PART I

HYDRAULICS OF TIDAL INLETS
SIMPLE ANALYTIC MODELS FOR THE ENGINEER[1]

By

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I. INTRODUCTION

Inlet-Bay Regime

Inlets are common coastal features around the world. Essentially an inlet connects a lagoon, a bay or an estuary to the ocean (or sea), and the flow through the inlet channel is primarily induced by the tidal rise and fall of water level in the ocean. When speaking of the hydraulics of an inlet, one is interested mainly in determining the flow through the inlet and the tidal variation in the bay, given the following:

(1) Inlet geometry

(2) Bay geometry

(3) Bottom sediment characteristics in the inlet

(4) Fresh water inflow into the bay (and out through the inlet)

(5) Ocean tide characteristics

A combination of all these factors can produce a rather complex situation. For example, a significant fresh water outflow through the inlet may result in two distinct layers of flow - a saline bottom layer underneath a fresh water surface layer. Such a situation will give rise to a stronger ebb than flood at

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the surface, and a stronger flood than ebb at the bottom. Waves entering through the entrance can complicate the events further by generating their own flow oscillations and associated sediment transport.

From an engineering point of view, it is convenient to approach the problem of inlet hydraulics with reference to the simplest possible system, i.e., an inlet connecting the ocean to a well defined bay, as shown in Fig. 1. The inlet is assumed to have a certain bottom friction, but sedimentary aspects will not be considered in what follows. Salinity induced stratification of the flow is also ignored. Some of the major contributors to the development of the hydraulics of such a system are - Brown (1925), O'Brien (1931), Keulegan (1951), Baines (1957), Keulegan (1967), van de Kreeke (1967), Mota Oliveira (1970), Huval and Wintergerst (1972), Dean (1971), O'Brien and Clark (1973), King (1974) and Escoffier (1975).

Returning to Fig. 1, it is noted that most inlets have a well defined throat section, i.e., a minimum flow cross-sectional area. This is analogous to the Vena Contracta of such flow measuring devices as the venturi meter. O'Brien (1931) and others have shown that the throat section is a characteristic feature of an inlet, and that measurement of the current at the throat can yield information on the size of the throat and also on the relationship between the flow and the ocean and bay tides. Another important aspect of an inlet is the fact that its cross-section resembles a wide channel, such that the hydraulic radius can be approximated by the depth. This fact is generally lost when one looks at the commonly distorted depiction of the cross-section. This is illustrated by the example shown in Fig. 2.

II. HYDRAULICS OF A SIMPLE INLET-BAY SYSTEM

Problem Formulation

The governing equations for a simple inlet-bay system will be derived

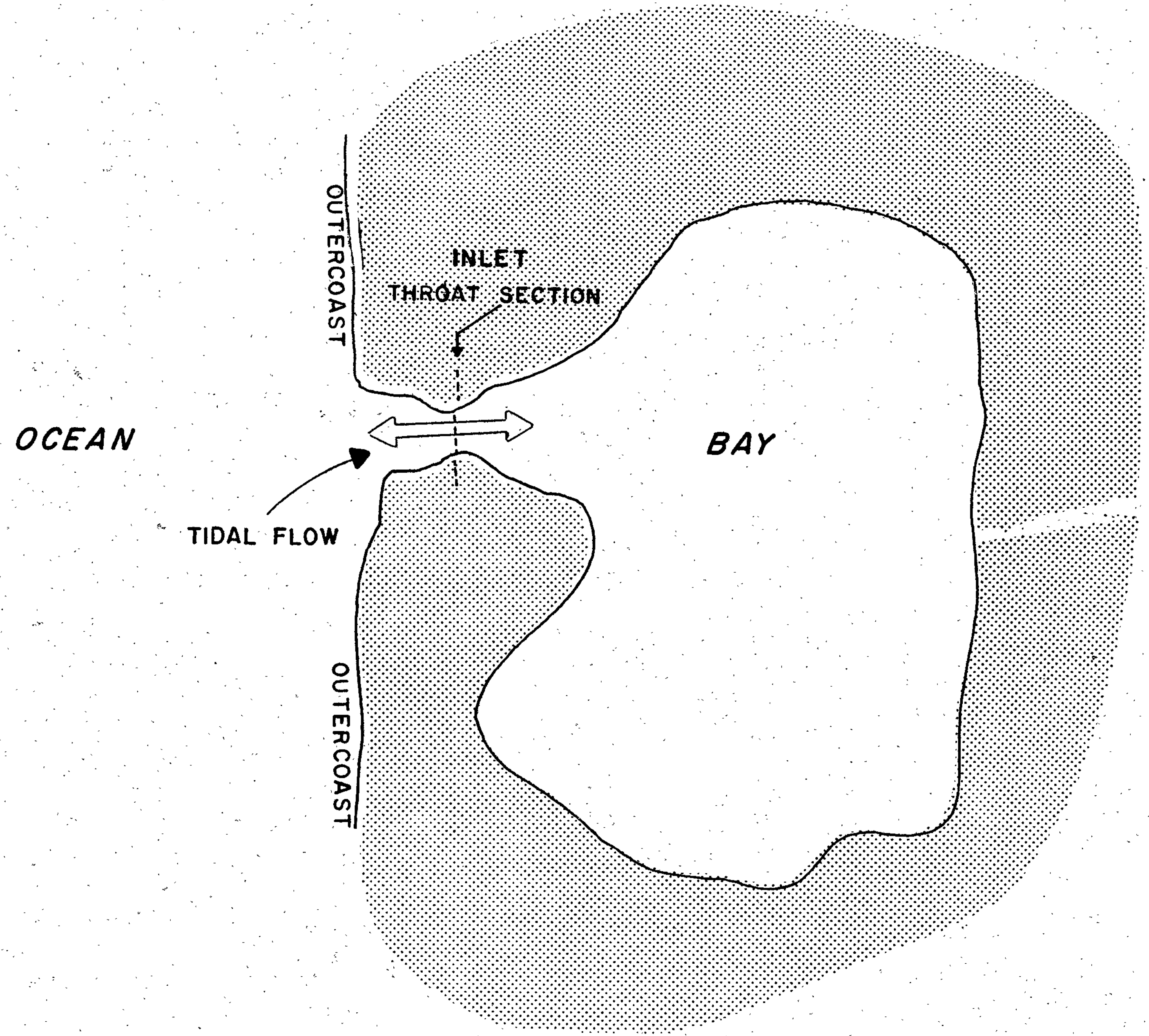

Fig. 1. A Simple Inlet-Bay System.

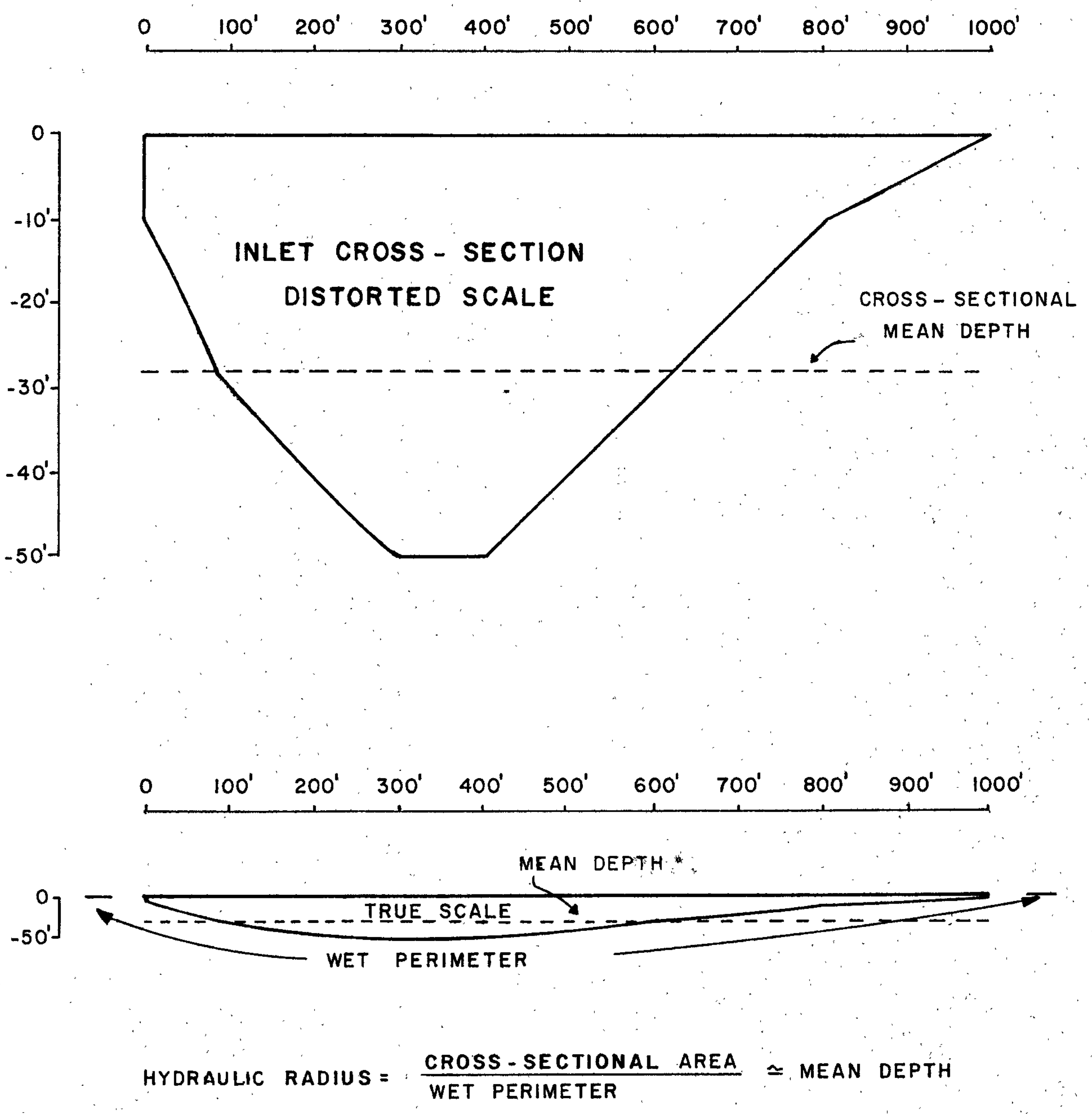

Fig. 2. Comparison between Distorted and Undistorted Depictions of Inlet cross-section.

subject to the following assumptions:

1. The inlet and bay banks are vertical.
2. The range of tide is small compared to the depth of water everywhere.
3. The bay surface remains horizontal at all times, i.e., the tide is "in phase" across the bay.
4. The meanwater level in the bay equals that in the ocean.
5. The acceleration of the mass of water in the channel is negligible.
6. No fresh water inflow into the bay.
7. No flow stratification due to salinity.
8. Ocean tide is represented by a sine curve.

The consequence of these assumptions is a deep bay connected to the ocean via an inlet of a short length L as shown in Fig. 3. Assumption 3 requires that the longest dimension of the bay be small compared to the time of travel of the tide through the bay.

With reference to the notation of Fig. 3, the Bernoulli equation can be written between the ocean and the bay,

$$\eta_o + \alpha_o \frac{V_o^2}{2g} = \eta_B + \alpha_B \frac{V_B^2}{2g} + \Delta h \qquad (2\text{-}1)$$

where

η_o = Ocean tide elevation with respect to meanwater level,

η_B = Bay tide elevation with respect to mean water level,

V_o = Ocean current velocity,

V_B = Bay current velocity,

α_o, α_B = Coefficients (greater than unity) which depend on the special distributions of V_o and V_B, respectively,

Δh = Total head loss between the ocean and the bay.

The ocean and bay are large bodies of water so that V_o and V_B can be

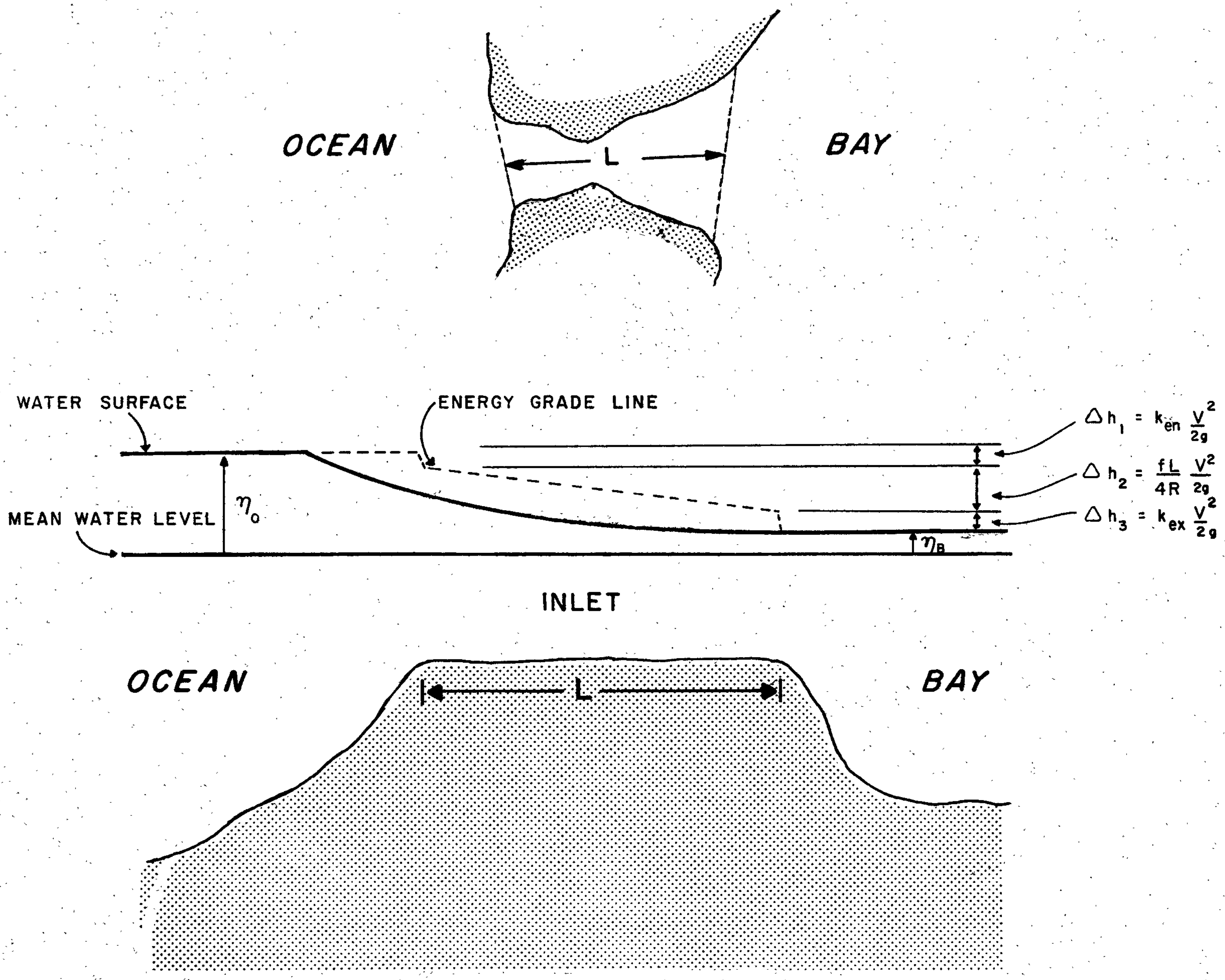

Fig. 3. Energy Losses Across an Inlet of length L.

considered to be negligible. Therfore Eq. (2-1) becomes

$$\Delta h = \eta_o - \eta_B \qquad (2\text{-}2)$$

As shown in Fig. 3, Δh can be considered to have three contributions, i.e.,

$$\Delta h = \Delta h_1 + \Delta h_2 + \Delta h_3 \qquad (2\text{-}3)$$

Here,

$$\Delta h_1 = k_{en} \frac{V^2}{2g} \qquad (2\text{-}4)$$

is the head loss at the flow entrance due to the convergence of the flow streamlines into the inlet, to generate a velocity V in the inlet and k_{en} is the entrance loss coefficient.

$$\Delta h_2 = \frac{fL}{4R} \frac{V^2}{2g} \qquad (2\text{-}5)$$

is the gradual head loss due to bottom friction in the channel of length L. R is the hydraulic radius and f is the Darcy-Weisbach friction factor.

$$\Delta h_3 = k_{ex} \frac{V^2}{2g} \qquad (2\text{-}6)$$

is the head loss at the exit due to the expansion of the flow out of the channel. k_{ex} is the exist loss coefficient.

Substitution of Eqs. (2-4), (2-5) and (2-6) into (2-3) and of (2-3) into (2-2) yields,

$$\frac{V^2}{2g} \left(k_{en} + k_{ex} + \frac{fL}{4R}\right) = \eta_o - \eta_B \qquad (2\text{-}7)$$

or taking square roots,

$$V = \sqrt{\frac{2g}{\left(k_{en} + k_{ex} + \frac{fL}{4R}\right)}} \sqrt{|\eta_o - \eta_B|} \cdot \text{sign}\,(\eta_o - \eta_B) \qquad (2\text{-}8)$$

Sign $(\eta_o - \eta_B)$ must be included since the current reverses in direction every one-half tidal cycle.

The rate of rise and fall of the bay tide is specified by the following continuity equation:

$$Q = VA = A_B \frac{d\eta_B}{dt} \qquad (2\text{-}9)$$

where Q is the flow rate through the inlet, A the inlet flow cross-sectional area and A_B the bay surface area. Eliminating V between Eqs. (2-8) and (2-9) leads to,

$$\frac{d\eta_B}{dt} = \frac{A}{A_B} \frac{\sqrt{2g}}{\sqrt{k_{en} + k_{ex} + \frac{fL}{4R}}} \sqrt{|\eta_o - \eta_B|} \, . \, \text{sign}\,(\eta_o - \eta_B) \qquad (2\text{-}10)$$

we introduce the dimensionless quantities

$$\eta_o' = \eta_o/a_o \, , \; \eta_B' = \eta_B/a_o \, , \; \theta = 2\pi T/t \qquad (2\text{-}11)$$

where a_o is the ocean tide amplitude (one-half the tidal range) and T is the tidal period. Substitution into Eq. (2-10) gives,

$$\frac{d\eta_B'}{d\theta} = K \sqrt{|\eta_o' - \eta_B'|} \, . \, \text{sign}\,(\eta_o' - \eta_B') \qquad (2\text{-}12)$$

where

$$K = \frac{T}{2\pi a_o} \; \frac{A}{A_B} \; \frac{\sqrt{2ga_o}}{\sqrt{k_{en} + k_{ex} + \frac{fL}{4R}}} \qquad (2\text{-}13)$$

K is referred to as the "coefficient of filling or repletion" as defined by Keulegan (1951). Note that Eq. (2-12) is a first order differential equation for η_B', the solution for which must be expected to be a function of K and θ only, for a known sinusoidal variation of the ocean tide η_o (or η_o').

A definition sketch for the tides and current is shown in Fig. 4.

Solution

Selecting

$$\eta_o = a_o \sin(\theta - \tau) \qquad (2\text{-}14)$$

Eq. (2-12) can be solved for η_B as a function of a_o, θ and K. Note that τ represents the time lag as indicated in Fig. 4. Because of the non-linearity of Eq. (2-12), η_B is not a sine curve, but has higher harmonics.

To an engineer, three aspects regarding the hydraulics of inlet-bay are of greatest significance.

1. Lag of Slack Water ε, After HW and LW in the Ocean

Slack water is the time of zero current just prior to current reversal. According to the simple case depicted in Fig. 4, this occurs when the ocean and bay tide curves intersect, i.e., There is no head difference necessary for flow. The time lags (in radians) of slack after HW and after LW are observed to be the same, in this idealized case.

Keulegan's solution for ε in degrees as a function of the repletion coefficient K is presented in Fig. 5. Note that the time of slack water is also the time of maximum bay elevation according to this model, as seen in Fig. 4. As the lag ε increases, the bay tide becomes smaller until ε approaches 90° when there is no tidal fluctuation in the bay. This limiting situation occurs when $K \to 0$, which can occur when the bay is so large that $A/A_B \to 0$, or when the friction term under the square root sign in the denomenator (Eq. 2-13) tends to be very large.

The other limiting situation is when $K \to \infty$, when $\varepsilon \to 0°$. This is the case of a very wide inlet (A/A_B large) or negligible friction. Fig. 4 shows that in this case the bay tide approaches the ocean tide.

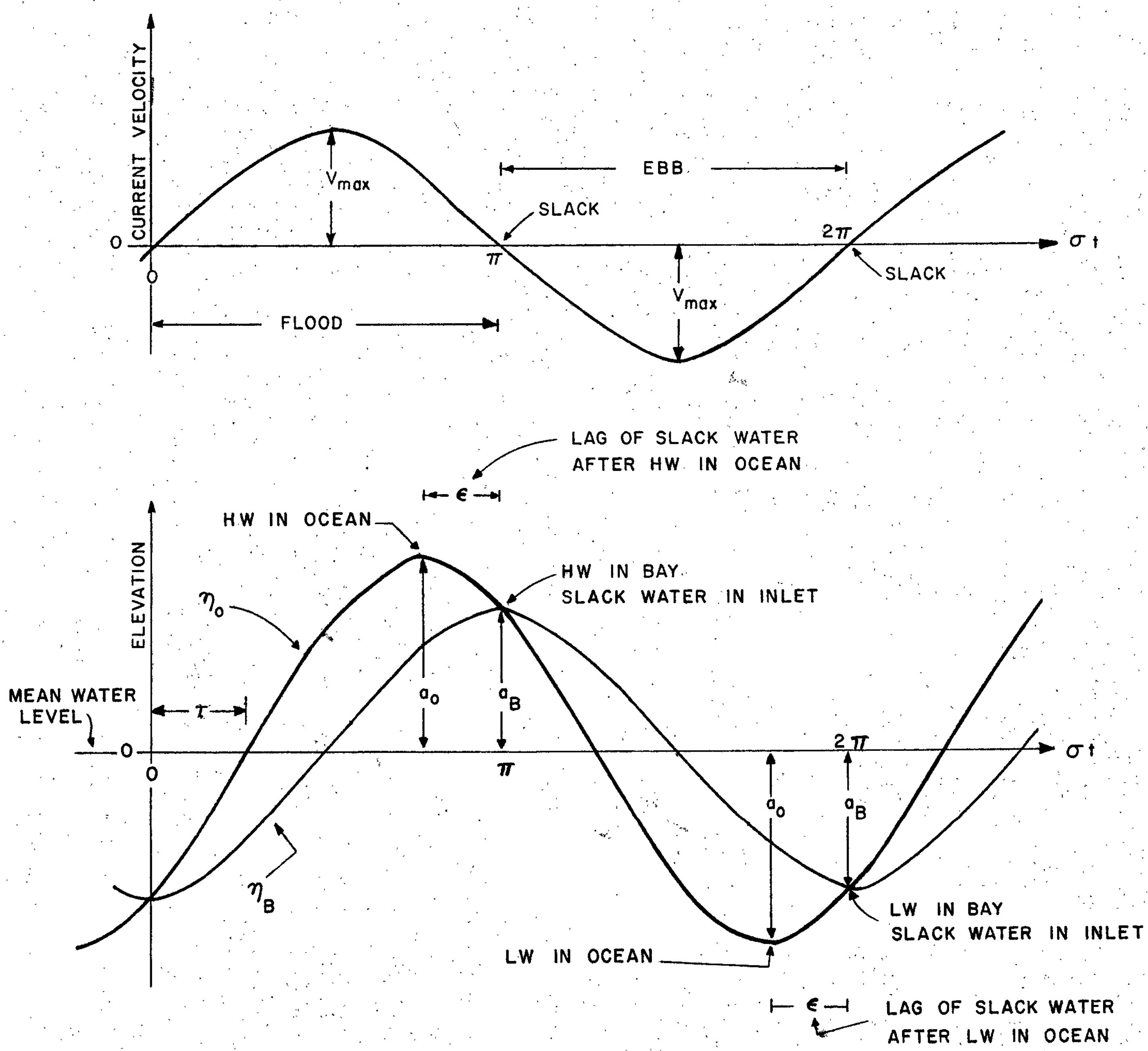

Fig. 4. Ocean Tide, Bay tide and Current Through the Inlet as Functions of Time σt in Radians.

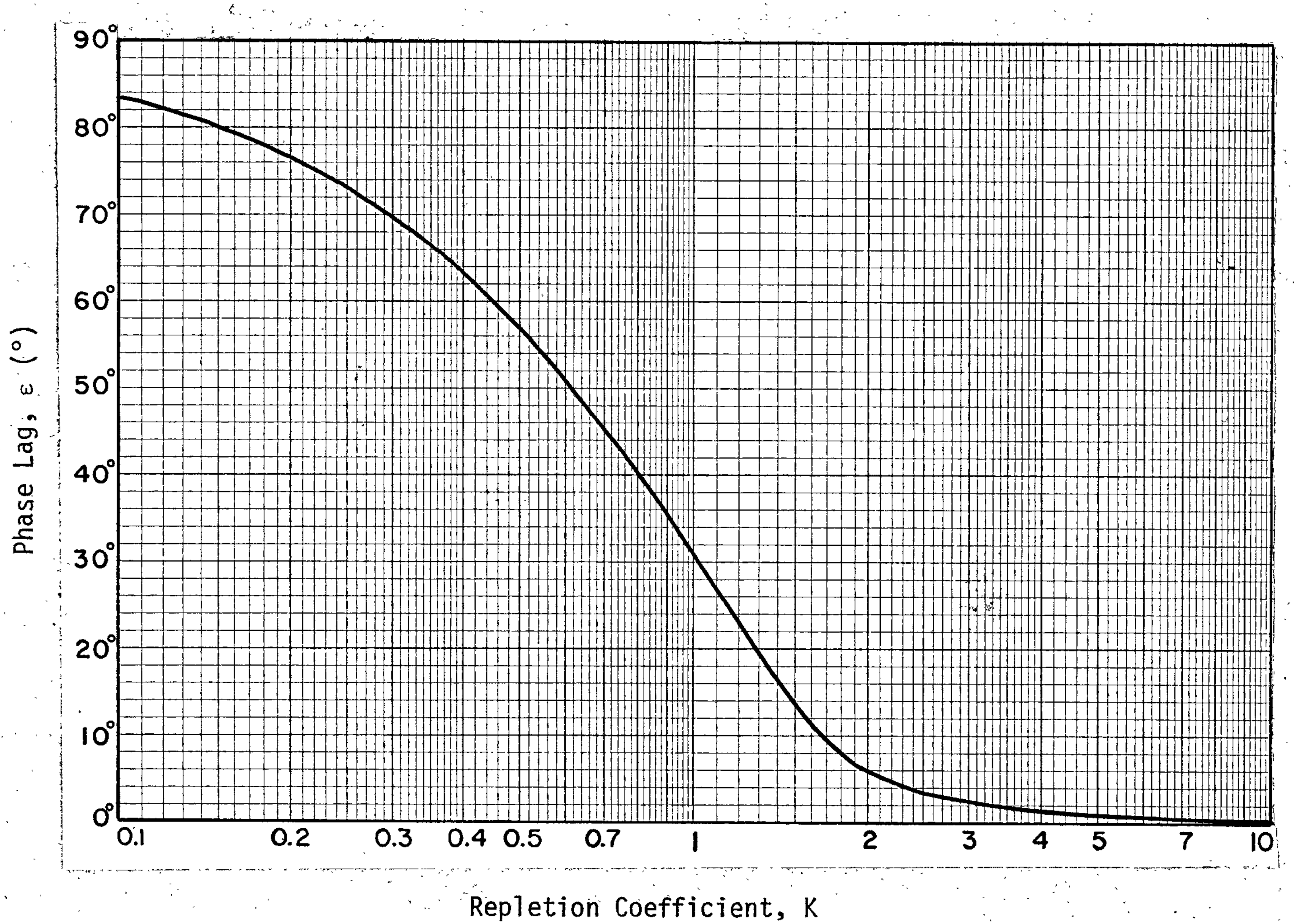

Fig. 5. Lag ε in Degrees as a Function of Keulegan's Repletion Coefficient K.

2. Maximum Bay Tide Range

As noted, this occurs at the time of slack water in the inlet channel. Keulegan's solution of the ratio a_B/a_o, where a_B is the bay tide amplitude, as a function of K is presented in Fig. 6.

At this stage it is worthwhile to look at a result obtained by Dean (1971) using a linearized approach. Dean assumed an approximation of Eq. (2-8) which resulted in a linear relationship between the velocity V and the head difference $\eta_o - \eta_B$, and the definition of a "linear discharge coefficient" C_{DL} defined as,

$$C_{DL} = \frac{1}{\sqrt{k_{en} + k_{ex} + \frac{fL}{4R}}} \cdot \frac{\sqrt{a_o}}{\sqrt{(\eta_o - \eta_B)_{max}}} \qquad (2\text{-}15)$$

where $(\eta_o - \eta_B)_{max}$ is the maximum head difference across the inlet. Omitting calculations, it can be shown, using this approach, that,

$$\frac{a_B}{a_o} = \frac{(C_1/\sigma)}{\sqrt{1 + (C_1/\sigma)2}} \qquad (2\text{-}16)$$

where $\sigma = 2\pi/T$ is the tidal frequency in radians and

$$C_1 = C_{DL} \frac{\sqrt{2g}}{\sqrt{a_o}} \frac{A}{A_B} \qquad (2\text{-}17)$$

Eq. (2-16) allows calculcation of a_B if an estimate of C_{DL} is available, in addition to a_o, A, A_B and σ. We will return to Eq. (2-16) later.

3. Maximum Current

The maximum flood and ebb currents, V_{max}, are defined in Fig. 4. V_{max} is also referred to as the strength of current. Fig. 7 shows Keulegan's solution presented as a dimensionless maximum current V'_{max} as a function of K, where

$$V'_{max} = \frac{T}{2\pi a_o} \frac{A}{A_B} V_{max} \qquad (2\text{-}18)$$

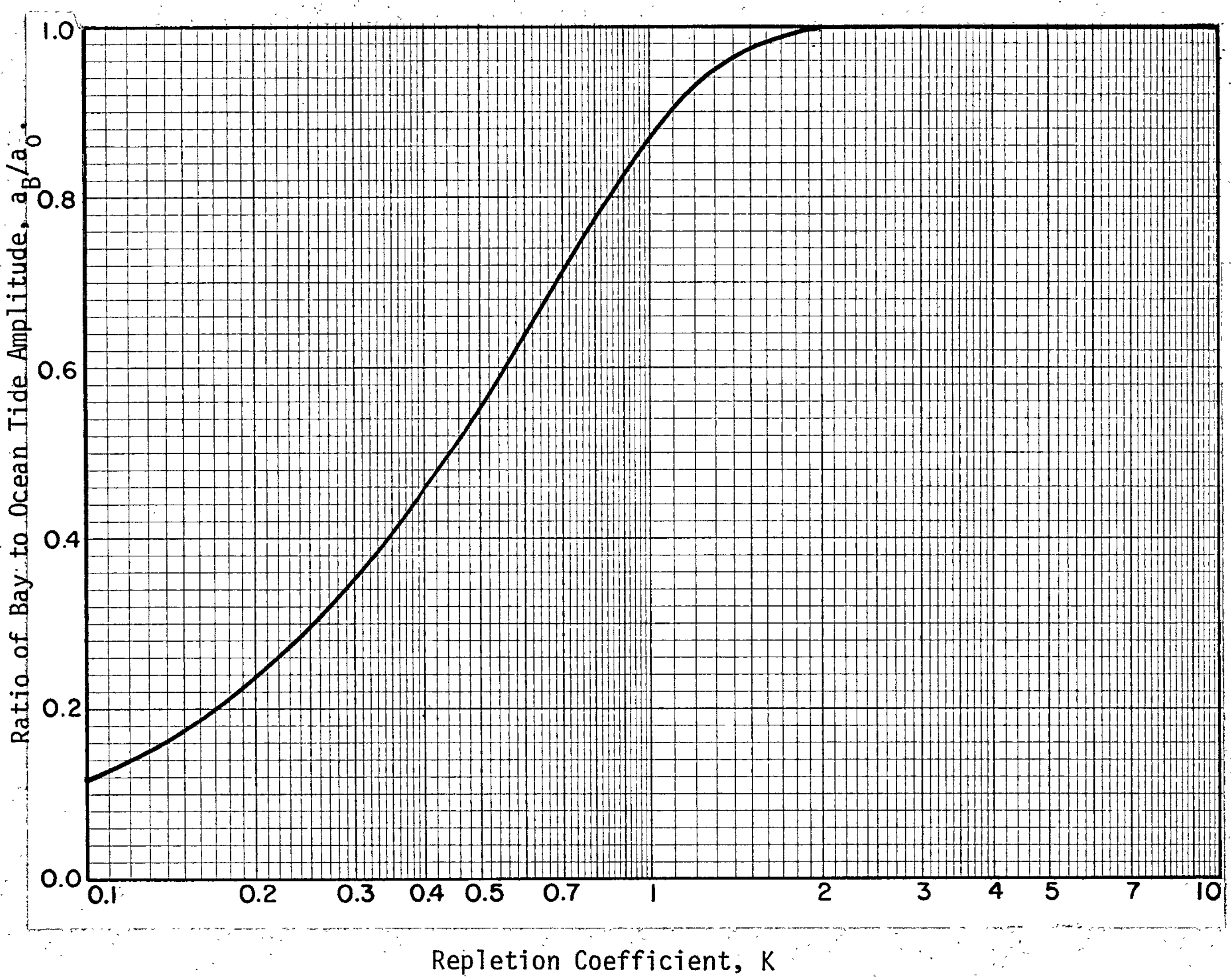

Fig. 6. Ratio of Bay Tide Amplitude to the Ocean Tide Amplitude, a_B/a_0, as a Function of Keulegan's Repletion Coefficient K.

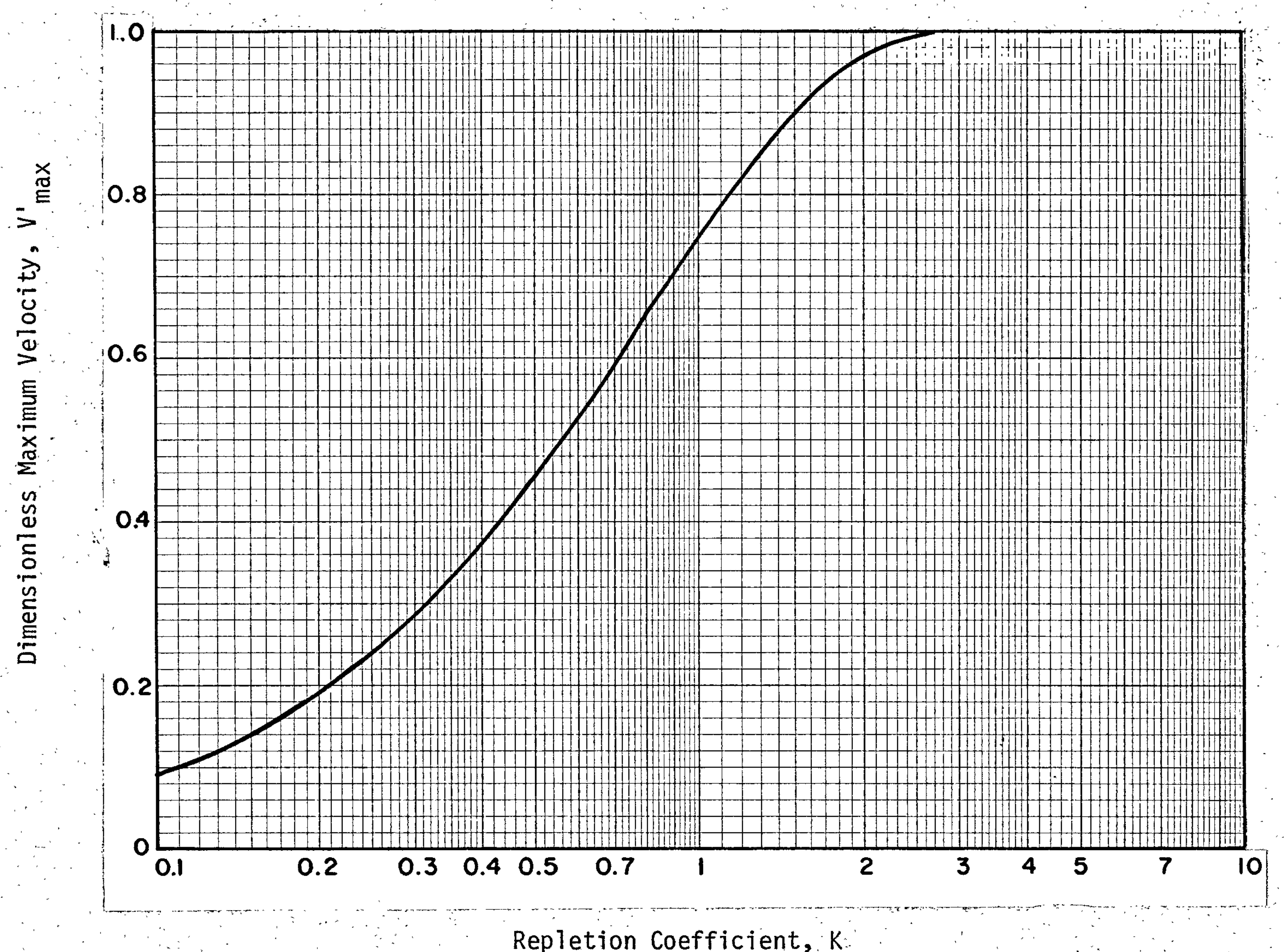

Fig. 7. Dimensionless Maximum Velocity, V'_{max}, as a Function of Keulegans Repletion Coefficient.

Note that V_{max} occurs when the head difference $\eta_o - \eta_B$ is a maximum. This happens when the bay tide is at mean sea level, before HW in the ocean and again before LW in the ocean.

Impedance and Equivalent Channel

In the term,

$$k_{en} + k_{ex} + \frac{fL}{4R} \tag{2-19}$$

the value of k_{ex} is usually taken as unity, whereas k_{en} depends on the shape of the entrance. Generally, k_{en} ranges from 0.05 to 0.25 (Dean, 1971). From an engineering point of view, however, it is sufficiently accurate to assume the sum $k_{en} + k_{ex}$ to be equal to unity.

A problem with real inlets is that they do not have a constant flow cross-section, and therefore the velocity V changes from point to point along the channel. In order to overcome this problem, O'Brien and Clark (1973) suggested defining a channel with a constant cross-sectional area equal to the throat area A_c at mean sea level, with the additional requirement that the total head loss Δh_2 be equal in the two channels. Omitting details, this leads to the definition of a length L_c of the equivalent channel as, (See Fig. 8(a), (b))

$$L_c = R_c A_c^2 \int_0^{x_L} \frac{dx}{R_x A_x^2} \tag{2-20}$$

where

A_c = Throat cross-sectional area at mean sea level,

A_x = Channel cross-sectional area at mean sea level as a function of distance x,

R_x = Channel hydraulic radius at mean sea level as a function of distance x,

x = Distance measured along the deepest part of the channel,

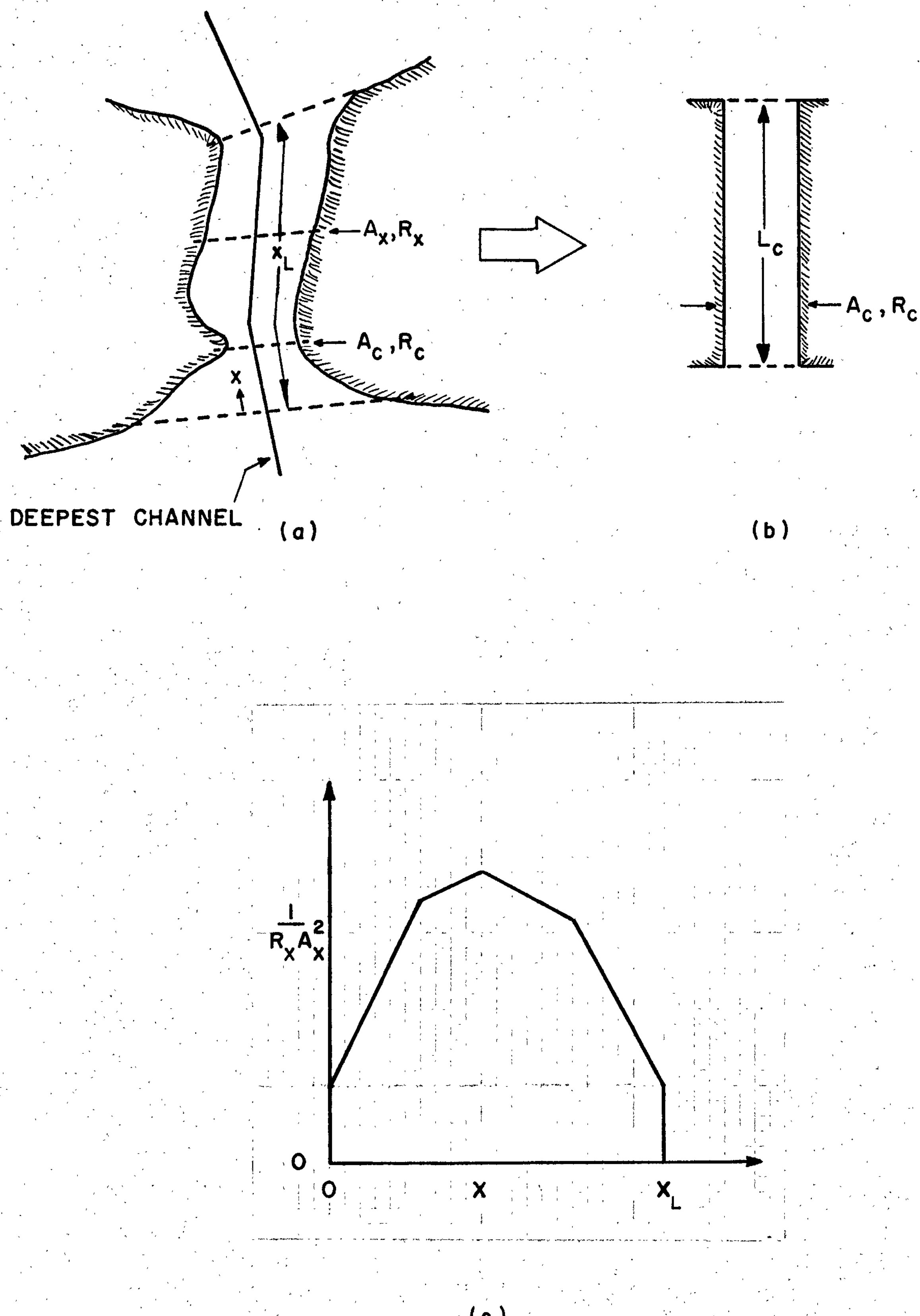

Fig. 8. Definition and Computation of Equivalent Length of an Inlet.

x_L = Length of the channel (maximum value of x), along the deepest part of the channel.

In view of the above, one may write,

$$F = k_{en} + k_{ex} + \frac{fL_c}{4R_c} \qquad (2\text{-}21)$$

$$= 1 + \frac{fL_c}{4R_c} \qquad (2\text{-}22)$$

Where F may be referred to as the "impedance" of the inlet-bay system. We will now redefine the repletion coefficient as,

$$K = \frac{T}{2\pi a_o} \frac{A_c}{A_B} \sqrt{\frac{2ga_o}{F}} \qquad (2\text{-}23)$$

Also A in Eq. (2-18) should be replaced by A_c, and V by V_c in Eq. (2-8) and elsewhere. V_{max} in Fig. 7 is now the maximum cross-sectional average current at the throat.

Note that in the absence of geometric data on the channel, i.e., $R_x = R_x(x)$ and $A_x = A_x(x)$, it is best to select the length of the inlet L between the entrance and exit, to compute F according to Eq. (2-22).

Example 2-1

Consider the case of Rockaway Inlet on the south shore of Long Island (Mehta and Hou, 1974). This inlet connects Jamaica Bay to the Atlantic Ocean. The procedure is as follows:

1. Planimetering Jamaica Bay[3] on National Ocean Survey (NOS) chart 1215 gives $A_B = 6.20 \times 10^8$ ft^2.
2. From NOS chart 542, the following data on the channel cross-sections are obtained:

[3] To be consistent with the definition of the tidal prism as defined in the Section III, the bay should include the body of water bayward of the throat section, rather than the section at the bayward end of the inlet.

Section	x (ft)	R_x^* (ft)	A_x (ft^2)
1	0	17	1.85×10^5
2	6000	16	8.88×10^4
3	10000	14	9.11×10^4
4	15600	26	7.18×10^4
5	24600	26	1.25×10^5

*Hydraulic radius assumed to be equal to mean depth.

3. The throat section is observed to be at section 4.

   $A_c = 7.18 \times 10^4$ ft^2

   R_c = 26 ft

   x_L = 24600 ft

   Graphical integration using the above data according to Eq. (2-20)[4] gives L_c = 20240 ft.

4. Select f = 0.022 (this will be discussed in Section IV). This gives F = 5.2 according to Eq. (2-22).

5. Since the tide gage at an inlet records a tide which is affected by the flow through the inlet, the effective "ocean" tide range $2a_o$ must be obtained by interpolation between two outer coast tide gage station, one to the left and the other to the right of the inlet. NOS Tide Tables give the following data:

Station	Mean Tide Range
Coney Island	4.70
Long Beach, Outer Coast	4.50

[4] Compute $1/R_x A_x^2$, and plot it on y axis, for the corresponding x on the x-axis. Join the points by straight lines and determine the area under the curve thus obtained by a planimeter. Multiply this area by $R_c A_c^2$ (See Fig. 8 (c)).

Linear intropolation between these two stations at the location of Rockaway Inlet gives $2a_o = 4.65$ ft.

6. For a semi-diurnal range of tide T = 12.4 hours, Eq. (2-23) gives the repletion coefficient

$$K = 1.9$$

7. From Fig. 5,

$$\varepsilon = 8°$$

$$= 8° \times \frac{T}{360°} = \underline{0.27} \text{ hour}$$

From Fig. 6,

$$a_B/a_o = 0.99$$

$$2a_B = 0.99 \times 4.65 = \underline{4.60} \text{ ft.}$$

From Fig. 7,

$$V'_{max} = 0.93$$

$$V_{max} = 0.93 \times \frac{2\pi a_o}{T} \cdot \frac{A_B}{A_c} = \underline{2.62} \text{ ft/sec}$$

III. THE TIDAL PRISM

Hydraulically Determined Prism

The tidal prism is the volume of water that enters the bay during flood, and leaves during ebb. The prism is significant because it is a measure of the rate at which waters in the bay are renewed by oceanic flushing. By definition the prism P_H is,

$$P_H = \int_0^{T/2} Q dt \qquad (3\text{-}1)$$

If we assume that the flow rate Q is expressed as a sinefunction with an amplitude

Q_{max}, i.e.,

$$Q = Q_{max} \sin \sigma t \qquad (3\text{-}2)$$

then substituting Eq. (3-2) in (3-1) gives,

$$P_H = \frac{Q_{max}\ T}{\pi} \qquad (3\text{-}3)$$

Keulegan (1967) noted that in order to account for the non-linearities in the actual flow rate Q, Eq. (3-3) must be divided by a coefficient C_K. It was found that C_K varies with the repletion coefficient K and ranges from 0.81 to 1. From an engineering point of view, an average value of $C_K = 0.86$ will suffice. Thus, noting that $Q_{max} = V_{max}\ A_c$, Eq. (3-3) becomes,

$$P_H = \frac{V_{max} T\ A_c}{\pi C_K} \qquad (3\text{-}4)$$

where P_H may be referred to as the hydraulically computed tidal prism.

<u>Volumentrically Determined Prism</u>

Another definition of the prism is

$$P_V = 2a_B A_B \qquad (3\text{-}5)$$

This is the volumetrically determined prism, and for a single inlet-bay system, P_H and P_V are clearly identical.

<u>Example 3-1</u>

Consider the inlet connecting O'Brien's Lagoon to the Gulf of Mexico, at Treasure Island, Florida (Sedwick and Mehta, 1974; Sedwick, 1974). Given:

$$A_B = 9.1 \times 10^5\ ft^2$$

$$2a_B = 2.0\ ft$$

Therefore, according to Eq. (3-5),

$$P_V = \underline{1.82 \times 10^6} \ ft^3$$

Given:

$$V_{max} = 1.90 \text{ ft/sec}$$

$$A_c = 42 \text{ ft}^2$$

T = 18 hours (average of diurnal and semi-diurnal periods)

Therefore, according to Eq. (3-4),

$$P_H = \underline{1.91 \times 10^6} \ ft^3$$

which compares reasonably with P_V.

When there are more than one connections between the bay and the ocean, the bay is filled by all the inlets, and therefore P_V becomes the sum of the tidal prisms of each inlet. Thus for N inlets,

$$\sum_{i=1}^{i=N} P_{Hi} = P_V = 2a_B A_B \qquad (3\text{-}6)$$

also,

$$\sum_{i=1}^{i=N} P_{Hi} = \frac{T}{\pi C_K} \sum_{i=1}^{i=N} V_{maxi} A_{ci} \qquad (3\text{-}7)$$

The range of tide in the bay may be different from point to point. For engineering estimation, it is generally sufficient to assume a range which is the average of these values.

Example 3-2

John's Pass and Blind Pass connect the Gulf of Mexico to north Boca Ciega Bay. Current measurements have yielded the following values for the average hydraulically determined tidal prisms (Mehta and Adams, 1975).

$$P_{HJohn's\ Pass} = 3.60 \times 10^{8}\ ft^{3}$$

$$P_{HBlind\ Pass} = 2.20 \times 10^{7}\ ft^{3}$$

$$\sum_{i=1}^{i=2} P_{Hi} = P_{HJohn's\ Pass} + P_{HBlind\ Pass} = \underline{3.82 \times 10^{8}}\ ft^{3}$$

For north Boca Ciega Bay,

$2a_B$ = 1.7 ft. (average of tide at several points around the bay)

$$A_B = 2.21 \times 10^{8}\ ft^{3}$$

$$P_V = \underline{3.76 \times 10^{8}\ ft^{3}}$$

ΣP_{Hi} and P_V are thus observed to be in reasonable agreement.

Prism-Area Relationship

O'Brien (1931) spent a summer studying Pacific Coast inlets and made the observation that the ratio of the tidal prism P_s based on the spring range of tide divided by the throat section A_c is either constant or a slowly varying function of the tidal prism, depending on whether the inlets have no jetties or two jetties, respectively. This observation pertains to inlets which are in stable sedimentary equilibrium, and has found worldwide applicability[5]. Recent studies have also shown that a similar rule may be used with reasonable accuracy if P_s is replaced by the prism P_m based on the mean range of tide.

The ratio P_s/A_c has been found to depend on the number of jetties (zero, one or two) and on whether the inlet is on the Atlantic, Gulf or Pacific

[5] It is not applicable to newly cut or improved inlets.

Coast. This adds up to nine relationships, and nine more for P_m/A_c. Furthermore, in some cases, different investigators have proposed slightly different relationships for the same category (e.g., two jettied Pacific Coast inlets). Ignoring these details, the following relationships give first order accuracy for engineers (P_m in cubic feet, A_c in square feet):

$$P_m = 5.3 \times 10^4 A_c \text{ ft (unimproved)} \tag{3-8}$$

$$P_m = 1.3 \times 10^4 A_c^{1.1} \text{ ft (one and two jetties)} \tag{3-9}$$

Eq. (3-9) was proposed by Johnson (1973). A convenient approximation for converting P_s to P_m or <u>vice</u> <u>versa</u> is,

$$\frac{P_m}{P_s} = \left(\frac{a_{om}}{a_{os}}\right)^{1/2} \tag{3-10}$$

where a_{om} is the mean and a_{os} the spring ocean tide amplitudes. Eq. (3-10) is obtained by combining Eqs. (2-16) and (2-17) with (3-5), (Mehta, et al., 1975).

Considering Eq. (3-8), upon substitution into Eq. (3-4) yields,

$$\frac{V_{max} T}{\pi C_k} = 5.3 \times 10^4 \text{ ft}$$

with $T = 12.4$ hours and $C_k = 0.86$

$$V_{max} = 3.2 \text{ ft/sec}$$

Eq. (3-8) is therefore essentially a statement that under semi-diurnal tides, maximum cross-sectional average currents at the throats of unimproved inlets are of the order of 3 feet per second. Indeed it has been found that in general maximum currents do infact range from 2.5 to 3.5 feet per second.

IV. DEVIATIONS FROM THE SIMPLE SYSTEM

A Semi-Empirical Approach

Although the model described in Section II is based on sound hydrodynamic principles, the assumptions inherent in its derivation limit its applicability to relatively small inlet-bay systems. O'Brien and Clark (1973) therefore used the available data on ocean tides in NOS Tide Tables, and inlet currents in NOS Tidal Current Tables, to examine the behavior of a number of real inlets. They considered the bay tide to be approximated by a sine curve (no higher harmonics) which results in the following two simple relationships,

$$V_{max} = \sqrt{\frac{2a_o g \sin\varepsilon}{F}} \qquad (4\text{-}1)$$

$$\frac{a_B}{a_o} = \cos\varepsilon \qquad (4\text{-}2)$$

Because of the asymmetry of real tides and currents, unlike Fig. 4, the strengths V_{max} of flood and of ebb are not in general equal in magnitude, nor are the lags ε. Furthermore, because the ocean tide range $2a_o$ changes somewhat from day to day, V_{max} and ε also vary correspondingly, and F, as computed from Eq. (4-1) is found to vary as well. O'Brien and Clark however noted that two week averages, $\bar{\varepsilon}$ and $\bar{F}$, of the lag ε and impedance F, respectively, for a given inlet are nearly constant. We may thus write Eq. (4-1) as

$$V_{max} = \sqrt{\frac{g \sin\bar{\varepsilon}}{\bar{F}}} \sqrt{2a_o} \qquad (4\text{-}3)$$

or

$$V_{max} = C_D \sqrt{2a_o} \qquad (4\text{-}4)$$

where $C_D = \sqrt{g \sin\bar{\varepsilon}/\bar{F}}$ may be referred to as a characteristic velocity coefficient of the inlet. Once C_D is evaluated for a given inlet, it may be used to approximately estimate V_{max} under a given range of ocean tide $2a_o$.

Before proceeding, a point should be noted with reference to the strength of current reported in the Current Tables. This generally is not a cross-sectional average, but is close to what may be measured near the surface in the center of the channel at the throat. If this value is V_{maxs}, then it has been found to be reasonable to obtain V_{max} (cross-sectional average) from

$$V_{max} = 0.85\ V_{maxs} \tag{4-5}$$

where the 0.85 factor is an empirically determined constant.

Example 4-1

Consider Indian River Inlet, connecting the Atlantic to Indian River Bay, Delaware. NOS Tide Table and Current Table data may be combined to yield the following biweekly average values (O'Brien and Clark, 1973),

$$\bar{\varepsilon} = \underline{65.8°}$$

$$\bar{F} = \underline{15.3}$$

This gives $C_D = 1.39$, so that Eq. (4-4) for this inlet is ($g = 32.2$ ft/sec^2)

$$V_{max} = 1.39\sqrt{2a_o} \tag{4-6}$$

Eq. (4-6) can now be used to approximately determine V_{max} under any given range $2a_o$ (in feet) at Indian River Inlet. It is interesting to note that Keulegan (1967) obtained the same value of C_D using his method and his data. For larger inlet-bay systems, however, Keulegan's results deviate from predictions based on measurements. Note that Eq. (4-2) gives $a_B/a_o = 0.41$. Tide Tables yield $a_B/a_o = 0.21$. The agreement is not very close.

If one is interested in differences in magnitude between the strengths of flood and ebb, Eq. (4-3) may be evaluated for the flood and ebb phases separately.

Example 4-2

Consider Chesapeake Bay Entrance. Tide Tables and Current Tables data, and application of Eq. (4-1) yields for the flood phase (subscript F) and ebb phase (subscript E)

$$\bar{\varepsilon}_F = \underline{110.2°}$$

$$\bar{\varepsilon}_E = \underline{113.8°}$$

$$\bar{F}_F = \underline{51.6}$$

$$\bar{F}_E = \underline{20.7}$$

thus

$$V_{maxF} = 0.76\sqrt{2a_o} \qquad (4\text{-}7)$$

$$V_{maxE} = 1.19\sqrt{2a_o}$$

Note that the lags exceed 90°. Eq. (4-2) can not therefore be used to predict the range of tide in the bay.

The approach described in this section is useful for determining C_D in Eq. (4-4), provided ocean tide and inlet current records, atleast two weeks long, are available. Tide Tables are generally sufficient for tides, because predictions at a large number of outer coast stations are available. However, current predictions are not available at all the inlets. It is moreover expensive and often cumbersome to install a current meter at an inlet. Times of slack water can however be recorded by an onshore observer without difficulty. This yields ε values and the average, $\bar{\varepsilon}$. To calculate C_D we also need F. This can be estimated from Eq. (2-22) if the friction-factor f is available in addition to L_c and R_c.

f values in inlets have been generally found to be larger than those for rivers, and therefore, friction factors or Manning's n for rivers can not be used for inlets. To estimate f in inlet, the procedure described in this section was used to calculate the average impedance $\bar{F}$ and lag $\bar{\varepsilon}$ for a number of inlets (e.g., Rockaway Inlet by Mehta and Hou (1974)). Eq. (2-22) then gave f, which was converted to the Chezy coefficient C according to,

$$C = \sqrt{\frac{8g}{f}} \qquad (4\text{-}8)$$

An observation that larger inlets have higher Chézy coefficients led to the empirical plot of Fig. 9, where C has been plotted against L_s/R_c. The straight line relationship expressed according to this plot is similar to the relationship between C and A_c proposed by Bruun (1966), but yields comparatively lower C values. L_s is defined in Fig. 10 as the distance along the inlet channel between the outer bar and the point where the channel ends or is divided by the inner shoal. L_s can be estimated from charts or surveys with relative ease. Clearly, because of the nature of actual inlet channels, a degree of engineering judgement is often required in such an estimation. Fig. 9 is then entered with the ratio L_s/R_c and C determined on the vertical axis. Then $f = 8g/C^2$.

Variable Bay Area

Often the bay has gently sloping rather than vertical banks, in which case the surface area at low water may be significantly smaller than at high water.

Assume,

$$A_B = A_{BMWL}\,(1 + \beta\eta_B) \qquad (4\text{-}9)$$

where A_{BMWL} is the bay surface area at mean water level in the bay and β is a coefficient which depends on the bay geometry. The continuity expression is,

$$Q = \frac{d}{dt}\,(A_B\eta_B) \qquad (4\text{-}10)$$

Substitution of Eq. (4-9) into (4-10) gives,

$$Q = A_{BMWL}\,(1 + 2\beta\eta_B)\,\frac{d\eta_B}{dt} \qquad (4\text{-}11)$$

This analogous to Eq. (2-9). Following a procedure similar to Section II,

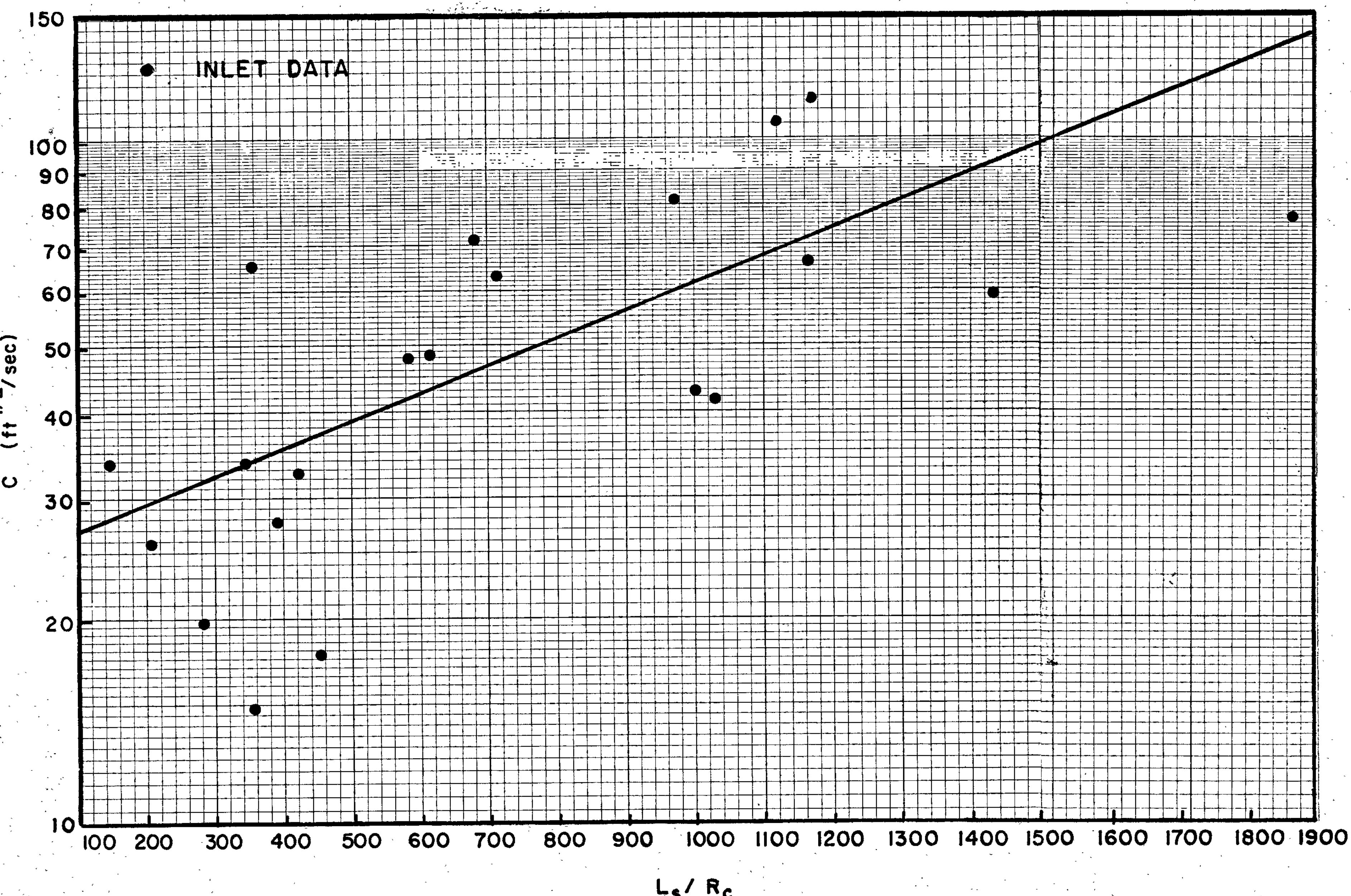

Fig. 9. Empirical Relationship between the Chezy Coefficient C ($ft^{1/2}$/sec) and the Ratio L_s/R_c.

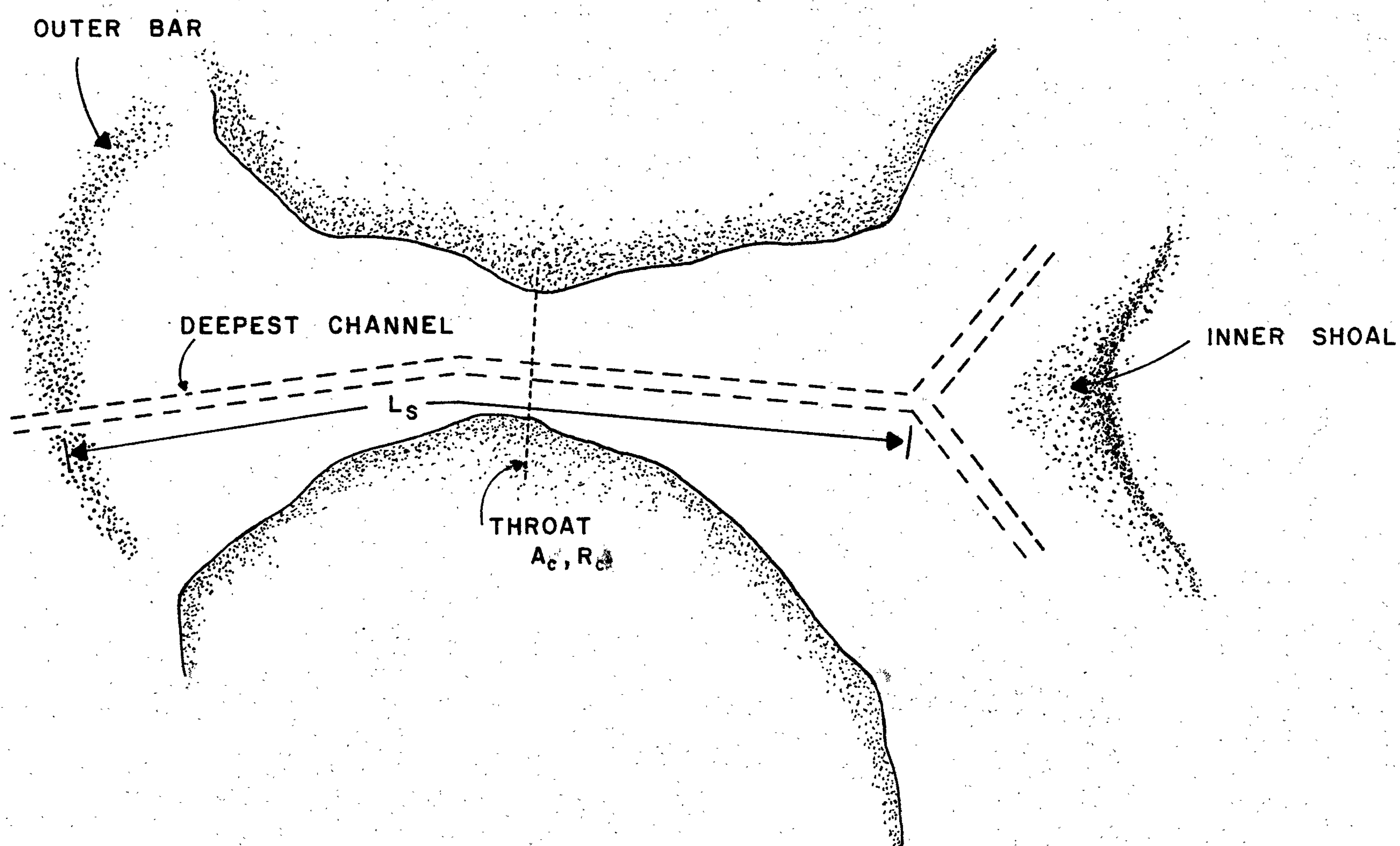

Fig. 10. Definition of the Length L_s to be used in Fig. 9.

$$\frac{d\eta_B'}{d\theta} = \frac{K}{(1 + 2a_o\beta\eta_B')} \sqrt{|\eta_o' - \eta_B'|} \cdot \text{sign}(\eta_o' - \eta_B') \quad (4\text{-}12)$$

Here, in the repletion coefficient, K, $A_B = A_{BMWL}$. The hydraulics of the inlet are observed to depend on the parameter β, in addition to K. In general, Eq. (4-12) must be solved on a computer. Huval and Wintergest (1972), Mota Oliveira (1970), King (1974) and others have treated this problem.

Effect of Inertia

Under certain conditions, as for example when the inlet channel is very long, the acceleration of the mass of water in the channel may have a measurable effect on the hydraulics. One such effect is to make the bay tide range larger than the ocean range. This case of a_B/a_o greater than unity is beyond the scope of the described methods. For example, in Example 2-1, $a_B/a_o = 0.99$ is predicted for Rockaway Inlet. Tide Tables however yield $a_B/a_o = 1.12$.

The inclusion of the acceleration term $\partial V/\partial t$ in the momentum equation leads to,

$$V = \sqrt{\frac{2g}{F}} \sqrt{\left|\eta_o - \eta_B - \frac{L_c}{g}\frac{\partial V}{\partial t}\right|} \cdot \text{sign}\left(\eta_o - \eta_B - \frac{L_c}{g}\frac{\partial V}{\partial t}\right) \quad (4\text{-}13)$$

At slack water, V = 0, and

$$(\eta_o - \eta_B)_{slack} = \frac{L_c}{g}\left.\frac{\partial V}{\partial t}\right|_{slack} \quad (4\text{-}14)$$

Where $\partial V/\partial t$ at slack is also its maximum value[6]. Usually, this head difference is negligible.

King (1974) has defined a number

$$K_1 = \frac{a_o F}{2L_c} \frac{A_B}{A_c} \quad (4\text{-}15)$$

He notes that when $K_1 < 100$, the inertia term assumes significance. King has solved Eq. (4-13) along with the continuity equation on an analog computer, and has presented a family of curves analogous to that in Fig. 6. Recently,

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[6]Unlike the Keulegan model, in this case the ocean and bay will become horizontal (no head) after slack water in the inlet.

Escoffier (1975) has solved the same equations analytically.

A Numerical Approximation of the Basic Equations

There are times when one is interested in obtaining the bay tide and the flow discharge as functions of time, rather than merely the maximum values from the plots of Figs. 6 and 7. This becomes tedious when the ocean tide is not sinusoidal as for example when the "ocean" is the Gulf of Mexico. In such a situation, Eqs. (2-8) and (2-9) should be solved on a computer.

We have,

$$\eta_o = \eta_o (t) \tag{4-16}$$

which is the known ocean tide input.

$$\Delta\eta_B^n = \frac{A_c}{A_B} \sqrt{\frac{2g}{F}} \sqrt{|\eta_o{}^n - \eta_B^n|} \;.\; \text{sign} (\eta_o{}^n - \eta_B^n) \;.\; \Delta t \tag{4-17}$$

which is Eq. (2-10) in difference form. η_o and η_B are at time level n. The time level shift is carried out through,

$$\eta_B^{n+1} = \eta_B^n + \Delta\eta_B^n \tag{4-18}$$

It is sufficient to select, as initial condition, at n = 0,

$$\eta_B^0 = 0 \tag{4-19}$$

since the solution, eventually after about two tidal cycles, becomes independent of the effects of this initial condition. van de Kreeke (1967) has solved a similar system of equations with an additional contribution of a time varying fresh water outflow through the inlet.

Eqs. (4-17) and (4-18) must be solved successively, with the input Eq. (4-16), on a computer, for a selected time increment Δt. Δt = 5 minutes may be selected as a reasonable first choice. Solution of the above equations essentially yields η_B^{n+1} at the end of each time increment. Eq. (2-9) may be approximated according to,

$$V_c^{n+1} = \frac{A_B}{A_c \Delta t} (\eta_B^{n+1} - \eta_B^n) \tag{4-20}$$

$$Q^{n+1} = V_c^{n+1} A_c \tag{4-21}$$

At each time level, Eqs. (4-20) and (4-21) give the throat velocity V_c^{n+1} and the flow rate Q^{n+1}.

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