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ECONOMIC INTEGRATION AND AGGLOMERATION IN A CUSTOMS UNION IN THE PRESENCE OF AN OUTSIDE REGION

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ABSTRACT: New Economic Geography (NEG) models do not typically account for the presence of regions other than the ones involved in the integration process. We explore such a possibility in a Footloose Entrepreneur (FE) model aiming at studying the stability properties of long-run industrial location equilibria. We consider a world economy composed by a customs union of two regions (regions 1 and 2) and an “outside region” which can be regarded as the rest of the world (region 3). The effects of economic integration on industrial agglomeration within the customs union are studied under the assumption of a constant distance between the customs union itself and the third region. The results show that higher economic integration does not always implies the standard result of full agglomeration of FE models. This incomplete agglomeration outcome is due to the fact that the periphery region keeps a share of industrial activities in order to satisfy a share of “external demand”. That is, the deindustrialization process brought about by economic integration in the periphery of the union is mitigated by the demand of consumers living in the rest of the world. In general, the market size of the third region affects the number of the long-run equilibria, as well as their stability properties. In addition to the standard outcomes of FE models, we describe the existence of two asymmetric equilibria characterised by unequal distribution of firms between regions 1 and 2, with no full agglomeration though. Interestingly, these equilibria are stable and therefore can be regarded as a likely long-run equilibrium state of the economy.

Keywords: industrial agglomeration, three-region NEG models, footloose entrepreneurs.

JEL classification: C62, F12, F2, R12.

1. Introduction

Will further economic integration increase or reduce regional disparities? This is one of the core questions in European policy discussions. The analysis of integration areas is one of the classical topics in trade theory (for an overview see, e.g., Krishna, 2008); however, in the following we apply a New Economic Geography (NEG) framework, since with its emphasis on endogenous agglomeration processes it seems to be particularly suited to analyse the effects on regional disparities. Economic integration – represented by a reduction in trade cost – influences the balance between centripetal and centrifugal forces and depending upon which force prevail the long-run the spatial distribution of economic activity may differ. Typically, NEG models allow for two standard long-run outcomes: an equal distribution of the manufacturing activity between the regions or, with a sufficient reduction of trade costs, full agglomeration in one of the two regions (see, for reviews, Fujita et al., 1999; and Baldwin et al., 2003). The analytic structure of NEG models is intrinsically complex, therefore many NEG models are actually confined to the analysis of two regions. However, for a comprehensive study of integration areas this is not sufficient: one has to differentiate at least between two regions inside the integration area and one region outside. So far a small strand of literature has developed three-region models though. Within this literature, Paluzie (2001) shows that a reduction in the external trade cost strengthens the agglomerative forces in the home country with two regions. Similar results are put forward by Alonso-Villar (1999, 2001), and Monfort and Nicolini (2000). In contrast, Krugman and Livas Elizondo (1996) argue that a reduction in the international transport cost may favour dispersion of economic activity between the two regions in the home country (in their model the domestic dispersion force is due to land rent and commuting costs and is thus exogenous and independent of trade costs). Brülhart et al. (2004) and Crozet and Koenig-Soubeyran (2004) introduce more geographical structure into the analysis, as they assume that one of the home regions is a border region, i.e. that it has lower transport cost wrt the outside region than the other home region. Also in this framework, a reduction of the international transport cost favours agglomeration in the two-regions home country; in particular (but not always), agglomeration in the border region. Interestingly, Brülhart et al. (2004) point out that the size of the third region matters for the results.

Taking a closer look to the above mentioned contributions reveals that these studies only address one part of the issues at hand as they only analyse the effects of a closer integration with the rest of the world; however, neglecting the effects of a closer integration within the integration area. In our paper, we are focussing on the latter issue while deliberately neglecting the former.

We explore the effects of trade integration between two (symmetric, home) regions (1 and 2) in the presence of a third region (region 3), that is upon construction a mere outside region. As a first result we show that our construction implies that transport cost with the third region do not matter. Nevertheless, the third country is important as (outside) market and its size influences the balance between centripetal and centrifugal forces between the two countries/regions inside the integration area. To put it differently: The effects of a trade liberalisation between the two countries inside the integration area will depend upon the size of the third region, upon the importance of the outside links for the integration area.

In particular, we show that with an increase of the size of third region the symmetric equilibrium between the two regions/countries within the integration area loses stability at a lower value of the trade freeness inside the integration area. To put it differently: Stronger outside links favour agglomeration within the integration area.

In addition, and this is our main result, we show that the size of the third region also influences what happens, if the symmetric equilibrium is unstable. With instability, factor mobility sets in leading to asymmetries in the factor allocation; this asymmetry changes itself the strength of the agglomerative and deglomerative forces and we show analytically that for a smaller third region the agglomerative forces outweigh the deglomerative forces leading to full agglomeration in one of the regions inside the integration area; instead, for a bigger third region, asymmetries weaken the agglomerative processes and strengthen the deglomerative forces – and interior asymmetric equilibra can be established.

This result is important, first, because it shows that even if with a reduction of trade costs with the integration area the symmetric equilibrium loses stability (as found also in the papers reviewed above), the long-run outcome need not be full agglomeration, but may also be partial agglomeration (thus a reduction of trade costs does not lead to extreme regional disperion). Second, the result is important because it is one of the rare examples in the NEG literature that produces partial agglomeration as the outcome of an endogenous process.

The remainder of the paper is organized as follows. Section 2 presents the basic framework of the model. In section 3 we characterize the short-run equilibrium. Section 4 deals with our complete dynamical model, whose local stability properties are studied in section 5. Section 6 reports preliminary results on global dynamics. Section 7 concludes.

2. Basic framework

The economic system is composed of three regions (r = 1, 2, 3) and two sectors. The traditional or agricultural sector (A) is located in all three regions, whereas the manufacturing sector (M) can be localised at most in two regions (s = 1, 2). Production involves the use of two factors of production, unskilled labour (L) and entrepreneurs (N). In the overall economy, the amount of unskilled labour is L: a share ℓ is equally distributed between the manufacturing regions 1 and 2 and the rest is located in region 3 (the region without manufacture), i.e. $\ell_1 = \ell_2 = \frac{\ell}{2}$ and $\ell_3 = 1 - \ell$. Unskilled labour does not migrate; the N existing entrepreneurs, instead, are mobile between regions 1 and 2.

Assuming that unskilled workers and entrepreneurs possess the same tastes, we write the representative consumer's utility function as follows:

$$U = C_A^{1-\mu} C_M^{\mu} \tag{1}$$

where C_A and C_M correspond to the consumption of the homogeneous agricultural good and of a composite of manufactured goods:

$$C_M = \sum_{i=1}^{n} d_i^{\frac{\sigma-1}{\sigma}} \tag{2}$$

where d_i is the consumption of good i, n is the total number of manufactured goods and $\sigma > 1$ is the constant elasticity of substitution; the lower σ, the greater the consumers' taste for variety. The exponents in the utility function $1 - \mu$ and μ indicate, respectively, the invariant shares of disposable income devoted to the agricultural and manufactured goods, with $0 < \mu < 1$.

Only labour is used in the production of the homogeneous agricultural good. One unit of (unskilled) labour is used to produce one unit of the agricultural output, so that constant returns prevails. Moreover, we assume that none of the regions has enough labour to engage exclusively in the production of the agricultural good, the so-called 'non-full-specialization condition'.

The manufacturing sectors involve monopolistic competition as modelled by Dixit-Stiglitz (1977). In our context, each firm requires a fixed input of an entrepreneur to operate and β units of unskilled labour for each unit produced. Since one entrepreneur is needed for each firm, the total number of firms always equals the total number of entrepreneurs. Moreover, because of consumers' preference for variety and increasing returns in production, a firm would always produce a variety different from those produced by other firms. It follows that the number of varieties always equals the number of firms. Denoting the share of entrepreneurs located in region 1 in period *t* by λ_t and by *N* the total number of entrepreneurs, the number of regional varieties produced in period *t*in region 1 and 2 are

$$n_{1,t} = \lambda_t N \qquad n_{2,t} = (1-\lambda_t)N \tag{3}$$

where $0 \leq \lambda_t \leq 1$ and where, by assumption, no manufacturing activity occurs in region 3.

Transportation of the agricultural product between regions is costless. Transport costs for manufactures take an iceberg form: if one unit is shipped from region *s* to region *r* only $1/T_{rs}$ arrives, where $T_{rs} \geq 1$, $r = 1, 2, 3$ and $s = 1, 2$.

Region 1 and 2 are involved in a trade agreement whereas the economic integration with region 3 is less deep. We model this spatial arrangement as follows: the three regions are located on the vertices of a isosceles triangle

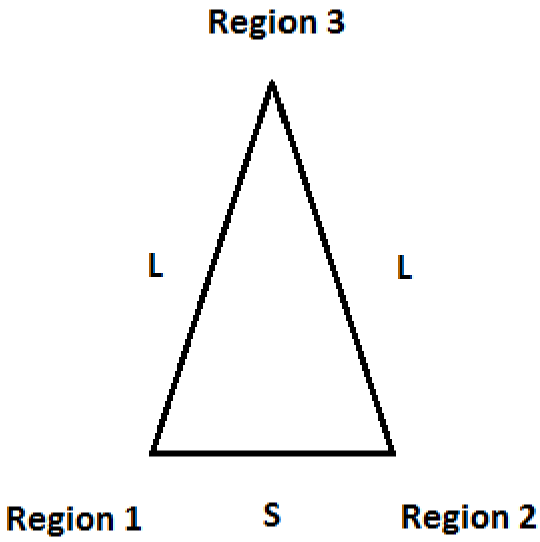


Figure 1

The "distance" (trade barriers) between regions 1 and 2 is *S* (short); the distance between 1 and 3 and 2 and 3 is the same and is equal to *L* (long). Transport costs between regions 1 and 2 are

$$T_{12} = T_S ,$$

and between regions 1 and 3 and regions 2 and 3 are

$$T_{13} = T_{23} = T_L$$

where $T_L > T_S \geq 1$. Finally, in order to simplify the notation, we introduce the following "trade freeness" parameters: $\phi_{12} = \phi_S$, $\phi_{13} = \phi_{23} = \phi_L$, where $\phi_S \equiv T_S^{1-\sigma}$ and $\phi_L \equiv T_L^{1-\sigma}$ and where $\phi_L < \phi_S \leq 1$.

3. Short-run general equilibrium

The short-run equilibrium in period *t* is characterized by a given spatial allocation of entrepreneurs across the regions, $\lambda_{r,t}$. In a short-run general equilibrium, which is established instantaneously in each period, supply equals demand for the agricultural commodity and each manufacturer meets the demand for its variety. As a result of Walras's law, simultaneously equilibrium in the product markets implies equilibrium in the regional labour markets.

With zero transport costs, the agricultural price is the same across regions. Denoting by Y the income of the overall economy, that (as confirmed below) is invariant over time, total expenditure on the agricultural product is $(1-\mu)Y$. Assuming $(1-\mu)Y > \max\left(\ell L, \left(1-\frac{\ell}{2}\right)L\right)$ all regions produce the agricultural commodity, whereas $(1-\mu)Y > \max\left(\frac{\ell}{2}L, \left(1-\ell\right)L\right)$ implies that no single region is able to satisfy all the demand for the agricultural good. Since competition results in zero agricultural profits, the short-run equilibrium nominal wage in period t is equal to the agricultural product price and therefore is always the same across regions. Setting this wage/agricultural price equal to 1, it becomes the numeraire in terms of which the other prices are defined. Facing a wage of 1, each manufacturer has a marginal cost of β. Each maximizes profit on the basis of a perceived price elasticity of $-\sigma$ and sets a local (mill) price p for its variety, given by

$$p = \frac{\sigma}{\sigma - 1}\beta \tag{4}$$

The effective price paid by consumers in region r for a variety produced in region s is pT_{rs}. The regional manufacturing price index facing consumers in region r is given by

$$P_{r,t} = \left(\sum_{s=1}^{2} n_s p^{1-\sigma} T_{rs}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

Under our assumptions, we can write

$$P_{1,t} = \left(n_1 p^{1-\sigma} + n_2 p^{1-\sigma} T_S^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$
$$P_{2,t} = \left(n_1 p^{1-\sigma} T_S^{1-\sigma} + n_2 p^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$
$$P_{3,t} = \left(n_1 p^{1-\sigma} T_L^{1-\sigma} + n_2 p^{1-\sigma} T_L^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

and therefore

$$P_{r,t} = \Delta_{r,t}^{\frac{1}{1-\sigma}} N^{\frac{1}{1-\sigma}} p \tag{5}$$

where $\Delta_{1,t} = \lambda_t + \phi_S (1-\lambda_t)$, $\Delta_{2,t} = \phi_S \lambda_t + 1 - \lambda_t$, $\Delta_{3,t} = \phi_L$.

The demand facing a producer located in region *s* is

$$d_{s,t} = \left(\sum_{r=1}^{3} \mu Y_{r,t} P_{r,t}^{\sigma-1} T_{rs}^{1-\sigma} \right) p^{-\sigma} = \left(\sum_{r=1}^{3} s_{r,t} P_{r,t}^{\sigma-1} T_{rs}^{1-\sigma} \right) \mu Y p^{-\sigma} \tag{6}$$

We can write:

$$\begin{aligned}
d_{1,t} &= \left(s_{1,t} P_{1,t}^{\sigma-1} + s_{2,t} P_{2,t}^{\sigma-1} T_{21}^{1-\sigma} + s_{3,t} P_{3,t}^{\sigma-1} T_{31}^{1-\sigma} \right) \mu Y p^{-\sigma} \\
&= \left(\frac{s_{1,t}}{\Delta_{1,t}} + \frac{s_{2,t}}{\Delta_{2,t}} T_S^{1-\sigma} + \frac{s_{3,t}}{\Delta_{3,t}} T_L^{1-\sigma} \right) \frac{\mu Y p}{N} \\
&= \left(\frac{s_{1,t}}{\Delta_{1,t}} + \frac{s_{2,t}}{\Delta_{2,t}} \phi_S + \frac{s_{3,t}}{\Delta_{3,t}} \phi_L \right) \frac{\mu Y p}{N} \\
&= \left(\frac{s_{1,t}}{\Delta_{1,t}} + \frac{s_{2,t}}{\Delta_{2,t}} \phi_S + s_{3,t} \right) \frac{\mu Y p}{N} \\
d_{2,t} &= \left(s_{1,t} P_{1,t}^{\sigma-1} T_{12}^{1-\sigma} + s_{2,t} P_{2,t}^{\sigma-1} + s_{3,t} P_{3,t}^{\sigma-1} T_{32}^{1-\sigma} \right) \mu Y p^{-\sigma} \\
&= \left(\frac{s_{1,t}}{\Delta_{1,t}} T_S^{1-\sigma} + \frac{s_{2,t}}{\Delta_{2,t}} + \frac{s_{3,t}}{\Delta_{3,t}} T_L^{1-\sigma} \right) \frac{\mu Y p}{N} \\
&= \left(\frac{s_{1,t}}{\Delta_{1,t}} + \frac{s_{2,t}}{\Delta_{2,t}} \phi_S + \frac{s_{3,t}}{\Delta_{3,t}} \phi_L \right) \frac{\mu Y p}{N} \\
&= \left(\frac{s_{1,t}}{\Delta_{1,t}} + \frac{s_{2,t}}{\Delta_{2,t}} \phi_S + s_{3,t} \right) \frac{\mu Y p}{N}
\end{aligned} \tag{7}$$

where $Y_{r,t}$ represents income and expenditure in region *r* in period *t*, $s_{r,t} \equiv Y_{r,t} / Y$ denotes region *r*'s share in expenditure in period *t* and *r* = 1, 2, 3.

Short-run general equilibrium in region *s* requires that each firm meets the demand for its variety. For a variety produced in region *s*,

$$x_{s,t} = d_{s,t} \tag{8}$$

where $x_{s,t}$ is the output of each firm located in region *s*. From equation (4), the short-run equilibrium operating profit per variety/entrepreneur in region *s* is

$$\pi_{s,t} = px_{s,t} - \beta x_{s,t} = \frac{px_{s,t}}{\sigma} \tag{9}$$

Since profit equals the value of sales times $1/\sigma$ and since total expenditure on manufacturers is μY, the total profit received by entrepreneurs is $\mu Y/\sigma$. Total income is $Y = L + (\mu/\sigma)Y$, so that

$$Y = \frac{\sigma L}{\sigma - \mu} \tag{10}$$

Total profit is therefore $\mu L/(\sigma - \mu)$. Equation (10) confirms that total income is invariant over time. From (10), $(1-\mu)Y > \max\left(\ell L, \left(1 - \frac{\ell}{2}\right)L\right)$ is equivalent to $\min\left(\ell\mu + (1-\ell)\sigma - \mu\sigma, \left(2-\ell\right)\mu + \ell\sigma - 2\mu\sigma\right) > 0$; and $(1-\mu)Y > \max\left(\frac{\ell}{2}L, \left(1-\ell\right)L\right)$ is equivalent to $\min\left(\ell\mu + (2-\ell)\sigma - 2\mu\sigma, \left(1-\ell\right)\mu + \ell\sigma - \mu\sigma\right) > 0$.The former is a (sufficient) non-full-specialization condition and the latter is a necessary one, where both are expressed in terms of the utility parameters.

Using (4) to (10), the short-run equilibrium profit in region *s* is determined by the spatial distribution of entrepreneurs and the regional expenditure shares:

$$\pi_{s,t} = \left(\sum_{r=1}^{3} \mu Y_{r,t} P_{r,t}^{\sigma-1} T_{rs}^{1-\sigma}\right) \frac{p^{1-\sigma}}{\sigma} = \left(\sum_{r=1}^{3} s_{r,t} P_{r,t}^{\sigma-1} \phi_{rs}\right) p^{1-\sigma} \frac{\mu Y}{\sigma}$$

Under our assumptions on trade costs across regions, we can write

$$\pi_{1,t} = \left(s_{1,t}P_{1,t}^{\sigma-1} + s_{2,t}P_{2,t}^{\sigma-1}\phi_S + s_{3,t}P_{3,t}^{\sigma-1}\phi_L\right) p^{1-\sigma} \frac{\mu Y}{\sigma}$$

$$\pi_{2,t} = \left(s_{1,t}P_{1,t}^{\sigma-1}\phi_S + s_{2,t}P_{2,t}^{\sigma-1} + s_{3,t}P_{3,t}^{\sigma-1}\phi_L\right) p^{1-\sigma} \frac{\mu Y}{\sigma}$$

or, alternatively:

$$\pi_{1,t} = \frac{\mu}{\sigma}\frac{Y}{N}\left(\frac{s_{1,t}}{\Delta_{1,t}} + \phi_S \frac{s_{2,t}}{\Delta_{2,t}} + \phi_L \frac{s_{3,t}}{\Delta_{3,t}}\right) = \frac{\mu}{\sigma}\frac{Y}{N}\left(\frac{s_{1,t}}{\lambda_t + \phi_S(1-\lambda_t)} + \phi_S \frac{s_{2,t}}{\phi_S\lambda_t + 1 - \lambda_t} + s_{3,t}\right) \quad (11)$$

$$\pi_{2,t} = \frac{\mu}{\sigma}\frac{Y}{N}\left(\phi_S \frac{s_{1,t}}{\Delta_{1,t}} + \frac{s_{2,t}}{\Delta_{2,t}} + \phi_L \frac{s_{3,t}}{\Delta_{3,t}}\right) = \frac{\mu}{\sigma}\frac{Y}{N}\left(\phi_S \frac{s_{1,t}}{\lambda_t + \phi_S(1-\lambda_t)} + \frac{s_{2,t}}{\phi_S\lambda_t + 1 - \lambda_t} + s_{3,t}\right) \quad (12)$$

Regional incomes/expenditures are

$$Y_{1,t} = \frac{\ell}{2}L + \lambda_t N \pi_{1,t} \quad (13)$$

$$Y_{2,t} = \frac{\ell}{2}L + (1-\lambda_t) N \pi_{2,t} \quad (14)$$

$$Y_{3,t} = \left(1-\ell\right)L \quad (15)$$

$$\frac{Y_{1,t}}{Y} = s_1 = \frac{\ell L}{2Y} + \frac{\lambda_t N \pi_{1,t}}{Y} \quad (16)$$

$$\frac{Y_{1,t}}{Y} = s_{1,t} = \frac{\ell L}{2Y} + \frac{\lambda_t N \pi_{1,t}}{Y} = \frac{\ell}{2}\frac{\sigma-\mu}{\sigma} + \lambda_{1,t}\frac{\mu}{\sigma}\left(\frac{s_{1,t}}{\Delta_{1,t}} + \phi_S \frac{s_{2,t}}{\Delta_{2,t}} + s_{3,t}\right) \quad (17)$$

$$\frac{Y_{2,t}}{Y} = s_{2,t} = \frac{\ell L}{2Y} + \frac{(1-\lambda_t) N \pi_{2,t}}{Y} = \frac{\ell}{2}\frac{\sigma-\mu}{\sigma} + (1-\lambda_t)\frac{\mu}{\sigma}\left(\phi_S \frac{s_{1,t}}{\Delta_{1,t}} + \frac{s_{2,t}}{\Delta_{2,t}} + s_{3,t}\right) \quad (18)$$

$$\frac{Y_{3,t}}{Y} = s_{3,t} = \frac{(1-\ell)L}{Y} = (1-\ell)\frac{\sigma-\mu}{\sigma} \quad (19)$$

Using (17) to (19) and taking into account that $s_{3,t} = 1 - s_{1,t} - s_{2,t}$, the shares in $s_{1,t}$ and $s_{2,t}$, can be expressed in terms of λ_t:

$$s_{1,t} = \frac{\dfrac{\sigma-\mu}{2} + \dfrac{\mu\phi_S\lambda_t}{\Delta_{2,t}} - (1-\ell)\left(\dfrac{\sigma-\mu}{2\sigma}\right)\left[\sigma - 2\mu\lambda_t\left(1-\dfrac{\phi_S}{\Delta_{2,t}}\right)\right]}{\sigma - \mu\lambda_t\left(\dfrac{1}{\Delta_{1,t}} - \dfrac{\phi_S}{\Delta_{2,t}}\right)} \tag{20}$$

$$s_{2,t} = \frac{\dfrac{\sigma-\mu}{2} + \dfrac{\mu\phi_S(1-\lambda_t)}{\Delta_{1,t}} - (1-\ell)\left(\dfrac{\sigma-\mu}{2\sigma}\right)\left[\sigma - 2\mu(1-\lambda_t)\left(1-\dfrac{\phi_S}{\Delta_{1,t}}\right)\right]}{\sigma - \mu(1-\lambda_t)\left(\dfrac{1}{\Delta_{2,t}} - \dfrac{\phi_S}{\Delta_{1,t}}\right)} \tag{21}$$

Given that the agricultural price is 1, the real income of an entrepreneur in region *s* is:

$$\omega_{s,t} = \pi_{s,t}P_{s,t}^{-\mu} \tag{22}$$

Given that the three regions shares in total expenditure do not depend on ϕ_L, from (11) and (12) – taking into account (19), (20) and (21) –, we can derive the following proposition:

Proposition 1. *Profit differentials are not affected by the distance of region 1 and region 2 from region 3.*

Therefore, a change of the distance of region 1 and/or region 2 from region 3 has no impact on operating profits. This is because the demand for the manufactured goods is unitary elastic: the change in trade costs, via ϕ_L, determines a proportional change in the price index in region 3 and a similar but inversely proportional change in the quantity demanded, so the overall change of expenditures on manufacturing in this region is zero. This is also because, since region 3 does not produce manufactured varieties a change in ϕ_L does not impact on price indices in region 1 and 2. This is a result that follows from our simple set-up, the assumptions of a CES subutility function for manufactured goods and no manufacturing production in region 3: a change in transport costs towards the outside region has no effect.

4. The entrepreneurial migration hypothesis and the complete dynamical model

The central dynamic equation is based on the replicator dynamics, widely used in evolutionary game theory:

$$M\left(\lambda_t\right)=\lambda_t\left[1+\gamma\frac{\omega_{1,t}-\left[\lambda_t\omega_{1,t}+\left(1-\lambda_t\right)\omega_{2,t}\right]}{\lambda_t\omega_{1,t}+\left(1-\lambda_t\right)\omega_{2,t}}\right]=\lambda_t\left[1+\gamma\left(1-\lambda_t\right)\frac{T\left(\lambda_t\right)}{1+\lambda_t T\left(\lambda_t\right)}\right] \quad (23)$$

where γ represents the migration speed and where $T\left(\lambda_t\right)=\frac{\omega_{1,t}}{\omega_{2,t}}-1$. According to (23), the share of entrepreneurs in region 1, $M\left(\lambda_t\right)$, depends on a comparison between the real income gained in that region and the weighted average of the incomes in region 1 and 2. Taking into account the constraint, $0\le\lambda_t\le 1$, the complete dynamical model is represented by the following piecewise smooth one-dimensional map:

$$\lambda_{t+1}=\Lambda(\lambda_t)=\begin{cases}0 & if \quad M(\lambda_t)<0\\ M(\lambda_t) & if \quad 0\le M(\lambda_t)\le 1\\ 1 & if \quad M(\lambda_t)>1\end{cases} \quad (24)$$

A long-run stationary equilibrium involves $\Lambda(\lambda^*)=\lambda^*$, where λ^* represents a so-called fixed point of the map (25). There are three types of fixed points:

i) the Core-Periphery equilibria are characterized by full agglomeration of manufacturing in one region. These are: $\lambda^{CP(0)}=0$, corresponding to complete agglomeration in region 2, which gives $M(0)=0$; and $\lambda^{CP(1)}=1$, corresponding to full agglomeration in region 1, which gives $M(1)=1$.

ii) the symmetric equilibrium is characterized by an equal split of the manufacturing sector between regions 1 and 2: $\lambda^*=\frac{1}{2}$, that gives $M\left(\frac{1}{2}\right)=\frac{1}{2}$ and $T\left(\frac{1}{2}\right)=0$;

iii) the asymmetric equilibria are characterized by incomplete agglomeration in one of the two regions of the customs union, with some industry still present in the other region. The following cases are possible:

Case 1: no asymmetric fixed point exists.

Case 2: two asymmetric fixed points exist which are symmetric around 1/2: λ^a, $1-\lambda^a$;

Case 3: four asymmetric fixed points exist that are symmetric two by two around 1/2: λ^a, $1-\lambda^a$ and λ^b, $1-\lambda^b$.

These equilibria are obtained by solving $M(\lambda^i)=\lambda^i$ and $M(1-\lambda^i)=1-\lambda^i$, corresponding to $T(\lambda^i)=0$ and $T(1-\lambda^i)=0$, where *i* = *a*, *b*.

5. Existence and local stability of stationary equilibria

In this section, we explore the local stability analysis of the fixed points listed above. Due to the symmetry of the map $\Lambda(\lambda_t)$, a general property is that each equilibrium (stationary, periodic or aperiodic) is symmetric to itself or another equilibrium exists that is symmetric to such equilibrium around to $\frac{1}{2}$; similarly, the basins of attraction of each equilibrium, as well as any other invariant set, also enjoy this symmetric property. In what follows, this symmetric rule is applied to the fixed points (stationary equilibria) of the map $\Lambda(\lambda_t)$.

We find the local stability properties of the CP equilibria $\lambda^{CP(1)}=1$ and $\lambda^{CP(0)}=0$, by evaluating the one-side eigenvalues of the map $M(\lambda_t)$ in correspondence of these equilibria:

$$-1<M'(1)=1-\gamma\frac{T(1)}{1+T(1)}<1,\ 0<M'(0)=1+\gamma T(0)<0$$

From which

$$0<T(1)<\frac{2}{\gamma-2},\ \ -\frac{2}{\gamma}<T(0)<0,$$

We explore the stability properties of the CP equilibrium $\lambda^{CP(1)}=1$. The same results apply to the other CP equilibrium by symmetry.

We have that $-1 < M'(1) < 1$ for

$$\frac{\gamma-2}{\gamma}\kappa(\phi) < \phi^{\frac{\mu-\sigma+1}{\sigma-1}} < \kappa(\phi) \tag{26}$$

where, for convenience, we set:

$\phi_S \equiv \phi$ and $\kappa(\phi) = \dfrac{2\sigma}{(\sigma-\mu)[\ell + 2\phi(1-\ell)] + [\mu(2-\ell) + \ell\sigma]\phi^2}$; and where $\kappa(\phi) > 1$ for $0 \le \phi < 1$.

for $1 < \sigma < 1 + \mu < 2$ and $0 \le \phi < 1$, the right hand side inequality in (26) is always satisfied;

for $1 < 1 + \mu < \sigma$ it can be shown that the right hand side inequality in (26) is satisfied for sufficiently high values of ϕ and violated for low values (hint: we are dealing with two monotonically decreasing functions of ϕ, the first tends to infinity for $\phi \to 0$ and it is equal to 1 at $\phi = 1$, the second is positive (and larger than 1) but finite at $\phi = 0$ and it is equal to 1 at $\phi = 1$, since at $\phi = 1$ the first derivative of the first function is smaller in absolute value than the derivative of the second function, the two functions necessarily cross at some $\phi = \phi^T$, where $0 < \phi^T < 1$).It is not possible to specify the corresponding bifurcation value for the trade freeness parameter $\phi = \phi^T$ explicitly.

Note that for $\sigma > 1 + \mu$, as ϕ crosses ϕ^T from left to right, the map undergoes a so-called border collision bifurcation: the CP equilibrium $\lambda^{CP(1)} = 1$ meets the upper branch of the asymmetric equilibrium gaining stability. Symmetrically, $\lambda^{CP(0)} = 0$ meets the lower branch of the asymmetric equilibrium gaining stability. From this, we infer that the asymmetric equilibrium must have always the same local stability properties in the neighborhood of the CP equilibria (see Figures 2 and 4).

Finally, the left hand side inequality in (26) is satisfied for a sufficiently small value of γ:

$$\gamma < \frac{2}{1 - \phi^{\frac{\mu-\sigma+1}{\sigma-1}}\kappa(\phi)^{-1}} \tag{27}$$

When this latter condition does not hold, $\lambda^{CP(1)}=1$ is stable for $0<\tilde{\phi}_1<\phi<\tilde{\phi}_2<1$, where $\tilde{\phi}_1$ and $\tilde{\phi}_2$ can only be obtained numerically by solving (27) with an equality sign. [$\tilde{\phi}_1$ and $\tilde{\phi}_2$ correspond to two flip bifurcation points, which are not visible due to the constraints of the map $\Lambda(\lambda_t)$].

Moving on to the symmetric equilibrium, its local stability requires that the eigenvalue of the map $M(\lambda_t)$ (which coincides to that of the map $\Lambda(\lambda_t)$) evaluated at this equilibrium should lie within the interval $(-1,1)$:

$$-1<M'\left(\frac{1}{2}\right)=1+\frac{\gamma}{4}T'\left(\frac{1}{2}\right)<1,$$

which implies

$$-\frac{8}{\gamma}<T'\left(\frac{1}{2}\right)<0 \tag{28}$$

Concerning the inequality on the right hand side of (28), it is satisfied for

$$\phi<\frac{(\sigma-\mu)\left[\ell(\sigma-1)-\mu\right]}{(\sigma-1)(\sigma-\mu)\ell+\mu(\mu+3\sigma-2)}\equiv\phi^P<1$$

for $1<\sigma<1+\dfrac{\mu}{\ell}$, it follows that $\phi^P<0$. Therefore, this inequality can never be satisfied;

for $\sigma>1+\dfrac{\mu}{\ell}$, as ϕ crosses ϕ^P from left to right, the map $\Lambda(\lambda_t)$ undergoes a so-called pitch-fork bifurcation.

A first interesting result we may highlight is that ϕ^P depends positively on ℓ:

$$\frac{\partial\phi^P}{\partial\ell}=\frac{2\mu(\sigma-\mu)(2\sigma-1)(\sigma-1)}{\left[(\sigma-1)(\sigma-\mu)\ell+\mu(\mu+3\sigma-2)\right]^2}>0$$

which implies that the local stability of the symmetric equilibrium depends upon the dimension of the outside region as follows: increasing the size of the third region (reducing ℓ) has a destabilizing effect on this equilibrium and tends to favor agglomeration.

In order to study in detail the properties of the pitchfork bifurcation, we first redefine our central map to highlight the control parameter we interested in, trade freeness ϕ, and we verify how these may change when another crucial parameter, the size of the third region ℓ, changes. We could replicate the same analysis for any other parameter. The redefined map is

$$M(\phi, \lambda_t) = \lambda_t \left[1 + \gamma(1 - \lambda_t) Z(\phi, \lambda_t)\right],$$

where $Z(\phi, \lambda_t) = \dfrac{T(\phi, \lambda_t)}{1 + \lambda_t T(\phi, \lambda_t)}$.

From the theory of dynamical systems (see Wiggins, 1990), in correspondence of a pitchforfork bifurcation, that is, when $\phi = \phi^P$ and $\lambda^* = 1/2$, the following conditions must hold:

(i) $\quad \dfrac{\partial M}{\partial \phi}\left(\phi^P, \dfrac{1}{2}\right) = 0$;

(ii) $\quad \dfrac{\partial^2 M}{\partial \lambda_t^2}\left(\phi^P, \dfrac{1}{2}\right) = 0$;

(iii) $\quad \dfrac{\partial^2 M}{\partial \lambda_t \partial \phi}\left(\phi^P, \dfrac{1}{2}\right) \neq 0$

(iv) $\quad \dfrac{\partial^3 M}{\partial \lambda_t^3}\left(\phi^P, \dfrac{1}{2}\right) \neq 0$.

Moreover, the sign of the following expression can be used to determine on which side of ϕ^P the two branches of asymmetric equilibria, at least initially, lie:

(v) $$-\frac{\dfrac{\partial^3 M}{\partial \lambda_t^3}\left(\phi^P, \dfrac{1}{2}\right)}{\dfrac{\partial^2 M}{\partial \lambda_t \partial \phi}\left(\phi^P, \dfrac{1}{2}\right)} > or < 0$$

We have a supercritical pitchfork bifurcation when this expression is larger than zero and a subcritical pitchfork bifurcation when it is less than zero.

We have that:

condition (i), which corresponds to $\frac{\partial Z}{\partial \phi}\left(\phi^P, \frac{1}{2}\right) = 0$, is verified due to the fact that at the symmetric equilibrium $\omega_{1,t} = \omega_{2,t}$ and $\frac{\partial \omega_{1,t}}{\partial \phi}\left(\phi, \frac{1}{2}\right) = \frac{\partial \omega_{2,t}}{\partial \phi}\left(\phi, \frac{1}{2}\right)$ for any ϕ;

condition (ii) corresponds to $\frac{\partial^2 T}{\partial \lambda_t^2}\left(\phi^P, \frac{1}{2}\right) - \frac{\partial T}{\partial \lambda_t^2}\left(\phi^P, \frac{1}{2}\right)^2 = 0$. It can be checked (via calculation) that this equality holds for any ϕ;

condition (iii) corresponds to $\frac{\partial^2 T}{\partial \lambda_t \partial \phi}\left(\phi^P, \frac{1}{2}\right) \neq 0$. After calculation we obtain the following result:

$$\frac{2\mu(2\sigma-1)\left[(\sigma-1)(\sigma-\mu)\ell+\mu(\mu+3\sigma-2)\right]^2}{(\sigma-\mu)(\sigma-1)^2(\mu+\ell\sigma)\left[(\sigma-1)(\sigma-\mu)\ell+\mu(\mu+\sigma-1)\right]} > 0$$

which is always satisfied;

condition (iv) corresponds to $\frac{\partial^3 T}{\partial \lambda_t^3}\left(\phi^P, \frac{1}{2}\right) \neq 0$. After calculation we obtain the following result:

$$-\frac{32\mu^4(2\sigma-1)^3\left\{3(\sigma-1)^2(\mu-\sigma)\ell^2+\left[(3-2\sigma)\mu^2+2(\sigma-1)^2(\sigma-3\mu)\right]\ell+\mu(\mu+\sigma-1)\left[\mu+2(\sigma-1)\right]\right\}}{(\sigma-1)^3(\mu+\ell\sigma)\left[(\sigma-1)(\sigma-\mu)\ell+\mu(\mu+\sigma-1)\right]^3}$$

This expression could be negative, positive or zero depending on parameter values. If it is different from zero, given that $0<\mu<1<\sigma$, the sign of (v) corresponds to that of the following expression:

$$3(\sigma-1)^2(\mu-\sigma)\ell^2+\left[(3-2\sigma)\mu^2+2(\sigma-1)^2(\sigma-3\mu)\right]\ell+\mu(\mu+\sigma-1)\left[\mu+2(\sigma-1)\right] \tag{29}$$

This allows us to state the following proposition 2:

Proposition 2: *It is possible to show that it exists a* $\ell = \overline{\ell}$ *such that (29) is positive for* $\ell < \overline{\ell}$ *and it is negative for* $\ell > \overline{\ell}$ *, with* $0 < \overline{\ell} < 1$ *for* $\sigma > 1 + \mu$.

Proof.

First we rewrite (29) as

$$A\ell^2 + B\ell + C$$

where:

$$A \equiv 3(\sigma - 1)^2 (\mu - \sigma) < 0,$$

$$B \equiv (3 - 2\sigma)\mu^2 + 2(\sigma - 1)^2 (\sigma - 3\mu) \geq (<) 0$$

$$C \equiv \mu(\mu + \sigma - 1)\left[\mu + 2(\sigma - 1)\right] > 0.$$

Therefore (29) admits one positive and one negative solution. In order to have real roots, it must be:

$$\Delta \equiv B^2 - 4AC = \left(12\sigma - 8\sigma^2 - 3\right)\mu^4 + 4\left(10\sigma^2 - 12\sigma + 3\right)(\sigma - 1)^2 \mu^2 + 4\sigma^2 (\sigma - 1)^4 > 0$$

$\Delta = 0$ is a quartic equation that admits 4 solutions of which only two at most are real (or none).

Define $x \equiv \mu^2$. We have that:

$$\Delta = \left(12\sigma - 8\sigma^2 - 3\right)x^2 + 4\left(10\sigma^2 - 12\sigma + 3\right)(\sigma - 1)^2 x + 4\sigma^2 (\sigma - 1)^4 = 0$$

This is now a second degree equation whose solutions, x_a *and* x_b, are real since

$$\tilde{\Delta} = 144(3\sigma - 1)(2\sigma - 1)^2 (\sigma - 1)^5 > 0$$

Moreover, these solutions are both negative for $1 < \sigma < \frac{3+\sqrt{3}}{4} \cong 1.183$ and they are one positive x_a and one negative x_b (with the positive larger than the negative) for $\sigma > \frac{3+\sqrt{3}}{4} \cong 1.183$.

Therefore:

for $1 < \sigma < \frac{3+\sqrt{3}}{4} \cong 1.183$ $\Delta > 0$ always.

For $\sigma > \frac{3+\sqrt{3}}{4} \cong 1.183$ $\Delta > 0$ for $0 < x < x_a > 1$. Therefore also in this case $\Delta > 0$ for all relevant values of μ.

Therefore $A\ell^2 + B\ell + C = 0$ admits two real solutions one positive and one negative. Let's call the positive solution $\overline{\ell}$, where $\overline{\ell} \equiv -\frac{B+\sqrt{\Delta}}{2A}$. Given that A < 0, the expression (29) is positive for $0 \leq \ell < \overline{\ell}$ and it is negative for $\ell > \overline{\ell}$.

Finally, notice that the condition $\overline{\ell} < 1$ corresponds to

$$12(\sigma-1)^2(\sigma+\mu)(\mu-\sigma+1)(\sigma-\mu) < 0$$

That can be further reduced to $\sigma > 1+\mu$.

Q.E.D.

At this stage it could be interesting two consider two simpler cases:

First case: $\ell = 1$. By setting $\ell = 1$, we are back to the standard FC model. Expression (29) becomes:

$$(\mu+\sigma-1)(\mu-\sigma+1)(\sigma+\mu)$$

which is negative for $\sigma > 1+\mu$. That is, for this case, only a subcritical pitchfork bifurcation can occur. As shown in Fig. 2(a), the curve of asymmetric equilibria lies on the left of ϕ^P.

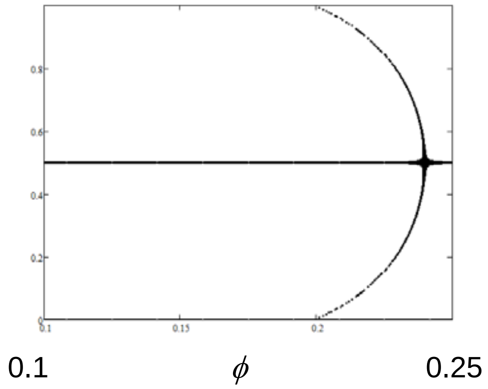
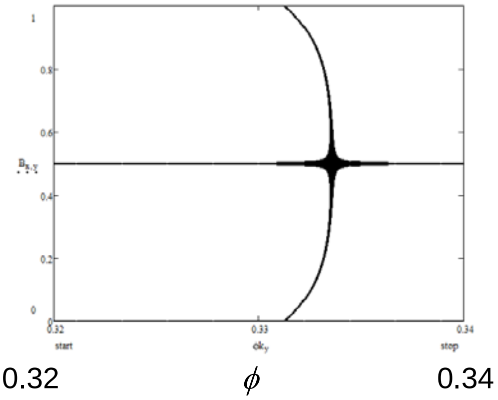
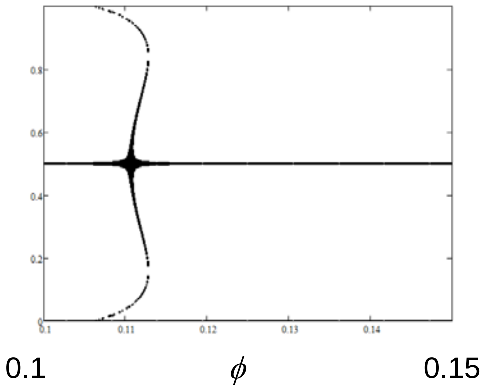
(a)  	$\sigma = 2$ $\mu = 0.45$ $\ell = 1$ Size of third region: $1 - \ell = 0$
(b)  	$\sigma = 3$ $\mu = 0.45$ $\ell = \frac{2}{3}$ Size of third region: $1 - \ell = \frac{1}{3}$
(c)  	$\sigma = 2$ $\mu = 0.45$ $\ell = \frac{2}{3}$ Size of third region: $1 - \ell = \frac{1}{3}$

Figure 2

The two existing asymmetric equilibria λ^a and $1-\lambda^a$ are unstable. That is, for any such equilibrium $M'(1-\lambda^a)>1$ and $M'(\lambda^a)>1$ that correspond to $T'(\lambda^a)>0$ and $T'(1-\lambda^a)>0$. This can be verified only numerically as the Figure 3 (so-called "wiggle" diagram) below shows:

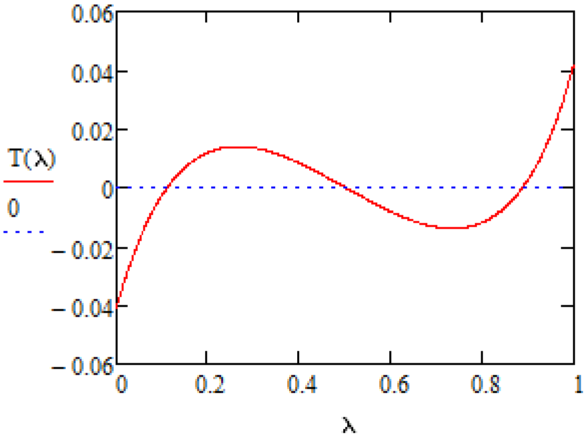


Fig. 3: Wiggle diagram, showing the stability of equilibria for $\ell=1$, $\mu=0.45$ $\phi=0.22$ and $\sigma=2$

As shown in Fig. 3, plotting $T(\lambda)$ with respect to λ, for the given parameter configuration, the symmetric equilibrium is locally stable stable since $T'\left(\frac{1}{2}\right)<0$. At the same time, also the CP equilibria are attracting given the boundary conditions in (24), which give $\Lambda'(0)=\Lambda'(1)=0$ (that is, due to the presence of borders the CP equilibria are 'superstable', i.e. the first derivative of the map evaluated at those equilibria is equal to zero). Instead, the two asymmetric equilibria are unstable, given that the slope of $T(\lambda)$ in correspondence of these equilibria is positive. λ^a and $1-\lambda^a$ separate symmetrically within the unitary interval (0, 1), the basins of attraction of the symmetric equilibrium, $\left(\lambda^a,1-\lambda^a\right)$, of the CP equilibrium $\lambda^{CP(0)}=0$, $\left[0,\lambda^a\right)$, and of the CP equilibrium $\lambda^{CP(1)}=1$, $\left(1-\lambda^a,1\right]$.

Second case: $\ell=2/3$. This case corresponds to an equal distribution of the agricultural sector among the three regions. If $\ell=2/3$, expression (29) can be rewritten as

$$-\frac{\mu}{3}\left[2\sigma^2-\left(4+5\mu\right)\sigma+2-3\mu\left(1-\mu\right)\right] \tag{30}$$

Solving for σ, we obtain:

$$\sigma_i = 1 + \frac{5\mu \pm \sqrt{\mu(49\mu + 16)}}{4} \quad i = 1,2$$

with $0 < \sigma_2 < 1 < \sigma_1$. We can disregard σ_2 and conclude that (30) is larger than zero for $1 < \sigma < \sigma_1$ and it is less than zero for $\sigma > \sigma_1 > 1$. In Figure 2(b), we set $\sigma > \sigma_1 > 1$, therefore (30) is negative. As ϕ crosses ϕ^P a subcritical bifurcation emerges. The curve of asymmetric equilibria lies entirely on the left of ϕ^P. The two existing asymmetric equilibria are unstable. See Figure 4.

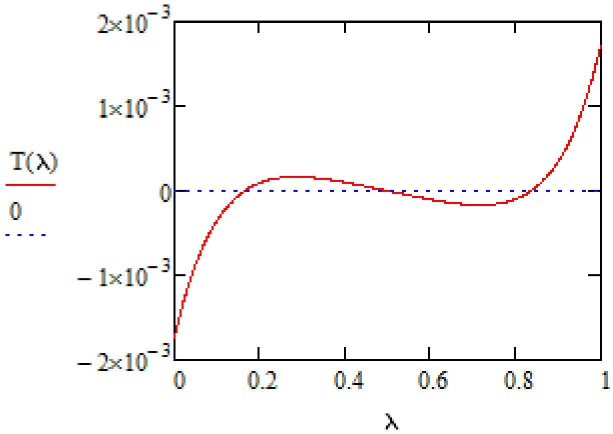


Fig. 4: Wiggle diagram showing the stability of equilibria for $\ell = 2/3$, $\mu = 0.45$ $\phi = 0.333$ and $\sigma = 3$

In Figure 2(c), we set $1 < \sigma < \sigma_1$, therefore (30) is positive. As ϕ crosses ϕ^P from left to right, a supercritical pitchfork bifurcation emerges. The curve of asymmetric equilibria lies, at least initially, on the right of ϕ^P. Four asymmetric equilibria may exist. The two external equilibria, λ^b and $1-\lambda^b$, are unstable. This is due to the fact that due to the border collision bifurthe CP equilibria gain stability (see above). Instead, the two interior asymmetric equilibria, λ^a and $1-\lambda^a$, are stable. This is due to the fact that in the neighborhood of the symmetric equilibrium the pitchfork bifurcation must be supercritical. The disappearance of the four asymmetri equilibria occurs via a so-called fold bifurcation. Typically according to such type of bifurcation by varying a parameter (in our case by reducing ϕ) two equilibria emerge (one stable and one unstable). In our case, due to symmetry, this occurs both below and above $\lambda^* = \frac{1}{2}$.

To check further on the stability properties of the asymmetric equilibria see Figure 5, where, for $1 < \sigma < \sigma_1$, we have plotted $T(\lambda)$ for different values of the trade freeness parameter. For $\phi = 0.109$ and $\phi = 0.11$, only the external asymmetric equilibria coexist with the symmetric and the CP equilibrium; the external asymmetric equilibria delimit the basins of attraction of the stable equilibria: a situation analogous to Fig. 4. By increasing slightly ϕ, in the example up to $\phi = 0.112$, the two stable interior asymmetric equilibria emerge, for which $M'(\lambda^a) < 1$ and $M'(1-\lambda^a) < 1$, that correspond to $T'(\lambda^a) < 0$ and $T'(1-\lambda^a) < 0$.

The basins of attraction are now given by $\left[0, \lambda^b\right)$ for the CP equilibrium $\lambda^{CP(0)} = 0$, $\left(\lambda_b, 0.5\right)$ for the interior asymmetric equilibrium λ^a, $\left(0.5, 1-\lambda^b\right)$ for other stable asymmetric equilibrium λ^a and finally, $\left(1-\lambda^b, 1\right]$ for the CP equilibrium $\lambda^{CP(1)} = 1$. Notice that the symmetric equilibrium, which is unstable after crossing the bifurcation value ϕ^P, separates the basins of attraction of the two interior asymmetric equilibria.

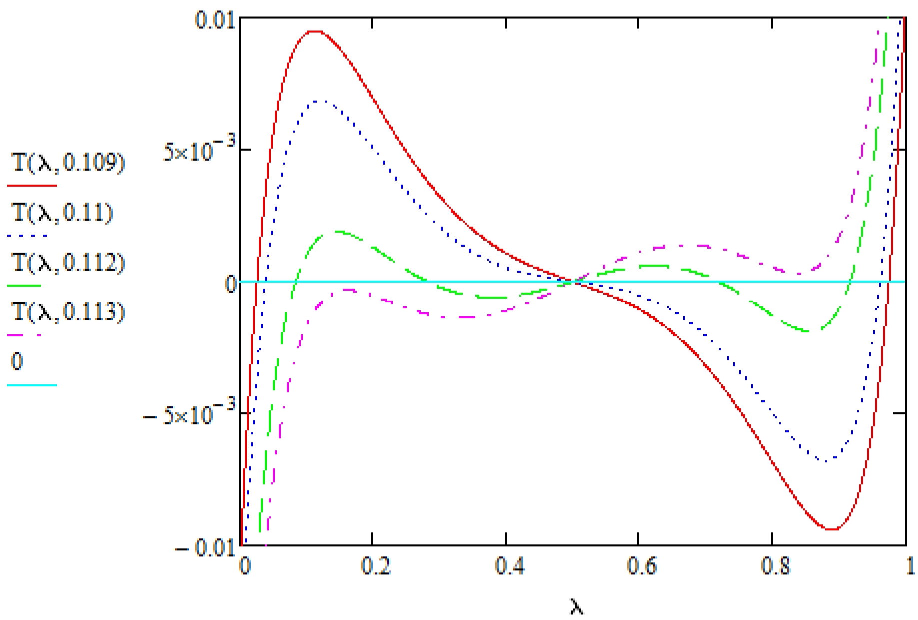


Fig. 5: Wiggle diagram showing the stability of equilibria for $\ell = 2/3$, $\mu = 0.45$ and $\sigma = 3$ and for different values of the trade freeness parameter: $\phi = 0.109$ $\phi = 0.11$ $\phi = 0.112$ and $\phi = 0.113$

Finally, in Fig. 6 we present the general case $0 < \ell < 1$. These simulations that the negative impact of the size of the third region on ϕ^P is significant.

Finally, concerning the inequality on the left hand side of (28), it holds for a sufficiently small value of γ or for a sufficiently high value of ℓ (see also below).

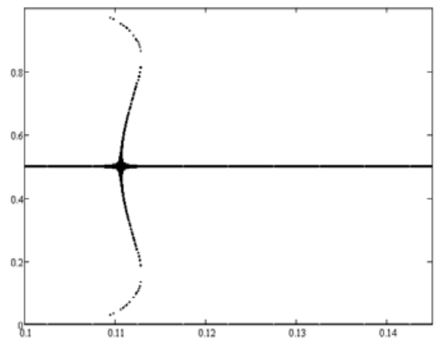
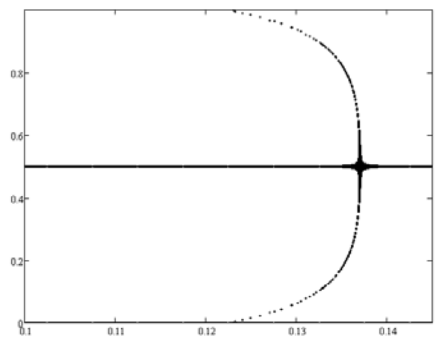
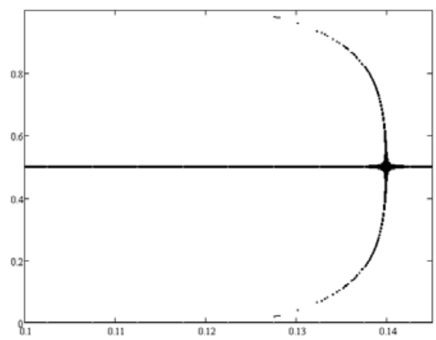
	$\sigma = 2$ $\mu = 0.45$ $\ell = \frac{2}{3}$ Size of third region: $1 - \ell = 1/3$
	$\sigma = 2$ $\mu = 0.45$ $\ell = \frac{2}{3} \cdot 1.09$ Size of third region: $1 - \ell = 0.273$
	$\sigma = 2$ $\mu = 0.45$ $\ell = \frac{2}{3} \cdot 1.1$ Size of third region: $1 - \ell = 0.267$

Figure 6

6. Preliminary results on global dynamics

As it is stated in the previous sections, the map Λ has two CP-fixed points λ=0 and λ =1, the symmetric fixed point λ^*=1/2, and it can also have four more, asymmetric, fixed points, λ^a, 1-λ^a and λ^b, 1- λ^b (which are symmetric with respect to λ^* by pairs).

Let us write down the expressions of the bifurcation curves of the fixed point λ^*=1/2, related to its eigenvalue ± 1. The eigenvalue the map Λ evaluated at λ^* can be written as

$$M'(\lambda^*) = \eta \equiv 1 + \frac{\gamma(1-\phi)}{(1+\phi)}\left(\frac{\mu}{(\sigma-1)} + \frac{2\mu\phi - \ell(\sigma-\mu)(1-\phi)}{\sigma(1+\phi)-\mu(1-\phi)}\right).$$

The value η = 1 corresponds to the pitchfork bifurcation. This bifurcation holds if

$$\ell = \ell_{pf} \equiv \frac{\mu}{(\sigma-\mu)(1-\phi)}\left(\frac{\sigma(1+\phi)-\mu(1-\phi)}{(\sigma-1)} - 2\phi\right).$$

The value η = -1 is related to the flip bifurcation. The flip bifurcation occurs if

$$\ell = \ell_{fl} \equiv \frac{(\sigma(1+\phi)-\mu(1-\phi)}{(\sigma-\mu)(1-\phi)}\left(\frac{\mu}{(\sigma-1)} + \frac{2(1+\phi)}{\gamma(1-\phi)}\right) + \frac{2\mu\phi}{(\sigma-\mu)(1-\phi)}.$$

To get an idea about the global dynamics of the map Λ let us fix μ=0.45, γ=20 and consider the (ϕ, ℓ)-parameter plane for different values of σ.

First, let σ=2. In Fig.7 we present the 2Dim bifurcation diagram in the (ϕ, ℓ)-parameter plane where different colours correspond to attracting cycles of different periods (up to the period equals 33). The two curves $\ell = \ell_{pf}$ and $\ell = \ell_{fl}$ related to the pitchfork and, respectively, the flip bifurcation of the fixed point λ^* are plotted using the related equations. To get this 2Dim diagram only one initial condition was used, so, multistability cannot be observed in such a case. In order to clarify the dynamics let us consider 1Dim bifurcation diagrams.

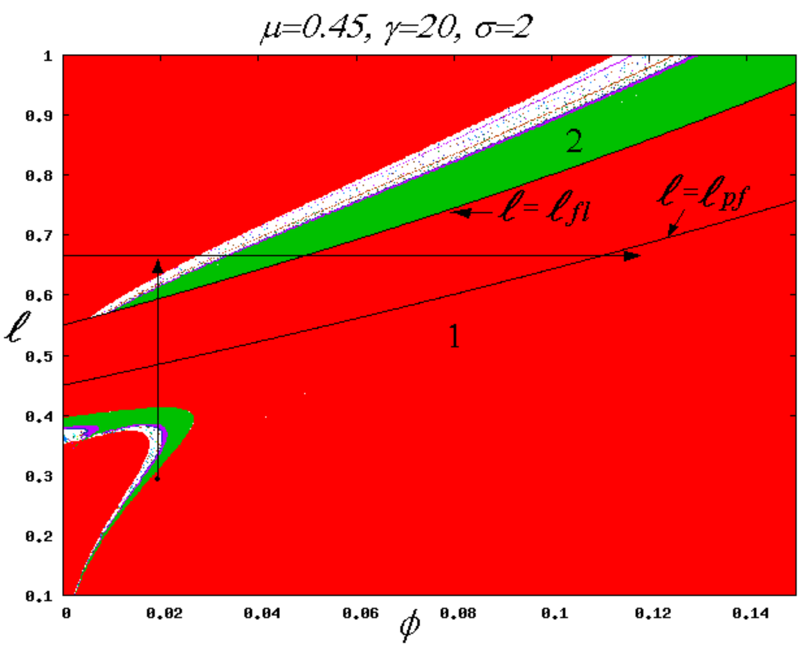


Figure 7 2Dim bifurcation diagram in the (ϕ, ℓ)-parameter plane at σ=2, μ=0.45, γ=20.

The 1Dim diagrams related to the straight lines with arrows are shown in Fig. 8 (horizontal line) and Fig. 9 (vertical line).

First, let us fix the value $\ell = 2/3$ and vary the parameter ϕ in the range (0, 0.15) (the related parameter pass is shown in Fig.7 by a horizontal line with an arrow). The corresponding 1Dim bifurcation diagram is shown in Fig.8 (a) together with its two enlargements, in (b) and (c). Let us comment on the bifurcation sequence starting from the attracting fixed point λ^* (e.g., at ϕ = 0.055) and will decrease the value of ϕ (see Fig.8 (b)). At $\phi = \phi_3$ a supercritical period-doubling bifurcation occurs leading to the attracting 2-cycle denoted g_2; then at $\phi = \phi_2$ this cycle undergoes a supercritical pitchfork bifurcation resulting in two more (coexisting) attracting 2-cycles. Each of these cycles undergoes a cascade of period-doubling bifurcations following the 'logistic' scenario up to the pairwise merging of two coexisting 2-cyclic attractors in one 2-cyclic attractor due to the homoclinic bifurcation of the 2-cycle g_2. At $\phi=\phi_1$ the chaotic interval has a contact with its basin of attraction, bounded by the two repelling CP fixed points, after which these two fixed points become stable.

Now consider the enlargement of Fig.8 (a) shown in (c). At $\phi=\phi_4$ a border collision bifurcation leads to the stabilisation of the CP fixed points and to the appearance of two more repelling fixed points λ^a, 1- λ^a. At $\phi=\phi_5$ the fixed point λ^* undergoes a supercritical pitchfork bifurcation leading to two more fixed points, λ^b and 1- λ^b. Then at $\phi=\phi_6$ we observe a reverse fold bifurcation due to which the four fixed points λ^a, 1- λ^a and λ^b, 1- λ^b merge by pairs and disappear.

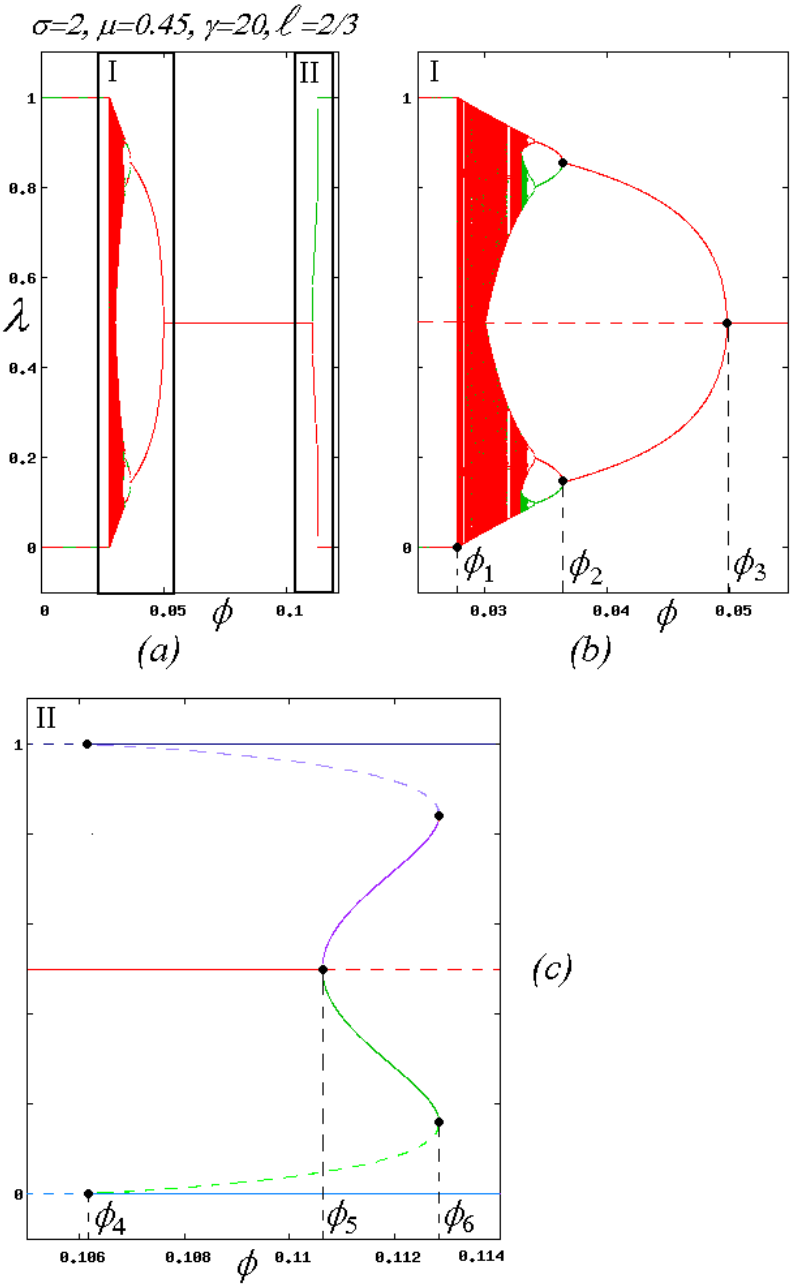


Figure 8 In (a): 1Dim bifurcation diagram at μ=0.45, σ=2, γ=20, $\ell = 2/3$, $\phi \in (0, 0.15)$; In (b) and (c): enlargements of two windows indicated by rectangles marked I and II in (a).

Let us come back to the 2Dim diagram in Fig. 7, fix ϕ = 0.02 and will vary the value of the parameter ℓ. The 1Dim bifurcation diagram for ϕ= 0.02 and $\ell \in (0.3, 0.65)$ is shown in Fig. 9. We observe that at $\ell = \ell_1$ the fixed point λ^*=1/2 undergoes the supercritical pitchfork bifurcation (for decreasing ℓ) leading to two fixed points $\lambda^*{}_1$=λ^a and $\lambda^*{}_2$=1-λ^a. If we continue to decrease the value of ℓ at $\ell = \ell_2$ each of these fixed points undergoes a supercritical period-doubling bifurcation and then it follows the 'logistic' bifurcation scenario. For the values of ℓ less than certain value this scenario is observed in the reverse order up to the period-doubling bifurcation at $\ell = \ell_3$ leading to the attracting fixed point. Thus, for example, at $\ell = 0.3$ we are back to the two attracting fixed points λ^*_1 and λ^*_2.

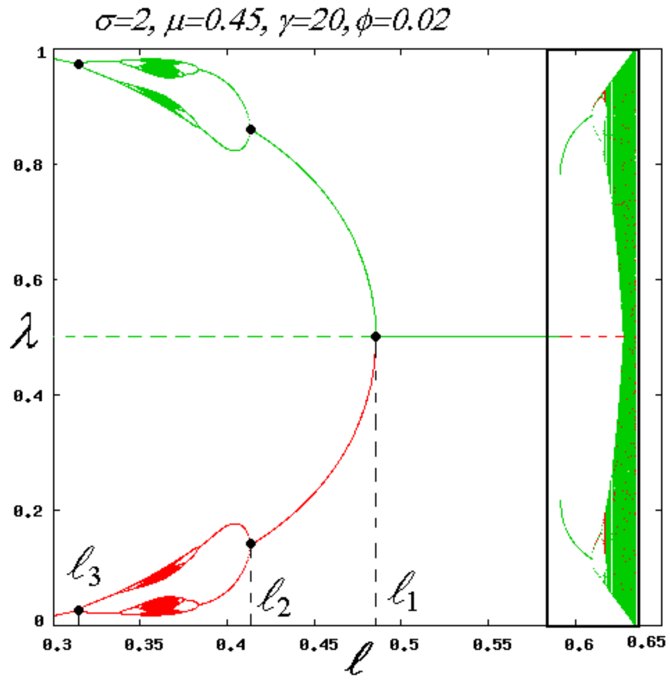


Figure 9 1Dim bifurcation diagram at μ=0.45, σ=2, γ=20, ϕ=0.02, $\ell \in (0.3, 0.65)$.

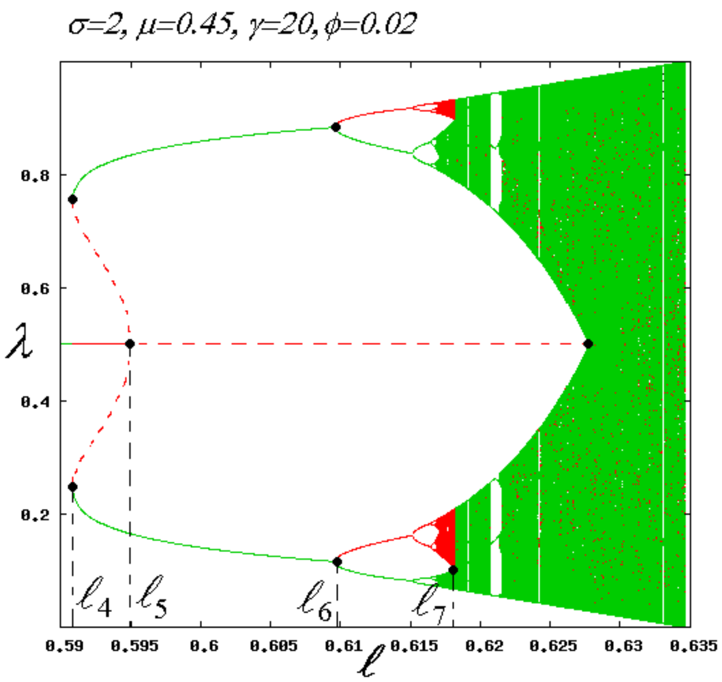


Figure 10 An enlargement of the window indicated in Fig. 9.

Starting again from the attracting fixed point λ^*=1/2 (see Fig. 9) and increasing the value of ℓ one can see that at $\ell = \ell_5$ the fixed points λ^* undergoes a subcritical period-doubling bifurcation that can be seen in Fig. 10 which shows an enlargement of the Fig. 9. Here the dashed colour lines are related to the points of a repelling cycle of period 2 denoted g_2' born at $\ell = \ell_4$ due to the fold bifurcation together with an attracting cycle g_2 of period 2. The parameter range $\ell \in (\ell_4, \ell_5)$ corresponds to coexisting attracting fixed point λ^* and 2-cycle g_2. Then, at the subcritical period-doubling bifurcation the points of g_2' merge with the fixed point λ^* and after this fixed point becomes repelling so that the only attractor is the 2-cycle g_2. If we continue to increase the value of ℓ, at $\ell = \ell_6$ the 2-cycle g_2 undergoes a supercritical pitchfork bifurcation leading to two new 2-cycles $g^a{}_2$ and $g^b{}_2$. Each of these cycles undergoes a 'logistic' sequence of bifurcations (for $\ell \in (\ell_6, \ell_7)$ we again have coexisting attractors) up to the moment of homoclinic bifurcation of the 2-cycle g_2 leading to merging of the attractors. After this bifurcation the attractor is unique. It exists up to the contact with its basin of attraction bounded by the repelling CP fixed points. As a result these fixed points are stabilised.

Note that the period-doubling bifurcation of λ^* observed in Fig. 8 (b) is supercritical and it is subcritical in Fig. 10, while the pitchfork bifurcation of λ^* in Fig. 8 (b) is subcritical and it is supercritical in Fig. 9.

To compare the bifurcation structure shown in Fig. 7 with the one for a larger value of σ, we show in Fig.11 the 2Dim bifurcation diagram in the (ϕ, ℓ)-parameter plane at σ=8, μ=0.45, γ=20. The basic sequence of bifurcations is similar, but we leave its complete characterization, as well as more detailed investigation of the dynamics of the map, for future work.

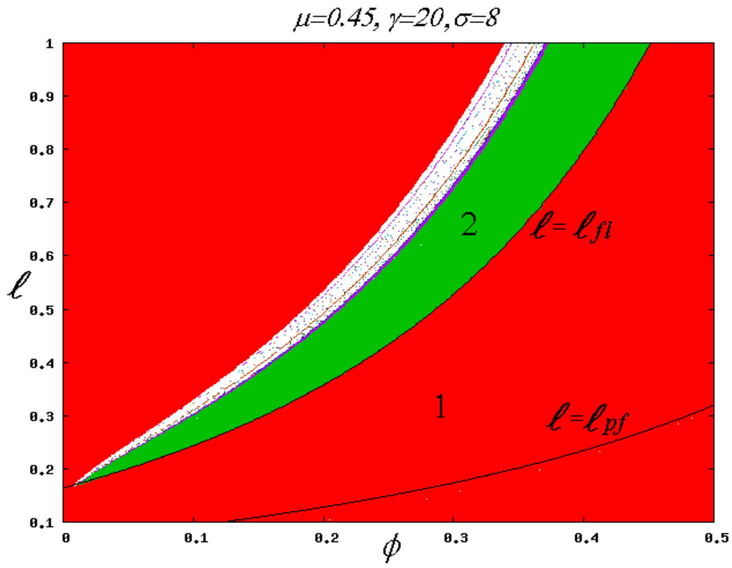


Figure 11 2Dim bifurcation diagram in the (ϕ, ℓ) -parameter plane at σ=8, μ=0.45, γ=20.

7. Final remarks

The (scant) NEG literature on three-region models has dealt so far with the impact of trade policy with a third region on industrial agglomeration in a two-region home country. In contrast, the FE-NEG model presented in this paper has delivered results on how economic integration between two regions (1 and 2) participating in a customs union impacts on the distribution of industrial firms within the union, holding constant the distance between the union itself and the rest of the world (region 3). We have shown that:

1) because our simple set up, a change in trade policy with the rest of the world does not impact on profit differentials between regions 1 and 2, thus leaving unaffected the distribution of industrial firms within the union;
2) it is the size (of the market) of the third region that matters for the balance between centripetal and centrifugal forces between the two countries/regions inside the integration area. This leads to the general conclusion that the effects of a trade liberalisation between the two countries inside the integration area strictly depends upon the size of the market external to the union. Such a result holds looking at both stable and unstable symmetric equilibria as summarized in the following two points;
3) an increase of the external demand coming from the third region, leads the symmetric equilibrium between region 1 and 2 to lose stability at a lower value of the trade free-

ness inside the integration area. That is, as shown in Brülhart et al. (2004), stronger outside links favour agglomeration within the integration area;

4) in addition, and this is our main result, the size of the third region also influences what happens, if the symmetric equilibrium is unstable. For a smaller third region the agglomerative forces outweigh the deglomerative forces leading to full agglomeration in one of the regions inside the integration area. Instead, for a bigger third region, asymmetries weaken the agglomerative processes and strengthen the deglomerative forces – and interior asymmetric equilibra can be established.

This latter results is important for two reasons: a) because it shows that even if with a reduction of trade costs with the integration area the symmetric equilibrium loses stability (as found also in the papers reviewed above), the long-run outcome need not be full agglomeration, but may also be partial agglomeration (thus a reduction of trade costs does not lead to extreme regional dispersion); b) because it is one of the rare examples in the NEG literature that produces partial agglomeration as the outcome of an endogenous process.

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