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Sunk Costs, Depreciation, and Industry Dynamics

Adelina Gschwandtner[*] and Val E. Lambson[**]

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Abstract: Two of the most robust results from dynamic competitive models of industrial organization suggest that higher sunk cost industries should exhibit (1) higher intertemporal variability in the market value of their firms, and (2) lower intertemporal variability in the size of their industries. These predictions have done well empirically. This paper argues on theoretical and empirical grounds that depreciation generates countervailing effects.

Keywords: Sunk costs, depreciation, entry and exit, irreversible investment.

JEL Classification: L00

1. Introduction

The study of competitive industry dynamics has contributed substantially to our understanding of industry behavior and its determinants. The theoretical and empirical strands of this literature complement each other nicely.[1] An important component of this literature is the effect of sunk costs. Two of the most robust theoretical results are that higher sunk cost industries should exhibit greater variability in the value of their firms over time and smaller variability in industry size over time. These results are intuitive. Sunk costs are naturally defined as the difference between entry costs and scrap values. Natural equilibrium conditions require that values be bounded above by entry costs (because higher values would provoke more entry) and bounded below by scrap values (because lower values would provoke more exit). These definitions immediately deliver the result that higher sunk costs allow a greater range of values. Furthermore, higher sunk costs make entry and exit more expensive, reducing the amount of entry and exit and the variability of industry size.

This paper extends previous analysis by introducing depreciation, a phenomenon that has received little attention in this corner of the literature.[2] On the theory side, the paper contributes a formal analysis of the industry-level effects of depreciation. Potential difficulties arise because when there is depreciation the value of a firm depends on the age distribution of its various capital components. Under the assumption that depreciation is geometric (for which there is some empirical support), we argue that a change in depreciation rates at the industry level is analogous to a change in the discount factor. This makes the analysis tractable, and generates the result

that depreciation partially reverses the effects of sunk costs. On the empirical side, we construct depreciation indexes from existing estimates of depreciation rates and use them to test the theoretical predictions.

Depreciation attenuates the effects of sunk costs by moving firms back toward their initial conditions. This attenuation can be illustrated by comparing two extreme cases: the complete absence of depreciation and very rapid depreciation. Without depreciation, small shocks are absorbed entirely by changes in the capitalized value of existing firms; only larger shocks trigger entry and exit. By contrast, with rapid depreciation, firms essentially start from scratch each period. Thus the equilibrium path exhibits constant market values equal to entry costs and all adjustment is accomplished by entry and exit. As the comparison of these extreme cases suggests, faster depreciation is associated with lower intertemporal variability of market values and higher intertemporal variability of industry size.

Section 2 heuristically describes the model that is formally described in the mathematical appendix. Section 3 describes the measures of depreciation employed in the empirical analysis. These measures, constructed from estimates collected from the existing literature, are derived mostly from market prices of second-hand capital goods, and hence are arguably more reliable than those based on accounting data. Section 4 presents the empirical findings. Section 5 concludes.

2. Theory

Let $(\xi_1,...,\xi_l)$ be the list of expenditures on each input required to create a unit of capital in a particular industry.[3] With $\xi := \Sigma_i \xi_i$ defined to be the cost of constructing a unit of capital and with χ defined as the scrap value of a unit of capital, $(\xi - \chi)$ is a natural definition of sunk cost. Let λ_i be the depreciation rate for the ith capital input. Assume these depreciation rates can be aggregated into a single depreciation rate, say λ, for the industry. Consistent with empirical observation by Jorgenson (1996) and others, assume capital depreciates at a constant rate. Specifically, if new capital (of age $\tau = 0$) is worth V per unit, then a unit of capital of age τ is worth $V(1 - \lambda)^\tau$. In this context, $V \in [\chi, \xi]$ is a natural equilibrium restriction, because if $V > \xi$ then there is an incentive to build capital while if $V < \chi$ then there is an incentive to scrap capital.

This aspect of equilibrium directly generates the first class of empirical implications. These concern the intertemporal variability of firm value. Specifically, if the value of a firm is approximately equal to the value of its capital, then the intertemporal range of a firm's value, denoted R(V), is approximately

(2.1) $\quad R(V) = max_t \sum_\tau V_t x_{t\tau} (1 - \lambda)^\tau - min_t \sum_\tau V_t x_{t\tau} (1 - \lambda)^\tau,$

where V_t is the unit value of new capital at time t and $x_{t\tau}$ is the amount of the firm's capital that is τ periods old at time t. Normalizing (2.1) with respect to labor yields:

(2.2) $\qquad R\left(\frac{V}{L}\right) = max_t V_t \left[\frac{\sum_\tau x_{t\tau}(1-\lambda)^\tau}{L_t}\right] - min_t V_t \left[\frac{\sum_\tau x_{t\tau}(1-\lambda)^\tau}{L_t}\right],$

where R(V/L) is the range of value per worker and L_t is the amount of labor the firm employs in period t. This normalization fulfills two purposes: (1) it controls for size, thus distinguishing sunk cost and depreciation explanations of variability from other, size-related explanations, and (2) with some additional structure, it mimics an investment-level analysis. Specifically, suppose the technology is Leontief or, alternatively, suppose the technology is homothetic and factor prices are stable. This implies a constant capital-labor ratio, which in turn implies that the maximum and the minimum in (2.2) are respectively achieved when $V_t = \xi$ and $V_t = \chi$. Then (2.2) can be manipulated as follows:

$$(2.3) \quad [R\left(\frac{V}{L}\right)] = max_t V_t \left[\frac{\sum_\tau x_{t\tau}(1-\lambda)^\tau}{L_t}\right] - min_t V_t \left[\frac{\sum_\tau x_{t\tau}(1-\lambda)^\tau}{L_t}\right]$$

$$= \xi\left[\frac{\sum_\tau x_{t\tau}(1-\lambda)^\tau}{L_t}\right] - \chi\left[\frac{\sum_\tau x_{t\tau}(1-\lambda)^\tau}{L_t}\right]$$

$$= (\xi - \chi)\left[\frac{\sum_\tau x_{t\tau}(1-\lambda)^\tau}{L_t}\right]$$

Converting to natural logs generates

$$(2.4) \quad \ln\left[R\left(\frac{V}{L}\right)\right] = \ln(\xi - \chi) + \ln\left[\frac{\sum_\tau x_{t\tau}(1-\lambda)^\tau}{L_t}\right].$$

Equation (2.4) predicts a positive correlation between the range of normalized firm value and sunk costs. It also predicts a negative correlation between the range of normalized firm value and depreciation. More specifically, (2.4) predicts the following effects:

$$(2.5) \quad \frac{\partial \ln R(V/L)}{\partial(\xi-\chi)} = \frac{1}{(\xi-\chi)} > 0 \quad \text{and}$$

$$(2.6) \quad \frac{\partial \ln R(V/L)}{\partial\lambda} = \frac{-\sum_\tau[\tau x_\tau(1-\lambda)^{\tau-1}]}{\sum_\tau[x_\tau(1-\lambda)^\tau]} = -\mu(\tau)/(1-\lambda) < 0$$

5

where $\mu(\tau)$ stands for the weighted average age of the firm's capital stock. Instead of

assuming a stable capital-labor ratio, one can assume a stable capital-output ratio, which

suggests the interpretation of capital as capacity. Then similar reasoning implies

$$(2.7) \qquad \frac{\partial \ln R(V/S)}{\partial(\xi-\chi)} > 0 \quad \text{and}$$

$$(2.8) \qquad \frac{\partial \ln R(V/S)}{\partial \lambda} < 0,$$

where S is the firm's sales.

The second class of empirical implications concerns the intertemporal variability

of industry size. Lambson (1992, Theorem 4.4) provides theoretical support for the

proposition that the intertemporal range of industry size is decreasing in ξ and increasing

in χ. The effects of higher ξ are intuitive: they directly weaken the incentive to invest in

good times and they indirectly weaken the incentive to disinvest in bad times. The

indirect effect is due to the future protection from entry afforded by the higher sunk

costs. Both the direct and indirect effects work in the same direction: to reduce the

range of industry size over time. The effects of higher χ are the opposite of the effects of

higher ξ.

The effects of increasing depreciation rates are also intuitive. They are even more

readily apparent in the context of comparing the extreme case of no depreciation with

the extreme case of very rapid depreciation. In the first case, small economic shocks are

absorbed by changing market values within the set of values between entry costs and

scrap values. Larger economic shocks generate entry or exit as well. These outcomes

can be compared with those of the opposite extreme case wherein depreciation is very rapid, market values are pinned down by entry costs, and all adjustment takes the form of changing industry size. This suggests that depreciation rates are positively correlated with variability of industry size.

Even so, the conclusions for size variability are not as robust as the conclusions for value variability. First, entry costs and scrap values are not identified separately in the data. Second, similar changes in sunk costs, $(\xi - \chi)$, wrought by different combinations of changes in entry costs, ξ, and scrap values, χ, may not exhibit the same effects. Increasing ξ and χ by the same amount does not change sunk costs, $(\xi - \chi)$, but it can affect industry size. Finally, analysis of range, which is what is most natural in this context, is necessarily the analysis of changes in two distinct points. The significance of these matters is illustrated by the following example.

Suppose there are two market conditions, one good and the other bad, and assume that they follow an i.i.d. process. Let $\pi(k,g)$ and $\pi(k,b)$ be current profits in good times and bad times, respectively, when the industry is of size k.[4] Suppose that when market conditions change they change enough to provoke entry or exit. Then in good times, which occur with probability ρ, the value of a unit of new capital is ξ, there are N units of capital, and

(2.9) $\pi(N,g) + \delta(1\text{-}\lambda)[\rho\xi + (1 - \rho)\chi] = \xi$

where $\delta \in (0,1)$ is a discount factor. In bad times, which occur with probability $(1 - \rho)$, the value of a unit of new capital would be χ and the amount of capital, X, would satisfy

(2.10) $\pi(X,b) + \delta(1-\lambda)[\rho\xi + (1 - \rho)\chi] = \chi$.

Note that everything in (2.9) and (2.10) is exogenous except the industry sizes (number

of firms), N and X, which are determined thereby. Finally, subtracting (2.10) from (2.9)

yields

(2.11) $\pi(N,g) - \pi(X,g) = \xi - \chi$.

Now note that increases in λ must induce reductions in N and X to reestablish (2.9) and

(2.10). These reductions will automatically satisfy (2.11). Also note that the effect on the

relative sizes of the industry in good times and bad, N - X, is ambiguous, and will depend

on the concavity or convexity properties of π.   Still, the analysis of the extremes

establishes a tendency for a positive correlation between depreciation and the variability

of market size, and this can be taken to data.

Summary of Theoretical Results:

Value Results: $\dfrac{\partial \ln R\left(\frac{V}{L}\right)}{\partial(\xi-\chi)} > 0$ $\dfrac{\partial \ln R\left(\frac{V}{L}\right)}{\partial \lambda} < 0$ $\dfrac{\partial \ln R\left(\frac{V}{S}\right)}{\partial(\xi-\chi)} > 0$ $\dfrac{\partial \ln R\left(\frac{V}{S}\right)}{\partial \lambda} < 0$

Industry Size Results: $\dfrac{\partial \ln R(y)}{\partial(\xi-\chi)} < 0$ $\dfrac{\partial \ln R(y)}{\partial \lambda} > 0$ $\dfrac{\partial \ln R(k)}{\partial(\xi-\chi)} < 0$ $\dfrac{\partial \ln R(k)}{\partial \lambda} > 0$

where R(.), V, L, S, y, and k denote, respectively, range, value, employment, sales,

number of firms, and capital.

3. Measuring Depreciation

Jorgenson (1996) and Fraumeni (1997) discuss the empirical literature on depreciation. We agree with their view that economically relevant measures of depreciation are determined by the workings of resale markets for capital assets. Such measures are more likely to be economically relevant than, for example, accounting measures.

We require two different measures of depreciation: a firm-level measure to test the implications regarding the variability of value over time, and an industry-level measure to test the implications regarding the variability of size over time. To construct either measure requires estimates of depreciation for the capital inputs. We have taken these from Hulten and Wykoff (1981) as summarized by Jorgenson (1996) Table II. Hulten and Wykoff apply the Box–Cox power transformation to prices of used assets in order to estimate the rate and form of economic depreciation. This allows them to statistically discriminate between various patterns of depreciation (such as geometric, linear and one-hoss-shay depreciation patterns). They find that the observed depreciation patterns are approximately geometric. In a later paper, Hulten and Wykoff (1996) revise and extend these measures to include the effect of obsolescence, defined as the decline in price resulting from the introduction of new vintages of capital. As a result, the revised rates are somewhat higher than the initial Hulten-Wykoff depreciation rates. The two sets of depreciation indexes based on these estimates will be referred to as HW1 and HW2, respectively. Finally, the Bureau of Economic Analysis (BEA) publishes

its own estimates of depreciation rates for use in the National Income and Product

Accounts. The estimates employed in the national accounts differ from the other two

depreciation measures in that they incorporate information about lifetimes and salvage

values of assets and accounting formulas permitted for tax purposes. The economic

depreciation rates for nonresidential structures estimated by Hulten and Wykoff are

much lower than those employed in the U.S. national accounts. The BEA depreciation

rates can be found, for example, in Jorgenson and Sullivan (1981). More recent, but not

very different, depreciation rates can be found in Fraumeni (1997).

We constructed the firm-level estimate of depreciation as a weighted sum of the

depreciation rates of the capital inputs used by the firm. The weights are estimates of

the firm's expenditures for the respective capital inputs. Specifically, the depreciation

index for firm f in industry F is

(3.1) $\Lambda_{Ff} = \sum_i \lambda_i \, P_{iF} \, (S_f/S_F)$

where S_f is firm f's average sales over time, λ_i is the depreciation rate of input i, P_{iF} is the

aggregate expenditure on input i in industry F, and S_F is industry F's average sales over

time. Of course, it would be preferable to have a direct firm-level measure of the usage

of the various inputs. However, if usage is roughly proportional to labor or sales, then

the index will generate fairly good estimates of firm-level use. For the industry's

expenditures on input i, P_{iF}, we used the capital flows table constructed by the Industry

Economics Division (IED) of the Bureau of Economic Analysis (BEA) of the United States

Department of Commerce. The capital flows table is a supplement to the benchmark

input-output accounts and it shows purchases of new structures, equipment, and software by industry. Specifically, the capital flows table lists the capital inputs used in each industry; we multiplied these by the depreciation rates for the respective industries in which the inputs were produced and then summed over inputs.[5]

We used similar methods to construct our depreciation measures for the analysis of the intertemporal size of an industry. In contrast to the analysis of intertemporal firm value, where each observation corresponds to a firm, here an observation corresponds to an industry. Of course, defining industries is seldom without difficulties. We assigned firms to industries according to SIC (NAICS) codes at the 2-4 digit level. If expenditure per unit of output (sales) is interpreted as a cost of capacity, then

$$(3.2) \qquad \Lambda_F = \sum_i \lambda_i \, P_{iF} / S_F$$

is roughly interpretable as the depreciation rate of capacity in industry F.[6]

4. Evidence

The first regressions test the theoretical propositions from Section 2 that the range of normalized firm value is positively correlated with sunk costs and negatively correlated with the rate of depreciation of its capital inputs. The database used for these purposes contains information on up to 4628 publicly traded manufacturing companies in the United States observed between 1970 and 2008. There is obvious selection bias in the firms that survive, but it is of little practical importance because the theory makes predictions about surviving firms. Most of the database was compiled from Standard and

Poor's Compustat. As is commonly done, we measured firm value as the sum of stock market capitalization and total liabilities. Stock market capitalization is calculated as the year-end closing price of common shares times the number of common shares outstanding. The closing price is the closing trade price for shares traded on a national stock exchange and the closing bid price for shares trading over-the-counter. We measured intertemporal variability in two ways: range and variance. Although theory favors range as the appropriate measure, variance is less sensitive to data problems that result from outliers. It turns out that one may remain agnostic as to which is the better measure: the regressions with range and the regressions with variance yield similar results. We normalized the dependent variable and the sunk costs proxy by the number of employees and by sales.

The regressions include both a sunk cost proxy as in our prior studies (namely, investment in property, plants and equipment) and a depreciation rate (explained above in Section 3). The results for the complete data set (1970-2008) are in Tables 1 and 2. The coefficient of the depreciation rate is negative, as predicted, and highly significant. The coefficient of the sunk cost proxy is positive, also as predicted, and highly significant. Concerned that 39 years might be too long to expect a firm to remain similar, we also divided the sample into ten year subsamples (1970-79, 1980-1989, 1990-99, 2000-2008). The results are in Tables 3 and 4. Our conclusions were unaffected. In addition to what we have reported, we tried several other specifications of independent variables as robustness checks, including the number of employees, capital expenditures, capital intensiveness (as measured by the capital-labor ratio), and new capital expenditures.

None of these variants had any significant effect on our conclusions. To summarize, the empirical analysis of the value conclusions, which are the theoretically better founded of the two classes of results, is strongly supportive of the theory.

The second class of theoretical results concerns the effects of sunk costs and depreciation on the variability of industry size over time. We used three different measures of industry size: employment, the number of firms, and the capital stock.

Employment variability is the best behaved of the three dependent variables relative to its theoretically predicted behavior. The results are in Table 5. The coefficients of both the depreciation and the sunk cost proxies are significant and of the predicted sign.

The number of firms is another possible measure of industry size. With sponsorship from the US Small Business Administration (SBA), the Census Bureau collects annual data on entry and exit by industry for the United States as a whole and for each state.[7] This database contains information about entry, exit, and employment from 1999-2006 for each included industry.[8] Our data do not include the number of firms. They do include annual data on entry and exit. Fortunately, this is adequate to calculate the range of the number of firms because, assuming there is enough variability over time and a long enough observation period to observe the extremes, the range of the number of firms is

$$R(y) = \max_\tau \{y_0 + \sum_{t=1}^{\tau} [n_t - x_t]\} - \min_\tau \{y_0 + \sum_{t=1}^{\tau} [n_t - x_t]\}$$

$$= \max_\tau \left\{ \sum_{t=1}^{\tau} [n_t - x_t] \right\} - \min_\tau \left\{ \sum_{t=1}^{\tau} [n_t - x_t] \right\}$$

where n_t and x_t are respectively entry and exit in period t. Since the initial number of

firms, y_o, cancels, not observing it poses no problem. The results are in Table 6. The

various specifications all exhibit coefficients of the predicted sign, but the results are

weaker. The depreciation coefficients are all strongly of the predicted sign, but the

coefficients of the sunk cost proxies (capital expenditures normalized by labor and by

sales), though of the predicted sign, are not statistically different from zero.[9]

Finally, the results from using capital as a measure of industry size are in Table 7.

The depreciation coefficients are statistically significant and of the predicted sign but the

sunk cost proxies, though of the predicted sign, are not statistically significant.

To summarize, the results of the empirical analysis of the size conclusions are not

as strongly supportive as the analysis of the value conclusions. The weaker results are

arguably due to a small sample size. When the estimates are significantly different from

zero, they always have the predicted sign.

5. Concluding Remarks

The field of industrial organization began as the study of imperfect competition.

Differences in profit rates across industries, a very well documented phenomenon, were

taken to be evidence that competition was imperfect. However, Lambson (1992)

showed that differing profit rates across industries are consistent with perfect

competition, even in the long run. Since maximizing average profits is not the firms'

objective, the market provides no mechanism to equalize them. Rather, if firms attempt

to maximize the expected present value of investments, then it is the value of a marginal

dollar of investment that will tend to equalize across investments. Under these

circumstances, it seems likely that any robust empirical implications will be inherently

dynamic. This paper has focused on some of these dynamic implications relating to the

effects of sunk cost and depreciation on intertemporal variability of capital values and

intertemporal variability of industry size, showing them to be broadly consistent with the

data.

6. Mathematical Appendix

Index the countably infinitely many time periods by the positive integers, $t \in$

$\{1,2,3,...\}$. The set of market conditions is M. A market condition, $m \in M$, describes the

values of the relevant exogenous variables such as factor prices and demand conditions.

Market conditions follow an exogenous stochastic process known to the firms. Let H_t be

the set of conceivable *t*-period histories of market conditions and define $H = \cup_t H_t$ as the

set of all finite-period histories. For $h \in H_t$ and $g \in H_s$, where *s>t*, let $\rho(g|h)$ be the

probability that the history g is realized given that the history h is realized. If g is the

history of market conditions through time t and k is the aggregate capital stock in the

industry at time t, then the current profit from owning a unit of capital is denoted $\pi(k,g)$

and can be interpreted as having been derived from the static competitive equilibrium,

with aggregate capital stock k, when market conditions are described by the last market

condition in g. An exit rule is a set of nodes such that no node in the set precedes any

other; an exit rule is interpreted as a set of nodes that would provoke exit if reached. If

the (endogenous) stochastic process governing the capital stock is $K = \{k_h\}_{h \in H}$, then the

expected present value of a unit of capital in the last period of the history $h \in H_t$ is

$V(K,h) = \pi(k,h) +$

$$sup_{\gamma \in \Gamma} \sum_{s=t+1}^{\infty} [\delta(1-\lambda)]^{s-t} \{\sum_{g \in \eta(\gamma,s)} \rho(g|h)\, \pi(k,g) + \sum_{g \in Hs \cap \gamma} \rho(g|h)\, \chi$$

where $\delta \in (0,1)$ is the discount factor, $\lambda \in (0,1)$ is the depreciation rate, Γ is the set of all

exit rules and $\eta(\gamma,s)$ is the set of s-period histories such that a firm using the exit rule γ

would not scrap any capital before or during the period s. For $h \in H_t$, let $h^{-1} \in H_{t-1}$ be the

history comprised of the first $t-1$ market conditions of h. An equilibrium is a stochastic

process K such that, for all t and all $h \in H_t$,

$V(K,h) = \xi$ if $k(h) > k(h^{-1})$

$V(K,h) \in [\chi,\xi]$ if $k(h^{-1}) = k(h)$

$V(K,h) = \chi$ if $k(h^{-1}) > k(h)$

Informally, firms build more capital only if the expected present value of the marginal

unit is not less than its cost, and firms scrap capital only if its expected present value

does not exceed its scrap value.

Two technical assumptions are useful. A1 allows proof of the existence of

equilibrium by backward induction in finite truncations, simple intermediate value

theorems and taking limits. The variable ϕ_h can be interpreted as a fixed production cost.

A2 implies that there is no incentive for investment if there is no possibility of future profitable production.

A1: For all *h*, $\pi(k,h)$ is continuous and decreasing in k with $\lim_{k \to 0} \pi(k,h) = \infty$ and $\lim_{k \to \infty} \pi(k,h) = -\phi_h < 0.$

A2: For all t and all $h \in H_t$, $\overline{V}(h) < \xi$ where

$$\overline{V}(h) = -\phi_h + \sup_{\gamma \in \Gamma}\left\{ \sum_{s=t+1}^{\infty} [\delta(1-\lambda)]^{s-t} \left[\sum_{g \in \eta(\gamma,s)} \rho(g \mid h)(-\phi_g) + \sum_{g \in \gamma \cap H_s} \rho(g \mid h)\chi \right] \right\}$$

The theorem establishes that an equilibrium exists and can be characterized by a stochastic sequence of ordered pairs.

Theorem: There exists a stochastic sequence of ordered pairs, {N_h,X_h} such that the stochastic sequence K satisfying k_h = min{X_h,max{N_h,k_{h-1}}} for all h∈H is an equilibrium.

Proof: For all T, all t ≤ T, and all h∈H_t, define

$$w_T(k,h) = \chi \quad \text{if } h \in H_T$$

$$w_T(k,h) = \pi(k,h) + \delta(1-\lambda) \sum_{g \in H_{t+1}} \rho(g \mid h) w_T(k_{gT}, g) \quad \text{otherwise,}$$

where $k_{gT} = \min\left\{ x_{gT}, \max\left\{ N_{gT}, k \right\} \right\}$. The pairs $\left\{ N_{gT}, X_{gT} \right\}$ are defined by backward induction as follows.

For $g \in H_{T-1}$, $N_{g\tau}$ is the smallest value that satisfies $w_\tau\left(N_{g\tau}, g\right) = \xi$. This is well

defined because by A1

$$\lim_{k \to 0} w_\tau(k, g) = \lim_{k \to 0} \pi(k, g) + \delta(1 - \lambda)\chi > \xi$$

while A1 and A2 imply

$$\lim_{k \to \infty} w_\tau(k, g) = -\phi_g + \delta(1 - \lambda)\chi < \xi$$

Since $w_\tau(k, g)$ inherits continuity in k from π, $N_{g\tau}$ exists.

For g∈H_g, X_gτ is the smallest value that satisfies. $w_\tau\left(x_{g\tau}, g\right) = \chi$. If $\eta(\gamma, s) \subset H$

$-\phi_g + \delta(1 - \lambda)\xi < \chi$ then reasoning similar to the previous paragraph implies that $X_{g\tau}$ is

finite. Otherwise $X_{g\tau}$ is infinite. Make the induction hypothesis that $w_\tau(k, g)$ is

continuous and negatively dependent on k and that $N_{g\tau}$ and $X_{g\tau}$ are well defined for

$g \in H_s$ if $t + 1 \le s \le T$. Then A1 implies $\lim_{k \to 0} w_t(k, h) = \lim_{k \to 0} \pi(k, h) + \delta(1 - \lambda)\xi > \xi$ while A1

and A2 together imply $\lim_{k \to \infty} w_\tau(k, h) \le -\phi_h + \delta(1 - \lambda)\xi < \xi$. So $N_{h\tau}$ is well defined for

$h \in H_t$. Similar arguments establish $X_{h\tau}$ is well defined (although possibly infinite).

Next note that N_{hT} and X_{hT} are monotonically increasing in T. This follows

because $w_\tau(k, h) \le w_{\tau+1}(k, h)$ for all k and all h∈H $h \in H_t$. Then an application of A1

establishes that N_{hT} is bounded in T. So $N_h = \lim_{\tau \to \infty} N_{h\tau}$ and $X_h = \lim_{\tau \to \infty} X_{h\tau}$ are well defined

(although X_h may be infinite) and satisfy the requirements. QED

18

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Table 1

Dependent variable: Logarithm of the range of firm values 1970-2008, normalized by labor.

Depreciation Measure	HW1	HW2	BEA
Log Depreciation	-0.172***	-0.172***	-0.181***
	(0.041)	(0.040)	(0.036)
Log Sunk Costs	0.576***	0.579***	0.586***
	(0.084)	(0.085)	(0.0834)
Years in Sample	0.0498***	0.0498***	0.0514***
	(0.004)	(0.004)	(0.0039)
R-Squared	0.452	0.452	0.457

Notes: N=1,661 industry over time observations. Standard errors in parentheses are robust and cluster corrected by industry. One, two and three stars denote significance at the 10%, 5% and 1% levels, respectively. HW1, HW2 and BEA denote the depreciation index calculated from (3.1) for the various data sources. The proxy for sunk costs is the firm's average value over time of property, plant and equipment.

Table 2

Dependent variable: Logarithm of the variance of firm values 1970-2008, normalized by labor.

Depreciation Measure	HW1	HW2	BEA
Log Depreciation	-0.366***	-0.367***	-0.383***
	(0.0796)	(0.0786)	(0.0696)
Log Sunk Costs	1.170***	1.176***	1.191***
	(0.160)	(0.162)	(0.159)
Years in Sample	0.0652***	0.0651***	0.0684***
	(0.00845)	(0.00823)	(0.00756)
R-Squared	0.451	0.451	0.456

Notes: N=1,661 industry over time observations. Standard errors in parentheses are robust and cluster corrected by industry. One, two and three stars denote significance at the 10%, 5% and 1% levels, respectively. HW1, HW2 and BEA denote the depreciation index calculated from (3.1) for the various data sources. The proxy for sunk costs is the firm's average value over time of property, plant and equipment.

Table 3

Dependent variable: Logarithm of the range or variance of firm values, normalized by labor.

	Dependent Variable: Range		Dependent Variable: Variance	
Subsample	**Log of HW1 Depreciation Measure**	**Log of Sunk Costs**	**Log of HW1 Depreciation Measure**	**Log of Sunk Costs**
1970-1979	-0.141*	0.595***	-0.307**	1.195***
	(0.07)	(0.0835)	(0.141)	(0.162)
1980-1989	-0.118***	0.550***	-0.295***	1.110***
	(0.0203)	(0.069)	(0.0411)	(0.140)
1990-1999	-0.130***	0.456***	-0.272***	0.925***
	(0.0254)	(0.0609)	(0.0529)	(0.114)
2000-2008	-0.143***	0.434***	-0.286***	0.902***
	(0.0169)	(0.0685)	(0.035)	(0.129)

Notes: Numbers in parentheses are robust standard errors cluster corrected by industry. One, two and three stars denote significance at the 10%, 5% and 1% levels, respectively. HW1, HW2 and BEA denote the depreciation index calculated from (3.1) for the various data sources. The proxy for sunk costs is the firm's average value over time of property, plant and equipment.

Table 4

Dependent variable: Logarithm of the range or variance of firm values, normalized by sales.

Subsample	Dependent Variable: Range		Dependent Variable: Variance	
	Log of HW1 Depreciation Measure	Log of Sunk Costs	Log of HW1 Depreciation Measure	Log of Sunk Costs
1970-1979	-0.273***	0.350***	-0.563***	0.698***
	(0.0522)	(0.103)	(0.103)	(0.199)
1980-1989	-0.275***	0.478***	-0.603***	0.958***
	(0.023)	(0.110)	(0.0406)	(0.225)
1990-1999	-0.296***	0.308***	-0.597***	0.626***
	(0.0271)	(0.127)	(0.0522)	(0.245)
2000-2008	-0.311***	0.259***	-0.607***	0.538***
	(0.0201)	(0.108)	(0.0391)	(0.197)

Notes: Numbers in parentheses are robust standard errors cluster corrected by industry. One, two and three stars denote significance at the 10%, 5% and 1% levels, respectively. HW1, HW2 and BEA denote the depreciation index calculated from (3.1) for the various data sources. The proxy for sunk costs is the firm's average value over time of property, plant and equipment.

Table 5

Dependent variable: Logarithm of the intertemporal range of labor, normalized by the intertemporal mean of labor in an industry.

	HW1	HW2	BEA
Log Depreciation Measure	0.081***	0.081***	0.084***
	(0.028)	(0.028)	(0.028)
Log Sunk Costs	-0.317***	-0.317***	-0.316***
	(0.071)	(0.071)	(0.072)
R-squared	0.334	0.333	0.341

Notes: N=50 industry-level observations. Robust standard errors are in parentheses.

***,**, and * represent significance at the 0.01, 0.05, and 0.1 levels respectively. The proxy for sunk costs is the total of deflated capital expenditures for each industry.

Table 6

Dependent variable: Logarithm of the intertemporal range of the number of firms.

Depreciation Measure	HW1	HW2	BEA
Log Depreciation	0.212***	0.210***	0.199**
	(0.056)	(0.056)	(0.056)
Log Sunk Costs	-0.132	-0.134	-0.150
	(0.166)	(0.167)	(0.176)
R-Squared	0.227	0.224	0.207

Notes: N=50 industry-level observations. Robust standard errors are in parentheses.

***, **, and * denote significance at the 0.01, 0.05, and 0.10 levels, respectively. The

proxy for sunk costs is the total of deflated capital expenditures for each industry.

Table 7

Dependent variable: Logarithm of the intertemporal range of capital divided by the intertemporal mean of capital in an industry.

Depreciation Measure	HW1	HW2	BEA
Log Depreciation	0.138***	0.138***	0.142***
	(0.030)	(0.030)	(0.029)
Log Sunk Costs	-0.023	-0.024	-0.023
	(0.124)	(0.124)	(0.123)
R-Squared	0.226	0.226	0.240

Notes: N=50 industry-level observations. Standard errors are in parentheses. ***, **, and * denote significance at the 0.01, 0.05, and 0.10 levels, respectively. The proxy for sunk costs is the total of deflated capital expenditures for each industry.

Table 8a

Summary Statistics: Tables 1 - 4	Mean	Median	SD
Independent Variables			
HW1	0.0665403	6.14E-06	1815.566
HW2	0.06764	6.04E-06	0.699071
BEA	0.687839	5.89E-06	0.572084
Sunk Costs	5.351372	56.72	1.809835
Years in Sample	18.25	15.00	11.21
Dependent Variables			
Range of Firm Values	6.437661	229.45	1.933361
Variance of Firm Values	10.77109	3.7824	3.782412
Sales	628.8336	58.86	2564.703
Employees	3.557827	0.331	20.51242
Sample Size			
Firms	1661		
Independent Variables			
HW1	0.00002898	9.086E-06	0.00006123
HW2	0.00002902	8.921E-06	0.00006055
BEA	0.00002763	9.016E-06	0.00004859
Sunk Costs	10907.723	7932.7803	8267.9856
Dependent Variables			

Normalized Range of Labor	0.174	0.165	0.116
Normalized Range of Number of Firms	0.00342	0.00207	0.00362
Normalized Range of Capital	0.874	0.604	0.775
Sample Size			
Industries	50		

Notes: **Normalized Range of Capital (in millions of current dollars) is defined as**

$\frac{\text{Max}_K - \text{Min}_K}{\mu_K}$. **Normalized Range of Labor is defined as** $\frac{\text{Max}_L - \text{Min}_L}{\mu_L}$. **Normalized Range of**

Number of Firms is defined as $\frac{\text{Max}_{\text{Firms}} - \text{Min}_{\text{Firms}}}{\mu_{\text{Firms}}}$. **Sunk Costs are measured as capital**

expenditures (in millions of current dollars) divided by the average number of

employees in the industry. Firm Value is measured in millions of dollars. Labor is

measured by the number of Employees (unit=1000).

End Notes

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[1] Examples of primarily empirical work include Deutsch (1984), Dunne, Roberts, and Samuelson (1989), Geroski, Gilbert, and Jacquemin (1990), Geroski and Schwalbach (1991), Siegfried and Evans (1992,1994), Audretsch (1995), Lambson and Jensen (1995,1998), Asplund (2000), Ramey and Shapiro (2001), Gschwandtner and Lambson (2002,2006), Disney, Haskel, and Heden (2003), Melitz (2003), and many others. Examples of primarily theoretical work include Jovanovic (1982), Ericson and Pakes (1989), Dixit (1989), Sutton (1991), Lambson (1991,1992), Hopenhayn (1992), Cabral (1995), Caballero and Pindyck (1996), Asplund and Nocke (2006),and many others.

[2] There are, of course, exceptions, such as Kessides (1990), Farinas and Ruano (2005), and Ghosal (2007). Lambson and Jensen (1998) asserted informally that depreciation dampens the effects of sunk costs.

[3] Our earlier work refers to firms or plants rather than capital. When a firm is identified with a unit of capital and depreciation is ignored, this is natural. Depreciation makes the value of a firm depend on the age distribution of its various capital units, so exposition at the level of those units is more natural.

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[4] This definition of π, where the second argument is a market condition involves a slight abuse of notation relative to the appendix where the second argument is a finite history of market conditions. Context makes clear which is meant.

[5] Data for the Input-Output Accounts are here:

http://www.bea.gov/industry/io_annual.htm. More specific capital-flow data are here:

http://www.bea.gov/industry/capflow_data.htm.

[6] We used value of shipments data from the U.S. Census Bureau to approximate the industry's average sales S_F. These data are here:

http://www.census.gov/econ/census02/data/comparative/USCS.HTM

[7] These data are available at: http://www.sba.gov/advo/research/data.html.

[8] More specific information is at: http://www.sba.gov/advo/research/dyn_us_n4.txt.

[9] Ideally, also measures of other sunk costs and controls for industry-wide factors such as demand swings, R&D and advertising expenditures could have been used. Unfortunately, no such data were available to us.