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The Impact of Regulation and Economic Conditions on the Dynamics of Financial Markets

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Abstract

This dissertation encompasses four studies on selected topics in financial regulation and financial stability. The first paper asks whether there is empirical evidence of cyclicality in regulatory capital requirements prescribed by Basel regimes. This much debated issue was until then only addressed in theoretical papers, or simulation studies. While we do not find evidence on cyclicality in the Basel I or Basel II Standardized Approach, we find statistically and economically significant evidence concerning Basel II IRB portfolios. The second paper implements an agent based model to simulate an artificial asset market. This setup is then used to assess the impact of (i) a short selling ban, (ii) a Tobin Tax like transaction tax, (iii) mandatory Value-at-Risk limits and (iv) arbitrary combinations of these. I present results that show that while reducing volatility, a short selling ban nurtures market bubbles, and a Tobin Tax increases the variance of the returns. In this model a mandatory risk limit is beneficial from all stability perspectives taken. I examine the robustness of the model regarding its initial parameterization and show that high levels of a Tobin Tax lead to substantial market turbulence. The third paper considers the question which macroeconomic variables are linked to a time series of special interest from a financial stability perspective: firm defaults. Furthermore, we evaluate the empirical evidence of a hidden credit cycle by adding a latent factor to our models. We conclude that there is no empirical support of a hidden credit cycle in Austria once sufficient regressors are included and industry sectors differ in their respective macro drivers. The forth paper extends this work by implementing Bayesian Model Averaging (BMA) — a modern technique to counter model uncertainty. Furthermore we enrich this statistical approach by combining BMA with Bayesian ridge regression. We draw the conclusion that BMA is indeed a powerful tool to counter model uncertainty. Interest rates and components of inflation are distilled as major drivers for firm failures in Austria.

Kurzbeschreibung

Diese Dissertation umfasst vier Studien, die sich jeweils bestimmten Themen der Finanzregulierung und der Finanzmarktstabilität widmen. Die erste Arbeit geht der Frage nach empirischen Befunden betreffend die Zyklizität von Eigenmittelerfordernissen unter den Basel'er Regulierungen nach. Diese oft debattierte Fragestellung wurde bis dato nur in theoretischen Arbeiten, sowie in auf Simulationsansätzen beruhenden Arbeiten untersucht. Wenngleich die Ergebnisse keine Befunde für zyklische Eigenmittelanforderungen unter Basel I und Basel II Standardansatz zeigen, ist die Zyklizität in Basel II IRB Portfolien statistisch und ökonomisch signifikant. Die zweite Studie implementiert ein Agent Based Modell zur Simulation eines künstlichen Finanzmarktes. In Folge wird dieses Modell dazu genutzt die Effekte von (i) einem Leerverkaufsverbot, (ii) eine Tobin Steuer ähnlichen Transaktionssteuer, (iii) einem verpflichtenden Value-at-Risk Limit und (iv) einer beliebigen Kombination dieser Maßnahmen zu untersuchen und auszuwerten. Die Ergebnisse belegen zwar eine Volatilitätsreduktion bei Einführung eines Leerverkaufsverbotes, doch zeigen auch, dass dabei die Bildung von Marktblasen gefördert wird. Dementgegen führt eine Tobin Steuer tendenziell zu höherer Volatilität. In diesem Modell wirkt ein verpflichtendes Value-at-Risk Limitsystem als förderlich aus allen in Betracht gezogenen Perspektiven der Marktstabilität. Ferner werden Robustheitschecks hinsichtlich der Wahl der Eingangsparameter durchgeführt, die ergeben, dass höhere Sätze einer Tobin Steuer mit erheblichen Marktturbulenzen einhergehen. Die dritte Arbeit beschäftigt sich mit der Frage, welche makroökonomischen Variablen mit der aus Sicht der Finanzmarktstabilität bedeutenden Zeitreihe verknüpft sind: Firmenausfällen. Darüber hinaus werden den angewandten Modellen ein latenter Faktor hinzugefügt, um damit empirisch der Existenz eines verdeckten Kreditzykluses nachzugehen. Sofern genügend Regressoren in den Modellen enthalten sind, ergibt sich kein empirischer Beleg für einen verdeckten Kreditzyklus. Weiters lassen die Ergebnisse den Schluss zu, dass Sektorspezifika bei der Menge der Risikotreiber eine wichtige Rolle spielen. Die vierte Arbeit erweitert diese Uberlegungen, indem sie eine moderne statistische Technik bei Präsenz von Modellunsicherheit, Bayesian Model Averaging (BMA) implementiert. Darüber hinaus wird diese statistische Methodologie durch Bayesian Ridge Regression komplementiert. Zinssätze und Subkomponenten von Inflation destillieren sich als wesentliche Risikotreiber für Firmenausfälle in Osterreich.

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Chapter I

Motivation and Overview

This cumulative dissertation comprises four working papers on the effects of financial regulation and the effects of a changing economic environment from the perspective of financial stability. Chapters II to V constitute each one of these essays. Beforehand, the current chapter provides a motivation for the work (Section 1) as well as a summarizing overview of the papers (Section 2).

1 Motivation

Motivating research about financial stability is a "walk in the park" considering the current market turmoil. However, by making use of the financial crisis for motivation one risks giving the impression that only the (coincidental) occurrence of the crisis brought about scientific thought in the field of financial stability and regulation — an otherwise less important topic. In contrast, I argue that research in the field is of immense value irrespective of the financial crisis. Over the past decades banking and finance has become a major economic sector in developed industries. The fraction of GDP stemming from the financial sector has risen by more than 15% from 1995 to 2010 in the European Union¹. The trend of a growing importance of the financial sector reaches back several decades. While in the seventies levels of about 15% of GDP were stemming from financial activities,

¹This holds for any definition of European Union, EU 15, EU 25 and EU 27. Source: Eurostat. For data concerning the United States see Shin (2010, page 167-170).

current levels lie around 25%, with Luxembourg well above 40%.

This growing industry is highly regulated. And not without reason: The list of financial crises, bank runs and defaults is not only a long one, these events regularly caused severe and lasting damage in the economy as a whole. Economic theory and political reality alike have responded, as thousands of research papers and pages of regulatory restrictions on banks, funds and insurers testify. In a historic perspective, a long term issue in banking regulation was avoiding sun-spot-bank-runs, as theoretically pinned down by the famous model of Diamond and Dybvig (1983). A central bank as lender of last resort together with a deposit insurance scheme was largely seen as a *the* remedy to prevent runs. However, it turned out that such an intervention causes moral hazard on the side of banks: "...if a bank's liabilities are insured at a premium that is independent of portfolio risk, then banks hold the riskiest portfolios they are allowed to hold" (Kareken and Wallace, 1978). Again partly due to this finding, interest in risk management techniques — the wish to accurately quantify risk — has since then steadily increased. In the eighties the combination of financial shocks and advances in the area of risk management, most notably the propagation of the Value-at-Risk concept, led to a response of regulators in order to tackle the problem of moral hazard in banking described above. What followed is today labelled the Basel regime.

Despite the thick rule book banks have to follow since, the recent events in financial markets proved to be disastrous. The cumulative dissertation at hand addresses mainly two issues in this context:

First, similar to the introduction of a deposit insurance to tackle bank runs, capital rules in the Basel regime might have mitigated moral hazard in the banking sector while bringing about its own side effects, e.g., the issue of procyclicality, as argued by e.g., Kashyap and Stein (2004). A good motivation is given by Shin (2010, page 13):

"Risk management is an essential part of the operation of a financial institution,

and the value-added of a good risk management system can indeed be substantial. But there may be a divergence of interests between an individual firm and the system as a whole. Exploring exactly how the divergence of interests plays out in the economy is an urgent modelling task for economists."

History shows a more profound knowledge of the effects of current regulation as well as the impact of proposed regulation as a response to crisis is of high importance.

Second, banks are still prone to failure. In order to have a better understanding of risk, we ask how economic conditions influence the resilience of the financial sector via the defaults of clients. We aim for a better knowledge of the macro economic factors that drive aggregate credit risk in portfolios. Related to this question is a statistical research question, namely what are appropriate tools to identify such linkages.

2 Overview

In this section I provide an overview of the papers that follow. It is intentionally kept simple to be understandable to a broader audience. Advanced readers familiar with the topics may refer to the introduction sections in the respective chapters.

The first paper is entitled "Quantifying Cyclicality of Regulatory Capital: First Evidence From Austria" and is joint work with Michael Sigmund². Let me briefly outline the background. January 2007 marked the introduction of Basel II, a new framework of regulation for banks. For the first time, banks were allowed to calculate their regulatory capital requirement for credit risk — a central figure in the Basler framework — according to their internal risk models. In the prelude of the introduction of Basel II academic debate erupted if capital requirements that are not only risk-sensitive across banks' assets

²Oesterreichische Nationalbank, Otto-Wagner-Platz 3, A-1090 Vienna, Austria.

but also across time³ would increase capital requirements in economic downturns (for a constant stock of assets). It was hypothesized that banks faced with (self-) increasing capital requirements would react by reducing credit supply. Hence, it was feared that such a regulatory regime would lead to a procyclical effect via the banks lending channel. This question did not remain a theoretical one: With the financial crises starting in 2008 banks' equity was severely reduced all around the globe and the question of potentially cyclical capital requirements attracted attention in economic policy and public debate.

The first research paper, Chapter II, addresses this question.⁴ We ask if there is statistical evidence of a comovement of economic conditions and banks' capital requirements in Austria. The scientific contribution of the paper consists in studying this relationship not in a theoretical model or based on simulated data, as previous research has done, but to use realized data. We draw on the Austrian regulatory reporting system which provides detailed accounts of all banks operating in Austria. The advantage of using observations rather than simulated results is that we can avoid making sensitive assumption on the risk models in use or on the reaction of banks' management. A further contribution consists in the fact that we are able to distinguish between various regulatory regimes, that is Basel I and the two approaches for credit risk in Basel II, first the use of external ratings, called the Standardized Approach, and second the internal ratings based approach, IRB. We set up a panel model that accounts for these conditions and include interaction terms to measure potential cyclical variation in levels of capital requirements. Our findings show no evidence of cyclicality under Basel I or in the Standardized Approach of Basel II but substantial and statistically significant effects in IRB portfolios.

The second paper's title is "**Regulatory Medicine Against Financial Market Instability: What Helps And What Hurts?**". It is not co-authored. As the title suggests, the study draws on the analogy between financial markets and organic systems. In both

³In fact this might not be limited to internal models but also to the use of external ratings from rating agencies.

⁴Note that we solely focus on the question if capital requirements increase under worsening economic conditions, not if this in turn affects the credit supply.

cases the complex environment and the manifold interactions make it hard to predict what consequences a particular drug or regulation will have. Therefore, without proper prior testing, side effects and unwanted outcomes may appear. With the steep climb in the availability of computing power over the recent decades a new way of modelling such interactions emerged: agent based models. In a nutshell, agent based models mimic the actions of numerous agents in a computer simulation. Recent work in this field (see the references in Chapter III) shows that these models are able to replicate stylized facts of financial markets.

In this paper we introduce leveraged agents trading a single stock according to a mispricing signal. We find that the baseline model exhibits fat tails and clustered volatility in its time series of returns. Subsequently, we implement three regulatory measures, a transaction tax on trading (a "Tobin Tax"), a ban of short selling and a Value-at-Risk limit. All three measures constitute currently discussed or already implemented regulation. However, there is no agreement on the effects (and side effects) of the measures. Concerning the Tobin Tax proponents argue that such a measure would drive uninformed investors out of the market, while critics fear a drop in market liquidity would increase market volatility. Studying the effects of a mandatory Value-at-Risk framework has a similar motivation as the first paper. In times prices fall and risk models indicate increased variance in the returns risk limits may force traders to sell, thus amplifying the downtrend. In our setup we are able to identify the single effects of these measures and thus provide scientific content to the debate. Moreover, we provide insights on the interaction of measures. The fact that pharmaceuticals sometimes have negative or unforeseen effects when administered together is a big issue in modern medical research. In economics, this topic has received only little if any attention. This study therefore also investigates the joint effect of regulatory medicine. The results identify a mandatory risk limit as the only measure surveyed that is beneficial from every perspective. A short selling ban on the other hand — though reducing volatility — increases tail risk. The contrary holds true for a Tobin Tax: it reduces the occurrence of crashes but increases volatility. Furthermore, the interplay of measures matters: measures block each other and a well-chosen combination can mitigate unforeseen side effects. Concerning the Tobin Tax the findings indicate that an overdose can do severe harm.

The third paper, "What Drives Aggregate Credit Risk?", is co-authored by Michael Sigmund. This paper as well as the forth explore the driving macroeconomic factors of aggregate credit risk. In other words, we want to explain the time series of firm failure rates, which we use as a proxy for corporate defaults. Indeed, credit risk, i.e., the default of borrowers, is the largest risk category according to the classical categorization taken in the Basler framework. Thus, a deep understanding of its drivers is valuable for financial institutions as well as for regulators from multiple viewpoints. For example, conducting meaningful stress tests requires the translation of macroeconomic scenarios into portfolio losses. The same applies when financial institutions and supervisors are interested in forecasting the credit quality of portfolios on an aggregated level. Both in the field of macro prudential supervision and strategic risk management a knowledge of the determinants of aggregated defaults is crucial.

Traditional approaches to estimate aggregate credit risk (ACR) consider macroeconomic variables as drivers. However, recent literature suggests the existence of a latent risk factor influencing ACR, which is regularly interpreted as the latent credit cycle. In the third paper we explicitly model this latent factor by adding an unobserved component to our models which already include macroeconomic variables. We make use of insolvency rates of Austria to model realized probabilities of default. To better understand sector specific characteristics, we model six corporate industry sectors separately. The contribution of the paper to the literature on ACR risk is threefold. First, in order to cope with the lack of theory behind ACR drivers, i.e., to cope with model uncertainty, we implement state-of-the-art variable selection algorithms to draw from a rich set of macroeconomic variables. Second, we add an unobserved risk factor to a state space model which we estimate via a Kalman filter in an expectation maximization algorithm. Third, we analyze whether the consideration of an unobserved component indeed improves the fit of the estimated models.

Interestingly, we find that enlarging a macro-to-probability of default model by incorporating a latent risk factor only improves the model significantly if the model is allowed to select from a small number of possible predictors. We show that this finding is not explained by the selection procedure applied but is attributable to a limited set of variables. The limited number of included variables also explains why some of the relevant literature finds strong support for including unobserved risk factors in macro-to-probability of default models. Using our enlarged dataset of macroeconomic predictors the results indicate that models of sizes of about eight regressors are not significantly improved by the addition of a latent factor. Moreover we find several variables which drive ACR simultaneously in a number of sectors and are thus particularly crucial for modeling ACR. These variables include interest rates, inflation and (negative) credit growth. However, there are also considerable sectoral differences in the selected variables. Among the sector-specific variables we find e.g., the oil price and exports in the transportation sector, investment in equipment in the trade sector and short-term interest rates in the service sector.

The forth paper is entitled "Model Uncertainty and Aggregated Default Probabilities: New Evidence from Austria". It is joint work with Paul Hofmarcher and Kurt Hornik⁵, Bettina Grün⁶ and Michael Sigmund. This essay shares much of the motivation with the third one, although we apply a more modern approach in dealing with model uncertainty. This approach, labeled "Bayesian Model Averaging" (BMA), has only recently gained popularity in econometric modeling. Thereby, one refrains from assuming that there is one "true" model but instead averages over a huge number of potential models. Sampling from the set of regressors BMA then computes several thousands of models, which are weighted by their marginal likelihood and subsequently averaged. This simple procedure reveals important determinants of the dependent variable and their respective

⁵Paul Hofmarcher and Kurt Hornik: Department of Finance, Accounting and Statistics, WU Wirtschaftsuniversität Wien, Augasse 2–6, A-1090 Vienna, Austria.

⁶Department of Applied Statistics, Johannes Kepler University Linz, Altenbergerstraße 69, A-4040 Linz, Austria.

coefficients. In order to explicitly deal with highly correlated candidate regressors present in our dataset, we enhance BMA with ridge regression. Our findings suggest that factor prices like short term interest rates and energy prices constitute major drivers of default rates, while firms' profits reduce the expected number of failures. Finally, we show that the results of our baseline model are fairly robust to the choice of the prior model size.

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Chapter II

Quantifying Cyclicality of Regulatory Capital: First Evidence from Austria

Abstract

With the financial crisis spreading to the real economy, the discussion about potential procyclical implications of Basel II received a surge of attention. While existing research approaches the topic either from a theoretical perspective or from an empirical perspective that draws on simulated data, we are first in studying the cyclicality of risk weights on the basis of realized data. Furthermore, we are able to differentiate not only between Basel I and Basel II, but also between the Standardized Approach (StA) and the internal ratings-based (IRB) approach. We argue that without knowledge of these approaches' presumably distinct cyclicality of risk weights, any measure to dampen procyclicality is premature. For this purpose, we first study which banks opt for implementation of the IRB approach and then set up a panel model to quantify the cyclicality of capital requirements. While we find no evidence of cyclicality in portfolios subject to the Basel II StA, we find economically substantial and statistically significant effects in IRB portfolios.

Keywords: Basel accord, procyclicality, business cycles.

JEL Classification: E44, G28.

1 Introduction and Motivation

In the face of the ongoing crisis, interest in the discussion about potential procyclical implications of the current regime of financial regulation, Basel II, has increased. In a nutshell, it is argued that in economically bad times higher regulatory capital requirements induce banks to reduce their lending activities, thus hampering aggregate demand (and vice versa in good times). In the respective literature this procyclical effect is referred to as the "bank capital channel" (see Drumond, 2008, for a synthesis). In this study we empirically analyze the link between economic conditions and increases in regulatory capital requirements — we refer to this link as "cyclicality of capital requirements." At least from an empirical point of view, potential procyclicality effects — a further economic downturn stemming from reduced lending activities due to the cyclicality of capital requirements — are exceptionally complex to identify. Even if one controls for all relevant factors that affect bank lending and takes banks' capital constraints into account, bank lending might be procyclical even without capital requirements. So it remains unclear how to distinguish between (additional) procyclicality induced by cyclical capital requirements and reduced lending due to decreased demand or lending opportunities.

As Kashyap and Stein (2004) point out, capital constraints are more binding in a recession. That is, the scarcity of bank capital relative to positive net present value lending opportunities is more severe in such an economic environment. From a bank's perspective, two effects lead to more binding capital constraints in times of crisis:

- (i) Banks suffer losses, and these losses directly reduce equity. One can refer to this as "contraction in the numerator," as the capital base relative to risk-weighted assets shrinks due to a smaller capital base.
- (ii) The risk underlying banks' assets increases; under the assumption of a regulatory system that maps risk via an increasing function into risk weights, capital requirements also rise in economically difficult times. Basel II clearly aims at providing

such a function; in fact this function constitutes the key change compared to Basel I (Drumond 2008). One can refer to this as "expansion in the enumerator."

To complete the picture, we add one further factor:

(iii) Capital constraints are more binding during a crisis because the possibility of raising new capital erodes under such circumstances. Although it seems that the difficulty of raising new capital was neglected before the crisis,¹ its presence as well as its high correlation with the two effects mentioned above are now generally acknowledged. Many banks' assumption of unchanged funding sources in times of crisis proved to be terribly wrong.

To sum it up, the two effects lead to tighter capital constraints for banks and therefore to reduced lending,² which in turn has a negative impact on the real economy. In fact (*i*) is somehow a "natural" outcome of the crisis, while (*ii*) is regulatorily induced. Therefore studies on the procyclicality of regulatory system focus on the second effect.

The issue of an economic cycle-amplifying effect due to volatile capital requirements has been much debated in financial literature. On the theoretical side, we find papers by Catarineu-Rabell et al. (2005), Heid (2007) and recently Pederzoli et al. (2010), who model the effects of business cycle fluctuations on capital requirements. Empirical studies on the other hand generally use data on rating migrations to simulate the effects of a downturn on regulatory capital requirements. Among those we find e.g., the works of Kashyap and Stein (2004), Gordy and Howells (2006) and Repullo et al. (2009) among many others³. Although the hypothesis that Basel II induces additional cyclicality of

¹E.g., Aguiar and Drumond (2007) address this effect via a varying liquidity premium on equity, Markovic (2006) via the introduction of the adjustment cost channel, the default risk channel and the capital loss channel. Nevertheless, the fact that the possibility to raise new capital is not included in theoretical models has rather been seen as a drawback than a feature of the model.

²See Blum and Hellwig (1995) for a simple Aggregate Demand-Aggregate Supply model on how capital requirements affect aggregate demand. Aliaga-Diaz (2005) incorporates capital requirements into a Dynamic Stochastic General Equilibrium (DSGE) model. Drumond (2008) provides an extensive literature review of this link from a theoretical perspective. Peek and Rosengren (1995) and Jackson et al. (1999) present empirical support.

³See Kashyap and Stein (2004) and Lowe (2002) for an overview.

capital requirements is generally supported, a high level of uncertainty remains. There are two main reasons for this: One is that all of the studies mentioned base their research on simulated data rather than observed outcomes of capital requirements.⁴ Lowe (2002) states that due to structural changes, the effects of Basel II cannot be assessed adequately under the regime of Basel I, which can be seen as a version of the Lucas Critique. The wide range of results of empirical studies reflects the sensitivity of critical assumptions about the construction of simulated data. Reviewing the literature on this topic, one finds differing assumptions about management reactions, rating migration, rerating frequency, severeness of the downturn, etc.

The second reason for the high level of uncertainty is that there is very little or no evidence on how the cyclicality of capital requirements differs between regulatory regimes, i.e., Basel I, Basel II StA and Basel II IRB.⁵ In fact, many empirical studies focus solely on IRB and therefore do not allow a comparison. While it seems obvious that Basel II takes more sensitive risk weights into account than Basel I, irrespective of the approach, the comparison of StA and IRB is not clear from an ex ante perspective. Furthermore, we argue that without knowledge of these approaches' presumably distinct cyclicality, any measure to dampen procyclicality suggested by the literature is premature.

The contribution of our study is therefore twofold. First, we examine the cyclicality of capital requirements based on realized, not simulated data. Our observation period covers an entire business cycle from the year 2000 to 2009, thus including the recent crisis. Second, we provide first evidence on the question so far unanswered in existing literature of whether risk weights show more cyclicality under the StA or under IRB. To measure the extent to which Basel II contributes to cyclicality, we set up a panel model. The regulatory reporting system, which provides us with detailed and frequent information on

⁴This is because most studies were conducted prior to or at an early stage of implementation of Basel II. However, there is research studying the determinants of capital ratios subject to Basel I that makes use of realized data. See Francis and Osborne (2009) and the references therein.

⁵See www.oenb.at/en/presse_pub/period_pub/baselII/basel_ii.jsp for a comprehensive overview of Basel II, including a detailed description of the differences between Basel II StA and Basel II IRB.

the Austrian banking sector, serves as a data source. Drawing on this source, we hope to find answers to the question of how capital requirements evolve in crisis periods, and to differentiate between Basel I, Basel II StA and Basel II IRB.

The remainder of the paper is structured as follows. Section 2 examines IRB more closely, focusing on banks' and regulators' motivation for introducing this regulatory approach. Section 3 presents the modeling approach to quantify cyclical effects, whose results are presented in Section 4. Section 5 concludes with an outlook on how the cyclicality of capital requirements can be embedded in the economic and political discussion about procyclicality. In particular, we highlight some areas of future research.

2 IRB Implementation

In this section we give a brief overview of IRB to better understand its role in the cyclical behavior of regulatory capital requirements and to address the question of which banks are able and willing to switch to IRB.

From a bank's perspective, the benefit of an IRB approach lies mainly in reduced capital requirements, as intended by the BIS.⁶ Furthermore, the possibility of calculating own risk weights for certain bank assets without relying on the fixed Basel II tabularized weights can be seen as a major incentive. Banks subject to IRB are required to estimate their risk parameters based on a time series of at least five years. However, under certain circumstances, this time period may temporarily be reduced to two years.⁷ In any case, this time span allows probability of default (PD) and loss given default (LGD) estimations to be conducted over the horizon of an economic boom phase during which estimates may be favorable with regard to minimizing risk weights.

⁶Compare the Quantitative Impact Studies (Bank for International Settlements 2006).

⁷See EU Directive 2006/48 Annex VII Part 4, 66 and 71.

On the cost side, the design and implementation of an IRB approach requires a certain amount of resources and know-how that only larger banks are likely to have at their disposal. Moreover, to counteract any incentives for banks to minimize their risk-weighted assets excessively, banks are only allowed to implement a certified model subject to regulatory supervision. From the regulator's view, the reduced capital requirements are compensated for by a higher risk sensitivity, leading to more sophisticated coverage and a deeper awareness of the risks a bank is exposed to.

To econometrically analyze the decision-making process, we conduct a series of probit regressions that try to incorporate the above arguments. A few theoretical papers (i.e., Ruthenberg and Landskroner 2008 and Hakenes and Schnabel 2006) use bank size as a proxy for the ability to carry out large initial investments in risk management technologies that are necessary to comply with the regulatory requirements for such models. Aside from bank size (measured in total assets), variables that indicate the portfolio composition are used as explanatory variables.

In our models we find that bank size has a significant positive effect on the probability of adopting the IRB approach. On the benefits side, we could not clearly identify significant variables related to portfolio structure and quality. However, we believe that these inconclusive results are possibly related to the fact that IRB banks have not yet implemented the IRB for their entire portfolio.

3 Model Specification and Data

Following the argumentation of the previous section, we now turn to the modeling of the panel model to assess cyclical capital requirements⁸ in banks. The capital requirement of bank *i* at time *t*, $CR_{i,t}$, can be expressed as

 $^{^8 {\}rm For}$ the remainder of the work "capital requirements" will exclusively refer to regulatory capital requirements of credit risk.

$$\mathbf{CR}_{\mathbf{i},\mathbf{t}} = f \begin{pmatrix} rr_{i,t}, \\ ee_t, \\ bank \operatorname{size}_{i,t}, \\ other \operatorname{factors}_{i,t} \end{pmatrix}.$$
 (1)

Under $rr_{i,t}$, "regulatory regime", we identify whether bank *i* is subject to Basel I or Basel II, uses the IRB approach to determine its regulatory capital requirement, etc., at time *t*, while under the term ee_t , "economic environment", we identify general financial or macroeconomic conditions at time *t*. As the latter are assumed to be identical for all banks at a given time *t*, there is no subscript *i*. In this study the focus lies on

$$E\left(\frac{\partial CR_{i,t}}{\partial ee_t} \mid rr_{i,t} = X\right).$$
(2)

E denotes the mathematical expectation parameter. Clearly, the hypothesis is that the relation between capital requirements and economic environment is subject to the regulatory regime a bank has to follow.

3.1 Data Description

In order to determine the dependence of capital requirements on economic conditions, we set up a panel model. In the next step, we present the data input needed to model Function (1). We use quarterly data from all banks active in the Austrian market between March 2000 and March 2009. To the authors' knowledge, so far there has been no attempt to answer the discussed questions with a dataset of comparable size. The number of data points available totals $26,604.^9$ The bulk of the data stems from the Austrian reporting system which obliges banks to regularly provide certain data, especially solvency related data. Consequently, information on banks' regulatory capital requirements (*CR*) and on their respective regulatory regime are available on a monthly basis. Clearly, *CR* is the dependent variable, while we use data on the regulatory regime to construct (*i*) a dummy

 $^{^{9}}$ This is less than 850 banks times 4 quarters times 9 years (i.e., 30,600), as not every banks reports non-zero numbers for the whole period. In such cases the respective data points have been eliminated.

variable equal to one if the bank reports under the Basel II regulation,¹⁰ and *(ii)* a variable which measures the share of the risk-weighted assets a banks calculates using the IRB approach.¹¹ These time series will be denoted $B2D_{i,t}$ and $IRB_{i,t}$ for the remainder of the study.

A priori, many variables would be suited to quantifying economic conditions, e.g., gross domestic product, unemployment, credit spreads, asset price indices, interest rates, to name just a few. Fortunately, we can draw on intensive literature concerning this selection process in Austria. Kalirai and Scheicher (2002) and Boss (2002) study the influence of several macroeconomic factors on provisions for credit losses or respectively on the probability of default in the Austrian financial sector. Reviewing these studies, certain factors are found to have a high explanatory power of the relevant exogenous variable in both studies.¹² Among these are asset price indices, exports, GDP, nominal short-term interest rates and industrial production.

Following these findings, we use Austrian real exports and Austrian real GDP to summarize economic conditions.¹³ Thus, EE_t refers to either exports or GDP. With respect to bank size, we use total assets, denoted $TA_{i,t}$.

3.2 Estimation

Having presented the data, we now turn to details of the model specification. As changes in economic conditions or in the size of a bank obviously affect its capital requirements in relative terms, the variables enter the model in logarithms. Furthermore, in order to capture $\partial CR_{i,t}/\partial ee_t$ conditional on the regulatory regime (see Equation (2)) dependences are modeled by including interaction terms.

¹⁰This is the case for no bank before January 2007 and for all banks after January 2008.

¹¹Therefore *IRB* equals zero for all banks not making use of the IRB approach.

 $^{^{12}}$ In the case of Kalirai and Scheicher (2002) it is the sum of write-offs and in the case of Boss (2002) sector wide average PDs.

¹³We also calculate respective estimations for nominal terms.

Hence, Equation (1) is modeled via

$$\log CR_{i,t} = \alpha_{0i} + \alpha_1 \log TA_{i,t} + \alpha_2 B2D_{i,t} + \alpha_3 IRB_{i,t} + \sum_{j=0}^{p} \beta_j \log EE_{t-j} + \sum_{j=0}^{p} \gamma_j (\log EE_{t-j} \times B2D_{i,t}) + \sum_{j=0}^{p} \eta_j (\log EE_{t-j} \times IRB_{i,t}) + u_{i,t}$$
(3)

As already stated, $TA_{i,t}$ denotes total assets and is therefore a measure of the size of the bank, $B2D_{i,t}$ a dummy variable indicating the switch to Basel II, $IRB_{i,t}$ the share of risk weighted assets calculated by IRB and EE_t either real GDP or real exports. $u_{i,t}$ is the usual error term, thus including "other factors". We include lags up to two, p := 2, in order to additionally incorporate the dependence on lagged explanatory variables.

Problematically, Equation (3) contains two issues that must be dealt with when estimated. First, individual time constant effects, α_{0i} , are unobserved and estimating them would lead to a severe reduction in degrees of freedom. Second, several variables in Equation (3) are candidates for containing unit roots, which would render an estimation inconsistent. To closer examine the matter, we apply the *panel unit root test* suggested by Hanck¹⁴ and find strong evidence for unit roots especially in the time series CR and TA, as well as in the time series for economic environment.

However, both issues can easily be dealt with by first differencing over time. This yields

$$\Delta \log CR_{i,t} = \alpha_1 \Delta \log TA_{i,t} + \alpha_2 \Delta B2D_{i,t} + \alpha_3 \Delta IRB_{i,t} + \sum_{j=0}^p \beta_j \Delta \log EE_{t-j} + \sum_{j=0}^p \gamma_j \left(\Delta \log EE_{t-j} \times B2D_{i,t}\right) + \sum_{j=0}^p \eta_j \left(\Delta \log EE_{t-j} \times IRB_{i,t}\right) + \Delta u_{i,t}.$$
(4)

Note that the individual time-constant effects have disappeared. Furthermore, we find no evidence of unit roots in the differenced time series. In our case, first differencing has additional appeal compared to fixed-effects or random effects estimation. Applying a test

¹⁴This panel test is based on the Simes' multiple test. See Hanck (2009) for details.

suggested by Wooldridge (2002, see section 10.6.3), we cannot reject the hypothesis of serial correlated errors in the model specified in levels, but find strong evidence against serial correlated errors in differences.

The parameters β_j 's measure the influence of the economic environment on capital requirements under Basel I. Under this regime, there was little or no risk sensitivity. Therefore, we expect these parameters to be indistinguishable from zero. In the subsequent sections the parameters of highest interest will be γ_j 's and η_j 's, as they measure the procyclicality of capital requirements under Basel II and IRB, respectively. A negative sign of these parameters would mean that in times of deteriorating economic conditions, capital requirements increase (on average) while the opposite would hold true for an upswing.

As the main distinctive criterion between Basel I and its successor Basel II is that the latter aims at increasing the sensitivity of capital requirements to the risk of banks' assets (Drumond 2008), one could expect the *long-run propensity* of additional cyclicality of Basel II StA, $\bar{\gamma} := \sum_{j=0}^{p} \gamma_j$, to be negative. This would indicate a more pronounced cyclical movement of capital requirements than under Basel I. However, as already stated, most literature on procyclicality focuses on IRB, not on the StA. In fact, the StA assigns risk weights to all instruments that carry credit risk. These risk weights are either fixed (if no external rating exists) or subject to a mapping process of international rating agencies, which, according to Cantor (2004) run *through the cycle* (TTC) models.¹⁵

Consulting existing literature on that matter, we find mixed results. Amato and Furfine (2003) and Catarineu-Rabell et al. (2005) find little or no cyclicality in TTC models, while Bangia et al. (2002) using migration matrices of Standard & Poors find substantial dependence of rating migrations on the business cycle. As a consequence of the mixed results, it is not clear ex ante whether the long-run propensity of Basel II StA is in fact negative. Likewise, the question concerning the long-run propensity of Basel II

¹⁵As discussed in Cantor and Mann (2003) and Fons (2002), agency ratings are stable because they are intended to measure the default risk over long investment horizons.

IRB, $\bar{\eta} := \sum_{j=0}^{p} \eta_j$, is far from clear-cut. Although the simulation studies of Gordy and Howells (2006) and Kashyap and Stein (2004) indicate a pronounced movement of capital requirements under IRB, reality could still show distinct behavior due to management actions, rating model specifications, etc. As a matter of fact, using IRB offers more flexibility in calculating risk weights and therefore the possibility to avoid increasing capital requirements. Furthermore, IRB models are also generally allowed to be TTC.¹⁶ Therefore, we must conclude that ex ante there is again no agreed opinion whether the long-run propensity of IRB will in fact be negative, indicating additional cyclicality of IRB compared to StA and Basel I.

Estimating Equation (4) provides us with the parameters γ_j and η_j , which may be used for calculating the long-run propensities of interest, $\bar{\gamma}$ and $\bar{\eta}$, as they are defined as the sum of the individual parameters. However, Equation (4) does not provide us with estimates of their uncertainty, i.e., their standard errors, as the long-run propensities are not directly estimated. Therefore, we rewrite the model specified in Equation (4) using the definitions of $\bar{\gamma}$ and $\bar{\eta}$ from above and adding $\bar{\beta} := \sum_{j=0}^{p} \beta_j$ to get

$$\Delta \log CR_{i,t} = \alpha_1 \Delta \log TA_{i,t} + \alpha_2 \Delta B2D_{i,t} + \alpha_3 \Delta IRB_{i,t} + \bar{\beta} \Delta \log EE_t + \sum_{j=1}^p \beta_j \left(\Delta \log EE_{t-j} - \Delta \log EE_t\right) + \bar{\gamma} \left(\Delta \log EE_t \times B2D_{i,t}\right) + \sum_{j=1}^p \gamma_j \left(\Delta \log EE_{t-j} \times B2D_{i,t} - \Delta \log EE_t \times B2D_{i,t}\right) + \bar{\eta} \left(\Delta \log EE_t \times IRB_{i,t}\right) + \sum_{j=1}^p \eta_j \left(\Delta \log EE_{t-j} \times IRB_{i,t} - \Delta \log EE_t \times IRB_{i,t}\right) + \Delta u_{i,t}$$
(5)

This way, we can calculate usual standard errors of the long-run propensities, as they are directly estimated.

¹⁶As the IRB banks in our sample do not differ in the degree of *through the cycle* vs *point in time*, we can not make a distinction here. Generally, the models are said to be neither clear TTC nor PIT, but rather a mixed approach.

4 Results

In this section we turn to the results of the estimation of Equation (5). We present the estimates in Table II.A and II.B with White's robust estimates of standard errors and respective p-values. Table II.A shows the outcome using real exports to indicate economic environment while Table II.B uses real GDP. The corresponding tables for nominal exports and GDP can be found in the Appendix.

The main focus of our interest lies in the parameter $\bar{\beta}$, $\bar{\gamma}$ and $\bar{\eta}$ representing the cyclical effects of Basel I, Basel II StA and IRB. A negative sign of these coefficients indicates cyclicality, meaning that once economic conditions worsen and exports or GDP move down, capital requirements go up and vice versa. Hence, the estimates of parameter $\bar{\beta}$ indicate that there was no cyclicality under Basel I. This result is in line with expectations (see Section 3 for a discussion thereof), as Basel I had no integrated risk sensitivity. The fact that the coefficient has a positive sign may stem from banks' investing in riskier customer segments in good times. However, considering the small size of the estimate of $\bar{\beta}$, the economic importance of this effect is rather low.

More surprisingly, we find no evidence of cyclicality under Basel II StA either. Depending on the specification of the model, we find either a negative or a positive sign of the estimate of $\bar{\gamma}$. Moreover, the estimate is not significant regardless of the way in which current economic conditions are modeled. In accordance with these findings we conclude that under Basel II StA there seem to be only little or no cyclical effects.

Interestingly, the case of IRB is much different. The cyclicality of capital requirements under IRB measured by $\bar{\eta}$ is large and statistically different from zero under usual significance levels. This finding is in line with prior empirical research as in Kashyap and Stein (2004), Gordy and Howells (2006) and Repullo et al. (2009). The estimated parameter of around -1.5 indicates that a fall of exports or GDP of one percent translates

coefficients		estimates	std. error	p-values	
D I D M					
Basic Effects					
α_1	Elasticity of Total Assets	0.749280	0.051782	0.000000	
α_2	Elasticity of Basel II Introduction	-0.122964	0.013648	0.000000	
$lpha_3$	Effect of IRB Implementation	-0.209455	0.210095	0.318797	
Long Run Business Cycle Elasticities:					
$ar{eta}$	underlying - Basel I	0.052185	0.015655	0.000859	
$ar{\gamma}$	(additional) of Basel II	-0.024417	0.157141	0.876522	
$ar\eta$	(additional) of IRB	-1.669019	0.279067	0.000000	

Table II.A: Estimation results using real exports to indicate current economic environment.

on average in an 1.5% increase in regulatory capital requirements of IRB portfolios.

The reported parameters of introduction of Basel II and IRB, α_2 and α_3 , as well as the elasticity of total assets, α_1 , are as expected. The introduction of Basel II and IRB lower regulatory capital requirements of credit risk, while total assets clearly have an increasing effect.

The inclusion of GDP or exports to represent current economic conditions shows (see Table II.A and II.B) that most parameters are robust to this question. Additionally, regarding the use of nominal terms instead of real terms (see Table II.C and II.D) we find that the estimates are in line with the one derived using real variables.

5 Conclusions

Building on these results, we conclude that cyclicality of capital requirements is a major issue for IRB banks but appears to be less important for StA banks. However, one should bear in mind that the cyclical behavior of capital requirements — as analyzed in this study — is *after* management action. Therefore, possible cyclical movement of capital requirements under Basel II StA might trigger counter measures on the side of the management that might not show up in the regression.

coefficients		estimates	std. error	p-values
Basic Effects				
α_1	Elasticity of Total Assets	0.751603	0.050840	0.000000
α_2	Elasticity of Basel II Introduction	-0.136265	0.011876	0.000000
$lpha_3$	Effect of IRB Implementation	-0.275248	0.203722	0.176676
Long Run Busi	iness Cycle Elasticities:			
$\bar{\beta}$	underlying - Basel I	0.020535	0.005983	0.000599
$ar{\gamma}$	(additional) of Basel II	0.121632	0.103403	0.239490
$ar\eta$	(additional) of IRB	-1.572507	0.197363	0.000000

Table II.B: Estimation results using real GDP to indicate current economic environment.

As the cyclicality of capital requirements is the basis for potential procyclicality, it is important to distinguish between IRB and StA in policy analysis. Numerous suggestions for adequate measures to address procyclicality have been made in the respective literature (see Drumond (2008) section 4.3 for an overview). Although the discussion of these proposals would go beyond the scope of this text, our empirical study provides a quantitative foundation for the ongoing discussion. For the next step in the procyclicality discussion, further research on the empirical influence of cyclicality requirements on future lending activities and economic growth is necessary.

Appendix A

coefficients		estimates	std. error	p-values		
Basic Effects						
α_1	Elasticity of Total Assets	0.748007	0.051674	0.000000		
$lpha_2$	Elasticity of Basel II Introduction	-0.130055	0.012441	0.000000		
$lpha_3$	Effect of IRB Implementation	-0.230273	0.207561	0.267259		
Long Run Business Cycle Elasticities:						
$ar{eta}$	underlying - Basel I	0.170104	0.045620	0.000193		
$ar{\gamma}$	(additional) of Basel II	0.166658	0.271407	0.539185		
$\bar{\eta}$	(additional) of IRB	-2.591295	0.395711	0.000000		

Table II.C: Estimation results using nominal exports to indicate current economic environment.

coefficients		estimates	std. error	p-values			
Dagia Effects							
Dasic Ellects							
α_1	Elasticity of Total Assets	0.749526	0.051911	0.000000			
$lpha_2$	Elasticity of Basel II Introduction	-0.122294	0.013137	0.000000			
$lpha_3$	Effect of IRB Implementation	-0.220497	0.211959	0.298220			
Long Run Busi	Long Run Business Cycle Elasticities:						
$ar{eta}$	underlying - Basel I	0.050780	0.018181	0.005226			
$ar{\gamma}$	(additional) of Basel II	-0.037176	0.069250	0.591381			
$ar\eta$	(additional) of IRB	-2.369288	0.321060	0.000000			

Table II.D: Estimation results using nominal GDP to indicate current economic environment.

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Chapter III

Regulatory Medicine Against Financial Market Instability: What Helps And What Hurts?

Abstract

Do we know if a short selling ban or a Tobin Tax results in more stable asset prices? Or do they in fact make things worse? Just like medicine regulatory measures in financial markets aim at improving an already complex system, cause side effects and interplay with other measures. In this paper an agent based stock market model is built that tries to find answers to the questions above. In a stepwise procedure regulatory measures are introduced and their implications on market liquidity and stability examined. Particularly, the effects of (i) a ban on short selling (ii) a mandatory risk limit, i.e., a Value-at-Risk limit, (iii) an introduction of a Tobin Tax, i.e., a transaction tax on trading, and (iv) any arbitrary combination of the measures are observed and discussed. The model is set up to incorporate non-linear feedback effects of leverage and liquidity constraints leading to fire sales. In its unregulated version the model outcome is capable of reproducing stylized facts of asset returns like fat tails and clustered volatility. Introducing regulatory measures shows that only a mandatory risk limit is beneficial from every perspective, while a short selling ban — though reducing volatility — increases tail risk. The contrary holds true for a Tobin Tax: it reduces the occurrence of crashes but increases volatility. Furthermore, the interplay of measures is not negligible: measures block each other and a well-chosen combination can mitigate unforeseen side effects. Concerning the Tobin Tax the findings indicate that an overdose can do severe harm.

Keywords: Tobin Tax, transaction tax, short selling ban, Value-at-Risk limits, risk management herding, agent based models.

JEL Classification: E37, G01, G12, G14, G18.

1 Motivation

"It's remarkable that while any new technical device or medical drug has extensive testing for efficiency, reliability and safety before it ever hits the market, we still implement new economic measures without any prior testing."

— Dirk Helbing, Swiss Federal Institute of Technology Zürich in Buchanan (2009).

The financial crisis spurred the discussion about further regulations in asset markets. However, the consequences of imposing a transaction tax, a short selling ban or mandatory risk limits are unknown to a large extent. "Prior testing" is both: hardly feasible and absolutely necessary. Hardly feasible, because the large sums of money handled, the interconnectedness of actions etc. do not allow for lab experiments. Only within newly emerged agent based models this task seems doable. At the same time, prior testing is absolutely necessary. Like the human body the financial market is an enormously complex organism and like medicine regulatory measures aim at improving it. Side effects or unforeseen interactions of measures require prior testing, as the quotation above demands. This paper provides evidence if indeed imposing regulatory measures makes markets more stable. To do so, an agent based model framework is set up, which shares basic ideas of Thurner et al. (2009). Subsequently, this baseline model is modified by the introduction of regulatory measures.

Why Agent Based Models?

The dynamics of financial markets pose a challenge to research just as they pose a threat to financial stability. Typically, asset returns are characterized by so called stylized facts including fat tails and clustered volatility (see Cont 2001). Classic economic theory fails to predict such behavior. Recent literature, however, has shown that by the incorporation of leverage, fire sales, escape dynamics and liquidity constraints stylized facts occur (Friedman and Abraham 2009). The financial crisis confirmed the importance of taking such effects into account. However, modeling interaction effects in an analytical framework soon becomes intractable. Therefore, the dramatic increase of computational power over the past decades gave rise to agent based models. Within such models one is allowed to move away from the classical modeling approach featuring the representative agent but to model the action of each and every actor, thus integrating non-linear feedback dynamics.¹

While typical agent based models feature heterogeneous agents who dynamically optimize seemingly irrational strategies², Thurner et al. (2009) recently showed that even under the assumption of relatively rational value-investors fat tails occur when feedback effects of leverage are incorporated. The baseline model introduced in Section 2 draws on Thurner et al. (2009). It models leveraged agents who trade a single asset according to a mispricing signal. In its unregulated version the model reproduces fat tails and clustered volatility.

Regulatory Measures

The focus is then shifted to the question of interest, the impact of regulatory measures. The financial crisis has amplified voices demanding a stronger regulatory framework of asset markets. Among the cloud of demands the following are picked for closer examination:

¹Numerous papers give witness to the popularity agent based models gained over the past two decades, see LeBaron (2001) and LeBaron (2006) for extended literature discussions.

²Compare for instance trend followers in models of Lux (1998) and De Grauwe and Grimaldi (2006) or the Minority Game literature, e.g., Challet et al. (2001) and Satinover and Sornette (2007).

- (i) a ban of short selling,
- (ii) a mandatory risk limit and
- (*iii*) a Tobin Tax, i.e., transaction tax on trading³.

In each of the three cases a high level of uncertainty concerning the consequences of an introduction prevents a fact-led discussion. This is probably best seen by reading the following two citations of U.S. Securities and Exchange Commission Chairman Christopher Cox. The first quote was made at the time of introduction of the short selling ban in US-stock markets in September 2008 (New York Times 2008) and the second only three months later in December 2008 (Reuters 2008):

"The emergency order temporarily banning short selling of financial stocks will restore equilibrium to markets."

"While the actual effects of this temporary action will not be fully understood for many more months, if not years, knowing what we know now, I believe on balance the commission would not do it again. ... The costs appear to outweigh the benefits."

In their empirical study Marsh and Niemer (2008) find "no strong evidence that (short selling bans) have been effective in reducing share price volatility or limiting share price falls." Further studies based on observed data (e.g., Lobanova et al. 2010, Boehmer et al. 2009 and Beber and Pagano 2009) find rather negative effects of short selling restrictions on market liquidity and increasing effects on volatility.

Similarly, the adoption of a Tobin Tax, i.e., transaction tax on trading, has led to controversy within the field of academics as well as within politics. Originally proposed by Tobin (1978), the tax now enjoys great popularity as a potential means to reduce market

³While the Tobin Tax was originally suggested only for foreign exchange rate markets, the term is now regularly applied to mean a tax on financial transactions in general. This paper uses the terms transaction tax and Tobin Tax synonymously.

volatility and as source for tax revenues. In fact, merely naming supporters and opposers of the tax would be way out of the scope of this paper. However, a clear reflection of the popularity can be grasped by the length of the respective article in Wikipedia (2010), which also provides a comprehensive list of the numerous supporters and opposers in politics. In the academic world, studies come to mixed conclusions. While a negative effect on trading volume is generally agreed upon, the impact on price volatility is less clear cut and even contrary, leading Hanke et al. (2010) to infer that "in sum, the literature on the effects of a Tobin tax on market efficiency arrives at opposite ends. ... there is no general agreement on the consequences of a Tobin tax on price volatility." While some argue that a transaction tax reduces the trading of rather uninformed actors, thus leading to more efficient and less volatile markets, others argue that a transaction tax prevents flexible price adjustment to new information and therefore rather leads to price jumps and higher volatility (see also the debate in Hanke et al. 2010). Westerhoff (2003), Ehrenstein et al. (2005), Pellizzari and Westerhoff (2009) and Mannaro et al. (2008) study the imposition of a transaction tax within the framework of agent based models. While the latter conclude that volatility rises with the imposition of a Tobin Tax, the other papers find that the effects depend on the liquidity of a market and on the magnitude of the tax.

The third regulatory measure, titled mandatory risk limits, may seem less debated, but is in fact already in place for many of the larger market participants like banks via the Basel regime. In such a regime, agents are obliged to quantify their risk and relate this risk to their own funds, thus keeping their theoretical default probability below a certain threshold. Insurers and hedge funds are as well required to run risk managements techniques – a regulation that is currently intensified. Irrespective of the regulatory framework, risk quantification and risk limiting has become a general practice among the major market participants, i.e., funds and banks (see e.g., chapter 1.1 in McNeil et al. 2005). While a sound risk management is without doubt for the benefit of the single institution, its consequences for systemic risks are ambiguous. To see this, imagine an agent close to its risk limit when stocks decline. The decline not only shrinks her own funds but may also increase the risk quantified for the same position. This may in turn lead to fire sales, thus amplifying the initial shock. Such phenomena combined with strategy herding could potentially lead to severe downturn momentum.⁴

This paper not only discusses the implications of the three regulatory measures, but further provides evidence on their potential interplay. While the interplay of drugs and their side effects is a pervasive topic in medical research, it is much less debated in the context of financial markets. Thus, this paper aims at giving answers to the interplay of the regulatory measures.

To conclude, the contribution of the paper is threefold. Firstly, while the effects of a short selling ban and of a transaction tax have already been studied, there remains a level of uncertainty that requires further research. Furthermore, the effects of risk limits — though beneficial on the individual level — may have negative side effects, which seem to have been neglected in the scientific discussion. Secondly, existing literature approaches the questions usually either from an empirical view using observed data or a reduced form theoretical model, but not within the framework of an agent based model⁵. Thirdly, to the author's knowledge this paper is first in examining the combination of these regulatory measures.

The remainder of the paper is structured as follows. Section 2 introduces the baseline model with no regulations in place. Subsequently, Section 3 presents adjustments due to the regulatory measures. Finally, Section 4 discusses the results and outlines potential shortcomings of the approach, thus suggesting ways of further research, while Section 5 concludes.

⁴Strategy herding is in fact a major driver for market crashes in agent based Minority Games. See e.g., the work of Satinover and Sornette (2007).

⁵Note the exceptions concerning the Tobin Tax: Westerhoff (2003), Ehrenstein et al. (2005), Pellizzari and Westerhoff (2009) and Mannaro et al. (2008).

2 The Baseline Model

This section describes the baseline model representing the unrestricted market. The description starts at the most general level and successively works downwards explaining the model in more detail.

At the top level there is the market clearing equation defining that at each timestep t total demand, as sum over the N^a individual demands $D_{i,t}$, must equal the total number of shares N^s , therefore ensuring that supply meets demand and the market clears.

$$\sum_{i=1}^{N^{a}} D_{i,t}(p_{t}) = N^{s}$$
(1)

As described below, demand of each single agent is a function of price p_t among others. By solving Equation (1) one obtains the price.⁶ At each timestep agents choose the fraction of their total wealth $W_{i,t}$ to be invested in cash $C_{i,t}$ and in shares, therefore

$$W_{i,t} = C_{i,t} + p_t D_{i,t}(p_t).$$
(2)

Before turning to the demand equations, note that when $D_{i,t} < 0$ agents take a short position and when $D_{i,t} > 0$ they are long. To fund their actions agents can leverage themselves up to a maximum leverage of λ^{max} .⁷ As long as leverage is not at its maximum, agents' demand is a linear function of the perceived mispricing signal. This mispricing signal is the difference between the current price and the perceived fundamental value, thus $m_{i,t} := p_{i,t}^{perc} - p_t$. This leads to the demand functions:

⁶This set–up is more sophisticated than the one used by usual agent based models, in which price is a (linear) function of "excess demand", which implies a linear response to market movements (e.g., Friedman and Abraham 2009). It comes, however, at the cost of more complex computational demands.

⁷According to modern standard, leverage is defined as the asset side (of the balance sheet) divided by own funds, therefore $\lambda_{i,t}^{long} := p_t D_{i,t}(p_t)/W_{i,t}$ and $\lambda_{i,t}^{short} := (W_{i,t} - p_t D_{i,t}(p_t))/W_{i,t}$.

$$D_{i,t} = \begin{cases} (1 - \lambda^{max}) W_{i,t}/p_t & \text{if } m_{i,t} < m_{i,t}^{crit,short} \\ \lambda^{max} W_{i,t}/p_t & \text{if } m_{i,t} > m_{i,t}^{crit,long} \\ \beta_i m_{i,t} W_{i,t}/p_t & \text{otherwise,} \end{cases}$$
(3)

where β_i represents a parameter denoting the aggressiveness of the agent, that is how fast he reacts to price signals and $m_{i,t}^{crit}$ the mispricing signal which would lead to the use of the maximum leverage. Thus, $m_{i,t}^{crit,short} = (1 - \lambda^{max})/\beta_i$ if agent *i* is in a short position and $m_{i,t}^{crit,long} = \lambda^{max}/\beta_i$ if she is long. While the first two lines of Equation (3) simply limit the demand to its maximum leverage, the third specifies demand in the unbounded case as a linear function of the mispricing signal and the aggressiveness of the agent, β_i .⁸ Price and wealth in Equation (3) ensures that at a given mispricing signal two equally aggressive agents will invest the same fraction of their wealth.⁹

Until now, the perceived fundamental value of the share, $p_{i,t}^{perc}$, was left unspecified. In this model, agents' perceptions follows a discrete Ornstein-Uhlenbeck process that guarantees the perceived values to wander around but mean revert to the fundamental value.

$$\log p_{i,t}^{perc} = \rho \, \log p_{i,t-1}^{perc} + (1-\rho) \, \log V + \epsilon_{i,t}, \tag{4}$$

where V denotes the *true* fundamental value, $\epsilon \sim N(0, \Sigma)$ and $0 < \rho < 1$. In order to mirror market wide misjudgment and herding ϵ correlates across agents. Finally, in each round t before each market participant i computes his demand according to Equation (3) and the price p_t is derived according Equation (1)¹⁰ the wealth $W_{i,t}$ is updated according to

$$W_{i,t} = W_{i,t-1} + D_{i,t-1} (p_t - p_{t-1}).$$
(5)

⁸In fact, the underlying utility function would be (subscripts omitted): $U(D,C) = D^{\beta m} C^{1-\beta}$. See also Thurner et al. (2009).

⁹As in practice only a fraction of agents actually take short positions, for simulation purpose define τ as the fraction of agents who avoid taking short positions even in the baseline model.

¹⁰Note that Equation (3) and Equation (1) have to be solved simultaneously as both depend on the other.

In line with Thurner et al. (2009), agents default if their wealth, $W_{i,t}$, decreases below 10% of their initial wealth and are reintroduced after 100 timesteps. This fact is primarily necessary in order to avoid having agents with diminishingly little wealth — and therefore no relevance — in the market. In reality, agents' defaults would impact other agents if their holdings would not be pure assets but bets on the long or short side. The consideration of this mechanic in the model would, however, require a matching algorithm between agents as well as an algorithm to determine if the defaulting agent holds pure assets or derivatives. Both requirements would increase model complexity even more. Additionally, the size of the effect can be considered as small, as the results (see Section 4) show only a limited number of defaults (i.e., wealth sinks below 10% of initial wealth) and even a smaller number of negative wealth. Consequently, we assume for simplicity that agents' defaults do not impact other agents.

With the model specifications outlined, we are ready to run the model in its unregulated version. Figure III.a displays the implied characteristics of the returns calibrated according to Table III.C (see the Appendix, page 61). While plot a and b display excess kurtosis present in the implied time series of returns as well as a gain/loss asymmetry, plot c and d provide evidence on the absence of autocorrelation among returns but non-zero autocorrelation among squared returns, i.e., clustered volatility is present (see Cont 2001). The emergence of these characteristics is endogenous considering the normal iid distribution of $\vec{\epsilon_t}$ in Equation (4).

So, the patient shows symptoms, but which medicine help?

3 Short selling ban, risk limits and transaction tax

Having specified the unregulated model, this section presents the amendments for each regulatory measure in sequence.



Figure III.a: Statistical Properties of Returns: (a) display of kernel density estimates of the returns compared to the normal distribution with respective standard deviation, (b) Q-Q plot of returns against normal and student t distributions with 7 df, (c) plot of a typical draw of returns, (d) autocorrelation function for returns and squared returns with 95% confidence intervals.

Short selling ban

In the unregulated market, agents' demand of shares can be positive or negative alike. In the latter case, the agent goes short. To implement a short selling ban the demand Equation (3) has to be adjusted to cap the demand at zero.

$$D_{i,t} = \begin{cases} 0 & \text{if } m_{i,t} \leq 0\\ \lambda^{max} W_{i,t}/p_t & \text{if } m_{i,t} > m_{i,t}^{crit,long}\\ \beta_i m_{i,t} W_{i,t}/p_t & \text{otherwise.} \end{cases}$$
(6)

Note that in comparison to Equation (3) only the first line changed.

Value-at-Risk limits

As discussed in the introduction, Value-at-Risk is now a widely applied concept in risk management. Hereby, one quantifies the risk of a given position according to a quantile of the estimated loss distribution. While different methods are applied in practice, this paper sticks to the popular and straight forward variant called variance-covariance approach. Hence, in a first step market participants calculate their individual Value-at-Risk for holding one unit of the asset at each timestep:

$$VaR_{i,t} = \mu_{i,t} - \alpha * \sigma_{i,t}, \tag{7}$$

where $\mu_{i,t}$ and $\sigma_{i,t}$ are empirical estimates of mean and standard deviation of asset returns. More precisely, agents compute $\mu_{i,t}$ and $\sigma_{i,t}$ out of past observations of the (endogenous) time series of returns. Note the agent specific subscript indicating that agents use different look-back periods for calculations (see the Appendix, page 61, for details on the calibration). Furthermore, in line with the assumptions of the variance-covariance approach $\alpha = \Phi^{-1}(0.99)$ represents the 99%-quantile of the normal distribution in Equation (7).

In a second step, the VaR determined by Equation (7) feeds into the demand function.

To adjust the demand equation of the baseline model, define $m_{i,t}^{crit,var} := (\beta_i \operatorname{VaR}_{i,t})^{-1}$ as the critical mispricing signal, at which the unbounded demand would be higher than the maximum Value-at-Risk.¹¹ Consequently, Equation (3) changes to

$$D_{i,t} = \begin{cases} (1 - \lambda^{max}) W_{i,t}/p_t & \text{if } m_{i,t} < m_{i,t}^{crit,short} \\ -W_{i,t}/(p_t \operatorname{VaR}_{i,t}) & \text{if } m_{i,t} < -m_{i,t}^{crit,var} \\ \lambda^{max} W_{i,t}/p_t & \text{if } m_{i,t} > m_{i,t}^{crit,long} \\ W_{i,t}/(p_t \operatorname{VaR}_{i,t}) & \text{if } m_{i,t} > m_{i,t}^{crit,var} \\ \beta_i m_{i,t} W_{i,t}/p_t & \text{otherwise,} \end{cases}$$
(8)

while in case more than one restriction hits, the one that satisfies $\min(|D_{i,t}|)$ is in effect. Comparing the baseline model of Equation (3) with Equation (8) above one finds that simply two new lines have emerged holding the implied risk in check. Thus, agents subjected to a Value-at-Risk limit are not only bound by the maximum leverage constraint but by a maximum portfolio Value-at-Risk as well, which aims at reducing the default probability of agents below a certain threshold. Consequently, it may be that agents have to unwind part of their position, i.e., decrease their demand from t to t + 1, solely due to an increase in the estimated volatility, $\sigma_{i,t}$.

The simultaneous reign of a short selling ban and a Value-at-Risk limit would bind the demand to zero if $m_{i,t} \leq 0$, while the remainder of Equation (8) would hold.

Transaction Tax

In its core, a transaction tax reduces the expected return of an investment by $(twice)^{12}$ the tax level applied. Agents will therefore require a higher expected payoff for the same level of investment, i.e., their demand in the asset. Reviewing the unrestricted model yields that

 $^{^{11}{\}rm The}$ deviation is simple, considering that the Value-at-Risk concept limits the portfolio VaR, $D_{i,t}$ \cdot p_t \cdot $\operatorname{VaR}_{i,t}$, to equal $W_{i,t}$ at maximum. ¹²The tax is applied at buying and selling the asset.

the average realized pay-off of an investment is a positive function of the absolute value of the mispricing signal. Clearly, the larger the mispricing signal, the higher the certainty that the asset is over- or underpriced. Consequently, in a Tobin Tax environment agents will keep their current demand unchanged, if they are subjected to only a minor mispricing signal, or in other words they will only change their demand if the mispricing signal is strong enough, so that expected pay-offs of the trade are positive. Hence, under the regime of a transaction tax, agents compute their demand according to

$$D_{i,t} = \begin{cases} D_{i,t-1} & \text{if } |D_{i,t}^* - D_{i,t-1}| < \Gamma \\ (1 - \lambda^{max}) W_{i,t}/p_t & \text{if } m_{i,t} < m_{i,t}^{crit,short} \\ \lambda^{max} W_{i,t}/p_t & \text{if } m_{i,t} > m_{i,t}^{crit,long} \\ \beta_i m_{i,t} W_{i,t}/p_t & \text{otherwise,} \end{cases}$$
(9)

where Γ is a threshold for the mispricing signal, and $D_{i,t}^*$ is the demand that would result without incorporation of the Tobin Tax, i.e., without the first line of Equation (9). Note that only this first line is new and in case more than one restriction hits, the one that satisfies min($|D_{i,t}|$) is in effect. This ensures that changes in demand due to shifts in the mispricing signal, $m_{i,t}$, require a defined magnitude. In the simulation the threshold is chosen in order to keep (empirically determined) expected returns of trading positive.

Combination of Regulatory Measures

For any arbitrary combination of the three regulatory measures one has to merge the formulas from above. As in each case restrictions are added to the original demand, merging them is straight forward. For instance, under a short selling ban and a Value-at-Risk limit demand is limited to zero or positive values, while at the upper bound the leverage and Value-at-Risk limit bind demand against becoming excessive. As the final set of equation for each combination of regulatory measures is somewhat lengthy, their display is omitted here.

4 Results

This section turns now to the simulated impacts of the regulatory measures on (i) market liquidity, (ii) market volatility, (iii) market stability, i.e., the risk of tail events, and (iv) probability of default of an agent. To measure market liquidity, first define market volume by

$$volume := \frac{1}{(N^t - 1)} \sum_{t=2}^{N^t} \sum_{i=1}^{N^a} |D_{i,t} - D_{i,t-1}|, \qquad (10)$$

i.e., the average amount of shares traded per timestep. N^t denotes the number of timesteps in one simulation run. In line with Amihud (2002) and references therein we define market illiquidity as the ratio of market volatility to market volume. The intuition behind this measure is that it quantifies how sensible prices react to a single unit of the asset changing hands and thus how liquid a market is. To measure market volatility the standard deviation of returns is evaluated and for market stability its respective (excess) kurtosis, as a measure for extreme shifts in the price.¹³ Finally, the number of defaults of each run is evaluated.

As a first result, Figure III.b visualises the resulting distribution of the relevant metrics. On the x-axis the respective regulatory regime is coded: zero represents a regulation not being in effect and one indicates it is. The first number corresponds to the short selling ban, the second to a transaction tax and the third to the Value-at-Risk limit (VaR limit). On the y-axis the respective measures of liquidity and stability are shown. Studying Figure III.b one can see that market illiquidity shows strong dependence on both short selling restrictions and the Tobin Tax. Not only does market illiquidity rise, also its volatility across runs is affected. The volatility of returns depicted in Figure III.b seems to be mitigated by a short selling ban, while also a mandatory VaR limit contributes. Concerning the kurtosis, there is less clear cut evidence. Its distributions are strongly skewed to the

 $^{^{13}}$ As large upswings do not pose a threat to financial stability, the kurtosis is evaluated from the distribution in which negative returns are flipped at zero and positive ones are ignored. It turned out that none of the results depend on the flipping and the new measure is correlated by more than 0.89 with the standard kurtosis.

right – in fact, a few points lie far in the extreme tail, even overreaching 50. The number of defaults is obviously affected by a short selling ban, which reduces their numbers strongly.

To assess the effects of regulatory measures more closely, illiquidity, standard deviation, kurtosis and number of defaults are regressed on the exogenous dummy variables short selling ban, VaR limit, Tobin Tax and respective interaction terms indicating the regulatory measure to be in effect or not. As indicated by Figure III.b – especially the illiquidity plot -, heteroskedasticity is an issue. Hence, the regression was conducted using feasible GLS¹⁴. Table III.A displays the regression results with stars indicating statistical significance. With an adjusted R squared of 0.94 the liquidity model manages to explain a relatively high fraction of the endogenous variance. Note that all coefficients of the liquidity regression were multiplied by 100 to enhance readability. Interestingly, all regulatory measures reduce market liquidity, i.e., increase market illiquidity. The largest reduction in market liquidity stems from an introduction of a short selling ban, while the introduction of a Tobin Tax ranks second. According to the GLS coefficients, with a transaction tax of $0.3\%^{15}$ enabled a single stock traded causes a higher impact on prices, measured in the standard deviation of returns by 0.00363.¹⁶ However, also surprising is the fact that the combined introduction of a transaction tax and a short selling ban reduces liquidity additionally to the individual liquidity effects.

As anticipated from Figure III.b, both, a short selling ban and a VaR limit, temper market movements by reducing market volatility. The fact that VaR reduces market volatility is an interesting finding since — as outlined in the introduction — VaR might also have downside effects, as recently emphasized by e.g., Adrian and Shin (2008). On the one

¹⁴Alternatively, one can look at the quantiles of the resulting distributions. Find the respective quantile regression results in the Appendix, Table III.D, page 62. In short, the picture modeling the median is very similar both in terms of magnitude and significance of the parameters compared to the feasible GLS regression results displayed in this section.

¹⁵See Table III.C on page 61 in the Appendix for the calibration used. Within the regression model all exogenous variables are dummy variables.

 $^{^{16}}$ More easily interpretable is probably the respective coefficient for market volume (regression not depicted here), which shows that a transaction tax reduces the average amount of traded assets of a single agent by 0.13 per timestep.



Standard Deviation of Returns

Figure III.b: Distributions of market characteristics under different regulatory regimes. The x-axis codes the regulatory measures being in effect or not by assigning 1 or 0.

hand a VaR forces agents to unwind exposures potentially triggering market turbulence. On the other hand, however, in a different market environment it helps mitigating market turbulence as it prevents traders to take excessive risk positions in the first place. In this set up the regression coefficient in Table III.A tells us that the latter effect is obviously more dominant. Nonetheless, a closer look into the results reveals that the first increasing effect on volatility is also present in our model: in simulation runs with the same random seed, the one in which agents were subject to a VaR limit yielded in (only) 70%¹⁷ of the runs a lower volatility. In the other 30% the measure led to higher market volatility.

More surprising is probably the fact that a Tobin Tax *increases* market volatility by a statistically significant amount. In line with volatility, obligatory VaR limits remedies huge swings in markets, as can be seen in the column of kurtosis. Likewise, tail events occur more seldom when a transaction tax is introduced. Both effects are statistically and economically significant. By contrast, a short selling ban positively influences the probability of market crashes, via a prior build up of market bubbles. The fact that a short selling ban reduces volatility while increasing the likelihood of tail events emerges due to the absence of critical investors. Bubbles are nurtured in a calm environment of low volatility, which leads to crashes when resolved. Put in medical terms: a short selling ban seems to suppress the immune system. To visualize this interesting finding,

¹⁷ The	se figures	are totall	y, i.e., ove	er all regu	ilatory reg	gimes. C	Considerin	g no d	other re	gulation	the fig	ure is
68%, on	ly a shor	t selling b	an being	active 70	0%, only a	a transa	ction tax	being	active	74% and	l both	being
active 7	3%.											

illiqui	dity	volatil	ity (sd)	kurto	osis	defaul	\mathbf{lts}
2.169	***	2.677	***	2.16	***	310.935	***
0.007	***	-0.01	***	-0.186	***	-9.745	***
0.373	***	-0.046	***	0.344	***	-84.836	***
0.363	***	0.022	***	-0.4	***	-3.225	***
-0.003		0		-0.08		3.883	***
0.005		-0.003		-0.008		-1.03	
0.055	***	0.005	**	-0.165	**	5.635	***
0		0.001		0.06		0.161	
0.936		0.157		0.031		0.773	
	illiquie 2.169 0.007 0.373 0.363 -0.003 0.005 0.055 0 0 0.936	illiquidity 2.169 *** 0.007 *** 0.373 *** 0.363 *** -0.003 . 0.005 . 0.055 *** 0.9366 -	illiquidity volatil 2.169 *** 2.677 0.007 *** -0.01 0.373 *** -0.046 0.363 *** 0.022 -0.003 0 0 0.005 . -0.003 0 0.055 *** 0.055 *** 0.005 0 0.001 0.936 0.157	illiquidity volatility (sd) 2.169 *** 2.677 *** 0.007 *** -0.01 *** 0.373 *** -0.046 *** 0.363 *** 0.022 *** -0.003 0 -0.003 0 0.005 -0.003 . 0 0.055 *** 0.005 ** 0 0.001 -0.003 .	illiquidityvolatility (sd)kurto 2.169 *** 2.677 *** 2.16 0.007 *** -0.01 *** -0.186 0.373 *** -0.046 *** 0.344 0.363 *** 0.022 *** -0.4 -0.003 0 -0.08 0.005 . -0.003 . 0.055 *** 0.005 ** 0 0.001 0.066 0.936 0.157 0.031	illiquidityvolatility (sd)kurtosis 2.169 *** 2.677 *** 2.16 *** 0.007 *** -0.01 *** -0.186 *** 0.373 *** -0.046 *** 0.344 *** 0.363 *** 0.022 *** -0.4 *** -0.003 0 -0.08 -0.008 0.005 . 0.005 . -0.003 . -0.165 ** 0 0.001 0.06 0.031	illiquidityvolatility (sd)kurtosisdefau2.169*** 2.677 *** 2.16 *** 310.935 0.007 *** -0.01 *** -0.186 *** -9.745 0.373 *** -0.046 *** 0.344 *** -84.836 0.363 *** 0.022 *** -0.4 *** -3.225 -0.003 0 -0.08 3.883 0.005 . -0.003 . -0.008 -1.03 0.055 *** 0.005 ** -0.165 ** 5.635 0 0.001 0.06 0.161 0.773

Significance Codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1

Table III.A: Results of feasible GLS regression.

Figure III.c displays a typical asset price movement under three regimes. At first, there is hardly any difference in the price level and volatility is low. At a certain point prices start to increase steeply. This is also the point where they start to move away across regimes. While in the VaR regime and even in the unrestricted model price rises are more modest, with short selling prohibited the price rises extraordinarily quickly. The following fall comes certain and costs the default of four agents. Consequent wealth effects cause a lower average price level in the following periods compared to the other regimes. However, regarding the respective GLS coefficients of the interaction terms in Table III.A, this dynamic is mitigated, when a short selling ban is combined with a Tobin Tax.

The number of defaults is negatively associated with all of the regulatory measures. By far the strongest reduction comes from a short selling ban. This is easily explainable: with short selling present in the market, the level of risk is much higher, as agents in a short position would otherwise have no exposure. Additionally, they require a counterpart for their position. Hence, the level of risk across the system is higher, therefore leading to substantially more defaults.

While the result that a Tobin Tax increases volatility and reduces tail risk is interesting in itself, one might be interested in how this conclusion changes when the tax level varies (i.e., deviates from its standard value of 0.003). Indeed, e.g., Westerhoff (2003) finds a dependence of the results on the level of the tax. Consequently, simulations with a Tobin Tax of 0.1%, up to a level of 5% were run. Table III.B and Figure III.d display the results. While there is only a modest increase in volatility noticeable up to a level of 1%, volatility increases substantially above 1%. At 5% average market volatility outreaches 10%, a substantial increase from its initial value. At the same time, market volume is constantly reduced. As already noted above, a Tobin Tax has a mitigating effect on tail risk in the model applied. This can also be seen in Table III.B, where the kurtosis of returns is reduced by a Tobin Tax. However, there is — indeed — a certain threshold, at which the medicine is overdosed: At a level of 2% the av-



Figure III.c: Price dynamics display higher kurtosis under a short selling ban.

erage kurtosis overreaches 20 and market symptoms worsen at any higher level of the tax.¹⁸

Level of								
Tobin Tax	0	0.001	0.003	0.005	0.01	0.02	0.03	0.05
Volatility (sd)								
mean	0.027	0.027	0.027	0.027	0.029	0.039	0.061	0.118
median	0.027	0.027	0.027	0.027	0.029	0.031	0.036	0.046
75%-quantile	0.027	0.027	0.027	0.028	0.029	0.032	0.085	0.176
Kurtosis								
mean	2.700	2.078	2.197	2.766	2.377	27.504	67.261	28.912
median	1.847	1.746	1.392	1.023	0.912	1.933	6.237	6.364
75%-quantile	2.836	2.707	2.207	1.638	1.151	2.297	>100	>100

Table III.B: Statistics of returns under different levels of Tobin Tax.

What can we now learn from the results? First, the results show that in the chosen setting a mandatory risk limit is the only measure that is beneficial from all perspectives¹⁹. A ban on short selling reduces market volatility, but comes with an increase in tail risk. On the other hand, a Tobin Tax reduces the occurrence of bubbles while at the same time makes markets more volatile. However, when increased over a certain thresholds results turn and

¹⁸For a display of results across different regulatory regimes see Table III.E in the Appendix, page 63.

¹⁹Liquidity is omitted here.



Figure III.d: Market characteristics under different levels of Tobin Tax. The values are means across runs.

a Tobin Tax clearly contributes to market instability from all perspectives.

Second, the interplay of measures does play a role in judging on the regulatory medicine to be prescribed. When a mandatory risk limit or a Tobin Tax is present, a ban of short selling has significantly lower impact on tail risk than without. The column concerning the number of defaults in Table III.A also indicates that regulatory measures can block each other to some extent. While the interplay of these measures should not be left unconsidered when deciding on their implementation, there is no evidence that they turn individual effects in a different direction.

Of course, a relevant question is the robustness of the results. In fact, the model's high dimensionality of input parameters brings about the question, if the results are stable to a different calibration. To find out, we run further simulation exercises varying key input parameters, among them the number of agents and the distribution of the aggressiveness parameter, β . The details about the parameter space of the modified variables can be found in the Appendix, Table III.C, page 61 under the heading "Robustness Check". The investigation of whether the results described above hold was done by again regressing market illiquidity, volatility, kurtosis and defaults on the regulatory dummy variables and — additionally — on the input parameters varied.²⁰ The respective regression output is depicted in Table III.F in the Appendix, page 64^{21} . Summarizing, we find largely robust results: still illiquidity is positively affected by all regulatory measures, market volatility is still positively affected by a transaction tax, but negatively by a VaR or a short selling ban. Concerning the kurtosis of returns, we see that while the coefficients associated with a VaR and a short selling ban show their known sign, the transaction tax now displays a positive sign, which is not in line with the primary findings shown in Table III.A, but explained by the analysis of a transaction tax above 0.3% (see Table III.B). The number of defaults shows the same dependence to the three regulatory measures as in the benchmark model. Due to the higher number of observations, the p-values have generally decreased impacting primarily the interaction terms. These show again similar behavior, with the exception of the combined short selling ban and a transaction tax which now offsets part of the negative individual liquidity effects (decreases illiquidity). For brevity, as concerns the other regressors please refer to Table III.F.

Despite these promising results from the robustness exercise, one should bear in mind that the model setup is an abstraction and that while it provides a range of features, certain shortcomings remain: E.g., one could ask if there would be a change in results in case the risk limits would only be applied by the largest agents or if their conception would be more homogeneous across agents. Likewise, the question arises what if not the whole market would be subjected to a Tobin Tax but tax havens are present. Furthermore, with a short selling ban in place market participants might anticipate the absence of short sellers and incorporate it in their demand decisions, thus mitigating the risk of bubbles.

 $^{^{20}}$ None of the conclusions drawn below depend on the fact that the varied input parameters were included the regression.

²¹When comparing Table III.A and Table III.F, note that in the latter Tobin Tax is not coded as dummy $\{0, 1\}$ but in levels $\{0, 0.001, 0.003, 0.005, 0.01, 0.03, 0.05\}$ leading to a substantially different magnitude of the respective coefficients.

5 Conclusions

This paper introduces an artificial market where agents trade a single asset. The conception of relatively rational agents allows for a straight forward implementation of regulatory measures. These are a short selling ban, a Tobin Tax, a mandatory Value-at-Risk limit and any arbitrary combination of these. In its unregulated version, the model is capable of reproducing stylized facts of financial markets, most notably fat tails and clustered volatility.

Introducing regulatory measures constitutes an intervention into a complex system, whose consequences, side effects and joint interplay are ex ante unclear. The results described in Section 4 constitute a reduction of market liquidity under each of the regulatory regimes. A finding less surprising than the one concerning market stability: the results indicate that only a mandatory risk limit is beneficial from every perspective, while a short selling ban — though reducing volatility — increases tail risk. The contrary holds true for a Tobin Tax: it reduces the occurrence of crashes but increases volatility — an outcome that shows the importance of prior testing. However, when increased over a certain threshold results turn and a Tobin Tax clearly contributes to market instability from all perspectives. Furthermore, the interplay of measures is not negligible. Regression analyses show that measures can block each other and a well-chosen combination can mitigate unforeseen side effects.

However, further research is indeed needed to test the implications of regulatory measures under a different model set up. The high complexity of financial markets makes this a challenging task, but a *feasible* one considering the power of agent based simulation and a *worthy* one considering the necessity for prior testing, as Dirk Helbing would argue.

Appendix A

Table III.C presents the values used for simulation. The model was calibrated to fit roughly weekly data of stock markets. Where possible, values from Thurner et al. (2009) were used. Each run composes 4000 timesteps. One draw of $\vec{\epsilon_t}$ was used for every regulatory regime in sequence.

Table III.D depicts the analogon to Table III.A, but instead of feasible GLS quantile regression is used to model the medians of the market characteristics. Standard errors of coefficients were obtained using bootstrapping methods. The values are strikingly close to the ones of the feasible GLS regression.

Table III.E shows the results of a varying degree of Tobin Tax and varying regulatory regimes in place (i.e., a short selling ban and/or a mandatory risk limit). For the aggregated view (across all regimes) see Table III.B in Section 4, page 56.

The simulation was set up in R programming language (R Development Core Team 2011).

Parameter	Comment	See	Benchmark Model	Robustness Check
N^a	number of agents	Equ. (1)	150	50-300
N^s	number of assets	Equ. (1)	$N^a * 3$	$N^a * 3$
eta_i	aggressiveness of agents	Equ. (3)	10-50	1-100
$W_{i,0}$	initial wealth of funds	Equ. (2)	2	2
λ^{max}	maximum leverage	Equ. (3)	10	10
Τ	fraction of agents never	Footn. (9)	0.95	0.85 - 0.98
	taking short positions			
θ	persistence of perc.	Equ. (4)	0.99	0.99
	fundamental values			
N	fundamental value	Equ. (4)	1	1
Σ^2	covariance matrix for perc.	Equ. (4)	diag. elements 0.025^2 ,	diag. elements 0.025^2 ,
	fundamental values		off-diag. $0.4 * (0.025)^2$	off-diag. 0 to $0.8 * (0.025)^2$
$\mu_{i,t}$	mean of returns for	Equ. (7)	emp. mean of last	emp. mean of last
	VaR-calc.		$\max(\beta_i) * 10/\beta_i \text{ obs.}$	$\max(\beta_i) * 10/\beta_i$ obs.
$\sigma_{i,t}$	sd of returns for	Equ. (7)	emp. sd of last	emp. sd of last
	VaR-calc.		$\max(\beta_i) * 10/\beta_i \text{ obs.}$	$\max(\beta_i) * 10/\beta_i$ obs.
Tobin Tax	Tobin Tax	Equ. (9)	0.003	0.001-0.05
Γ	Threshold for Tobin Tax	Equ. (9)	$\beta_i/0.14*$ Tobin Tax	$\beta_i/0.14*$ Tobin Tax
	Ë	o III C. Voluce	ببدمط لمع منسبنامدنمه	

simulation.
for
used
Values
II.C:
Table I

exogenous	illiqui	dity	volatil	ity (sd)	kurto	osis	defa	ults
Intercept	2.172	***	2.676	***	1.821	***	308	***
VaR	0.007	**	-0.01	***	-0.199	***	-9	***
ssban	0.368	***	-0.046	***	0.283	***	-83	***
TT	0.363	***	0.021	***	-0.407	***	-3	***
(VaR*ssban)	-0.003		0		-0.051		3	**
(VaR*TT)	0.004		-0.004		0.024		-1	
$(ssban^{*}TT)$	0.055	***	0.006	**	-0.125	**	6	***
(VaR*TT*ssban)	0.001		0.002		0.024		0	

Significance Codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1

 Table III.D: Results of quantile regression for the median.

Level of								
Tobin Tax	0	0.001	0.003	0.005	0.01	0.02	0.03	0.05
Volatility (sd)								
			SS	ban=off	& VaR=	off		
mean	0.0268	0.0268	0.0270	0.0276	0.0291	0.0311	0.0335	0.0424
median	0.0268	0.0268	0.0270	0.0275	0.0291	0.0311	0.0335	0.0424
75%-quantile	0.0271	0.0271	0.0273	0.0279	0.0294	0.0314	0.0340	0.0431
			SS	ban=on	& VaR=0	off		
mean	0.0263	0.0263	0.0266	0.0272	0.0288	0.0308	0.0333	0.0424
median	0.0263	0.0263	0.0266	0.0272	0.0288	0.0308	0.0333	0.0424
75%-quantile	0.0267	0.0266	0.0269	0.0275	0.0291	0.0312	0.0338	0.0431
			ss	ban=off	& VaR=	on		
mean	0.0267	0.0267	0.0269	0.0273	0.0288	0.0463	0.0890	0.1915
median	0.0267	0.0266	0.0268	0.0273	0.0286	0.0312	0.0836	0.1756
75%-quantile	0.0270	0.0270	0.0272	0.0277	0.0289	0.0683	0.1025	0.2133
			SS	ban=on	& VaR=	on		
mean	0.0263	0.0262	0.0265	0.0270	0.0285	0.0462	0.0898	0.1967
median	0.0262	0.0262	0.0264	0.0270	0.0284	0.0309	0.0859	0.1757
75%-quantile	0.0265	0.0265	0.0268	0.0273	0.0286	0.0681	0.1015	0.2182
Kurtosis								
			SS	ban=off	& VaR=	off		
mean	2.9987	2.0928	2.3827	4.7605	2.4309	1.9394	3.2479	4.9251
median	1.8204	1.7082	1.4136	1.0377	0.9532	1.8795	3.2058	4.8683
75%-quantile	2.7932	2.6700	2.2109	1.6856	1.2171	2.1333	3.5810	5.4066
			SS	ban=on	& VaR=	off		
mean	2.8150	2.2957	2.2823	3.2105	2.7176	1.9308	3.3086	4.9295
median	2.1041	1.9867	1.5727	1.1959	0.9412	1.8772	3.2650	4.8795
75%-quantile	3.1662	3.0241	2.4672	1.8577	1.1714	2.1391	3.6347	5.3922
			SS	ban=off	& VaR=	on		
mean	2.4366	1.8568	2.0085	1.1941	2.2026	>100	>100	>100
median	1.6215	1.5430	1.2379	0.8922	0.8696	1.9902	>100	>100
75%-quantile	2.5234	2.4760	1.9687	1.4305	1.0994	>100	>100	>100
			SS	ban=on	& VaR=	on		
mean	2.5490	2.0652	2.1160	1.8971	2.1578	>100	>100	>100
median	1.8530	1.7365	1.3694	0.9849	0.8784	1.9899	>100	>100
75%-quantile	2.8225	2.6501	2.1724	1.4909	1.1164	>100	>100	>100

 Table III.E: Statistics of returns under different levels of Tobin Tax.

us illiquid	lity	volatili	(ty (sd))	kurto	\mathbf{sis}	default	S
9.617	* * *	2.260	* * *	11.300	* * *	183.318	* * *
0.090	* * *	-0.032	* * *	-0.229	* * *	-12.353	* * *
0.437	* * *	-0.049	* * *	0.258	* * *	-84.709	* * *
464.662	* * *	22.061	* * *	53.912	* * *	-4684.473	* * *
-0.038		0.000		-0.013		4.793	* * *
23.574	* * *	33.600	* * *	372.148	* * *	1411.457	* * *
-11.747	* * *	1.210	***	-5.614	* *	2729.318	* * *
15.613	*	0.449		-6.453		-357.371	* * *
-0.023	* * *	-0.001	* * *	-0.006	* * *	1.686	* * *
3.246	* * *	0.837	* * *	-5.914	* * *	98.519	* * *
-6.442	* * *	0.058	*	-7.556	* * *	-250.435	* * *
0.012	* * *	0.002	* * *	-0.012	* * *	-0.363	* * *
0.001		4e-4	* * *	0.032	* * *	1.765	* * *
0.675		0.492		0.102		0.770	

Table III.F: FGLS results of the robust analysis. Note that Tobin Tax is not coded as dummy $\{0,1\}$ but in levels $\{0, 0.001, 0.003, 0.005, 0.01, 0.03, 0.05\}$ and that Σ^2 corresponds to the correlation (not the covariance) between the innovations in the perceived fundamental value (see Table III.C).

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Chapter IV

What Drives Aggregate Credit Risk?

Abstract

A deep understanding of the drivers of credit risk is valuable for financial institutions as well as for regulators from multiple viewpoints. The systemic component of credit risk drives losses across portfolios and thus poses a threat to financial stability. Traditional approaches consider macroeconomic variables as drivers of aggregate credit risk (ACR). However, recent literature suggests the existence of a latent risk factor influencing ACR, which is regularly interpreted as the latent credit cycle. We explicitly model this latent factor by adding an unobserved component to our models, which already include macroeconomic variables. In this paper we make use of insolvency rates of Austrian corporate industry sectors to model realized probabilities of default. The contribution of this paper to the literature on ACR risk is threefold. First, in order to cope with the lack of theory behind ACR drivers, we implement state-of-the-art variable selection algorithms to draw from a rich set of macroeconomic variables. Second, we add an unobserved risk factor to a state space model, which we estimate via a Kalman filter in an expectation maximization algorithm. Third, we analyze whether the consideration of an unobserved component indeed improves the fit of the estimated models.

JEL classification: E44, E32, E37.

Keywords: credit risk, unobserved component models, state space, Kalman filter, stress testing.

1 Introduction and Motivation

The enormous rise in the number of publications on credit risk over the last decades bears testimony to an increasing interest in this topic. From a systemic perspective, the level of aggregate credit risk (ACR) is of major interest as - in contrast to idiosyncratic (borrowerspecific) credit risk - it cannot be diversified away and is therefore a potential source of financial instability. Although the nature of ACR suggests that it is primarily of concern to regulators, central banks and supervisory authorities, more and more commercial banks and other financial institutions seek a deeper understanding of ACR as this is essential to managing risk, maintaining a sound capital planning process and applying meaningful stress testing programs as well as a consistent approach to designing an adequate rating model philosophy¹. The value of structured products, or of any portfolio with non-zero credit risk, is largely determined by their inherent systemic component - an important point that should be clear after the 2008/2009 financial crisis.

In addition, the growing relevance of forecasting ACR is evident from the numerous stress tests carried out by central banks around the world, as ACR forecasts constitute a precondition for stress-testing. To be able to perform efficient system-wide stress testing, central banks or any other supervisory authorities need a structured approach to forecasting ACR. Hence, a profound understanding of ACR drivers is of high relevance for banks and supervisors alike. Numerous papers have addressed this topic in recent years; inter alia Nickell et al. (2000), Koopman & Lucas (2005) and Couderc & Renault (2005). However, any approach to finding significant drivers of ACR faces two major challenges:

1. Given the lack of a clear-cut theoretical framework explaining the causes and driving

¹See the distinction between *point in time* and *through the cycle models* e.g., in Heitfield (2005).

factors of ACR in a financial system, a long list of macroeconomic variables is a priori available for explaining ACR. Selecting among them becomes even more challenging when taking the possible dynamic lag structure of these macroeconomic variables into account.

2. At the same time, there is mounting evidence of latent factors driving (aggregate) credit risk, as emphasized recently by Lown and Morgan Lown & Morgan (2004), Jimenez & Mencia (2009), Koopman et al. (2009) and Bruche & Gonzalez-Aguado (2010). With no directly measurable metric at hand, the question is how to incorporate this evidence into an econometric model.

In this paper we present an approach that deals with both of the above issues in a state-of-the-art fashion. In order to manage the high number of possible explanatory variables for ACR, we make use of advanced variable selection techniques (Hastie et al. 2009). We cope with the second issue by following the approach of Jimenez & Mencia (2009) and Koopman et al. (2009), treating the credit cycle as a latent factor.

Since Kalman (1960) described a recursive solution to the discrete data linear filtering problem (Kalman-Bucy filter), the idea of incorporating an unobserved state variable into a state space model has led to an extensive amount of literature in various fields of science. In economics, state space models are used as a very flexible tool in time series analysis.² Harvey & Koopman (2009) give a short introduction into the various applications of state space models in economics and finance. The most prominent applications are macroeconomic models used to identify the natural rate of unemployment, permanent consumption, the output gap or the expected rate of inflation, and time series models such as trend-cycle decomposition and seasonal component models (Burmeister et al. 1986).

Only recently, state space models have drawn attention in credit risk-related research. The respective papers aim at exploring the so-called "hidden", "unobserved" or "latent" credit

²For example, to estimate their parameters, autoregressive moving average models, dynamic stochastic general equilibrium models and time-varying coefficient models can be put into a state space form.

risk factors. In this paper, we use the expression latent risk factor to refer to the general idea of including additional unobserved predictors in various models. In our models, latent risk factors are added as unobserved components.

Crowder et al. (2005), Bruche & Gonzalez-Aguado (2010), Koopman et al. (2008) and Banachewicz et al. (2008) aassume that the state variable (latent risk factor) is discrete and the number of states is at least two (a "good" and a "bad" state). The resulting models are commonly referred to as hidden Markov models.³ By contrast, Koopman & Lucas (2005), Jimenez & Mencia (2009) and McNeil & Wendin (2007) choose a more general approach in terms of state space by modeling it as a continuous state variable.⁴ This setup-up leads to the classical state space model described by Kalman (1960).

Nevertheless, there is no common theoretical view on the source and/or definition of latent factors. They could be related to (a mixture of) general credit market conditions such as the leverage and/or solvency ratios of creditors, collateral and other asset values or it could, via the lending channel, supply adjunct criteria such as banks' capital buffers and lending criteria, etc. In any case, the latent factor should be a variable that is unobserved (at least in our dataset), but still has a significant and persistent impact on credit risk. In view of the recent financial crisis, one promising idea would be to relate the credit cycle to the leverage cycle, as explicitly defined by Geanakoplos (2010) and Fostel & Geanakoplos (2008). In their papers, they argue that a small initial drop in the value of assets and collateral causes a big drop in the wealth of leveraged "optimists" which is then amplified by forced sales and uncertainty.

A second credit cycle theory assumes the following relation between credit standards, banking competition and the phase of the business cycle. In a nutshell, empirical studies report that (too) lenient credit standards during an economic upturn result in the build-up

 $^{^{3}}$ See Rabiner (1989) for details on recursions and filter techniques which extract the sequence of hidden in which only the state-dependent output variables are observed.

⁴A continuous state variable leads to more restrictions on other model assumptions, in particular on the transition equation. See Minka (1999) for more details.

of high credit risk, which materializes in the ensuing economic downturn.⁵ As analyzed by Ruckes (2004), such behavior can be supported by banks' profit-maximizing strategies in a simple game theoretic setting. In line with his model, credit standards vary anti-cyclically and therefore might enhance the influence of the macroeconomy on ACR. Third, the credit cycle could be explained by the theory of *cyclical default correlation* (Giesecke 2004) which can be understood as a partly systemic risk factor founded in the existence of direct ties (e.g., financial, legal or client-supplier links) between firms.⁶

Our paper tests whether there is evidence for a latent effect on ACR. It builds on previous work by Boss (2002) and Boss et al. (2009) describing the current OeNB macro-to-probability of default models. We extend these OeNB models in two ways. First, we add a new dimension to the discussion about the link between the macroeconomy and credit risk measures by enlarging the set of possible macroeconomic predictors.⁷ We apply advanced variable selection algorithms to find the best macroeconomic predictors for a given model size. Second, we integrate an unobserved factor into the models via a state space formulation, thus enriching them by explicitly modeling the hypothesized cycle. In a next step, we interpret the sector-specific results. Finally, we evaluate the results by comparing the state space model output with the output obtained from the traditional models that are based on macroeconomic factors only.

2 Model Specifications

In this section we outline the econometric theory and estimation procedures behind the models used to explain ACR. In terms of data, we use - in line with previous work by Boss (2002) and Boss et al. (2009) — quarterly default frequencies rates from 1985 to 2011Q1

⁵See e.g., Lang & Nakamwa (1995) and Bonfim (2009).

⁶Such direct ties could lead to contagion effects that describe the default dependence between interconnected corporates. See e.g., Eisenberg & Noe (2001).

⁷In a classical multivariate framework, this boils down to re-examining the trade-off between the bias and variance of estimated results.
provided by *Kreditschutzverband von 1870* to approximate sectoral corporate probabilities of default in Austria. These default frequency rates are calculated by dividing the number of quarterly defaults by the total number of firms. The corporate sectors in question are construction, production, trade, transport, tourism and services.⁸ The macroeconomic variables used to construct our design matrix are taken from the OeNB's macroeconomic database. The set of explanatory variables $\{x_j\}_{j=1}^k$ is extended by one to six lags of each time series which multiplies the pool of candidate predictors.

ACCRONYM	MEANING	Transformation
ATX	Austrian Traded Index	YoY-Log-Difference
CPNReal	Credit, private, amount outstanding, real	YoY-Log-Difference
DDR	Domestic Demand, real	YoY-Log-Difference
GONReal	Gross operating surplus, real	YoY-Log-Difference
HIC	Harmonised index of consumption prices	YoY-Rel-Difference
IER	Equipment investment, real	YoY-Log-Difference
IOR	Other investment, real	YoY-Log-Difference
LTIReal	Long-term real interest rate	No Transformation
MTR	Imports, real	YoY-Log-Difference
PCR	Private consumption, real	YoY-Log-Difference
POIL	Oil price in EUR	YoY-Log-Difference
PRO	Average labour productivity	YoY-Log-Difference
PSNReal	Private sector savings, real	YoY-Log-Difference
PYR	Private sector disposable income, real	YoY-Log-Difference
STIReal	Short-term real interest rate	No Transformation
URX	Unemployment rate	YoY-Rel-Difference
WURYD	Compensation per employee, real	YoY-Log-Difference
XTR	Exports, real	YoY-Log-Difference
YER	GDP expenditure, real	YoY-Log-Difference

Table IV.A: Possible explanatory macroeconomic variables. Up to 6 quarterly lags are also considered for each variable.

As a starting point for modeling ACR, we look at the linear observable macroeconomic factor model:

$$y_{i,t} = \beta_{0,i} + \sum_{j=1}^{k} x_{j,t} \,\beta_{j,i} + \epsilon_{i,t},$$
(1)

where y_i is the logit-transformed sectoral default frequency rates⁹ ($i \in \{0, 1, 2, ..., 7\}$), k is

⁸Corporate sectors are classified according to NACE Rev. 2, the classification of economic activities applied throughout the European Union. See Zeller et al. (2008) for more details.

⁹The logit transformation ensures that the default frequency rates used remain within the interval (0; 1). A probit transformation would serve the same purpose. Other popular approaches to modeling dependent ratios without transforming them include the fractional logistic regression by Papke & Wooldridge (1996)

the number of macroeconomic predictors and $t \in \{1, 2, .., T\}$ constitutes the time index. x_j is the j^{th} transformed macroeconomic predictor.

How to Select Explanatory Variables?

Let us now address the first issue raised in the introduction: As, in our opinion, general equilibrium literature on credit markets does not provide the sufficient theoretical background for deriving explanatory variables, the list of candidate predictors is extensive and, as a consequence, candidate predictors might even outnumber observations. In previous work on the topic, regressors have been selected by mere qualitative reasoning (e.g., Jimenez & Mencia 2009 and Koopman et al. 2008). Boss et al. (2009) group the variables into thematic sets and allow only one variable from each set to be selected. In order to deal with the high variance-versus-low bias trade-off in a nonheuristic way, we depart from these qualitative approaches and consider a data-driven subset selection mechanism.

One of the available subset selection algorithms is the so-called *Best Subset Selection*¹⁰ which selects for each $k \in \{0, 1, 2, .., p\}$ the subset of size k that gives the smallest residual sum of squares. The variance-versus-bias trade-off is directly linked to the choice of k and is therefore a discrete mechanism. With respect to model interpretation, Best Subset Selection offers the choice of k input variables from the set of p variables. However, a severe drawback is the computational cost of this method. The fact that the number of possible models increases exponentially with p, puts a relatively low bound on feasible values of p (p < 50) even with a fast algorithm such as the leaps and bounds procedure at hand.¹¹ Consequently, the application of Best Subset Selection would require a preselection of the variables considered above, especially when one wants to account for a dynamic lag structure.

and beta regression models (Ferrari & Cribari-Neto 2004). A discussion of the advantages and disadvantages of the different approaches to modeling dependent ratios would be beyond the scope of this paper.

 $^{^{10}}$ See Hastie et al. (2009) for details.

¹¹See Furnival & Wilson (1974) for details.

Alternatives to this approach are Forward- and Backward-Stepwise Selections¹². Forward-Stepwise Selection starts with an intercept and sequentially adds the regressors which contribute most to an improvement of the fit (as measured e.g., by the Bayesin information criterion, BIC) until k variables are selected (Hastie et al. 2009). Backward-Stepwise Selection starts with the full model and sequentially drops the least important variables in terms of model fit until k variables are reached. While not as computationally demanding as Best Subset Selection, these algorithms might not select the "optimal" set from the perspective of the minimal residual sum of squares. A comparison between Best Subset Selection and Forward Stepwise Selection applied to different subsamples of our dataset shows that the two mechanisms produce relatively similar results. As Backward Stepwise Selection requires the number of candidate predictors to be smaller than the number of observations, p < T, a preselection of variables - as in the case of Best Subset Selection would still be necessary to make the selection procedure applicable.

As a third alternative selection procedure Shrinkage Methods¹³ appear to be promising. In contrast to subset selection, shrinkage methods do not retain or discard a variable but "shrink" the regression coefficients by imposing a penalty on their size. For example, the elastic net procedure proposed by Zou & Hastie (2005) is a shrinkage method which uses a convex combination of the L1 (lasso) and the L2 (ridge regression) norm as the penalty restriction in the standard minimization of the sum of residual squares (with respect to the vector β) to estimate Equation (1). While promising at first sight, the combination of shrinkage methods with the estimation of latent factors (see below) requires a largely revised estimation procedure and is beyond the scope of this paper.

By way of summary, we find that Best Subset Selection and Backward Stepwise Selection both require a preselection of variables, while shrinkage methods do not, in general, allow for including latent factors within the state space framework.¹⁴ There-

¹²See Hastie et al. (2009) for details.

 $^{^{13}}$ See Hastie et al. (2009) for details.

¹⁴The question of how to combine the elastic net algorithm with an unobserved component in a Bayesian framework is currently being examined in an ongoing research project.

fore, we will use Forward Stepwise Selection, which does not require any form of variable preselection and shows a promising performance in simulation exercises (Hastie et al. 2009).

How to incorporate the unobserved credit cycle?

In a next step we extend our macroeconomic factor model by "latent risk factors". Motivated by the discussion presented in Section (1) we add an unobserved risk factor to the framework of Equation (1) and will refer to this new equation as the measurement equation *Measurement Equation* (2). We explicitly model the latent credit cycle as an autoregressive state process that evolves through time and refer to this specification as the *State Equation* (3).

$$y_{i,t} = X_{i,t} \Gamma_i + z_{i,t} \lambda_i + v_{i,t} \qquad v_{i,t} \sim \mathcal{N}(0, r_i)$$
(2)

$$z_{i,t} = z_{i,t-1} \phi_i + w_{i,t} \qquad w_{i,t} \sim \mathcal{N}(0, q_i)$$
 (3)

In addition to the previous notation, λ_i , Γ_i , ϕ_i , q_i and r_i are parameters to be estimated, $z_{i,t}$ is the unobserved factor, and $v_{i,t}$ and $w_{i,t}$ are error terms. Capital letters denote vectors or matrices and small letters scalars. Moreover, we assume that $Cov(v_{i,t}, w_{i,t}) = 0$ and that there are no cross correlations in the state and measurement equation between the sectors i, $Cov(w_{j,t}, w_{i,t}) = 0$ and $Cov(v_{i,t}, v_{i,t}) = 0$ for any $i \neq j$.

We estimate the Equation System (2) and (3) via an expectation maximization algorithm (EM algorithm)¹⁵. Based on an initial set of parameters $(\lambda_i, \Gamma_i, \phi_i, q_i \text{ and } r_i)$ the unobserved component is extracted via the Kalman filter in the expectation step. Given the unobserved component z_i the likelihood of Equation (2) is maximized with respect to the parameter set. These steps are repeated until convergence.¹⁶ However, the state space representation of a given dynamic system might not be uniquely defined by a given

 $^{^{15}\}mathrm{See}$ McLachlan & Thriyambakam (1996) for details.

 $^{^{16}\}mathrm{See}$ Shumway & Stoffer (2006) and Holmes (2010) for details.

parameter set $\lambda_i, \Gamma_i, \phi_i, q_i, r_i$ without restricting some of these parameters. This can be seen from the fact that the likelihood function of the equation system would remain unchanged when multiplying Equation (3) with any non-zero factor or nonsingular matrix — it would simply measure the unobserved factor on a different scale.¹⁷ Consequently, we fix the metric of the unobserved variable by restricting $q_i = 1$ without loss of generality.

3 Results

In this section we present evidence of the relevance of the latent factor in our dataset as well as an analysis of the most frequently selected variables. For this purpose we estimate models for each of the corporate sectors under review with a varying number of explanatory variables. The explanatory variables are chosen by applying the Forward Stepwise Selection method described in Section 2. For each number of explanatory variables ranging from 1 to 15, we estimate the top five models according to their explained sum of squares, which results in 75 models per sector.¹⁸ Additionally, to gain insight into the importance of latent factors for explaining ACR, we estimate these models with and without an unobserved component. To compare the respective results, we follow Koopman et al. (2009) and conduct a likelihood ratio test defined by

$$2(l_u - l_r) \stackrel{a}{\sim} \chi_m^2,\tag{4}$$

where l_u represents the likelihood of the unrestricted model with the latent factor and l_r the restricted models without this factor and m the number of restrictions implemented. In our case, the only imposed restriction is $\lambda_i = 0$.

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 $^{^{17}}$ For more details see Hamilton (1994) and Carro et al. (2010).

¹⁸Thus, models of different sizes do not compete with each other, and applying any selection criteria such as the Akaike information criterion (AIC) or the Bayesian information criterion (BIC) would result in the same selection of variables.

To judge whether latent factors are statistically significant, Figure IV.a plots the likelihood ratio statistics for all models per sector, with the x axis representing the number of included predictors per model. Note that for each given number of explanatory variables, five models are estimated. The horizontal line in each plot represents the 99%-critical value of the χ^2 -distribution.¹⁹ Thus, values above the line indicate a statistically significant contribution of the latent factor to the model fit and can thus be interpreted as evidence for the existence of an unobserved component. The results shown in Figure IV.a are quite surprising: While there is evidence for a latent factor in smaller models, i.e., models with about 1 to 7 explanatory variables, this evidence clearly vanishes when considering models of larger size.²⁰

This behavior is similar in all sectors with the exception of construction. Especially in the production sector, any significant contribution of the estimated unobserved component series is lost early (in terms of model size). As the model fit obtained by the variables selected by the algorithm alone is already rather high, it cannot be significantly improved by the unobserved component. A similar pattern is visible for the service, trade, transportation and tourism sectors. The construction sector constitutes an exception in this context since here, including a latent factor results in a more persistent significant improvement of the model fit. However, for model sizes beyond a certain threshold the improvement of the model fit is insignificant in this case as well. We relate this finding to the fact that the construction sector mainly consists of corporates working in *structural* and *civil* engineering. While the main customers in structural engineering are households, a large portion of orders in civil engineering is publicly assigned and could thus cause the behavior of this sector to differ from that of other sectors.

All in all, the results described above are somewhat surprising. On the one hand, it is

¹⁹From a strictly statistical point of view the results, especially the critical values resulting from the LR test, have to be taken with caution. First, they are only asymptotically valid. Second, we treat the likelihoods of the restricted models without explicitly conditioning them on the model selection criterion.

 $^{^{20}}$ To a very large extent, our LR test results can be confirmed by applying the BIC, which explicitly takes the length of the time series into account.



Figure IV.a: Likelihood ratio statistic (y-axis) versus number of included explanatory variables (x-axis) for all corporate sectors (varying y-scale!). The grey horizontal line represents the 99%-critical value of the χ^2 -distribution.

obvious that the inclusion of more variables reduces the space that a time series estimated by the Kalman filter technique can fill. On the other hand, the model sizes discussed here are far from "large" and there is ample literature underlining the importance of the inclusion of a latent factor in the model (e.g., Lown & Morgan 2004, Jimenez & Mencia 2009, Koopman et al. 2009 and Bruche & Gonzalez-Aguado 2010). One important distinction between our approach and e.g., the approach followed by Jimenez & Mencia (2009) and Koopman et al. (2009) is that they selected variables by mere qualitative reasoning. The set of macroeconomic candidate predictors considered in previous work is generally smaller than in our models. Jimenez & Mencia (2009) for instance, only consider real GDP growth, interest rates and, in an enlarged set-up, also bond spreads and a sector-specific additional variable, while Bruche & Gonzalez-Aguado (2010) only consider real GDP growth.

In a closer examination of the difference between previous findings in the literature and our findings, we set up a downsized macroeconomic environment in which we only include real GDP growth, short- and long-term interest rates and inflation — all up to six lags. With this much smaller macroeconomic variable set, we conduct Best Subset Selection²¹ for model sizes from 1 to 15 for each sector. Figure IV.b presents the results, which are easily summarized: In all sectors there is substantial evidence of a significant improvement when considering the Kalman series *irrespective of the size of the model*. Clearly, our results show that an enriched dataset combined with a modern selection technique like Forward Stepwise Selection is able to capture dynamics that are otherwise deemed *unobserved*.

Which Fundamentals Drive Aggregated Defaults?

An additional question is which macroeconomic variables are selected by the forward selection algorithm. For this purpose, we point to Figure IV.c and Table IV.B. Figure IV.c presents the frequency with which estimated models contain a certain explanatory variable

²¹We chose Best Subset Selection as it is computationally feasible for this smaller set of explanatory variables and superior to Forward Stepwise Selection since Best Subset Selection finds the optimal model among all possible models.

or its lagged cousin, thus indicating its importance in explaining aggregated defaults in the individual sectors. The respective black bar represents the fraction in which this variable has a positive coefficient.²² Hence, in the construction sector, for instance, the variable HIC²³ (inflation) - or any of its lags - was selected in about 90% of all models and nearly always had a positive sign.

A closer look at Figure IV.c reveals interesting results. In all sectors but construction, funding costs such as the real short-term interest rate (STIReal), the real long-term interest rate (LTIReal) but also inflation/ (HIC) and real private credit growth (CPNReal) play an important role.

First, the explanatory variable LTIReal appears very frequently in models explaining defaults in the production, trade and tourism sectors. The sign of its coefficient is positive in the majority of cases, indicating rising defaults when LTIReal is high. Clearly, a higher interest rate raises the cost of funding in these sectors. In contrast, the service and transportation sectors seem to be affected by STIReal. An intuitive explanation for this finding is that these sectors rather tend to be financed by short-term lending and are thus more vulnerable to STIReal. While this interpretation seems plausible for the service sector, the negative signs of coefficients for the transportation sector suggest a different background: STIReal might be a timely indicator of economic activity. Hence, a reduction of STIReal, which is highly correlated with the central bank's target rate, might be a first indicator of an economic downturn, which would increase the default rate in the transportation sector.

Furthermore, in the same five sectors (all but construction) HIC has a positive influence on aggregate defaults in the majority of cases. As stated by Qu (2008), the role of inflation in firm defaults can be examined from two perspectives: first, the perspective of prices that

²²In cases in which the algorithm chose a dynamic lag structure, i.e., the variable appeared more than once in one equation due to the lag specification, the black bar shows the number of models for which the sum of the respective coefficients is positive.

 $^{^{23}}$ Abbreviations as quoted in Table IV.A denote the variables transformed as indicated in the right-hand column of the table.



Figure IV.b: Likelihood ratio statistics (y-axis) versus number of included explanatory variables (x-axis) for all corporate sectors (varying y-scale!). The grey horizontal line represents the 99% critical value of the χ^2 distribution. It is important to note that we only include four possible candidate predictors (STIReal, LTIReal, HIC and YER).



Figure IV.c: Frequency of selected variables. Black bars show the fractions assigned to positive coefficients for the particular macroeconomic variable.

companies charge for their goods and services and second, the perspective of factor prices. Higher prices of goods and services ceteris paribus increase earnings and thereby improve a company's creditworthiness. Higher factor prices lead to increased production costs and tend to weaken creditworthiness — a fact which implies an increase in credit risk. Additionally, higher inflation is also a proxy of economic uncertainty. In our dataset, the second effect obviously dominates the first, leading to positive coefficients in the majority of models.

Additionally, in all six sectors CPNReal has a solely negative coefficient. Even in the construction sector, the model inclusion probability is above 30%. Although this result is in line with Bonfim (2009), many studies on credit risk especially in developing economies search for a positive coefficient of credit growth. The theoretical assumption is that rapid credit growth in boom phases might lead to higher defaults in immediately following downturns.²⁴ With regard to the Austrian corporate credit market, we clearly cannot support this hypothesis. However, we do not include dummies for rapid credit growth and/or consider lags up to several years as other studies do.²⁵ The negative sign in our results can be interpreted as follows: In good times, productive investment projects arise and companies might at least meet their short-term payment obligations - a circumstance which, ceteris paribus, reduces the number of insolvencies.

Aside from the above, the variable YER (real GDP growth), is frequently selected with a negative sign in the production sector.²⁶ Moreover, the variable GONReal (real gross operating surplus growth), surprisingly, enters more than 70% of the production models with a positive sign. In the construction sector, the selection algorithm selected the variable PCR (real private consumption) with the expected negative sign in about 90% of all models. This highlights the influence of housing construction, a segment of construction

²⁴See Jimenez & Saurina (2006) and Bank for International Settlements (2010).

 $^{^{25}\}mathrm{See}$ Foos et al. (2010) and Berger & Udell (2004) among many other papers.

²⁶Interestingly, YER seems to be of importance only in the production sector. However, in many other sectors direct subcomponents of YER, such as XTR (real export growth) or PCR, are selected and indicate that the additional information contained in YER does not significantly contribute to explaining aggregate credit risk.

whose main customers are households. Second, the variable POIL (oil price) enters over 90% of the models with a positive sign.

A particularly interesting finding is that the oil price also constitutes an important driver of defaults in the transportation sector as it defines the price of the main input good. In line with findings for other sectors, PCR is selected with the expected negative sign in more than 90% of the models. In addition, a further transportation-specific variable emerges: XTR (real export growth) proves to be important in the transportation sector. Clearly, more exports lead to more business activity and thus reduce the level of risk.

The aggregate insolvency rates in the service sector are influenced by a couple of variables, which reflects the fact that services consist of 38 different NACE sectors.²⁷ Aside from the general variables (STIReal, LTIReal, HIC and CPNReal), the most prominent additional variables are the real growth of compensation per employee (WURYD) as well as real other investment growth (IOR). The negative sign for WURYD indicates that households' income growth is a good proxy for more corporate revenues that lead to lower credit risk. Additional variables in the trade sector are real equipment investment (IER) growth, real other investment (IOR) growth and real domestic demand (DDR) growth. As Figure IV.c shows, investment growth (IER, IOR) appears to be more important in the trade sector than in other sectors. In most models, the expected negative sign can be observed.

Finally, tourism is the only sector in which real private disposable income growth (PYR) is selected with a negative coefficient in more than 70% of the models. This clearly shows that households spend their higher disposable income on holiday activities, which causes revenues in the tourism sector to go up and insolvency rates to go down. Summing up, we find a number of variables which drive ACR across multiple sectors and are thus particularly crucial for understanding ACR. These variables include inflation, interest rates and (negative) credit growth. Furthermore, we identify sector-specific variables, such as

 $^{^{27}\}mathrm{See}$ Zeller et al. (2008) for more details.

	Pr	od.	Ser	vice	Cor	nstr.	Tra	ade	Tra	nsp.	Tou	rism
	rel.	+	rel.	+	rel.	+	rel.	+	rel.	+	rel.	+
ATX	0.33	0.33	0.11	0.09	0.03	0.03	0.01	0	0.22	0.61	0.44	0.45
CPNReal	0.84	0	0.89	0	0.29	0	0.76	0	0.89	0	0.97	0
DDR	0.39	0.39	0	0	0	0	0.73	0.75	0.03	0.03	0	0
GONReal	0.76	0.75	0.81	0.82	0.61	0	0.03	0.03	0.03	0	0.08	0.04
HIC	0.91	0.91	0.99	1	0.01	0	0.97	1	0.88	0.88	0.87	0.89
IER	0.01	0	0	0	0.04	0	0.89	0.29	0.48	0	0	0
IOR	0.07	0	0.71	0	0	0	0.6	0	0.01	0	0.44	0
LTIReal	1	1	0.07	0.07	0.04	0.04	0.85	0.88	0.01	0.01	0.92	0.95
MTR	0.24	0.01	0.12	0.12	0.08	0.08	0	0	0.11	0.11	0.04	0.04
PCR	0.03	0.01	0.44	0.01	1	0	0.03	0.01	1	0	0.03	0
POIL	0.15	0.15	0.21	0.22	0.92	0.92	0	0	0.77	0.99	0.23	0.23
PRO	0.65	0.65	0.61	0.62	0.83	0.83	0.33	0.34	0.24	0.03	0.79	0.81
PSNReal	0	0	0.01	0.01	0.28	0.28	0.36	0.37	0.31	0.01	0.04	0.01
PYR	0	0	0.03	0	0.09	0.08	0	0	0.04	0	0.71	0
STIReal	0.03	0	0.77	0.78	0.24	0.24	0.11	0.05	0.49	0	0.03	0.03
URX	0.01	0.01	0.36	0.36	0.56	0.56	0	0	0	0	0.29	0.3
WURYD	0.09	0	0.68	0.03	0.35	0	0.05	0.03	0.72	0.68	0.63	0.64
XTR	0.07	0.03	0.49	0.38	0.56	0.56	0.01	0	0.55	0.03	0	0
YER	0.72	0	0.01	0	0.01	0	0.2	0.01	0.03	0	0.08	0

Table IV.B: Frequency of Selected Variables and Respective Fraction of Positive Coefficients.

exports in the transportation sector or investment in equipment in the trade sector, which highlight the importance of taking sectoral differences into account when analyzing ACR in corporate sectors.

4 Conclusions

This paper focuses on the determinants of aggregate credit risk (ACR). On the one hand, we explicitly measure the importance of latent risk factors via a state space system for different corporate sectors and model sizes. On the other hand, we evaluate the influence of observable macroeconomic variables in different corporate sectors by analyzing the choices of the Forward Stepwise Selection procedure.

We find that enhancing a macro-to-probability of default model by incorporating a latent risk factor only improves the model considerably if the model is allowed to select from a small number of possible predictors. We show that this finding is not explained by the selection procedure applied but is attributable to a limited set of variables. The limited number of included variables also explains why some of the relevant literature finds strong support for including unobserved risk factors in macro-to-probability of default models.

As pointed out in the introduction, the literature has not yet agreed upon a meaningful economic interpretation of the credit cycle as an unobserved credit risk factor. Mainly on the basis of the likelihood ratio tests performed, we conclude that the significance of the explanatory value of the unobserved factor depends on the number and quality of the macroeconomic variables that are selected as predictors. Since the results for the construction sector show that influential observable predictors might not always be available, there is (state) space open to different credit cycle theories. At the same time, the inclusion of an unobserved component into an ACR model comes at little methodological costs. When forecasting aggregate levels of credit risk, it therefore seems to be prudent to work with a state space model.

Coming back to the credit cycle theories mentioned in the introduction, we think that the second theory, which assumes that (too) lenient credit standards during an economic upturn result in the build-up of high credit risk which then materializes in the ensuing economic downturn, seems not to apply to the highly competitive Austrian banking sector.²⁸ This view is supported by the negative coefficient of credit growth (CPNReal) in all corporate sectors observed. Since this paper analyzes the Austrian corporate sector, we are not in the position to judge whether the credit cycle can be interpreted as the leverage cycle, which would require the modeling of ACR for mortgage loans in the retail sector. Finally, among the above-mentioned credit cycle theories the cyclical default correlation hypothesis seems to be the most promising option in support of our findings. The persistent importance of the unobserved factor in the construction sector for different models sizes underpins this argument as direct ties between firms in the construction

²⁸High competition in the lending market generally results in low net interest margins. These, in turn, require strict lending standards which generally rule out subprime lending.

sector are often observed.

Moreover, we find several variables which drive ACR simultaneously in a number of sectors and are thus particularly crucial for modeling ACR. These variables include interest rates, inflation and (negative) credit growth. However, there are also considerable sectoral differences between the selected variables. Among the sector-specific variables we find e.g., the oil price and exports in the transportation sector, equipment investment in the trade sector and short-term interest rates in the service sector. Most of the selected variables show the expected sign in the regressions performed and can be explained by general economic theory and/or by specific sectoral economic conditions. Overall, our analysis suggests that only an enlarged set of macroeconomic variables can explain ACR in a comprehensive way - and a comprehensive explanation of ACR is without doubt crucial for the development of macroeconomic scenarios for stress-testing exercises.

Our findings also clearly indicate that taking model uncertainty into account is of high importance in a field where, a priori, many regressors constitute candidate predictors for explaining ACR. We accounted for model uncertainty by estimating 75 models for each corporate sector. However, there are more sophisticated statistical methods to perform model averaging. In particular the concept of Bayesian model averaging could be a promising advancement for future research projects.

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Chapter V

Model Uncertainty and Aggregated Default Probabilities: New Evidence from Austria

Abstract

Understanding the determinants of aggregated default probabilities is crucial for both financial institutions and supervisors and thus has attracted substantial research over the past decades. This study addresses a major difficulty in understanding the determinants of aggregate probabilities of default: Researchers must choose among many potential models and decide which regressors to include or exclude. Due to the lack of clear guidance from theory, researchers must apply expert knowledge. Thus, model uncertainty is present blurring the view on default drivers. This paper presents Bayesian Model Averaging (BMA) as a powerful tool that overcomes this difficulty. Furthermore, we supplement BMA with ridge regression to mitigate multicollinearity arising from the consideration of lagged explanatory variables. This framework allows us to account for the complex relationship between the macroeconomy and firms' defaults by drawing from a rich set of macroeconomic variables. After introducing the statistical methods we apply our approach to an Austrian dataset. Our findings suggest that factor prices like short term interest rates and energy prices constitute major drivers of default rates, while firms' profits reduce the expected number of failures. The advantage of BMA becomes evident as we identify drivers of defaults which constitute a component of another macro variable, like investment as component of GDP or energy prices as component of inflation — variables which might not have passed the expert's inclusion decision. Finally, we show that the results of our baseline model are fairly robust to the choice of the prior model size.

JEL Classification: E44, C52, E37.

Keywords: Bayesian Model Averaging, model uncertainty, ridge regression, credit risk, firm defaults, stress testing.

1 Introduction

Understanding the driving factors of aggregated probabilities of corporate defaults is an important topic both for financial institutions and supervisors. For example, conducting meaningful stress tests requires the translation of macroeconomic scenarios into portfolio losses. The same applies when financial institutions and supervisors are interested in forecasting the credit quality of portfolios on an aggregated level. Both in the field of macro prudential supervision and strategic risk management knowledge of the determinants of aggregated defaults is crucial.

Consequently, estimating the link between macroeconomic variables and probabilities of defaults has been a long-standing topic in research, as numerous papers testify (see below). However, the classical approach of regression faces a major challenge: Due to the sparse theoretical framework of how firm defaults are linked to specific macroeconomic variables, researchers are compelled to draw on their intuition which macro variables to include or not. Such a procedure neglects the uncertainty in the model choice and might end up with wrong conclusions. This challenge, commonly known as model uncertainty, is a problem shared with many other empirical fields of research. In what follows we present a state-of-the-art statistical approach of dealing with model uncertainty, a combination of Bayesian Model Averaging and ridge regression which we then apply to Austrian data.

Motivated by the high interest in the topic from industry and supervisors, there is a growing body of literature examining the relationship between firm defaults and economic conditions. Altman (1983) uses augmented distributed lags to demonstrate the effect of GNP, money supply and corporate profits on firms' ability to survive. Altman (1984) presents a survey discussing different business failure models that have been tested and developed outside the United States. Liu & Wilson (2002) use a time-series model to construct measures showing that interest rate and insolvency legislation are important variables in explaining firm bankruptcy. Similarly, Virolainen (2004) regresses Finnish sector-specific default rates on macroeconomic indicators like GDP, interest rates and levels of corporate indebtedness. Liu (2004) uses an error-correction model to investigate the macroeconomic determinants of UK corporate failure rates. Liu (2009) extends this research by implementing a vector error-correction model specifically accounting for policyinduced changes in the macroeconomy, concluding that macro variables like the interest rate and inflation impact firm failures. Simons & Rolwes (2009) use macroeconomic-based models for estimating default probabilities using a Dutch dataset. Additionally, they compare their results with Austrian data. They conclude that for both countries their model delivers different results, deducing that their provided model is country specific. Further contributions are Koopman & Lucas (2005) who analyze the co-movement of credit and macro cycles in the US and Foglia et al. (2009) who examine Italian default frequencies per sector.

Screening the literature reveals that authors have to rely on expert knowledge when deciding upon the inclusion or non-inclusion of macro variables. To the best of our knowledge, uncertainty about the correct model specification for aggregated probabilities of default has not explicitly been addressed yet. The approach we present here refrains from assuming that there is one "true" model but instead averages over a huge number of potential models. This approach is known as Bayesian Model Averaging (BMA) (see Hoeting et al., 1999). Thereby, the researcher controls the model size via a prior model inclusion probability for each variable¹. Sampling from the set of regressors BMA then computes a huge number of models, which are weighted by their marginal likelihood and subsequently averaged. This simple procedure reveals important determinants of the dependent variable and their respective coefficients.

As noted above, BMA is becoming a central tool applied in dealing with model uncertainty, or in general settings with large numbers of potential regressors and relatively limited numbers of observations (see Ley & Steel, 2009). In the literature on growth determinants Fernandez et al. (2001) and Sala-I-Martin et al. (2004) propose BMA to identify robust drivers of countries' average growth. Wright (2008) and Avramov (2002) use BMA to forecast exchange rates and stock returns respectively. Empirical results have shown that BMA might outperform single model in prediction (see Hoeting et al., 1999).

However, at least in our case highly correlated candidate variables (multicollinearity) constitute an issue to be accounted for. To some extent this fact arises due to the inclusion of lagged explanatory variables, which display a particularly high correlation. To explicitly deal with this correlation structure we supplement BMA with a shrinkage method, *ridge regression* (Hoerl & Kennard, 1970b,a). Ridge regression aims at avoiding the commonly observed characteristic upon inclusion of highly correlated variables: coefficients display high absolute magnitudes which are canceled out by coefficients of correlated cousins of comparable magnitude with reversed sign. By adding a penalty term dependent on the size of coefficients ridge regression indeed overcomes this issue.

The remainder of the paper proceeds as follows. Section 2 presents the methodological approach outlined above, i.e., BMA and ridge regression. We then apply this approach in

¹The approach we follow here attaches the same prior inclusion probability to each variable (see Section 2). However, in general the researcher could attach a higher probability to variables deemed to be of special relevance.

Section 3 and Section 4, whereby the former presents the dataset and the latter the results. Finally, Section 5 concludes and provides discussion on further research.

2 Model Specification and Estimation

In the following subsections we give a brief overview of the methods we apply. First, we highlight the advantage of ridge regression. Second, we refine our methodology by introducing the spike and slab approach, a specific BMA technique, to account explicitly for model uncertainty.

In order to introduce the methodological approach presented in this paper, we start with the familiar framework of linear regression. Here, we assume that the relationship between the *logit transformed* aggregated default rates, as response variable, \mathbf{y} ($N \times 1$) and the design matrix of the explanatory variables (here the macro variables) \mathbf{X} ($N \times K$) is given by the linear regression

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon},\tag{1}$$

where $\epsilon \sim N(0, \sigma^2 \mathbf{I}_N)$. The vector $\boldsymbol{\beta}$ denotes the parameter vector of interest. Assuming that the explanatories X are highly correlated the standard OLS estimator $\hat{\boldsymbol{\beta}} = (\boldsymbol{X'X})^{-1}\boldsymbol{X'y}$ might be ill-conditioned (multicollinearity). In particular, at least one of the eigenvalues η_k of $\boldsymbol{X'X}$ will move towards zero, inflating the variance of the OLS estimator $E((\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})'(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})) = \sigma^2 \sum_K \eta_k^{-1}$.

2.1 Ridge Regression and Bayesian Ridge Regression

Ridge regression (see Hoerl & Kennard, 1970b) belongs to the class of shrinkage methods in the context of linear regression models. In contrast to well known subset selection algorithms (e.g., Forward Stepwise Selection) it does not retain a subset of predictors and discard the rest but shrinks the size of predictors proportionally in accordance with their importance (Friedman et al. 2009). To see why this is so valuable, imagine the usual setup of highly correlated variables in the design matrix leading to large positive and negative coefficients and thus to unreliable results. Indeed, multicollinearity may result in poorly determined parameters. One way to deal with multicollinearity is the use of ridge regression. From a frequentist point of view, ridge regression solves the optimization problem

$$\hat{\boldsymbol{\beta}}^{\text{ridge}} = \underset{\boldsymbol{\beta}}{\operatorname{argmin}} \left\{ (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) + \lambda \sum_{j=1}^{K} |\beta_j|^2 \right\}.$$
(2)

The Lagrangian parameter λ defines how much the classical OLS- β s are shrunk. If λ moves towards 0 then the constraint is not binding and one arrives at the OLS solution. As for OLS, it is possible to give a closed solution of the ridge regression problem, which is given by

$$\hat{\boldsymbol{\beta}}^{\text{ridge}} = (\mathbf{X}'\mathbf{X} + \lambda \mathbf{I})^{-1}\mathbf{X}'\mathbf{y}.$$
(3)

The ridge regression solution is very similar to the OLS solution (except for the term $\lambda \mathbf{I}$) and is linear in the response variable \mathbf{y} . The proportional shrinkage of the ridge parameters via the L_2 norm in Equation (2) provides the ability to cope with correlated variables as large coefficients are penalized. Clearly, a precondition of ridge regression is the standardization of regressors in order to treat variables measured on different scales equally. An analogous approach to ridge regression is available in a Bayesian setting. *Bayesian ridge regression* was first introduced by Hsiang (1975). Keeping the assumptions of linear regression and setting $\lambda = \sigma^2/\tau^2$ one implements the following hierarchical Bayesian model:

$$\mathbf{y}|\boldsymbol{\beta}, \sigma^2 \sim N(\mathbf{X}\boldsymbol{\beta}, \sigma^2 \mathbf{I}_n),$$
 (4)

where the prior specifications of the coefficients β is given by

$$\boldsymbol{\beta}|\tau^2 \sim \prod_{j=1}^P N(0,\tau^2),\tag{5}$$

with proper priors² for the variances σ^2 and τ^2 .

²Any inverted gamma prior for σ^2 and τ^2 would maintain conjugacy. Here we use the limiting improper priors $\frac{1}{\sigma^2}$, respectively $\frac{1}{\tau^2}$.

The prior on β conditional on τ and the fact that $\boldsymbol{y} \sim N(\boldsymbol{X}\beta, \sigma^2 \boldsymbol{I}_n)$ allows for the use of Markov Chain Monte Carlo (MCMC) sampling to estimate the posterior distribution of interest.

2.2 Model Uncertainty

As outlined in the introduction, an important task in statistical modeling is the choice of an optimal model from the set of all possible models. With K potential explanatory variables, one faces 2^{K} possible combinations of regressors. Selecting the best model out of 2^{K} linear models is a challenging task. In addition, several models with similar performance might arise which does not allow for an unambiguous single best choice. Thus, the uncertainty associated with a selected model is an important aspect, especially when it comes to forecasting (see Steel, 2011). One natural way to deal with model uncertainty is to pool over the considered models — as BMA does. Thereby, weights of the single models depend on how much the data support each model via the posterior distribution. An excellent review of BMA is given in Hoeting et al. (1999). Using BMA, one obtains the distribution of some quantity of interest β , e.g., the effect of a macro-variable, by averaging inference over all models M_k

$$P(\boldsymbol{\beta}|\boldsymbol{Z}) = \sum_{l=1}^{2^{K}} P(\boldsymbol{\beta}|M_{l}, \boldsymbol{Z}) P(M_{l}|\boldsymbol{Z}),$$
(6)

where $P(M_k|\mathbf{Z})$ is the posterior probability of model M_k given the whole dataset \mathbf{Z} (\mathbf{X} and \mathbf{y} combined) and is derived by

$$P(M_k | \mathbf{Z}) = \frac{P(\mathbf{Z} | M_k) P(M_k)}{\sum_l P(\mathbf{Z} | M_l) P(M_l)},$$
(7)

where $P(M_k)$ is the prior probability of model M_k and $P(\mathbf{Z}|M_k)$ is the marginal or integrated likelihood of model M_k obtained by integrating over the parameters (see Hoeting et al., 1999). Suitable choices of prior inclusion probabilities $P(M_k)$ allow to control the expected model size, i.e., the number of included parameters. In order to sample different models M_k of varying size and average across them, we make use of *spike and slab* priors (Mitchell & Beauchamp, 1988; George & McCulloch, 1993, 1997).

2.2.1 Model Uncertainty via the Spike and Slab Approach

The central point in using spike and slab priors is to assign each coefficient a prior which is a mixture of a point mass at zero and a specified "slab" distribution. This allows to exclude variables from the regression. In this sense spike and slab constitutes an optimal supplement to ridge regression which alone does not provide variable selection. Formally, we modify the prior defined in Equation (5) and use for all considered regressions discussed in this work a *coefficient prior* of the form

$$P(\beta_j | c_j, \tau, \sigma^2) \sim (1 - c_j) I_0 + c_j \pi(\tau),$$
 (8)

where c_j is a binary random variable with success probability $\gamma = P(c_j = 1)$ (which we set to the same value for all candidate regressors j). $\pi(\tau)$ is the prior distribution of β_j defined by Equation (5).

The Posterior Inclusion Probability ($P(c_j = 1 | \mathbf{Z})$ or PIP³) of each variable j contains valuable insights about the importance of variable j. In particular, the PIP is of high value as it displays the fraction of models visited in which variable j was selected, $P(c_j = 1 | \mathbf{Z})$.⁴ PIP can thus be understood as a measure of "posterior importance" of a given variable and is a widely used measure in Bayesian Model Averaging (see Sala-I-Martin et al., 2004).

2.2.2 Model size

We have not yet discussed in detail the specification of the prior variable inclusion probabilities used by the spike and slab approach in Equation (8). One possible approach would be to assign each variable β_j an uninformative inclusion probability of $\gamma = 0.5$, i.e., c_j is drawn from a Bernoulli distribution Be(0.5). This has the odd and troubling implication that we assume the number of included variables to be very large (see Sala-I-Martin et al., 2004). In particular, the expected model size, $E[M_j]$, equals $K \times 0.5$, where K is the number of

³For convenience we omit subscripts to PIP throughout this paper.

⁴See Mitchell & Beauchamp (1988).

candidate regressors. In our case, as explained below, we have 160 candidate regressors to choose from, K = 160, which would results in a very large prior model size, $E[M_j] = 80$. Models of this size are uncommon as researchers and practitioners prefer smaller models. Therefore, instead of choosing one value for the prior model size, we specify a range of values for prior mean model sizes \bar{k} , with each variable having a prior inclusion probability of $\gamma = \bar{k}/K$, independent of the inclusion of other variables. We estimated our models for 9 different expected prior model sizes, $\bar{k} \in \{5, 7, 9, 11, 16, 22, 28, 40, 80\}$ resulting in the prior inclusion probabilities shown in Table V.A.

\bar{k}	5	7	9	11	16	22	28	40	80
γ	0.031	0.044	0.056	0.069	0.1	0.14	0.18	0.25	0.5

Table V.A: Prior model size and associated prior inclusion probabilities for the single variables.

We follow Sala-I-Martin et al. (2004) in assuming that most researchers strongly prefer models containing a large number of variables so we will concentrate on models with prior model sizes between 5 and 16 variables. This is also in line with the fact that most empirical models on aggregated default rates (see Simons & Rolwes, 2009; Liu, 2009) use moderate numbers of explanatory variables. Our benchmark model will have the prior model size of $\bar{k} = 7$. While we calculate results for large models as well, we will not focus our attention on these cases when it comes to interpretation.

2.3 Estimation

In order to estimate our models, we used Markov Chain Monte Carlo (MCMC) methods. In particular, the Gibbs sampler ran for 200,000 iterations, using a thinning of 10. The first 10,000 draws were discarded as burn-in period. This results in 19,000 draws from the posterior for each parameter of interest. All the computations are done using JAGS (Just another Gibbs sampler) and its R (see R Development Core Team, 2011) interface packages **rjags** (see Plummer, 2011). MCMC diagnostic is done with the package **coda** (see Plummer et al., 2010).

3 Data

We now apply the presented framework of BMA with ridge regression to analyze aggregate default probabilities in Austria. A common approach taken in the literature (see e.g. Simons & Rolwes, 2009; Foglia et al., 2009, among many others) is to use firm default frequencies as proxy for default probabilities. We follow this line by basing our analysis on quarterly corporate insolvency frequencies for the period between January 1987 and April 2011. These insolvency rates are aggregated over all Austrian corporate sectors and are calculated by dividing the number of quarterly defaults by the total number of firms, which results in quarterly aggregated default rates, pd. The number of firm defaults and the total number of firms were obtained from the Austrian creditor association Kreditschutzverband von 1870. As noted above, we transform default rates via the logit function, i.e.,

$$\boldsymbol{y} := \operatorname{logit}(\boldsymbol{pd}).$$

The set of potential explanatory variables contains 32 different macroeconomic variables which are taken from the database of Oesterreichische Nationalbank (OeNB).⁵ These macroeconomic variables are part of the Austrian Quarterly Forecast Model (AQM) and are used for forecasting by the OeNB twice a year. As this dataset reflects the variable set of a macroeconomic forecasting model, our results can be used to integrate the time-series of credit defaults into the macro-model, or implement a stress testing framework building on the respective macroeconomic forecasts.

Another advantage of using this dataset is that the list of candidate regressors covers multiple aspects of the economic environment. We consider financial regressors, like interest rates, the stock index and credit amount outstanding, private sector indicators, e.g., private consumption and disposable income, as well as general and external trade related variables, like GDP, exports and investment. Additionally, various price indicators, like the harmonized consumer price index or the oil price, are included.

⁵The only exception is the ATX, Austrian Traded Index, which was taken from Datastream.

This large set is even further increased by adding lags up to 4 quarters of each candidate regressor, hence resulting in a design matrix X containing 160 explanatory variables each with 97 quarterly observations. The variable names, the applied transformation as well as two of their autocorrelation coefficients are illustrated in Table V.B. The variables included were transformed as indicated in column 2 in Table V.B to ensure stationarity of the timeseries. "YoY-Log-Difference" equals a transformation of the original time-series, X_t , by $\log X_t - \log X_{t-4}$, "YoY-Difference" by $X_t - X_{t-4}$ and "YoY-Rel-Difference" by $(X_t/X_{t-4}) -$ 1 where t is the time indicator in quarters.⁶

4 Results

In this section we present the results from the combined approach of BMA with ridge regression described in Section 2 applied to the Austrian dataset. To assess variable importance, we calculate the posterior inclusion probabilities (PIP). These are a central quantity within BMA to measure a variable's importance (see Sala-I-Martin et al., 2004). In line with prior research (and intuition), we focus on variables with a higher PIP than their prior inclusion probability, i.e., variables that are deemed more important after consideration of the data. Additionally, means and standard deviations of the coefficients — conditional on model inclusion — are displayed.

4.1 Macroeconomic predictors of firm failure rates: Baseline estimation

We are now ready to present the baseline estimation results with a prior model size⁷ of 7. We find a posterior mean of 10.12, which is clearly above the prior model size and suggests that the posterior puts more importance on models with more explanatories.⁸

⁶Note that this transformation is followed by a standardization (subtraction of mean and division by standard deviation) within the ridge regression.

⁷Note that as described in Section 2.2.2 a prior model size of 7 does not mean each model includes exactly 7 variables, but that each candidate regressor has a probability of inclusion, which yields on average a model size of 7.

⁸For the sake of completeness we provide here posteriors related to the shrinkage parameter (see Section 2.1). We find for the shrinkage parameter $\lambda = \sigma^2/\tau^2$ a posterior mean of 0.72012, whereby flat

NAME	MEANING	TRANSFORMATION	MEAN	SD
PD	Aggregated default rates	logit	-5.309	0.186
		No Transformation	0.005	0.001
			LAG1	LAG4
ATX	Austrian Traded Index	YoY-Log Difference	0.797	0.041
CPN	Nominal private credit, amount outstanding	YoY-Log-Difference	0.891	0.401
DDR	Real domestic demand	YoY-Log-Difference	0.923	0.454
GEI	Government interest payment	YoY-Log-Difference	0.881	0.033
GON	Nominal gross operating surplus	YoY-Log-Difference	0.862	0.001
HEG	HIC for energy	YoY-Difference	0.750	-0.263
HEX	HIC for non-energy	YoY-Difference	0.863	0.437
HIC	Harmonized index of consumption prices	YoY-Difference	0.830	0.164
IEN	Nominal equipment investment	YoY-Log-Difference	0.876	0.003
IER	Real equipment investment	YoY-Log-Difference	0.884	0.042
ION	Nominal other investment	YoY-Log-Difference	0.881	0.040
IOR	Real other investment	YoY-Log-Difference	0.881	0.040
ITR	Real total investment	YoY-Log-Difference	0.861	0.042
LNN	Total employment	YoY-Difference	0.868	0.200
LTI	Nominal long-term interest rate	No Transformation	0.970	0.834
MTN	Nominal imports	YoY-Log-Difference	0.887	-0.004
MTR	Real imports	YoY-Log-Difference	0.909	0.120
PCN	Nominal private consumption	YoY-Log-Difference	0.958	0.638
PCR	Real private consumption	YoY-Log-Difference	0.957	0.669
POIL	Oil price in EUR	YoY-Difference	0.702	-0.411
PRO	Average labor productivity	YoY-Log-Difference	0.869	0.093
PSN	Nominal private sector savings	YoY-Log-Difference	0.788	-0.082
PYN	Nominal private sector disposable income	YoY-Log-Difference	0.896	0.379
PYR	Private sector disposable income, real	YoY-Log-Difference	0.849	0.167
STI	Nominal short-term interest rate	No Transformation	0.969	0.780
URX	Unemployment rate	YoY-Rel-Difference	0.779	-0.205
WIN	Nominal total compensation to employees	YoY-Log-Difference	0.959	0.685
WURYD	Real compensation per employee	YoY-Log-Difference	0.866	0.302
XTN	Nominal exports	YoY-Log-Difference	0.871	-0.037
XTR	Real exports	YoY-Log-Difference	0.880	0.020
YEN	Nominal GDP	YoY-Log-Difference	0.914	0.291
YER	Real GDP	YoY-Log-Difference	0.892	0.188

Table V.B: Set of candidate regressors. Additionally, up to four quarter lags are considered for each variable. The columns LAG1 and LAG4 display the autocorrelation coefficients for one and four quarters.

Table V.C presents the results of our analysis: The first column reports the PIP of the variables within the applied BMA framework. We sorted the variables in decreasing order of PIP and print only those which have a PIP greater than the prior inclusion probability.

From Table V.C we infer that the 3-month interest rate lagged by four quarters (STI L4)

⁽uninformative) hyperpriors on τ^2 and σ^2 were assumed. The posterior means of the variances σ^2 and τ^2 are 0.11893 and 0.16516 respectively.

has the highest PIP. Its posterior mean coefficient (P.MEAN) is positive and in line with standard economic theory that higher costs of funding imply higher PDs. Similarly, higher interest rates do not only increase the cost of funding but also prevent firms form receiving further funding due to bank lending standards, thus triggering firm failures. This finding is in line with previous literature. Vlieghe (2001), Liu & Wilson (2002) and Liu (2009) among many others report this strong and positive dependence between firm defaults and interest rates.

Interestingly, the second most important variable is the unlagged short term interest rate, STI, which has a negative posterior mean. The fact that the unlagged short term interest rate is negatively related to firm defaults is to the authors' knowledge a common puzzle in empirical works on aggregate credit risk (see e.g., Ali & Daly 2010 or Divino et al. 2008). However, there is an economic interpretation for this result. STI are usually highly correlated with central bank fund rates and these tend to be raised in economic boom phases to avoid overheating. Thus, higher short term interest rates are a timely measure for economic activity. Clearly, in economic good times PDs tend to decline.

On the third rank we find energy prices with a lag of one year, HEG L4. Energy prices constitute an essential determinant of factor prices and thus obviously pose a very relevant risk factor from the perspective of firms. Its posterior mean of 0.084 indicates a positive relationship between defaults and rising energy prices. This finding illustrates the power of BMA. While numerous papers have identified inflation as a determinant of aggregated default rates (see e.g. Foglia et al., 2009; Virolainen, 2004), we find a *component* of inflation, energy price rises, as one major factor. Owing to the application of BMA one is able to include components of indicators instead of the aggregates such as inflation or GDP yielding more precise conclusions.

The fourth most important variable according to its PIP is nominal import growth (MTN). To find imports among the top ranks is surprising as respective literature usually refrains to include it. However, the positive sign of the coefficient can be supported by several arguments. Firstly, imports by corporates are expenses. Ceteris paribus higher expenses increase the default probability. Secondly, more imports by private households could substitute domestic products which decrease the average revenue of domestic corporates. Thirdly, the time-series of imports might also catch exchange rate fluctuations to some extent, which in turn appear in papers as in Foglia et al. (2009) and Bhattacharjee et al. (2009).

The fifth (and tenth) highest PIPs can be observed for PRO L1 (PRO), the log difference of the average labor productivity. While the interpretation is less straight forward, it might be that an increase in labor productivity drives those firms out of the market which can not adopt such a productivity shock in their business strategy.

Furthermore, on the following ranks we find GON, gross operating surplus, and WIN, total compensation to employees. Both variables have the expected sign of the posterior mean. GON measures profits of firms which intuitionally lower firm defaults and is also reported in previous findings (see e.g. Liu, 2004; Liu & Wilson, 2002). WIN is the aggregate sum of wages paid out and according to our findings reduces the probability of a firm's default. It is important here to stress the difference to the variable WURYD, real compensation per employee, which appears on rank 13 with a lag of one year and a *positive* coefficient. While WURYD measures compensation per employee, WIN is the total sum across the economy. While seemingly related, there are important distinctions which also come apparent when regarding their opposite signs of posterior means. First, WURYD is measured *per employee* making it inversely related to the general employment level — or put differently, WIN is positively related to the general employment level, which constitutes another important variable at rank 19, LNN. Also, the finding of a positive coefficient on real compensation per employee confirms the results presented in Vlieghe (2001). Another difference is the time index with which both variables enter. While WIN enters without lag, thus reflecting more contemporary conditions, WURYD enters with a lag of 4.

On the ranks 8 to 9 we find government interest payments (GEI) and investment (ION). Both variables were selected in approximately 30% of the visited models. As the negative sign of ION indicates, investments reduce the number of defaults in the economy as also reported by e.g., Boss et al. (2007). Two channels may be responsible for this fact. First, investments reduce the number of firm defaults as they are a proxy for fresh equity induced into corporates. Second, it may also be that in times few firms default, managers decide to invest more, which results in a mutual dependence of both variables.⁹ However, the fact that investment enters with a lag of 2 (compare Table V.C) speaks in favor of the first channel. The appearance of GEI is less anticipated. The positive sign of its posterior mean (together with a relatively small posterior standard deviation) tells us that in times of high interest payment from the side of the government defaults tend to increase. Potentially, this finding reflects the increased economic uncertainty when sovereign spreads rise. Koopman & Lucas (2005) report a positive dependence of default rates with aggregated corporate spreads. As such a variable is missing in our dataset, GEI potentially acts as a proxy.

Beyond the "top 10", variables already discussed like other nominal investment, ION, and total employment, LNN, appear. In the majority of cases variables as well as posterior means are plausible from an economic perspective. However, private consumption, PCN, lagged by 4 quarters enters in most models with a positive sign. This puzzling finding may be explained by the fact that its unlagged cousin, private real consumption, PCR, enters with a negative sign at rank 16 (and further on rank 21 and 22). High private consumption one year ago might cause too optimistic turnover predictions on the side of firms, which begin to falter once stock levels do not sell. Such an interpretation is supported by the fact that contemporaneous (real) private consumption enters with the expected negative sign.

At the same time it is not only interesting to look at variables that were selected frequently, but also at variables that were *not* selected. Among those we find for example ATX, the

⁹In that case endogeneity of the variable would be present rendering the interpretation as a driver for (fewer) firm defaults incorrect.

Austrian Traded (stock market) Index. As the majority of firms are small enterprises, little (or no) dependence on stock market returns is plausible. Less anticipated is the fact that classical macroeconomic variables, especially GDP or disposable income, play also a minor role. In our model setting the data do not support their inclusion which confirms the findings of Simons & Rolwes (2009). However, this underlines the existence of model uncertainty and therefore the need for averaging over sets of possible models. Indeed, as noted above, by applying BMA we find components of general indicators, like investment of GDP and energy prices of inflation, as major risk drivers. BMA thus allows for a deeper insight into the matter of firm default determinants.

4.2 Model size robustness

So far, we presented the results of our baseline estimation with a prior model size of $\bar{k} = 7$. Although we believe that models with 7 expected variables are reasonable, this choice is somewhat arbitrary and the effects of using different prior model sizes need to be explored. For the 30 most substantial variables in the baseline model, Tables V.D and V.E present the PIP and posterior means given inclusion of different prior model sizes. The prior inclusion probabilities are simply given by the choice of \bar{k} divided by the number of possible variables, K = 160. For each prior model size, variables which appear within the 10 most substantial variables are printed in bold. Variables that are substantial in the baseline model but not when other priors are in use are printed in italics.

In total we find three variables which appear within the 10 most substantial variables for all considered model sizes. These are STI L4, WIN and GEI. From Table V.E we can infer that the signs of the variables are consistent across the different model sizes. Solely for two variables, PCR L4 and WURYD, we find for the prior model size $\bar{k} = 80$ controversial signs compared to the other considered models¹⁰. A summary of Table V.D is also displayed in Figure V.a where we show PIPs of the top variables in the benchmark model across the

 $^{^{10}}$ These variables are ranked 52 respectively 35 in the baseline model and appear therefore not in Tables V.D and V.E.


Figure V.a: PIPs for top variables in the benchmark model (in black) and less important variables (in grey) across varying prior model sizes. The dotted line illustrates the prior inclusion probabilities γ . For the x-axis a logarithmic scale is used.

range of prior model sizes.

Variables tending to lose importance when increasing prior model size. Four variables tend to lose importance — by dropping out of the "top 10" — when increasing the prior model size. These are STI, MTN, PRO L1 and GON. Moreover, PRO L1 even becomes unsubstantial for prior model sizes above 40. This suggests that such variables could be acting as "catching-all" for various other effects (see Sala-I-Martin et al., 2004). That means in smaller models these variables capture several effects and mechanisms in a combined form, while in larger models, these effects are broken up as more regressors are added. As a matter of fact, this in turn implies that when interpreting coefficients one has to focus even more on the *partial* character of the coefficient, i.e., measuring the effect given the inclusion of other regressors.

A good example is STI, unlagged short term interest rates, which becomes less substantial as

we increase the model size (see Figure V.a). For prior model sizes $\bar{k} = 40$ respectively $\bar{k} = 80$ STI appears on the 34th respectively 44th rank, while for our benchmark model it appears on the 2nd rank. On the other hand, variables like PCN (nominal private consumption, lagged 4 quarters), PCR (real private consumption), ION (nominal investment) and LNN (total employment) become more important for larger prior model sizes. This nourishes the hypothesis that short term interest rates might be a "catch all substitute" for private consumption and investment. The fact that PCR, ION and LNN enter without lag, i.e., in their contemporaneous form, also supports the interpretation mentioned before – that STI is a proxy of economic activity in smaller models.

Variables becoming "top 10" when increasing prior model size. Within the most substantial variables we find some variables which do not appear within the "top 10" set of the baseline model, but seem to become "top 10" when changing the prior model size. Nevertheless, all these variables are substantial in the baseline model (that is, show a higher PIP than prior inclusion probability) and are mostly ranked between the 11th and 20th rank in the baseline model $\bar{k} = 7$. These are nominal private consumption (PCN L4), real compensation per employee (WURYD L4), both lagged by one year, total employment (LNN), total compensation to employees (WIN L3) lagged by 3 quarters, private consumption rate (PCR) and nominal investment (ION). Additionally, we find the variables real domestic demand (DDR), its one year lagged values¹¹ (DDR L4) and nominal total compensation to employees (WIN L4), lagged by one year, appearing as "top 10" variable for some considered prior model sizes.

5 Conclusion and Discussion

In this paper we propose a fully Bayesian approach combining ridge regression and BMA to determine which macroeconomic variables are substantially related to aggregated probabilities of default. Compared to the literature which mainly focuses on one single model,

¹¹DDR L4 is ranked 39 for our baseline model and ranked 10 for the model $\bar{k} = 80$ with a PIP of 0.6193.

our approach addresses the problem of model uncertainty. Additionally, we propose ridge regression to deal with multicollinearity, an immanent problem in case lagged variables are included. In our benchmark model the most frequently selected candidate regressors indicate that firms' factor prices play a key role in determining defaults. Energy prices and interest rates lagged by one year are positively related to defaults in nearly all sectors. On the other hand, indicators of economic activity, like investment and contemporaneous short term interest rates, are associated with fewer firm defaults. As expected, firms' profits reduce the expected number of failures. Interestingly, classical macroeconomic variables, like GDP or disposable income, are less frequently selected. This finding underlines the need for an approach capable of dealing with model uncertainty — a feature Bayesian Model Averaging perfectly provides.

Finally, we show that the results of our baseline model are fairly robust to the choice of the prior model size. More precisely, when increasing the prior model size, variables do not change the sign of their posterior mean (with only 2 exceptions in 54 substantial variables considered). Moreover, most of the "top 10" variables remain within the 20 most important variables for other estimated prior model sizes. However, the relative importance of some regressors does change. This finding suggests that some variables being of high relevance in smaller models act as proxy for multiple effects combined which can be successively split into its components when considering models of larger size.

Further research is needed to better understand the dynamics of firm failures, a highly relevant time-series for regulators and banks alike. On one hand, the application of statistical approaches robust to model uncertainty should be applied on a dataset of wider geographical coverage. In line with the findings of Simons & Rolwes (2009) country specific circumstances need to be analyzed. Also, our methodological framework allows for the considerations of (even) more candidate regressors. On the other hand, form a methodological perspective our approach could be revamped in a way that allows the examination of common sets of variables. This would allow analyzing substitutional and complementary effects between the explanatories. That is, asking not only which variables were selected, but also which variables were selected *together*.

P.SD	0.07424	0.03197	0.08042	0.03486	0.03973	0.04922	0.04811	0.03893	0.02815	0.12336	0.04667	0.14971	0.07839	0.07677	0.04427	0.03397	0.06636	0.05714	0.06654	0.05737	0.05292	0.05501	0.08215	0.05974	0.08160	0.03107	0.04717	
P.MEAN	-0.12546	0.06891	0.12613	0.06540	-0.06694	-0.06964	0.08202	0.07033	-0.05976	-0.13917	0.07822	0.16775	0.11667	-0.08299	0.07200	0.05129	0.08118	0.07965	-0.10317	-0.06871	0.08072	-0.03837	0.07022	0.08639	-0.12006	0.05605	0.07015	
PIP	0.10055	0.09220	0.08680	0.07895	0.07675	0.07365	0.07205	0.06620	0.06325	0.06205	0.06205	0.05860	0.05790	0.05780	0.05720	0.05690	0.05640	0.05590	0.05185	0.05160	0.05070	0.04900	0.04890	0.04795	0.04590	0.04500	0.04380	
NAME	YEN	WURYD L1	MTN L1	HEG L3	IOR L1	IOR	HEG L 2	WURYD	CPN L3	STI L1	HIC L4	DDR L4	MTR L1	WIN L1	HEX L4	GEI L1	MTR	\mathbf{XTR}	PCR L3	GON L1	DDR L2	IOR L2	STIL3	NTN	PCR L4	PRO L4	ITR L1	
Я	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	
P.SD	0.10891	0.07130	0.02716	0.06266	0.03609	0.04303	0.05773	0.02431	0.02744	0.04294	0.06898	0.03077	0.04346	0.05158	0.06706	0.04336	0.03721	0.02442	0.03024	0.03964	0.04514	0.05754	0.05970	0.04509	0.07090	0.08902	0.06439	
P.MEAN	0.28701	-0.24265	0.08353	0.19201	0.12603	-0.14711	-0.14142	0.06928	-0.08130	0.11102	0.16323	-0.08711	0.10794	0.12784	-0.13192	-0.09409	0.08565	-0.06264	-0.07927	-0.08061	-0.09098	-0.11796	-0.11908	0.10109	0.14892	-0.16907	-0.08806	
PIP	0.84800	0.56610	0.41610	0.38210	0.37815	0.34515	0.31540	0.30640	0.29725	0.20840	0.20200	0.20145	0.18160	0.16495	0.15040	0.14005	0.13775	0.13020	0.12955	0.12670	0.12265	0.12205	0.12170	0.11715	0.11510	0.11000	0.10510	
NAME	STIL4	\mathbf{STI}	HEG L 4	MTN	PRO L1	GON	MIN	GEI	ION L2	PRO	PCN L4	ION L1	WURYD L4	DDR L1	WIN L3	PCR	PRO L 2	CPN L4	LNN	ION	PCR L1	PCR L2	WIN L2	ITR	DDR	WIN L4	ION L3	
R	-	2	°°	4	Ŋ	9	7	∞	6	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	

Table V.C: Baseline model for all 54 variables with a PIP greater than the prior inclusion probability γ of 7/160 = 0.0437. P.MEAN and P.SD denote the coefficient's mean and standard deviation given its model inclusion.

STIL4	0.8977	0.8480	0.7716	0.7525	0.7376	0.7561	0.7478	0.7620	0.8301
\mathbf{ITS}	0.6434	0.5661	0.4334	0.3777	0.2894	0.2434	0.2637	0.2938	0.5110
HEG L 4	0.5004	0.4161	0.4031	0.2821	0.2786	0.3287	0.3650	0.4589	0.6931
NTM	0.5003	0.3821	0.3830	0.2432	0.2477	0.2827	0.3063	0.3778	0.5773
PRO L1	0.4746	0.3781	0.3261	0.2151	0.1351	0.1547	0.1762	0.2245	0.4718
GON	0.4624	0.3452	0.3396	0.2202	0.2056	0.2630	0.2877	0.3715	0.5605
MIN	0.3115	0.3154	0.4320	0.4496	0.5112	0.5301	0.5524	0.5873	0.7291
GEI	0.2124	0.3064	0.2926	0.4116	0.4652	0.4625	0.4965	0.5425	0.6961
ION L2	0.3962	0.2973	0.2917	0.2040	0.1804	0.1900	0.2284	0.2973	0.5243
PRO	0.1605	0.2084	0.1615	0.1935	0.2145	0.2480	0.2690	0.3317	0.5613
PCN L4	0.1137	0.2020	0.3283	0.4061	0.5248	0.5644	0.6192	0.6407	0.7210
ION L1	0.1202	0.2014	0.1966	0.1973	0.2496	0.2624	0.2861	0.3356	0.5424
VURYD L4	0.0702	0.1816	0.2873	0.3939	0.5983	0.7018	0.8087	0.8923	0.9527
DDR L1	0.0672	0.1650	0.1363	0.1727	0.1930	0.2235	0.2441	0.3200	0.5372
WIN L3	0.0983	0.1504	0.1263	0.2204	0.3073	0.3534	0.4057	0.4584	0.6277
PCR	0.1677	0.1401	0.2170	0.2561	0.2958	0.3374	0.3663	0.4351	0.6478
PRO L 2	0.0973	0.1378	0.1492	0.1961	0.2399	0.2435	0.2691	0.3281	0.5444
CPN L4	0.0573	0.1302	0.1050	0.1610	0.2037	0.2424	0.2687	0.3141	0.4890
LNN	0.0282	0.1295	0.1593	0.2505	0.4216	0.5112	0.6061	0.7140	0.8555
ION	0.1541	0.1267	0.1953	0.2354	0.2950	0.3623	0.3838	0.4503	0.6117
PCR L1	0.0864	0.1226	0.1185	0.1542	0.1943	0.2410	0.2718	0.3648	0.5813
PCR L2	0.0470	0.1221	0.1086	0.1828	0.2170	0.2751	0.3332	0.3979	0.5917
WIN L2	0.0801	0.1217	0.1252	0.1694	0.2147	0.2766	0.3140	0.4021	0.6189
ITR	0.0901	0.1172	0.1556	0.1893	0.2575	0.2998	0.3181	0.3805	0.5955
DDR	0.1691	0.1151	0.2393	0.2614	0.3001	0.3321	0.3612	0.4008	0.5663
WIN L4	0.0521	0.1100	0.1766	0.2235	0.2965	0.3076	0.3438	0.3833	0.5508
ION L3	0.0725	0.1051	0.0961	0.1238	0.1642	0.1978	0.2215	0.2846	0.5123
YEN	0.0585	0.1006	0.0836	0.0825	0.1154	0.1704	0.2035	0.2783	0.5395
VURYD L1	0.0812	0.0922	0.1184	0.1509	0.1910	0.2281	0.2402	0.3040	0.5232
MTN L1	0.0625	0.0868	0.0732	0.1127	0.1382	0.1681	0.2152	0.2713	0.5088

k = 80	0.0669	-0.0201	0.0446	0.0335	0.0174	-0.0329	-0.0531	0.0406	-0.0260	0.0323	0.0532	-0.0292	0.0737	0.0263	-0.0417	-0.0408	0.0300	-0.0236	-0.0580	-0.0360	-0.0337	-0.0356	-0.0405	0.0359	0.0334	-0.0289	-0.0233	-0.0273	0.0275	0.0212
k = 40	0.1053	-0.0453	0.0576	0.0614	0.0303	-0.0611	-0.0828	0.0516	-0.0420	0.0498	0.0922	-0.0497	0.0885	0.0482	-0.0732	-0.0589	0.0478	-0.0389	-0.0698	-0.0582	-0.0529	-0.0589	-0.0649	0.0567	0.0660	-0.0654	-0.0400	-0.0420	0.0417	0.0404
k = 28	0.1320	-0.0750	0.0627	0.0758	0.0441	-0.0755	-0.1004	0.0571	-0.0517	0.0617	0.1134	-0.0624	0.0944	0.0630	-0.0955	-0.0694	0.0581	-0.0455	-0.0733	-0.0685	-0.0616	-0.0736	-0.0790	0.0710	0.0867	-0.0942	-0.0509	-0.0558	0.0511	0.0579
k = 22	0.1523	-0.0973	0.0679	0.0905	0.0565	-0.0860	-0.1150	0.0592	-0.0572	0.0722	0.1234	-0.0694	0.0964	0.0725	-0.1057	-0.0771	0.0665	-0.0492	-0.0745	-0.0746	-0.0670	-0.0791	-0.0928	0.0817	0.1016	-0.1086	-0.0579	-0.0715	0.0578	0.0644
k = 16	0.1798	-0.1404	0.0727	0.1042	0.0698	-0.0957	-0.1274	0.0644	-0.0661	0.0809	0.1446	-0.0787	0.1037	0.0861	-0.1207	-0.0851	0.0750	-0.0549	-0.0761	-0.0785	-0.0740	-0.0919	-0.1065	0.0887	0.1167	-0.1390	-0.0607	-0.0651	0.0630	0.0822
k = 11	0.2324	-0.1931	0.0770	0.1443	0.1066	-0.1201	-0.1438	0.0672	-0.0770	0.0966	0.1587	-0.0836	0.1105	0.1041	-0.1302	-0.0938	0.0829	-0.0569	-0.0778	-0.0831	-0.0838	-0.1070	-0.1171	0.0943	0.1406	-0.1723	-0.0785	-0.0774	0.0676	0.1095
k = 9	0.2643	-0.2311	0.0841	0.1859	0.1223	-0.1409	-0.1512	0.0659	-0.0801	0.1013	0.1653	-0.0866	0.1135	0.1120	-0.1115	-0.1055	0.0855	-0.0550	-0.0734	-0.0858	-0.0861	-0.1070	-0.1112	0.0935	0.1580	-0.1771	-0.0721	-0.0965	0.0674	0.0879
k = 7	0.2870	-0.2426	0.0835	0.1920	0.1260	-0.1471	-0.1414	0.0693	-0.0813	0.1110	0.1632	-0.0871	0.1079	0.1278	-0.1319	-0.0941	0.0856	-0.0626	-0.0793	-0.0806	-0.0910	-0.1180	-0.1191	0.1011	0.1489	-0.1691	-0.0881	-0.1255	0.0689	0.1261
k = 5	0.3073	-0.2608	0.0850	0.2062	0.1316	-0.1526	-0.1490	0.0702	-0.0825	0.1166	0.1563	-0.0891	0.1083	0.1200	-0.1248	-0.1032	0.0889	-0.0573	-0.0546	-0.0919	-0.0915	-0.0824	-0.1158	0.1086	0.1776	-0.1522	-0.0813	-0.1159	0.0707	0.1362
	STIL4	ILS	HEG L4	NTN	PRO L1	GON	WIN	GEI	ION L2	PRO	PCN L4	ION L1	WURYD L4	DDR L1	WIN L3	PCR	PRO L2	CPN L4	TNN	ION	PCR L1	PCR L2	WIN L2	ITR	DDR	WIN L4	ION L3	YEN	WURYD L1	MTN L1



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