–98 value.

For the 2002 bycatch data, initial parameter estimates for the gamma distribution were made with the moment estimators described earlier. These initial estimates were refined by using the maximum of the marginal likelihood also described earlier. The final MML estimates were $\hat{\alpha}$=0.603 and $\hat{\beta}$=0.030.

For 2002, the resulting EB bycatch rate estimates per vessel, $\tilde{\lambda}_i$ (Table 2), differed little from conventional ML estimates per vessel. A similar result occurred in the 1997–98 analysis. However, vessel 28 (Table 2), showed a large adjustment between the ML and EB estimates. It is apparent from Table 2 that this vessel had unusually low effort (τ_{28}=34) and a relatively large seabird bycatch (y_{28}=8). This adjustment towards the aggregate mean is a predictable EB adjustment for situations where individual stratum data are weak. For vessel 28, the predicted EB estimate of seabird bycatch rate per thousand hooks was 4.4, whereas the actual observed bycatch rate was 8 (Table 2).

When the 2002 seabird bycatch rates were used to predict the 2003 seabird bycatch rates for individual vessels, neither the ML or EB estimates provided a significant correlation (ρ=0.036, n=38). In contrast, when the 1997 bycatch rates were used to predict the 1998 bycatch rates, there was a significant correlation for the one-tailed test ρ=0.324, n=33, P=0.033).

Discussion and conclusion

Empirical Bayes estimators are superior to Bayes estimators in the sense that prior distributions can be estimated rather than assumed. If one prefers the Bayes method, one would counter that noninformative priors make the assumption of priors relatively benign, whereas for the empirical Bayes model, the assumption of the family of priors may be quite critical.

The empirical Bayes method can be applied even when the marginal distribution is analytically intractable, by substituting numerical integration for analytical integrals. However, the computational intensity required by using numerical integration can appear daunting even with the current speed of desktop computers (Laslett et al., 2002).

Nevertheless, if the prior family is properly selected, the empirical Bayes method can provide very precise estimates. For our Poisson-gamma simulation, the empirical Bayes method provided uniformly superior estimates of the Poisson λ_i for a wide range of $\tau_{\max}$ values. Although the ratio values in Table 1 indicate that the EB estimator is most useful when $\tau_{\max}$ is large, the greatest benefit of the EB method is probably on the opposite end of the scale when individual stratum sampling is relatively weak. Note that the bias in $(\hat{\alpha},\hat{\beta})$ when $\tau_{\max}$ = 5000 (Table 1) may be simply bias in marginal maximum likelihood estimates because maximum likelihood estimators are not generally unbiased. Another possibility is that bias was caused by computational error in calculating the marginal likelihood when the τ_i's were large, even though the marginal likelihood was calculated on the log-scale.

In the seabird bycatch analysis, results show that in almost all cases estimates of bycatch rates at the individual vessel level were not significantly affected by using the EB method. These results may indicate that individual vessel sampling levels (i.e., τ_i) are at a sufficiently high level that ML estimates are already precise estimates of seabird bycatch rates. For the prediction of the 2003 bycatch rate of seabirds from the 2002 analysis, the *TMSE* of the ML estimator of λ_i was reduced a minor amount from 0.0007904 to 0.0007339 by using the EB estimator of λ_i. However, the important issue is that neither the ML nor EB estimates for 2002 significantly correlated with the observed 2003 bycatch rates of seabirds. From the point of view of the EB method, this lack of correlation indicates that the Poisson catch rate, λ_i, was not a characteristic of individual fishing vessels, but is largely due to chance in any one year. Vessels 28 and 36 did not fish in 2003 and were excluded in the comparison. More detailed studies with current data should be carried out to determine if this is a valid conclusion.

As in 2002, the EB estimates for 1997 did not improve on stratum ML estimates for estimating seabird bycatch rates per vessel. However, ML and EB estimates of individual vessel bycatch rates in 1997 were found to be significantly positively correlated with observed bycatch rates for 1998. In this era of high bycatch rates of seabirds, bycatch rates were found to be more dependent on the practices of individual vessels.

Table 2

Bycatch of seabirds from the 2002 longline groundfish fishery in the eastern Bering Sea, where τ_i refers to the number of hooks observed, y_i is the observed number of birds caught, $\hat{\lambda}_i$ is the maximum likelihood (ML) estimate of the seabird bycatch rate, $\tilde{\lambda}_i$ is the empirical Bayes (EB) estimate of the seabird bycatch rate, and $\tilde{\lambda}_i\,\tau_i$ is the EB estimate of the expected observed number of birds caught.

Vessel number	Hooks (1000's) τ_i	Birds caught (numbers) y_i	ML vessel estimate $\hat{\lambda}_i$	EB estimate $\tilde{\lambda}_i$	EB estimate of birds caught $\tilde{\lambda}_i\,\tau_i$
1	1042	11	0.0106	0.0108	11.2
2	3895	22	0.0057	0.0058	22.4
3	917	16	0.0175	0.0175	16.0
4	1487	5	0.0034	0.0037	5.5
5	1272	12	0.0094	0.0097	12.3
6	2446	93	0.0380	0.0378	92.3
7	1477	0	0.0000	0.0004	0.6
8	1016	4	0.0039	0.0044	4.5
9	1758	6	0.0034	0.0037	6.5
10	253	0	0.0000	0.0021	0.5
11	1380	39	0.0283	0.0280	38.7
12	199	0	0.0000	0.0026	0.5
13	233	0	0.0000	0.0023	0.5
14	1212	13	0.0107	0.0109	13.2
15	2304	162	0.0703	0.0696	160.3
16	1408	30	0.0213	0.0212	29.9
17	2292	5	0.0022	0.0024	5.5
18	559	2	0.0036	0.0044	2.5
19	1332	26	0.0195	0.0195	26.0
20	753	17	0.0226	0.0224	16.9
21	2255	37	0.0164	0.0164	37.1
22	699	0	0.0000	0.0008	0.6
23	2907	5	0.0017	0.0019	5.5
24	3308	8	0.0024	0.0026	8.5
25	2221	10	0.0045	0.0047	10.4
26	1797	15	0.0084	0.0085	15.3
27	1413	13	0.0092	0.0094	13.3
28	34	8	0.2334	0.1271	4.4
29	659	101	0.1533	0.1468	96.7
30	1738	16	0.0092	0.0094	16.3
31	1527	4	0.0026	0.0030	4.5
32	2278	49	0.0215	0.0215	48.9
33	1332	19	0.0143	0.0144	19.1
34	2059	17	0.0083	0.0084	17.3
35	3089	2	0.0007	0.0008	2.6
36	1435	7	0.0049	0.0052	7.4
37	625	11	0.0176	0.0176	11.0
38	5447	14	0.0026	0.0027	14.5
39	1598	8	0.0050	0.0053	8.4
40	757	29	0.0383	0.0375	28.4

The empirical Bayes method indicates that sampling levels aboard individual vessels are sufficient to support individual vessel estimates of seabird bycatch rates. This seabird bycatch study was more an observational rather than a controlled study. It was not known which vessels used bird avoidance gear, or how this gear was deployed, or what other relevant onboard practices were taking place. Nevertheless, this seabird bycatch study illustrates how EB methods can provide alternative estimators and evaluation methods for a variety of sampling problems.

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